Logicism and Theory of Coherence in Bertrand Russell's Thought

John Owen E. Adimike
Don Bosco Institute of Philosophy
Ogungbade, Off New Ife Road
Ibadan
joweneadimike@gmail.com
+2349130032134

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Abstract

Logicism is the thesis that all or, at least parts, of mathematics is reducible to deductive logic in at least two senses: (A) that mathematical lexis can be defined by sole recourse to logical constants [a definition thesis]; and, (B) that mathematical theorems are derivable from solely logical axioms [a derivation thesis]. The principal proponents of this thesis are: Frege, Dedekind, and Russell. The central question that I raise in this paper is the following: 'How did Russell construe the philosophical worth of logicism?' The argument that I build in response to this is that Russell perceived an inverse proportion between a logical reduction of mathematics and the certitude of non-novel mathematical theorems – such that the more we reduce mathematics to logic, the more certain we become of our mathematical theorems; this was portrayed through a presentation of mathematical knowledge as coherent. Therefore, I set out to sketch Russell's coherence theory and appraise it in relation to the presence discourse: that is, in relation to logicism and mathematical certainty.

Keywords: Logicism, Epistemic Coherence, Mathematical Theorems, Logical Theorems.

1.0 Introduction

Logicism is the thesis that all or, at least parts, of mathematics is reducible to deductive logic in at least two senses: (A) that mathematical lexis can be defined by sole recourse to logical constants [a definition thesis]; and, (B) that mathematical theorems are derivable from solely logical axioms [a derivation thesis]. Among the principal proponents of this thesis are Gottlob Frege, Richard

Dedekind, and Bertrand Russell. Had Russell's logicism been successful, it would [have] demonstrated how mathematical theorems [afterwards, MTs] feature appropriate epistemic properties typically ascribed to logical theorems [subsequently, LTs].

A tacit implication of the logicist thesis is the inheritance of relevant epistemic properties of logical concepts and theorems by mathematical ones. Should mathematical claims be in fact, descriptions or apocopes for logical ones and are themselves derivable from logical ones, then MTs necessarily feature the relevant epistemic characters attributed to LTs. A possible interpretation would be to say that MTs are necessary, potentially knowable, *a priori*, certain, and self-evident. Thus, by this fact, MTs obtain the properties of LTs by some kind of hereditary accretion or, transferal of property. These properties are called Epistemic Transferred Properties [ETPs] and this accretive procedure is called Epistemic Property Transfer [EPT] thesis.

Russell may have very well motioned to sustain the EPT thesis even as he upheld logicism; the EPT was a strong motivating factor for Russell's logicism.[1] Lugubriously, the EPT opposes Russell's logicist stance – or so it seems. Russell himself states:

> … the chief reason in favour of any theory on the principles of mathematics must always be inductive, i.e. it must lie in the face that the theory in question enables us to deduce ordinary mathematics. In mathematics, the greatest degree of self-evidence is usually not to be found quite at the beginning, but at some later point; hence, the early deductions, until they reach this point, give reasons rather for believing the premises because true consequences follow from them, than for believing the consequences because they follow from the premises.[2]

Given this and seeing that Russell 'apparently' never believed in EPT, did Russell ascribe significant epistemic consequences to

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[1] Martin Godwyn and Andrew Irvine, "Bertrand Russell's Logicism," in *The Cambridge Companion to Bertrand Russell*, ed. Nicholas Griffin (Cambridge: Cambridge University Press, 2003), 171-202.

[2] Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, vol. 1, 2nd ed. (Cambridge: Cambridge University Press, 1962).

logicism? Did logicism hold significance for mathematical epistemology? Russell did grant relevance to logicism – because it enhances mathematical explanation and facilitates discovery. Russell's key motivations of logicism include: (1) portraying how apparently differing theorems and their proofs can be derived from a common core of axioms; (2) revealing methods for proving already-stated theorems that are otherwise unproven; and, (3) suggesting novel concepts and theorems in mathematics.

