UNDERSTANDING AND MODALITY IN DISTINCTIVELY MATHEMATICAL EXPLANATIONS OF COMPLEX SYSTEMS

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Abstract

Various accounts of distinctively mathematical explanations (DMEs) of complex systems have been proposed recently which bypass the contingent causal laws and appeal primarily to mathematical necessities constraining the system. These necessities are considered to be modally exalted in that they obtain with a greater necessity than the ordinary laws of nature. This paper focuses on DMEs of the number of equilibrium positions of pendulum systems, and considers three different DMEs which bypass causal features — (D1): that there are four or more equilibrium positions of any double pendulum system because its configuration space obtains a torus with at least four critical points (Lange 2016, p. 27); (D2): that k-th Betti number of the configuration space of any n-uple pendulum yields k-stable directions of the pendulum system; and (D3): that any pendulum system has 2^n number of equilibrium positions. It then argues that there is a tension between the modal strength of these DMEs and their epistemic hooking, and we are forced to choose between (a) a purported DME with greater modal strength and wider applicability but poor epistemic hooking, or (b) a narrowly applicable DME with lesser modal strength but with the right kind of epistemic hooking. It also aims to show why some kind of DMEs despite their strong modality are unappealing for working scientists. The broader goal is to show why such tensions weakens the case for DMEs for pendulum systems in general.

A number of distinctively mathematical explanations (DMEs) have been proposed in the literature (Baker 2012; Lange 2016; Mancosu 2008; Steiner 1978) where the explanatory power of the explanans derives not by accurately describing the causal nexus of a target system but by appealing to some modally exalted mathematical facts that seem to constrain the system with a necessity surpassing that of the ordinary laws of nature. This paper focuses on Lange's (2016) version of these DMEs for complex systems such as the double pendulum. Lange shows that a constraint on the number of equilibrium positions of a double pendulum can be obtained bypassing the causal features of the pendulum system. He argues that any

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double pendulum system, whether simple or complex, has four or more equilibrium positions and this fact is modally constrained by some distinctively mathematical facts which appeal only to the configuration space of the system (a torus) irrespective of the contingent laws governing these systems (2016, p. 31). Such DMEs are then termed non-causal 'explanations by constraint' because the explanatory features of the target system, i.e. the number of equilibrium positions, remains invariant even if contingent laws of nature were to change. A number of desideratum have been recently proposed to evaluate these DMEs (Baron 2016; Craver & Povich 2017; Povich 2019), including: the (1) *modal desideratum*: which evaluates the modal strength of the DME; and (2) *the distinctiveness desideratum*: which segregates the parts of the mathematical explanation that are distinctively mathematical from the parts that are merely doing representational work or cause-tracking work such as in the flagpole-shadow explanation or the train example.[1] I propose a new desideratum to evaluate DMEs - the *epistemic desideratum*, which illuminates the hooking between the DME and the target system. (I will use an intuitive way of judging the epistemic desideratum which will be clear from a detailed discussion of the case studies in this paper.) I am going to argue that the *epistemic desideratum* is in conflict with the *modal desideratum* and/or the *distinctiveness desideratum* for the chosen case study of double pendulums, which I will also suitably extend to n-uple pendulums. I evaluate the modal strength and epistemic hooking of the DME for double pendulums by introducing counterpossibles, both mathematical and physical, covering a related family of complex systems. I then pose a dilemma for the proponents of DMEs of choosing between (a) a purported DME with greater modal strength and wider applicability but poor epistemic hooking, or (b) a narrowly applicable DME with lesser modal strength but with the right kind of epistemic hooking. I show this by expanding the pool of DMEs by considering three of these candidates:

(D1) There are four or more equilibrium positions of any double pendulum.

(D2) For any n-uple pendulum, the number of equilibria with k stable directions is equal to the k-th Betti number of its configuration space (n-dimensional torus), where a stable direction implies a pendulum rod pointing downward.

