The Graphical Method for Finding the Optimal Solution for Neutrosophic linear Models and Taking Advantage of Non-Negativity Constraints to Find the Optimal Solution for Some Neutrosophic linear Models in Which the Number of Unknowns is More than Three

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Abstract:
The linear programming method is one of the important methods of operations research that has been used to address many practical issues and provided optimal solutions for many institutions and companies, which helped decision makers make ideal decisions through which companies and institutions achieved maximum profit, but these solutions remain ideal and appropriate in If the conditions surrounding the work environment are stable, because any change in the data provided will affect the optimal solution and to avoid losses and achieve maximum profit, we have, in previous research, reformulated the linear models using the concepts of neutrosophic science, the science that takes into account the instability of conditions and fluctuations in the work environment and leaves nothing to chance. While taking data, neutrosophic values carry some indeterminacy, giving a margin of freedom to decision makers. In another research, we reformulated one of the most important methods used to solve linear models, which is the simplex method, using the concepts of this science, and as a continuation of what we did in the previous two researches, we will reformulate in this research. The graphical method for solving linear models using the concepts of neutrosophics. We will also shed light on a case that is rarely mentioned in most operations research references, which is that when the difference between the number of unknowns and the number of constraints is equal to one, two, or three, we can also find the optimal solution graphically for some linear models. This is done by taking advantage of the conditions of non-negativity that linear models have, and we will explain this through an example in which the difference is equal to two. Also, through examples, we will explain the difference between using classical values and neutrosophic values and the extent of this's impact on the optimal solution.

Keywords: linear programming; Neutrosophic science; Neutrosophic linear models; Graphical method for solving linear models; Graphical method for solving neutrosophic linear models.

Introduction:

A continuation of what we have done in previous research, the purpose of which was to reformulate some operations research methods using the concepts of neutrosophic science. See [1-14] The science that made a great revolution in all fields of science, which grew and developed very quickly, as many topics were reformulated using the concepts of this Science, and we find neutrosophic groups, neutrosophic differentiation, neutrosophic integration, and neutrosophic statistics ... [14-16], and given the importance of the graphic method used to find the optimal solution for linear models, which is a graph of the model and is one of the easiest ways to solve linear programming problems, but it is not sufficient to address All linear programming problems because they often contain a large number of variables, and the use of the graphical method is limited to the following cases:

- The number of unknowns is n = 1 or n = 2 or n = 3.
- In linear models whose constraints are equal constraints, if the number of unknowns and the number of equations meet one of the following conditions: n – m = 1 or n – m = 2 or n – m = 3.Here we can transform the model into a function of one variable or two variables or three variables, respectively, by taking advantage of the non-negativity constraints that the variables of the linear model have. In this research, we will present a reformulation of the graphical method for solving linear models using the concepts of neutrosophic, as well as the graphical method for solving linear models that Its restrictions are equal restrictions, and the difference between the number of unknowns and the number of restrictions is equal to one, two, or three.

Discussion:

The graphical method is one of the important ways to find the optimal solution for the linear and nonlinear models, so in the research [3] we reformulated it for the neutrosophic nonlinear models, and in this research we will present the graphic method to find the optimal solution for the neutrosophic linear models that were presented in the research [1], we know that the model the script is written in the following abbreviated form:

$$Z = \sum_{j=1}^{n} c_j x_j \rightarrow (Max \;\; or \;\; Min\,)$$

Restrictions:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \qquad ; i = 1,2, \dots, m$$

$$x_j \geq 0 \quad ; j = 1,2, \dots, n$$

If at least one of the values c_j , a_{ij} , b_i , is a neutrosophic value then the linear model is a neutrosophic linear model.

