Truthmaker Semantics in Linguistics

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Abstract: Truthmaker semantics is a recent development in formal and philosophical semantics, with similar motivation and scope to possible worlds semantics. The technical background is rather different, however, and results in a more fine-grained *hyperintensional* notion of content, allowing us to distinguish between classically equivalent propositions. After briefly introducing the main ideas, this entry will describe the technical apparatus of *state spaces* and the central notions of *content* and *partial content*. It will then outline applications of truthmaker semantics in language, logic, and other areas of philosophy.

1 Introduction

Truthmaker semantics is a recent development in formal and philosophical semantics, with similar motivation and scope to possible worlds semantics. The technical background is rather different, however, and results in a more fine-grained *hyperintensional* notion of content, allowing us to distinguish between classically equivalent propositions. After briefly introducing the main ideas, this entry will describe the technical apparatus of *state spaces* (§2) and the central notions of *content* (§3) and *partial content* (§4). §5 describes applications of truthmaker semantics in language, logic, and philosophy.

Truthmaker semantics evaluates sentences relative to *states*. A model assigns certain states to be the *truthmakers* and *falsitymakers* of a given sentence. States are typically incomplete: a given state may make certain sentences true and certain others false, but will have nothing to say either way about the rest. Intuitively, *that Anna is knitting* makes the sentence 'Anna is knitting' true and makes the sentence 'Anna is doing nothing' false, but says nothing either way about what Lily is doing. Possible worlds, by contrast, are usually assumed to be complete, settling the truth-value of each and every sentence of the language.

Central to truthmaker semantics is the notion of *exact* truthmaking, which requires a state to be wholly relevant to the sentences it makes true. Thus, the state *that Anna is knitting and Lily is sleeping* will not count as an exact truthmaker for 'Anna is knitting', for it does too much to count as wholly relevant to that truth. Nevertheless, that state has a part, the state *that Anna is knitting*, which is wholly relevant to the truth of 'Anna is knitting'. To capture these ideas formally, the domain of states is structured by a part-whole relation, $\sqsubseteq$. This allows us to understand an operation of *fusion* on states, whereby states s and u form a whole, $s \sqcup u$, of which both are parts. Fusion is central to the truthmaker interpretation of conjunction, as the example just given suggests. To accommodate this, truthmaker semantics typically requires that the fusion of arbitrary states always exists, even if the result is an impossible state.

Truthmaker semantics has antecedents in the semantics of relevant and substructural logics. One early example is Nelson's (1930, 444) understanding of $A \wedge B$ to mean 'A and B [as] a unit or whole, not simply an aggregate', which 'expresses the joint force of A and B'. The first explicit statement of truthmaker semantics was given by van Fraassen (1969) in his semantics for **FDE**, the logic of *first degree entailment*. Something similar is mentioned in passing by Fine (1975), in the context of counterfactuals, and (again in passing) by Restall (1996) and Jago (2009), in the context of discussion of the metaphysics of truthmaking. Development of a unified semantic approach, however, was almost wholly due to Kit Fine (2012a; 2014; 2016; 2017a; 2017b) and Stephen Yablo (2014; 2016; 2018). More recent work is collected in Faroldi and Van De Putte 2023. Fine (2017c) surveys the approach in its early days; Champollion (2024) gives a recent overview of truthmaker semantics aimed at linguists. A maintained bibliography of work on truthmaker semantics can be found at `https://truthmakersemantics.github.io/publications/`.

2 State Spaces and Models

Truthmaker semantics is built around the notion of a *state space*: a set of states S with a part-whole structure $\sqsubseteq$ on it. $\sqsubseteq$ is a *complete partial order*: it is reflexive, transitive, and antisymmetric, and for each subset T of S, there exists a *least upper bound* $\bigsqcup T \in S$. (A least upper bound of a set T is the least (w.r.t. $\sqsubseteq$) state $s \in S$ for which $t \sqsubseteq s$ for each $t \in T$. A least upper bound will be unique, so we are entitled to call it *the* upper bound and write $\bigsqcup T$.) Each complete partial order is bounded, meaning that it has a least element $\square$ and a greatest element $\blacksquare$ in S, such that $\square \sqsubseteq s \sqsubseteq \blacksquare$ for every state $s \in S$. These can be defined as $\bigsqcup \varnothing = \bigsqcap S = \square$ and $\bigsqcup S = \bigsqcap \varnothing = \blacksquare$.