I certainly believe that Russell ascribed relevance to logicism. Thus, I argue that Russell upheld the belief that the reduction of mathematics to logic steps up the certainty of MTs – even the rudimentary facts of basic arithmetic. My discussion is not so much as concerned with expending much philosophical rigour on the grandeur of demonstrating the reduction of MTs as it is of justifying LTs in order to ground the certainty of MTs. I maintain that this certainty does not derive from any transferal of ETPs; rather, it is owed to the coherent structure of mathematical knowledge. I therefore structure this paper into two sections; in the first, I argue Russell's logicism and whether or not, he upheld the EPT thesis; and in the second, I discuss Russell's theory of coherence and its mark on logicism. I do not consider the fluidity of Russell's thought any much alarming because of the relative constancy of his thought on coherence from 1907 through the 1950s. I therefore encourage the reader to muster courage in the analysis that I urge below.

2.0 Russell's Logicism and the EPT

"The most obvious and easiest things in mathematics", says Russell, "are not those that come logically at the beginning; they are things that, from the viewpoint of logical deduction, come somewhere in the middle".[3] This passage explains that Russell did not take mathematical axioms as homogeneously self-evident.[4]

In *The Principles of Mathematics* (1903), Russell committed himself to establish the logicist view that "all pure mathematics deals

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[3] Bertrand Russell, *The Basic Writings of Bertrand Russell*, ed. John G. Slater (Routledge: New York, 2009), 2.

[4] Russell, *The Basic Writings of Bertrand Russell*, 279.

exclusively with concepts definable in terms of a very small number of fundamental logical concepts, and that all its propositions are deducible from a very small number of logical principles"[5] as well as to expound "the fundamental concepts which mathematics accepts as indefinable".[6] The *Principia Mathematica* encapsulates some heavy utility of symbolism in the argumentation of the logicist conviction. Of it [i.e., the *Principia*], Bede Rundle excogitates a striking conceptual overlap in theoretical terrain with that "covered by Frege in his *Grundgesetze der Arithmetik*, a work to which the authors acknowledge their chief debt on questions of logical analysis; in some respects, such as the demarcation between logical and metalogical theses, *Principia Mathematica* falls short of the standards of rigor observed in Frege's masterpiece".[7]

As essentially formulated by Russell in the *Principia Mathematica* as well as the *Principles*, Russell takes logicism to assert that the enterprise of mathematics can be reduced to symbolic logic—i.e., as a claw of symbolic logic. Grounded upon symbolic logic as well as logical axioms [such as Peano's axioms of arithmetic, and that of reducibility], it is then possible to express mathematical truths and concepts in some formal language. With respect to this, it must be conceded that Russell's views on formal language typically represented in the theory of descriptions, held a significant impact on his conception and eventual articulation of this 'formal language' which expresses mathematics.[8] Similarly, Russell's debt to Alexius Meinong is seen in that Russell's realist position [which he had taken up due to G. E. Moore] was strengthened by "an extreme form of the referential theory of meaning, the view that in order for a

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[5] Bertrand Russell, *The Principles of Mathematics*, 2nd ed. (New York: W.W. Norton & Company, 1938), xv.

[6] Russell, *The Principles of Mathematics*, xv.

[7] Bede Rundle, "History of Modern Logic: From Frege to Gçdel", in *Kabbalah to Marxist Philosophy*, vol. 5 of *Encyclopedia of Philosophy*, ed. Donald M. Borchert, (New York: Thomson Gale, 2006), 466.

[8] William Demopoulos and Peter Clark, "Logicism: Frege, Dedekind, and Russell," in *The Oxford Handbook of Philosophy of Mathematics and Logic*, ed. Stewart Shapiro (Oxford: Oxford University Press, 2005), 154-159.

linguistic expression to have a meaning there must be something that it means, something to which it refers".[9]

Needless to say, Russell's perception of the logicist project was marked by optimism. Also, there was some tinge of ambition to the project because, Russell hoped that the project would eradicate the paradoxes and problems which had so long bedevilled the enterprise of mathematics.[10] By means of the logicist project, it was Russell's aim to restore certainty and clarity to the mathematical enterprise.

