

# Comparison of $H^\infty$ and $\mu$ -synthesis Control Design for Quarter Car Active Suspension System using Simulink

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## Abstract:

To improve road dealing with and passenger consolation of a vehicle, a suspension system is supplied. An active suspension system is taken into consideration better than the passive suspension system. In this paper, an active suspension system of a linear quarter vehicle is designed, that's issue to exclusive disturbances on the road. Since the parametric uncertainty within the spring, the shock absorber and the actuator has been taken into consideration, robust control is used.  $H^\infty$  and  $\mu$ -Synthesis controllers of are used to improve using consolation and road dealing with potential of the vehicle, in addition to confirm the sturdy stability and overall performance of the system. In the  $H^\infty$  design, we designed a driving force for passenger consolation and to preserve the deflection of the suspension small and to reduce the disturbance of the road to the deflection of the suspension. For the  $\mu$  synthesis system, we designed a controller with hydraulic actuator and uncertainty model. We designed a MATLAB / SIMULINK model for the active suspension system with the  $H^\infty$  and  $\mu$ -synthesis controllers we tested the use of 4 road disturbance inputs (bump, random, sinusoidal pavement and slope) for deflection of the suspension, body acceleration and body travel for passive, active suspension with controller and active suspension without controller. Finally, we evaluate the  $H^\infty$  and  $\mu$ -synthesis controllers with a Simulink model for suspension deflection, body acceleration and body travel simulation, and the result suggests that both designs offer correct overall performance, however the  $H^\infty$  controller has superior overall performance as compared to the  $\mu$ -synthesis controller.

**Keywords** —Quarter Car Active Suspension System,  $H^\infty$  Controller,  $\mu$ -Synthesis Controller, Robust Performance, Robust Stability

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## I. INTRODUCTION

At present, the arena's leading automotive agencies and studies establishments have invested huge human and material assets to increase a value-effective vehicle suspension system, to be widely used within the vehicle. The principal cause of the suspension system is to isolate the vehicle body from the irregularities of the road to maximize passenger consolation and preserve continuous road

wheel touch to provide avenue grip. Many studies have proven that vibrations because of abnormal avenue surfaces have an effect of draining energy in drivers, affecting their physical and intellectual health [1]. The needs for extra driving consolation and manage capacity of road, consisting of vehicles, have encouraged the improvement of recent styles of suspension systems, consisting of active and semi-active suspension systems. These electronically managed suspension structures can

probably enhance riding comfort, in addition to riding the vehicle on the road. An active suspension system has the capability to constantly adjust to converting road situations. By converting its character to reply to variable road situations, the active suspension offers advanced dealing with, road experience, responsiveness and protection.

An active suspension system has the potential to constantly regulate to changing road conditions. By converting its body or wheel to reply to extraordinary road situations, the active suspension gives advanced managing, road feel, responsiveness and safety.

Active suspension structures respond dynamically to changes in the road profile because of their capability to supply strength that may be used to produce relative movement among the body and the wheel.

Typically, active suspension systems include sensors to measure suspension variables, such as body velocity, suspension displacement and wheel pace, and wheel and body acceleration.

An active suspension is one wherein the passive components are augmented through actuators that offer extra forces. These extra forces are decided by means of a feedback manage regulation that uses sensor statistics connected to the vehicle.

The current active suspension system is inefficient if there are changes inside the system or actuator parameters, then controlling the suspension system will become a massive hassle. Therefore,  $H_{\infty}$  and  $\mu$ -synthesis control strategies of are used. The  $H_{\infty}$  and  $\mu$ -synthesis manipulate of successfully suppresses car vibrations inside the touchy frequency variety of the human body. The desired robust overall performance and strong stability are carried out inside the closed loop system for an quarter vehiclesystem within the presence of structured uncertainties.

## II. MATHEMATICAL MODELS

### A. Passive Suspension System Mathematical Model

The design of the vibration control system have to start with the status quo of the mathematical model of the system after which decide the design necessities and the formal description of the system. And then choose one or extra layout techniques to

design the control system, and then connect it to the simulation or model experiments to become aware of that the manipulate system is designed to satisfy the performance requirements. Therefore, the establishment of the mathematical models of the system is a prerequisite for all manipulate designs and the layout of the manipulate system is intently related to the satisfactory assessment model of the system.

As with other engineering control systems, a mathematical version of the suspension controlsystem refers back to the formal version, to abbreviate the mathematical model. These models are typically based at the dynamic precept so as to be derived or through a number of the dynamics assessments of the system. Then it experiencesmathematical simulation and optimization, or statistical approach. The key to develop a mathematical model system is to offer a description of the model form and decide its parameters.

In the area of vibration control, there are three maximum popular sorts of fashions to explain the form, the description of the popularity area, the description of the transfer function and the description of the weight function. According to the implementation of time continuous manipulate and discrete timecontrol of various traits, each of them is divided into a time continuous and discrete mathematical description over.

