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Speed Control of Ward Leonard Layout System using H ∞ Optimal Control

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Abstract

In this paper, modelling designing and simulation of a Ward Leonard layout system is done using robust control theory. In order to increase the performance of the Ward Leonard layout system with H ∞ optimal control synthesis and H ∞ optimal control synthesis via γ-iteration controllers are used. The open loop response of the Ward Leonard layout system shows that the system needs to be improved. Comparison of the Ward Leonard layout system with H ∞ optimal control synthesis and H ∞ optimal control synthesis via γ-iteration controllers to track a desired step speed input have been done. Finally, the comparative simulation results prove the effectiveness of the proposed Ward Leonard layout system with H ∞ optimal control synthesis controller in improving the percentage overshoot and the settling time.

Keywords: Ward Leonard layout, H ∞ optimal control synthesis controller, H ∞ optimal control synthesis via γ-iteration controller

1. Introduction

Ward Leonard layout, additionally referred to as the Ward Leonard Drive system, become a widely used DC motor speed manipulate system added by way of Harry Ward Leonard in 1891. It was applied to railway locomotives utilized in World War I , and become utilized in anti- aircraft radars in World War II . Connected to automated anti-aircraft gun administrators, the monitoring motion in two dimensions needed to be extraordinarily smooth and particular. The MIT Radiation Laboratory decided on Ward-Leonard to equip the well-known radar SCR-584 in 1942. The Ward Leonard layout become widely used for elevators till thyristor drives have become available inside the Nineteen Eighties, because it supplied easy velocity control and steady torque. Many Ward Leonard control structures and versions on them stay in use.

2. Mathematical Modelling of the Ward Leonard layout

The Ward Leonard layout system is shown in Figure 1 below.

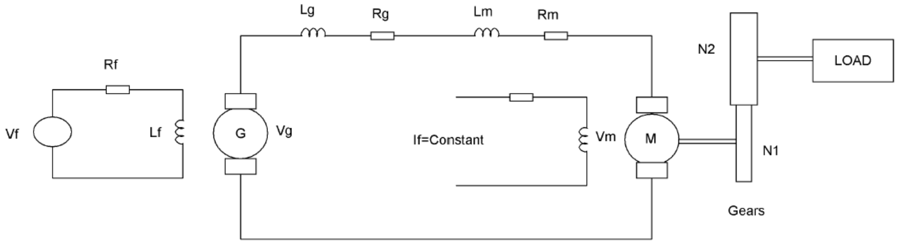

Figure 1 Ward Leonard layout

The equations of the Ward–Leonard layout are as follows. The Kirchhoff's law of voltages of the excitation field of the generator G is

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \qquad (1)$$

The voltage vg of the generator G is proportional to the current if , i.e.,

$$V_g = K_1 i_f$$

The voltage vm of the motor M is proportional to the angular velocity ω_m , i.e.,

$$V_m = K_2 \omega_m$$

The differential equation for the current ia is

$$\left(R_g + R_m\right) i_a + \left(L_g + L_m\right) \frac{di_a}{dt} = V_g - V_m = K_1 i_f - K_2 \omega_m \qquad (2)$$

The torque Tm of the motor is proportional to the current ia, i.e.,

$$T_m = K_3 i_a$$

The rotational motion of the rotor is described by

$$\left(J_m + \left(\frac{N_1}{N_2}\right)^2 J_L\right) \frac{d\omega_m}{dt} + \left(B_m + \left(\frac{N_1}{N_2}\right)^2 B_L\right) \omega_m = K_3 i_a \qquad (3)$$

Here, Jm is the moment of inertia and Bm the viscosity coefficient of the motor: likewise, for JL and BL of the load. From the above relations, we can determine the transfer function of the Ward–Leonard (WL) layout (including the load):

$$G_{WL}(s) = \frac{\omega_L}{V_f(s)} = \frac{K_1 K_2 \frac{N_1}{N_2}}{\left(L_f s + R_f\right)\left[\left(\left(L_g + L_m\right)s + \left(R_g + R_m\right)\right)\left(\left(J_m + \left(\frac{N_1}{N_2}\right)^2 J_L\right)s + \left(B_m + \left(\frac{N_1}{N_2}\right)^2 B_L\right)\right) + K_2 K_3\right]}$$

Where

$$\omega_y = \frac{N_1}{N_2}\omega_m$$

The parameters of the system is shown in Table 1 below.