I must concede at this moment, a fundamental distinction between Russell's perception and the Fregean [and neo-Fregean] towards the project of logicism. While the latter construe theirs to be in essence, a matter of epistemology – i.e., dealing with the security of the epistemological footing of our beliefs in arithmetical truths, demonstrating their entailment from logical truths as well as analytic principles with an equivalent epistemological status of definitions – Russell construed his as being a matter of application of philosophical method.[11]

The methodology spoken of in the previous paragraph, consists of a two-phase process. Kevin Klement explains this quite aptly:

> In the first phase, one begins with a certain theory, doctrine or collection of beliefs, which is thought to be more or less correct, but is taken to be in certain regards vague, imprecise, disunified, overly complex or in some other way puzzling. The aim is to work backwards from these beliefs, taken as a kind of "data", to a certain minimal stock of undefined concepts and general principles which might be thought to underlie the original body of knowledge. The second phase consists in rebuilding or reconstructing the original body of knowledge in terms of the results of the first phase. More specifically, one defines those elements of the original doctrine deemed worth preserving in terms

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[9] Paul Edwards, William Alston, and A. N. Prior, "Bertrand Russell," in *Shaftesbury to Zubiri*, vol. 9 of *Encyclopedia of Philosophy*, ed. Donald M. Borchert, 2nd ed. (New York: Thomson Gale, 2006), 541.

[10] Irving Copi, *The Theory of Logical Types* (London: Routledge & Kegan Paul, 1971), 1.

[11] Kevin Klement, "Neo-Logicism and Russell's Logicism," in *Russell: The Journal of Bertrand Russell Studies*, 32 (Winter 2012-2013): 144.

> of the "minimum vocabulary" identified in the first phase, and derives or deduces the main tenets of the original theory from the basic principles or general truths so identified.[12]

Russell describes this procedure as *analysis*,[13] which, rather than being a process of discovering "'what we meant all along' by a given collection of statements,"[14] is one that sees to the provision of "a *replacement* for the original doctrine, something that preserves what was desirable about the original, but taking a new form in which connections between various concepts are made clear, the logical interrelations between various theses of the theory are explicit, and vague or unclear aspects of the original terminology are eliminated".[15]

Given Russell's perception of logicism, we shall now examine Russell's logicism and the EPT. Logicism, as is generally put, entails the hereditary accretion of LTs by MTs. This is confuted however, by Russell. He also denied the certainty of logical principles: "In mathematical logic, it is the conclusions that have the greatest certainty: the closer we get to the ultimate premises, the more uncertainty and difficulty we find".[16] Russell further claims that the axioms of infinity and multiplicity are non-necessary; similarly, he denies the logical veracity of any existential postulate: "Among 'possible worlds', in the Leibnizian sense, there will be one having one, two, three …, individuals. There does not even seem any logical necessity why there should be even one individual-why, in fact, there should be any world at all".[17]

Lastly, Russell argues that the justification of mathematical axioms is based on a spuriously inductive method: whether they can be used to develop theorems of arithmetic and analysis. Thus, it is non-clear that Russell's axioms are potentially knowable *a priori*.

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[12] Klement, "Neo-Logicism and Russell's Logicism," 144.

[13] Here, one familiar with Russell literature may be quick to observe some equivocation. While Russell refers to the methodology itself as analysis, he also refers to the first part of the procedure as the stage of 'analysis', and the second as that of 'synthesis' or, as the synthetic stage.

[14] Klement, "Neo-Logicism and Russell's Logicism," 144.

[15] Klement, "Neo-Logicism and Russell's Logicism," 144.

[16] Russell, *The Basic Writings of Bertrand Russell*, 285.

[17] Russell, *The Basic Writings of Bertrand Russell*, 203.

As regards the existential axioms, Russell points out that "Existence theorems, where individuals are concerned, are now theorems as to existence in the philosophical sense; hence it is natural that they should not be demonstrable *a priori*".[18]

There is still further reason to believe that Russell never ratified the EPT thesis. Regardless of his importunity with respect to the pursuit of the logicist programme, the *Principia Mathematica* fails to adequately argue out the claim that all MTs are derivable from solely logical axioms. Russell states that:

> We have sufficiently defined the character of the primitive ideas in terms of which all the ideas of mathematics can be defined, but not of the primitive propositions from which all the propositions of mathematics can be deduced. This is a more difficult matter, as to which it is not yet known what the full answer is. We may take the axiom of infinity as an example…, though it can be enunciated in logical terms, cannot be asserted by logic to be true.[19]

Russell therefore, is uncertain as to whether or not MTs can be proven by sole virtue of pure logical axioms. If the claim of the derivation thesis, namely that 'all MTs are derivable from solely logical axioms' were true, then there must be proof of their logicality – and dear Russell has explicitly denied this in some of the citations above. Russell's reluctance in endorsing the derivation thesis suggests his perception of logicism. Firstly, he appears more concerned about the definition thesis of logicism; secondly, the axiomatization of the entire enterprise of mathematics has an independent worth even if the necessary axioms for this are not purely logical.

