Was Łukasiewicz Wrong?*

Three-valued Logic and Determinism

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Abstract

Łukasiewicz has often been criticized for his motive for inventing his three-valued logic, namely the avoidance of determinism. First of all, I want to show that almost all of the critcism along this line was wrong. Second I will indicate that he made mistakes, however, in constructing his system, because he had other motives at the same time. Finally I will propose some modification of his system and its interpretation which can attain his original purpose in some sense.

1 Łukasiewicz's motive for his three-valued logic

Łukasiewicz writes:

> ⋯the proposition "I shall be present in Warsaw at noon on 21 December of the next year", can at the present time be neither true nor false. For if it were true now, my future presence in Warsaw would have to be necessary, ⋯If it were false now, my future presence in Warsaw would have to be impossible⋯[1]

As this passage shows, it cannot be denied that Łukasiewicz had in mind the problem of determinism when he originated his three-valued logic. However, as Z. Jordan notes, his main motive was more general, in that he tried to reduce the logic of modality to extensional logic.[2] First he gave the following three theorems which he believed modal logic ought to have:

1. If it is not possible that p, then not-p. (CNMpNp)
2. If it is supposed that not-p, then it is(on this supposition) not possible that p. (CNpCNpNMp,which is equivalent with CpCpLp.)
3. For some p: it is possible that p and it is possible that not-p. (Σ pKMpMNp)

Then he proved that they were incompatible on the principle of bivalence and that we could not define any modal operator satisfying the requirements of those three theorems, if we used two-valued truth tables.[3] *This* was the reason he abandoned the principle of bivalence.

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[1](Łukasiewicz, 1930)p.165.

[2](Jordan, 1945)p.389.

[3](Łukasiewicz, 1930) pp.161-164. He retracted the second theorem later. (Lukasiewicz, 1953) p.353, (Łukasiewicz, 1954) p.396f.

It was not until this step that he came into contact with determinism. It was natural for Aristotle's sea fight argument to occur to him next, because it had been interpreted as the place where Aristotle had denied bivalence. Moreover, it pertained to the matter of modality, that is, the possibility of free acts and the contingency of future events. Łukasiewicz may well have found there the source of intuitive interpretation of modality for his non-bivalent extensional modal logic.

Considering these circumstances, almost all the attacks made against Łukasiewicz prove to be wrong. The leading type of criticism to him was the one depending on truth theory. There are two versions of this type, though they are often mixtured; the one finds its ground in the redunduncy of truth, while the other in the timelessness of truth.

Kneale and Kneale write:

> By introducing the phrase 'it is true that' we make no assumption about determinism which is not made by use of the simple sentence in the future tense.[4]

According to R. D. Bradley, logical determinism results from misunderstanding of the expression 'always true(false)'; logical determinists don't understand that it really means 'timelessly true', that is, truth does not allow any temporal specification.[5]

I doubt if 'use of the simple sentence in the future tense' 'makes no assumption about determinism'. I also wonder whether 'genuine' timelessness, which excludes any relation with time, is really compatible with the expression 'always true(false)'. Moreover, setting these problems aside, both arguments have more fundamental defects; they are common in denying that bivalence has any relation with determinism. But this way of criticism is off the point, because it does not succeed in establishing that we *can* not employ bivalent logic in a way that does relate to determinism. What Łukasiewicz sought was the logic of modality and it was his point that determinism would follow, *if we used bivalent logic as the logic of modality.*

In other words, his argument can be construed as a 'proposal' of a concept of truth that does relate to determinism or modality. This is also confirmed by the fact that he used his concept of truth in his *definition* of determinism.[6] His three-valued logic was a logic that did not lead to determinism despite such a concept of truth. Ironically speaking, it may be true that redundunt or timeless truth will not lead to determinism even if we use it with the principle of bivalence, which, however, is the very reason Łukasiewicz abandoned such a concept of truth.

Another type of criticism against Łukasiewicz is the one based on modal theory, which blames him for having made a modal mistake. This type also has two versions: the one which interprets his concept of necessity as alethic modality and the other as causal modality.

S. Haack insists that he made the modal mistake of deriving the consequence 'if p, then necessarily q(CpLq)' from the premise 'Necessarily, if p then q(LCpq)'. This shows that she interpreted his concept of modality as alethic

[4](Kneale and Kneale, 1962)p.51.
[5](Bradley, 1959)p.200.
[6](Łukasiewicz, 1946)p.113.

modality, since only in that case it is obvious that this inference is really a mistake. In other words, it may be a valid inference for some other modal concepts.[7]

R. D. Bradley admits that the truth of a future-tense proposition in some sense 'necessitates' a future event. But in his view it is a logical(or semantical) necessity, which Łukasiewicz mistook as causal necessity.[8]

Here again, however, we can assume that Łukasiewicz proposed a third concept of necessity which was neither alethic nor causal. If so, we should elucidate his concept of necessity. This task is what I will do in the last section along with the elaboration of his concept of truth, though I will do it in a way which should rather be called 'reconstructive' than 'interpretative'.

