

Against the Unrestricted Applicability of Disjunction Elimination*

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Abstract

In this paper I argue that the disjunction elimination rule presupposes the principle that a true disjunction contains at least one true disjunct. However, in some contexts such as supervaluationism or quantum logic, we have good reasons to reject this principle. Hence, disjunction elimination is restricted in at least one respect: it is not applicable to disjunctions for which this principle does not hold. The insight that disjunction elimination presupposes the principle that a true disjunction contains at least one true disjunct is applied to two arguments which argue for this very principle. I show that these arguments are rule-circular since they rest on disjunction elimination. I claim that rule-circularity better explains why the arguments fail than the explanations provided by [Rumfitt \(2015\)](#), which, for instance, rely on controversial principles about truth.

1 Introduction

The protagonists of this paper are two principles about disjunction.

- (1) TRUE DISJUNCT: If a disjunction is true, then at least one of its disjuncts is true.¹
- (2) DISJUNCTION ELIMINATION (henceforth $\vee E$):

$$\frac{\begin{array}{ccc} \Gamma & \Gamma, [A]^1 & \Gamma, [B]^1 \\ \vdots & \vdots & \vdots \\ A \vee B & C & C \end{array}}{C} \vee E_1$$

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¹To be entirely correct, TRUE DISJUNCT must be phrased as follows: If a disjunction is *valid*, then at least one of its disjuncts is *valid*. For the sake of convenience, I shall stick to the less general concept of truth.

To put it differently: Given that from the premise set Γ we can derive $A \vee B$, $A \rightarrow C$ and $B \rightarrow C$, this allows us to infer C .

Life for our protagonists will turn out to be hard. I argue for the following: There are contexts in which we have good reasons to deny TRUE DISJUNCT as a generally valid principle. When I speak of denying TRUE DISJUNCT, then I mean that a disjunction can be true *without* either of its disjuncts being true. As it turns out, $\vee E$ presupposes the truth of TRUE DISJUNCT. Thus, $\vee E$ is not unrestrictedly applicable in contexts where TRUE DISJUNCT does not hold.

As a side remark, note that denying TRUE DISJUNCT requires one to also deny the principle of bivalence according to which every statement is either true or false. Why? Consider the disjunction $A \vee B$. As we reject TRUE DISJUNCT it may be the case that $A \vee B$ is true without A or B being true individually. Furthermore, A and B cannot be false because if both of them were false, then the disjunction $A \vee B$ would be false.² Thus, if $A \vee B$ violates TRUE DISJUNCT, then at least one of its disjuncts is a statement for which the principle of bivalence does not hold.

Why is this paper necessary? It seems that some philosophers, at least [Rumfitt \(2015\)](#), are not aware of the link between $\vee E$ and TRUE DISJUNCT, or do not take it seriously enough. In his recent book *The Boundary Stones of Thought*, Rumfitt's main project is to argue for classical logic without classical semantics (by rejecting the principle of bivalence). He also rejects TRUE DISJUNCT. But apparently he does not see the connection to $\vee E$: He criticizes arguments *for* TRUE DISJUNCT, which explicitly apply $\vee E$, *not* because they apply $\vee E$. Instead he criticizes these argument by means of, for instance, introducing controversial principles about truth. He does not recognize that it is mere rule-circularity which is the flaw in these arguments (they argue for TRUE DISJUNCT and, via $\vee E$, already make use of TRUE DISJUNCT).

In this paper I proceed as follows. First, I present two cases where TRUE DISJUNCT is rejected and, as a consequence, also $\vee E$: supervaluationism and quantum logic. Second, I flesh out an account of why exactly rejecting TRUE DISJUNCT forces us to also reject the

²I think this is straightforward. Note that nonetheless one may doubt why we should accept that $A \vee B$ is false in case both A and B are false, given that we reject TRUE DISJUNCT.

universal applicability of $\vee E$. Third, I criticize the way Rumfitt rejects the arguments for TRUE DISJUNCT and argue that my solution – pointing out their rule-circularity – does a better job in explaining why the arguments fail.

2 The two Principles in Supervaluationism and Quantum Logic

In this section, I present supervaluationism and quantum logic as two examples where TRUE DISJUNCT is rejected and, as a consequence, also $\vee E$. This evidences (i) that TRUE DISJUNCT and $\vee E$ do not hold *in all reasonable logical systems* and (ii) that there might be a distinctive connection between these two principles.

