The authors define the notion of directed linguistic graphs and linguistic-weighted graphs. Chapter two of the book discusses all types of linguistic graphs, linguistic dyads, linguistic triads, linguistic wheels, complete linguistic graphs, connected linguistic graphs, disconnected linguistic graphs,and linguistic components of the graphs. Further, we define the notion of linguistic subgraphs of a linguistic graph. The third chapter also introduces the notion of Linguistic Cognitive Maps (LCMs) analogous to Fuzzy Cognitive Maps (FCMs). We also propose Linguistic Relational Maps, which are analogous to Fuzzy Relational Maps. These models find applications in problems whose data is unsupervised, but the values are linguistic.

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Linguistic Graphs and their Applications

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Chapter Three

PREFACE

In this book, authors, systematically define the new notion of linguistic graphs associated with a linguistic set of a linguistic variable. We can also define the notion of directed linguistic graphs and linguistic-weighted graphs. Chapter two discusses all types of linguistic graphs, linguistic dyads, linguistic triads, linguistic wheels, complete linguistic graphs, linguistic connected graphs, disconnected linguistic graphs, linguistic components of the graphs and so on. Further, we define the notion of linguistic subgraphs of a linguistic graph.

However, like usual graphs, we will not be able to arbitrarily connect any two linguistic words of a linguistic set associated with a linguistic variable. They can be related or adjacent depending on the linguistic variable associated with the linguistic set. This is an exceptional feature of a linguistic graph.

Further, the direction in general of a linguistic-directed graph is special. For if the linguistic set corresponds to the linguistic variable height of a person and if we wish to represent it by a linguistic-directed graph, then it is mandatory that we can have a directed line from short to tall and never a directed line from the node tall to node short for in nature the height of a person never decreases from tall to short only a directed line can be drawn from short to tall. This is yet another striking feature of linguistic-directed graphs in general.

Finally, we discuss the linguistic adjacency matrices associated with the linguistic graphs. As the entries of the linguistic adjacency matrix must have linguistic notations, we use in the case of linguistic-undirected graphs the linguistic terms {e, ϕ}, where e denotes the linguistic edge exists, and ϕ denotes that there is no linguistic edge between two linguistic nodes.

{e, ϕ} ↔ {1, 0}. 1 is represented by e in the linguistic graph (that is, there is an edge) 0 is represented by ϕ, which is no edge.

Thus the entries of the linguistic adjacency matrices associated with the undirected linguistic graph will be only e and ϕ; in this case, the matrices will be symmetric. In the case of the linguistic-directed graph, the linguistic adjacency matrices will have entries from {e, ϕ} but will not be symmetric as in the case of an undirected linguistic graph. This is the only difference between directed linguistic graphs and undirected linguistic graphs. However, in the case of linguistic edge-weighted graphs, the linguistic adjacency matrices will have entries as linguistic terms together with the ϕ the empty edge. However, in the case of undirected weighted linguistic graphs, the linguistic adjacency matrices will be symmetric with entries from a linguistic set together with {ϕ}. However, in the case of directed linguistic edge-weighted graphs, the linguistic adjacency matrices will have linguistic entries from a linguistic set together with ϕ. The third chapter also introduces the notion of Linguistic Cognitive Maps (LCMs) analogous to Fuzzy Cognitive Maps (FCMs).

These Linguistic Cognitive Maps (LCMs) can be applied to problems whose data is unsupervised, but the values are linguistic. Illustrative examples to this effect are given in the last chapter of this book. In each chapter of this book, the authors suggest some problems for the reader to solve. Finally, this book has over 200 illustrative linguistic graphs.

So that the reader has a clear idea about these linguistic graphs when they try to apply them in LCMs models, these LCM models will be a boon to non-mathematicians at large.

We acknowledge Dr K.Kandasamy with gratitude for his continuous support.

W.B.VASANTHA KANDASAMY
K. ILANTHENRAL
FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

1.1 Introduction

In this chapter, we define the new notion of linguistic sets associated with a linguistic variable. We use only linguistic words in these linguistic sets.

We do not use the notion of linguistic sentences to be present in our linguistic words (or sets or terms). For linguistic sentences may combine two or more linguistic words, for we try to build structures like graphs etc. We do not, at this stage use the notion of linguistic sentences for the linguistic sets which we associate with any linguistic variable. So, with a very simple modification from Zadeh [26-7] definition of a linguistic variable, we define the linguistic variable as a variable whose associated values are words in an artificial or a natural language. We do not include as per the researcher Zadeh the linguistic variable as a variable whose values are sentences or words or a fuzzy membership.

We see height is a linguistic variable, age is a linguistic variable and so on. We wish to state if someone wants a rough estimate of the age of a person who is just in front of us or in shop or a park so on. One can easily use any linguistic words, young or very young or just young or old or very old or middle aged or so on.

No one is natural senses say 35 years and 2 months or 45 years or 10 years or so on. So, what first occurs in the mind of a person, be it illiterate or a literate to describe the age only in linguistic term or words. So undoubtedly, linguistic terms are more natural than numerical values.

Here we do not say numerical values should not be used or it is in any way less than linguistic values, what we wish to emphasize is in general linguistic values are in many places handy in comparison with numerical values.

So, an honest trial is made how the study of linguistic terms can influence the human thinking while studying or analyzing a problem. Or to be more precise, we try to build linguistic value as a possible analogue to numerical values whenever the use of them is explicit.

So, we build simple concepts on linguistic sets (terms) like ordering operations like min or max [24]. Using these, we further develop structural concepts like linguistic graphs and their application.

This chapter has four sections. Section one is introductory in nature. Section two describes the notion of linguistic variables as a linguistic set or words or terms. The ordering operations are defined on these linguistic sets in section three. The final section defines some operations on these linguistic sets.

1.2 Linguistic Variables as Words or Terms

Here for the first time, we describe linguistic variables only by words, develop and define them.

We illustrate linguistic variables in terms of linguistic words by some examples.

Example 1.2.1. Let us consider the linguistic variable “speed of a vehicle on road”. Let S be the linguistic set associated with this variable,

S = {very fast, fast, just fast, fastest, medium speed, slow, just medium, very slow, just slow, ϕ} here the term ϕ means the empty linguistic word/term corresponding to zero 0 of the number system, that is no words.

Here if we take any two linguistic terms from S we can compare them like slow and fast where

“slow is a lesser speed than fast”;

slow ≤ fast

medium and just slow

just slow ≤ medium and so on.

Example 1.2.2. Let us consider the linguistic variable “colour” and not shades of a particular colour.

Let S be the linguistic set associated with the linguistic variable colour.

S = {black, blue, green, white, brown, red, orange}.

We cannot compare any of the two colours.

Comparison of colours is possible only if shades of a colour say like dark blue, blue, light blue, white shaded blue and so on.

Next, we give yet another example.

Example 1.2.3. Let us consider the linguistic variable height of persons. The linguistic set or linguistic terms associated with height is as follows.

S = {tall, very tall, very very tall, just tall, short, shortest, just short, very short, medium, just medium}.

We study the linguistic set S later and define operations on them.

Example 1.2.4. Let us consider the linguistic variable overgrowth (height, colour of leaves and so on) of paddy plants. Let S be the linguistic set associated with it.

S = {good, average, very good, bad, not up to the mark, very bad, just bad, just good and so on}.

Example 1.2.5. Let us consider the linguistic variable temperature of water while heating from ice state. The linguistic terms S associated with this variable is as follows.

S = {high, very low, highest, lowest, ϕ (corresponds to 0 degree), low, just low, and so on}.

Now we can also have linguistic variables in social problems like school dropouts, unemployment as a societal problem, symptom disease model, etc.

Example 1.2.6. The social problem of unemployment among the educated youth is the linguistic variable. The linguistic terms associated with it are

S = {educated criminals, frustration, taking up to drugs, taking to violence, distress, mental stress, emptiness of life and so on}.

Example 1.2.7. Let us consider the linguistic variable school dropouts in children. Let S be the linguistic term,

S = {bonded labourers, runaways from home, family problems (parents in fault) rag pickers, bonded labourers}. This social problem can be given linguistic representation.

Example 1.2.8. Now we deal with the diagnostic problem of disease symptom model for lung infection, which is taken as the linguistic variable.

Let S be the set of linguistic terms.

S = {l_1 - temperature in the evening

l_2 - cold and fever

l_3 - only cough

l_4 - cough and fever

l_5 - breathlessness

l_6 - reduction of weight

l_7 - pain in the chest

l_8 - cold and cough with phlegm

l_9 - tiredness}

describing the linguistic variable "lung infection".

1.3 Ordering or Partial Ordering on Linguistic Sets / Terms

In this section, we introduce the notion of partial or total order on linguistic sets [23, 24]. It is important to note that linguistic terms or sets are of 3 types.

i) Not partially or totally orderable

ii) Only partially orderable

iii) Totally orderable

We will define and describe the notion of all the 3 concepts.

When we say not orderable, we cannot compare even any two of them. For take the linguistic set, S = {Black, Brown, Green, Blue, Red, White, Orange, Pink, Violet}

related with the linguistic variable colours and not shades of colour.

So, this set S is not orderable for we do not have even a single pair of distinct elements which are comparable.

So, working with not orderable linguistic sets is a problem in such cases one can seek fuzzy membership techniques or making a power set using that S so that the linguistic power set P(S) of S becomes a partially orderable set under the relation of containment relation.

Now we discuss about partially ordered linguistic sets.

Suppose we have a linguistic variable colours and shades of colours of the persons / people all over the world. The corresponding linguistic set or terms

S = {white, black, dull black, dark black, brown, light brown, dark brown, very dark brown, very light brown, yellow, wheatish yellow and so on}.

Now we see S is a partially orderable set and not a totally orderable set.

We see white and black cannot be ordered under shades of colour whereas dull black, black and dark black can be ordered as dull black is some shades less than black so we can order as

dull black ≤ black

Similarly black is somewhat some shades less than dark black so that we can order it as

black ≤ dark black

Now we see dull black is many shades less than dark black so dull black ≤ dark black.

In fact, we have

dull black ≤ black ≤ dark black

Thus, we see in this linguistic set S some of the linguistic terms are not comparable and some of them are comparable we call this linguistic set as a partially ordered set.

We will give the definition in a technical way.

Definition 1.3.1. *Let S be a linguistic set (set of linguistic terms) associated with some linguistic variable. We say this linguistic set S is partially orderable if there exist at least two distinct linguistic terms in S which are comparable, and we generally denote that comparable relation by '≤'. We call (S, ≤) as a partially ordered linguistic set.*

We give one more example of a linguistic set which is only a partially orderable linguistic set.

Example 1.3.2. Let S be a linguistic set associated with some linguistic variable which is not orderable. Let P(S) be the power set of S which we call as linguistic power set of S. We show P(S) is a partially orderable set under the order; inclusion of subsets of a set S.

Let S = $\{s_1, s_2, s_3, s_4\}$ be the linguistic set. Let P(S) be the linguistic power set of S;

P(S) = $\{\phi, \{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}, \{s_1, s_2\}, \{s_1, s_3\}, \{s_1, s_4\}, \{s_2, s_3\}, \{s_2, s_4\}, \{s_3, s_4\}, \{s_1, s_2, s_3\}, \{s_1, s_2, s_4\}, \{s_1, s_3, s_4\}, \{s_2, s_3, s_4\}, S = \{s_1, s_2, s_3, s_4\}\}$.

We see $\{s_1, s_3, s_4\} \subseteq S = \{s_1, s_2, s_3, s_4\}$, $\{s_1\} \subseteq \{s_1, s_3\}$, $\{s_1, s_3\} \subseteq \{s_1, s_3, s_4\}$.

So $\{s_1\} \subseteq \{s_1, s_3\} \subseteq \{s_1, s_3, s_4\} \subseteq \{s_1, s_3, s_2, s_4\}$.

However, $\{s_1, s_2\}$ and $\{s, s_3\}$ cannot be ordered under any containment relation.

$\{s_4, s_3\}$ cannot be ordered under the containment relation with $\{s_1, s_2, s_3\}$ and so on.

Thus $\{P(S), \subseteq\}$ is only a partially ordered linguistic set.

Now we provide an example of a totally ordered linguistic set S.

Example 1.3.3. Let

S = {tall, short, very very short, very tall, just tall, just short, very short, tallest shortest, just medium, medium}

be a linguistic set associated with the height of some persons.

Now when we visualize the height of a person in general.

We know in terms of numbers or numerical values height of a person in general varies from one foot to say a maximum of 8 feet. We denote (say) the person with height as one foot as the shortest and the person who measures 8 feet as the tallest and all other persons vary in that interval [1, 8] this can be categorically put as

{very very short, very short, short, just short, medium, tall, very tall, very very tall and so on}.

Since the linguistic set S associated with height and the numerical values of height are comparable or totally orderable, we can say any two terms in S are comparable under the '≤' less than or equal to relation.

In view of this we can order this S related with height as follows.

shortest ≤ very very short ≤ very short ≤ short ≤ just short ≤ just medium ≤ medium ≤ just tall ≤ tall ≤ very tall ≤ tallest I

We see every pair of elements in S is linguistically comparable. In fact we can form a chain of the form given in I. We call this set as the totally ordered linguistic set.

We define the totally ordered set as follows.

Definition 1.3.2. *S be a linguistic set associated with some linguistic variable. If every pair of elements in S is comparable, that is for any a, b ∈ S, a ≤ b (or b ≤ a) where '≤' means they are linguistically comparable then we define (S, ≤) to be a totally ordered set.*

In fact, every totally ordered set S has a chain associated with it which is unique.

Now having seen the concept of partial order, total order and not ordered we proceed on define some operations on linguistic sets in the following section.

1.4 min and max Operations on Linguistic Sets

In this section, we define the notion of two operations min and max on totally ordered set and linguistic power set of a linguistic set S. We will first illustrate this situation by some examples.

Example 1.4.1. S = {old, oldest, very old, just old, young, very young, youngest, just young, middle age, adult, just adult}

be the linguistic terms associated with the linguistic variable age.

We see S is totally ordered set the linguistic chain associated with S is as follows.

youngest ≤ just young ≤ young ≤ very young ≤ just adult ≤ adult ≤ middle age ≤ just old ≤ old very old ≤ oldest I

We define min and max on the linguistic set S.

Suppose we take {just adult, very old};

just adult ≤ very old.

min {just adult, very old} = just adult and max {just adult, very old} = very old.

So we can define min and max operations on S.

Thus, S is closed with respect to both operations min and max.

Now on a partially ordered linguistic set we cannot define min or max operations for min or max is defined on every pair of linguistic terms.

Now we define on the linguistic power set P(S) of a linguistic set S the notion of ∪ and ∩, where S is only a linguistic set which is not a totally ordered linguistic set.

Example 1.4.2. Let S = {fast, slow, medium speed, very slow} be a linguistic set associated with the linguistic variable speed of a car in a highway.

Let P(S) = {{ϕ}, {fast}, {slow}, {medium speed}, {very slow}, {fast, slow}, {fast, medium speed}, {fast, very slow}, {slow, medium speed}, {slow, very slow}, {medium speed, very slow}, {fast, slow, medium speed}, {fast, slow, very slow}, {fast, medium speed, very slow}, {slow, very slow, medium speed}, S = {fast, slow, medium speed, very slow}}

be the linguistic power set of the linguistic set S.

Now P(S) is closed under both the operations ∩ and ∪ P(S) is only a partially ordered set under the set inclusion relation '⊆'.

{P(S), ∩} is a closed operation on the linguistic power set P(S). For take two subsets from

P(S) say

{fast, medium speed}, {fast, very slow, slow} ∈ P(S),

we see

{fast, medium speed} ∩ {fast very slow, slow} = {fast}.

We find {fast, medium speed} ∪ {fast, very slow, slow}

= {fast, slow, very slow, medium speed}

Consider ϕ and {slow, fast, very slow} ∈ P(S)

ϕ ∪ {slow, fast, very slow} = {slow, fast, very slow} and ϕ ∩ {slow, fast, very slow} = ϕ.

We see {P(S), ∩} and {P(S), ∪} are closed under the operations ∩ and ∪.

Now we define two other operations min and max on linguistic subsets of P(S).

Let

A = {slow, fast, very slow}

and

B = {medium, slow} ∈ P(S)

We find

min {A, B} = min{{slow, fast, very slow}, {medium, slow}} = {min{slow, medium}, min{slow, slow}, min{fast, medium}, min{fast, slow}, min{very slow, medium}, min{very slow, slow}} = {slow, medium, very slow} I

Now we find

max{A, B} = max{{slow, medium}, {slow, fast, very slow}}

= {max{slow, slow}, max{slow, fast}

max{slow, very slow}, max{medium, slow}, max{medium, fast}, max{medium, very slow}}

= {slow, fast, medium} II

Clearly, I and II are distinct they are very different operations on P(S).

In fact, ∩, ∪ min and max are 4 distinct different operation on P(S).

Having defined operations on P(S) and S we now proceed on to suggest some problems for the reader in the following section.

1.5. Suggested Problems

In this section, we suggest a few problems on linguistic sets associated with the linguistic variable, total order or partial order on linguistic sets and finally, four operation min, max, $\cup$ and $\cap$ on linguistic sets.

1.5.1 Find the linguistic set S associated with the linguistic variable 'weight of people'.

1.5.2 For the linguistic variable prediction of rain for a week, give the associated linguistic set S.

1.5.3 For the linguistic variable colour of eyes of people describe the associated linguistic set S.

i) Is S a totally ordered set or a partially order set or neither?

1.5.4 Let S be a linguistic set associated with the linguistic variable growth of plants in a ragi field.

i) Is S a totally ordered linguistic or a partially ordered linguistic set?

Justify your claim.

1.5.5 Let the symptoms suffered by a patience be the linguistic variable. The linguistic set S associated with the patience problems are as follows.

S = (a_1) - fever

(a_2) - vomiting

(a_3) - chills on and off

(a_4) - sweating

(a_5) - no appetite

(a_6) - pain on the right side of abdomen

i) Prove S is not totally orderable.

ii) Is S a partially orderable set?

iii) Find the linguistic power set P(S) of S and define the two operations $\cup$ and $\cap$.

iv) Is it possible to define min or max on P(S).

1.5.6 Let S = {good, bad, fair, very bad, just bad, just good, best, very good} be the linguistic set related with the linguistic variable performance of an employee in a factory.

i) Is S a totally ordered set?

ii) Is P(S) the linguistic power set of S a partially ordered set or totally ordered set?

1.5.7 Let A = {good, bad, very good, just bad}.

B = {good best, just good, fair, very bad} $\in$ P(S)

Find (a) $A \cap B$

(b) $A \cup B$

(c) min {A, B} and

(d) max{A, B}

Are they distinct or some of them are identical?

i) Does S have a linguistic chain of length 8?

ii) How many linguistic chains of length 7 exist in P(S)?

iii) What is the length of the largest linguistic chain in P(S)?

iv) Does the sub set P = {{ϕ}, {best}, {good, best}, {bad, best, good}, {best, good, bad, just bad}, {best, good, bad, just bad, very bad}, {best, good, bad, just bad, very good, very bad} $\subseteq$ P(S)

form a linguistic chain?

Obtain the chain formed by the subset P $\subseteq$ P(S).