2 the problems of his three-valued logic

Before doing that, however, I also want to point out his mistakes. Although I believe that his motive for three-valued logic was well grounded, it does not necessarily mean that his system was really appropriate for achieving his purpose. In fact, the problem of how to interprete it intuitively has been repeatedly investigated. The principal ones are as follows:

1. In his system the law of excluded middle and that of contradiction do not hold, while the law of identity does.

2. His interpretation of implication makes it true whenever its both parts have the value 1/2(Undetermined).

3. We cannot understand intuitively his following definitions of disjunction and possibility:[9]

   Apq =df. CCpqq Mp =df. CNpp

In my view, all these problems result from 2: the valuation of 1(Truth) when both parts of implication have the value 1/2. Since he did not state explicitly the reason for that, we have to conjecture it.

Perhaps, as A. Urquhart notes, he needed three-valued tautologies, because he believed that his three-valued logic had a revolutionary significance of relativizing bivalent logic; if implication were valuated as 1/2 in the case above, not only the law of identity(Cpp) but every formula that does not include any modal operator fails to be a three-valued tautology.[10]

Or his concern with probability may have affected his choice.[11] He explains as follows:

> ⋯if values other than "0" and "1" are interpreted as "the possible", only two cases can reasonably be distinguished: either one assumes that there are no variations in degree of the possible and

[7](Haack, 1978)p.209. A. N. Prior also emphasized that it was because Łukasiewicz's modality was not alethic that he was able to define it truth-functionally.(Prior, 1953)p.324.

[8](Bradley, 1959)p.205.

[9]Of course, we can understand them via Łukasiewicz's truth tables. Here I mean that we cannot relate them with our ordinary usage of these words.

[10](Urquhart, 1986)p.72f. Moreover, His Fregean realism concerning truth-values may be one of the reasons which made him stick to the law of identity. (Łukasiewicz, 1921)p.89f.

[11]He once called probability by the name 'truth value'. (Łukasiewicz, 1913)p.17.

> consequently arrives at the three-valued system; or one assumes the opposite, in which case it would be most natural to suppose (as *in the theory of probabilities*) that there are many degrees of possiblity, which leads to the infinite-valued propositional calculus. I believe that the latter system is preferable to all others.[12](italic by the present author)

Of course, he clearly distinguished infinte-valued logic from the calculus of probabilities. However, his definitions of the logical operators of infinite-valued logic bears some traces of probability. He defined them as follows:

$$\mathrm{Cpq}{=}1 \text{ for } \mathrm{p} \leqq \mathrm{q}$$
$$\mathrm{Cpq}{=}1 - \mathrm{p}{+}\mathrm{q} \text{ for } \mathrm{p} > \mathrm{q}$$
$$\mathrm{Np}{=}1 - \mathrm{p}$$

And he interpreted the matrix of three-valued system as a special case in which truth values were restricted to three: 0, 1 and 1/2.[13]

These definitions make his definition of possibility a little more understandable. For these definitions make CNpp(which is equivalent with Mp) true iff $1 - \mathrm{p} \leqq \mathrm{p}$, that is, $\mathrm{p} \geqq 1/2$, which means that p is possible iff the truth-value of p is not less than that of not-p. However, this probability-like conception of possibility somehow goes against his view that we get three-valued logic when we assume that there are *no variations in degree of possibility*. In my view, he was right in distinguishing three-valued logic from infinite-valued logic and taking only this distinction as reasonable, but mistaken in going only halfway, in that he interpreted the former as a special case of the latter.

3 some modification of his three-valued logic

Anyway if he had restricted his concern exclusively to the problem of future contingency, he would not have had to define implication in his way. So if we define it as material implication instead, making use of his definitions of disjunction(or conjunction) and negation, we get Kleene's strong three-valued logic(or we may call it partial logic). If we have to add modal operators as Łukasiewicz defined, P. Woodruff's three-valued logic may be one of the most approximate systems.[14]

In fact, all those three problems of Łukasiewicz's three-valued logic dissolve by this modification; as for the problem 1(let alone the problems 2 and 3), the law of identity, that of excluded middle and that of contradiction become equivalent. Though none of them are three-valued tautologies, we can take them as 'hedged' tautologies, meanig that they never become false.[15]

If we modify Łukasiewicz's system in the way stated above, we can regard the matter of determination as one version of 'presupposition'. That is, as we can presuppose the existence of denoted things or sortal correctness for a proposition

[12](Łukasiewicz, 1920)p.173.

