

Classicality Lost: K3 and LP after the Fall

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Abstract

It is commonly held that the ascription of truth to a sentence is intersubstitutable with that very sentence. However, the simplest subclassical logics available to proponents of this view, namely K3 and LP, are hopelessly weak for many purposes. In this paper, I argue that this is much more of a problem for proponents of LP than for proponents of K3. The strategies for recapturing classicality offered by proponents of LP are far less promising than those available to proponents of K3. This undermines the ability of proponents LP to engage in public reasoning in classical domains.

1 Introduction

Gottlob Frege famously held that “nothing is added to [a] thought by ... ascribing to it the property of truth” (1956, 293). This idea is commonly expressed with the slogan that truth is *transparent*: ϕ and $Tr(\ulcorner \phi \urcorner)$ ¹—the sentence that says that ϕ is true—are fully intersubstitutable in extensional contexts. Unfortunately, in classical logic, the law of excluded middle, i.e. $\phi \vee \neg\phi$, and the rule of explosion, i.e. $\phi, \neg\phi \therefore \psi$, allow us to derive any sentence from the liar sentence if we have transparency. It’s tempting to put the blame on transparency here. However, it isn’t entirely obvious what to replace transparency with.² That is why a number of authors have instead blamed classical logic. Saul Kripke (1975), Robert Martin and Peter

¹“ $\ulcorner \phi \urcorner$ ” is a term for ϕ in the object language. Note that the corner quotes here are Gödel quotes, not Quine quotes. To be fully precise, I would need to use both kinds of quotes; however, since corner quotes are commonly used for both, I allow myself the usual use-mention sloppiness here and throughout.

²See McGee (1990) and Halbach (2011) for surveys of some of the options.

Woodruff (1975), and Peter Woodruff (1984) have shown how we can preserve transparency if we dispense with either excluded middle or explosion. The most conservative deviation from classical logic that gives up excluded middle is Kleene’s Strong 3-valued Logic (K3), and the most conservative non-explosive logic is the Logic of Paradox (LP).³ Semantically, both of these logics are *three-valued* logics: they introduce a third truth value in addition to the classical truth values of truth and falsity. Proponents of K3 are called *gap theorists* because they interpret the third truth value as a truth value gap—neither true nor false—, while proponents of LP are called *glut theorists*, because they interpret the third truth value as a truth value glut—both true *and* false.

However, the move to such subclassical logics has serious drawbacks. In giving up modes of reasoning that are central to classical logic, we are left with logics where “nothing like sustained ordinary reasoning can be carried on,” to quote Solomon Feferman (1984, 95).⁴ As a partial remedy for this, Hartry Field (2008) and Jc Beall (2009) have recently devised stronger logics that build on K3 and LP, respectively, but that are still weak enough to be compatible with transparency.

Despite their virtues, Field’s and Beall’s logics have significant drawbacks of their own. Most notably, they are vastly more complex than classical logic, K3, and LP. For the latter three logics, we have algorithms for deciding whether a sentence of propositional logic is classically valid. By contrast, the task of deciding whether a sentence is valid in Field’s logic, which has been more deeply studied than Beall’s, is vastly more complex even than deciding whether a sentence of the language of arithmetic is true, the latter of which is already far beyond what’s humanly possible.⁵ So while Field’s logic may come close to classical logic in terms of strength, it is doubtful whether it meets Feferman’s challenge of being able to allow

³See Kleene (1950) for K3 and Asenjo (1966) and Priest (1979) for LP.

⁴At the time, Feferman was talking about K3, but he (2012, 190) has since observed that the same holds for LP.

⁵As McGee (2010, 430-31) shows by building on work due to Welch (2008), the set of valid sentences in Field’s logic is complete Π_2^1 .

for sustained ordinary reasoning if it is humanly impossible to determine whether a given sentence is a theorem of the logic or not.⁶

More may be said on behalf of Field (2008), and also on behalf of Beall (2009), but the foregoing suggests that attempting to find a logic that's stronger than K3 or LP that meets both transparency and Feferman's challenge remains elusive. It may therefore be worth taking another look at K3 and LP to see how much we can get out of them. This is the topic of the present paper, where I set aside the more sophisticated subclassical logics.