By and large, there are two significant implications characteristic of Russell's logicist project that bear on mathematical epistemology: (1) in the logicist attempt, better explanations of mathematical theorems are acquired through a harmony of proofs in a single axiomatic set; (2) novel concepts and theorem-proving methods are learned. However, as Russell's logicism does not entail the hereditary accretion of such logical features as necessity, and *a*

[18] Bertrand Russell, "The Paradox of the Liar," unpublished manuscript in *The Bertrand Russell Archives* (McMaster: McMaster University), 65.
[19] Russell, *The Basic Writings of Bertrand Russell*, 206.

priority by MTs from LTs, so Russell still does not provide a defense of/for the EPT thesis.

In the final analysis, is there then, perhaps, any manner wherein the logicist manifesto proliferates the certainty of MTs? For Russell, the answer is in the affirmative (even though there has been so far, a rather sparse textual evidence). The second section of this paper shall demonstrate why the affirmative obtains.

3.0 Russell's Theory of Coherence and the Epistemic Significance of Logicism

Russell recognized the existence of *a priori*, necessary, and self-evidential truth; an example of such is the logical law of thought of non-contradiction. This law states that P cannot simultaneously be true and false; $P \wedge -P$ cannot obtain concurrently. As such, for any set of propositions implying a contradiction, at least one member of the set must be false. This assertion is still supported by Russell even when significantly modified. Represented, we have the following:

(A): For any set of propositions, *S*, if *S* implies a contradiction, then there is at least one proposition *p* that is false.

(A'): For any set of propositions, *S*, if *S* does not imply a contradiction then *S* has evidence for its assertions.

Similarly, Russell writes that: "The proof of a logical system is its adequacy and its coherence. That is: (1) the system must embrace among its deductions all those propositions which we believe to be true and capable of deduction from logical premises alone … and (2) the system must lead to no contradictions".[20]

Taking 'proof of a logical system' to mean 'truth of the axioms', then Russell's conclusion appears peculiar. There are quite a number of consistent mathematical axiomatizations and in the absence of an argument where the logical constancy of a set of propositions is support enough for their veridicality.

Russell means more than 'logical constancy' when he speaks of coherence. For Russell, when a number of propositions bear a

[20] Whitehead and Russell, *Principia Mathematica*, 12.

relation *R* to one another, the set becomes more 'obvious', 'credible' than the individual propositions. It is the collation of all such relations *R* that Russell calls 'coherence'. In a 1912 paper, Russell introduced the terminology of coherence: "In regard to probable opinion, we can derive great assistance from coherence, which we rejected as the definition of truth, but may often use as a criterion. A body of individually probable opinions, if they are mutually coherent, become more probable than any one of them would be individually".[21]

Then, he calls this view the 'coherence theory of probability': "I do not accept the coherence theory of *truth* but there is a coherence theory of probability which is important and which I think valid"; he goes ahead to explain the theory: "suppose you have two facts and a causal principle which corrects them, the probability of all three may be greater than the probability of any one, and the more numerous and complex the inter-connected facts and principles become, the greater is the increase of probability derived from their mutual coherence".[22]

Russell's repeated discussions of the coherence theory and the great chunk of temporal slices claimed in the process, suggest the importance that Russell attached to this theory. What I maintain is that by his logicist venture, Russell was indeed endeavouring, at least to some extent, to demonstrate the possibility of coherent organization in mathematical knowledge – thus, strengthening one's evidence for the certainty of mathematical claims. To clarify my position, I shall elucidate three key ideas: (1) what Russell means by coherence; (2) the relations coherent propositions bear to one another; and, (3) the epistemic properties of a set of propositions that coherence provides support for.

Russell succinctly defines coherence in these words: "Two propositions are coherent when both may be true, and are incoherent when one at least must be false".[23] In the instance of more than two propositions, we should revert to Russell's 'dream

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[21] Bertrand Russell, *The Problems of Philosophy*, ed. John Perry (Oxford: Oxford University Press, 1997), 140.
[22] Bertrand Russell, *Human Knowledge: Its Scope and Its Limits* (New York: Simon and Schuster, Inc., 1948), 395.
[23] Russell, *Human Knowledge*, 120.

example'[24] where he uses coherence in reference to a computable measure of the evidential support a proposition offers another. Rephrasing the second question, we ask what properties of a set of propositions, Ω, defines its coherence. To answer this, some certain questions pop up:

(I) the number of propositions in Ω;
(II) the complexity of propositions in Ω;
(III) the possibility of deriving some propositions in Ω from other propositions in Ω;
(IV) the question of the satisfiability of Ω; and
(V) the safety of Ω from contradiction.

