A Semantics for Weak, Question-Sensitive Belief

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Abstract

Recent work in epistemology defends the unorthodox theses that belief is (1) an evidentially weak, and (2) question-sensitive attitude, and (3) that forming beliefs is sometimes a matter of guessing. What motivates these theses are examples of rationally permissible belief-ascriptions that exhibit these traits. The main aim of this paper is to outline a semantic account of categorical and conditional belief-ascriptions that captures the motivating data. We then survey some consequences of the proposed semantics, particularly with respect to the question of whether closure under rules of inference is rationally required for weak, question-sensitive belief.

1 Introduction

Recent work in epistemology defends the unorthodox theses that belief is (1) an evidentially weak, (2) question-sensitive attitude, and that (3) rationally permitted belief is sometimes a matter of guessing. (1) Belief is weak: the evidential standards for believing a proposition are undemanding; specifically, one can believe p without knowing p and without being highly confident in p [13, 26, 16]. (2) Belief is question-sensitive: agents believe propositions against the backdrop of salient partitions of the logical space provided by a question under discussion (QUD, [25]) [34, 7, 14, 15]. (3) Beliefs are best guesses: in situations of uncertainty, one's belief just is one's best guess, where one's guess concerns a QUD, and a particular guess can dominate other guesses despite one's evidence assigning it an arbitrarily low probability [16, 5].[1]

First, we outline a semantics for belief-ascriptions with categorical and conditional contents (Section 3). On our approach, belief is treated as a modal sensitive to information states with probabilistic and inquisitive structure. A categorical φ is believed relative an information state just in case φ is the most probable complete answer to the underlying question. A categorical φ is believed *conditionally* on ψ w.r.t. an information state just in case φ is believed w.r.t. an information state appropriately updated with ψ. Finally, a conditional $\varphi \to \psi$ is believed w.r.t. an information state just in case it is more probable than any alternative $\varphi \to \chi$, where χ is any other complete answer to the underlying question.

Second, we outline some consequences of the semantics pertaining to the question of whether rational weak, question-sensitive belief is closed under rules of inference (Section 4). Our semantics allows us to reinforce a recent finding [24] that the target notion of belief is not closed under rules of inference like Modus Tollens, Disjunctive Syllogism, and others. We argue that this prediction is not unwarranted due to the semantic equivalence of believing $\varphi \to \chi$ and believing χ conditional on φ, and conjecture that any relevant failure of closure under rule X matches the failure of X to be truth-preserving in the presence of modalised sentences.

2 Data and Desiderata

We first introduce the theses that belief is weak, question-sensitive, and a matter of guessing via examples, and outline some desiderata for a semantic account of belief-ascriptions.

[1] Theses (1)-(3) fit naturally, but not necessarily. See [22] for a partition-sensitive account of *strong* belief.

Consider an urn containing 100 marbles, with 40 blue, 30 green, 15 red, and 15 yellow marbles inside. A marble is randomly chosen from the urn. Supposing one is in a context where questions demand only complete answers, (1) appears fine, while (2) appears odd w.r.t. Q:

Q: What is the colour of the chosen marble?

(1) I {believe}{think} that the chosen marble is blue.
(2) I {believe}{think} that the chosen marble is non-blue.

Q asks for a complete guess concerning the colour of the chosen marble, and only (1) delivers on this while respecting the chances of picking each colour. Relative to Q', however, the situation is reversed:

Q': Is the chosen marble blue or not?

(1) I {believe}{think} that the chosen marble is blue.
(2) I {believe}{think} that the chosen marble is non-blue.

Q' asks whether the chosen marble is blue, and in the reversed situation only (2) delivers on this in accordance with the chances. As one's credence that the chosen marble is blue should be 0.4 in light of the evidence, the proposition believed w.r.t. Q is not the most probable on one's evidence *overall*. In both scenarios, moreover, one permissibly believes a proposition despite not knowing it. Finally, one's answer to both questions seems to be an (informed) guess.

Agents hold beliefs not only toward propositions with categorical content, but also propositions with conditional content. Consider the case of rolling a fair, seven-sided die. Relative to Q", both (3) and (4) appear fine, while (5) appears odd:

Q": Did the die land on even or odd?

(3) I {believe}{think} that the die landed on odd.
(4) I {believe}{think} that if the die landed on a composite, it landed on even.
(5) I {believe}{think} that if the die landed on a composite, it landed on odd.