We can also define a *greatest lower bound* $\bigsqcap T$ for each subset T of S, by setting $\bigsqcap T = \bigsqcup \{s \mid s \sqsubseteq t \text{ for all } t \in T\}$. (The greatest lower bound of a set T is the greatest state $s \in S$ for which $s \sqsubseteq t$ for each $t \in T$. Again, a greatest lower bound will be unique.) For pairs of states, we write $s \sqcup u$ for $\bigsqcup\{s, u\}$, the *fusion* of s and u, and $s \sqcap u$ for $\bigsqcup\{s, u\}$. $\langle S, \sqcup \rangle$ and $\langle S, \sqcap \rangle$ are *complete semilattices*, with identity elements $\square$ and $\blacksquare$, respectively: $s \sqcup \square = s$ and $s \sqcap \blacksquare = s$ for each $s \in S$. $\sqcup$ and $\sqcap$ are commutative, associative, and idempotent, and $s \sqcup u = u$ iff $s \sqcap u = s$ iff $s \sqsubseteq u$. It is sometimes required in addition that spaces are *distributive*, such that, for any $s, t_1, t_2, \ldots \in S$:

$$s \sqcap (t_1 \sqcup t_2 \sqcup \cdots) = (s \sqcap t_1) \sqcup (s \sqcap t_2) \sqcup \cdots$$

(The remainder of the entry will ignore this requirement and $\sqcap$ more generally.)

This entry will consider only a very simple propositional language:

$$p \qquad | \qquad \neg A \qquad | \qquad A \wedge B \qquad | \qquad A \vee B$$

There are two main approaches to interpreting sentences, which differ on the treatment on negation. On the *unilateral* approach, truthmakers for $\neg A$ are understood in terms of truthmakers for A, for example, as states which somehow *exclude* or *preclude* truthmakers for A (Champollion and Bernard 2024; Fine

2017a). The *bilateral* approach, by contrast, assigns *falsitymakers* as well as truthmakers to sentences, with truthmakers for $\neg A$ then understood as the falsitymakers for A. Most authors have adopted the bilateral approach, as we shall here.

State spaces $\langle S, \sqsubseteq \rangle$ are expanded to models by adding *positive* and *negative valuation* functions:

Definition 1 (Models)**.** A *model* $\mathcal{M}$ is a quadruple $\langle S, \sqsubseteq, V^+, V^- \rangle$, where $\langle S, \sqsubseteq \rangle$ is a state space and $V^+, V^- : \mathcal{P} \longrightarrow 2^S$ are functions from sentence letters to nonempty subsets of S.

Exact truthmaking ($\Vdash$) and falsitymaking ($\dashV$) relations are then defined as follows:

Definition 2 (Exact truthmaking and falsitymaking)**.** Given a model $\mathcal{M}$ (which we leave implicit), *exact truthmaking* $\Vdash$ and *exact falsitymaking* $\dashV$ relations are defined by double recursion as follows:

$$
\begin{array}{rcl}
s \Vdash p & \text{iff} & s \in V^+ p \\
s \dashV p & \text{iff} & s \in V^- p \\
s \Vdash \neg A & \text{iff} & s \dashV A \\
s \dashV \neg A & \text{iff} & s \Vdash A \\
s \Vdash A \wedge B & \text{iff} & \exists tu (s = t \sqcup u \;\&\; t \Vdash A \;\&\; u \Vdash B) \\
s \dashV A \wedge B & \text{iff} & s \dashV A \text{ or } s \dashV B \\
s \Vdash A \vee B & \text{iff} & s \Vdash A \text{ or } s \Vdash B \\
s \dashV A \vee B & \text{iff} & \exists tu (s = t \sqcup u \;\&\; t \dashV A \;\&\; u \dashV B)
\end{array}
$$

These are said to be *exact* relationships in that $s \Vdash A$ says that s is wholly relevant to A's truth (and $s \dashV A$ says that s is wholly relevant to A's falsity). This requirement leads to the unusual truthmaking clause for conjunction: a truthmaker for $A \wedge B$ is the fusion of a truthmaker for A and a truthmaker for B. That state may itself not be *wholly* relevant to A's (or B's) truth and so, in general, truthmakers for conjunctions will not be truthmakers for their conjuncts. (Similar remarks apply to falsitymaking for disjunctions.)

That state will nevertheless be *sufficient* for (if not wholly relevant to) the truth of the conjuncts. This is the notion of *inexact truthmaking*, defined as follows.