The automobile is a complex vibration system, it should be simplified depending on the hassle evaluation. There are numerous approaches to simplify motor vehicles, but in line with the convenience of the have a look at, we simplify in this paper to a system model as shown in Figure 1 below:

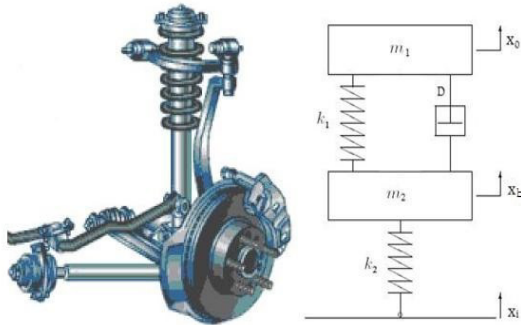


Fig. 1. Quarter Model of Passive suspension system

Figure 1 shows a quarter vehicle model of the passive suspension system. The suspended mass  $m_1$  represents the body of the car, and the non-suspended mass  $m_2$  is an axle and wheel meeting. It guarantees that the tire comes into touch with the road floor while the vehicle is in movement, and is modeled as a linear spring with stiffness. The linear damper, whose common damping coefficient is  $D$ , and the linear spring, whose common stiffness coefficient is  $k_1$ , include the passive factor of the suspension system. The state variables  $x_0(t)$  and  $x_b(t)$  are the vertical displacements of the sprung and unsprung masses, respectively, and  $x_i(t)$  is the vertical road profile.

This is a version of dual mass vibration system of the body and the wheel of the car. From this version, we will examine the dynamics of the automobile suspension system and establish degree of freedom of differential equations of motion. Its equilibrium role is the beginning of the coordinates; we can get the equations as follows (1) and (2).

$$m_1 \ddot{x}_0(t) + D \left[ \dot{x}_0(t) - \dot{x}_b(t) \right] + k_1 [x_0(t) - x_b(t)] = 0 \quad (1)$$

$$m_2 \ddot{x}_b(t) - D \left[ \dot{x}_0(t) - \dot{x}_b(t) \right] + k_1 [x_b(t) - x_0(t)] + k_2 [x_b(t) - x_i(t)] = 0 \quad (2)$$

Letting

$$W = x_0 - x_b$$

$W$  Suspension Deflection

$X_i$  Road Disturbance

If we make a Laplace transformation to the above equation, we can get equation (3):

$$\frac{W}{X_i} = \frac{M_1 K_2}{M_1 M_2 s^2 + (M_2 + K_2) D s + (M_2 K_1 + M_1 K_1 + M_2 K_2 + K_1 K_2)} \quad (3)$$

Table I: Parameters of quarter vehicle model

Model parameters	symbol	symbol Values
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Vehicle body mass	$m_1$	300 Kg
Wheel assembly mass	$m_2$	40 Kg
Suspension stiffness	$k_1$	15,000 N/m
Tire stiffness	$k_2$	150,000 N/m
Suspension damping	$D$	1000 N-s/m

The passive suspension system  $P_{C1}(s)$  transfer function is

$$P_{C1}(s) = \frac{4.5 \times 10^9}{12000s^2 + 1.5 \times 10^8 s + 2.261 \times 10^9}$$

**B. Active Suspension System Mathematical Model**

The mathematical model and the simulation accomplished within the following sections handiest discuss the amount of force created by using the active suspension. Active suspensions permit the designer to balance these goals the usage of a hydraulic actuator with comments controller that is pushed by using a motor among the chassis and the wheel assembly. The force  $U$  implemented between the body and the wheel meeting is controlled by comments and represents the active element of the suspension system.

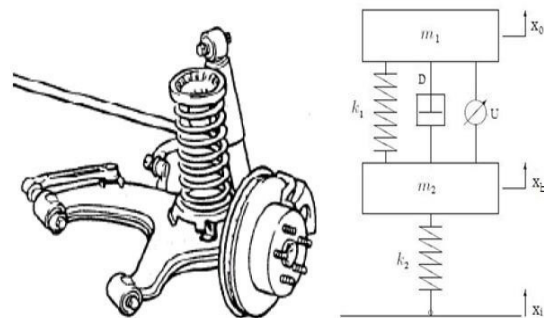


Fig. 2. Quarter Model of active suspension system with actuating force (u) between sprung and unsprung mass.

Figure 2 shows a quarter vehicle model of the active suspension system. The mass  $m_1$  (in kilograms) represents the vehicle chassis (body) and the mass  $m_2$  (in kilograms) represents the wheel assembly. The spring  $K_1$  and the shock absorber  $D$  represent the passive spring and the surprise absorber placed among the vehicle body and the wheel assembly. The  $K_2$  spring fashions the compressibility of the tire. The

variables  $x_0$ ,  $x_b$  and  $x_i$  (all in meters) are the body travel, wheel travel, and road disturbance, respectively. The actuator pressure  $f_s$  (in Kilo Newton's) implemented between the body and the wheel assembly is controlled through feedback and represents the active component of the suspension system.