As the chance of the die landing on an even [odd] given that it lands on a composite are $\frac{2}{3}$ [$\frac{1}{3}$], (4) [(5)] appears permissible [impermissible]. Moreover, note that (4) and (5) appear to communicate the same as (4') and (5'):[2]

(4') If the die landed on a composite, I {believe}{think} that it landed on even.

(5') If the die landed on a composite, I {believe}{think} that it landed on odd.

Call ascriptions in the form of (4) beliefs in conditionals, and those in the form of (4') conditional beliefs.

We seek to develop a semantics for belief-ascriptions of the kind captured by these examples. We thus intend to capture the predictions of the theses surveyed in Section 1. Some desiderata for our semantics, judging by the examples above:

- The semantics should capture belief-ascriptions with both categorical and conditional contents.
- Belief-ascriptions should be sensitive to QUDs and contextually salient 'chances'.
- Beliefs in conditionals and conditional beliefs should be semantically equivalent.

[2] Variants of sentences above with 'maybe' or 'might' also sound pairwise indistinguishable [9]. The same goes for 'probably', and plausibly other epistemic modals and similar expressions [4, pp. 603-5]. Conditionals are also said to 'commute' with negation [30, Ch. 3]. Nevertheless, note that embedding non-doxastic attitudes like 'hope', 'hate', and 'regret' is not pairwise indistinguishable from holding those attitudes to the corresponding conditionals [6, 2].

3 The Semantics

Let $\mathscr{L}$ be a propositional language with negation, conjunction, and a belief operator (B). Let P be the set of atomic sentences of $\mathscr{L}$, W the set of possible worlds (LOGICAL SPACE), and $[\![\cdot]\!]$ be the function $P \longrightarrow \mathscr{P}(W)$. (In what follows, we use $\phi, \psi, \ldots$ as placeholders for atomic sentences.) Some definitions needed for the semantics:

Definition 3.1. An INFORMATION STATE s is a set of possible worlds, i.e., $s \in \mathscr{P}(W)$. Let $S = \mathscr{P}(W)$ be the set of all information states on W.

Definition 3.2. A QUESTION is a partition of s, i.e., a division of $s \in S$ into subsets $\{X_i\}_{i \in I}$ s.t. (a) $\bigcup_{i \in I} X_i = s$, (b) $X_i \cap X_j = \emptyset$ for all $i \neq j \in I$, and (c) $X_i \neq \emptyset$ for all $i \in I$. [12, 19, 11]

Definition 3.3. A QUESTION SPACE on s, $\mathscr{Q}(s)$ is the set of all admissible questions on s; we reserve the formal clarification of $\mathscr{Q}(s)$ for a footnote.[3]

Definition 3.4. A PROBABILITY DISTRIBUTION is a function Pr on the elements of a Boolean algebra $\mathscr{A}$ over W s.t. (a) $Pr(W) = 1$ and (b) for any $s, s' \in S$ s.t. $s \neq s'$, $Pr(s \cup s') = Pr(s) + Pr(s')$.

Definition 3.5. A PROBABILISTIC-INQUISITIVE (PI) INFORMATION STATE i is a triple $\langle s, Pr(\cdot \mid s), Q \rangle$, where $Q \in \mathscr{Q}(s)$. (Cf. sharp information states in [32].)

Sentences of $\mathscr{L}$ are evaluated w.r.t. worlds and PI information states. The truth-conditions for the Boolean fragment of $\mathscr{L}$ are standard and only sensitive to worlds. The truth-conditions for categorical belief ascriptions, or sentences of the form $B\phi$ are as follows:[4]

$$[\![B\phi]\!]^{w,i} \text{ is true iff } s \cap [\![\phi]\!] \in Q \text{ and } Pr(s \cap [\![\phi]\!]) > \max_{s \cap [\![\phi]\!] \neq X \in Q} Pr(X) \tag{6}$$

Intuitively, our account of categorical belief-ascriptions checks that the believed proposition is a cell in a partition of the logical space (a complete answer to some Q) and that Pr assigns a higher probability to the believed proposition than to other cells in the partition (with Q and Pr specified in the relevant i). Onto conditional contents.