Recently, Beall (2011, 2013b, 2015a) has taken some first steps towards determining how much we can get out of K3 or LP. Following Gilbert Harman (1986, ch. 2), Beall distinguishes between *logic* and *reasoning*. Certain inferences may be justified by certain standards of reasoning, even though they don't involve moving from premises to a conclusion that logically follows from the premises. And, in fact, Beall argues, in most domains of inquiry, applications of excluded middle and explosion are perfectly legitimate, even though they are *logically* invalid according to K3 and LP, respectively. Therefore, goes Beall, if we're reasoning about a classical domain we may be confident that reliance on classical modes of reasoning won't lead us astray.

In this paper, I argue that glut theorists have a much harder time recapturing classical reasoning in select domains than gap theorists. I do so by investigating the feasibility of two prominent proposals of how glut theorists can recapture classicality, namely by way of shriek rules and by way of conversational implicatures. What's attractive about these proposals is that they only appeal to ordinary, well-understood resources. If glut theorists are able to recapture classical reasoning using only such resources, then they may come close to meeting Feferman's challenge. Of course, there will remain areas where classical reasoning is inappropriate, but such is the price of transparency. Unfortunately, as I will argue, the glut theorist's attempt at recapturing classical reasoning using either of these resources is ill-fated when it comes to public reasoning.

⁶It may be worried that we're being too demanding here. After all, classical first-order logic is also undecidable. However, validity in classical first-order logic, unlike Field's logic, is at least semi-decidable, and thus axiomatizable, since it is complete Σ_1^0 .

Another proposal of how glut theorists can recapture classicality, defended most prominently by Graham Priest (2006, 290–5), involves so-called *denial*. Contrary to Frege (1918) and many others, denial is said to be a *sui generis* speech act, not to be reduced to assertion of a negation. However, even fellow glut theorist David Ripley thinks that such a speech act would first have to be invented: “I don’t know of any phenomenon studied outside the realm of philosophical logic that could fill the theoretical role occupied by denial in our philosopher’s theories” (Ripley 2015, 292).⁷ And even if it exists, *sui generis* denial is surely a much less ordinary and well-understood phenomenon than rules of inference and conversational implicatures. Thus, it would be a disappointment from the perspective of Feferman’s challenge if glut theorists had to resort to such an extraordinary speech act. Therefore, for the purposes of this short paper, I set aside any discussion of denial.

2 Classical recapture for gap and glut theorists

Give a person a fish and you feed them for a day; teach a person to fish and you feed them for a lifetime.

Suppose you and I are subclassical logicians, and suppose you want to teach me classical mathematics. One way to teach me would be to simply assert any mathematical statement you’ve proven using classical reasoning. However, a more sustainable method would be to teach me how to prove theorems myself. One way to do so would be to carry out some derivations and hope that I’ll catch on. But upon inspecting your proofs, I am baffled. Your proofs are grossly fallacious; almost none of them are underwritten by the subclassical logic we both adhere to.

If we’re both paracomplete proponents of K3, this problem is easily overcome. When you present a proof of some theorem of, say, classical set theory you simply add as an additional premise the claim that $\forall x \forall y (x \in y \vee \neg x \in y)$, i.e. the claim that set theory is *complete* in the sense that any two sets are such that either the first is a member of the second or it isn’t. Since K3 becomes classical logic when we add the law of excluded middle as an axiom, this

⁷See also Murzi and Carrara (2015a) for a criticism of Priest’s appeal to denial.

guarantees that your set theoretic proof, which is classically valid, becomes valid according to K3. Thus, by asserting the relevant instances of the law of excluded middle, K3 logicians can recapture classical reasoning in select domains.

Unfortunately, things aren't so simple if instead we're *paraconsistent* proponents of LP. As a glut theorist, you need to fill the gaps in your proof not by communicating to me that mathematics is complete but that mathematics is *consistent*. Since glut theory is of course compatible with the non-classicality of mathematics, and indeed there are glut theorists who claim that mathematics is inconsistent,⁸ there will be no way for me to infer that you take mathematics to be consistent from the fact that you're a glut theorist. It's tempting to think that you can communicate to me that mathematics is consistent by asserting what might be thought to be the dual of the above, i.e. $\neg\exists x\exists y(x \in y \wedge \neg x \in y)$. However, $\neg\exists x\exists y(x \in y \wedge \neg x \in y)$ is actually logically equivalent to $\forall x\forall y(x \in y \vee \neg x \in y)$ in both LP and K3 (as well as in classical logic), and so this attempt at expressing consistency just amounts to an assertion of excluded middle again. And unlike in K3, excluded middle is logically valid in LP, so I wouldn't learn anything from your assertion of $\neg\exists x\exists y(x \in y \wedge \neg x \in y)$ that I couldn't already figure out on my own.