1.5.8 Let yield of a certain crop consider maize be the linguistic variable

i) Form the linguistic set S formed by it

ii) Is it a totally ordered set?

iii) Is it a partially ordered set?

iv) Is S a finite or an infinite linguistic set?

v) Can we say forming the linguistic set S is a flexible one depending on the expert? Justify or substantiate your claim.

1.5.9 Let S be the linguistic set associated with the linguistic variable intelligence of students in a classroom.

i) Is S a totally ordered set?

ii) Is S a partially ordered set?

iii) Find the linguistic power set P(S) of the set S and prove P(S) is closed under the four operations, $\cap$, $\cup$, min and max.

iv) Prove all the four operations mentioned in (iii) are distinct.

v) Obtain any other special features associated with this problem.

1.5.10 Let S be the linguistic set associated with the linguistic variable all shades of the three colours yellow blue and red.

i) Is S a finite or infinite set?

ii) Prove S is not a totally orderable set.

iii) Prove S is a partially ordered set.

iv) Find P(S) the power set of S and prove {P(S), min} and {P(S), max} are not closed operations on P(S)

v) Prove {P(S), $\cup$} and {P(S), $\cap$} are closed operation on P(S).

1.5.11 Give one example of a linguistic variable other than colour whose associated linguistic set is not orderable.

1.5.12 Can we say all time dependent linguistic variables are always totally orderable? Justify your claim.

1.5.13 Can be say time independent linguistic variables have linguistic sets which are not partially orderable?

1.5.14 Can we say the linguistic variable the intensity of rain in the rainy season will give a linguistic set which is a totally ordered set?

1.5.15 Can we say the emotions of a person as a linguistic variable yields a linguistic set which is a totally ordered set?

1.5.16 List out all emotions related with a novel and order the related linguistic set S. Will S be of infinite cardinality or be only of finite cardinality.

1.5.17 Give some linguistic set which is associated with emotions of a person of finite cardinality.

1.5.18 Can we order the linguistic term angry and happy by some order? Justify or substantiate your claim.

1.5.19 Can the feelings depressed and unhappy be ordered? Justify your claim.

1.5.20 Suppose we study the growth of a plant for a period of one month.

i) Can we say the growth can always be an increasing order of height? Justify it.

ii) Do a practical study of this problem?

1.5.21 Find out a method by which one can tackle the linguistic sets which are not ordered.

1.5.22 Spell out some very special features enjoyed by the linguistic set.

1.5.23 Will linguistic set concept be easily understood by a illiterate person than the numerical values?

Substantiate your claim!

1.5.24 Can this study make a non-mathematician more confident than operations with number system?

1.5.25 Which do you think is in the easy grasp of school children under 10 linguistic set approach or number theoretic approach? Justify your claim.

1.5.26 Can we say operations like min and max on linguistic sets which are totally orderable easier than sum and product of numbers in the age groups less than 7 years?

1.5.27 Can we say this linguistic approach will make them like mathematics than numerical approach?

1.5.28 Will this linguistic approach build more confidence in school children?

1.5.29 Will linguistic approach make a researcher in fields like sociology etc. (that is non mathematicians at large) a easy way to apply them in real world problem?

1.5.301 Obtain any other advantages of linguistic sets in the place of number systems.

1.5.31 List out the disadvantages and limitations of linguistic set approach.

Chapter Two

BASIC LINGUISTIC GRAPHS

2.1 Introduction

In this chapter we for the first time introduce basic concepts about linguistic graphs. These linguistic graphs are different from linguistic line, linguistic planes and curves built using linguistic terms. Basically, linguistic graphs like classical graphs will be associated with a set of linguistic vertices and relations connecting linguistic vertices. When we say linguistic nodes / vertices they are nothing but elements from the linguistic set of words or which we call as linguistic terms associated with a linguistic variable L.

These basic notions about linguistic variables and the associated linguistic terms and the notion of ordering etc, etc; has been introduced in chapter I of this book to make this book a self contained one.

Before we proceed onto define the concept of linguistic graphs we proceed onto describe them by some nice examples.

We will describe three types of linguistic graphs in this chapter. The chapter has five sections. Section 1 is introductory in nature. Section 2 introduces linguistic graphs which are not directed and the edges are arbitrary. Section 3 introduces the notion of directed linguistic graphs. The notion of linguistic weighted linguistic graphs are introduced in section four. The final section supplies some problems which will make the student / researcher well versed with the concept of linguistic graphs.

2.2 Linguistic Graphs in General

In this section we introduce the notion of linguistic graphs using the linguistic set which is associated with the linguistic variable L.

We find the number of possible linguistic graphs given a finite linguistic set S.

We discuss various properties associated with linguistic graphs in general and in particular the linguistic subgraphs associated with them.

Example 2.2.1. Let

S = {good, best, bad, very good, very bad, fair}

be the linguistic set / term or word associated with the linguistic variable; performance of a student in the classroom.

We give them some possible linguistic graphs associated with S.

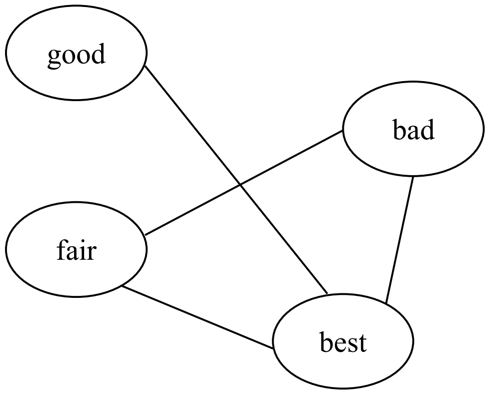


Figure 2.2.1

If we have a edge or a relation between any two linguistic terms / words we denote it by e. If there is no edge or a relation between two linguistic terms / words we denote it by ϕ meaning empty linguistic edge / relation.

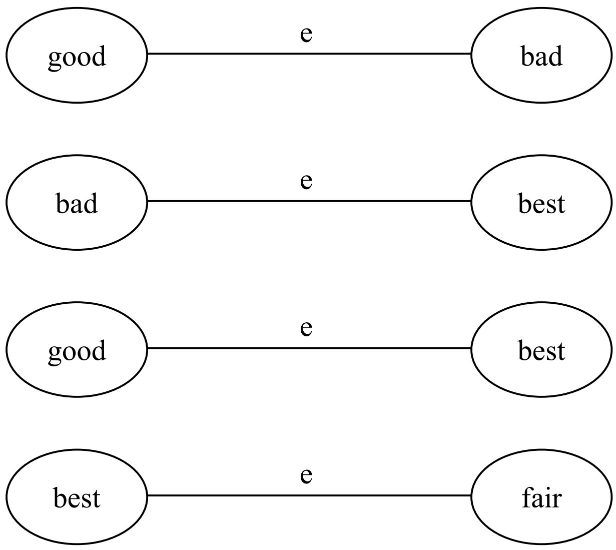


Figure 2.2.2

These are some of the linguistic graphs with two nodes. Usually, they will be called as linguistic dyads.

In fact using the set S we can have $6C_2 = 15$ such linguistic dyads. For from the linguistic set of order n we can in general have nC_2 number of linguistic dyads.

A few of the linguistic this dyads using set S given in example 2.2.1 is given in Figure 2.2.2.

Figures 2.2.3(a) to 2.2.3(d) represents empty linguistic graphs.

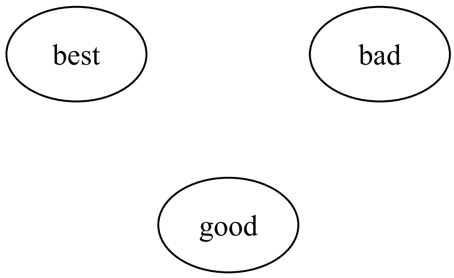


Figure 2.2.3(a)

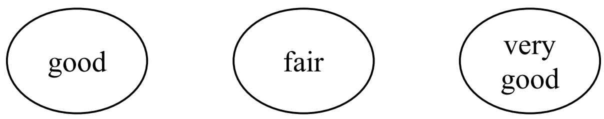


Figure 2.2.3(b)

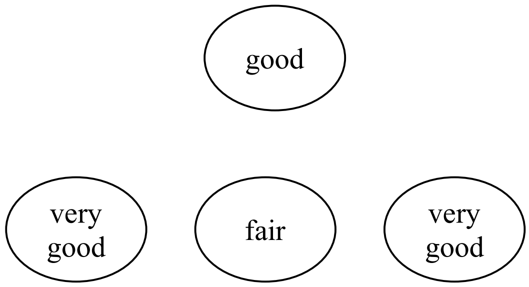


Figure 2.2.3(c)

Figure 2.2.3(d)

We do not include in these empty linguistic graphs or the null linguistic graphs with only linguistic node / term / word with node linguistic relation or edge.

There are 6 + $C6_2$ + $6C_3$ + $6C_4$ + $6C_5$ + 1 number of empty linguistic graphs with one linguistic node, two linguistic nodes, three linguistic nodes, four linguistic nodes, five linguistic nodes and six linguistic nodes respectively.

Next, we give examples of linguistic triads and disconnected linguistic graphs.

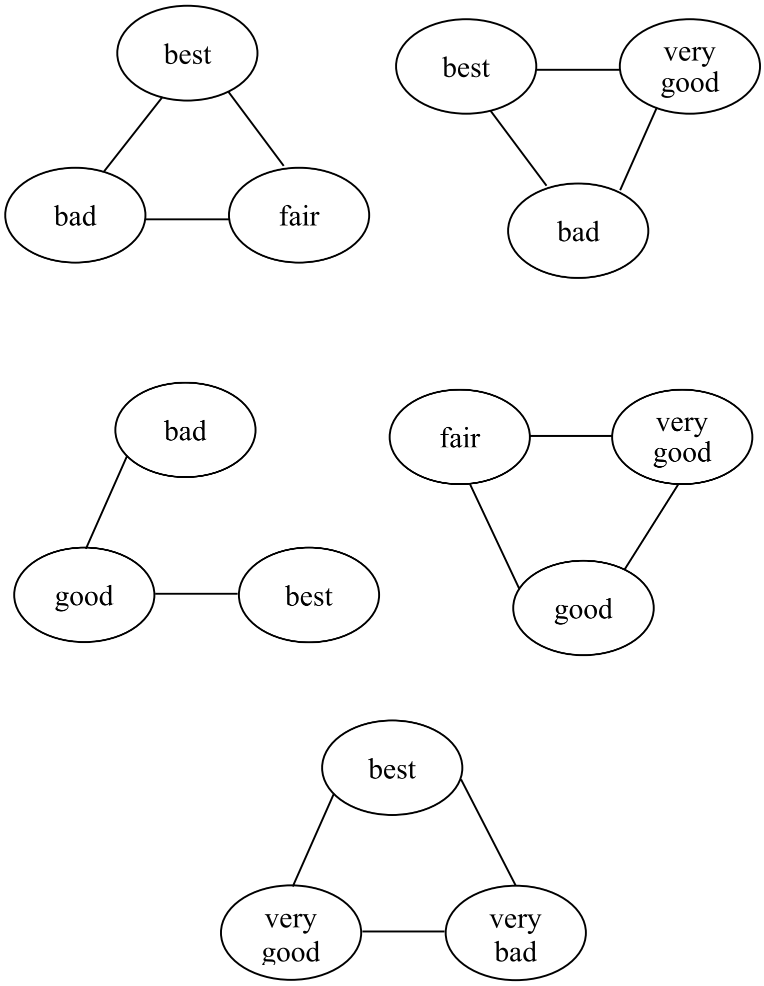


Figure 2.2.4

The Figures 2.2.4 gives us some of the linguistic graphs of order three with linguistic nodes from the linguistic set S.

Some of them are linguistic triads or linguistic complete graphs if all the three nodes are connected by linguistic edges /

relation. If one linguistic edge is left and there are only two linguistic edges we call them as linguistic incomplete triads.

There can be several such in complete linguistic triads however we have only $6C_3 = 20$ number of complete linguistic triads using the linguistic set S.

We say linguistic graphs as in case of usual graphs to be disconnected if we have at least one linguistic node which is not connected with any of the other linguistic nodes for the given set of nodes or in the linguistic graph.

Definition 2.2.1. *A linguistic graph G is disconnected or not connected, if we have at least one linguistic node which is not connected with any of the other linguistic nodes for the given set of nodes or in the linguistic graph.*

We will illustrate this situation by some examples using the linguistic set S.

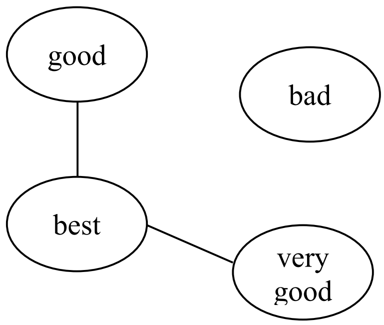


Figure 2.2.5(a)

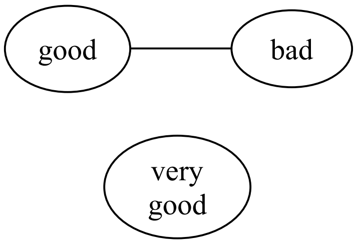


Figure 2.2.5 (b)

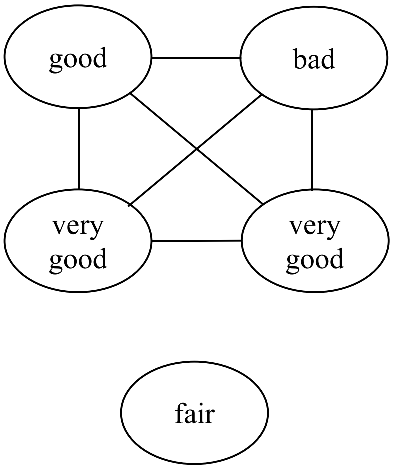


Figure 2.2.5 (c)

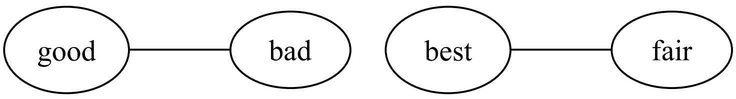


Figure 2.2.5 (d)

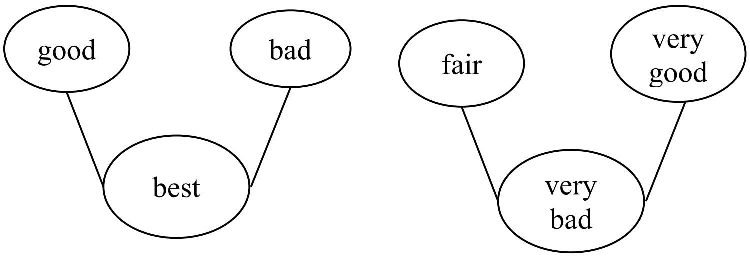


Figure 2.2.5 (e)

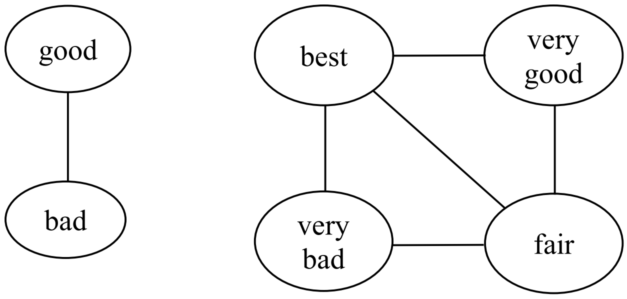


Figure 2.2.5 (f)

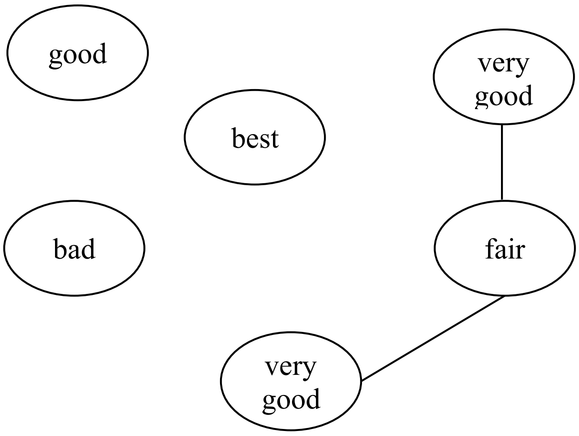


Figure 2.2.5 (g)

The Figures 2.2.5 (a) to 2.2.5 (g) give several linguistic graphs which are disconnected linguistic graphs using the linguistic set S.

Now we provide some illustrations of complete linguistic graphs.

Before we give examples, we provide the abstract definition of the complete linguistic graph.

Definition 2.2.2. *A linguistic graph G is said to be complete if every linguistic node in that linguistic graph is connected with every other linguistic node of that linguistic graph.*

We give some examples.

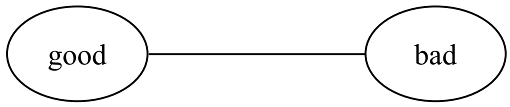


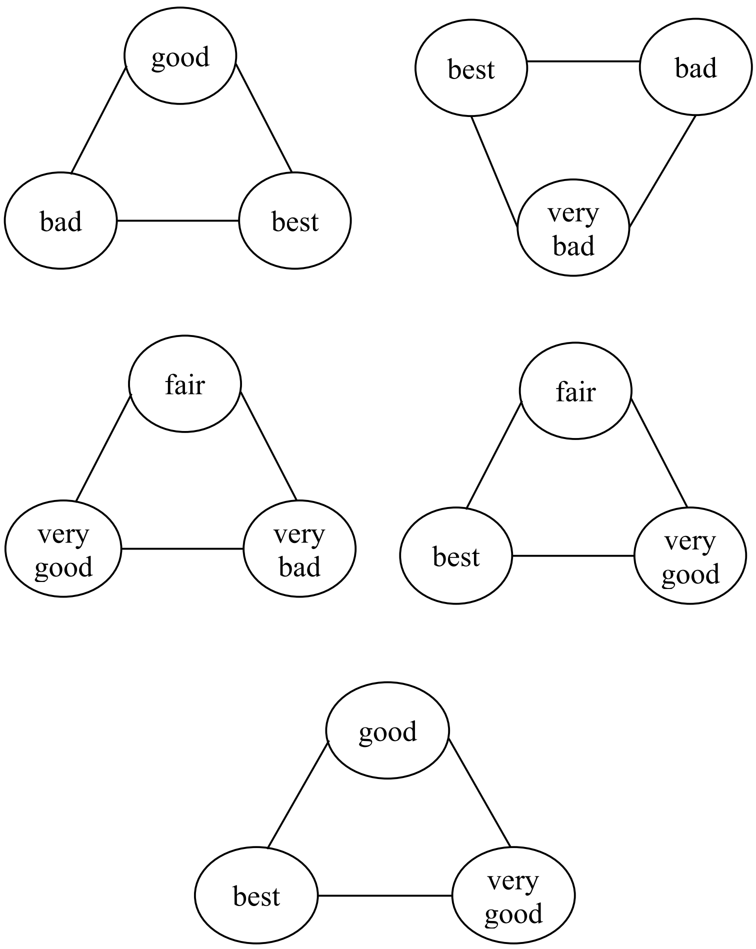
Figure 2.2.6

This a linguistic dyad which is a complete linguistic graph of order two or with only two nodes.

The dyads are foundation of any graph so are the linguistic dyads in case of linguistic graphs.

There are $6C_2$ = 15 such complete linguistic graphs of order 2.