Let $\mathscr{L}^+$ be an extension of $\mathscr{L}$ that adds an indicative conditional operator $\rightarrow$. We adopt a path semantic account of the indicative conditional, as given in [27]. A path semantic account provides a combination of an informational [21, 31, 8] and a selectional [28] account of conditionals. Intuitively, each path comes pre-equipped with an ordering on worlds; as such, paths break up the logical space into finer units than worlds. Some background on path semantics:

Definition 3.6 (Paths)**.**

1. A PATH is a sequence of worlds without repetitions.
2. A path is of LENGTH n iff the path is a sequence of n members; let p_n be the n^{th} member of a path.
3. The SET OF PATHS on a state $s = \{w_1, \ldots, w_n\}$ is $\text{PATH}(s) = \{\langle w'_1, \ldots, w'_n \rangle \mid \{w'_1, \ldots, w'_n\} = s, w'_1 \neq w'_n$ for all $i \neq j\}$.
4. p' is a PERMUTATION of p ($p' * p$) iff $p' \in \text{PATH}(s)$.
5. A PATH p'S STATE $\mathbf{S}(p)$ is $\{p'_1 \mid p' * p, p'_1 \neq p_1\}$
6. p' is a SUBSEQUENCE of p ($p' \leq p$) iff $\forall w \in \mathbf{S}(p)$, if $w \in \mathbf{S}(p')$, then $n(p', w) \leq n(p, w)$ (where $n(p, w) = i$ iff $p_i = w$).

[3]There exists a bijection between the set of partitions of S and the set of equivalence classes on S. For any equivalence relation $\approx_i$, the set of all equivalence classes in S generated by $\approx_i$ is $\{x \in S | [x]_{\approx_i}\}$, or the quotient set of S for $\approx_i$, $S/\approx_i$. Where $\mathscr{E}(S)$ is the set of all equivalence relations on S, $\mathscr{Q}(S) = \{S/\approx_i \mid \approx_i \in \mathscr{E}(S)\}$ is the set of all quotient sets on S. $\mathscr{Q}(S)$ just is the set of possible partitions on S. See [17] for a similar account.

[4]In case $\{X \in Q \mid X \neq s \cap [\![P]\!]\} = \emptyset$, we set $Pr(X) = 0$.

We also need definitions of update to capture the semantics of indicative conditionals:

Definition 3.7 (Update)**.**

1. A PATH UPDATE of p with q, $p+q$, is the largest member of the set $\{p' \leq p \mid \forall p'' \text{ if } p'' * p', \text{ then } p''_1 \in [\![q]\!]\}$.
2. An INFORMATION STATE UPDATE of s with q, s_q, is $s \cap [\![q]\!]$.
3. A QUESTION UPDATE of Q with q, Q_q, is $\{X_i \cap [\![q]\!] \mid X_i \in Q \text{ and } X_i \cap [\![q]\!] \neq \emptyset\}$.
4. A PROBABILITY DISTRIBUTION UPDATE of Pr with q, Pr_q, is $Pr(\cdot \mid q)$.

Updating a PI information state $i = \langle s, Pr, Q\rangle$ with q results in $i_q = \{s_q, Pr_q, Q_q\}$.

In path semantics, antecedents of indicative conditionals update the path and PI information state at which the consequent is evaluated. If the consequent is Boolean, the truth-value of the conditional depends on the truth-value of the consequent at the first world in the updated path; if the consequent is modal, the truth-value of the conditional depends on the truth-value of the consequent at the updated PI information state. Let P and $[\![\cdot]\!]$ be as with $\mathscr{L}$.

Definition 3.8. A PATH interpretation function $\|\cdot\|$ maps sentences and paths to truth-values so that, for any $q \in P$, $\|q\|^p$ is true iff $[\![q]\!]^{p_1}$ is true.