First: The graphical method for solving linear models: [17-20]

Through the studies presented according to classical logic in many references, we know that to find the optimal solution for linear models in which the number of variables is one, two, or three graphically, we represent the area of common solutions for the constraints in one of the spaces $R \; or \; R^2 \; or \; R^3$This depends on the number of variables in this sentence, for example if the number of variables is two, i.e. the solution is in space R^2 (where work is done on models that contain one or three variables with the same steps)

We find the optimal solution according to the following steps:

1. We determine the half-planes defined by the inequalities of the constraints by drawing the straight lines resulting from converting the inequalities of the constraints into equals. The drawing is done by specifying two points that satisfy the constraint, and then we connect the two points to obtain the straight line corresponding to the constraint. This straight line divides the plane into two halves in order to determine the half-plane that satisfies the constraint. We choose A point at the top of the mapping from one of the two half-planes. We substitute the coordinates of this point into the inequality. If it is satisfied, then the region in which this point is located is the solution region. If it is not achieved, then the opposite region is the solution region.
2. We define the common solutions region, which is the region resulting from the intersection of the halves of the levels defined by constraint inequalities. This region must be non-empty so that we can proceed with the solution.
3. In order to represent the objective function, we note that its relationship contains three unknowns, Z , x_1 , x_2 . Therefore, we must know a value for Z, which is unknown to us. Here we assume a value, let it be $Z_1 = 0$, draw the equation of the objective function Z_1 give another value, let it be Z_2, and represent the equation. We get a line parallel to the first line, and by continuing, we obtain a set of parallel lines representing the target function.
4. We draw ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$where $c_1$is coefficient of $x_1$and c_2 is coefficient of $x_2$in the objective function statement, and the direction of its increasing function is the direction of ray$\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, and the direction of its decreasing function is the opposite direction. This ray, that is, the drawing is done according to the type of objective function (maximization or minimization). In clearer terms, we find the optimal solution point by pulling the line representing Z_1parallel to itself towards the ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$to find the maximum value of the objective function, (And reverse this direction to find the smallest value), until it passes through the last point of the common solutions region and this point is the optimal solution point, which is located on the borders of the common solutions region and any other displacement, no matter how small, takes it out of it.

The graphical method for finding the optimal solution for neutrosophic linear models:
From the definition of neutrosophic linear models, we find that we can apply the same previous steps to obtain the optimal solution, which is a neutrosophic value suitable for all conditions. We explain the above through the following example:

Example 1:
A company produces two types of products A_1 , A_2 and uses three types of raw materials B_1 , B_2 , B_3 in the production process, if the available quantities of each of the raw materials are B_i ; $i = 1,2,3$, and the quantity needed to produce one unit of each products A_j ; $j = 1,2$, and the profit accruing from one unit of each of the products A_1 , A_2 is shown in the following table:

products / raw materials	A_1	A_2	available quantities
B_1	**6**	**4**	**36**
B_2	**2**	**3**	**12**
B_3	**5**	**0**	**10**
profit	$[6, 8]$	$[2, 4]$	

Table Issue data

Required:
Determine the quantities that must be produced of each products A_j ; $j = 1,2$ so that the company achieves maximum profit:
the solution :
Let x_j be the quantity produced from product j , where $j = 1,2$, then we can formulate the following neutrosophic linear mathematical model:

$$Z = [6,8]x_1 + [2,4]x_2 \rightarrow Max$$

Restrictions:

$$6x_1 + 4x_2 \leq 36 \quad (1)$$

$$2x_1 + 3x_2 \leq 12 \quad (2)$$

$$5x_1 \leq 15$$

$$x_1, x_2 \geq 0$$

The previous model is a linear neutrosophic model because there is indeterminacy in variables coefficients in objective function. To find the optimal solution for the previous model, we will use the graphical method according to the following steps:

The first constraint: We draw the straight line representing the first constraint:

$$6x_1 + 4x_2 = 36$$

We impose:

$$x_1 = 0 \Rightarrow 4x_2 = 36 \Rightarrow x_2 = 9$$

We get the first point: $A(0,9)$.