Table 1 System parameter

No	Parameter	Symbol	Value
1	Motor coil inductance	L_m	18 H
2	Motor coil resistance	R_m	20 ohm
3	Moment of inertia of the motor	J_m	66
4	Damping coefficient of the motor	B_m	28
3	Moment of inertia of the Load	J_L	23
4	Damping coefficient of the Load	B_L	18
5	Generator Coil inductance	L_g	16 H
6	Generator coil resistance	R_g	28 ohm
5	Generator field inductance	L_f	10 H
6	Generator field resistance	R_f	18 ohm
7	Generator voltage constant	K_1	8
8	Motor voltage constant	K_2	16
9	Motor torque constant	K_3	18
10	Gear one	N_1	64
11	Gear two	N_2	32

The transfer function of the system becomes

$$G(s) = \frac{1}{209.8s^3 + 806.7s^2 + 971s + 357.8}$$

And the state space representation becomes

$$\dot{x} = \begin{pmatrix} -9.9242 & -16.1380 & -5.9529 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 41.6667 \end{pmatrix} x$$

3. The Proposed Controllers Design

3.1 H ∞ Optimal Control Synthesis Controller Design

H ∞ optimal control synthesis solve the small-gain infinity-norm robust control problem; i.e., find a stabilizing controller F(s) for a system P(s) such that the closed-loop transfer function satisfies the infinity-norm inequality

$$\left\|T_{y_1u_1}\right\|_{\infty} \overset{\Delta}{\equiv} \sup \sigma_{\max}\left(T_{y_1u_1}\left(j\omega\right)\right) < 1$$

The block diagram of the system with **H** ∞ optimal control synthesis controller is shown in Figure 2 below

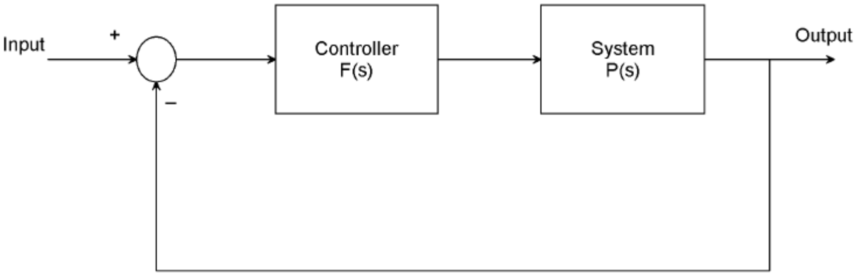

Figure 2 Block diagram of the system with **H** ∞ optimal control synthesis controller

An important use of the infinity-norm control theory is for direct shaping of closed-loop singular value Bode plots of control systems. In such cases, the system P(s) will typically be the plant augmented with suitable loop-shaping filters

The **H** ∞ optimal control synthesis controller transfer function is

$$F\left(s\right) = \frac{0.269s^3 + 1.034s^2 + 1.245s + 0.4587}{s^4 + 3.856s^3 + 4.669s^2 + 1.755s + 0.01709}$$

3.2 H ∞ Optimal Control Synthesis via γ-iteration Controller Design

H ∞ optimal control synthesis via γ-iteration compute the optimal **H**∞ controller using the loop-shifting two-Riccati formulae. The output is the optimal "γ" for which the cost function can achieve under a preset tolerance.

$$\left\|\begin{bmatrix}\gamma T_{y_1u_1}\left(ga\min d,;\right)\\ T_{y_1u_1}\left(otherind,;\right)\end{bmatrix}\right\|_{\infty} \le 1$$

The search of optimal γ stops whenever the γ relative error between two adjacent stable solutions is less than the tolerance specified. For most practical purposes, the tolerance can be set at 0.01 or 0.001. The block diagram of the system with **H** ∞ optimal control synthesis via γ-iteration controller is shown in Figure 3 below