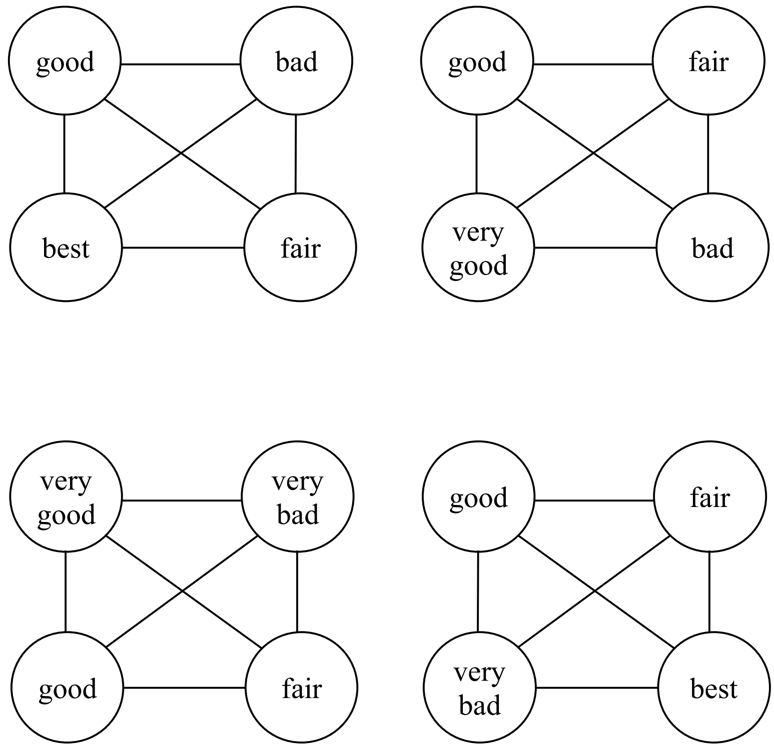
Consider the following linguistic graphs.



Figures 2.2.7

All these linguistic triads are complete linguistic graphs of order three. In fact, there are $6C_3$ = 20 such linguistic complete graphs of order three.

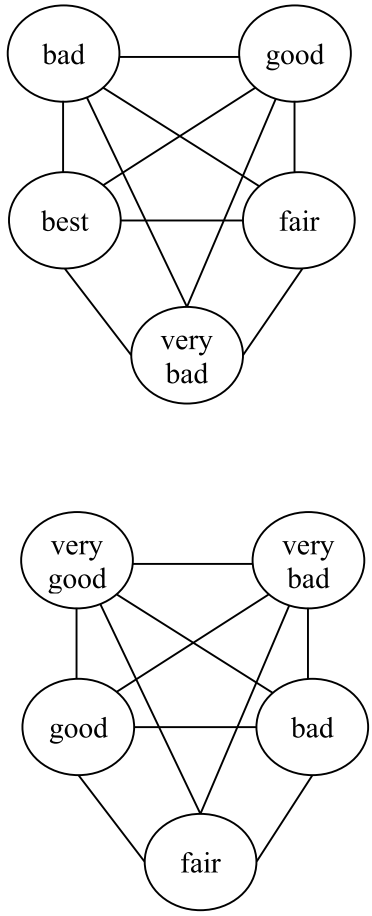
Now we provide examples of linguistic complete graphs of order four.



Figures 2.2.8

The linguistic graphs given in Figures 2.2.8 are complete linguistic graphs of order four. There are 15 linguistic complete graphs of order four.

Next, we proceed on to describe linguistic graphs of order five which is complete in the following.



Figures 2.2.9

Figures 2.2.9 gives the linguistic complete graphs with 5 linguistic nodes using the linguistic set S.

In fact, we have 6 such complete linguistic graphs with 5 linguistic nodes using the linguistic set S.

Now we see there is only one linguistic complete graph using all the six linguistic terms from S.

This is given in the following Figure 2.2.10.

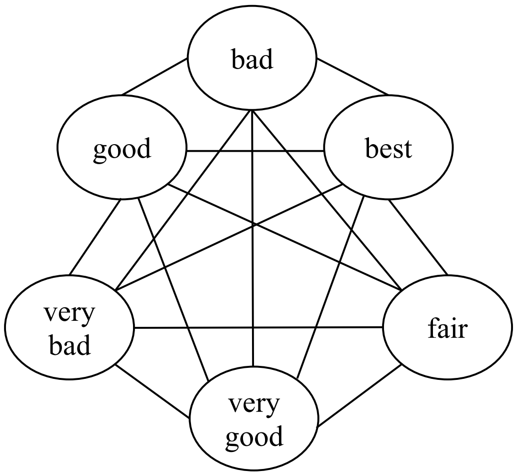


Figure 2.2.10

Now we proceed onto give examples of linguistic subgraphs of a graph.

We use the linguistic set S associated with the age of a person.

Example 2.2.2. Let

S = {old, young, very young, just young, youngest, very old, just old, middle age, just middle age, not that young, not that old, very very young, oldest} be the linguistic set / terms associated with the linguistic variable age.

We give some linguistic graphs using them and obtain their linguistic subgraphs in the following.

Let G be the linguistic graph with its linguistic nodes taken from the linguistic set S.

G =

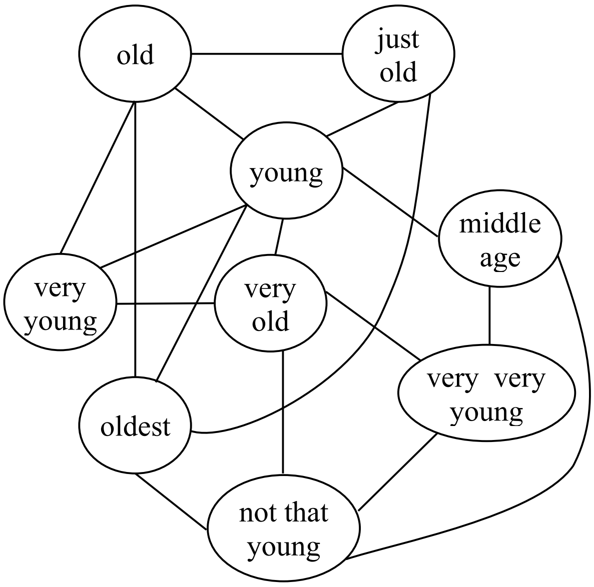


Figure 2.2.11

We first give linguistic subgraphs of G.

The notion of linguistic subgraph of G is just as in case of classical subgraphs of a graph G.

Consider

H_1 =

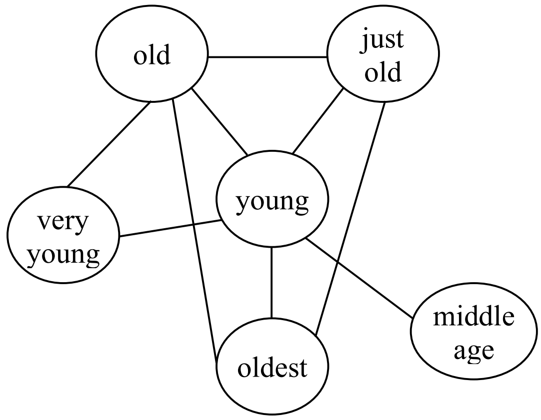


Figure 2.2.12

is a linguistic subgraph of the linguistic graph G.

It is noted for the subset of linguistic vertices taken by the linguistic vertices of G all the edges which are connected in G ought to remain connected in H_1 also.

Consider

H_2 =

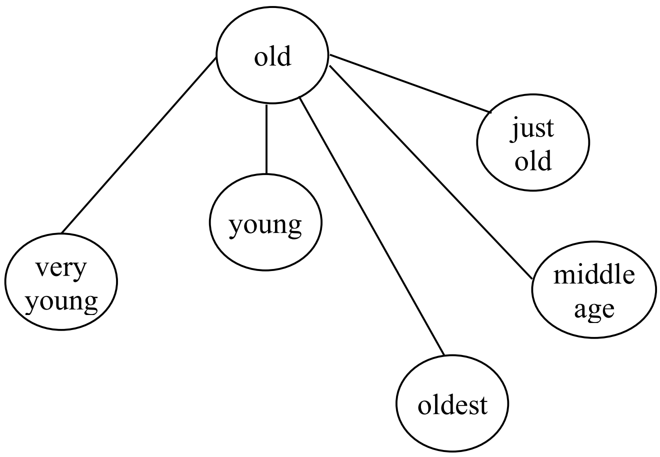


Figure 2.2.13

Clearly from Figure 2.2.13 is not a linguistic subgraph of G as the edges connecting

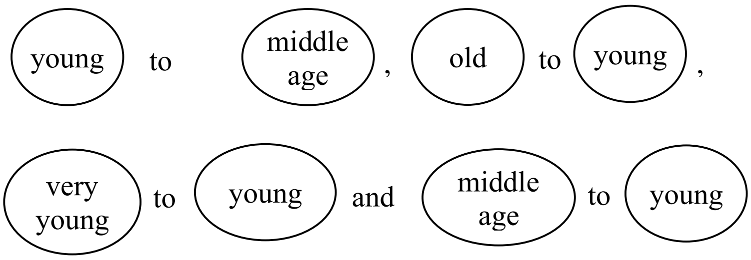


Figure 2.2.14

are missing; hence the claim.

Now consider the linguistic subgraph given by the following Figure.

$H_3 =$

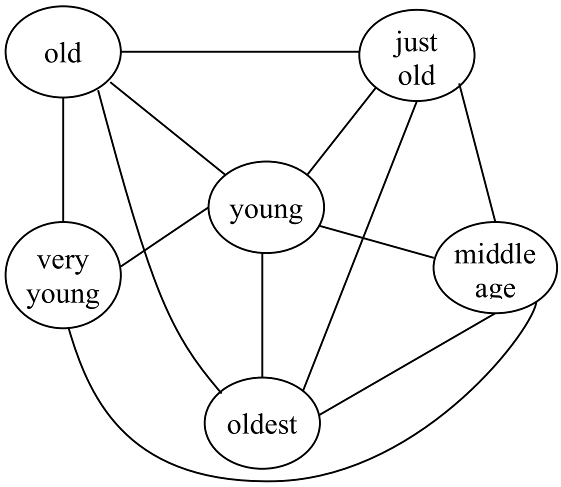


Figure 2.2.15

Consider the Figure 2.2.15, H_3 is not a linguistic subgraph of G as the edges from

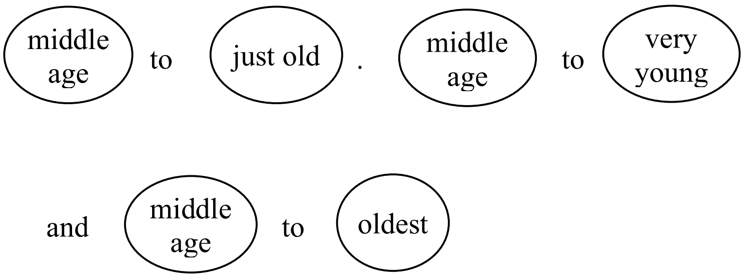


Figure 2.2.16

are not in the original linguistic graph G.

Thus we define a linguistic subgraph H to be a linguistic graph G if denoted by G = (V_L, E_L) where V_L denotes the linguistic nodes / vertices / terms from a linguistic set S and E_L denotes the linguistic relation (e or ϕ) from the linguistic vertices, then H = (W_L, F_L) where W_L is a proper subset of V_L ($W_L \subset V_L$) and F_L is also a proper subset of E_L ($F_L \subset E_L$).

Now we illustrate some more linguistic subgraphs of G.

Let

P_1 =

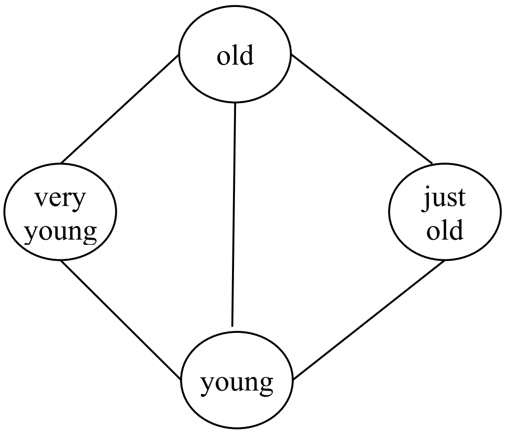


Figure 2.2.17

P_1 is a linguistic subgraph of G of order four.

P_2 =

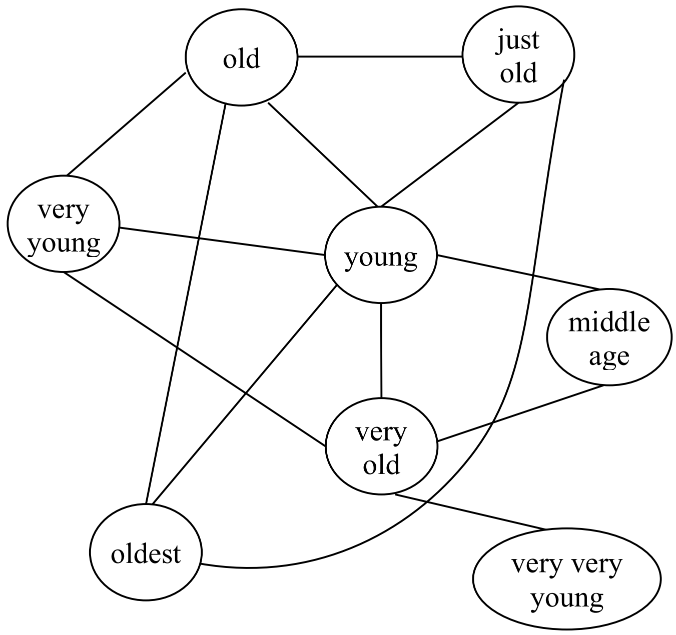


Figure 2.2.18

P_2 is a linguistic subgraph of G of order 8.

Consider P_3 the linguistic subgraph of G of order 5 given by the following Figure.

P_3 =

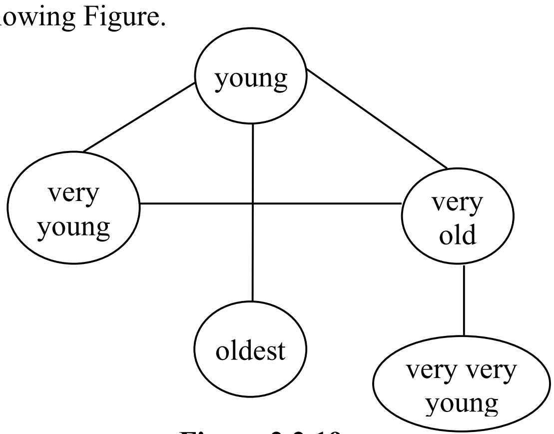


Figure 2.2.19

P_4 is a linguistic subgraph of G of order 3 which is not a complete triad given in the Figure 2.2.20.

P_4 =

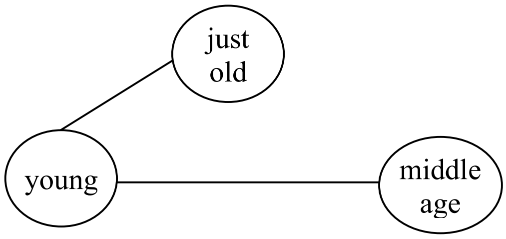


Figure 2.2.20

Let P_5 be a linguistic subgraph of G given by the following Figure.

P_5 =

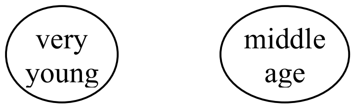


Figure 2.2.21

P_5 is just a disconnected linguistic subgraph of order two.

P_6 =

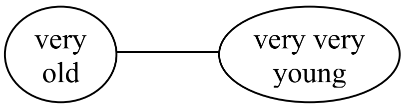


Figure 2.2.22

P_6 is a linguistic subgraph of G of order two which is a dyad.

Consider P_7 the following linguistic subgraph.

$P_7 =$

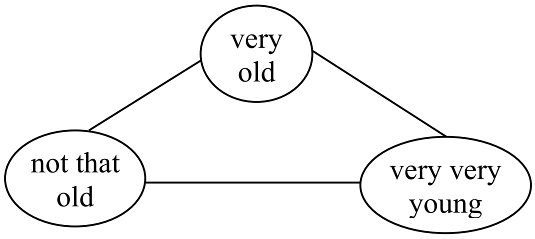


Figure 2.2.23

P_7 is a complete linguistic triad of the linguistic graph G.

$P_8 =$

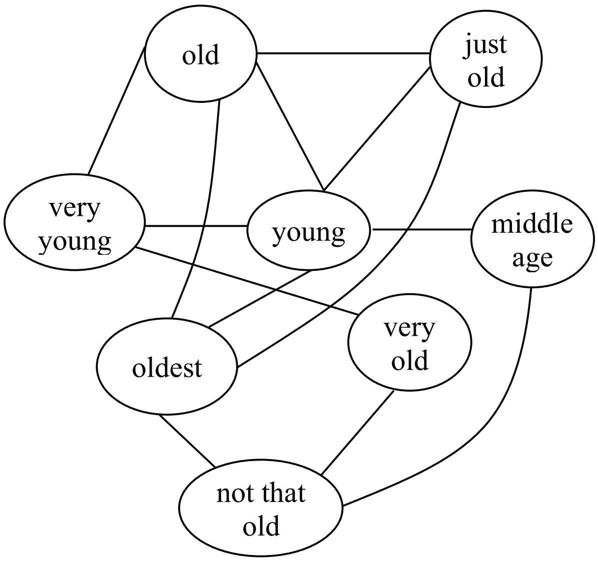


Figure 2.2.24

P_8 is a linguistic subgraph of the linguistic graph G of order 8 given in Figure 2.2.24.

P_9 =

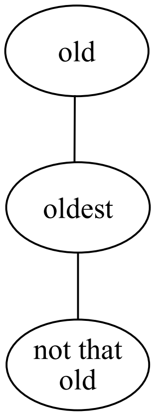


Figure 2.2.25

P_9 is a linguistic subgraph of G of order three in fact a linguistic line subgraph of G.

We give some more types of linguistic graphs using the same B.

Let

W =

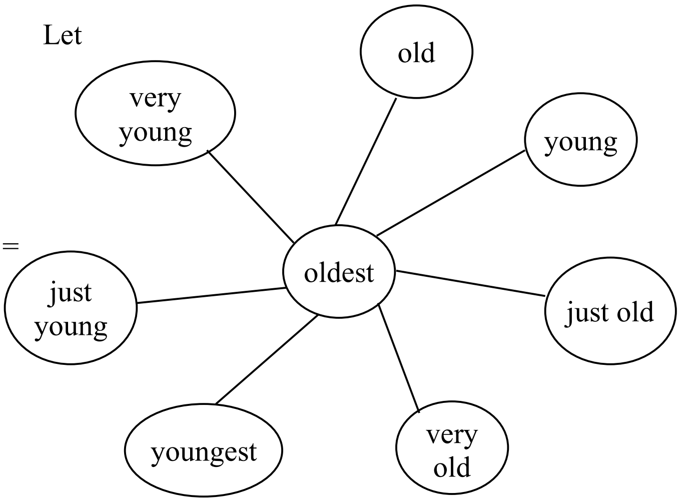


Figure 2.2.26

We call W as the linguistic star graph of order 8.