[13](Łukasiewicz, 1920)p.173, (Łukasiewicz, 1923)p.129.

[14](Woodruff, 1970)Woodruff interpreted the operator which corresponded to Łukasiewicz's necessity operator as truth operator and applied his system to the matter of presupposition.

[15]'Hedged tautolgy' is the terminology which Woodruff adopted. Besides the inference from LCpq to CpLq, which Haack took as a modal mistake, is valid according to Woodruff's definition of validity.

to have a truth value, so we can also presuppose the determination(positively or negatively) of the event asserted in a proposition for its having a truth-value.[16] In that case, the concept of truth becomes a partial and temporal one which depends on the circumstances of determination at the time of assertions.

This modification may come to find the significance of Łukasiewicz's three-valued logic in the *partialization* of bivalent logic rather than its relativization. I believe this is the true reason he was able to make a system of extensional modal logic and the true ground to distinguish his (modified) three-valued logic from his infinite-valued logic.

As for the concept of necessity, we can interpret it as something like 'determined positively'.[17] Indeed this concept has modal characters; it has the opposite which corresponds to impossibility', namely, 'determined negatively', and they form trichonomy together with their intermediate: 'undetermined', which corresponds to contingency. And these are really 'modalities' of events(or propositions); the same event can be determined to occur (have occured, be occurring) or not to occur or neither. This kind of necessity is also temporal in the same sense as above.

It cannot be denied that Łukasiewicz mainly had in mind causal determination as the source of temporal necesssity. But it is also true that he never explicitly *defined* his conept of temporal necessity as causal necessity. He gave causal determination only as an example of temporal necessity. And that, he clearly distinguished two arguments in favour of determinism: the one based on logical principle and the other based on the physical principle of causality.[18] Moreover he counted among the present facts not only the future facts causally determined now, but also the past facts whose effects somehow exist now. This shows that even his concept of causal necessity does not always mean the usual causal determination which is future-directed.[19]

If we adopt this kind of necessity, we can count our intentions and obligations etc. among the sources of temporal necessity. That suggests its closer relevance with the matter of free acts, since its applicability extends to not only physical but also our moral and social factors.

References

1. Bradley, R. D. :1959, 'Must the Future be What It is Going to be?', *Mind 68*, pp.193-208.

2. Haack, S. :1978, *Philosophy of Logics*, Cambridge, Cambridge University Press.

3. Jordan, Z. :1945, 'The Development of Mathematical Logic in Poland between the Two Wars'. In McCall, S. (Ed):1967, *Polish Logic 1920-1939*,

[16]This presupposition is by no means *ad hoc*. By this presupposition we get what is called Peircean conception of future tense. And thanks to it we can distinguish true future-tense statements from 'lucky guesses'. Ryle stressed the importance of this distinction.(Ryle, 1954)p.18f.

[17]The concept 'inevitable' may be more familiar. But this concept tends to be applied only to future events. Though 'indefeasible' is rid of this fault, it is rather an epistemological concept.

[18](Łukasiewicz, 1946)p.114.

[19](Łukasiewicz, 1946)p.112, p.127f.

Oxford, Oxford University Press.

4. Kneale, M. & Kneale, W. :1962, The Development of Logic, Oxford, Oxford University Press.

5. Łukasiewicz, J. :1913, 'Logical Foundations of Probability Theory'. In Borkowski L. (Ed.):1970, *Jan Łukasiewicz Selected Works* (contracted to [SW] below), Amsterdam, North-Holland Publishing Company.

6. Łukasiewicz, J. :1921, 'Two-Valued Logic'. In [SW].

7. Łukasiewicz, J. :1923, 'A Numerical Interpretation of the Theory of Propositions'. In [SW].

8. Łukasiewicz, J. :1930, 'Philosophocal Remarks on Many-valued Systems'. In [SW].

9. Łukasiewicz, J. :1946, 'On Deteminsm'. In [SW].

10. Łukasiewicz, J. :1953, 'A System of Modal Logic'. In [SW].

11. Łukasiewicz, J. :1954, 'Arithmetic and Modal Logic'. In [SW].

12. Prior, A. N. :1953, 'Three-Valued Logic and Future Contingents', Philosophical Quarerly 3, pp.317-326.

13. Ryle, G. :1954, 'It was to be'. In *Dilemmas*, Cambridge, Cambridge University Press.

14. Urquhart, A. :1986, 'Many-Valued Logic'. In Gabbay, D. & Guenthner, F. (Eds):1986, *Handbook of Philosophical Logic Vol.3*, Dordrecht, D. Reidel Publishing Company.

15. Woodruff, P. :1970.'Logic and Truth-Value Gaps'. In Lambert, K.(Ed):1970, *Philosophical Problems in Logic*, Dordrecht, D. Reidel Publishing Company.