2.1 Supervaluationism

In the following I discuss supervaluationism about vagueness. Some predicates are vague in the sense that for some objects we seem to lack criteria for identifying whether they satisfy the predicate or not. The basic idea behind supervaluationism is that for a vague predicate we have not yet decided where to draw the exact boundary between objects that satisfy it and ones that do not. Consider the predicate ‘tall’. It is vague whether ‘tall’ applies to someone who is 1.76m. But, by supervaluationism, this just reflects that we have not yet decided on where to draw the boundary. Vague predicates allow for several *precisifications*; a precisification is a stipulated boundary. Given a precisification for a vague predicate F , all statements of the form ‘ a is F ’ can be then evaluated by means of a classical valuation function v . For example, ‘John (1.76m) is tall’ may be true given a precisification under which all human beings taller than 1.70 count as tall. However, the statement is not true under all admissible precisifications, as the boundary could have also been drawn at 1.80m. This means that ‘John (1.76m) is tall’ is not *supertrue*. A statement is supertrue iff it is true under all admissible precisifications. The notion of supertruth shows that supervaluationism rejects TRUE DISJUNCT (with regard to supertruth): Although ‘John (1.76m) is tall’ and ‘Not: John (1.76m) is tall’ are not supertrue, the disjunction ‘John (1.76m) is tall or Not: John (1.76m) is tall’ is supertrue.

What is interesting for our purposes is that supervaluationism not only rejects TRUE DISJUNCT, but also $\vee E$ (Williamson 1994; Keefe 2000; Williams 2008).³ To see why, let us consider the supervaluationist’s D operator, where D stands for ‘definitely’ or ‘it is determinately the case that’ (confer Humberstone 2011, 832).

Definition 2.1 (D operator). Let V be a set of admissible classical valuations v . The D operator is such that $v(D\varphi) = 1$ iff $\forall v \in V : v(\varphi) = 1$.

$D\varphi$ evaluates to 1 iff φ evaluates to 1 under all admissible classical valuation functions. Given the D operator we can now define *supervalidity* \vDash_{sv} (what Williamson (1994) also calls *global validity*): The supertruth of the premises guarantees the supertruth of the conclusion. Note that $p \vDash_{sv} Dp \vee D\neg p$, $\neg p \vDash_{sv} Dp \vee D\neg p$ as well as $\vDash_{sv} p \vee \neg p$ are valid. If $\vee E$ were valid, we could now infer $\vDash_{sv} Dp \vee D\neg p$. This, however, is not valid. Hence, $\vee E$ fails in supervaluationism with the D operator. Williamson nicely summarizes this point as follows:

“Argument by cases [that is, $\vee E$,] does not hold in the supervaluationist logic, for although the inference from ‘ p ’ to ‘Definitely p or definitely not p ’ is globally valid, as is that from ‘Not p ’ to the same conclusion, the inference from ‘ p or not p ’ to ‘Definitely p or definitely not p ’ is not globally valid.” (Williamson 1994, 152)

There are different reactions to the fact that supervaluationism forces one to reject certain classical rules of inference, such as $\vee E$. Williamson (1994) thinks that therefore supervaluationism is the wrong account of vagueness. Keefe (2000), on the other hand, holds that the resulting logical revisionism is unproblematic. For our purposes, it is not important which route we accept.⁴ The upshot is just that there is a link between TRUE DISJUNCT and $\vee E$.

³ Not only $\vee E$ fails, but supervaluationism seems to also “involve breakdowns of the classical rules of contraposition, conditional proof [...] and *reductio ad absurdum*” (Williamson 1994, 151).

⁴ For a discussion, see Williams (2008).

2.2 Quantum Logic

Quantum logic, developed by [Birkhoff and Von Neumann \(1936\)](#), rejects the classical laws of distribution. Here is an instance of one of these laws:

$$p \wedge (m_1 \vee m_2) \vdash (p \wedge m_1) \vee (p \wedge m_2) \quad (\text{Dis})$$

Here is the reason why quantum logic rejects (Dis), which reveals that quantum logic ultimately denies TRUE DISJUNCT. (Note that I can be only very sketchy here, but I think it suffices to illustrate my point.) Suppose that p describes a particle’s position and m_1, m_2 its (possible) momenta. Say the conjunction on the left is true because p is true and the disjunction $m_1 \vee m_2$ is true without either of its disjuncts being true. How can the latter be? According to Heisenberg’s uncertainty principle, we cannot measure both a particle’s position and its precise momentum at the same time. Therefore, we are not allowed to assume that either of the disjuncts is true.⁵ The disjunction as such, however, may be true. This is exactly why we cannot derive the right side of (Dis) because it presupposes that we can legitimately form the conjunction of a particle’s position and its momentum. This is not possible, again, because given p , neither m_1 nor m_2 may be true. How do we account for this substantive failure of (Dis) in our logic? Dummett proposes the restriction of $\vee E$.