Once in the linguistic subgraph W the term oldest is removed we get the following linguistic subgraph V_1

V_1 =

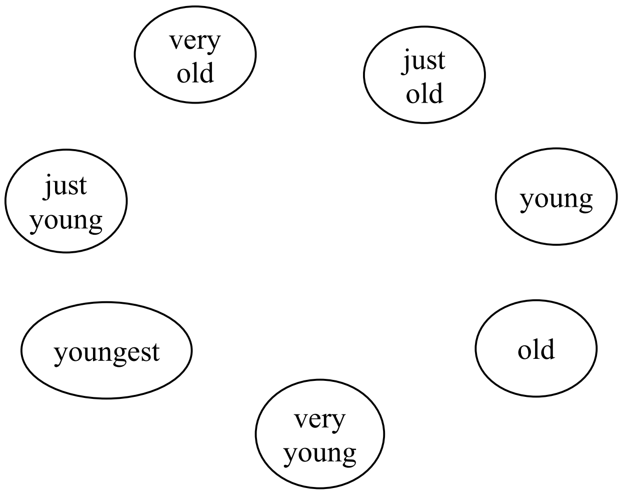


Figure 2.2.27

Consider the linguistic subgraph V_2 of W given by the following Figure 2.2.28.

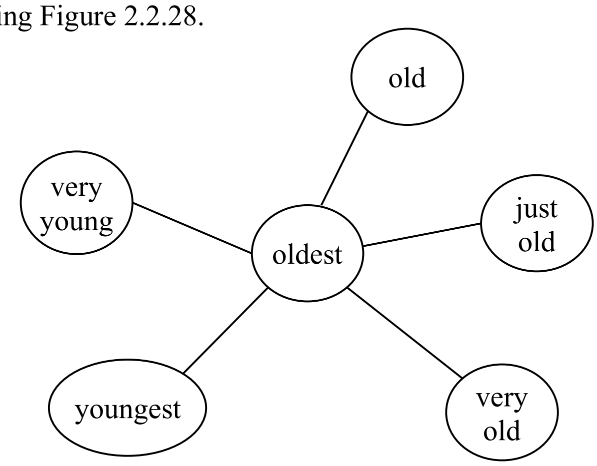


Figure 2.2.28

V_2 is also a star linguistic graph but its order is six.

Now consider the linguistic graph given by the following Figure 2.2.29.

X =

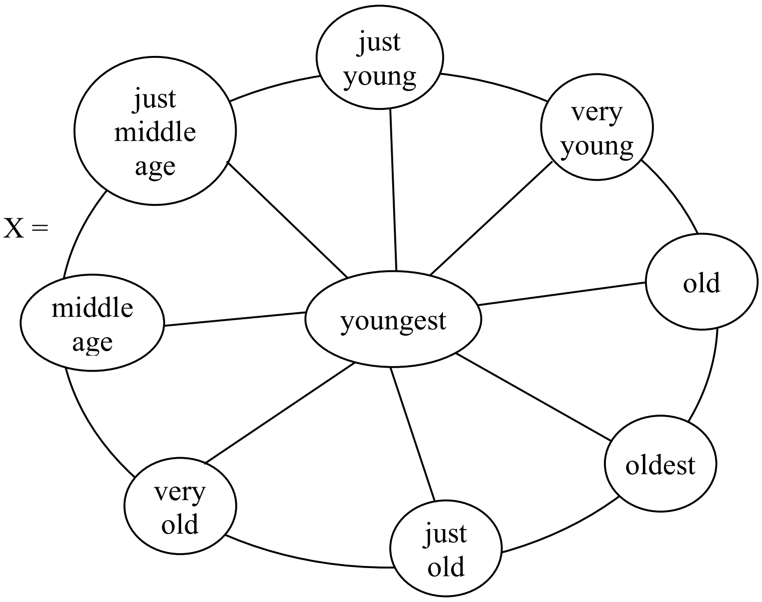


Figure 2.2.29

We see X is a wheel linguistic graph of order 9.

Let Y_1 be a linguistic subgraph of X given by the Figure 2.2.30.

$Y_1 =$

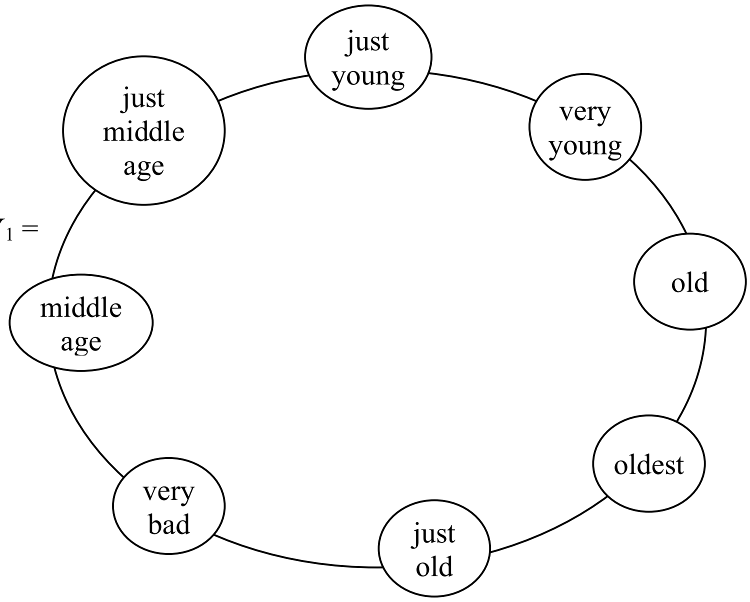


Figure 2.2.30

Clearly Y_1 is a circle linguistic subgraph of order 8. It is not a linguistic wheel.

Let Y_2 be a linguistic subgraph of X given by the Figure 2.2.31.

$Y_2 =$

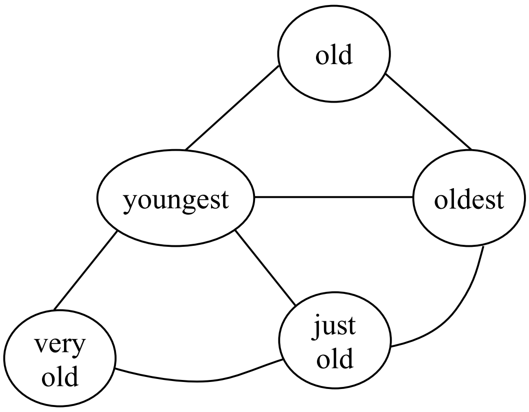


Figure 2.2.31

Y_2 is not a linguistic wheel graph or a linguistic circle graph.

Next, we prove some example of linguistic wheel graph using the same linguistic set S associated with the linguistic variable age of people.

R =

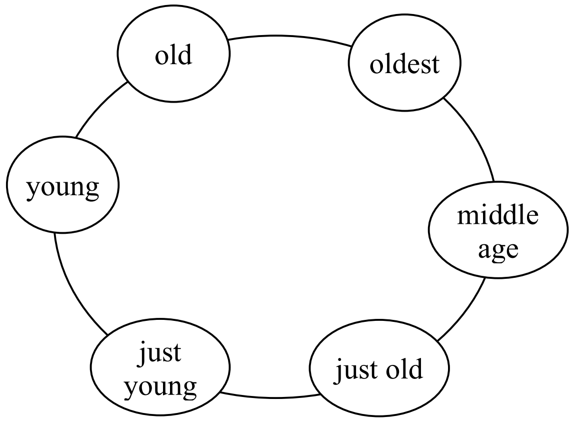


Figure 2.2.32

R is called as the linguistic circle graph. The linguistic subgraphs T_1, T_2, T_3 and T_4 of R are given in the following Figure 2.2.33 and Figures 2.2.34.

T_1 =

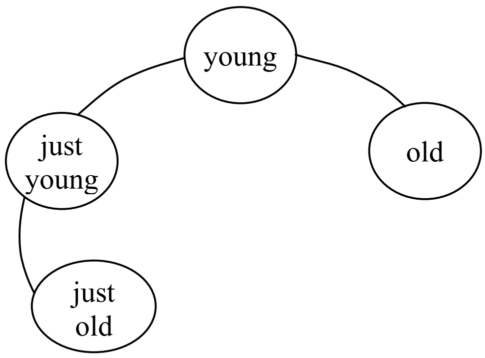
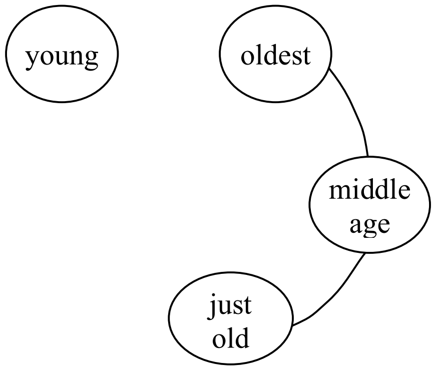
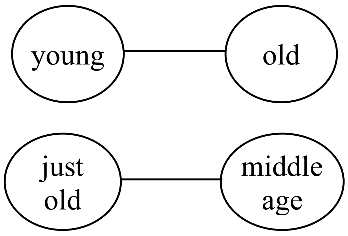


Figure 2.2.33

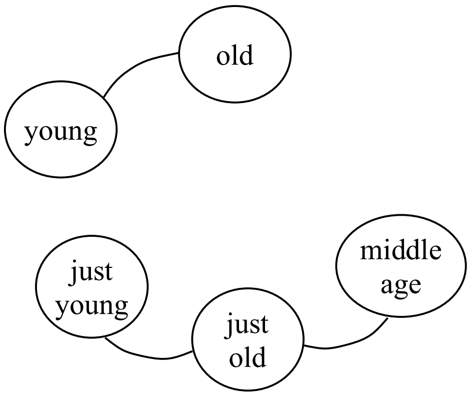
$T_2 =$



$T_3 =$



$T_4 =$



Figures 2.2.34

None of the linguistic subgraphs of R are linguistic circles.

Some of them are linguistic connected subgraphs and some of them disconnected linguistic subgraphs given in Figure 2.2.34.

Next we proceed onto define edge linguistic subgraphs.

We call a linguistic subgraph of a linguistic graph H to be a linguistic edge subgraph if from the linguistic graph G some of the edges are removed.

We will illustrate the situation by some example.

Example 2.2.3. Let L be a linguistic variable associated with the speed of the car on road.

Let S = {fastest, fast, just fast, slow, very slow, very fast, medium speed, just medium speed, very very slow, ϕ (ϕ = not running so empty speed), just slow, slowest}

be the linguistic set/word/term associated with speed of a car.

Let K be the linguistic graph given by the following Figure 2.2.35.

K =

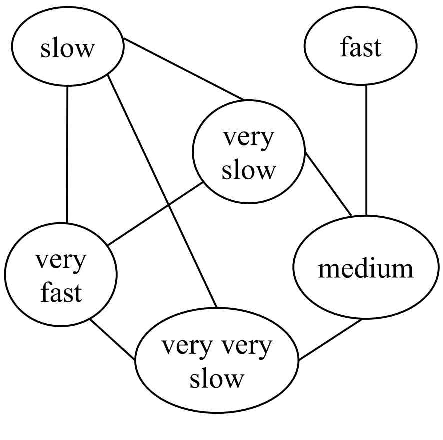


Figure 2.2.35

The linguistic edge subgraphs of K is as follows.

L_1 =

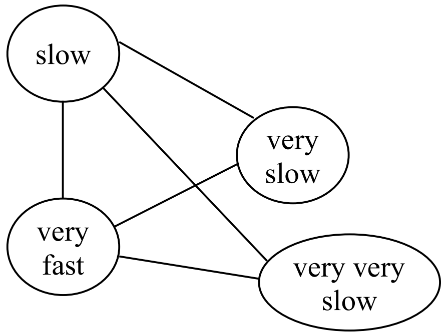


Figure 2.2.36

We have the edge linguistic subgraph got by removing the edges

L_2 =

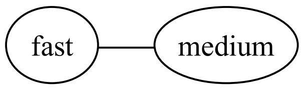


Figure 2.2.37

(which we will denote by (fast) (medium) (very very slow) medium and (very slow) medium.

When we remove the three edges we see the resultant edge linguistic subgraph also looses two linguistic nodes viz

L_3 =

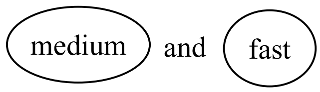


Figure 2.2.38

Let L_2 be the edge linguistic subgraph of K got by removing the edges (very fast) (slow), (very slow) (slow) and (very very slow) (slow) from K.

$L_4 =$

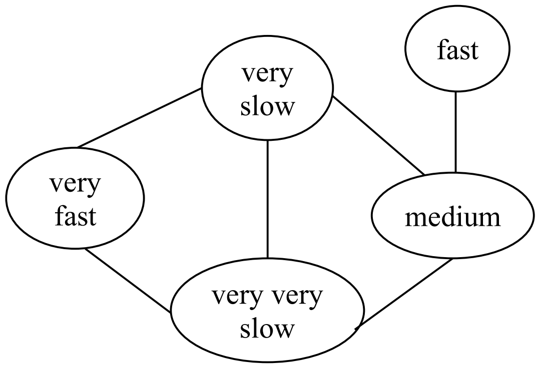


Figure 2.2.39

When the 3 edges are removed from K we get the edge linguistic subgraph with 5 vertices given in Figure 2.2.39.

We now give the edge linguistic subgraph L_5 got by removing the edges (very slow) (very fast), (very fast) (slow), (very slow) (medium) and (very very slow) (slow).

$L_5 =$

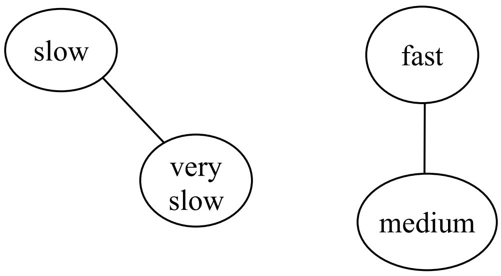


Figure 2.2.40

Clearly L_5 is a disconnected linguistic graph of order four, this has two linguistic dyads.

Thus, edge linguistic subgraphs play a vital role in the study of social information networks.

We provide yet another example of a linguistic graph M and find its edge linguistic subgraphs.

The nodes of M are from the same linguistic set S associated with the linguistic variable age.

The linguistic graph M is given in Figure 2.2.41.

M =

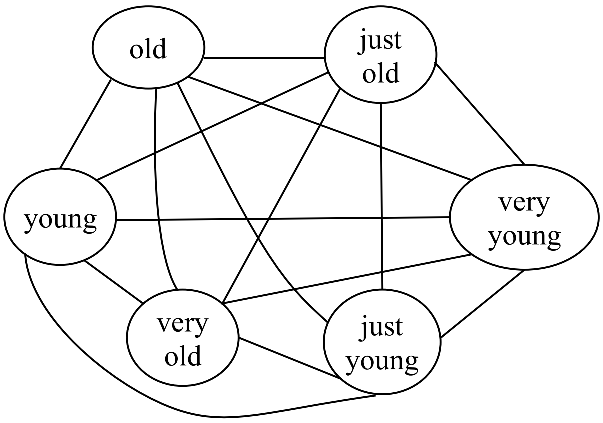


Figure 2.2.41

Clearly M is a complete linguistic graph of order six, so M is a connected linguistic graph. Removal of an edge will not result in a disconnected linguistic graph or with lesser number of nodes.

Only a maximum of 5 edges can result in a linguistic subgraph of lesser order that too when all 5 edges are removed from the same node.

However, to make a empty linguistic edge subgraph how many edges one should remove.

Study in this direction is not difficult; simple and is given as an exercise to the reader.

In the following section we describe directed linguistic graphs.

2.3 Linguistic directed graphs

In this section we proceed onto define 3 types of linguistic directed graphs.

i) The directions are given in these linguistic graphs very arbitrarily (when linguistic nodes cannot be ordered).

ii) The directions are from lowest or least to largest that is increasing relation of the linguistic nodes (in case the set of linguistic nodes are totally orderable).

iii) The directions of the linguistic graph are from largest to least that is in decreasing order (in case the set of ling nodes are totally orderable).

We will describe, develop and define linguistic graphs.

Here we assume mostly and take either partially ordered set or totally ordered set to develop these linguistic directed graphs for if the set is unordered we can have only empty linguistic graph of (ii) and (iii) mentioned in the beginning of this section. However we can have linguistic directed graphs mentioned in (i).

We will first describe these situations by examples.

Example 2.3.1. Let S = {brown, black, red, blue, orange, white, green, yellow}

be a linguistic set associated with colours.

Clearly S is neither orderable not partially orderable.

We can only have linguistic graphs whose relations or edges are arbitrarily marked.

Let G_1 be a linguistic graph with nodes from the linguistic set S. This G_1 is given by the Figure 2.3.1.

G_1 =

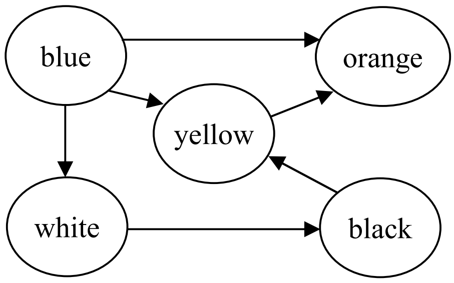


Figure 2.3.1

Using the same set of vertex nodes as in G_1 we get

G_2 =

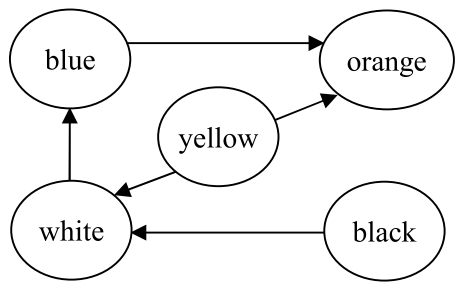


Figure 2.3.2

Clearly G_1 and G_2 are different.

Let G_3 be the linguistic directed graph with same set of 5 linguistic nodes.

G_3 =

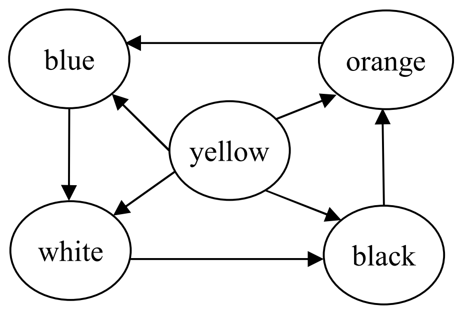


Figure 2.3.3

Let G_4 be a linguistic directed graph with same set of 5 linguistic nodes.

G_4 =

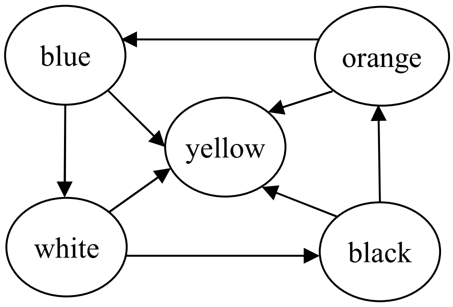


Figure 2.3.4

G_5 be a linguistic directed graph with same set of vertices as that of G_1

G_5 =

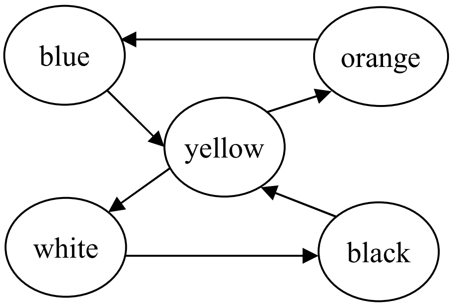


Figure 2.3.5

Let G_6 be the linguistic directed graph using the same set of linguistic nodes as in G_1.