We extend $\|\cdot\|$ so that it assigns truth-values to indicative conditionals as follows:

$$\|\phi \to \psi\|^{p_1,p} \text{ is true iff } \|\psi\|^{p_1,p+q} \text{ is true iff } [\![\psi]\!]^{(p+q)_1} \text{ is true.} \tag{7}$$

Let $I(p)$ map p to some $i = \langle s, Pr(\cdot \mid s), Q\rangle$ s.t. $s = \mathbf{S}(p)$. We account for categorical belief-ascriptions on analogy with (6):

$$\|B\phi\|^{p_1,p,I(p)} \text{ is true iff } s \cap [\![\phi]\!] \in Q \text{ and } Pr(s \cap [\![\phi]\!]) > \max_{s\cap[\![\phi]\!] \neq X \in Q} Pr(X) \tag{8}$$

Conditional beliefs are captured by updating the parameters at which the consequent is evaluated:

$$\|\phi \to B\psi\|^{p_1,p,I(p)} \text{ is true iff } \|B\psi\|^{p_1,p+q,I(p)_q} \text{ is true} \tag{9}$$

Accounting for beliefs in conditionals is more challenging. Conditionals, as per (7), are not true at possible worlds but at paths, and as such express sets of paths and not sets of possible worlds. This implies that the semantic content of conditional statements cannot serve the purpose of possible-world propositions in the foregoing account of belief. One way of sidestepping the issue is incorporating more fine-grained information states, and along with them more fine-grained partitions and probability distributions (as in [10]). We sidestep the issue in another way: when $A \to C$ is believed w.r.t. an i, we check that the probability assigned to $A \to C$ is greater than that assigned to $A \to X$, where X is any alternative to C in Q. To capture this, we define alternatives and probabilities of conditionals.

Definition 3.9. The set of ALTERNATIVES to ϕ in $I(p)$ is $\mathscr{A}_\phi^{I(p)} = \{X \in Q \mid X \neq [\![\phi]\!] \cap s\}$ (provided $[\![\phi]\!] \cap s \in Q$).

Definition 3.10. The probability of a conditional $A \to C$ w.r.t. an information state s, $C_s(A \to C)$, is $Pr(a \mid c \cap s)$, provided A and C are true at sets of possible worlds (a and c, respectively).[5]

[5] That the probability of a conditional is the probability of the consequent conditional on the antecedent is notoriously contested [23]. Selectional renditions of this thesis [29, 3, 1, 20] nevertheless fare better than other accounts. As the proofs in [10] show, the equation of Definition 3.10 provably holds on a path semantic account. We assume the equality in what follows, without spelling out the details on the lifting procedure that independently specifies the value of $C(A \to C)$ w.r.t. some s.

This allows us to account for beliefs in conditionals:

$$\|B(\phi \to \psi)\|^{p_1 p, I(p)} \text{ is true iff } s \cap [\![\psi]\!] \in Q \text{ and } C(\phi \to \psi) > \max_{X_i \in \mathscr{A}_r^{I(p)}} C(\phi \to \chi_i) \tag{10}$$

The core of our semantic proposal is given in (8)-(10). The semantics should predict the following:

Fact 3.11. (1) is true and (2) is false w.r.t. Q, and vice versa w.r.t. Q'.

Fact 3.12. (4) and (4') are true, and (5) and (5') are false w.r.t. Q".

Fact 3.13. $\|B(A \to C)\|^{p_1,p,p(I)}$ is true iff $\|A \to B(C)\|^{p_1,p,p(I)}$ is true.

We reserve the proofs for Appendix A.

4 Closure and Rules of Inference

One's beliefs can be used productively as premises in further reasoning. For instance, if I believe that it's raining if the ground is wet, and I believe that the ground is wet, I should plausibly also believe it's raining. Say that an attitude is *closed* under a rule of inference 'From Δ, infer γ' iff one cannot rationally hold an attitude to all sentences in Δ without holding the attitude to γ. If one's beliefs are to be used as premises in further reasoning, it is to be expected that belief obeys some such properties of closure. This section highlights some closure-related predictions of the foregoing semantics; a fuller treatment is left for future work.

Take as paradigmatic the rule of inference Modus Tollens (MT): 'From $\phi \to \psi$ and $\neg\psi$, infer $\neg\phi$.' For belief to be closed under MT, then, means that whenever $B(\phi \to \psi)$ and $B(\neg\psi)$ hold w.r.t to some $p_1, p, I(p)$, it also holds that $B(\neg\phi)$. For concreteness, consider a scenario (from [24]) in which Alice and Bob roll a fair, six-sided die, and Alice wins if the die lands 1–4, and Bob wins otherwise. A natural model of this scenario predicts that (11) and (12) hold w.r.t. Q, 'Who will win?'. (We omit the formal model for brevity.)

(11) I {believe}{think} that Alice will win.