More generally, the problem is this: in order for Beall's strategy of recapturing classical reasoning in select domains to be fully effective, gap and glut theorists need to coordinate among themselves which domains they take to be classical. After all, just like any other conversation, public reasoning operates against a background of shared assumptions, the so-called *common ground*.⁹ Gap theorist can add to the common ground and establish that a certain domain is classical by simply asserting the right instance of the law of excluded middle. How might glut theorists achieve the same thing?¹⁰ In the next two sections, I critically discuss two answers to this question that have been offered in the literature.

⁸See Priest (2006, esp. ch. 17), and Mortensen (2013) for an overview.

⁹See Stalnaker (2014).

¹⁰This issue is related to the so-called "just true" problem, which is the problem of how glut theorists can express that a sentence is just true and not also false. See Parsons (1984), Batens (1990, sect. 4), Parsons (1990, sect. 6), Simmons (1993, §4.4), Everett (1994, 1996), Bromand (2002), Olin (2003, ch. 2), Shapiro (2004), Littmann and Simmons (2004), Priest (2008, §6.3), Priest (2006, §20.4), Beall (2009, 2013a), Rossberg (2013), Berto (2014), Hughes (2015), and Murzi and Carrara (2015b) for more on the "just true" problem.

3 Shrieking

Beall (2013a) develops a device based on Priest’s (2006, §8.5) notion of “shrieking.”¹¹ He proposes that glut theorists may adopt an extra-logical rule of inference, called a *shriek rule*, that allows them to infer anything from a contradiction in classical domains such as mathematics. This rule says that if you have a sentence of mathematics ϕ_M , then you may infer anything from $\phi_M \wedge \neg\phi_M$.

It’s tempting to express the idea behind shrieking in terms of an extra-logical *axiom* instead of a rule. It’s tempting, that is, to think that a glut theorist can get the same effect by asserting $(\phi_M \wedge \neg\phi_M) \rightarrow \perp$, where \perp is some sentence that entails every other sentence. However, since *modus ponens* for the material conditional \rightarrow isn’t valid in LP, we may not conclude from a glut theorist’s assertion of ϕ_M and $(\phi_M \wedge \neg\phi_M) \rightarrow \perp$ that she regards ϕ_M as classical. In fact, in LP just as in classical logic, $(\phi \wedge \neg\phi) \rightarrow \perp$ is a logical truth, for any ϕ , and so its assertion by a glut theorist doesn’t tell us anything we didn’t already know.¹²

The difference between shriek rules and shriek axioms points to the more general fact that in LP, unlike in classical logic, there is a big difference between a rule that allows us to infer ψ from ϕ and the corresponding axiom $\phi \rightarrow \psi$.¹³ This is unfortunate. If $(\phi_M \wedge \neg\phi_M) \rightarrow \perp$ did indeed achieve its desired effect, then glut theorists would only need to assert it along with ϕ_M in order to express that they take ϕ_M to be classical. With a shriek *rule*, things aren’t as straightforward. But before we get to that, note first that shriek rules may well help glut theorists selectively recapture classicality *in thought*. But we’ve been discussing the issue of how two or more glut theorists can *coordinate* on the classicality of a certain domain.

You could attempt to express to me that you take, say, mathematics to be classical, by asserting that you’ve adopted the relevant rule. But absent a way for us to coordinate on

¹¹See also Field (2008, 388). See Murzi and Carrara (2015a) for some additional worries about shrieking than the ones discussed in this section.

¹²Priest (2006) as well as Beall (2009) have devised logics based on LP that contain a conditional that does validate *modus ponens*. So in those logics, a shriek axiom may do just as well as a shriek rule. (Though see Beall (2013a, §2.1) for worries about Priest’s version of shriek axioms.) But as mentioned in the introduction, the present focus is on determining how much we can get out of the simple logics K3 and LP.