“To say that the distributive law fails is, in effect, to say that the unrestricted rule of disjunction elimination fails: what follows from A together with $[\Gamma]$, and also follows from B together with $[\Gamma]$, need not follow from ‘ A or B ’ together with $[\Gamma]$. An understanding of ‘or’ under which disjunction elimination fails to hold generally is one which allows that it may be correct to say, ‘ A or B ’ although there is no answer to the question, ‘Which of A and B holds?’ – not just that we do not know it, but there is no answer to be known” ([Dummett 1978](#), 276).

Interesting for our purposes, Dummett recognizes an intimate connection between the failure of TRUE DISJUNCT and $\vee E$.⁶ Dummett’s ‘or’ for quantum logic is the classical \vee such that

⁵ One may wonder why exactly this is the case. Unfortunately, I cannot go into the philosophy of quantum mechanics, here. An advisable starting point is the debate between [Putnam \(1969\)](#) and [Dummett \(1978\)](#).

⁶ Also confer [Dummett \(1991, 77-78\)](#).

only a restricted $\vee E$ rule can be applied to it (Steinberger 2011, chapter 5). Let us denote this restricted rule $(\vee E)_{res}$. Here it goes (note that A , B and C are formulas, here).

$$\frac{\begin{array}{ccc} \Gamma & [A]^1 & [B]^1 \\ \vdots & \vdots & \vdots \\ A \vee B & C & C \end{array}}{C} \vee E_1$$

The difference to the classical $\vee E$ rule is that under the hypothesis A (and respectively B) no formulas from Γ may be used as ‘side premises’ (see also Humberstone 2011, 298-302). In order to see how $(\vee E)_{res}$ works, consider the proof for (Dis) as it would be valid under $\vee E$. We shall afterwards state which steps disqualify under $(\vee E)_{res}$.

1	$p \wedge (q \vee r)$							
2	p	$\wedge E, 1$						
3	$q \vee r$	$\wedge E, 1$						
4	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">q</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$p \wedge q$</td> <td style="padding-left: 10px;">$\wedge I, 2, 4$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$(p \wedge q) \vee (q \wedge r)$</td> <td style="padding-left: 10px;">$\vee I, 5$</td> </tr> </table>	q		$p \wedge q$	$\wedge I, 2, 4$	$(p \wedge q) \vee (q \wedge r)$	$\vee I, 5$	
q								
$p \wedge q$	$\wedge I, 2, 4$							
$(p \wedge q) \vee (q \wedge r)$	$\vee I, 5$							
5	$p \wedge q$	$\wedge I, 2, 4$						
6	$(p \wedge q) \vee (q \wedge r)$	$\vee I, 5$						
7	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">r</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$p \wedge r$</td> <td style="padding-left: 10px;">$\wedge I, 2, 7$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$(p \wedge q) \vee (q \wedge r)$</td> <td style="padding-left: 10px;">$\vee I, 8$</td> </tr> </table>	r		$p \wedge r$	$\wedge I, 2, 7$	$(p \wedge q) \vee (q \wedge r)$	$\vee I, 8$	
r								
$p \wedge r$	$\wedge I, 2, 7$							
$(p \wedge q) \vee (q \wedge r)$	$\vee I, 8$							
8	$p \wedge r$	$\wedge I, 2, 7$						
9	$(p \wedge q) \vee (q \wedge r)$	$\vee I, 8$						
10	$(p \wedge q) \vee (q \wedge r)$	$\vee E, 3, 4-6, 7-9$						

While $\vee E$ can be applied (and indeed is applied), $(\vee E)_{res}$ could not. This is because of line 5 and 8. In the hypothetical derivation from 4 to 6 (and respectively in that from 7 to 9) we draw on a premise other than the hypothesis q (or respectively r).

