THE CONTINUUM EAST AND WEST

Peter G. Jones

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This essay examines the relationship between mysticism, for which Buddhism's Middle Way doctrine would serve here as a defining example, and what, for want of better word, we call 'Western' philosophy. This is an issue of general interest to philosophers, since sooner or later in our investigations we must all decide whether the 'Western' kind of philosophy makes more or less sense to us than the 'Eastern' kind.

One obstacle we face in trying to make this decision is the difficulty of discerning clearly the defining characteristics of the two philosophies, those features that lead us to make such a final and definite distinction between them in the first place. We commonly speak of 'Eastern' and 'Western' philosophy, but are not so commonly able to say quite what we mean by this. The relevant issues are profound, mind-bending and probably inexhaustible. They need not be complicated, and they are often quite simple, but they are always immensely challenging.

One of these simple (stripped of the details) yet challenging issues would be the true nature of the continuum. The discussion that follows outlines the view of physicist, mathematician and philosopher Hermann Weyl. Weyl makes a careful distinction between the 'arithmetical' continuum, the continuum conceived of as an extended object, as it must be for the real numbers and space-time, and the 'intuitive' continuum, the empirical continuum of experience, which is not extended, and he demonstrates that when we set out to define what we mean by 'Eastern' and 'Western' philosophy, the foundations of analysis would be a good place to start. The interconnectedness of all the relevant issues at a foundational level, for all roads lead to Rome, means that we may as well start where we like, but mathematics takes us immediately to what might be the most clearly discernable and easily described difference between the two philosophies and worldviews, perhaps also the most general and profound, namely their entirely different conceptions of the continuum.

As there is just one source for each author quoted here I have not added numbered references but just tried to make it clear who is talking. Italics are always original.

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In *The Continuum: A Critical Examination of the Foundations of Analysis*, Hermann Weyl points out that the extended space-time of physics and ordinary perception is, in the same way as the number line, a construction of reason and not intuitively or empirically given. He addresses a problem that arises in different guises but with an equal vengeance in religion, physics, mathematics and metaphysics. It is the problem of modelling a continuum as an extended series of discrete locations or 'things', as we do must do for the number line, geometry and arithmetic, space and time, and even for our very concept of the continuum, when a series of discrete locations or 'things' is exactly and precisely what a continuum is not.

A continuum cannot be extended as a series of points or moments for the reasons Weyl gives below, and yet it must be in order for anything to be extended in space and time. This causes a problem in philosophy. It would be a 'first-order' metaphysical problem or 'antinomy', a straight choice between two ideas neither of which work. It would be closely connected with the question of how many angels can dance on the head of a pin, of how small we can make an angel before it becomes the 'ghost of a departed quantity'. The various problems and paradoxes to which the intellectually-constructed continuum of the arithmetical line gives rise has no impact on the usefulness of mathematics, which is wholly dependent on this conception, but it indicates that the continuum of space-time is not an equivalent case and is, rather, a true continuum. As such, it would not be a set of locations but a unity. A unity has no parts. This would suggest that space and time are conceptual imputations and that Reality, whatever is truly and independently-real amongst all the smoke and mirrors, is not in fact extended. This is a difficult idea but not a new one, and it is widely popular in religion. When theoretical physicists say 'distance is arbitrary', perhaps they are suggesting something similar. It might at least help to explain how a Big Bang can appear to have occurred before there is, was or ever will be a time or a place for it to have happened. For an ultimate view it would not have happened. If the continuum cannot have parts then all co-ordinate systems are emergent.

Here is Tobias Dantzig, a mathematician admired by Einstein, introducing the issues.

> Herein I see the genesis of the conflict between geometrical intuition, from which our physical concepts derive, and the logic of arithmetic. The harmony of the universe knows only one musical form - the *legato*; while the symphony of numbers knows only its opposite, - the *staccato*. All attempts to reconcile this discrepancy are based on the hope that an accelerated *staccato* may appear to our senses as *legato*. Yet our intellect will always brand such attempts as deceptions and reject such theories as an insult, as a metaphysics that purports to explain away a concept by resolving it into its opposite.