Definition 3 (Inexact truthmaking and falsitymaking)**.** In any model $\mathcal{M}$, $s \trianglerighteq A$ iff $u \Vdash A$ for some $u \sqsubseteq s$, and $s \trianglelefteq A$ iff $u \dashV A$ for some $u \sqsubseteq s$.

Inexact truthmaking, unlike its exact cousin, obeys the standard extensional clauses for conjunction and disjunction:

$$
\begin{array}{rcl}
s \trianglerighteq A \wedge B & \text{iff} & s \trianglerighteq A \text{ and } s \trianglerighteq B \\
s \trianglelefteq A \wedge B & \text{iff} & s \trianglelefteq A \text{ or } s \trianglelefteq B \\
s \trianglerighteq A \vee B & \text{iff} & s \trianglerighteq A \text{ or } s \trianglerighteq B \\
s \trianglelefteq A \vee B & \text{iff} & s \trianglelefteq A \text{ and } s \trianglelefteq B
\end{array}
$$

We may distinguish, within a model, a subset of *actual* states, downward-closed under parthood (so that all parts of an actual state are themselves actual) and under fusion (so that the fusion of any set of actual states is itself an actual state). We might think of the fusion of all actual states as the *actual world*. We can then say that A is *true in model* $\mathcal{M}$ when it is exactly made true by some actual state, or, equivalently, when it is inexactly made true by the actual world.

More generally, we may distinguish a set of *possible states* $S^\diamond$, downward-closed under $\sqsubseteq$ (so that any part of a possible state is itself a possible state). We may then think of the $\sqsubseteq$-maximal possible states as *possible worlds*. Under suitable conditions, we can then recover classical entailment and entailment in the 3-valued logics **K3** and **LP** (Fine 2017a).

3 Propositions and Subject Matter

Exact truthmaker and *exact falsitymaker* sets are defined as follows:

$$|A|^+ = \{s \in S \mid s \Vdash A\} \qquad |A|^- = \{s \in S \mid s \dashV A\}$$

We may lift $\sqcup$ to sets of states by setting:

$$T \sqcup U = \{t \sqcup u \mid t \in T, u \in U\}$$

This allows us to state the exact clauses in algebraic form:

$$\begin{array}{lll} |\neg A|^+ = |A|^- & |A \wedge B|^+ = |A|^+ \sqcup |B|^+ & |A \vee B|^+ = |A|^+ \cup |B|^+ \\ |\neg A|^- = |A|^+ & |A \wedge B|^- = |A|^- \cup |B|^- & |A \vee B|^- = |A|^- \sqcup |B|^- \end{array}$$

Sets of states are often understood as *propositions*. More precisely, a *unilateral proposition* is a set of states $P \subseteq S$, and a *bilateral proposition* is a pair $\mathbf{P} = \langle P^+, P^- \rangle$ of sets of states. The idea here is that P^+ contains $\mathbf{P}$'s truthmakers and P^- its falsitymakers. Given bilateral propositions $\mathbf{P} = \langle P^+, P^- \rangle$ and $\mathbf{Q} = \langle Q^+, Q^- \rangle$, we define bilateral Boolean operators as follows:

$$\begin{aligned} \neg\langle P^+, P^- \rangle &= \langle P^-, P^+ \rangle \\ \langle P^+, P^- \rangle \wedge \langle P^+, P^- \rangle &= \langle P^+ \sqcup Q^+, P^- \cup Q^- \rangle \\ \langle P^+, P^- \rangle \vee \langle P^+, P^- \rangle &= \langle P^+ \cup Q^+, P^- \sqcup Q^- \rangle \end{aligned}$$

We may in addition require one or more closure conditions on propositions:

Definition 4 (Closure Conditions)**.**

Closure ($\sqcup$): A proposition P is *closed* when $\bigsqcup Q \in P$ for any nonempty $Q \subseteq P$.

Convex closure ($\frown$): A proposition P is *convex* when, for any $t \in S$, if $s, u \in P$ and $s \sqsubseteq t \sqsubseteq u$, then $t \in S$ too.

Regular closure ($*$): A proposition P is *regular* when it is both closed and convex.

We write $P^{\sqcup}$, $P^{\frown}$, and P^{*} for the smallest closed, convex, and regular sets (respectively) that contain P.

