

Bourne on Future Contingents and Three-valued Logic*

Daisuke Kachi

Abstract

Recently, Bourne constructed a system of three-valued logic that he supposed to replace Łukasiewicz's three-valued logic in view of the problems of future contingents. In this paper, I will show first that Bourne's system makes no improvement to Łukasiewicz's system. However, finding some good motivations and lessons in his attempt, next I will suggest a better way of achieving his original goal in some sense. The crucial part of my way lies in reconsidering the significance of the intermediate truth-value so as to reconstruct Łukasiewicz's three-valued logic as a kind of extensional modal logic based on partial logic.

Keywords: Łukasiewicz, Bourne, three-valued logic, partial logic, truth-value gap, future contingents, alethic modality

1 Bourne's three-valued logic

Recently[1][2], Bourne proposed to replace Łukasiewicz's three-valued logic with his system of three-valued logic for the reasons that Bourne's system is 'a non-bivalent logic where classical laws remain intact[1, p.127]' contrary to Łukasiewicz's and that it can deal with future contingent propositions, which motivated Łukasiewicz's three-valued logic, in an appropriate way.

The principal alteration of Bourne's system to Łukasiewicz's system lies in his truth-functional definition of negation. He adopts the following definition of negation(Table 1) in place of Łukasiewicz's(Table 2):

P	\sim P
1	0
1/2	1
0	1

Table 1:

P	\sim P
1	0
1/2	1/2
0	1

Table 2:

He has four justifications for this modification:

*Reprinted from *Logic and Logical Philosophy*, 18-1, pp. 31-41, 2009.

- (1) He sees ‘no reason to think that [Lukasiewicz’s definition] is correct[1, p.124].’
- (2) ‘[G]iven that p is indeterminate, then it isn’t the case that p; so to say that it is not the case that p is clearly to say something true[1, p.124].’
- (3) His negation reserves more classical logical truths such as the law of excluded middle and the law of contradiction than Łukasiewicz’s, working with his definitions of conjunction and disjunction, which are the same as Łukasiewicz’s (Table 3, 4).
- (4) His system can deal with future contingent propositions appropriately by using his negation.

P&Q	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	0	0	0

Table 3:

P∨Q	1	1/2	0
1	1	1	1
1/2	1	1/2	1/2
0	1	1/2	0

Table 4:

As for (1), there are several logical reasons to adopt Łukasiewicz’s negation rather than Bourne’s. In the first place, Bourne’s negation loses the equivalence between a proposition and its double negation ($P \equiv \sim\sim P$), while the following equivalences hold: $\sim P \equiv \sim\sim\sim P$, $\sim\sim P \equiv \sim\sim\sim\sim P$, and so on. Besides, by this failure of equivalence concerning double negation, he also loses, as he admits[1, p.127], de Morgan duality ($P \& Q \equiv \sim(\sim P \vee \sim Q)$) and ($P \vee Q \equiv \sim(\sim P \& \sim Q)$), though the following equivalences hold: $\sim(P \& Q) \equiv \sim P \vee \sim Q$ and $\sim(P \vee Q) \equiv (\sim P \& \sim Q)$.

These failures of equivalence also reduce the persuasiveness of his justification(3), since it is doubtful whether it has any significance to retain the excluded middle and the law of contradiction in spite of losing these other basic classical laws.

In terms of (2), some may doubt whether we can really say that ‘given that p is indeterminate, then it isn’t the case that p’. According to some interpretation of future tensed propositions, even if the raining tomorrow is indeterminate now, it can be the case that it will rain tomorrow. Setting this doubt aside here, however, let’s consider the meaning of ‘p is indeterminate’. It is strongly arguable that we can paraphrase it into ‘p is neither positively nor negatively determinate’(cf. [8, p.165]). If so, we can again paraphrase it into ‘it is neither determined that p is the case nor p is not the case’. Therefore, we should say that ‘given that p is indeterminate, then *neither* it is the case that p *nor* it isn’t the case that p’ rather than saying that ‘given that p is indeterminate, then it isn’t the case that p’. It also follows that we cannot say that ‘so to say that it is not the case that p is clearly to say something true’; we should rather say that ‘so *neither* to say that it is the case that p *nor* to say that it is not the case that p is to say something true’, which is the very reason that p should be given the value ‘indeterminate’.