While a series of points serves perfectly well for the continuum of the number line and arithmetic, on examination it is a paradoxical idea that must be rejected in both metaphysics and physics as a model of space-time. The continuum of physics is, at this time, extended as a series of points and moments, and as such no sense can be made of it. Viewed as a real phenomenon a continuum so-defined would either be paradoxical or fail to qualify for the name. We have every right to define the continuum for mathematics as we currently do, and if our idea is paradoxical then it is only a problem when we investigate the foundations of analysis. When we define the continuum for mathematics we are not making a claim about the nature of Reality. Elsewhere it would be a different matter. In metaphysics we certainly cannot adopt *a priori* an arithmetical definition of the continuum. Insofar as it relates to metaphysics this might be the central message of Weyl's book. At the same time, physics and ordinary perception are heavily theory-laden, dangerously so. Our usual everyday theory is that time and space are extended in just the same way as is the number line, such that space- and time-points can be represented as locations in an extended co-ordinate system. But is there any evidence that space and time are extended objects? What is it that our wristwatch is actually measuring? Are we quite sure that our usual theory of extension, for which space-

time would be a ‘classical’ or Newtonian phenomenon, is *fundamentally* correct? Is it a metaphysical conjecture, a testable scientific theory, something we know from experience or a highly evolved misinterpretation? For our Western tradition of philosophy this would be a famously undecidable problem. Here the continuum appears to be paradoxical, for it cannot be extended *ex hypothesis*, and yet, by some magic, it is. Or it seems to be. For the Eastern tradition this everyday theory of space-time would be testable and it would fail the tests, being refutable in logic and falsifiable in experience. The continuum would be a unity, just as its name implies.

Weyl reduces the conceptually extended continuum of mathematics and traditional physics to what he calls the ‘true’ or ‘intuitive’ continuum, where the latter is carefully distinguished from the former. The intuitive continuum, the continuum as we experience it, is not extended as a series of moments or points. We do not experience time and space as consisting of moments and points, or, if we do, it is only ever the same moment and point. We are always here and now. What is more, there is actually something very odd about the idea that space and time are ‘grainy’ in this way. The length of ten thousand points would be equal to the length of one point, for a start, so no amount of points would be sufficient to construct basic geometry, let alone a piano. In the same way, no amount of moments would be sufficient to account for motion and change. Space and time are explanatory theories, Weyl proposes, generated by reason and imagination, not empirical phenomena.

For an orthodox view of space-time here is a passage from Wikipedia from the entry for Hermann Minkowsky.

> This new reality was that space and time, as physical constructs, have to be combined into a new mathematical/physical entity called 'space-time', because the equations of relativity show that both the space and time coordinates of any event must get mixed together by the mathematics, in order to accurately describe what we see. Because space consists of 3 dimensions, and time is 1-dimensional, space-time must, therefore, be a 4-dimensional object. It is believed to be a 'continuum' because so far as we know, there are no missing points in space or instants in time, and both can be subdivided without any apparent limit in size or duration. So, physicists now routinely consider our world to be embedded in this 4-dimensional Space-Time continuum, and all events, places, moments in history, actions and so on are described in terms of their location in Space-Time.

Dantzig explores the origins of this co-ordinate system.

> The notion of *equal-greater-less* precedes the number concept. We learn *to compare* before we lean *to evaluate*. Arithmetic does not begin with numbers; it begins with *criteria*. Having learnt to apply these criteria of equal-greater-less, man’s next step was to devise models for each *type* of plurality. These models are deposited in his memory very much as the standard meter is deposited at the Bureau of Longitudes in Paris. One, two, three, four, five …; we could just as well have said: I,

wings, clover, legs, hand … and, for all we know, the latter preceded our present form.

He goes on to observe that the staccato of the numbers is not empirical or intuitive, but a superimposition.

It is possible to assign to any point on a line a unique real number, and, conversely, any real number can be represented in a unique manner by a point on the line.

This is the famous *Dedekind-Cantor* axiom. This proposition, by sanctifying the tacit assumption on which analytical geometry had operated for over two hundred years, became the fundamental axiom of this discipline. It defines a new mathematical being, the *arithmetical line*. Henceforth the line - and consequently the plane, and space - ceases to be an intuitive notion and is reduced to *being a mere carrier of numbers*.

In the following passage Dantzig notes the paradoxical nature of the arithmetical line. This matters little in mathematics, but when the arithmetical line is taken to be a model of the true continuum it renders Reality paradoxical and causes philosophical havoc, in particular a deep rift between two quite different traditions of philosophy.

The axiom of Dedekind - "if all points of a straight line fall into two classes, such that every point of the first class lies to the left of any point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions" - this axiom is just a skilful paraphrase of the fundamental property we attribute to time. Our intuition permits us, by an act of the mind, to sever *all time* into the two classes, *the past* and *the future*, which are mutually exclusive and yet together comprise all of time, *eternity*: The *now* is the partition which separates all the past from all the future; any instant of the past was once a *now*, any instant of the future will be a *now* anon, and so any instant may itself act as such a partition. To be sure, of the past we know only disparate instants, yet, by an act of the mind we fill out the gaps; we conceive that between any two instants - no matter how closely these may be associated in our memory - there were other instants, and we postulate the same compactness for the future. This is what we mean by the flow of time.