G_6 =

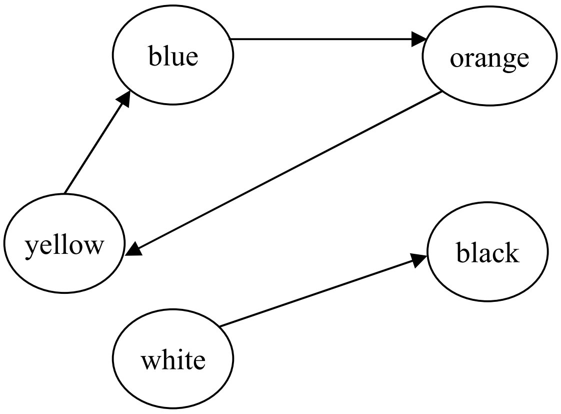


Figure 2.3.6

Let G_7 be a linguistic directed graph using the same set of 5 nodes.

G_7 =

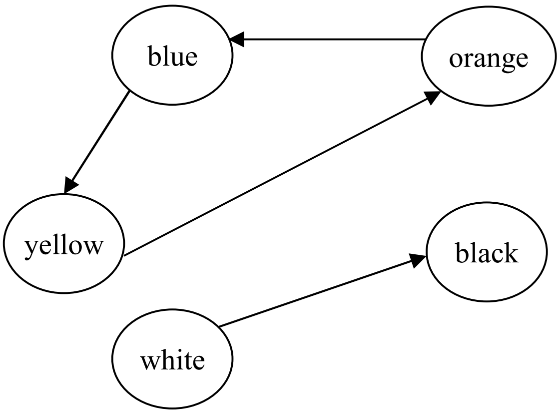


Figure 2.3.7

Let G_8 be the linguistic directed graph with same set of vertices as in G_1

G_8 =

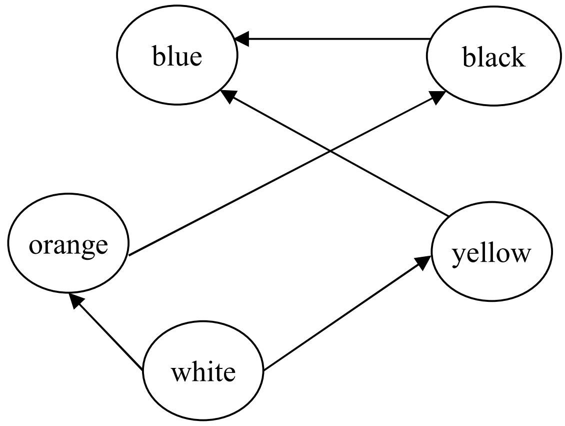


Figure 2.3.8

We give yet another linguistic directed graph G_9 using the same set of linguistic nodes as in G_1

G_9 =

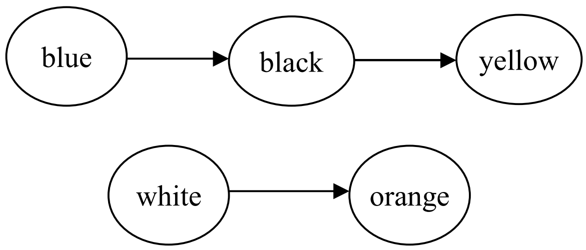


Figure 2.3.9

All the 9 linguistic directed graphs are distinct, but we have only empty linguistic directed graphs in case of increasing and decreasing relations between nodes as the linguistic nodes basically of the linguistic set S related to the linguistic variable colour which is neither totally ordered nor partially ordered. These are unordered linguistic set in this case. They do not fall under (ii) and (iii) mentioned in the beginning of this section.

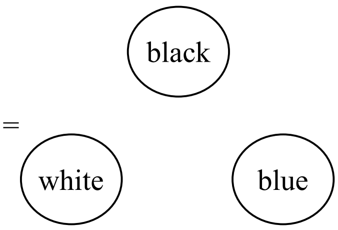
Further finding the number of such directed linguistic graphs cannot be easily found.

We leave it as a open problem to find out the number of such linguistic directed graphs given an unordered linguistic set.

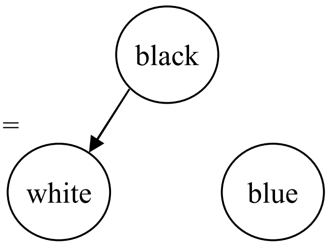
However, we work with 3 linguistic nodes, viz.,

{black, white, blue} and enumerate all directed linguistic graphs in the following.

B_1 =

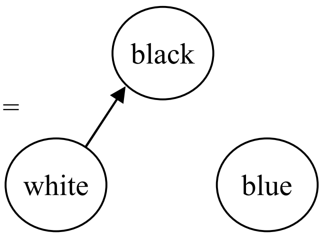


B_2 =

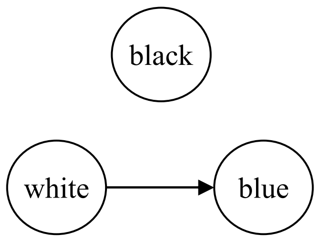


B_1 is a linguistic empty graph.

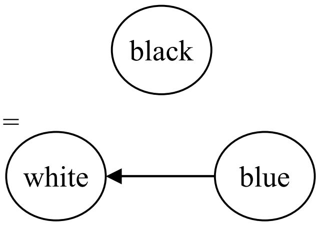
B_3 =



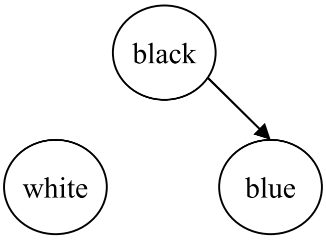
B_4 =



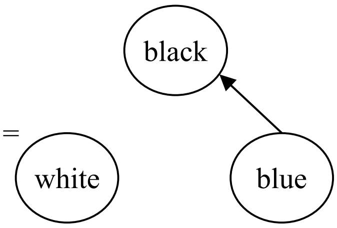
$B_5 =$



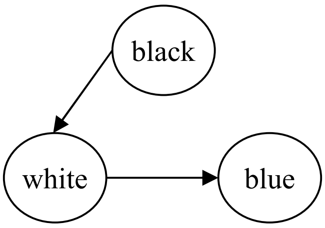
$B_6 =$



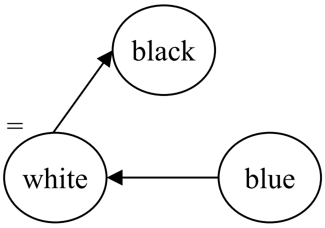
$B_7 =$



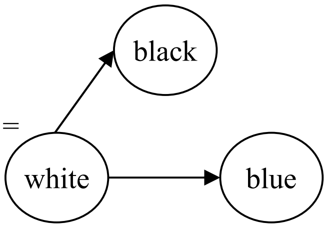
$B_8 =$



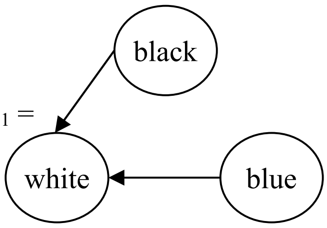
$B_9 =$



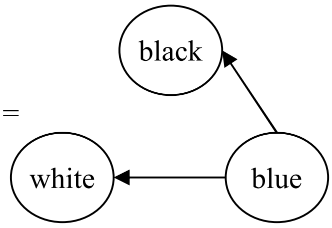
$B_{10} =$



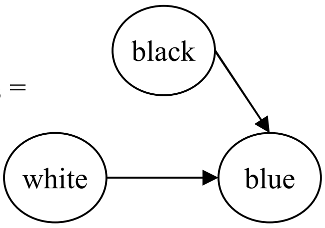
$B_{11} =$



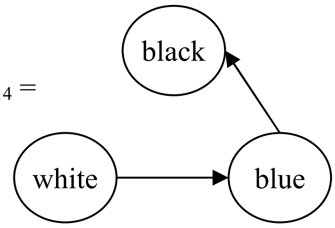
$B_{12} =$



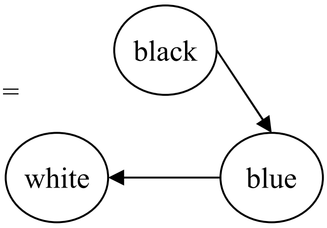
$B_{13} =$



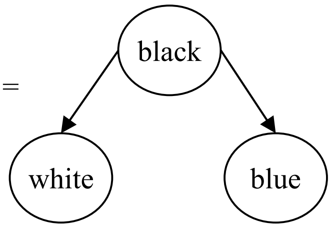
$B_{14} =$



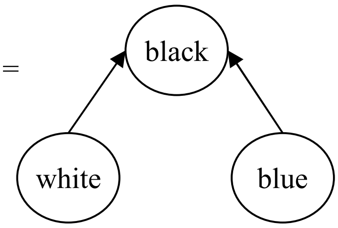
B_{15} =



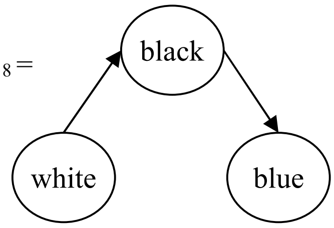
B_{16} =



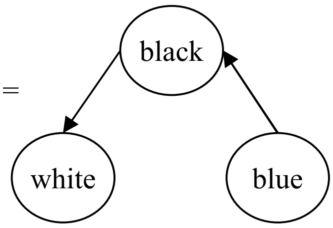
B_{17} =



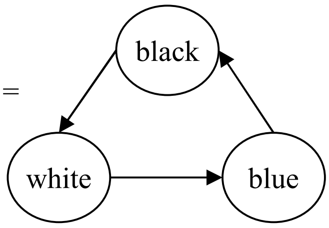
B_{18} =



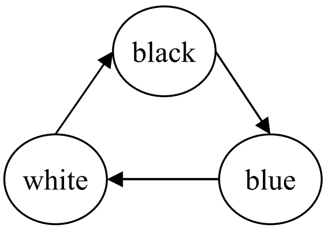
B_{19} =



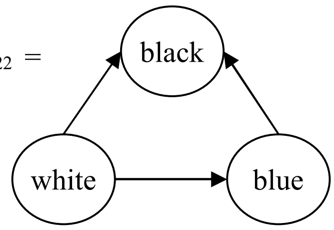
B_{20} =



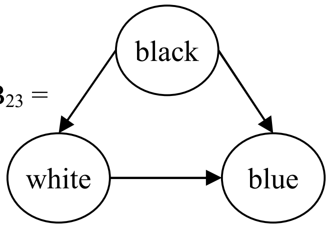
B_{21} =



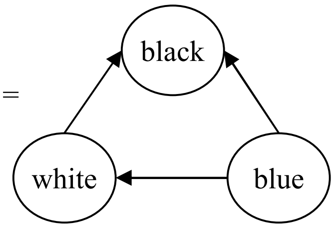
B_{22} =



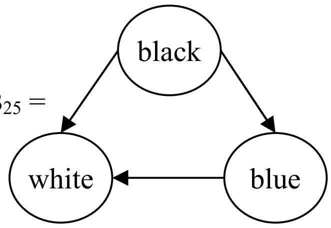
$B_{23} =$



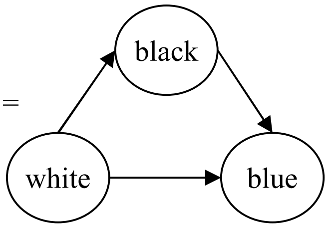
$B_{24} =$



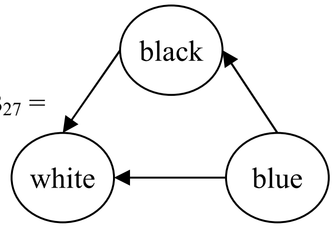
$B_{25} =$



$B_{26} =$



$B_{27} =$



Figures 2.3.10

Thus, we have mentioned the linguistic directed graphs using 3 linguistic nodes.

We have only one empty linguistic directed graph using increasing and decreasing relation of linguistic nodes.

Now we provide an example of linguistic directed graph in case of a totally ordered linguistic set.

Example 2.3.2. Let

S = {good, bad, fair, very good, very bad, just good, just fair, just bad, best, worst}

be the linguistic set associated with the linguistic variable performance of a teacher in a classroom.

Clearly the linguistic set S is totally orderable, we provide the linguistic increasing chain given in the following:

worst ≤ very bad ≤ bad ≤ just bad ≤ just fair ≤ fair ≤ just good ≤ good ≤ very good ≤ best …I

Let W_1 be a linguistic directed graph using the linguistic subset of S given by

T = {good, bad, very bad, best, worst, very good, fair}

W_1 =

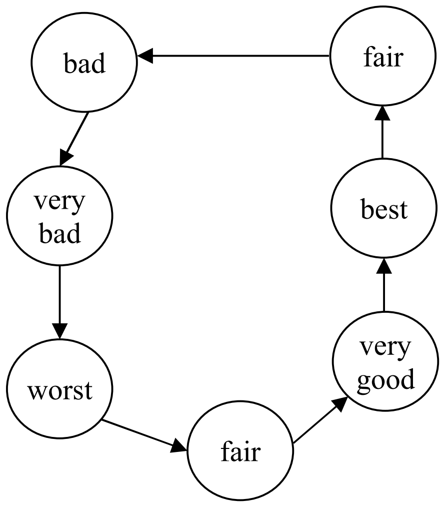


Figure 2.3.11

We see W_1 is a linguistic circle. However, we have not mapped all possible relations just very arbitrary but some sense!

Here we can have more number of such linguistic directed graphs using the linguistic $T \subseteq S$.

Let W_2 be a linguistic directed graph using the linguistic subset T of S.

$W_2 =$

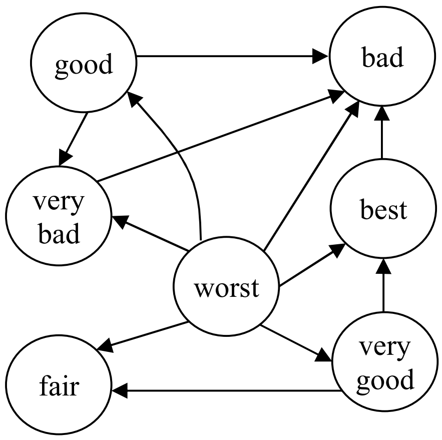


Figure 2.3.12

W_2 is also just a linguistic directed graph which is different from W_1.

Let V be the linguistic directed graph where the direction is in the increasing relation that if best and bad are to related.

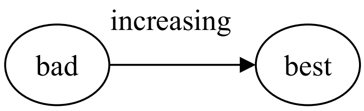


Figure 2.3.13

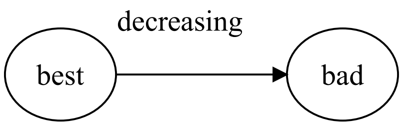
So we cannot have 

Figure 2.3.14

We have only one linguistic directed graph which is bound by the increasing relation.

V is given in Figure 2.3.15.

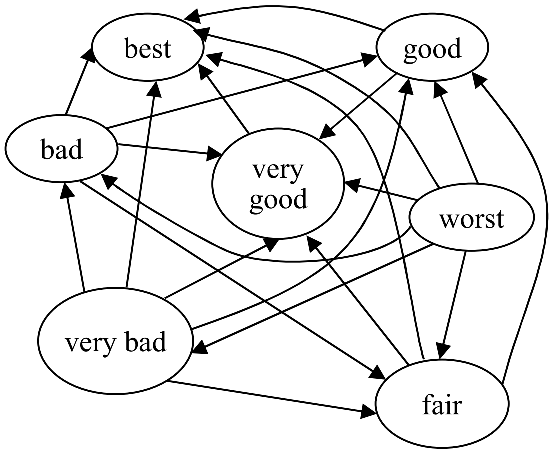
V = 

Figure 2.3.15

Clearly V is a directed linguistic complete graph given in the increasing direction.

This graph is unique once given the linguistic set and the order in which the directed edges occur.

Let U be the linguistic directed graph in the decreasing direction using the same linguistic subset T of S;

U is given by the following Figure 2.3.16.

U =

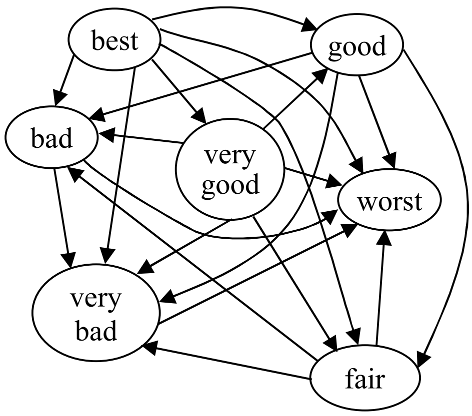


Figure 2.3.16

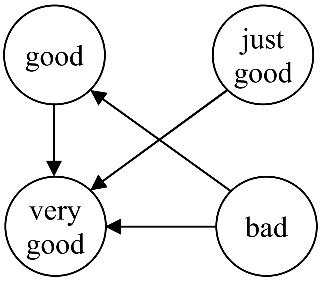
We see U is the only linguistic graph in the decreasing direction. They are unique once the set of linguistic nodes are supplied, unlike the linguistic directed graph whose edges are marked very arbitrarily.

Now take the linguistic subset

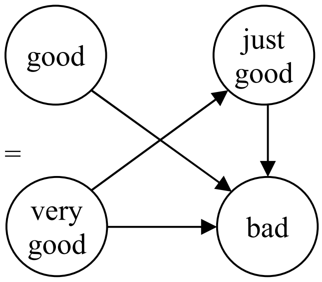
R = {good, just good, very good, bad} ⊆ S.

The linguistic directed graphs using the linguistic set R are as follows

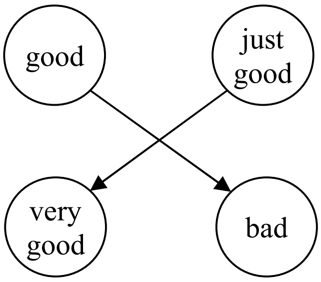
B_1 =



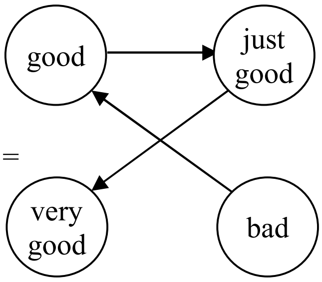
B_2 =



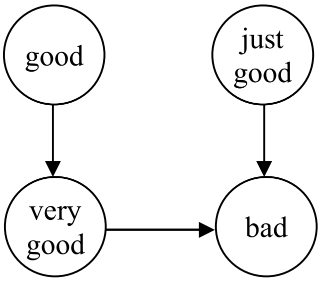
B_3 =



B_4 =



B_5 =



and so on

Figures 2.3.17

We can have several such linguistic directed graphs using the linguistic subset $R \subseteq S$.

However, if we use the increasing or decreasing relation linguistic directed graphs are two both are distinct but we have one and only one such linguistic directed graph in that case.