¹³For this reason, it also wouldn’t help to add $(\phi_M \wedge (\phi_M \rightarrow \psi_M)) \rightarrow \psi_M$ as an axiom, because this sentence is already a logical truth of LP. So perhaps Lewis Carroll’s (1895) Tortoise is a proponent of LP?

the classicality of rules and their adoption, I won't be able to rule out that it's also false that you've adopted the rule. You can't rule out, that is, that it's both true and false that you're committed to ψ_M . So while you can tell me that you're committed to the classicality of mathematics, you can't tell me that *that* is a classical truth, a truth that's not also false, that you are thus committed. Compare this again with how simple it is for gap theorists to unequivocally commit themselves to the classicality of mathematics: they simply need to assert the relevant instances of the law of excluded middle.

Might I be able to read off of your *behavior* that you take mathematics to be classical? In general, people's inferential behavior is not a good guide to what they accept as consequences of their beliefs.¹⁴ But perhaps the present case is different. Note that adding material *modus ponens* to LP suffices to recapture classical reasoning. So, perhaps you can infer ψ_M from $\phi_M \rightarrow \psi_M$ and ϕ_M often enough to indicate to me that you accept a version of *modus ponens* restricted to sentences of mathematics. Whether this may work can't be determined from the armchair. But note that this route to recapturing classicality would be much more circuitous compared to the ease with which gap theorists can commit themselves to the classicality of mathematics.

4 Whistling

It has also been suggested that glut theorists can express classicality by way of conversational implicatures. Perhaps I can compute the implicature that you take mathematics to be classical if you, the glut theorist, never assert both a mathematical sentence and its negation.¹⁵ My pragmatic reasoning might go like this: It is a maxim of conversation, called the *maxim of quantity*, that one ought to say just enough of what's relevant for the purposes of a conversation, not more and not less. It follows from this maxim that if you thought that the sentence you asserted was both true and false, you would have asserted it *and* its negation. Since you

¹⁴See Harman (1986, ch. 2) and Harman (2009). See also Scharp (2013, 82) for a similar observation.

¹⁵See Shapiro (2004, 339), Priest (2006, 291), Beall (2009, 51–2), and Armour-Garb and Priest (2005, 168). See Grice (1967) for the pragmatic theory that's in the background here.

didn't assert the negation, you must think that the sentence is classical. Do this often enough and I may be able to infer that you take mathematics to be classical.

However, in the present case, the maxim of quantity conflicts with another conversational maxim, the maxim that says to not assert anything that's already in the common ground, even if it's relevant to the topic under discussion. Since Robert Stalnaker (1978, 49) endorses this maxim, we'll call it *Stalnaker's maxim*. A result of Stalnaker's maxim is that speakers will often not assert things that they think the audience already believes. This creates trouble for the maxim of quantity because, in many conversations, glut theorists will believe that their audience already believes the *negation* of a certain sentence, and what they want to establish is that the sentence itself is true *in addition* to being false. So they will only assert the sentence, without its negation. Indeed, glut theory's core thesis, the thesis that there are sentences that are both true and false, is a case in point. According to glut theorists' own lights, this is a non-classical thesis in that it is both true and false. But glut theorists don't often go around asserting the falsity of their core thesis.¹⁶ That's plausibly because at least in conversations with classical logicians, the falsity of this claim is already common ground. As a result, your pragmatic reasoning would lead you astray if you inferred that I believe that the claim that there are sentences that are both true and false is classical from the fact that I don't go around asserting its negation.

Another problem with this pragmatic strategy is that the purported conversational implicatures, even if they were generated, wouldn't display the same behavior displayed by implicatures in general. There are two general features of conversational implicatures that are relevant here; purported implicatures of classicality exemplify the first one but not the second. The first feature, stressed by Grice, is that we can cancel implicatures. Here the supposed implicatures of classicality behave like ordinary implicatures: to cancel the supposed implicature that what you asserted is classical, you simply add an assertion of its negation. But there's another feature of implicatures that naturally goes with their cancelability: implicatures can usually be made explicit. For example, if you say of some people that they moved

¹⁶For an honorable exception, see Priest (1979, 239).

in together and adopted a puppy, thereby generating the implicature that the moving in occurred before the adoption, you can make that implicature explicit by saying that they moved in together *and then* adopted a puppy. If there's any doubt on the part of your audience about whether my chosen sentence order corresponds to the temporal order of the events your assertion is about, you can dispel that doubt by making the temporal order explicit. And the same holds for most conversational implicatures. The purposes of most implicatures isn't to express the inexpressible, but to cut corners.¹⁷ The same isn't the case for the supposed conversational implicatures of classicality. On the current proposal, glut theorists can only ever *suggest*, but never actually *say*, that something is classical. That a domain is classical therefore becomes one of those mysterious Tractarian truths that we can only allude to but never actually put into words, at least not unequivocally. What we thought was a mundane feature of many domains, namely that they are classical, takes on almost mystical qualities.¹⁸