3 The Link between the two Principles

Until now I have discussed supervaluationism and quantum logic as logics where TRUE DISJUNCT and $\vee E$ do not hold. In this section, I flesh out an account of why, if you deny TRUE DISJUNCT within a certain logic, you are forced to deny the unrestricted applicability of $\vee E$

in this logic as well. Note that the following argument is a philosophical rather than a formal one. This is because until now, I have not introduced a precise semantics.

My thesis is that $\forall E$ presupposes the truth of TRUE DISJUNCT. To see this, consider again the general mechanism of $\forall E$. Say you want to eliminate the disjunction $A \vee B$. You know that under the hypothesis that A you get C , and that under the hypothesis that B you also get C . Thus, you conclude that C must be the case. I claim that this last step is valid only if $A \vee B$ is such that TRUE DISJUNCT holds, that is, either A is true or B is true. What you do when you open the hypothesis that A is to assume A 's truth (ideally for the sake of argument). In other words, the truth of A is assumed as a possibility. But what we do when we finally apply $\forall E$ is to take this possibility seriously. We reason as follows: The truth of $A \vee B$ means that either A or B is true. Thus, if either of them guarantees the truth of C , then C must be the case. This is because either A or B is true (although you may not know which). To make a long story short: From (i) A 's truth guarantees C 's truth and (ii) B 's truth guarantees C 's truth and (iii) in fact, one of A or B is true, C 's truth is guaranteed. But this is only possible if we assume TRUE DISJUNCT. Thus, $\forall E$ presupposes TRUE DISJUNCT. In other words, endorsing the unrestricted applicability of $\forall E$ requires one to also endorse that a true disjunction contains at least one true disjunct. As a consequence, I suggest to restrict $\forall E$ as follows: In cases where for $A \vee B$ TRUE DISJUNCT does not hold, you cannot apply $\forall E$. (I think this is not in conflict with Dummett's $(\forall E)_{res}$. See subsection [4.1.2.](#))

4 Two Arguments against True Disjunct

In the rest of the paper I consider two arguments for TRUE DISJUNCT. The discussion about supervaluationism and quantum logic reveals the urgency of spotting the flaw in such arguments, since in some contexts TRUE DISJUNCT turns out to be false. For the first argument, the discussion about quantum logic will be particularly helpful, and for the second argument the one about vagueness. In the case of both arguments, I shall first introduce the argument, then illustrate how Rumfitt criticizes it, and finally show that his criticism is implausible.

After having discussed the second argument, I argue that my account of the connection between TRUE DISJUNCT and $\vee E$ does a better job in explaining why the arguments are flawed – they are merely rule-circular.

4.1 First Argument

The first argument for TRUE DISJUNCT works with a fairly uncontroversial principle about truth: From A we get $True(A)$. Here it goes ([Rumfitt 2015](#), 174-175).

1	$True(A \vee B)$			
2	$True(A \vee B) \rightarrow A \vee B$			
3	$A \vee B$	$\Rightarrow E$, 1, 2		
4	<table style="border-collapse: collapse; margin-left: 1em;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td></td> </tr> </table>	A		
A				
5	$A \rightarrow True(A)$			
6	$True(A)$	$\Rightarrow E$, 4, 5		
7	$True(A) \vee True(B)$	$\vee I$, 6		
8	<table style="border-collapse: collapse; margin-left: 1em;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">B</td> <td></td> </tr> </table>	B		
B				
9	$B \rightarrow True(B)$			
10	$True(B)$	$\Rightarrow E$, 8, 9		
11	$True(A) \vee True(B)$	$\vee I$, 10		
12	$True(A) \vee True(B)$	$\vee E$, 3, 4–7, 8–11		

I think the argument is straightforward. With the principle about truth, we get $A \vee B$ from $True(A \vee B)$. Our aim is to derive $True(A) \vee True(B)$. We do this by disjunction elimination. We establish that under the hypothesis that A (and respectively B) we can derive $True(A) \vee True(B)$. Thus, we can eliminate $A \vee B$ and conclude $True(A) \vee True(B)$.