His last justification (4) is the following [1, p.126]¹; take the proposition

(a) Dr Foster will go to Gloucester

and the proposition

(b) Dr Foster will not go to Gloucester.

Though it may be thought that if one assigns the value 1/2 to (a), then Bourne's negation assigns the value 1 to (b) in spite of (b) being also indeterminate, he insists that it really does not, because these are to be analyzed as follows:

(a*) $F(\text{Dr Foster goes to Gloucester})$

(b°) $F\sim(\text{Dr Foster goes to Gloucester})$

Analyzed in this way, (b°) is not the negation of (a*) and so one can also assign the value 1/2 to (b°). According to Bourne, the correct analysis of the negation of (a) is the following:

(b*) $\sim F(\text{Dr Foster goes to Gloucester})$

He insists that one can assign the value 1 to this proposition, because to say that (b*) is true is not to say that (b°) is true and so 'even if it turns out that Dr Foster does go to Gloucester, we should still be happy to assign truth to (b*).'

He also believes that he can keep the excluded middle intact in this way, because (b°) taken as a whole is not the negation of (a*) and so the disjunction of (a*) and (b°) should not be taken as ' $p\vee\sim p$ ' but just as ' $p\vee q$ '. To the contrary, the disjunction of (a*) and (b*) can be legitimately taken as ' $p\vee\sim p$ ' and has the value 1 so that it does not break the law of excluded middle.

However, it is evident that he also thinks that the disjunction of their original propositions (a) and (b) is not true, because he interprets them as (a*) and (b°) and both have the value 'indeterminate'[1, p.126]. It follows that the disjunction is assigned the value 1/2 by the truth table that he adopts for disjunction (Table 4). But that is nothing but what Łukasiewicz wanted to show with his three-valued logic. As a result, Bourne has to agree with Łukasiewicz in thinking that the following proposition is not true but indeterminate.

(c) Either Dr Foster will go to Gloucester or Dr Foster will not go to Gloucester.

However, Bourne also says the following [1, p.123]:

For suppose I say,

¹Bourne's original numbering of propositions has been changed for the present paper.

(d) Either I will buy a Ducati or I will not buy a Ducati.

Because there is no middle ground to be had—either I will or I will not buy a Ducati—we must agree that (d) is determinately true.

It is obvious that Bourne contradicts himself, which suggests that the combination of his justification (2) and (4) includes some deficiency.

Bourne's confusion originates in the ambiguity of his future-tense operator ' F '. As I quoted above, he thinks that (b^*) can be assigned the value 1 even if it turns out that Dr Foster does go to school. Why? His justification is that saying that (b^*) is true is *not* saying that (b°) is true. Then what *is* saying that (b^*) is true? What is the truth condition of (b^*) (and (a^*))?

Adopting a model of branching future, he defines two concepts of truth in a way that can be summarized as follows[2, pp.52-61]:

' Fp ' is true now iff p is true on some future branches.

' Fp ' is determinately true now iff p is true on every future branch.

Though it is not clear which he means when he says that (b^*) is true, the future-tense propositions that have the value 1 should be those that are 'determinately true' in his sense, as contingent future-tense propositions are assigned the value 1/2. In this case, of course (b^*) can be assigned the value 1 even if it turns out that Dr Foster does go to Gloucester, so long as there are some (non-actualized) future branches on which he does not go to Gloucester. Furthermore, this interpretation makes both (a^*) and (b°) indeterminate while it makes ' $Fp \vee \sim Fp$ ' necessarily true, conforming to Bourne's explications above.

However, what ' $Fp \vee \sim Fp$ ' expresses is totally different from his intended meaning of the disjunction (d). Under the present interpretation, ' $Fp \vee \sim Fp$ ' just expresses an obvious truth that either there are no future branches on which p is not true or there are such branches. In other words, it just says that there is a present possibility of p being not actualized in the future or there is no such possibility. On the other hand, I believe that what Bourne could mean by saying 'Because there is no middle ground to be had—either I will or I will not buy a Ducati—we must agree that (d) is determinately true' is that either I will or will not buy a Ducati *on any future branch*. Though he may insist that it can be expressed by the determinate truth of ' $F(p \vee \sim p)$ ', in that case he has to give up his explications of (c) by its translation into the disjunction of (a^*) and (b°) . It follows that he has failed in retaining the excluded middle in a way he hopes in order to deal with the problems of future contingents. What he has done is just selecting a proposition that describes the determination of a future fact as an instance of one of the conjuncts of the law of excluded middle.