From this model, we will examine the dynamics of the vehicle suspension system as a linear system model and set up 2 degree of freedom. The differential equations of movement could be as follows:

$$m_1 \ddot{x}_0(t) + D[\dot{x}_0(t) - \dot{x}_2(t)] + k_1[x_0(t) - x_2(t)] = u$$

$$m_2 \ddot{x}_2(t) - D[\dot{x}_0(t) - \dot{x}_2(t)] + k_1[x_2(t) - x_0(t)] + k_2[x_2(t) - x_1(t)] = -u$$

We can set:

$$x_1 = x_2(t), x_2 = x_0(t), x_3 = \dot{x}_2(t), x_4 = \dot{x}_0(t)$$

The system state space equation can be express as:

$$\frac{dX}{dt} = AX + BU$$

In this equation, state variable matrixes are:

$$X = (x_1 \quad x_2 \quad x_3 \quad x_4)^T$$

Constant matrixes A and B are shown as below:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_2} & \frac{k_1}{m_2} & -\frac{D}{m_2} & \frac{D}{m_2} \\ \frac{k_1}{m_1} & -\frac{k_1}{m_1} & \frac{D}{m_1} & -\frac{D}{m_1} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{k_2}{m_2} & \frac{1}{m_2} \\ 0 & -\frac{1}{m_1} \end{pmatrix}$$

The system input variable matrix will be:

$$U = (x_1(t) \quad u)^T$$

The vehicle suspension system output matrix equation will be:

$$Y = CX + DU$$

In above equation, the output variable matrix Y will be:

$$Y = (k_2[x_1(t) - x_2(t)] \quad \dot{x}_0(t) \quad x_0(t))$$

Y will also express as the following equation:

$$Y = (k_2[x_1(t) - x_2(t)] \quad \dot{x}_0(t) \quad x_0(t))$$

Constant matrixes C and D will be shown as below:

$$C = \begin{pmatrix} -k_2 & 0 & 0 & 0 \\ \frac{k_1}{m_1} & -\frac{k_1}{m_1} & \frac{D}{m_1} & -\frac{D}{m_1} \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} k_2 & 0 \\ 0 & -\frac{1}{m_1} \\ 0 & 0 \end{pmatrix}$$

### III. ROAD PROFILES

Four types of road disturbance input are used to simulate distinctive forms of road conditions. They are bump input signal, sine pavement input signal, random input signal and slope road input signal. These entries are the prerequisite for simulating the vehicles suspension system, and should correctly mirror the real circumstance of the road while a vehicle is touring on the street. The specific signal is important to the simulation result. We expect that the car is a linear system.

#### A. Bump Road Disturbance:

The bump input signal is a primary input to investigate the suspension system. It simulated a completely extreme pressure for a totally short time, along with riding a car thru a speed hump. This road disturbance has a maximum peak of 10 cm as shown in Figure 3.

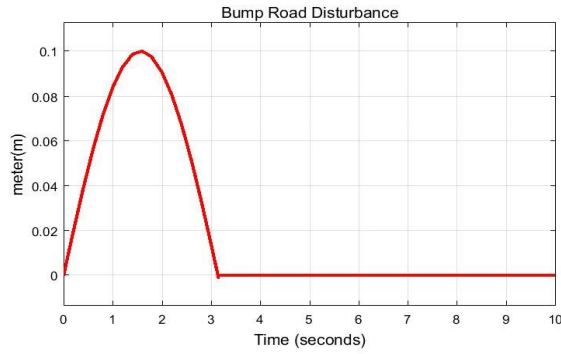


Fig. 3. Bump road disturbance

**B. Random Road Disturbance:**

Numerous investigations display that it's miles important to check a car with a random disturbance of the road to verify that the spring and the surprise reply quickly and effectively. The random disturbance of the road has a maximum peak of 10 cm and a minimum peak of zero cm as shown in Figure 4.

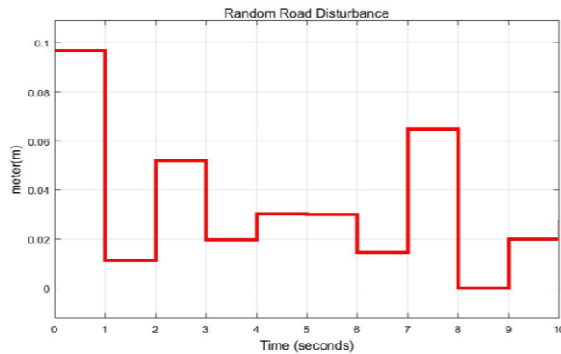


Fig. 4. Random road disturbance

**C. Sine Pavement Road Disturbance:**

The sine wave input signal can be used to simulate periodic pavement fluctuations. You can test the resilience of the vehicle's suspension system while the auto reviews periodic wave pavement. The sinusoidal access pavement test is executed by using all automotive industries before a brand new vehicle drives on the street. The alteration of the sinusoidal pavement road has a height of -10 cm to 10 cm, as shown in Figure 5.

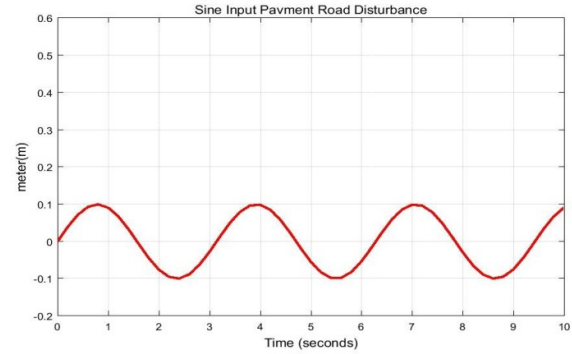


Fig. 5. Sine Input pavement road disturbance

**D. Slope Road Disturbance:**

The performance of the suspension is tested using the disturbance of the slope of the road through verifying the degree of elevation of the road that drives the suspension. The slope road disturbance at the has an elevation of 45 degree as shown in Figure 6.