We impose:

$$x_2 = 0 \Rightarrow 6x_1 = 36 \Rightarrow x_1 = 6$$

We get the second point: $B(6,0)$

If we take a point at the top of the designation from one of the two halves of the resulting plane after drawing the straight through the two points $A(0,9)$ and $B(6,0)$, let it be the point $O(0,0)$and substitute it in the inequality of the first entry, we find that the inequality is fulfilled, that is, the half of the plane that the point $O(0,0)$belongs to it, which is half the solution plane of the first-constraint inequality.

We proceed in the same way for the second and third restrictions and obtain the following graphical representation: Figure No. (1)

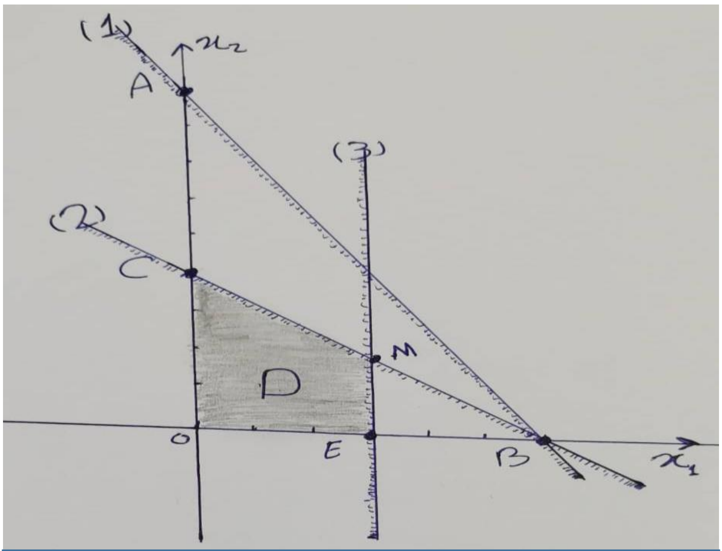

Figure No. (1) Graphic representation of the limitations of the linear model in Example 1

After representing the constraints, we notice that the common solution area is bounded by the polygon whose vertices are the points, $O(0,0)$, $E(3,0)$, M and $C(0,4)$.

The point M is the point of intersection, the second and third constraints we get their coordinates by solving the following two equations:

$$2x_1 + 3x_2 = 12$$

$$5x_1 = 15$$

We fiend**:** $M(3,2)$
Substituting the coordinates of the vertex points into the objective function expression, we get:

$$Z_O = 0$$
$$Z_E \in [12,16]$$
$$Z_M \in [22,32]$$
$$Z_C \in [8,16]$$

That is, the greatest value of the function Z is achieved at point $M(3,2)$, that is, the company must produce three units of the first product and two units of the second product, then it will achieve maximum profit.

$$Max\, Z = Z_M \in [22,32]$$

Note:
The process of substituting the objective function with the coordinates of the points of the vertices of the common solution area is possible when the number of points is small, as we can easily replace them in the objective function, and the point that gives the best value for the objective function represents the optimal solution, but when there are a large number of constraints, we get a large number From the vertical points located on the perimeter of the common solution region. In this case, the method of finding the coordinates of all these points and substituting them into the objective function becomes impractical, so we resort to representing the objective function and determining the optimal solution point as we mentioned previously.

Second: How to take advantage of the conditions of non-negativity to find the optimal solution for some neutrosophic linear models using the graphical method:

Example 2:
Find the optimal solution for the following linear neutrosophic model:

$$Z = x_1 - x_2 - 3x_3 + x_4 + [2,5]x_5 - x_6 + 2x_7 - [10,15] \rightarrow Max$$

Restrictions:

$$x_1 - x_2 + x_3 = 5 \qquad (1)$$

$$2x_1 - x_2 - x_3 - x_4 = -11 \qquad (2)$$

$$x_1 + x_2 - x_5 = -4 \qquad (3)$$

$$x_2 + x_6 = 6 \qquad (4)$$

$$2x_1 - 3x_2 - x_6 + 2x_7 = 8 \qquad (5)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