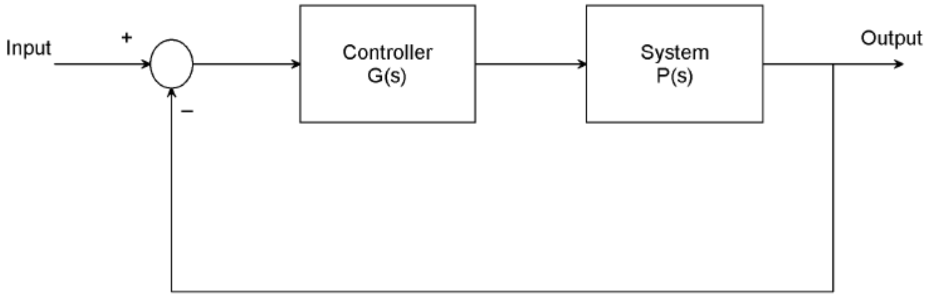

Figure 3 Block diagram of the system with **H** ∞ optimal control synthesis via γ-iteration controller

The **H** ∞ optimal control synthesis via γ-iteration controller transfer function is

$$G(s)=\frac{0.2585s^3+0.9939s^2+1.196s+0.4408}{s^4+3.856s^3+4.669s^2+1.755s+0.01709}$$

4. Result and Discussion

4.1 Ward Leonard layout System Open Loop Response

The Simulink model of the open loop Ward Leonard layout system and the simulation result of the system for a constant field voltage input of 100 volt is shown in Figure 4 and Figure 5 respectively.

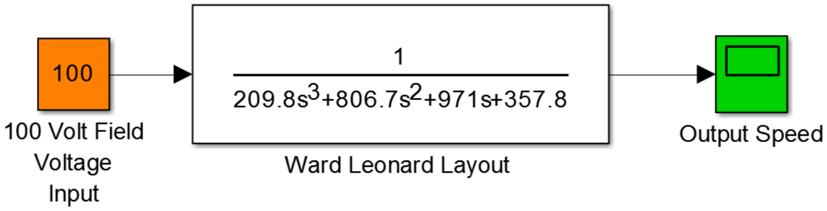

Figure 4 Simulink model of the open loop of Ward Leonard layout system

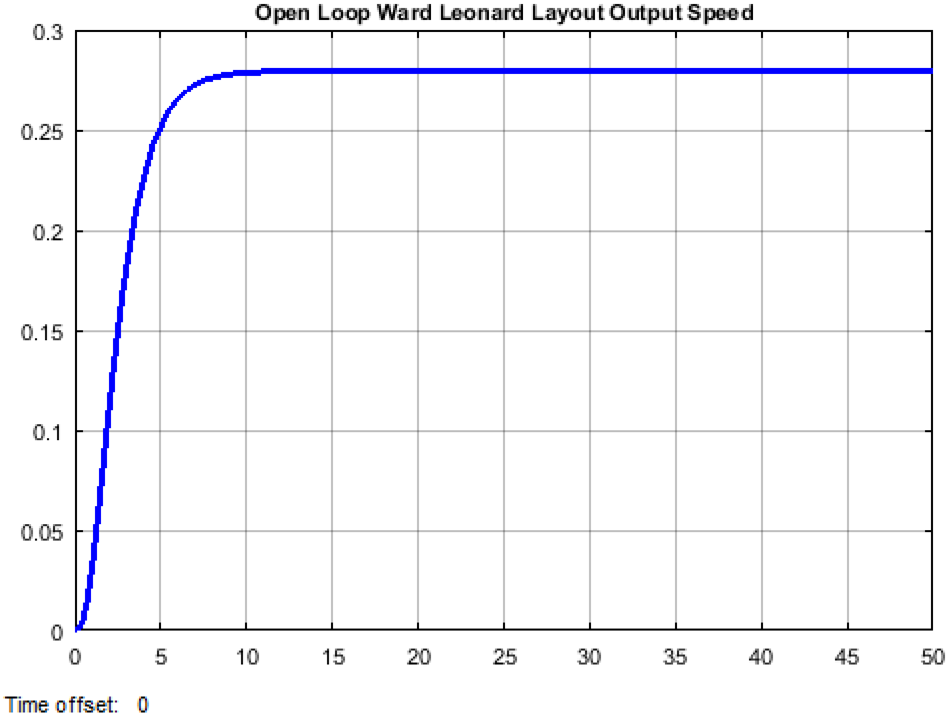

Figure 5 Simulation result

The simulation result shows that the Ward Leonard layout output speed is 0.75 rad/sec which needs a performance improvement.