Let A_1 be the linguistic directed graph which has its edges in the increasing direction using the linguistic set R.

A_1 =

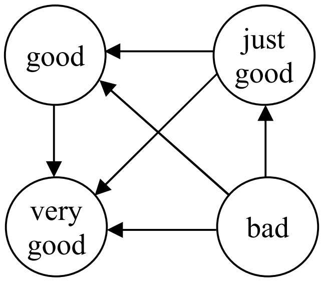


Figure 2.3.18

Let A_2 be the linguistic directed graph where the relations or edges are in the decreasing direction.

A_2 is given in Figure 2.3.19.

A_2 =

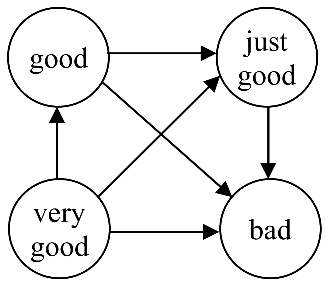


Figure 2.3.19

The linguistic directed graph with decreasing direction is unique and in relation it is just opposite of the linguistic directed graph with increasing direction and vice versa.

A_1 is also unique unlike the directed linguistic graphs which enjoys no condition of either decreasing or increasing.

Consider the set T we can have several directed linguistic star graphs given in the following Figures.

N_1 =

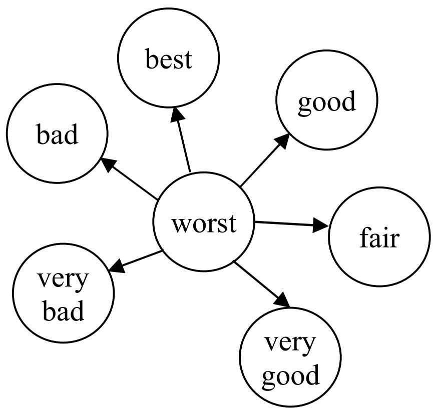


Figure 2.3.20

$N_2 =$

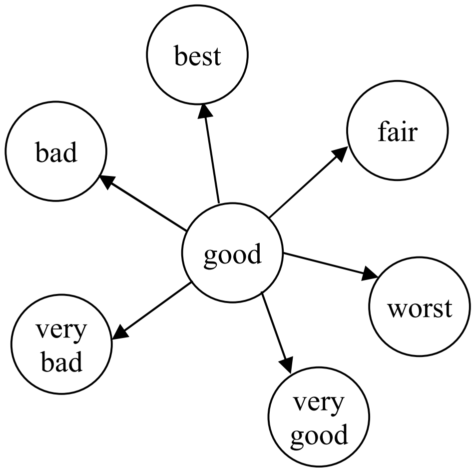


Figure 2.3.21

$N_3 =$

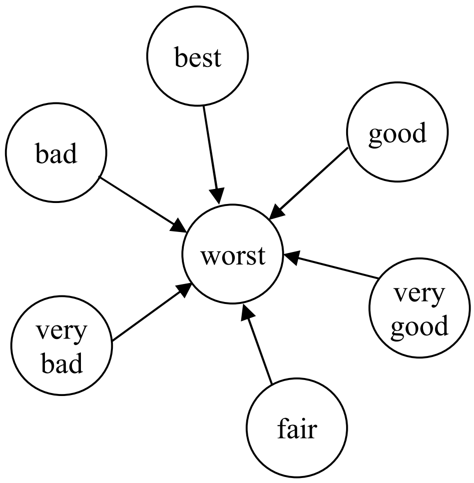


Figure 2.3.22

$N_4 =$

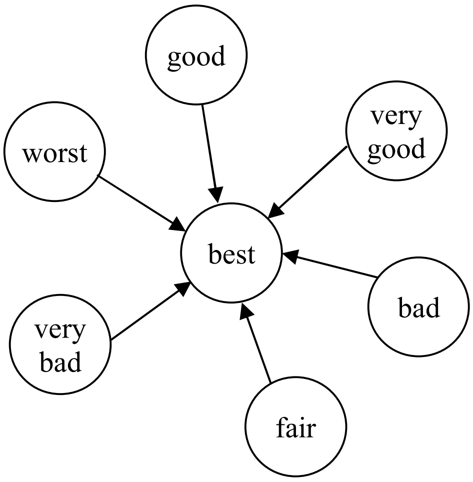


Figure 2.3.23

$N_5 =$

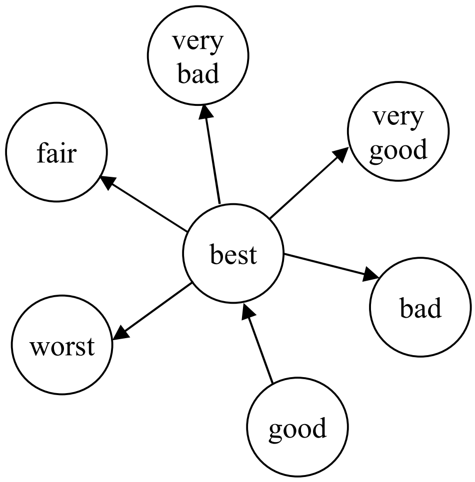


Figure 2.3.24

N_6 =

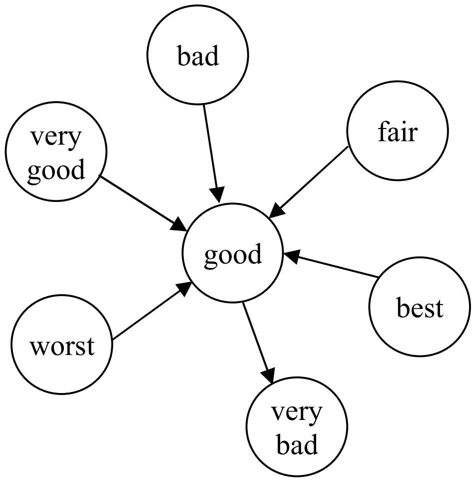


Figure 2.3.25

and so on and so forth.

All linguistic directed graphs with increasing or decreasing order of relations are only complete linguistic directed graphs and they can never be linguistic star graphs.

We can find linguistic directed subgraphs of all 3 types of graphs.

We find some linguistic directed subgraphs of W_2 which are given by the Figure 2.3.26.

C_1 =

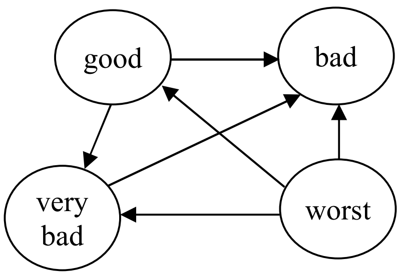


Figure 2.3.26

$C_2 =$

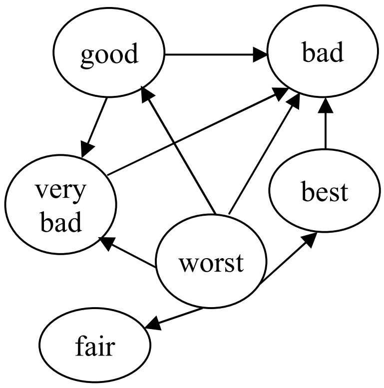


Figure 2.3.27

C_1 is a complete linguistic directed subgraph of W_2.

$C_3 =$

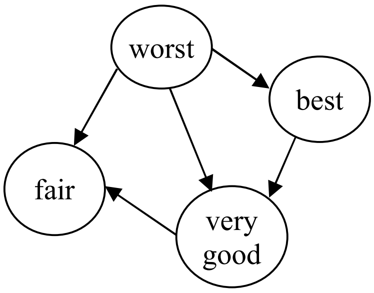


Figure 2.3.28

C_3 is not a complete linguistic directed subgraph of W_2

Let $C_4 =$

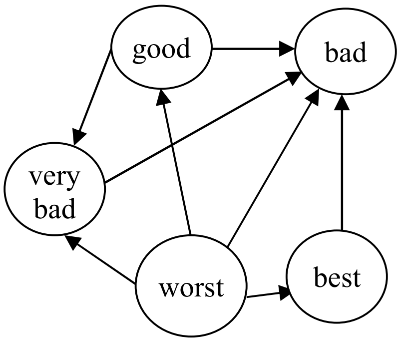


Figure 2.3.29

$C_5 =$

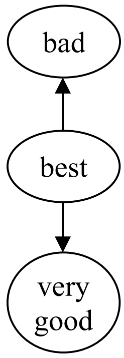


Figure 2.3.30

$C_6 =$

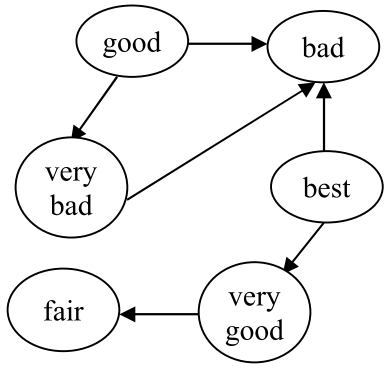


Figure 2.3.31

$C_7 =$

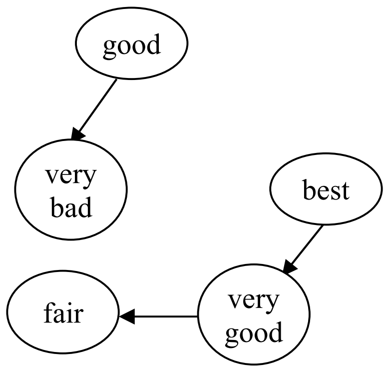


Figure 2.3.32

C_8 =

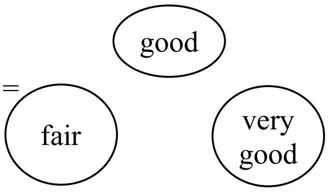


Figure 2.3.33

C_9 =

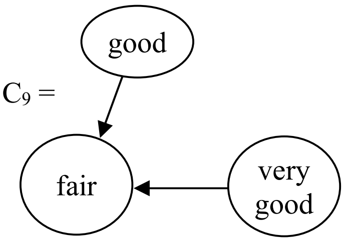


Figure 2.3.34

C_8 is a empty linguistic directed graph.

Now we see all the 9 linguistic directed subgraphs of the linguistic directed graph given in Figure 2.3.15 is as follows.

V_L =

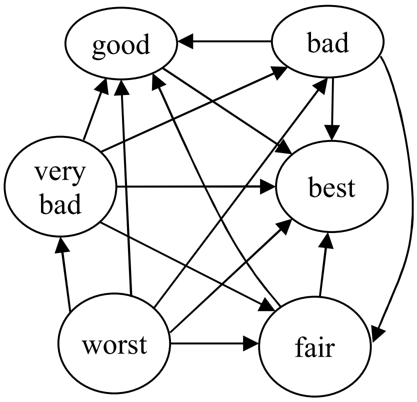


Figure 2.3.35

V_2 =

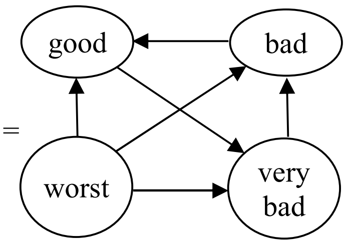


Figure 2.3.36

V_3 =

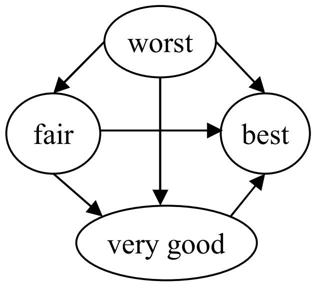


Figure 2.3.37

$V_4 =$

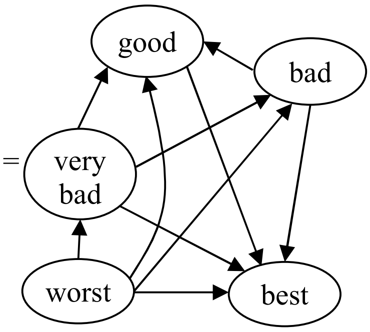


Figure 2.3.38

$V_5 =$

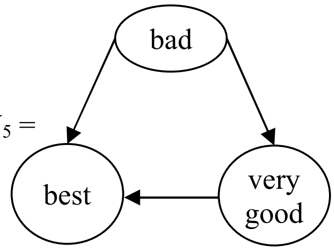


Figure 2.3.39

$V_6 =$

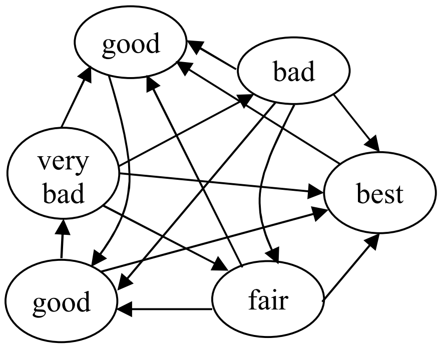


Figure 2.3.40

$V_7 =$

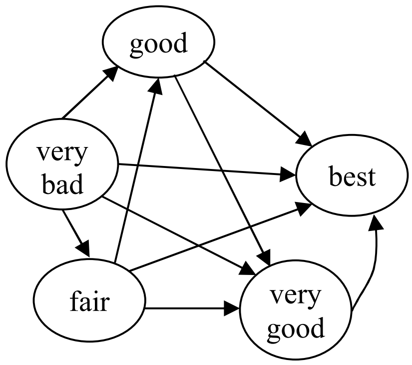


Figure 2.3.41

$V_8 =$

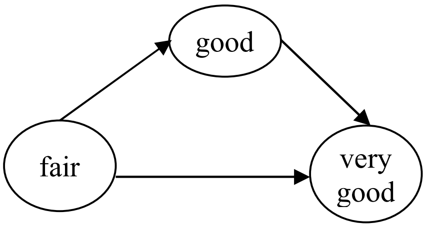


Figure 2.3.42

and

$V_9 =$

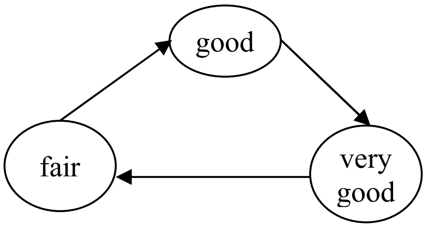


Figure 2.3.43

We have 9 linguistic directed subgraphs of V (increasing relation) and all of them are complete linguistic directed subgraphs. The Figures 2.3.35 to 2.3.43 gives 9 complete linguistic directed subgraphs of V given in Figure 2.3.15.

Now we give the same 9 linguistic directed subgraphs of U (decreasing direction of the edges).

$U_1 =$

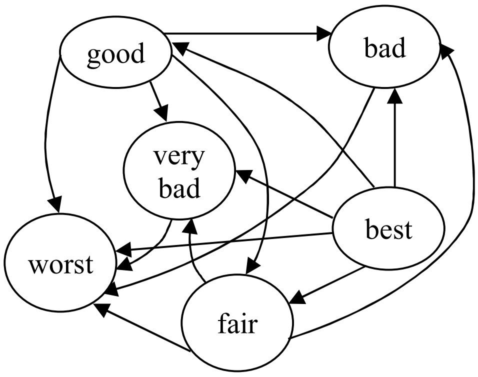


Figure 2.3.44

$U_2 =$

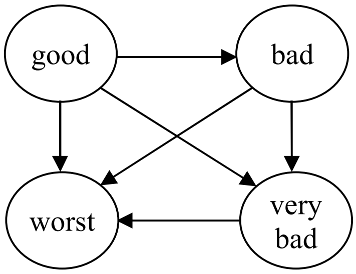


Figure 2.3.45

$U_3 =$

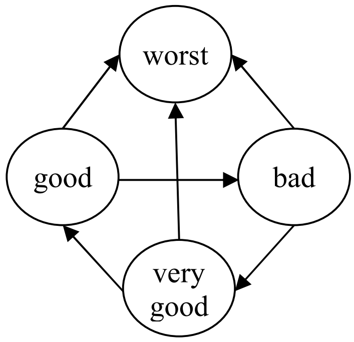


Figure 2.3.46

$U_4 =$

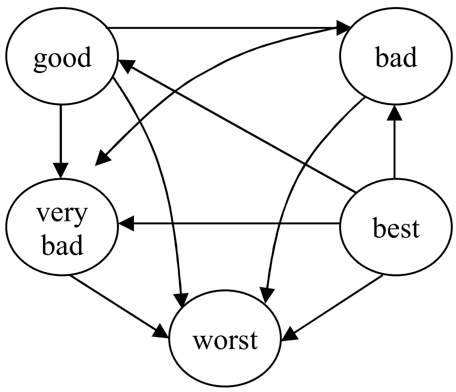


Figure 2.3.47

$U_5 =$

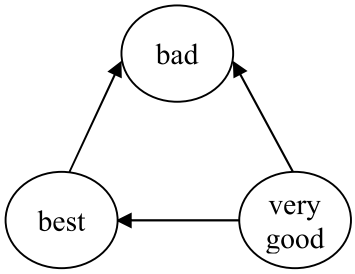


Figure 2.3.48

$U_6 =$

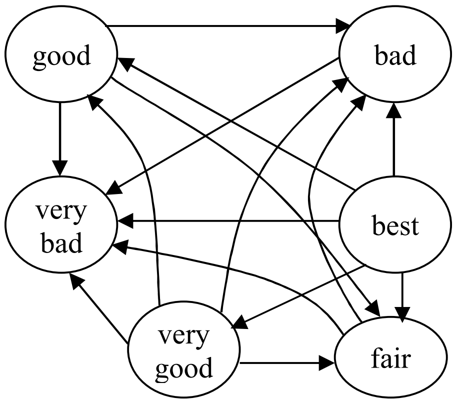


Figure 2.3.49

$U_7 =$

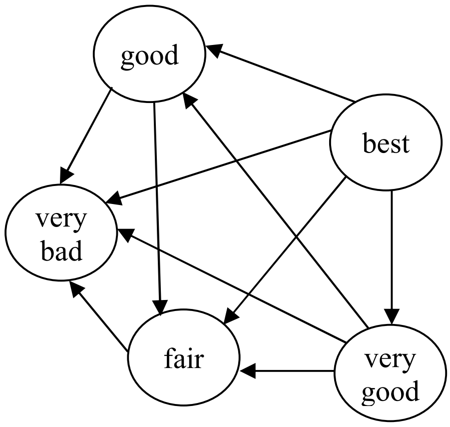


Figure 2.3.50

$U_8 =$

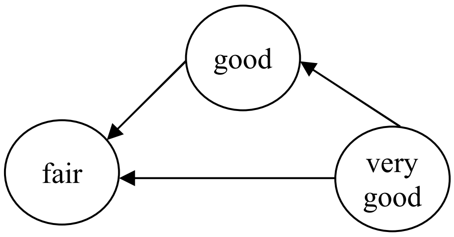


Figure 2.3.51

and

$U_9 =$

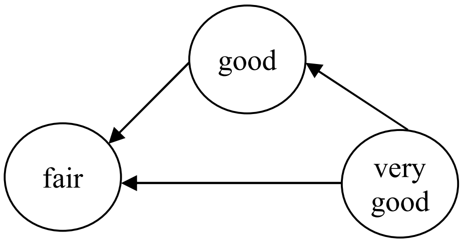


Figure 2.3.52

We see the nine linguistic directed subgraphs of V and U respectively are directed differently and further U_8 and U_9 (and V_8 and V_9) are identical though C_8 and C_9 are different.