(12) I {believe}{think} that, if the die lands high [> 3], Bob will win.

Let the sentences one believes w.r.t. Q be A and $H \to \neg A$ (i.e., $H \to B$). These sentences licence an inference to $\neg H$ via MT, i.e., the sentence 'The die will not land high'. Provided belief is closed under MT, the fact that $B(H \to \neg A)$ and $B(A)$ hold should imply that $B(\neg H)$ also holds. Nevertheless, the following is not permitted on our semantics:

(13) I {believe}{think} that the die will not land high.

As the chances of a fair die landing high are the same as those of it landing low, (13) seems to be a paradigm of *im*permissibility. As our theory predicts the permissibility of (11) and (12) and the impermissibility of (13), it implies that weak, question-sensitive belief is not closed under MT.[6] Is this a desirable prediction?

Pending further engagement, we offer the following perspective. Closure (or a weaker principle in the vicinity) is usually taken as a desideratum for attitudes like knowledge and belief as failures to follow up on the consequences of what one already knows or believes appears irrational. Nevertheless, once we admit a weaker sense of belief – one on which beliefs are tied to available information, partitions, and assignments of probabilities – it might result that failing to follow up on the consequences of what

[6] As the discussion in [24] suggests, our theory is expected to predict further failures of closure, for instance under Disjunctive Syllogism ('From $P \vee \neg P$, $P \to Q$, and $\neg P \to Q$, infer Q'), under Modus Ponens when the consequent is itself a conditional ('From $P \to (Q \to R)$, and P, infer $Q \to R$'), etc.

one so believes is not always irrational. Concretely, and without immediate reference to the foregoing account, beliefs (11) and (12) are pre-theoretically permissible in the given scenario, while (13) is not. If rationality is what counts, then these weak beliefs should not be jettisoned for not obeying closure. Rather, it is closure that should be jettisoned, at least in the context of weak belief, provided it makes unwarranted predictions.

Parenthetically, note that reasoning via a schema *resembling* MT *is* warranted in this scenario. In the initial setup, no outcome of the die-roll can be excluded from one's information state, so that one's information state includes worlds in which the die lands on any number from 1 to 6. Suppose, now, that a trusted informant asserts that Bob will not win (i.e., $\neg B$ / A). After updating, one's information state includes worlds in which the die lands $1-4$. But now, relative to the question 'Will the die land high or low?', our semantics predicts (13) to be permissible. In brief: after asserting (12) and being informed that Bob did not win, one *can* permissibly come to believe the MT-licensed conclusion that the die will not land high. As such, while the probabilistic guesses (11) and (12) by themselves do not permit one to guess whether the die lands high or low, relevant non-probabilistic input can make the guess in (13) permissible.

Even if these justifications are found wanting, it is useful to draw a parallel between failures of closure for weak belief and failures of truth-preservation for the corresponding rules of inference in the presence of modal vocabulary. We conjecture that each failure of closure under MT for weak, question-sensitive belief maps onto a failure of MT. The case above can serve as an example. Note that $B(A \to C)$ and $A \to B(C)$ are equivalent on our account (see Appendix A). We have that $H \to B(\neg A)$ and $\neg B(\neg A)$ are true at some p and $I(p)$. Let p be $\langle w_1, w_2, w_3, w_4, w_5, w_6 \rangle$, and let $b(w_1) = p$ without loss of generality. As $w_1 \notin [\![H]\!]$, H is false at w_1 while both $H \to B(\neg A)$ and $\neg B(\neg A)$ are true at $p = b(w_1)$. Intuitively speaking, this is a failure of MT.[7]

The parallel suggests the following. (We focus on MT for concreteness.) Failures of closure under MT for weak, question-sensitive belief share a common core with failures of truth-preservation for MT. In both cases considered above, what the conditionals do is restrict the consequent's domain of evaluation to antecedent-verifying worlds, so that when a belief is embedded in the consequent or when a conditional itself is believed, one's beliefs only hold *given some restriction*. However, one's beliefs with respect to restricted and unrestricted portions of the logical space do not entail the truth or falsity of the various available restrictions, as the restrictions are entertained suppositionally in the first place.

5 Conclusion

One aim of this paper was to outline a semantic account of belief that makes good on the claims that belief is weak and question-sensitive. On our approach, belief is a modal sensitive to information states with probabilistic and inquisitive structure. For ϕ to be believed w.r.t. such a state is, intuitively, for ϕ to be a probabilistically dominant complete answer to some question (where both the question and the probability distribution are specified in the information state).