The solution:

We note that the number of constraints $m = 5$ and the number of variables $n = 7$, meaning that $n - m = 2$. Therefore, we can, relying on the non-negativity constraints, find the optimal solution for the previous model using the graphical method according to the following steps:

1- We calculate five variables in terms of only two variables.
2- Since the variables of the linear model satisfy the non-negativity constraints, then we obtain from the variables that we calculated five inequalities of the type greater than or equal to.
3- Substituting the five variables into the objective function, we get an objective function with only two variables.
4- We write the new model, which is a linear model with two variables, so the optimal solution can be found graphically.

We apply the previous steps to Example 2:

We find:

$$x_3 = 5 - x_1 + x_2 \qquad (1)'$$

$$x_4 = 3x_1 - 2x_2 + 6 \qquad (2)'$$

$$x_5 = x_1 + x_2 + 4 \qquad (3)'$$

$$x_6 = 6 - x_2 \qquad (4)'$$

$$x_7 = 7 - x_1 + x_2 \qquad (5)'$$

Substituting in the objective function, we get:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25]$$

Since, $x_3, x_4, x_5, x_6, x_7 \geq 0$ from $(1)'$ ، $(2)'$ ،$(3)'$ ، $(4)'$ ، $(5)'$, we get the following set of constraints:

$$5 - x_1 + x_2 \geq 0$$
$$-3x_1 + 2x_2 - 3 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$
$$6 - x_2 \ \geq 0$$
$$7 - \ x_1 + x_2 \geq 0$$

Then the neutrosophic linear mathematical model becomes:

Find:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25] \rightarrow Max$$

Restrictions:

$$5 - x_1 + x_2 \ \geq 0$$
$$3x_1 - 2x_2 + 6 \geq 0$$
$$x_1 + x_2 + 4 \geq 0$$
$$6 - x_2 \ \geq 0$$
$$7 - \ x_1 + x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

The model has two variables, so the optimal solution can be found graphically by following the same steps mentioned in Example (1).
We obtain the representation. Figure No. (2) Is the required graphic representation:

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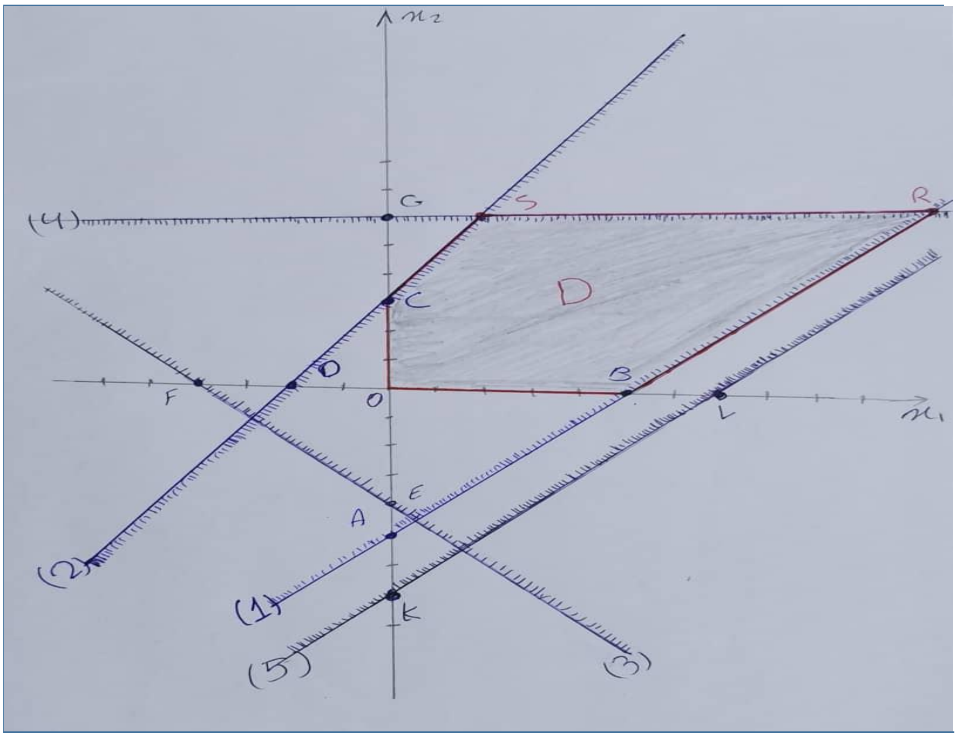