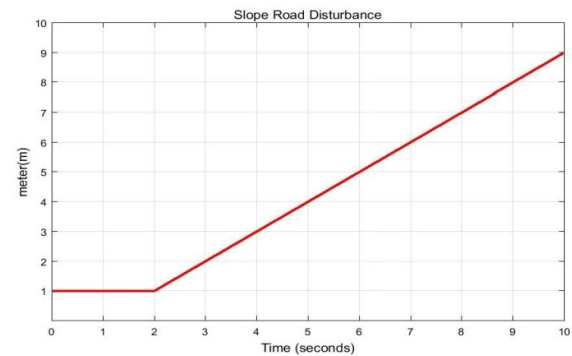


Fig. 6. Slope road disturbance

**IV. THE PROPOSED H ∞ CONTROL DESIGN**

**A. H infinity Controller Design  $G_{cl}(s)$ :**

The design of the active suspension system to offer consolation to passengers and driving on the road is advanced the usage of the design of the H ∞ controller. The essential objective of the controller design is to reduce suspension deflection, body acceleration and body travel. H ∞ synthesis is the approach used to design the proposed controller attaining the performance goal by means of minimizing the norm of the weighted transfer

function. The interconnected  $H_\infty$  design for the active suspension system is shown in Figure 7.

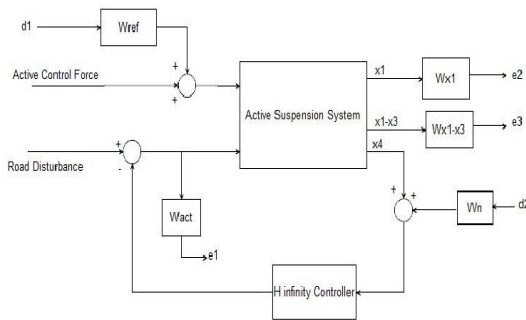


Fig. 7.  $H_\infty$  system interconnected block diagram

There are two purposes for the weighted function widespread: for a given general, there could be an immediate contrast for extraordinary overall performance targets and they may be used to recognize the frequency records incorporated in the analysis. The output or feedback signal is

$$y = (x_4 + d_2 * W_n) * H_\infty \text{ Controller}$$

The controller acts on the  $y$  signal and to produce the road disturbance signal. The  $W_n$  block modeled the sensor noise within the channel.  $W_n$  given a sensor noise of 0.05 m.

$$W_n = 0.05$$

$W_n$  is used to model the noise of the displacement sensor. The magnitude of the active control force is scaled using the  $W_{ref}$ . Let us assume the maximum control force is 0.1 Newton which means

$$W_{ref} = 0.1$$

The weighting function  $W_{act}$  is used to limit the magnitude and frequency content of the input road disturbance signal. Choosing

$$W_{act} = \frac{80}{11} \frac{s + 60}{s + 600}$$

$W_{x1}$  and  $W_{x1-x3}$  are used to keep the car deflection and the suspension deflection small over the desired range. The car body deflection  $W_{x1}$  is given as

$$W_{x1} = \frac{508.1}{s + 56.55}$$

The suspension deflection is used via weighting function  $W_{x1-x3}$ . The weighting function is given as

$$W_{x1-x3} = \frac{15}{0.2s + 1}$$

**B. The Proposed  $\mu$ -Synthesis Control Design**

In the active suspension system, the  $\mu$ -synthesis design included the dynamics of the hydraulic actuator. To recall the difference between the actuator model and the actual actuator dynamics, we use a first-order model of the actuator dynamics, as well as an uncertainty model. The block diagram of  $\mu$ -synthesis controller system interconnection for the active suspension system is shown in Figure 8.

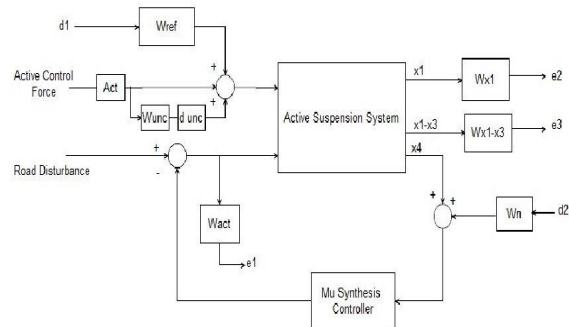


Fig. 8.  $\mu$ -synthesis interconnection block diagram

**C.  $\mu$ -Synthesis Controller Design  $S_{c1}(s)$ :**

The nominal model for the hydraulic actuator is

$$HYD_{act} = \frac{1}{\frac{1}{50}s + 1}$$

We describe the error of the actuator model as a hard and fast of feasible fashions that use a weighting functions because the actuator model itself is uncertain. The uncertainty of the model is represented with the aid of the  $W_{unc}$  weight that corresponds to the frequency variation of the uncertainty of the model and the object of uncertain LTI dynamics object  $4_{unc}$  which is  $Unc = Uncertain$  LTI dynamics "unc" with 1 outputs, 1 inputs, and gain less than 1.



$$W_{unc} = \frac{0.03s + 0.15}{0.001667s + 1}$$

The uncertain actuator model represents the hydraulic actuator model used for the control. A  $\mu$ -synthesis controller is synthesized using the D-K iteration. The D-K iteration approach is a method to synthesis that attempts to synthesize the controller. There are two control inputs: the road disturbance signal and the active control force. There are three measurement output signals, the suspension deflection, car body acceleration and car body travel.