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[1] For instance, a trigonometric identity $\tan \theta = h/l$, where l is the length of the shadow and h is the height of the flagpole, explains the height of the shadow, but this is not a distinctively mathematical explanation since the trigonometric identity merely tracks the causal features of the target system and does not constrain it (Povich 2019). Similarly, for the train example, the reason why a train covers a distance of 10 km in 1 hour when going at the speed of 10km/hr is not because the formula $time = distance/velocity$ is a constraint, rather it is only explicating physical constraints on the system (Baron 2016).
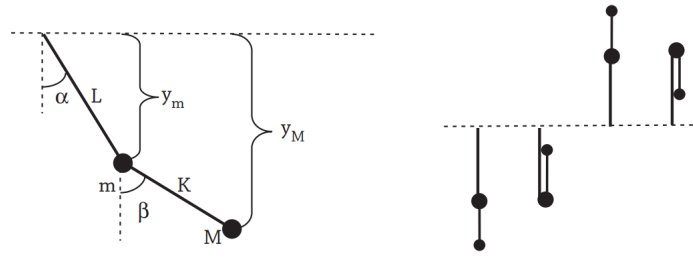
Figure 1: A double pendulum with stiff rods and its four equilibrium positions: from Lange (2016)

(D3) The number of equilibrium positions for any n-uple pendulum is 2^n.

The plan for the paper is as follows. In section 1, I discuss (D1), which is Lange's explanation of the double pendulum based on configuration spaces, and highlight its lack of appeal to working scientists when the strategy backing (D1) is extended to other kinds of pendulum systems. In section 2, I propose (D2) as an alternative strategy to extend the DME to any n-uple pendulum, and show how this strategy is modally stronger and more appealing to working scientists than (D1). In section 2.1, I discuss some methodological concerns with using configuration spaces to predict the constraint on the number of equilibrium positions for pendulum systems, and how such concerns deflate the modality but enhance the epistemic desideratum of (D1) and (D2). In section 2.2., I discuss a simpler version (D3) of these distinctively mathematical explanations and show why it fares better on the modal desideratum but poorer on the epistemic and distinctiveness desideratum. I then briefly discuss how a causal explanation of the number of equilibrium positions, having a better epistemic hooking but weaker modality, can subvert the problems faced by (D1), (D2) and (D3). The broader goal is to show why the peculiar kind of tension between modality, distinctiveness and epistemic hooking discussed in this paper weakens the case for DMEs for pendulum systems in general.

1 Concerns with Lange's account

In this section, I briefly sketch out the causal and non-causal version of the explanation or (D1) for the double pendulum given by Lange (2016). Then I show that the strategy of using the invariance properties of the configuration space in the DME is unappealing for other kinds of pendulum systems. This discussion paves way for the alternative strategy (D2) discussed in the next section.

The potential energy (P.E.) function for a double pendulum, in Figure 1, can be written as:[2]

$$U(\alpha, \beta) = -mgy_m - Mgy_M$$

$$y_m = Lcos\alpha$$

$$y_M = Lcos\alpha + Kcos\beta$$

$$U(\alpha, \beta) = -mgLcos\alpha - Mg(Lcos\alpha + Kcos\beta)$$

$$\frac{\partial U}{\partial \alpha} = mgLsin\alpha + MgLsin\alpha$$

$$\frac{\partial U}{\partial \beta} = mgKsin\beta$$

One way to find out the number of equilibrium positions of the pendulum is to find positions where the partial derivatives of $U(\alpha, \beta)$ are zero - these are positions where $sin\alpha$ and $sin\beta$ are zero: $(0, 0), (0, \pi), (\pi, 0)$ and (π, π). Lange (2016) calls this as a causal explanation since the explanation crucially relies on tracking the causal features of the system involving a change in the P.E. function. Another way to find out these equilibrium positions involves reasoning with the configuration space of the double pendulum which remain invariant despite any physical alterations to the pendulum system or a change in the contingent force laws acting on the system. (Newton's second law is here considered as a framework law which can do with any kinds of forces.)[3] The configuration space of the double pendulum obtains a torus of genus or $g = 1$. If we assume that the P.E. function remains finite and continuous, $U(\alpha, \beta)$ also obtains a torus (a distorted one though). If $U(\alpha, \beta)$ also satisfies a Morse function (having a non-degenerate Hessian matrix), then the Euler characteristic of the Morse function can be used to find a lower bound on the number of critical points of $U(\alpha, \beta)$. Since $N_{max} - N_{saddle} + N_{min} = 2 - 2g$ for a compact space and $g = 1$ for the torus, given there is at least one minima and one maxima, there must be at least two saddle points. This implies total four or more critical points for $U(\alpha, \beta)$. Lange maps this back to the pendulum claiming that the double pendulum must also have at least four or more critical or equilibrium points, and that it is is a non-causal explanation or 'explanation by constraint' because it only appeals to the configuration space of the double pendulum which is a torus. He also claims that this DME works for any kind of double pendulum, with stiff rods, non-stiff roads, complex pendulum and so on (p. 31).