Each (unilateral) proposition P has a *subject matter* $\mathbf{p}$ which, intuitively, is what the proposition is about. We define $\mathbf{p} = \bigsqcup P$. The subject matter of a bilateral proposition $\mathbf{P} = \langle P^+, P^- \rangle$ is defined as $\mathbf{p}^+ \sqcup \mathbf{p}^-$. This is not the only notion of bilateral subject matter available (see Fine 2017b for discussion), but it is a good option in that it captures the intuitive principle that negating a proposition does not affect its subject matter.

Sentences A, B are *exactly equivalent*, $A \equiv_e B$, when they express the same unilateral proposition. Where we impose no closure conditions on propositions, equivalence amounts to A and B having exactly the same truthmakers. But for technical reasons, it is sometimes preferable to insist that propositions be regular closed sets, so that $A \equiv_e B$ when $|A|^{+*} = |B|^{+*}$. This approach results in the familiar equivalences shown in figure 1. (Of these, the majority hold on the basic semantics, even without a distributive space. Idempotence for $\wedge$ requires that we define exact equivalence with respect to closed propositions, while Distributivity for $\vee$ requires regular propositions.)

Commutativity:	$A \wedge B \equiv_e B \wedge A$
	$A \vee B \equiv_e B \vee A$
Associativity:	$A \wedge (B \wedge C) \equiv_e (A \wedge B) \wedge C$
	$A \vee (B \vee C) \equiv_e (A \vee B) \vee C$
Distributivity:	$A \wedge (B \vee C) \equiv_e (A \wedge B) \vee (A \wedge C)$
	$A \vee (B \wedge C) \equiv_e (A \vee B) \wedge (A \vee C)$
Idempotence:	$A \wedge A \equiv_e A$
	$A \vee A \equiv_e A$
De Morgan:	$\neg(A \wedge B) \equiv_e \neg A \vee \neg B$
	$\neg(A \vee B) \equiv_e \neg A \wedge \neg B$
Double negation:	$\neg\neg A \equiv_e A$

Figure 1: Equivalences given regular closure

Relative to some fixed closure conditions c on propositions, (single-premise) *exact entailment* is defined as propositional inclusion: A exactly entails B when $|B|^{+c} \subseteq |A|^{+c}$. Thus when no closure conditions are imposed, exact entailment is preservation of exact truthmakers. On any notion of exact entailment, $A \wedge B$ does not exactly entail A, yet there remain important relationships between $A \wedge B$ and A. One is that $A \wedge B$ inexactly entails A (that is, any inexact truthmaker for $A \wedge B$ is thereby an inexact truthmaker for A), guaranteeing that A is true when $A \wedge B$ is. But there is also a tighter relationship, to which we now turn.

4 Partial Content and Partial Truth

There is an intuitive sense in which the content expressed by A is part of that expressed by $A \wedge B$, in a way that $A \vee B$'s content is not a part of A's content. Truthmaker semantics has a natural way to capture this relationship:

Definition 5 (Conjunctive parthood). P is a *conjunctive part* of Q, $P \leq Q$, when:

Up: Each $s \in P$ is part of some $u \in Q$ (i.e. $s \sqsubseteq u$); and

Down: Each $s \in Q$ has a part $u \in P$ (i.e. $u \sqsubseteq s$).

When the first condition holds, P *subserves* Q and when the second holds, Q *subsumes* P. We also say that Q *contains* P when $P \leq Q$.

Note that $\leq$ is a natural way to lift the parthood ordering $\sqsubseteq$ from states to sets of states, since for regular propositions P, Q, we have (Fine 2017a):

$$P \leq Q \text{ iff } P \wedge Q = Q$$

in parallel to the usual order-lattice equivalence on states ($s \sqsubseteq u$ iff $s \sqcup u = u$).

Containment is a natural complement to exact entailment, in the sense that $P \wedge Q$ contains but does not exactly entail P, whereas P exactly entails but does not contain $P \vee Q$. That truthmaker semantics distinguishes these relations is one of the factors that sets it apart from many other formal semantics. Classical, intuitionistic, and relevant logics all take any A to be equivalent to both $A \vee (A \wedge B)$ and $A \wedge (A \vee B)$ and, as a consequence, cannot distinguish a notion of entailment modelling *conjunction elimination* from one modelling *disjunction introduction*.