Furthermore, paradoxical though this may seem, *the present is truly irrational* in the Dedekind sense of the word, for while it acts as partition it is neither a part of the past nor a part of the future. Indeed, in an arithmetic based on pure time, if such an arithmetic was at all possible, it is the irrational which would be taken as a matter of course, while all the painstaking efforts of our logic would be directed toward establishing the existence of rational numbers.

In other words, the Dedekind sense of the word 'present' is irrational. Space-time cannot have the properties he assigns to the number line unless the Cosmos is irrational. This is the problem addressed by Weyl. He deals with it by making a clear distinction between the intuitive or experienced continuum, the intuition of the continuum that for all of us is an empirical phenomenon, and the intellectually constructed faux-continuum of Dedekind's arithmetical line. They could hardly be more different.

> To the criticism that the intuition of the continuum in no way contains those logical principles on which we must rely for the exact definition of the concept "real number," we respond that the conceptual world of mathematics is so foreign to what the intuitive continuum presents to us that the demand for coincidence between the two must be dismissed as absurd.

He points out that the usefulness of the arithmetical line has no bearing on its plausibility as a model of the space-time continuum.

> Whichever view of the relation of mathematics to nature one takes, there is no independent physical conception of the continuum on offer in all this, since all the mathematics is filtered through the real number system (or Hilbertian geometry as a surrogate). Moreover, I don't see that any argument can be made from the enormously successful applications of mathematics in natural science to the conclusion that one or another of the mathematical conceptions of the continuum surveyed above is uniquely singled out as the "real one". In any case, the work on the reach of predicative mathematics cited at the end of the preceding section shows that the properties of the continuum needed for its applications in natural science do not require it to have a definite reality in the platonistic sense.

Here is extract from an essay on Weyl and the continuum by John Bell.

> …Weyl regards the experienced *continuous flow* of phenomenal time as constituting an insuperable barrier to the whole enterprise of representing this continuum in terms of individual points, and even to the characterization of "individual temporal point" itself. As he says,
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> "The view of a flow consisting of points and, therefore, also dissolving into points turns out to be mistaken: precisely what eludes us is the nature of the continuity, the flowing from point to point; in other words, the secret of how the continually enduring present can continually slip away into the receding past.
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> Each one of us, at every moment, directly experiences the true character of this temporal continuity. But, because of the genuine primitiveness of phenomenal time, we cannot put our experiences into words. So we shall content ourselves with the following description. What I am conscious of is for me both a being-now and,

in its essence, something which, with its temporal position, slips away. In this way there arises the persisting factual extent, something ever new which endures and changes in consciousness."

We see here that an examination of the foundations of analysis leads us immediately into the realms of psychology, physics, metaphysics, religion, consciousness studies and more. Bell continues.

Weyl sums up what he thinks can be affirmed about "objectively presented time"— by which I take it he means "phenomenal time described in an objective manner"— in the following two assertions, which he claims apply equally, *mutatis mutandis*, to every intuitively given continuum, in particular, to the continuum of spatial extension:

1. *An individual point in it is non-independent, i.e., is pure nothingness when taken by itself, and exists only as a "point of transition" (which, of course, can in no way be understood mathematically);*

2. *It is due to the essence of time (and not to contingent imperfections in our medium) that a fixed temporal point cannot be exhibited in any way, that always only an approximate, never an exact determination is possible.*

The fact that single points in a true continuum "cannot be exhibited" arises, Weyl continues, from the fact that they are not genuine individuals and so cannot be characterized by their properties. In the physical world they are never defined absolutely, but only in terms of a *coordinate system*, which, in an arresting metaphor, Weyl describes as "the unavoidable residue of the eradication of the ego."

In particular, he found compelling the fact that the Brouwerian continuum is not the union of two disjoint nonempty parts— that it is, in a word, *indecomposable*. "A genuine continuum," Weyl says, "cannot be divided into separate fragments." In later publications he expresses this more colourfully by quoting Anaxagoras to the effect that a continuum "defies the chopping off of its parts with a hatchet."