Another bad news for Bourne is that Lukasiewicz's system also includes the truth-functional operator ' L ' that makes it possible to express Bourne's negation together with Lukasiewicz's negation (Table 5):

Consequently, we can reconfirm that the difference between Bourne's system and Lukasiewicz's lies just in the way of expressing two kinds of negation.

P	LP	$\sim LP$
1	1	0
1/2	0	1
0	0	1

Table 5:

Bourne adopted his negation and the intensional operator ‘ F ’, while Łukasiewicz chose his negation and the truth-functional operator ‘ L ’. Given the considerations so far, Łukasiewicz’s way seems much better than Bourne’s at least in retaining extensionality and so giving a clearer and simpler semantics to his operator ‘ L ’. Besides, Łukasiewicz’s negation has some characteristics preferable to Bourne’s from logical points of view, as I described in brief at the beginning of this paper. I will elaborate on them in the next section.

2 Three-valued logic as Partial Two-valued Logic

Though I believe that Bourne has failed in improving Łukasiewicz’s three-valued logic for the reasons described above, at the same time I find some notable points in his attempt itself. In the first place, he agrees with Łukasiewicz in affirming that future contingents really matter in terms of bivalence, at least when he says that ‘given that p is indeterminate, then it isn’t the case that p ’. As it is the contraposition of ‘given that it is the case that p , p is determinate, he has almost accepted Łukasiewicz’s following statements that were crucial for his motivation to invent three-valued logic and that have been criticized by many philosophers[8, p.165]:

—the proposition ‘I shall be in Warsaw at noon on 21 December of next year’, can at the present time be neither true nor false. For *if it were true now, my future presence in Warsaw would have to be necessary*, — [italic by the present author]

Of course they differ as to whether we should take the future fact in question as ‘determinate’ or ‘necessary’ when it is true, but at least Bourne’s attempt has made it plausible that there is *some* reason to introduce non-bivalence in the context of future contingents. In other words, he has shown that we *can* use non-bivalent logic in a way that does relate to the problems of future contingents, though it may not necessarily follow that we *should*.

Secondly, I also have sympathy with his trial to make Łukasiewicz’s three-valued logic somehow compatible with classical logic. Though I don’t think his way of achieving it by retaining only some classical laws that seem more important than others was appropriate, I believe that Bourne’s failure gives us good lessons about *how* he should have done it, rather than *what* he should have done. What I have in mind as a better way of preserving classical laws

in spite of accepting non-bivalence is taking the non-bivalence in the context of future-contingents not as a kind of total three-valuedness but as a kind of partial two-valuedness. In other words, we should adopt only Truth and Falsity as the genuine truth values that are not inclusive and so admit that there are cases in which a proposition has no truth-value, namely, the cases in which it falls in the so called ‘truth-value gap’. We can also characterize this strategy as retaining bivalence in a weaker sense that there are only two truth-values but giving up inclusiveness by allowing a truth-value gap, though it still retains exclusiveness, *contra* paraconsistent logic.

If we adopt this point of view that is based on partial logic, we can recognize the crucial factors of Bourne’s failures more clearly. As shown above, the most fundamental source of his failures lies in his negation. Beside the problem of double negation which I described above, Bourne’s negation is more deviant from classical negation than Lukasiewicz’s in being non-monotonic, or ‘irregular’ in Kleene’s sense[6]. That is, his negation makes it possible that some true propositions become false and *vice versa* by filling some truth-value gaps in the propositions. This means that the third value 1/2 does not just show some lack or incompleteness but has some positive status that can make some propositions true or false by itself. In other words, the third value does not just represent the truth-value gap but another genuine truth-value paralleled with Truth and Falsity. That makes Bourne’s system a genuine three-valued logic rather than a partial two-valued logic at the basic level.

The fact that Bourne’s negation is definable using Lukasiewicz’s negation and necessity underpins this characterization, for Lukasiewicz’s necessity operator is also non-monotonic. It supports the view that Lukasiewicz’s way of expressing two kinds of negation is legitimate in that he takes his monotonic negation as basic and the other non-monotonic negation as the one defined using a non-monotonic operation together. To the contrary, Bourne took the latter as fundamental and tried to define the former using his intensional operation shown by ‘*F*’ together.