The separation of these concepts in truthmaker semantics allows for a useful notion of *partial truth*. A proposition P is *partly true* when it contains a proposition Q which is true (in the usual sense of having a truthmaker). A partial truth P may thus be thought of conjunctively, as a proposition $P_1 \wedge P_2$ where P_1 is true. To say that Berkeley was an English philosopher is wrong but partly right, in that he was a philosopher. The notion of containment plays an important role in several applications of truthmaker semantics discussed in (§5), notably in the logical semantics for **AC** and in the philosophical account of verisimilitude.

5 Applications

This section briefly outlines some of the applications of truthmaker semantics. These fall into three broad areas: language; semantics for formal logics; and other philosophical topics.

Language

Truthmaker semantics was designed with linguistic applications in mind. An early application was to *scalar implicatures* (Fine 2017c, but first presented in Fine's 2010 Jack Smart lecture). In saying to the class,

(1) One of you passed,

this is taken to imply,

(2) Not all of you passed

But the implication is not the usual logical one, since the speaker may consistently add to (1) that, in fact, all passed. Fine's hypothesis is that the scalar implicature arises given the presupposition that the statements in question are *exactly true*, where to be exactly true is to be made true by the *relevant situation*, namely the actual facts restricted to the statement's subject matter. Under the presupposition that (1) is exactly true, the actual facts restricted to the subject matter *which students passed the test* must include just one instance of a student passing. So (assuming the class contains more than one student), (2) is true relative to the relevant situation. (This approach requires a *non-inclusive* approach to quantification, on which the truthmakers for $\exists x Ax$ are all truthmakers for some instance, Ac. On this approach, propositions are *not* taken to be closed, in the sense of definition 4.)

Another application of truthmaker semantics is to *conditionals* of various sorts (Fine 2012a; Santorio 2018; Yablo 2016). On Fine's (2012a) approach to counterfactual conditionals, for example, the state space is augmented with a *transition relation* on states. A counterfactual $A \mathrel{\Box\!\!\rightarrow} C$ is true when each A-state transitions to some C-state, relative to the world in question. This approach solves a long-standing technical issue. Counterfactuals seem to satisfy *Simplification*:

$$A \vee B \mathrel{\Box\!\!\rightarrow} C \vdash A \mathrel{\Box\!\!\rightarrow} C, B \mathrel{\Box\!\!\rightarrow} C$$

but not *Antecedent Strengthening*:

$$A \mathrel{\Box\!\!\rightarrow} C \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$$

Yet, substituting $(A \wedge B) \vee A$ for the logically equivalent A, *Simplification* implies *Antecedent Strengthening*. Truthmaker semantics offers an elegant solution since (quite aside from the counterfactual semantics) it invalidates the equivalence of $(A \wedge B) \vee A$ with A. Santorio (2018) and Yablo (2016) work with a rather different conception of truthmakers to the state-based one presented here, on which truthmakers are understood as sets of worlds.

Other linguistic applications (which we shall not detail here) include the analysis of *cases* (Moltmann 2021a), *imperatives* (Fine 2018a), *negation as exclusion* (Champollion and Bernard 2024), *negative events* (Champollion and Bernard 2024; Moltmann forthcoming), *presupposition* (Yablo 2014), *propositional attitudes* (Moltmann 2020; 2021b), *quantifiers* and *definite descriptions* (Fine 2017c; Fine and Jago Forthcoming), and *subject-matter* (Fine 2020; Yablo 2018; 2014; Fine 2017b).

Logical Semantics

One of the first applications of truthmaker semantics to appear in print was Fine's semantics for intuitionistic logic (Fine 2014). This approach sits mid-way between the Brouwer-Heyting-Kolmogorov (BHK) semantics and the Kripke semantics for intuitionistic logic. It involves states, as on Kripke's approach, but interprets the conditional in terms of a *transition relation* between states, in much the way the BHK semantics does. We can interpret states as proofs (or pieces of information) and so think of a truthmaker for $A \to C$ as a state which takes proofs of A into proofs of C. (The TMS treatment of conjunction in terms of fusion is also reminiscent of the BHK treatment.)

A significant development was Fine's (2016) semantics for Angell's system **AC**, the logic of *analytic entailment* (Angell 1989). Angell's idea was that A should entail C only when A's content contains C's. So $A \wedge B$ will entail A, but A will not in general entail $A \vee B$. This logic resisted semantic analysis for nearly 40 years, but can be understood very naturally in terms of partial content (§4), since A's content is part of $A \wedge B$'s, but $A \vee B$'s content is not in general part of A's or B's content. Weiss (2019) connects this approach to logic in ancient philosophy.