**V. Result and Discussion**

**A. Simulation of the Proposed Controllers**

In this subsection we simulate passive suspension system, active suspension system with  $G_{C1}(s)$  controller, active suspension system with  $S_{C1}(s)$  controller and active suspension system without controller for suspension deflection, body acceleration and body travel using bump, random, sine pavement and slope road disturbances.

**B. Simulation of a Bump Road Disturbance:**

The Simulink model for a bump input road disturbance and active control force input is shown in Figure 9. In this Simulink model, we simulate passive suspension system, active suspension system with  $G_{C1}(s)$  controller, active suspension system without controller for suspension deflection, body acceleration and body travel. Here in this Simulink we assign d1 and d2 as a random signal with amplitude of 0.001 and period of 10 seconds and there are three error signals named e1, e2 and e3 and there are three active control force for the active suspension with controller and without controller with a step function input.

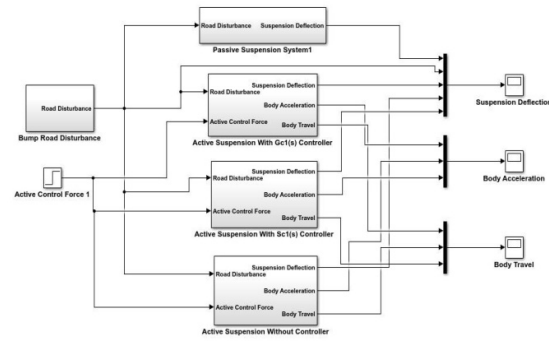


Fig. 9. Simulink model of a Bump road disturbance

The suspension deflection, body acceleration and body travel simulation output is shown in Figure 10, Figure 11 and Figure 12 respectively for a bump road disturbance and active control force inputs.

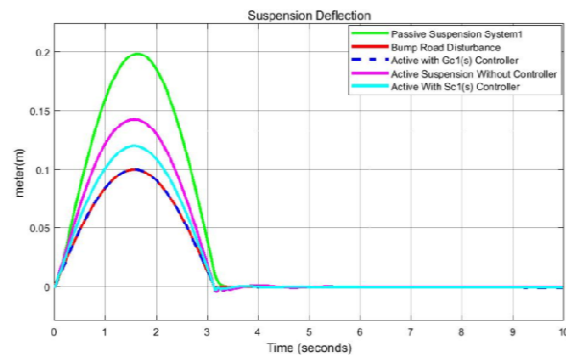


Fig. 10. Suspension deflection for Bump road disturbance

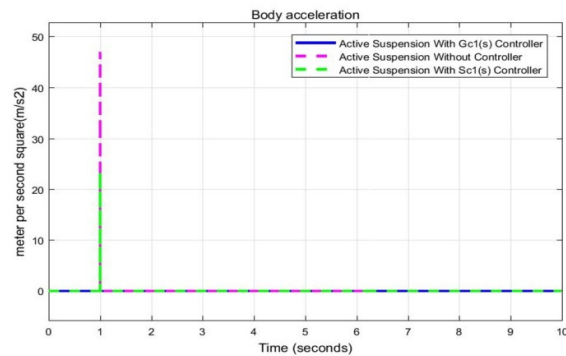


Fig. 11. Body acceleration for Bump road disturbance

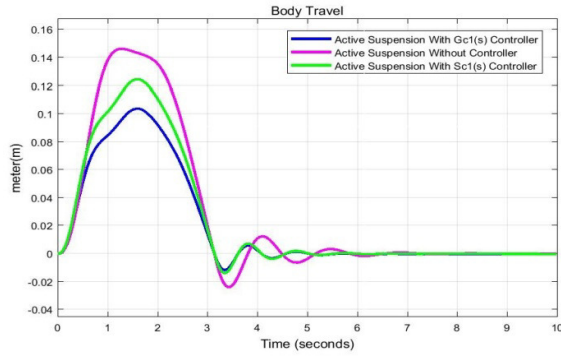


Fig. 12. Body travel for Bump road disturbance

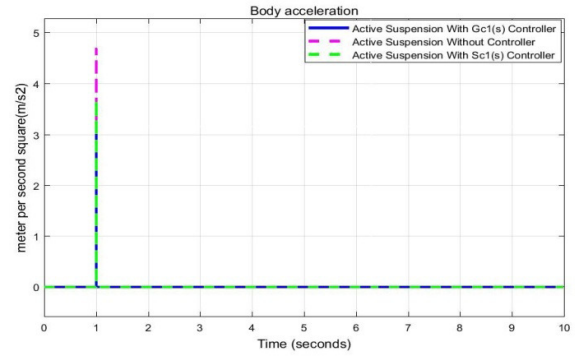


Fig. 15. Body acceleration for Random road disturbance

**C. Simulation of a Random Road Disturbance:**

The Simulink model for a random road disturbance and active control force inputs is shown in Figure 13. The suspension deflection, body acceleration and body travel simulation is shown in Figure 14, Figure 15 and Figure 16 respectively.