There are a couple of initial worries which although do not undermine Lange's thesis, they reveal certain presumptions that were not explicit earlier or show why his account is unappealing to a working scientist. First,

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[2]Borrowing the format from Lange (2016, pp. 26-27)

[3]See Ch. 2-4 in Lange (2016) for an elaborate discussion.

Lange is not quite right in stating that the explanation exploits only the configuration space of the pendulum, the explanation exploits the configuration space of the pendulum only given that the P.E. function remains 'linear'.[4] If the P.E. function was non-linear, the strategy fails because a non-linear P.E. function will not obtain a distorted torus as discussed above. Consider if the P.E. function was second-order (i.e. $U = mgL^2$), say in some exotic world, then $U(\alpha, \beta) = -mgL cos\alpha - Mg(L cos\alpha + K cos\beta)$ will instead be $U(\alpha, \beta) = -mgL^2 cos^2\alpha - Mg(L cos\alpha + K cos\beta)^2$. Lange's strategy falls flat here since now the torus can no longer be mapped on the P.E. function. One could show using partial derivatives approach that $U(\alpha, \beta)$ will now have only one critical point or equilibrium point.[5] So in an exotic world where the P.E. function is a second-order function, the standard DME of the double pendulum with four or more equilibrium positions fails. One might add this is not so much of a worry for Lange since he assumes that these DMEs only make sense when Newton's second law is applicable, and one can show, using the principle of least action and symmetry, that a non-linear P.E. function conflicts with Newton's second law of motion (in the form we know of).[6] So a non-linear P.E. function violates those very constraints that as per Lange supply the explanation by constraint. But the worry here is that the dependence of the DME on the linearity of P.E. function is not shown by Lange to be critical for the explanation. Given that it is a critical link in the explanatory dependence between the number of equilibrium positions and the invariant configuration space of the double pendulum, this should be added as an explicit constraint in his account. The explanatory dependence only arising from the configuration space of the torus is thus a misnomer.

The second more important worry is that the strategy of using the Euler characteristic of a Morse function cannot be extended in the same form to pendulums with higher number of members, say n-uple pendulums. The configuration space of a n-uple pendulum (and its linear P.E. function) obtains a n-torus with genus 1. Again, $N_{max} - N_{saddle} + N_{min} = 2 - 2g$ for a compact space, and $g = 1$ for the n-torus, there must be at least 4 or more

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[4]Saatsi (2018, p. 5) endorses a similar point about P.E. functions and argues that "Consider, for instance, changing the potential energy function so that it does not pull uniformly down, as in the case of a standard gravitational pendulum that Lange probably has in mind, but instead pulls symmetrically up above the centre of the pendulum, and down below it, so that there is a plane running through the centre where the potential energy vanishes. With such forces acting upon the pendulum it will have at least 8 equilibrium configurations." Lange (2018) replies to these objections stating that the P.E. function having a different feature which lends eight equilibrium positions to the pendulum is a different why-question, and still fails to account for the explanation why there are 'at least' four or more equilibrium positions for a double pendulum.

[5]The derivation is avoided here for brevity but can be easily obtained.