It may be wondered whether the above clash between the maxim of quantity and Stalnaker's maxim points to a general flaw with the Gricean approach to pragmatics and its reliance on multiple principles. Perhaps, that is, the problem lies not with the present proposal of how to express classicality, but rather with the theoretical framework within which it is proposed. An alternative approach to pragmatics that only relies on one principle of communication is the *relevant-theoretic* approach due to Dan Sperber and Deirdre Wilson (1995; 2012). Robyn Carston puts the *presumption of optimal relevance*, which is the principle at the core of this approach, as follows: "Speakers should not be, and are expected not to be, as explicit as possible. They should encode only what they cannot rely on their addressees to infer easily" (2002, 289). However, it's hard to see how the presence of this principle in communicative situations would lead to anything like an implicature of classicality. In fact, in our present situation, the principle seems to be entirely idle. After all, we've already seen

¹⁷It's difficult to find anyone explicitly endorsing this feature of implicatures in the literature. Perhaps that's because this feature is taken to be so deeply entrenched in the common ground that it's usually not worth emphasizing. However, Fox (2006, 7) endorses the more limited claim that all *scalar* implicatures can be made explicit; and the purported implicature of classicality would be at least a distant relative of scalar implicatures.

¹⁸See Wittgenstein (1922): "There is indeed the inexpressible. This *shows* itself; it is the mystical" (§6.522; emphasis in the original). Of course, Priest (2002, 2014a,b) thinks that there are ineffable things that we can nonetheless talk about. But according to him, these things are very much at the periphery of language.

that classicality can't be encoded; that's why there was a need for a pragmatic strategy to begin with. So, within the relevance-theoretic approach to communication, classicality can only ever be inferred by the addressee. But the whole problem of coordinating on classical domains rests on the premise that classicality can't be inferred easily.

At this point it may be protested that surely things can't be *that* bad. After all, glut theorists do say all the time that certain domains are classical while others aren't.¹⁹ Shouldn't we take them at their word, or at what their word implicates? Indeed, perhaps there is some pragmatic difference between an assertion of ϕ and an assertion $\phi \wedge \neg Tr(\neg^\Gamma \phi^\neg)$, even if, as per transparency, there is no logical difference. I don't take myself to have refuted this possibility once and for all. But glut theorists owe us a story about what exactly this pragmatic difference amounts to, and ideally this story would steer clear of mysticism. Note also that for Beall's brand of glut theory, the resources available to tell a story about the pragmatic difference between ϕ and $\phi \wedge \neg Tr(\neg^\Gamma \phi^\neg)$ are particularly limited. For, Beall's brand of glut theory comes with a strong form of deflationism about truth that says that the nature of truth is exhausted by the function played by the truth predicate as a device of generalization.²⁰ It's not obvious that this attitude towards truth leaves room for a pragmatic difference between ϕ and $\phi \wedge \neg Tr(\neg^\Gamma \phi^\neg)$.

5 Conclusion

Even if there turns out to be some pragmatic way for glut theorists to suggest that something is classical, this would fall well short of establishing that it's as easy for glut theorists to coordinate on classicality as it is for gap theorists. Rather, this would merely mean that things aren't *as* bad for glut theorists as they may initially seem, while leaving my claim that they *are* bad untouched.

One perhaps surprising upshot of our discussion is that when comparing the relative merits of different logics, it matters whether those logics are seen to be logics that govern

¹⁹See for example Priest (2008, ch. 3) and Beall (2009, §1.5).

²⁰See Beall (2009, §1.1) and Beall (2015b, §9.2). See also Armour-Garb and Beall (2005b) for a more general overview.

thought or logics that govern speech. As we've seen, the method of shrieking plausibly allows a solitary glut theorist to recapture classicality in thought, but it is much less promising as a way for multiple glut theorists to recapture classicality in speech. This is particularly significant in light of the fact that some authors such as Catarina Dutilh Novaes (2015) have recently argued that when asking about the normative role of logic, we should focus on dialogical interactions rather than on solitary thought.

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