Prior called such operations as shown by Lukasiewicz’s necessity operator ‘modal functions’ and characterized them as the functions that never take the third value, quoting Jordan[9, pp.323-324]. Indeed Bourne’s negation and conditional (Table 6), the latter being not defined as ‘ $\sim(P\vee Q)$ ’ but as ‘ $\sim(P\&\sim Q)$ ’, also satisfy this criterion.

$P\rightarrow Q$	1	1/2	0
1	1	0	0
1/2	1	1	1
0	1	1	1

Table 6:

This fact also reveals the true reasons that Bourne could retain classical laws such as the laws of excluded middle, non-contradiction, and identity; what he

did was just introducing modal functions in Prior's sense into the context of partial functions, so as to reduce some part of partial functions to two-valued total functions in a way that the logical laws selected by him turn out to be two-valued tautologies. Since the other parts still remain partial, the equivalences such as double negation and de Morgan duality are gone.

Moreover, this evaluation of Bourne's attempt from the partial logical point of view gives us another explanation of the failure of his justification (2). One of the most important requirements given by partializing truth functions is that we always have to take both of Truth and Falsity, or both of affirmative and negative propositions, into consideration at the same time. When bivalence is retained, we need not always take Falsity into consideration as well as Truth and *vice versa*, because Falsity can be just defined as non-Truth and *vice versa*. To the contrary, in the contexts of partial truth-functions, just telling about Truth or Falsity is doomed to be incomplete descriptions, since non-Truth does not necessarily mean Falsity and *vice versa*. Bourne's failure in terms of his justification (2) was taking only the positive case of determination and so saying that 'given that p is indeterminate, then it isn't the case that p', when he should have said that 'given that p is indeterminate, then *neither* it is the case that p *nor* it isn't the case that p'. This is a typical case of mistreatment of partial logical contexts. He should have recognized that there are two types of 'being not the case that p'; the one is the case in which it is the result of p having no (genuine) truth-value, namely neither Truth nor Falsity, and the other is the case in which it is the result of p having the truth-value of Falsity.

3 Partializing Lukasiewicz's three-valued logic

Now let me show a detailed way I have in mind to retain somehow classical laws by reconstructing Lukasiewicz's three-valued logic as a system based on partial logic. In fact, Lukasiewicz's three-valued logic also includes a non-monotonic operator among his basic operators in addition to his necessity operator; he adopted the conditional as one of the primitives that was assigned the definitions as shown by Table 7, and defined conjunction and disjunction as follows:

$$P \vee Q \equiv_{df} (P \rightarrow Q) \rightarrow Q, \quad P \& Q \equiv_{df} \sim(\sim P \vee \sim Q)$$

$P \rightarrow Q$	1	1/2	0
1	1	1/2	0
1/2	1	1	1/2
0	1	1	1

Table 7:

As a result, his conjunction and disjunction agree with those in Kleene's strong three-valued logic, as well as Bourne's. Though they are monotonic, his conditional is non-monotonic.

Probably, the reason Lukasiewicz adopted his non-monotonic conditional was, as Urquhart conjectures[10, pp.72-73], that he wanted the law of identity ‘ $P \rightarrow P$ ’ to be a three-valued tautology and so gave it the value 1 when both of its antecedent and consequent has the value $1/2^2$. However, from a partial logical point of view, this was an inappropriate move, because it makes the conditional operator non-monotonic.

Moreover, partial logic does not require three-valued tautologies, since we can take the propositions that have the third value just as a result of some of its subformulae *lacking* a truth-value. So we can admit the propositions that never have the value Falsity as a kind of what Woodruff called ‘hedged tautologies’[11] that represent logical truths in partial logic. It is to be noted here that this does not mean that we take both of Truth and the third value as designated values that are opposed to Falsity. For we should also take the propositions that never have the value Truth, namely that have either the value Falsity or the third value as ‘hedged contradictions’. This is one of the cases we should obey the rule of taking Truth and Falsity equally. Since the third value is not a genuine truth-value, neither can it be a designated nor anti-designated value.

So let’s replace Lukasiewicz’s non-monotonic conditional with the one defined by Kleene’s strong disjunction (or conjunction) and negation, both of which are the same as Lukasiewicz’s, in the following way:

$$P \rightarrow Q \equiv_{df} \sim P \vee Q \text{ [or } \sim(P \& \sim Q) \text{]}$$

Then it turns out to be a monotonic conditional that is shown in Table 8:

$P \rightarrow Q$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	1	$1/2$	$1/2$
0	1	1	1

Table 8:

Under this interpretation, all and only the classical tautologies and contradictions are preserved as the hedged tautologies and contradictions respectively in partial logic. Moreover, all the classical equivalences are also retained³.