Figure No. (2): Graphical representation of the constraints of the linear model in Example 2

Region D is the region of joint solutions and is defined by the polygon $OBRSC$, where, $O(0,0)$, $B(5,0)$, $C(0,3)$, and for the two points R , S we find:
Point R is the point of intersection of the first and fourth entries.
We obtain its coordinates by solving the set of equations:

$$5 - x_1 + x_2 = 0$$
$$6 - x_2 = 0$$

We get: $R(11,6)$

Point S is the point of intersection of the second and fourth entries.
We obtain its coordinates by solving the set of equations:

$$3x_1 - 2x_2 + 6 = 0$$
$$6 - x_2 = 0$$

We get: $S(2,6)$

Since the optimal solution is located at one of the vertices of the common solution region, we substitute the coordinates of these points with the objective function:

At point, $O(0,0)$

$$Z_O = 0$$

At point, $B(5,0)$

$$Z_B = [13,45]$$

At point , $R(11,6)$

$$Z_R \in [37,105]$$

At point , $S(2,6)$

$$Z_S \in [28,69]$$

At point , $C(0,3)$

$$Z_C \in [17,43]$$

The greatest value of the objective function is at the point,$R(11,6)$ that is, $x_1 = 11$ and $x_2 = 6$.

We calculate the values of the remaining variables by $(1)'$ ، $(2)'$ ،$(3)'$ ، $(4)'$ ، $(5)'$.

We find, $x_3 = 0$, $x_4 = 27$, $x_5 = 21$, $x_6 = 0$, $x_7 = 2$.

Substituting in the objective function of the original model we obtain the maximum value of the Z function, which is.

$$MaxZ \in [68,126]$$

Important Notes:

1- The graphical solution applies to a vertical point in space R^n.
2- The number of components of the ideal solution is non-existent because the ideal solution applies to a vertical point, and the vertical point is the result of the intersection of a number of lines or planes, and the number of non-existent components is at least $n - m$ components.
3- The model may include some conditions that do not play a role in the solution process.
4- The ideal solution may be a single point, or it may be an infinite number of points, when one of the sides of the common solution area that passes through the point of the ideal solution is parallel to the straight line $Z = 0$. Therefore, when the straight line representing the objective function is drawn, this straight line will apply to the

parallel side, and all the points of that side, the number of which are infinite, will be they are perfect solutions.

5- If the region of acceptable solutions is open in terms of increasing the function Z then we cannot stop at a specific ideal solution, and then we say that the objective function has an infinite number of acceptable solutions that give us greater and greater values of Z.

6- The state of not having an ideal solution (acceptable solution) when the conditions contradict each other and then the region of possibilities is an empty set (the problem is impossible to solve).

Conclusion and results:

In the previous study, we presented the graphical method for finding the optimal solution for neutrosophic linear models, and also a method that is rarely discussed in classical operations research references, which is how to take advantage of non-negativity constraints to find the graphically optimal solution for some neutrosophic linear models, but we must be aware that we may encounter neutrosophic linear models with two variables, but There may be difficulty in arriving at the common solution area, or there may be difficulty in determining the optimal solution after obtaining the common solution area. Therefore, it is preferable to use the Cymex neutrosophic method. As a result, the main goal is to obtain the optimal solution, so the researcher must determine the appropriate method for the model being solved.

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