Weyl's book on the continuum delves little further into metaphysical issues than is necessary for his examination of analysis. Elsewhere he says more, and we find a clear connection between his mathematico-philosophical views and Buddhism's theory of emptiness and doctrine of dependent origination. As far as it goes his book on the continuum could be read as a mathematical explanation of the universe of the perennial philosophy, and of how it differs from that of the Western tradition in at least one vital respect. Bell makes the correlation clear.

In *The Open World* (1932), Weyl provides an eloquent formulation of his philosophical outlook, which quickly moves beyond its initial echoes of Schopenhauer:

> "The beginning of all philosophical thought is the realization that the perceptual world is but an image, a vision, a phenomenon of our consciousness; our consciousness does not directly grasp a transcendental real world which is as it appears. The tension between subject and object is no doubt reflected in our conscious acts, for example, in sense perceptions. Nevertheless, from the purely epistemological point of view, no objection can be made to a phenomenalism which would like to limit science to the description of what is "immediately given to consciousness". The postulation of the real ego, of the thou and of the world, is a metaphysical matter, not judgment, but an act of acknowledgment and belief.
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> But this belief is after all the soul of all knowledge. It was an error of idealism to assume that the phenomena of consciousness guarantee the reality of the ego in an essentially different and somehow more certain way than the reality of the external world; in the transition from consciousness to reality the ego, the thou and the world rise into existence indissolubly connected and, as it were, at one stroke."

Any comparison of 'Eastern' and 'Western' approaches to philosophy must eventually end up here, examining the question of whether the continuum of space-time is arithmetical and paradoxical, or whether it would make more sense to say that spatio-temporal extension is an interpretation of appearances, a relationship between appearances, and not an empirical or even truly real phenomenon. Whichever way we decide this question, an examination of these issues will reveal a clear and crucial difference of opinion between East and West over the ultimate nature of Reality.

It is absurdly misleading to use the words 'Western' and 'Eastern' to describe two philosophical camps, and really it is dualism and nondualism that we are comparing here, both of which appear all over the world. Whatever words we use, mathematics can help us to pin down our definitions in important respects.

Weyl summarises his view as follows.

> The category of the natural numbers can supply the foundation of a mathematical discipline. But perhaps the continuum cannot, since it fails to satisfy the requirements [mentioned in Chapter1]: as basic a notion as that of the point in the continuum lacks the required support in intuition. It is to the credit of Bergson's philosophy to have pointed out forcefully this deep division between the world of mathematical concepts

and the immediately experienced continuity of phenomenal time.

The view of a flow consisting of points and, therefore, also dissolving into points turns out to be false. Precisely what eludes us is the nature of the continuity, the flowing from point to point; in other words, the secret of how the continually enduring present can continually slip away into the receding past….

…When our experience has turned into a real process in a real world and our phenomenal time has spread itself out over this world and assumed a cosmic dimension, we are not satisfied with replacing the continuum by the exact concept of the real numbers, in spite of the essential and undeniable inexactness arising from what is given. For, as always, there is more at work here than heavy-handed schematizing or cognitive economizing devised for fulfilling our practical tasks and objectives. Here we discover genuine reason which lays bare the "Logos" dwelling in reality (just as purely as is possible for this consciousness which cannot "leap over its own shadow"). But to discuss this further cannot be our business here. Certainly, the intuitive and the mathematical continuum do not coincide; a deep chasm is fixed between them….

…The reflections contained in this section are, of course, only a slightly illuminating surrogate for a genuine philosophy of the continuum. But since no penetrating treatment of this topic is at hand and since our task is mathematical rather than epistemological, the matter can rest there.

For a book on analysis it would have been inappropriate for Weyl to say more about this. If we are examining the pivotal questions on which Eastern and Western philosophies are divided, however, then the matter cannot rest here. The former philosophy makes a claim about the continuum that is denied point-blank by the latter. It may still be true that 'no penetrating treatment of this topic is at hand', at least outside of the 'mystical' literature, but this would not reflect on the importance of this topic across all of philosophy, and it need not prevent us from forming a view on which of these two philosophical approaches gives the most plausible description of space and time.

Is space-time extended or is it a continuum? Weyl suggest that we cannot have it both ways. Nagarjuna's Middle Way Buddhism, which is infuriatingly stubborn when it comes to endorsing extreme views on any topic, would say that the question is not quite answerable in this straightforward form. There would be a sense in which it is neither and a sense in which it is both. There is not a straightforward disagreement between East and West on the answer to this question, therefore, with the two sides adopting equal and opposite views. All the same, it seems true to say that the very different answers they give to this question reveal one of the most crucial and far-reaching differences between these two traditions of philosophical thought.

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