Thus, if we confine ourselves within the monotonic operators shown above, the resulting partial logic coincides with Kleene’s strong three-valued logic[6], so that it can be characterized as just a generalization of classical logic. By adding non-monotonic operators such as ‘ L ’, the system itself loses the qualification as partial logic. However, it retains partiality at a basic level and so we can take it as a kind of modal extension of partialized classical logic, since, as Prior suggested, its non-monotonic operators can be interpreted as expressions of a kind of alethic modality that is brought about by the partiality of

²So Bourne is wrong in saying that “ $P \supset P$ ” is true (*unlike Lukasiewicz and Bochvar’s full systems!*);[1, p.127, italic by the present author].

³As for other semantic definitions and theorems in partial logic, see my [4][5].

truth-functions⁴. (For example, the proposition ‘ $\sim Lp \& \sim L \sim p$ ’ tells that p is indeterminate, namely, that p has no (genuine) truth-value.) In that sense, we can call it ‘*extensional* alethic modality’, contrasted with the intensional alethic modality of modal logics, for the semantics of the present modality is truth-functional. In retrospect, that was what Łukasiewicz originally tried to characterize with the modal operators in his three-valued logic[8]⁵. Moreover, I believe that this extensional modality nicely represents the modality that is related to future contingents, since we can take it that the third value corresponds to the present lack of truth-makers of the propositions that tell about indeterminate future facts. Consequently, I believe that Łukasiewicz was well motivated in his attempt to cope with future contingents by the truth-functional modality in his three-valued logic and that our modified system based on partial logic inherits his spirit at a fundamental level.

References

- [1] BOURNE, C., ‘Future Contingents, Non-contradiction, and the Law of Excluded Middle Muddle’, *Analysis* 64:122–28, 2004.
- [2] BOURNE, C., *A Future for Presentism*, Cambridge University Press, Cambridge, 2006.
- [3] FONT, J. M., HÁJEK, P., ‘On Łukasiewicz’s Four-Valued Modal Logic’, *Studia Logica* 70:157–182, 2002.
- [4] KACHI, D., ‘Validity in Simple Partial Logic’, *The Annals of the Japan Association for Philosophy of Science* 10:139–153, 2002.
- [5] KACHI, D., ‘Tensed Ontology Based on Simple Partial Logic’, *Proceedings of Ninth International Symposium on Temporal Representation and Reasoning: TIME-02* :141–145, 2002.
- [6] KLEENE, S. C., *Introduction to Metamathematics*, North-Holland, Amsterdam, 1967.
- [7] ŁUKASIEWICZ, J., ‘On Three-valued Logic’, 1920, L. Borkowski (ed.), *Jan Łukasiewicz Selected Works*, 87-88, North-Holland, Amsterdam, 1970.
- [8] ŁUKASIEWICZ, J., ‘Philosophical Remarks on Many-valued Systems of Propositional Logic’, 1930, L. Borkowski (ed.), *Jan Łukasiewicz Selected Works*, 153-178, North-Holland, Amsterdam, 1970.
- [9] PRIOR, A. N., ‘Three-valued Logic and Future Contingents’, *Philosophical Quarterly* 3:317–326, 1953.

⁴Woodruff expressed similar points more explicitly than Łukasiewicz by calling the operator ‘ T ’, which has the same definition as Łukasiewicz’s ‘ L ’, ‘truth-operator’ in his System Q[11].

⁵Font and Hájek also stressed Łukasiewicz’s concern with modality in developing his many-valued logics[3].

- [10] URQUHART, A., 'Many-valued Logic', D. Gabbay and F. Guentner (eds.), *Handbook of Philosophical Logic, Vol.III*, 71-116, North-Holland, Amsterdam, 1970.
- [11] WOODRUFF, P. W., 'Logic and Truth-value Gaps', K. Lambert (ed.), *Philosophical Problems in Logic*, 121-142, D. Reidel, Dordrecht, 1970.

Daisuke Kachi
Faculty of Liberal Arts
Saitama University
255 Shimo-okubo Sakura-ku
Saitama-city, 338-8570, JAPAN
kachi@mail.saitama-u.ac.jp