Russell opines that some of the basic probes listed above might have probabilistic analogues.[25] For instance, in speaking of the 'interconnectedness of propositions', what Russell might very well be speaking of is whether or not, for any singular proposition $\alpha \in \Omega$ the credibility (or probability) of $\Omega \rightarrow \alpha \setminus (\alpha)$ is less than, equal to, or greater than, the credibility/probability of simpliciter.[26] As against what one would expect, Russell fails to succinctly speak on this matter and, notwithstanding the current method of probability is one way of making coherence exact, there are other rational and plausible methods one can explore.

Yet, with the method he advocates, Russell faces a major setback of vagueness. One glaring example of this vagueness is the example that the number of propositions in a set is a criterion for deciding the coherence of the set. In a formal expression, a finite amount of assertions can be unified; so, we can then ask if propositions can be defined so as to achieve a distinction between a finite collection of propositions and 'one' proposition. We can even go further to

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[24] Russell, *Human Knowledge*, 140.

[25] The later Russell held that on a field of non-demonstrative extrapolations, logical inferences are the natural endpoints. This is concordant with the views in: Russell, 383.

[26] Russell tacitly employs this concept of conditional probability to execute an explanation of the manner in which the credibility of a proposition ought to be defined by known probabilities.

question the appropriateness of cardinality as a measure of numbers of propositions in a non-finite set if we are to quantify the intricacy of the set.

The final idea in Russell's coherence theory which I shall clarify, is a claim regarding the epistemic properties of a set of propositions that coherence provides support for. I assert that what Russell truly intends by 'probability' in 'coherence theory of probability', is a measure of evidential support, non-satisfactory to current standards of axioms of probability theory. This is because of Russell's claim that "A body of individually probable opinions, if they are mutually coherent, become more probable than any one of them would be individually".[27] Literally interpreted, this assertion is diametrically opposed to the probabilistic rudiment that the conjunction of two or more propositions is less probable than either conjunct: i.e., $\forall A \forall B[p(A \wedge B) \leq p(A), p(B)$. For Russell, the reverse obtains: he seems to discern a rather overall probabilistic fact: if λ entails μ, then μ is less probable than λ. Russell therefore writes: "For the probability that Socrates is mortal is greater, on our data, than the probability that all men are mortal (this is obvious because if all men are mortal, so is Socrates; but if Socrates is mortal, then it does not follow that all men are mortal)".[28]

Perhaps the equivocation of 'probability' in his earlier writings, led Russell to show precisely, the difference between probability and credibility as is evident in *Human Knowledge* – Russell's most sustained discussion of probability and its construal readings. Though the coherence theory is pretty much incomplete, the above outline suffices for a discussion on its significance with regard to the logicist project. If the reduction of mathematics to logic erases contradictions, discovers logically simple principles, proliferates the number of propositions that one receives by constructing novel theorems and reveals 'derivability relations' between propositions, then the logical analysis of MTs makes mathematical knowledge more coherent, and for Russell, more probable.

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[27] Bertrand Russell, *The Problems of Philosophy*, ed. John Perry (Oxford: Oxford University Press, 1997), 140.
[28] Russell, *The Problems of Philosophy*, 80.

"Why then does coherence enhance the credibility of a set of propositions?", one may ask. The above readings show Russell's answer. The coherence of a set of properties is a sign of truth only when at the very least, some propositions are sustained independently by accessible suggestions. From the excerpts of Russell's work which we have reviewed in this section, Russell says the following to us [according to their order of appearance in the present paper]: (1) coherence is a sign of truth so long as one's beliefs are already 'individually probable'; (2) coherence portends truth among mathematical and logical propositions insofar as these propositions are by this time, 'obvious'; and, (3) the coherence of the propositions designated by Russell pass as support only because they are implied/entailed in inferences whose structure are valid and widely accepted as such.