More recently, other researchers have attempted to find truthmaker semantics for some of the most prominent logical families. Truthmaker semantics have been given for *deontic logics* (Anglberger et al. 2016; Faroldi 2019; Fine 2018b), *epistemic logics* (Hawke and Özgün 2023; Jago 2024), *modal logics* (Kim 2024; Rosella 2019; Saitta et al. 2022), and *relevant logics* (Jago 2020; Verdée 2023).

Much of this work has uncovered new logical systems, which 'live inside' those just mentioned. Whenever a truthmaker semantics is given, we have both an *exact* and an *inexact* notion of consequence (amongst other notions, such as those based on partial content). Often, it is the *inexact* notion which agrees with a preexisting notion of consequence (given that, on most notions of logical consequence, $A \wedge B$ entails A). On basic truthmaker models, inexact consequence coincides with the logic **FDE** (van Fraassen 1969), whereas the exact notion results in a wholly new logic (Fine and Jago 2019). This gives rise to investigation into the *exactification* of preexisting logics: given an inexact semantics for, say, intuitionistic logic (Fine 2014), what is the corresponding exact notion of consequence? At the time of writing, this is an open question.

A related area of investigation is into properties of the various notions of truthmaker entailment and equivalence (Fine and Jago 2019; Knudstorp 2023; Krämer 2024) and their proof systems (Korbmacher 2023).

Philosophy

Truthmaker semantics has also been put to use in attempting to solve important philosophical problems beyond those of language. One is the problem of *knowledge closure*: given an agent knows such-and-such, what else does she thereby know, as a matter of logic? The problem is closely related to *external world scepticism*: I know that, if I am typing this entry, then I am not a brain in a vat; but I do not know that I am not a brain in a vat; and so I do not know that I am typing this entry. It seems that I can have no knowledge of the external world. Closure is the crucial inference here: knowing both $A \to B$ and A entails knowing B. But it is plausible that closure be restricted to contents as understood on the truthmaker approach (Elgin 2021; Jago 2024; Yablo 2014). Thus, knowing A will not entail knowing $A \vee B$ and (depending on how the conditional is understood), knowing both $A \rightarrow B$ and A need not entail knowing B.

Another issue is the problem of *verisimilitude* or *truthlikeness*. We want to say that science makes progress towards the truth, whilst acknowledging that current theories are not wholly true. But what is it for one false theory to be closer to the truth than another? The standard logical approach (Popper 2014) faces a serious trivialization worry (Miller 1974). The truthmaker approach has a natural solution in the notion (§4) of partial truth (Fine 2021).

In philosophical action theory, action is differentiated from behaviour in terms of *intention*. 'What is left over if I subtract the fact that my arm goes up from the fact that I raise my arm?', asked Wittgenstein (1953, §621). More generally, what is it to *subtract* one content from another? What is left of the politician's speech, for example, once all the distracting statistics are removed? The idea may be understood precisely in terms of truthmaker semantics (Fine 2017b; Yablo 2014). Within suitable spaces, we may define a unique *remainder state* $s - u$, and then lift this notion to contents to obtain the remainder $P - Q$.

Truthmaker semantics has been applied to metaphysical issues surrounding *grounding* (Fine 2017b; Jago 2023; Krämer 2021) and *essence* (Hale 2020). In the case of grounding, Correia's (2010) notion of *worldly grounding* corresponds very closely to the truthmaker semantics for **AC**. Moreover, it can be shown that, for nonempty regular propositions, P weakly partially grounds Q, as defined in Fine's *pure logic of ground* (2012b), just in case $\mathbf{p} \sqsubseteq \mathbf{q}$ (Fine 2017b, lemma 21).

Truthmaker semantics has also been applied to issues in *philosophy of mind* (Elgin 2022), to the metaphysics of *propositions* (Jago 2017), to belief revision Krämer (2023), and to the attempt to give semantic foundations for philosophical analysis itself (Elgin 2023).

Summary

This entry has introduced the main technical and philosophical ideas involved in truthmaker semantics, focusing on the central concept of a *state space* and the distinction between *exact* and *inexact* truthmaking (§2). It then showed how to understand the philosophical concepts of *proposition* and *subject matter* (§3), and of *partial content* and *partial truth* (§4) in terms of truthmaker semantics. Finally, it discussed a number of applications of truthmaker semantics, in language, logic, and other areas of philosophy (§5).

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