4.1.1 Rumfitt’s Criticism of the Argument

Here is Rumfitt’s explanation of why the argument fails:

“The problem of this proof [...] is evident: its last step appeals to the *unrestricted* form of the \vee -elimination rule. Lines 5 and 9 may well be true, but they are not

logical truths and so must count as side premises in an application of that rule. Accordingly, the semantic principle that a true disjunction contains at least one true disjunct rests on a logical rule that is moot in the present context of debate.” (Rumfitt 2015, 175, original emphasis)

Obviously, his criticism is inspired by Dummett’s restricted disjunction elimination rule $(\vee E)_{res}$. In the hypothetical derivations, we use an additional premise from our premise set. Consequently, according to $(\vee E)_{res}$, we cannot eliminate the disjunction $A \vee B$.

4.1.2 My Response to Rumfitt’s Criticism

The restricted $\vee E$ rule that Dummett proposes is itself restricted to certain domains of application. At least, we should use it in quantum logic. In more ordinary contexts it is unclear why we should, as Rumfitt does, work with $(\vee E)_{res}$. Recall my own proposal: Other things being equal, we can use $\vee E$ if we know that $A \vee B$ is a disjunction where at least one disjunct is true. Suppose $A \vee B$ is such a disjunction. Then, the truth of the premise set Γ guarantees the truth of all derivable conclusions. Moreover, suppose we can derive C from Γ together with A , as well as from Γ together with B . Then, as at least either A or B is true, the truth of C is guaranteed and we can infer it. In this case, there seems to be no need for restricting $\vee E$. In the domain of quantum mechanics, however, we have a reason for doing so. We cannot guarantee that truth will be preserved if we combine a hypothesis with other premises. So $(\vee E)_{res}$ seems to be justified in the context of quantum mechanics. In the context of the above argument, it is unclear why we should refrain from using side premises in hypothetical derivations carried out for the sake of disjunction elimination. This is because the principle $A \rightarrow True(A)$ is true *and* we know that in this case it does not do any harm concerning truth-preservation.

Therefore, Rumfitt rejects the applicability of $\vee E$ *for the wrong reason*. A better reason for rejecting it is rather that we do not know whether $A \vee B$ is a disjunction for which TRUE DISJUNCT holds.

4.2 Second Argument

Let us turn to the second argument for TRUE DISJUNCT that Rumfitt criticizes (Rumfitt 2015, 257-258). He presents it in his discussion about vagueness, which bears similarities to our previous discussion about supervenience. Before presenting the actual argument, two preliminaries need to be clarified: Rumfitt’s polar semantics for vague predicates⁷ and the predicate $\text{Say}(u, P)$.

4.2.1 Preliminaries

First, let us focus on Rumfitt’s account of vagueness (Rumfitt 2015, chapter 8), which is inspired by Sainsbury (1995). The main idea is to treat vague predicates as polar predicates. An object satisfies a predicate iff it is maximally close to the respective paradigmatic pole. As an analogy, one may compare this to gravitational poles; whether an object is attracted by a certain pole depends on whether this pole is the closest one in the vicinity relative to the object. As an example, suppose there are 100 tubes of color, where the first tube, a_1 , is paradigmatically orange and the last tube, a_{100} , paradigmatically red. From a_1 onwards the tubes get constantly more and more red. Thus, a_{50} is a paradigmatic red-orange borderline case. Rumfitt holds that the statements ‘ a_{50} is red’ and ‘ a_{50} is orange’ are neither true nor false; a_{50} is equally close to the red *and* the orange pole. However, the statement ‘ a_{50} is red or a_{50} is orange’ *is* true according to Rumfitt. Thus, Rumfitt’s polar semantics is similar to supervenience in that it rejects TRUE DISJUNCT as a generally valid principle.

Second, before turning to the actual argument, we need to introduce the predicate $\text{Say}(u, P)$. It means that the utterance u says that P . Why do we need such a predicate instead of merely P ? The idea behind it goes back to Williamson (1994, 187-188). Williamson states that if we are concerned with bivalence – or issues related to it – we need to get clear on what we count as *truth-bearers*. According to him, truth-bearers are utterances. But, in addition to that, note that bivalence is restricted to cases where an utterance says that

⁷I just give the philosophical idea behind it. If you are interested in the formal details, see Rumfitt (2015, section 8.5). Rumfitt identifies the extension of vague predicates with regular open sets in a topological space. In order to circumvent TRUE DISJUNCT, he interprets a disjunction of, say, two color predicates A and B , as the interior of the closure of $R_A \cup R_B$. Here, R_A and R_B are the regular open sets associated with A and B .

something is the case (P). This explains why we need the predicate $\text{Say}(u, P)$. Rumfitt, following Williamson, suggests two rules concerning $\text{Say}(u, P)$, which he assumes to be at least *prima facie* plausible.