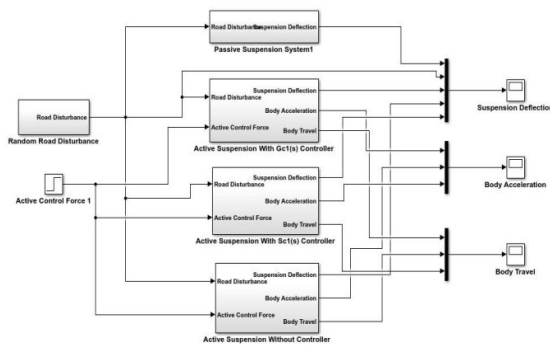


Fig. 13. Simulink model for a Random road disturbance

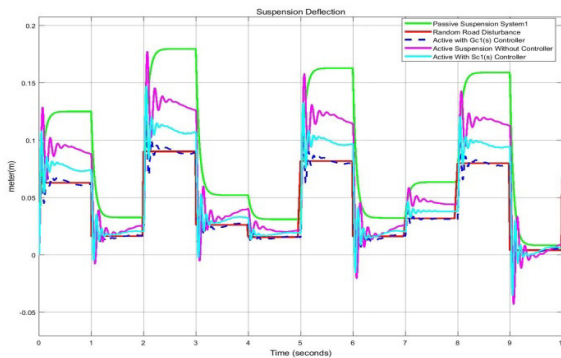


Fig.14.Suspension deflection for Random road disturbance

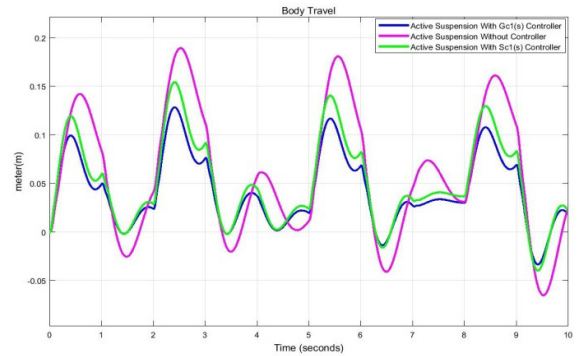


Fig. 16. Body travel for Random road disturbance

**D. Simulation of a Sine Input Pavement Road Disturbance:**

The Simulink model for a sine input pavement road disturbance and active control force inputs is shown in Figure 17. The suspension deflection, body acceleration and body travel simulation is shown in Figure 18, Figure 19 and Figure 20 respectively.