Recall we say two linguistic directed graphs are identical so they have same node set and the same set of directed edges (relations) V_8 and V_9 is an example of identical linguistic directed graphs similarly U_8 and U_9 are identical linguistic directed graphs.

The edge linguistic directed subgraph in case of (increasing or decreasing order relation) is not possible.

Only in case of linguistic directed graph which has neither increasing nor decreasing edge / relation the concept of edge ling sub graph is possible.

That is the directed linguistic graphs mentioned in (1) introduction of this section 2.3.

Example 2.3.3. Let

S = {tall, short, just short, very short, just tall, very tall, medium height, very very short, very very tall, shortest}

be the linguistic set associated with the linguistic variable height of a person.

Consider the linguistic directed graph G given by the following Figure 2.3.53.

G =

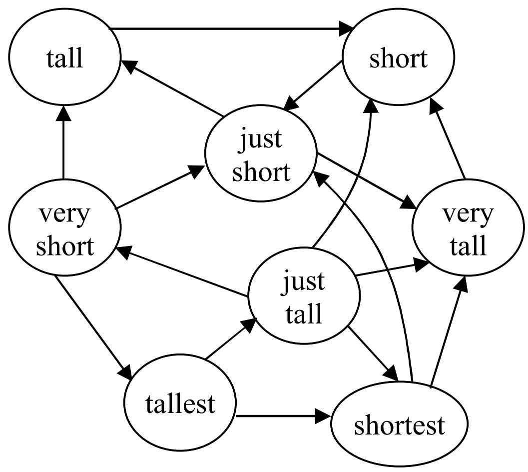


Figure 2.3.53

We give a few of the G edge linguistic directed subgraphs in Figure 2.3.54.

Remove the following linguistic edges in G,

$\overrightarrow{\text{edge (very short) (tall)}}$, edge $\overrightarrow{\text{(just short) (tall)}}$,

edge $\overrightarrow{\text{(tall) (short)}}$, edge $\overrightarrow{\text{(very short) (tallest)}}$ and

edge $\overrightarrow{\text{(tallest)(shortest)}}$

Let H_1 be the edge linguistic directed subgraph G.

H_1 =

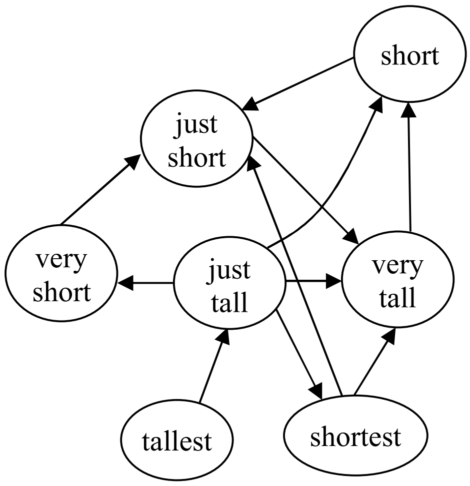


Figure 2.3.54

Let H_2 be the edge linguistic directed subgraph of G by removing the edges.

$\overrightarrow{\text{(just tall) (very short)}}$, $\overrightarrow{\text{(just tall) (short)}}$,

$\overrightarrow{\text{(just tall)(very tall)}}$, $\overrightarrow{\text{(just tall)(shortest)}}$,

$\overrightarrow{\text{(tallest)(just tall)}}$, $\overrightarrow{\text{(very short) (tallest)}}$

$\overrightarrow{\text{(tall)(short)}}$ and $\overrightarrow{\text{(very short) (tall)}}$

H_2 =

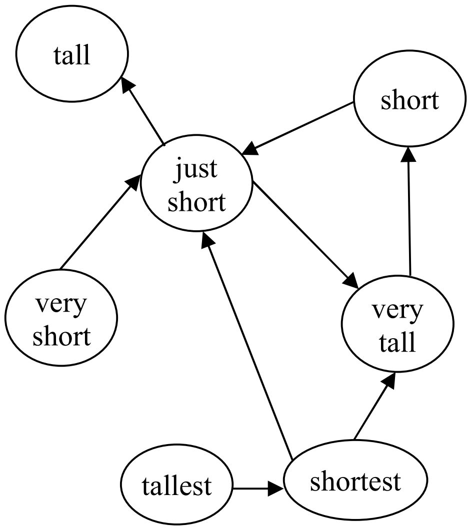


Figure 2.3.55

Let H_3 be the edge linguistic directed subgraph given in the following.

The edges removed are

$\overrightarrow{\text{(just short) (tall)}}$, $\overrightarrow{\text{(short) (just short)}}$, $\overrightarrow{\text{(just short) (very tall)}}$,

$\overrightarrow{\text{(shortest) (just short)}}$, $\overrightarrow{\text{(very short) (just short)}}$,

$\overrightarrow{\text{(just tall) (shortest)}}$ $\overrightarrow{\text{(tallest) (shortest)}}$ and

$\overrightarrow{\text{(shortest) (very tall)}}$.

The edge linguistic directed graph is as follows.

H_3 =

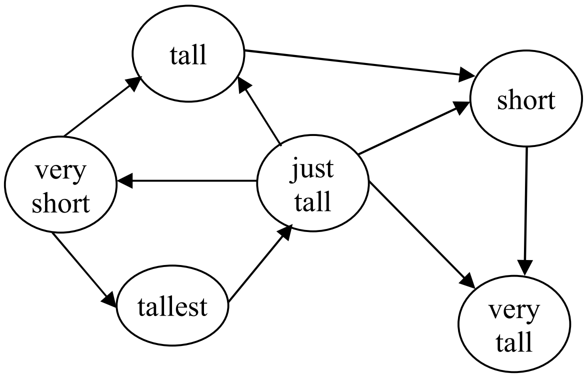


Figure 2.3.56

In the case of linguistic directed graphs (either) edges increasing (relation) or edges decreasing (relation)).

We cannot have the edge linguistic subgraphs for we have the following definition of subgraphs.

Definition 2.3.1. *Let G(V, E) be a linguistic directed graph (either the edges or relations are in increasing order or decreasing order). H is defined as the linguistic directed edge subgraph of G if some of the edges* $e \in E$ *are removed* $H = (V_1, F)$ *where* $V_1 \subseteq V$ *and* $F \subset E$ *(it is mandatory F is a proper subset of E).*

H may not in general be a linguistic directed graph in the usual sense for a relation should exist by definition of linguistic directed graph but is missing.

Thus, with this limitations and thus understanding only we can define linguistic directed subgraphs (increasing / decreasing relations) which may not have some edges as per definitions of linguistic directed graphs which are complete if the linguistic set under study is a totally ordered set.

If only partially ordered the definition may sometimes coincide with the classical definition.

We will illustrate this situation by some examples.

Example 2.3.4. Let S be as in example 2.3.3. Let G be a linguistic directed graph (increasing relation) is given below.

G =

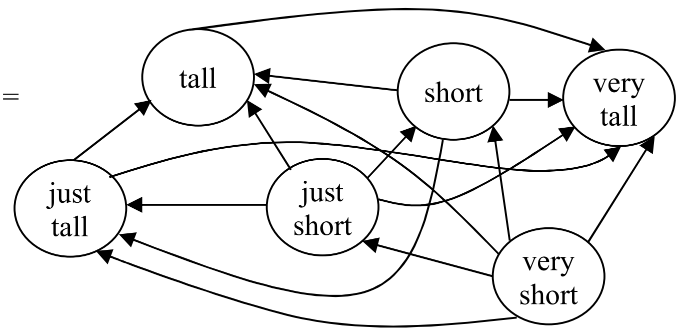


Figure 2.3.57

Remove the linguistic edges

$\overrightarrow{\text{(tall) (very tall)}}$, $\overrightarrow{\text{(just tall) (tall)}}$, $\overrightarrow{\text{(just short) (tall)}}$,

$\overrightarrow{\text{(very short) (tall)}}$ and $\overrightarrow{\text{(short) (tall)}}$

the resulting linguistic directed subgraph T_1 as follows;

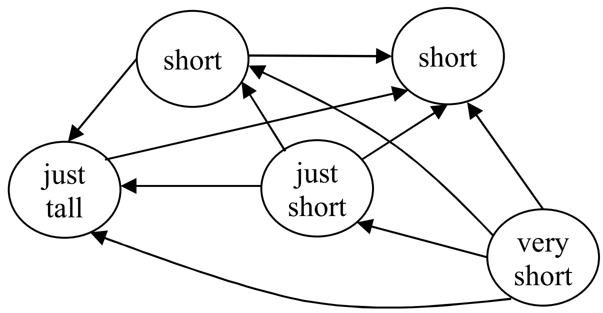
T_1 =


Figure 2.3.58

Clearly T_1 is a linguistic directed subgraph in fact T_1 is a complete linguistic directed subgraph of G.

Consider T_2 a linguistic directed subgraph of G got by removing the edges $\overrightarrow{\text{(tall) (very tall)}}$, $\overrightarrow{\text{(very short) (short)}}$ and

$\overrightarrow{\text{(just short) (tall)}}$ given in Figure 2.3.59.

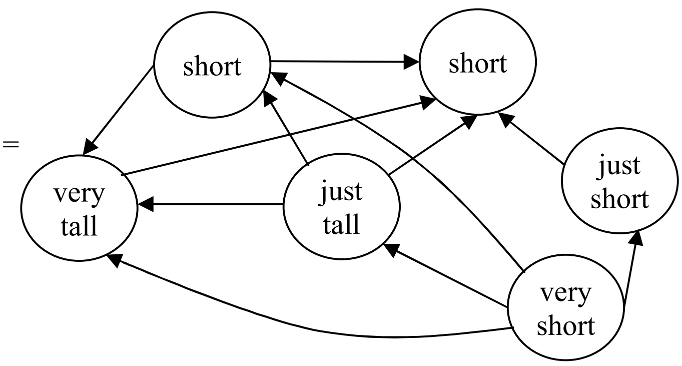
T_2 =


Figure 2.3.59

The edge linguistic directed subgraph T_2 is not a complete linguistic directed subgraph of G.

We proceed onto discuss linguistic directed graphs using partially ordered sets by some examples.

Let SE be any linguistic set; P(S) will denote the linguistic power set of S.

Clearly P(S) contain the empty linguistic set ϕ and S the whole linguistic set.

Example 2.3.5. Let S = {good, bad, fair, very good}

a linguistic set associated with the performance of a student.

P(S) = {ϕ, p_1 = {good}, p_2 = {bad}, p_3 = {fair}, p_4 = {very good}, p_5 = {bad, good}, p_6 = {bad, fair}, p_7 = {bad, very good}, p_8 = {good, fair}, p_9 = {good, very good}, p_{10} = {fair, very good}, p_{11} = {bad, good, fair}, p_{12} = {bad, good, very good}, p_{13} = {bad, fair, very good}, p_{14} = {fair, good, very good}, p_{15} = {good, bad, fair, very good} = S}

be the linguistic power set of S.

Now we give linguistic edge weights,

E = {i - improvement, gi - good improvement, ni - no improvement, js - just the same, vgi - very good improvement, si - some improvement}

for the linguistic graphs with linguistic nodes from P(S).

G_1 =

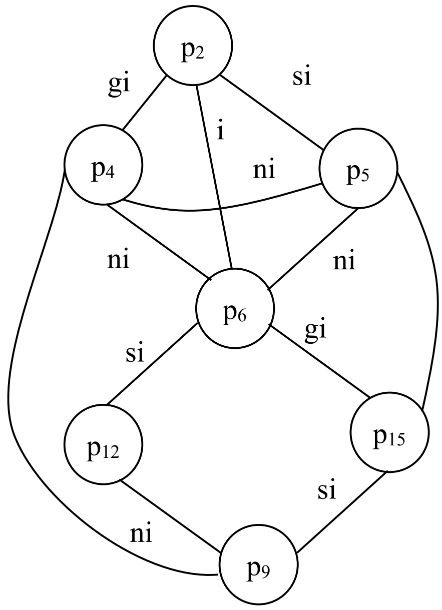


Figure 2.3.60

G_1 is a linguistic graph (not directed) using partially ordered set.

Thus, we can have undirected linguistic graphs using the subsets of the linguistic power set P(S) of a linguistic set S as nodes.

Let G_2 be yet another linguistic graph using the linguistic nodes of P(S).

G_2 is also not a directed linguistic graph but G_2 is also has edge weight or linguistic edge weights from E.

G_2 =

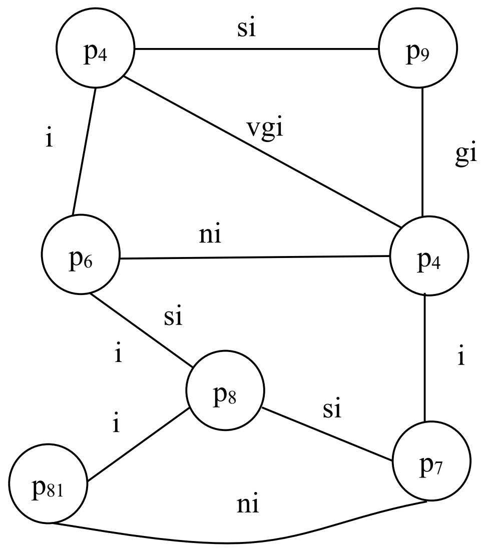


Figure 2.3.61

G_2 is a edge weighted linguistic graph of order 7.

Let G_3 be another edge weighted linguistic graph given by the following Figure 2.3.62.

G_3 =

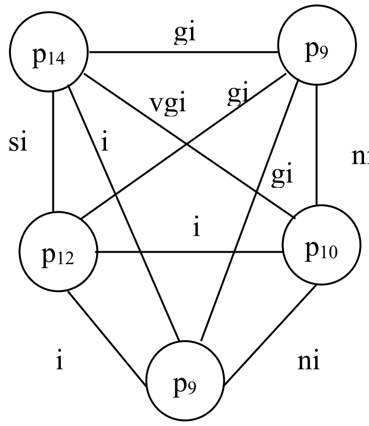


Figure 2.3.62

Now using the same linguistic set P(S) that is the linguistic power set of the linguistic set S which is only a partially ordered set we give a few linguistic directed graphs in the following.

Let H_1 be a linguistic directed graph using a subset of the linguistic power set P(S).

H_1 =

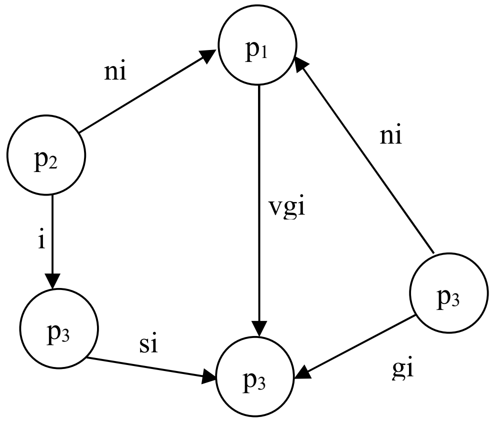


Figure 2.3.63

Let H_2 be the linguistic directed graph given by the following Figure 2.3.64.

H_2 =

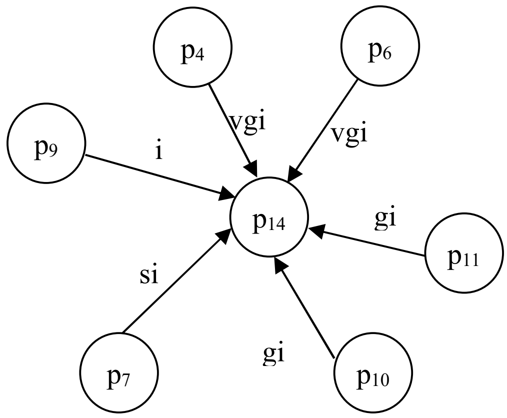


Figure 2.3.64

We see H_2 is a linguistic directed star graph with p_{14} as the linguistic central node.

Consider the linguistic directed graph H_3 with vertex set from the linguistic power set P(S).

$H_3 =$

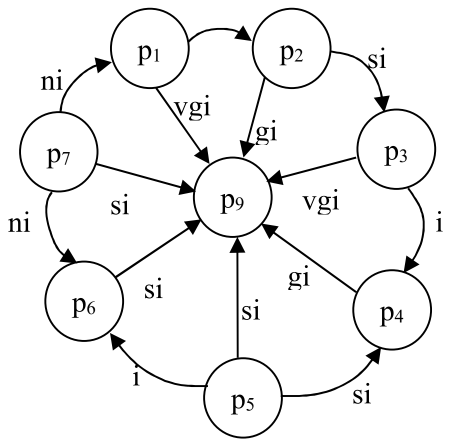


Figure 2.3.65

Clearly H_3 is a linguistic graph which has a linguistic subgraph that is a linguistic star graph with p_9 as the central node.

We see it is not a wheel as the directions are not in order.

Now let H_4 be the linguistic directed graph with entries from P(S) given by the following Figure 2.3.66.

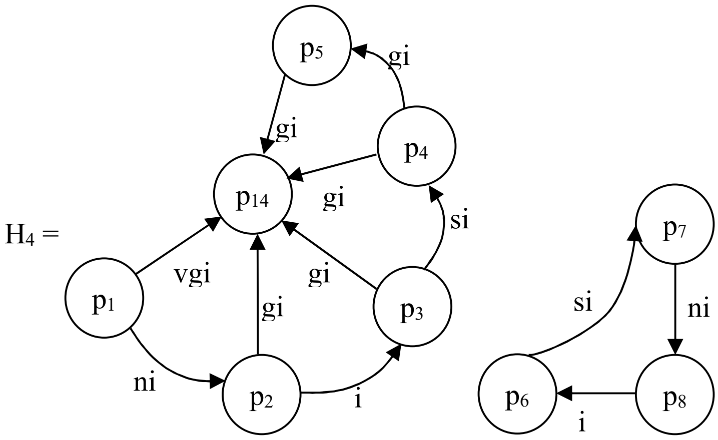


Figure 2.3.66

We see H_4 is a linguistic directed graph but is disjoint, it has 2 linguistic directed graphs as its components.

Now we have briefly described a few linguistic edge weighted subgraphs of linguistic edge weighted graphs.

Now we proceed onto give linguistic directed graph in increasing relations (here we take containment as the increasing relation if $A \subset B$ $A \to B$ if $B \subseteq A$ $B \to A$). However, we do not give the edge (linguistic) weights by the following Figure 2.3.67.

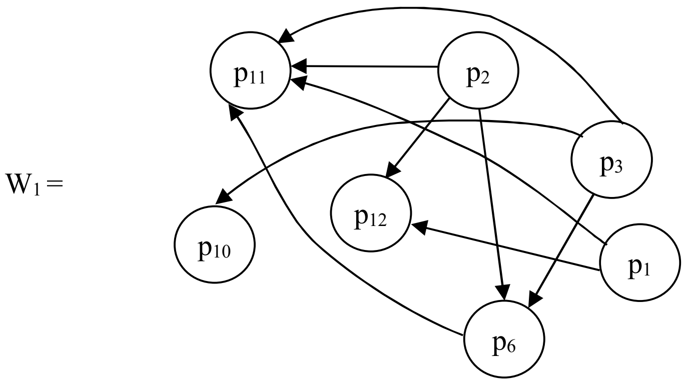


Figure 2.3.67

We see unlike the linguistic directed graph which are built using totally ordered sets are always complete linguistic directed graph (be it of increasing order or of decreasing order).