[6]This is because the Lagrangian $L = T(\dot{q}) - V(q)$ requires $V(q)$ to be linear in q where $T(\dot{q})$ is quadratic in $\dot{q}$, if it has to satisfy the Euler-Lagrange equation. A second-order function $V(q)$ in q will violate the Euler-Lagrange equations of motion. A detailed discussion will not be pursued here.

critical/equilibrium points for $U(\alpha, \beta)$ or the n-uple pendulum. But this result seems a bit trivial. A triple pendulum has 8 equilibrium positions, a quadruple-pendulum has 16 and so on, and yet all that the Euler characteristic tells us is there are 4 or more such positions. The explanation that every n-uple pendulum (with $n > 2$) has 4 or more equilibrium positions seems no superior to the explanation that every pendulum has at least 1 or more equilibrium position - something that may obtain from plain observation and does not even require tapping into modern algebraic topology. Lange might still claim that this does not render his explanation incorrect, but one may note that it does render the explanation lose much of its appeal for pendulums with higher number of members. Say Lange contests my claim and argues that his explanation was only applicable to double pendulums and not any other kind of pendulums; if so, then he faces a dilemma. This goes as follows. While Lange does not show this, his strategy can successfully explain the lower bound on the number of equilibrium positions of both spherical pendulums and simple pendulums, and not only double pendulums. The configuration space of a simple pendulum is a circle which has $g = 0$. Since a 2D surface like a circle cannot have a saddle point, and there must be at least one maxima and one minima, $N_{max} - N_{saddle} + N_{min} = 2 - 2g$ yields that there can only be exactly 2 equilibrium positions for a simple pendulum, which is correct. Reasoning similarly, for a spherical pendulum the configuration space is that of a sphere also with $g = 0$, Since a sphere has no saddles, there must be at least one minima and one maxima giving exactly 2 equilibrium positions for a spherical pendulum, which is also correct. The dilemma for Lange then is to either accept that the successful explanation in all these cases is merely by coincidence, which defeats his thesis, or to accept that using the Euler characteristic of the configuration space can be a general strategy of giving a lower bound on the number of equilibrium positions but this amounts to accepting that for other kinds of pendulums such as n-uple pendulums ($n > 2$) his strategy loses much of its appeal and cannot be of any interest to a working scientist. (The scientist will certainly want to know at least the exact lower bound on the number of equilibrium position.) Even if we step aside these worries, there are some serious problems with Lange's account. An improved account follows.

2   Alternative strategy and problems

Now I propose using an alternative strategy of reasoning with higher dimension configuration spaces that does not directly involve the Euler characteristic – (D2) – and show why this is a better strategy than Lange's in that (a) it applies to any n-uple pendulum system, and (b) it can give the exact number of equilibrium positions for such pendulum systems. But later I show why even this strategy (including Lange's strategy) is not only unap-
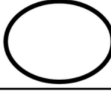
	•	◯	◯	◯
β_0	1	1	1	1
β_1	0	1	0	2
β_2	0	0	1	1
β_3	0	0	0	0
⋮	⋮	⋮	⋮	⋮

Figure 2: From Gholizadeh et. al. (2018) showing the k-th betti number for topological surfaces

pealing in some respects but also that reasoning with configuration spaces can be flawed and misleading in certain contexts such as non-Morse functions. This discussion then opens the floor for a much simpler and effective treatment of pendulum systems using (D3), which I discuss at the end of this section.

The Euler characteristic strategy can be suitably modified for the n-uple pendulums to use Betti numbers.[7] Betti numbers are topological invariant for compact topological surfaces that are Morse functions (See Figure 2). The k-th betti number shows the number of k-dimensional holes on a topological surface. (For any k-dimensional surface n-th betti number is always zero for any $n > k$.) This result can be connected with the number of stable positions obtaining in a pendulum system. (D2): for any n, the k-th Betti number (where $k < n$) of the n-dimensional torus can be shown to be equal to the number of equilibria of the n-uple pendulum with k stable directions where each stable direction implies a pendulum rod pointing downward. This is because the number of k-stable directions for an n-uple pendulum is $\binom{n}{k}$ which is equal to the k-th betti number of the n-torus. The reason why $\binom{n}{k}$ describes the k-th betti number of the n-torus is because of the total n number of dimensions one can choose k number of ways to travel from a given point, and if one reaches the same point after travelling then such a direction can be designated as a k-dimensional hole. As an illustration, consider this. A double pendulum obtains a torus. As shown in figure 2, for a torus, $\beta_0 = 1$ shows the number of connected surfaces, and $\beta_1 = 2$, and $\beta_2 = 1$ correspond to the number of k-th dimensional hole (i.e. 1D and 2D) in the torus. A double pendulum can have $\binom{2}{1}$ or 2 stable directions when only 1 rod is pointing down which is equal to $\beta_1 = 2$, or $\binom{2}{2}$ or 1 stable directions when both rods are pointing down which is equal to $\beta_2 = 1$, and also $\binom{2}{0}$ or 1 stable direction which is equal to $\beta_0 = 1$