Another aim of the paper was to outline some verdicts of the semantics with respect to the question of whether closure under valid rules of inference is rationally required for the attitude of weak, question-sensitive belief. First, we indicated that our semantics features failures of belief to be closed under valid rules of inference (e.g. Modus Tollens, Disjunctive Syllogism, etc.). Second, we argued that this prediction is warranted on account of the close confluence of each failure of closure under X for our target notion of belief and failures of X to be truth-preserving in the presence of modalised sentences.

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[7] The failure is akin to those observed with probability modals [33], desire-related modals [18], and other information-sensitive vocabulary embedded in consequents of conditionals. Other superficial failures of otherwise valid inference rules are expected, moreover, when dealing with attitudes embedded in consequents of conditionals [2].

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A Semantic Predictions

Model A.1. Let $W = s = \{b,g,r,y\}$, and the sentence B [G/R/Y/N] – 'The chosen marble is blue [green/red/yellow/non-blue]' – express the proposition b $[g/r/y/n]$ (with $n = g \cup r \cup y$). Let Q = $\{\{b\},\{g\},\{r\},\{y\}\}$ and Q' = $\{\{b\},\{n\}\}$. Let $Pr(b) = 0.4$, $Pr(g) = 0.3$, $Pr(r) = Pr(y) = 0.15$. Without loss of generality, let $p = \langle b,g,r,y\rangle$. Let $I(p) = \langle s,Pr,Q\rangle$ and $I'(p) = \langle s,Pr,Q'\rangle$.

Fact 3.11. In Model A.1, (1) is true [false] and (2) is false [true] relative to $p_1, p, I(p)[I'(p)]$.

$\|$I believe that the chosen marble is blue$\|^{p_1,p,I(p)}$is true
iff $\|$I believe that B$\|^{p_1,p,I(p)}$ is true
iff $\|Bb\|^{p_1,p,I(p)}$ is true
iff $s\cap b \in Q$ and $Pr(s\cap b) > \max_{s\cap b\neq X\in Q} Pr(X)$
iff $b \in Q$ and $0.4 > 0.3$ iff $\top$

$\|$I believe that the chosen marble is non-blue$\|^{p_1,p,I(p)}$is true
iff $\|$I believe that N$\|^{p_1,p,I(p)}$ is true
iff $\|Bn\|^{p_1,p,I(p)}$ is true
iff $s\cap n \in Q$ and $Pr(s\cap n) > \max_{s\cap n\neq X\in Q} Pr(X)$
iff $\perp$

$\|$I believe that the chosen marble is blue$\|^{p_1,p,I'(p)}$is true
iff $\|$I believe that B$\|^{p_1,p,I'(p)}$ is true
iff $\|Bb\|^{p_1,p,I'(p)}$ is true
iff $s\cap b \in Q$ and $Pr(s\cap b) > \max_{s\cap b\neq X\in Q} Pr(X)$
iff $b \in Q'$ and $0.4 > 0.6$ iff $\perp$

$\|$I believe that the chosen marble is non-blue$\|^{p_1,p,I'(p)}$is true
iff $\|$I believe that N$\|^{p_1,p,I'(p)}$ is true
iff $s\cap n \in Q'$ and $Pr(s\cap n) > \max_{s\cap n\neq X\in Q'} Pr(X)$
iff $n \in Q'$ and $0.6 > 0.4$
iff $\top$

Model A.2. Let $s' = \{w_1,\dots w_7\}$, and the sentence O [E/C/P] – 'The die landed on an odd [even/composite/ prime]' – express the proposition $o[e/c/p] = \{w_n \in W \mid n$ is odd [even/composite/prime]$\}$. Let Q" = $\{\{e\},\{o\}\}$, $Pr(w_n) = \frac{1}{7}$ for $1 \le n \le 7$, $Pr(o) = Pr(p) = \frac{4}{7}$, and $Pr(e) = Pr(c) = \frac{3}{7}$. Without loss of generality, let $p = \langle w_1,\dots,w_7\rangle$. Let $I(p) = \langle s,Pr,Q''\rangle$.

Fact 3.12. In Model A.2, (4) and (4') are true, and (5) and (5') are false relative to i_3.