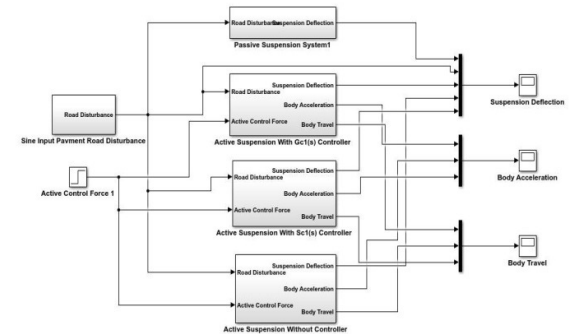


Fig. 17. Simulink model for a Sine input pavement road disturbance



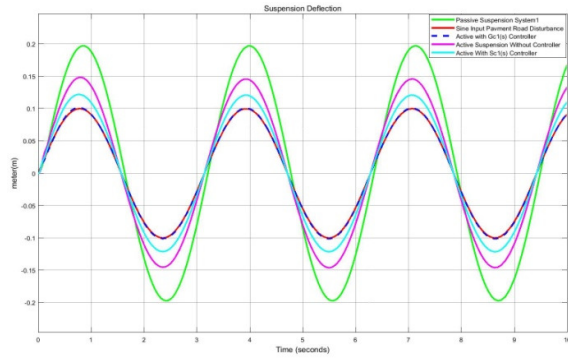


Fig. 18. Suspension deflection for Sine input pavement road disturbance

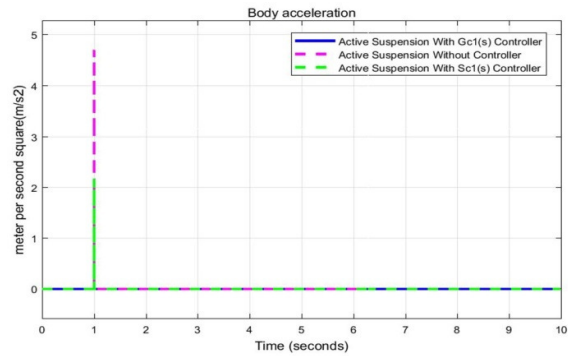


Fig. 19. Body acceleration for Sine input pavement road disturbance

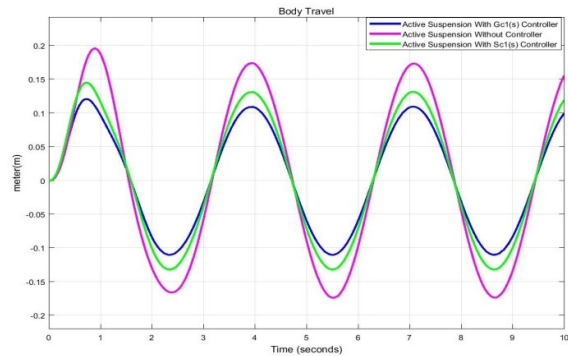


Fig. 20. Body travel for Sine input pavement road disturbance

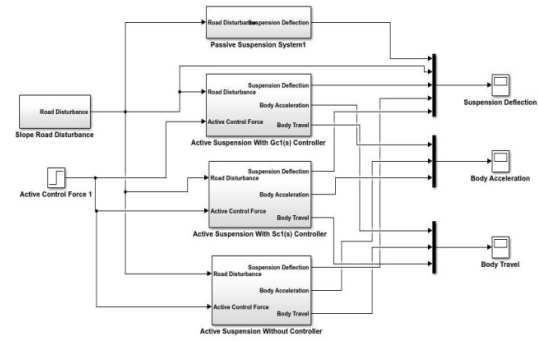


Fig. 21. Simulink model for a Slope road disturbance

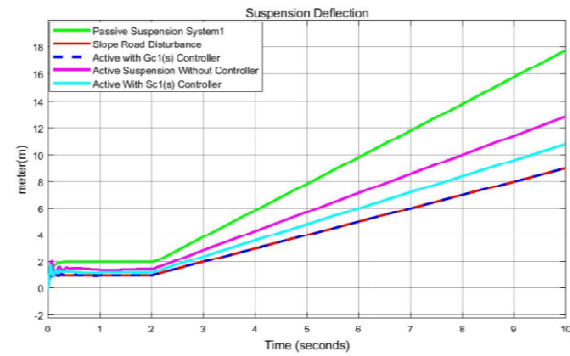


Fig. 22. Suspension deflection for Slope road disturbance

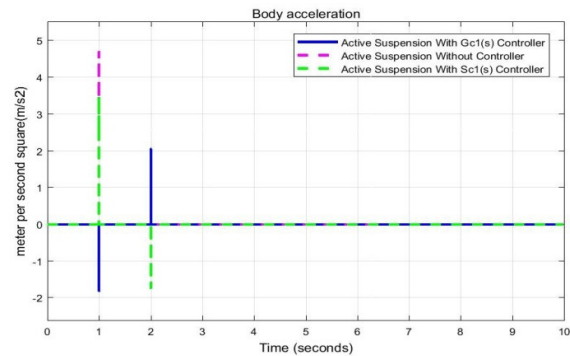
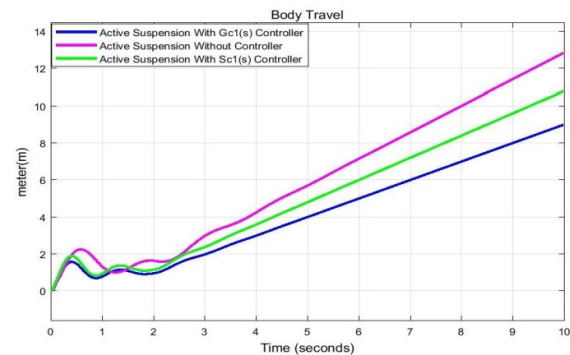


Fig. 23. Body acceleration for Slope road disturbance



**E. Simulation of a Slope Road Disturbance:**

The Simulink model for a slope road disturbance and active control force inputs is shown in Figure 21. The suspension deflection, body acceleration and body travel simulation is shown in Figure 22, Figure 23 and Figure 24 respectively.

Fig. 24. Body travel for Slope road disturbance

**F. Comparison of Active Suspension System With  $H^\infty$   $G_{C1}(s)$  and  $\mu$ -Synthesis  $S_{C1}(s)$  Controllers**

Here in this section, we compare active suspension system with  $H^\infty$  controller ( $G_{C1}(s)$ ) and  $\mu$ -synthesis controller ( $S_{C1}(s)$ ) for suspension deflection, body acceleration and body travel with bump, random, sine and slope road disturbances.

**G. Comparison for Bump Road Disturbance:**

In the suspension deflection simulation as shown in Figure 10, the active suspension system with  $S_{C1}(s)$  controller strokes are larger than the road surface wave amplitude while the active suspension system with  $G_{C1}(s)$  controller strokes fits the road surface wave amplitude. In the body acceleration as shown in Figure 11, the acceleration is effectively reduced in the active suspension system with  $G_{C1}(s)$  controller. In the body travel as shown in Figure 12, the vertical distance that the body travels is effectively reduced in the active suspension system with  $G_{C1}(s)$  controller. The reduction in overshoot value is shown in Table 2.