when 0 rods are pointing down. The same can be illustrated for a simple pendulum, spherical pendulum and for any higher order pendulum. Thus the difference between this strategy and Lange's strategy is evident in that using Betti numbers yields an exact number of stable equilibrium positions of any n-uple pendulum compared to the unappealing inexact number of lower bound suggested by Lange's strategy. Not only does this strategy give the number of stable positions for each k-th betti number, but also tells us the total number of equilibrium positions of any n-uple pendulum system. To find the total number of equilibrium positions, we add all the possible number of stable positions of the pendulum or all betti numbers of the configuration space. Using induction and pascal's identity for binomials, we obtain:

$$\sum_{k=0}^{k=n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n} = 2^n$$

The summation of all betti numbers thus gives 2^n as the total number of such positions, which is also equal to (D3) as suggested in the introduction. (But we come to the derivation of (D3) by a different way in the next subsection.) This strategy is not only modally superior to (D1), because it gives an exact number of equilibrium positions for any pendulum, but it is also epistemically valuable, i.e. satisfies our epistemic desideratum. The reason why the Betti numbers align well with the equilibrium positions of the pendulum system is because the P.E. function is a Morse function over the configuration space and the signature of the Hessian matrix tells us whether there is a critical point in the Morse function or not. Summing up all the critical points, we obtain the total number of equilibrium positions for the pendulum system. This is valuable since there is an explicit linkage being shown between the purported DME or mathematical constraint and the total number of equilibrium positions obtaining in the physical system, something that was not explicit in (D1) because it only gave a lower bound without really telling us how do they tie up with the exact number of equilibrium positions.

2.1 Why the strategy fails?

But why even this strategy is unappealing and flawed? I first discuss why this strategy fails and then examine in the next subsection, after discussing (D3), why this is also unappealing in some ways. One of the major presumptions in using the configuration space of such complex systems to predict the number of equilibrium positions of the system is that the P.E. function is a Morse function. This by no means is a trivial presumption, and I show how both (D1) and (D2) break down if the P.E. function is a non-Morse function.

Lange (2016, p.31) argues that the DME for a double pendulum applies to all kinds of double pendulums, whether with a stiff rod, or a non-stiff rod, or even a pendulum with a spring. This is because the configuration space of each of these systems is going to be a torus, and then *mutatis mutandis* the constraint on the equilibrium positions obtain. For Morse P.E. functions mapping onto the configuration space, this does not seem to be a problem, but when we perturb the system, the P.E. function no longer remains a Morse function thereby altering its Euler characteristic; the DME fails to apply generally thereby losing much of its modal strength. We look into perturbations in a simple pendulum first, since the analysis of a double pendulum can become quite complicated, but the lesson from the simple pendulum carries forward *mutatis mutandis* to n-uple pendulum systems.

Suppose a simple pendulum has non-stiff rods or is suspended by a non-linear spring. If one adds a non-linear perturbation α^2 (where α is the angle of inclination of the rod from the pivot) in the length of the rod, the P.E. function looks like:[8]

$$U(\alpha) = -mg(L + \alpha^2)cos\alpha$$

$$U(\alpha) = -mgLcos\alpha - \alpha^2 \cdot cos\alpha$$

$$U'(\alpha) = -mg(Lsin\alpha - 2a \cdot cos\alpha + a^2 sin\alpha)$$

$$U''(\alpha) = -mg(Lcos\alpha - a^2 \cdot cos\alpha + 4asin\alpha - 2cos\alpha)$$