$\|$I believe that C $\rightarrow$ E$\|^{p_1,p,I(p)}$ is true
iff $\|B(c\rightarrow e)\|^{p_1,p,I(p)}$ is true
iff $s\cap e \in Q''$ and $C(c\rightarrow e) > C(c\rightarrow o)$
iff $e \in Q''$ and $Pr(e \mid c) > Pr(o \mid c)$
iff $e \in Q''$ and
iff $\top$

$\|$If C, I believe that E$\|^{p_1,p,I(p)}$ is true
iff $\|c \rightarrow Be\|^{p_1,p,I(p)}$ is true
iff $\|Be\|^{p_1,p+c,I(p)_c}$ is true
iff $s_c\cap e \in Q''_c$ and $Pr_c(e) > \max_{s_c\cap e\neq X\in Q''_c} Pr_c(X)$
iff $\{2,4\} \in Q''_c$ and $\frac{2}{3} > \frac{1}{3}$
iff $\top$

$\|$I believe that C $\rightarrow$ O$\|^{p_1,p,I(p)}$ is true
iff $\|B(c\rightarrow o)\|^{p_1,p,I(p)}$ is true
iff $e \in Q''$ and $\frac{1}{3} > \frac{2}{3}$
iff $\perp$

$\|$If C, I believe that O$\|^{p_1,p,I(p)}$ is true
iff $s_c\cap o \in Q''_c$ and $Pr(o \mid c) > \max_{s_c\cap o\neq X\in Q''_c} Pr(X)$
iff $\{1\} \in \{\{2,4\},\{1\}\}$ and $\frac{1}{3} > \frac{2}{3}$
iff $\perp$

Fact 3.13. $\|B(A\rightarrow C)\|^{p_1,p,p(I)}$ is true iff $\|A\rightarrow B(C)\|^{p_1,p,p(I)}$ is true.

Proof. $\Rightarrow$ Suppose $\|B(A\rightarrow C)\|$ is true at some p_1, p, $I(p) = i = \langle s,Pr,Q\rangle$. By (10), $s\cap c \in Q$ and $C(a\rightarrow c) > \max_{X_i\in\mathscr{A}c^i} C(a\rightarrow Xi)$. By Definition 3.10, $Pr(c \mid a) > \max_{X_i\in\mathscr{A}c^i} Pr(Xi \mid a)$. By Definition 3.9, for all $X \in \mathscr{A}c^i$, it holds that $X \neq s\cap c$ and $X \in Q$. As such, $\max X_i \in \mathscr{A}c^i Pr(Xi \mid a) = \max_{s\cap c\neq X\in Q} Pr(X \mid a) = \max_{s_a\cap c\neq X\in Q_a} Pr(X)$. By Definition 3.7.4, $Pr(c \mid a) = Pr_a(c)$. By Definition 3.7.3, if $s\cap c \in Q$, then $s\cap c\cap a \in Q_a$. From this it follows that $\|Bc\|$ is true at $p_1, p+a, i_a = \langle s\cap a, Pr(\cdot \mid a), Q_a\rangle$. But then it holds that $\|a \rightarrow Bc\|$ is true at w,p,i. □

Proof. $\Leftarrow$ Suppose $\|a\rightarrow Bc\|$ is true at some p_1, p, $I(p) = i = \langle s,Pr,Q\rangle$, i.e., $\|Bc\|$ is true at some $p_1, p+a, I(p)a = \langle s_a, Pr(\cdot \mid a), Q_a\rangle$. Then, $sa\cap c \in Q_a$ and $Pr_a(c) > \max_{s_a\cap c\neq X\in Q_a} Pr_a(X)$. As above, $Pr_a(c) = Pr(c \mid a)$ and $\max_{s_a\cap c\neq X\in Q_a} Pr_a(X) = \max_{s\cap c\neq X\in Q} Pr(X \mid a) = \max_{X_i\in\mathscr{A}c^i} Pr(Xi \mid a)$. Then, $Cr(a\rightarrow c) > \max_{X_i\in\mathscr{A}c^i} Cr(a\rightarrow xi)$. By Definition 3.7.3, if $s_a\cap c \in Q_a$, then $s\cap c \in Q$. But then it holds that $\|B(a\rightarrow c)\|$ is true at w,p,i. □