Table II Reduction in overshoot value for bump road disturbance

Parameters	$S_{C1}(s)$	$G_{C1}(s)$	% Reduction in
Suspension Deflection	0.13 m	0.1 m	23.08 %
Body Acceleration	$24 \frac{m}{s^2}$	$5 \frac{m}{s^2}$	79.2 %
Body Travel	0.13m	0.11 m	15.38 %

**H. Comparison for Random Road Disturbance:**

In the suspension deflection simulation as shown in Figure 14, the active suspension system with  $S_{C1}(s)$  controller strokes have a larger amplitude than the active suspension system with  $G_{C1}(s)$  controller. In the body acceleration as shown in Figure 15, the acceleration is effective reduced in the active suspension system with  $G_{C1}(s)$  controller. In the body travel as shown in Figure 16, the vertical distance that the body travels is effectively reduced in the active suspension system with  $G_{C1}(s)$

controller. The reduction in overshoot value is shown in Table III.

Table III Reduction in overshoot value for random road disturbance

Parameters	$S_{C1}(s)$	$G_{C1}(s)$	% Reduction in
Suspension Deflection	0.18 m	0.13 m	27.78 %
Body Acceleration	$3.7 \frac{m}{s^2}$	$3 \frac{m}{s^2}$	19 %
Body Travel	0.16 m	0.13 m	18.75

**I. Comparison for Sine Pavement Road Disturbance:**

In the suspension deflection simulation as shown in Figure 18, the active suspension system with  $S_{C1}(s)$  controller strokes are larger than the road surface wave amplitude while the active suspension system with  $G_{C1}(s)$  controller strokes fits the road surface wave amplitude. In the body acceleration as shown in Figure 19, the acceleration is effectively reduced in the active suspension system with  $G_{C1}(s)$  controller. In the body travel as shown in Figure 20, the vertical distance that the body travels has a large amplitude in the active suspension system with  $S_{C1}(s)$  controller and is effectively reduced in the active suspension system with  $G_{C1}(s)$  controller. The reduction in overshoot value is shown in Table V.

Table V Reduction in overshoot value for sine pavement road disturbance

Parameters	$S_{C1}(s)$	$G_{C1}(s)$	% Reduction in
Suspension Deflection	0.13 m	0.1 m	23.08 %
Body Acceleration	$2. \frac{m}{s^2}$	$2. \frac{m}{s^2}$	4.56 %
Body Travel	0.14 m	0.12 m	14.3 %

**J. Comparison for Slope Road Disturbance:**

In the suspension deflection simulation as shown in Figure 22, the active suspension system with  $S_{C1}(s)$  controller slopes are larger than the road surface wave amplitude while the active suspension system

with  $G_{C1}(s)$  controller slope fits the road surface wave amplitude. In the body acceleration as shown in Figure 23, the acceleration is effectively reduced in the active suspension system with  $G_{C1}(s)$  controller. In the body travel as shown in Figure 24, the body travels has a large slope and vibration in the active suspension system with  $S_{C1}(s)$  controller and is effectively aligned with small vibration in the active suspension system with  $G_{C1}(s)$  controller. The reduction in overshoot value is shown in Table V.

Table V Reduction in overshoot value for slope road disturbance

Parameters	$S_{C1}(s)$	$G_{C1}(s)$	% Reduction in
Suspension Deflection	51.3 <sup>0</sup>	45 <sup>0</sup>	12.3 %
Body Acceleration	3.5 $\frac{m}{s^2}$	2 $\frac{m}{s^2}$	43 %
Body Travel	51.3 <sup>0</sup>	45 <sup>0</sup>	12.3 %

## VI. CONCLUSION

In this paper,  $H_{\infty}$  controller and  $\mu$  - synthesis controllers are successfully designed using MATLAB/SIMULINK for quarter car active suspension system. We design a Simulink model that represents the active suspension system with  $H_{\infty}$  controller,  $\mu$  - synthesis controller, without controller and passive suspension system and tested with bump, sine input pavement, random and slope road disturbances for suspension deflection, body acceleration and body travel. We compared the active suspension system with  $H_{\infty}$  controller and  $\mu$  - synthesis controller for the three parameters and we analyze the percentage reduction in overshoot of the two controllers.

The simulation results shows that the active suspension system with  $H_{\infty}$  controller is capable of stabilizing the suspension system very effectively than the active suspension system with  $\mu$  - synthesis controller for suspension deflection, body acceleration and body travel parameters with the

four road input disturbances. The system with  $H_{\infty}$  controller has a percentage reduction in overshoot than a system with  $\mu$  - synthesis controller.

We conclude that an active suspension system with  $H_{\infty}$  controller has the best performance with the different tests we made on the system and it achieves the passenger comfort and road handling criteria that it needed to make the active suspension system is the best suspension system.

## ACKNOWLEDGMENTS

First and foremost, I would like to express my deepest thanks and gratitude to Dr.Parashante and Mr.Tesfabirhan for their invaluable advices, encouragement, continuous guidance and caring support during my journal preparation.

Last but not least, I am always indebted to my brother, Taha Jibril, my sister, Nejat Jibril and my family members for their endless support and love throughout these years. They gave me additional motivation and determination during my journal preparation.

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