R, 1: From ‘ $\text{True}(u) \wedge \text{Say}(u, P)$ ’ we get ‘ P ’.

R, 2: From ‘ $\text{Say}(u, P) \wedge P$ ’ we get ‘ $\text{True}(u)$ ’.

4.2.2 The Argument

Given all that, consider now the following argument for TRUE DISJUNCT.

1	Say(u, a_{50} is red $\vee a_{50}$ is orange)			
2	Say(v, a_{50} is red)			
3	Say(w, a_{50} is orange)			
4	True(u)			
5	Say(u, a_{50} is red $\vee a_{50}$ is orange) \wedge True(u)	\wedge I, 1, 4		
6	a_{50} is red $\vee a_{50}$ is orange	R, 1		
7	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-bottom: 1px solid black; padding-right: 10px;">a₅₀ is red</td> <td></td> </tr> </table>	a ₅₀ is red		
a ₅₀ is red				
8	Say(v, a_{50} is red) \wedge a_{50} is red	\wedge I, 2, 7		
9	True(v)	R, 2		
10	True(v) \vee True(w)	\vee I, 9		
11	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-bottom: 1px solid black; padding-right: 10px;">a₅₀ is orange</td> <td></td> </tr> </table>	a ₅₀ is orange		
a ₅₀ is orange				
12	Say(w, a_{50} is red) \wedge a_{50} is orange	\wedge I, 3, 11		
13	True(v)	R, 2		
14	True(v) \vee True(w)	\vee I, 13		
15	True(v) \vee True(w)	\vee E, 6, 7–10, 11–14		

The first three lines stipulate what the utterances u , v and w say. Then, in line 4, we assume that u , which says that the tube is either red or orange, is in fact true. By our first rule we can now infer that this disjunction holds. Given that, the strategy is to eliminate the disjunction and to thereby derive at the claim that at least one of its disjuncts must be true. This is done by assuming either of its disjuncts and deriving precisely this (to be entirely correct,

we derive that at least one of the utterances, which say the disjuncts, is true). Having shown that this can be derived from either disjunct, we eliminate the disjunction and thereby have established that at least one of its disjuncts must be true. (Again, to be entirely correct, we have established that at least one of the utterances, that claim the respective disjunct, is true. This is not an important difference since by the first rule, the truth of an utterance yields that what is said by the utterance is the case.)

4.2.3 Rumfitt’s Criticism of the Argument

According to Rumfitt, the reason why the argument fails is rule R, 2. As Rumfitt diagnoses, in the above argument R, 2 “is applied to a case where tube a_{50} is a borderline case of redness, not a paradigm. In such a case the inference to ‘ v is true’ is not justified” (Rumfitt 2015, 258). It is not justified, according to Rumfitt, because v does not satisfy the predicate ‘true’. What is exactly is the satisfaction condition for ‘true’? At this point, Rumfitt extends his polar predicates approach to the predicate ‘true’. He thinks that ‘true’ is a polar predicate as well. But this requires specifying its poles against we assess whether an utterance satisfies ‘true’. One idea is that there are just the poles TRUE and FALSE. However, Rumfitt rejects this.

“If the only poles in the system are the TRUE and the FALSE, it would be correct to assess v as being as close to the TRUE as it is to the FALSE, just as tube a_{50} is as close to polar red as it is to polar orange. If, however, there is a third pole – let us call it the INDETERMINATE – then the inference from [line] 8 to [line] 9 takes us from an indeterminate premise to a false conclusion: if v is close to the pole of INDETERMINATE, then it is false to ascribe the contrary status of truth to it.” (Rumfitt 2015, 258)

So the INDETERMINATE pole explains why the second rule is not universally applicable. Thus, again, it is not $\forall E$ itself that, by Rumfitt, counts as the locus of failure in the argument.