4.2 Comparison of the Proposed Controllers for Tracking a Desired Step Speed

The Simulink model of the Ward Leonard layout system with **H** ∞ optimal control synthesis and **H** ∞ optimal control synthesis via γ-iteration controllers are shown in Figure 6 below

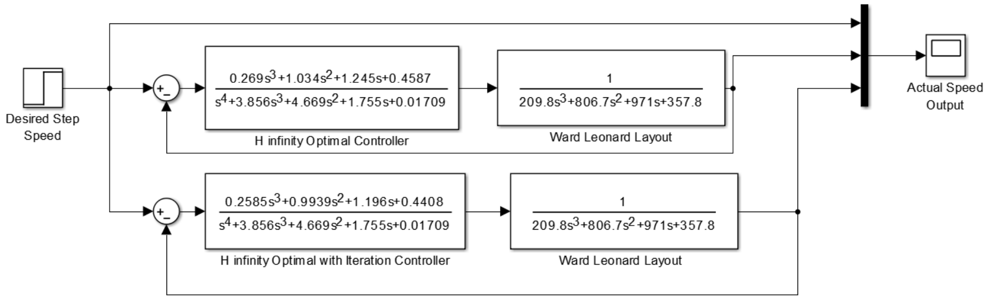

Figure 6 Simulink model of the Ward Leonard layout system with **H** ∞ optimal control synthesis and **H** ∞ optimal control synthesis via γ-iteration controllers

The simulation result of the Ward Leonard layout system with **H** ∞ optimal control synthesis and **H** ∞ optimal control synthesis via γ-iteration controllers for tracking a desired step speed (from 0 to 55 rad/sec) input is shown in Figure 7 below.

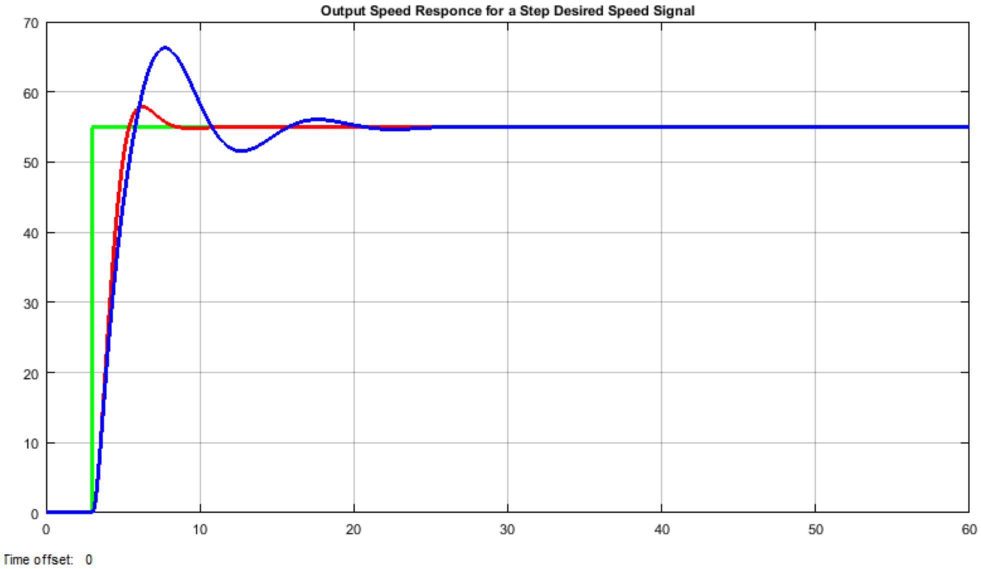

Figure 7 Simulation result

The performance data of the rise time, percentage overshoot, settling time and peak value is shown in Table 2.

Table 2 Step response data

No	Performance Data	**H** ∞ optimal	**H** ∞ optimal via γ-iteration
1	Rise time	3.8 sec	3.8 sec
2	Per. overshoot	3.63 %	21.8 %
3	Settling time	8 sec	26 sec
4	Peak value	57 rad/sec	67 ad/sec

5. Conclusion

In this paper, a Ward Leonard layout system is designed using a DC motor generator combination. In order to improve the performance of the system, a robust control technique with **H** ∞ optimal control synthesis and **H** ∞ optimal control synthesis via γ-iteration controllers are used. The open loop response of the system shows that the system needs improvement. The comparison of the proposed controllers is done to track a desired step speed and the results proves that the system with **H** ∞ optimal control synthesis controller improves the settling time and the percentage overshoot than the system with **H** ∞ optimal control synthesis via γ-iteration controller.

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