We want to obtain conditions where the perturbations render the P.E. function degenerate, i.e. $U' = U'' = 0$.[9] When $U' = 0$, we get $\tan\alpha = \frac{2\alpha}{\alpha^2+L}$. When $U'' = 0$, we get $\tan\alpha = -\frac{\alpha^2+L-2}{4\alpha}$. Eliminating the trigonometric identities by simultaneously solving U' and U'' we get the following polynomial:

$$\frac{2\alpha}{\alpha^2 + L} = -\frac{\alpha^2 + L - 2}{4\alpha}$$

$$\alpha^4 + \alpha^2(6 + 2L) + (L^2 - 2L) = 0$$

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[8]One may charge me for bringing the non-linear P.E. function in disguise here using non-linear perturbations, but this case is different from a law-like non-linear P.E. behaviour in that (a) we are not imposing perturbations on all kinds of rods - stiff rods are exempted and thus its P.E. function is also unaffected, and (b) a law-like non-linear P.E. function as discussed in the previous section is going to yield quite different number of equilibrium positions than a P.E. function that becomes non-linear by perturbations. One may verify this by deriving these cases independently by using a law-like non-linear P.E. function containing terms like $Mgl^2cos^2\alpha$ instead of a perturbation based non-linear P.E. function containing terms like $Mg(l + \alpha^2)cos\alpha$. A persual will consume space and is thus avoided here.

[9]For a double pendulum, we will have to obtain degeneracy conditions according to the Hessian matrix, but the larger point follows from here.

Let us take a peculiar value of L and see if this polynomial is solvable. For L=0.5, the polynomial becomes $\alpha^4 + 7\alpha^2 - 0.75 = 0$. There are 4 solutions to this equation: (-0.324, +2.665, +0.324, -2.66). For all these values of α, U' and U'' are zero and the P.E. function becomes degenerate, or fails to be a Morse function. There are general ways to arrive at this formulation of the problem by assuming that the perturbation is $f(\alpha)$ instead of α^2, and then deducing the conditions of degeneracy where $U' = U'' = 0$ holds, such as:

$$\frac{f'(\alpha)}{f(\alpha) + L} = -\frac{f(\alpha) + L - f''(\alpha)}{2f'(\alpha)}$$

It can thus be shown that there are several varieties of non-stiff rods where such perturbations yield a non-Morse function on the configuration space of the P.E. function and thus a general way of arriving at the number of critical points using Euler characteristic or Betti numbers (a topological invariant for Morse functions) fails. (The same could be shown for double pendulums and n-uple pendulums by assuming higher order perturbations but the derivations are being avoided here for paucity of space.) Morse-Bott theory or the stratified Morse theory can tackle non-degenerate cases but in these cases the situation becomes quite complicated since the topological invariant differs from case to case and there is no general solution that can be applied for obtaining a modally stronger and widely applicable DME. The reason why (D1) and (D2) fail in these cases because Euler characteristic and Betti numbers are the specific properties of Morse functions, they are not actually topological invariants that apply to both Morse and non-Morse functions, or to say that they are not topological invariants that apply regardless of the nature of the function involved. Since a topological invariance cannot be guaranteed in cases where perturbation is introduced, one can no longer rely on configuration spaces to extract a distinctively mathematical explanation; the larger strategy therefore fails. Therefore, for non-Morse function the modality of the (D1) and (D2) breaks down, or becomes restricted to Morse function. Introducing perturbations thus weakens the modality but improves our understanding of the epistemic hooking of (D1) and (D2) through cases where they fail to work.

2.2 A simpler DME?

But all these problems could be avoided by a simpler distinctively mathematical explanation (D3), that there for any n-uple pendulum, there are exactly 2^n number of equilibrium positions. One could arrive at this result inductively by considering the following. A simple pendulum has 2 or 2^1 equilibrium positions, i.e. when the rod is pointing down or pointing up. A double pendulum has two such rods. Both the rods can either point down or up. There are 2×2 or 2^2 ways of doing this so a double pendulum has