4.2.4 My Response to Rumfitt's Criticism

With his solution, Rumfitt commits himself to controversial philosophical claims. Here is why. It is unclear whether the polar predicate approach generalizes, and whether it especially generalizes to the predicate 'true'. The semantics may be successfully applied to some vague predicates such as color predicates, but consider for instance the vague predicate 'tall' (with respect to human beings). How tall is the paradigmatic tall person? Although we say things like: "That person is clearly tall", this does not mean that there is a person (or bunch of persons) who represents the pole of tallness. This indicates a striking dissimilarity between 'red' and 'tall' that may explain why 'red' can be regarded as a polar predicate whereas 'tall' cannot. There is a "natural limit of redness", what is not the case for 'tall'. To get clear on what I mean, compare this to gravitational poles. Here, we do have a (perhaps theoretical) local limit of gravity, the pole p , for which there is no space point y at which there is more gravity. This is analogous to 'red'. There is a certain shade of red and nothing that is more red than it. With regard to 'tall', however, we find that a person can always be taller than another one, and still counts as tall. Thus, for 'tall' there seems to be no pole. This observation casts doubt on the universality of the polar predicate approach since predicates must meet certain constraints in order to legitimately treat them as polar predicates. Here are two such constraints: (i) there must be a (possibly identifiable) pole and (ii) in order not to be a trivial account, objects that may fall under the predicate must possibly come in degrees. Why should 'true' be a polar predicate, then? Concerning (i) what could be identified as the truth pole? It is clearly true that $2 + 2 = 4$. However, it is unclear why it should count as a paradigmatic truth. Furthermore with regard to (ii), note that the predicate 'true' is about statements, while 'red' is about colored objects. It seems to be straightforward to determine the color-degree of objects, but how do we determine the truth-degree of statements? Even if there are such things like truth-degrees or paradigmatic truth, they may be highly depend on the kind of statement, the context of utterance etc. As a consequence, it would be difficult, if not impossible, to univocally identify truth-degrees and paradigmatic truth.

These considerations do not conclusively show that Rumfitt's line of reasoning fails, but that he commits himself to controversial substantive claims. I argue that my account of the

connection between TRUE DISJUNCT and $\forall E$ yields a better explanation of why the argument does not work. It circumvents a commitment to controversial claims – such as treating ‘truth’ as a polar predicate – and solves the problem by simply pointing to *rule-circularity*. As we saw, the derivation of TRUE DISJUNCT relies on $\forall E$, and thereby indirectly on TRUE DISJUNCT itself. Hence, the argument is merely rule-circular. It argues for a logical principle by using this logical principle in its derivation. One could object that sometimes rule-circular arguments may be legitimate. However, in a debate about which logical principles to adopt, they seem to be vicious; if you argue for a logical principle and use this very principle in your argument, the argument seems to be pointless. Rumfitt himself remarks earlier in the book that “such an argument cannot be expected to persuade an interlocutor who doubts or denies the universal applicability of [a certain logical principle]” (Rumfitt 2015, 4).

5 Conclusion

In this paper I first presented supervaluationism and quantum logic as two cases where we have good reasons to deny TRUE DISJUNCT and, as a result, also $\forall E$. I then tried to flesh out the exact connection between TRUE DISJUNCT and $\forall E$. As it turned out, $\forall E$ presupposes a commitment to TRUE DISJUNCT. Hence, you cannot apply $\forall E$ to cases where a disjunction is such that TRUE DISJUNCT does not hold. I pointed out that certain arguments for TRUE DISJUNCT rely on $\forall E$. Thus, they fail because of rule-circularity. Rumfitt does not account for that, and the philosophical machinery which he instead introduces lacks any motivation.

References

- Birkhoff, G. and Von Neumann, J. (1936). The logic of quantum mechanics. *Annals of mathematics*, pages 823–843.
- Dummett, M. (1978). *Truth and other enigmas*. Harvard University Press.
- Dummett, M. (1991). *The logical basis of metaphysics*, volume 5. Harvard University Press.
- Humberstone, L. (2011). *The connectives*. MIT Press.

- Keefe, R. (2000). *Theories of vagueness*. Cambridge University Press.
- Putnam, H. (1969). Is logic empirical? In *Boston studies in the philosophy of science*, pages 216–241. Springer.
- Rumfitt, I. (2015). *The Boundary Stones of Thought: An Essay in the Philosophy of Logic*. Oxford University Press.
- Sainsbury, R. M. (1995). Concepts without boundaries. *Filosoficky Casopis*, 43(2):186–205.
- Steinberger, F. (2011). What harmony could and could not be. *Australasian Journal of Philosophy*, 89(4):617–639.
- Williams, J. R. G. (2008). Supervaluationism and logical revisionism. *The Journal of Philosophy*, 105(4):192–212.
- Williamson, T. (1994). *Vagueness*. Routledge.