Russell's epistemological enterprise thus combines foundationalist and coherentist leitmotifs. Some justified beliefs are 'probable', 'obvious' or are conferred the status of 'facticity' either because they are self-evidential or directly inferred from observation; others obtain justificatory support from the introduction of coherence to a set of self-reliantly obvious/probable views. As such Russell awaited the opposition that coherence is not always indicative of the truth. Russell showed that only when some of the propositions in Ω are probable, self-evidential, or obvious, do the propositions in Ω get backing from the coherence of the entire set, Ω

Nonetheless, we are faced with another important question: If coherence does not generally point to truth, why then does it ever offer proof for any set of propositions, Ω? I think that we can provide a Russell-like response, employing his treatment of the criteria for good mathematical premises and causal inference. When one proves an intricate MT from non-complex [simple] premises, a greater proof for the veridicality of the theorem is acquired. Reasoning by way of symmetric relations, if we can prove that individual premises are indispensable for the deduction of evident arithmetical facts, this provides greater evidence for the premises and one has gained proof of the premises from an obvious arithmetical state of affairs. This is the interpretation of Russell's claim that a formal system (theory) ought to be judged by its 'adequacy'.

Well aware that a natural objection would be that MTs can be deduced from infinitely many different sets of axioms, and that there are seldom any axioms that can safely be proven essential for acquiring ordinary/conventional mathematics, for Russell, coherence is a measure of the necessity of the premises in an axiomatic system for deriving the consequences. It is for this reason that Russell lays emphasis on the 'interconnectedness of propositions' in a 'complicated deductive system' as a suggestion of their coherence.

For further clarity of this view, let us understand it in this manner: in inductive analysis, one can never make evident that a set of laws (or an individual laws) is needed to explain specific occurrences. However, one can recover confidence that if the true laws were to contrast with the conjectured ones in definite ways, then one should not have recorded the phenomena that s/he did. In the same manner, one can demonstrate that a certain axiom is necessary for a proof modulo other axioms. For instance, modulo ZF and classical logic, the axiom of choice is acknowledged as crucial to prove any number of elementary mathematical relationships. This suggests a relationship between Russell's views on simplicity and coherence. Summarily, the coherence of any set of propositions Ω can be truth-indicative because it meets up to what extent one's premises are 'roughly basic' for proving those facts taken as self-evident.

4.0 Conclusion

The factual significance of my paper is grasped within the context of twenty-first century trends of mathematical logic. With such attempts as those of Bob Hale, Crispin Wright, Otavio Bueno, Bernard Linsky, Kevin Klement, Alan Weir, Warren Goldfarb, Ed Nouri Zalta, and a host of other philosophers of logic and mathematics who are making revolutions in terms of the resuscitation of the logicist programmes [although with modifications of either programme], it becomes expedient that one gets to understand the proper purview and mind of the originators of the programme, before attempting a resuscitating project. While I do not make a case for either neo-Fregean logicism or neo-Russellian logicism in this paper, I am well aware of the bias towards Russell's brand of logicism as *supposedly* devoid of any useful import in terms of logic and epistemology. Hence, this paper has supplied a meaningful exposition of Russell's logicism and his outlook towards epistemic coherence.

In the first section, I argued that Russell committed himself to establish the logicist view that all pure mathematics deals exclusively with concepts definable in terms of a very small number of fundamental logical concepts, and that all its propositions are deducible from a very small number of logical principles. Russell took logicism to expound the fundamental concepts which mathematics accepts as indefinable. I have shown that Russell was uncertain as to whether or not MTs can be proven by sole virtue of pure logical axioms. If the claim of the derivation thesis, were true, then there must be proof of their logicality. Russell's reluctance in endorsing the derivation thesis suggests his perception of logicism.

We have also seen what Russell means by coherence. Russell means more than 'logical constancy' when he speaks of coherence. When a number of propositions bear a relation *R* to one another, the set becomes more 'obvious', 'credible' than the individual propositions. It is the collation of all such relations *R* that Russell calls 'coherence'. Thus, in Russell's view, in regard to probable opinion, we can derive great assistance from coherence, which we rejected as the definition of truth, but may often use as a criterion. A body of individually probable opinions, if they are mutually coherent, become more probable than any one of them would be individually.

Conclusively, we can say that Russell's theory of coherence offers a partial response to a need for certainty in mathematics: even if the logicist ambition is not successful, owing to the fact that certain axioms may not be purely logical, the enhanced structuration of mathematical knowledge results in a coherent corpus of theorems of superior security than the disjointed collection of mathematical subjects with which we began.