4 equilibrium positions. A triple pendulum has three such rods, so there are $2 \times 2 \times 2$ or 2^3 ways of doing this and so on. So for a n-uple pendulum, the number of equilibrium positions should be 2^n. This explanation not only gives an exact number of equilibrium positions but also bypasses the problems related to the topological invariance of Morse and non-Morse configuration spaces as in (D1) and (D2). It thus remains unaffected by any perturbations one may add to the linear P.E. function, the number of equilibrium positions remain 2^n. But this explanation exploits a less sophisticated topological property of the path space of the pendulum systems, that each equilibrium position of a rod is at a distance of π radians from each other. This topological property will remain invariant if (a) the P.E. function remains finite, continous and linear, and (b) Newton's second law of motion operates in its current form. (But it breaks down like (D1) or (D2) when P.E. function is non-linear.) So the modality is enhanced considerably compared to (D1) and (D2). This answers the question why (D2) is unappealing because there is a simpler way to arrive at the prediction. But notably the epistemic hooking is weakened because the explanation does not make clear why the distance between two equilibrium positions must be π radians. Or in other words, why does a path-space of 2π radians contains exactly two equilibrium points per rod with each of them π radians apart? That such a topological property holds in all other pendulum systems is inductively assumed under this strategy (D3). Further, it is not clear whether this explanation satisfies Craver & Povich's (2017) 'directionality desideratum' since the mathematics employed here may well be doing representational work rather than explanatory work, and may face the same problem cited with flagpole-shadow explanations that are sometimes cited to counter Lange's account of DMEs. The height of the flagpole constrains the length of the shadow and thereby explains it, but the length of the shadow does not constrain the height of the flagpole. Similarly, one could explain that every n-uple pendulum must have or is constrained to have 2^n number of equilibrium positions, but an explanation does not obtain in the opposite direction because a system that has 2^n such positions (or similar positions) is not constrained to be an n-uple pendulum. Further, the pattern 2^n can be found across a variety of other systems such as counting bits, finding sector size for disk drives or the number of sectors per track, and number of tracks per surface in such drives - these are all some power of two. Also, the logical block size in disk drives is nearly always some power of two. All these patterns occur in these systems without sharing any topological property with double pendulums or any other pendulums. That is, binary digits or logical blocks will not have any Betti numbers or Euler characteristic associated with them. The formalism supporting (D2) $\sum_{k=0}^{k=n} \binom{n}{k} = 2^n$ that gave us the total number of equilibrium positions for n-uple pendulums using Betti numbers will not make any sense for logical blocks or binary digits. So, both the directionality desideratum and the epistemic desideratum seem problematic here as (D3) fails

to satisfy them. This is despite (D3) having a modal force greater than (D1) and (D2), both of which had better epistemic hooking than (D3).

Also one may note here that the causal explanation involving only potential energy derivatives does not face the problem of directionality or poor epistemic hooking. We understand that a double pendulum has exactly four equilibrium positions because the partial derivatives of the P.E. function flatten out at the critical points, and by looking into the signature of the Hessian matrix one can find out the nature of these critical points (maxima, minima or saddle). If there is a change in the system, such as the perturbations we introduced in the previous section where (D1) and (D2) fail, the causal explanation can still provide an accurate answer while retaining the epistemic hooking. That is, the causal explanation works for both Morse and non-Morse functions or to say it covers both degenerate and non-degenerate critical points in the total number of equilibrium positions without sacrificing on the applicability of the explanation. But the causal explanation does not fare well on modality as Lange (2016, p.28) points out. No general account of such causal explanation can be used to reason that any kind of double pendulum system (irrespective of the kind of forces acting on it) is going to have four or more equilibrium positions. Such a fact will have to be derived on a case to case basis by considering each of the forces on the system and then finding the total number of critical points of the P.E. function - that such an exercise is always going to yield four or more critical points is not cognitively salient from the nature of the causal explanation. Thus, we face a general tension between the modal desideratum, directionality desideratum and the epistemic desideratum, in not only all the accounts of DMEs (D1, D2 & D3) but also in the causal explanation.

Conclusion

The upshot of this discussion is that there is a conflict between understanding and modality in distinctively mathematical explanations. The DMEs that seem modally stronger fare poorer on epistemic hooking, and the DMEs that fare better on the epistemic hooking are modally weaker. This paper highlights that the epistemic desideratum cannot be sidelined when evaluating DMEs since DMEs with strong modality but poor epistemic hooking are likely to fare poor on the directionality desideratum and thus face the charge of not being a true DME. Further, the epistemic desideratum is also important in that it allows us to look into the grounds on which the explanation may fall flat and thus improve our understanding of the modal desideratum. Finally, the conclusion that one cannot maintain modality, epistemic hooking and distinctiveness altogether shows that the modal necessity associated with certain DMEs does not seem to be as exalted as the case is made out to be.

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