If totally ordered linguistic set is replaced by a partially ordered linguistic set we see the resulting linguistic directed graph in general may not be a complete one.

Now we give some more examples of them.

W_2 =

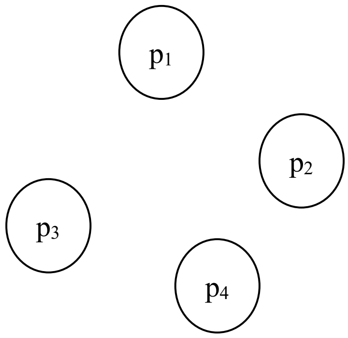


Figure 2.3.68

W_2 is just a linguistic directed empty graph as they cannot be related as each is a linguistic subset of order one.

Let W_3 be a linguistic directed graph given by the following Figure 2.3.69 using the linguistic subsets p_1, p_2, p_3, p_4 p_{15} and ϕ of P(S).

W_3 =

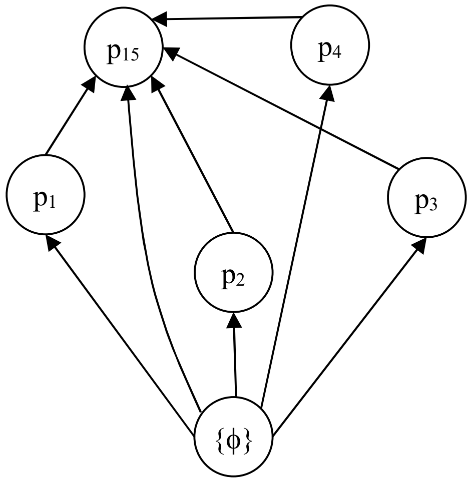


Figure 2.3.69

Let W_4 be a linguistic directed graph with entries from the linguistic power set P(S) (increasing order relation) given by the following Figure 2.3.70.

W_4 =

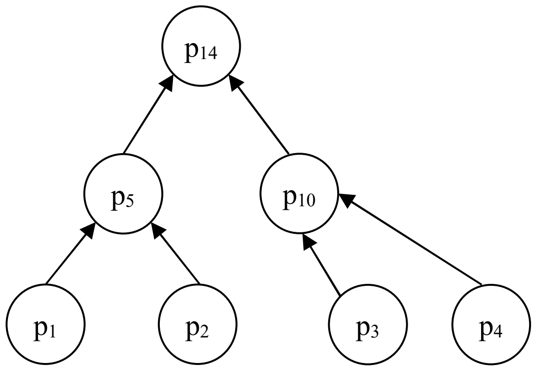


Figure 2.3.70

Clearly W_4 is a linguistic binary tree.

Now we provide some examples of linguistic directed graphs (of decreasing relation).

Let W_5 be a linguistic directed graph given in Figure 2.3.71.

W_5 =

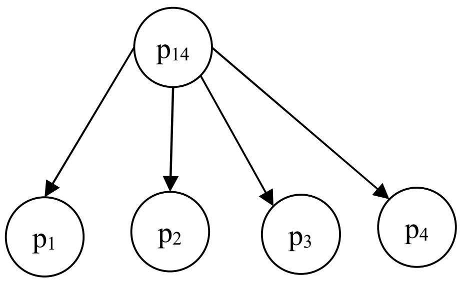


Figure 2.3.71

Let W_6 be a linguistic directed graph with entries from the linguistic power set P(S) of the linguistic set S given by the following Figure 2.3.72.

W_6 =

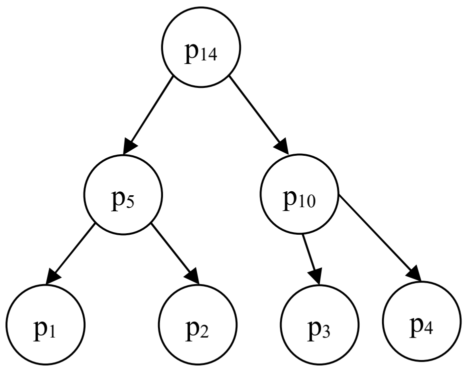


Figure 2.3.72

Let W_7 be the linguistic directed graph given by the following Figure 2.3.73.

$W_7 =$

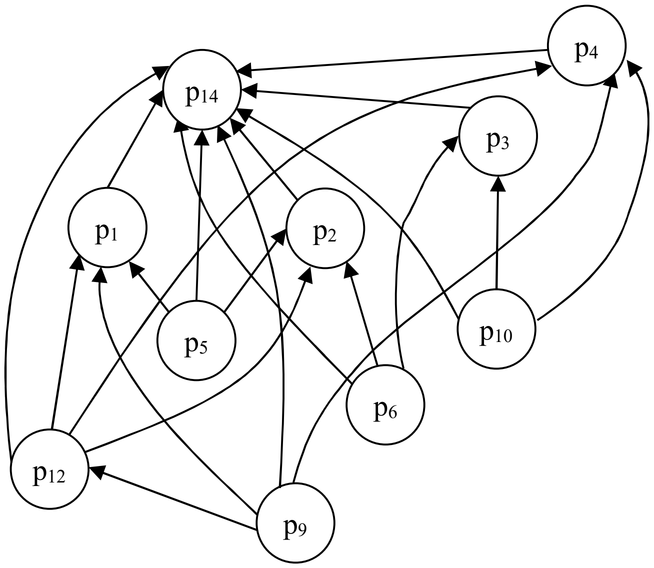


Figure 2.3.73

Let W_8 be the linguistic directed graph given by the following Figure 2.3.74.

$W_8 =$

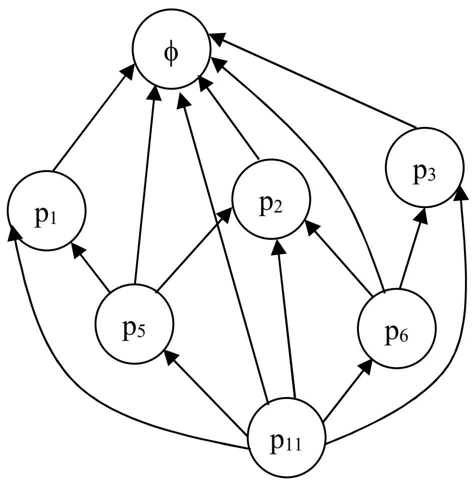


Figure 2.3.74

Clearly even W_8 is not a complete linguistic directed graph.

Consider W_9 the linguistic directed graph given by the following Figure 2.3.75.

W_9 =

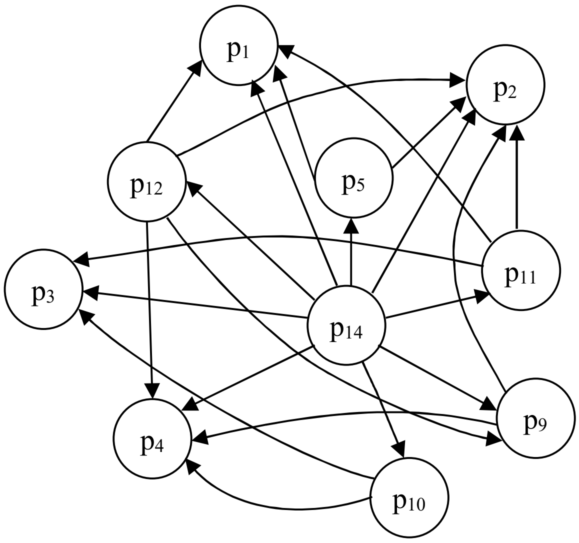


Figure 2.3.75

We see W_9 is also a linguistic directed graph which is not complete.

Now we provide linguistic subgraphs of linguistic graphs using partially ordered linguistic sets or subsets of linguistic power set P(S) of a linguistic set S.

We deal both linguistic subgraphs as well as edge linguistic subgraphs of these graphs. We take basically the linguistic power set P(S) given in example 2.3.5.

Let K be the linguistic graph given in the Figure 2.3.76.

K =

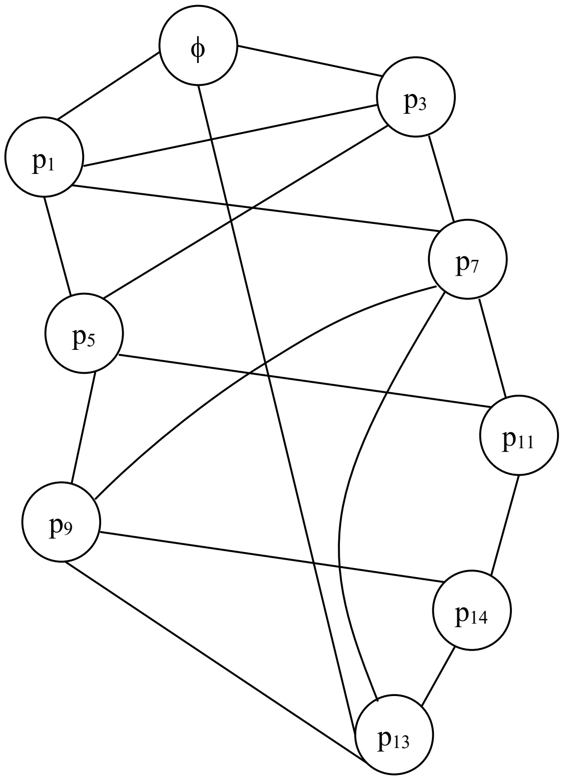


Figure 2.3.76

We give some of the linguistic subgraphs of K in the following.

B_1 =

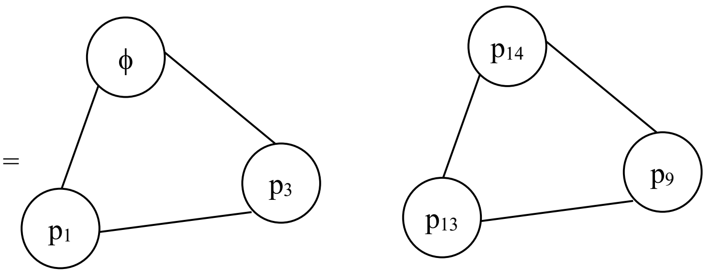


Figure 2.3.77

Clearly B_1 is not a connected linguistic subgraph of K.

B_1 is a disconnected linguistic subgraph with two components.

Let B_2 be a linguistic subgraph of K given by the following Figure 2.3.78.

B_2 =

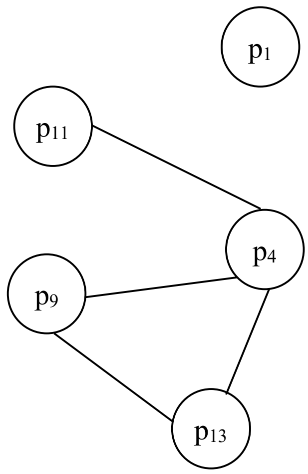


Figure 2.3.78

B_2 is also a disconnected linguistic subgraph of K.

The components of B_1 are different from the components of B_2 as B_2 has a linguistic vertex but in case of B_1 both are linguistic subgraphs.

Let B_3 be a linguistic subgraph of K given by Figure 2.3.79.

$B_3 =$

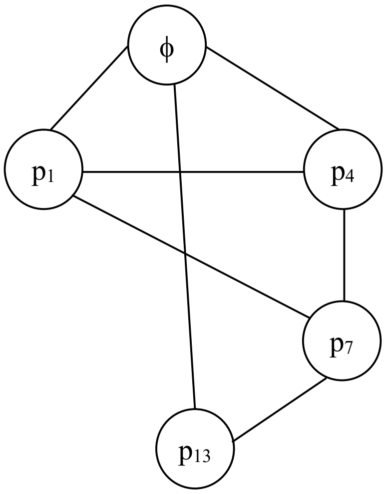


Figure 2.3.79

Now we give a few edge linguistic subgraphs of K.

Let C_1 be a edge linguistic subgraph of K obtained by removing the edges ϕp_3, ϕp_{13}, ϕp_1, p_1p_7, p_3p_7 and p_7p_{11}.

$C_1 =$

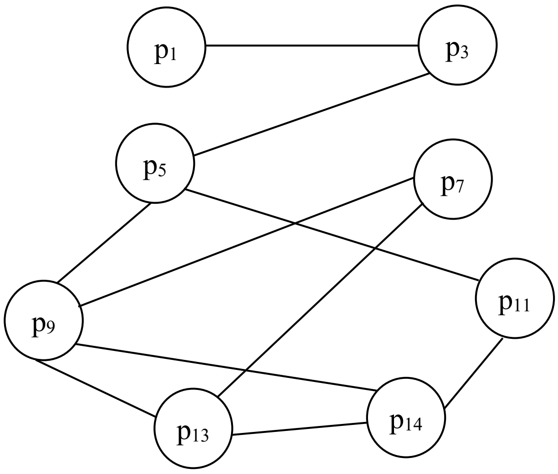


Figure 2.3.80

Let C_2 be the edge linguistic subgraph of K given by the following Figure for which these edges $p_1\phi$, p_1p_7, p_1p_3, p_1p_5, ϕp_{13}, p_7p_{11}, p_7p_{13}, p_7p_3, p_7p_9, p_9p_{13} and $p_{14}p_{13}$ and $p_{14}p_{13}$ are removed.

$C_2 =$

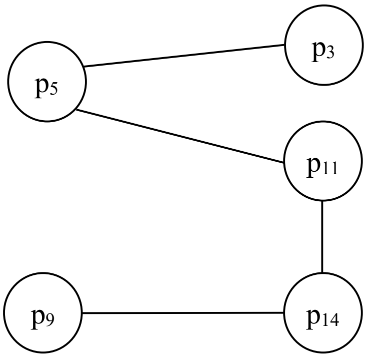


Figure 2.3.81

We see the edge linguistic subgraph is a line linguistic subgraph starting from p_3 and ending at p_9. That is

$C_3 =$

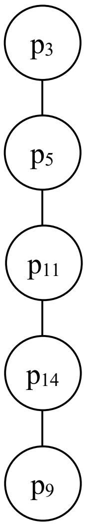


Figure 2.3.82

We can have several such linguistic subgraphs of K.

Now we find a few linguistic directed subgraphs of the linguistic directed graph M given by the following Figure 2.3.83.

M =

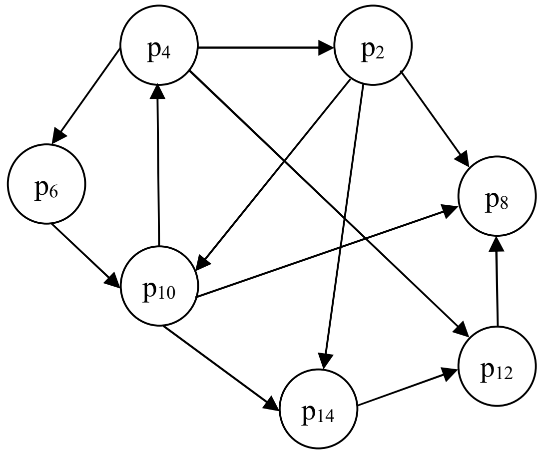


Figure 2.3.83

Let N_1 be the linguistic directed subgraph of M given by the following Figure 2.3.84.

N_1 =

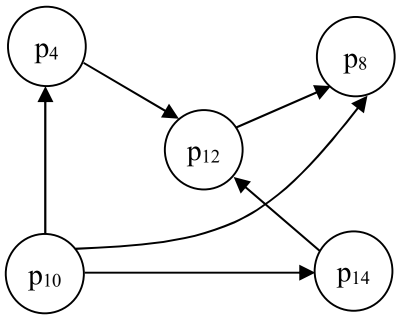


Figure 2.3.84

Let N_2 be the linguistic directed subgraph of M given by the Figure 2.3.85.

N_2 =

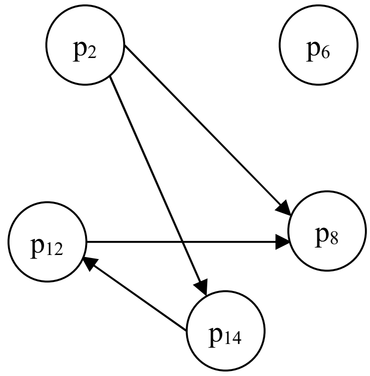


Figure 2.3.85

N_2 is just a linguistic directed subgraph of M which is disconnected with one component just the node p_6 and the other a proper linguistic directed subgraph of M.

Next we proceed onto give some edge linguistic directed subgraph R_1 of M given by the following Figure 2.3.86.

Let R_1 be constructed removing the following edges from M,

viz. $\overrightarrow{p_{10}p_{14}}$, $\overrightarrow{p_6p_{10}}$, $\overrightarrow{p_{10}p_4}$, $\overrightarrow{p_2p_{10}}$, $\overrightarrow{p_{10}p_8}$, $\overrightarrow{p_4p_2}$, $\overrightarrow{p_2p_{14}}$

and $\overrightarrow{p_2p_8}$.

$R_1 =$

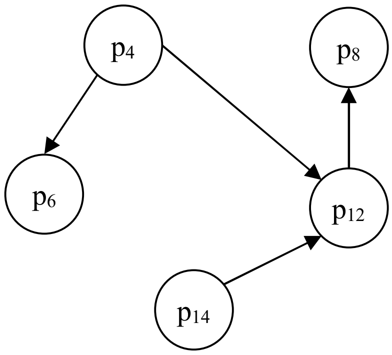


Figure 2.3.86

Let R_2 be a edge linguistic directed subgraph of M got by removing the following edges: $\overrightarrow{p_{14}p_{12}}$, $\overrightarrow{p_2p_{14}}$, $\overrightarrow{p_{12}p_8}$, $\overrightarrow{p_{10}p_{14}}$, $\overrightarrow{p_4p_2}$, $\overrightarrow{p_4p_6}$, $\overrightarrow{p_{10}p_4}$, $\overrightarrow{p_6p_{10}}$ and $\overrightarrow{p_4p_{12}}$ given in Figure 2.3.87.

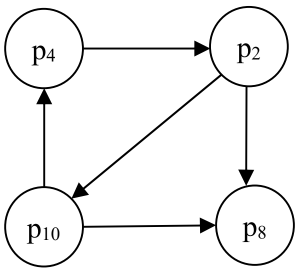


Figure 2.3.87

Now for the following linguistic directed graph Q (decreasing $p_{14} \to p_1$).

We find its linguistic directed subgraph and edge linguistic directed subgraphs of Q.

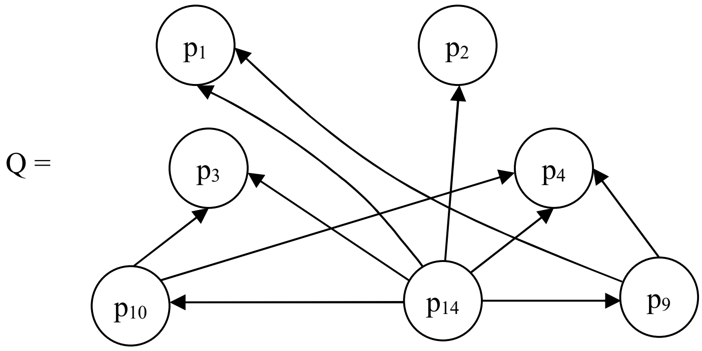


Figure 2.3.88

Let D_1 be the linguistic directed subgraph of Q given by the following Figure 2.3.89.

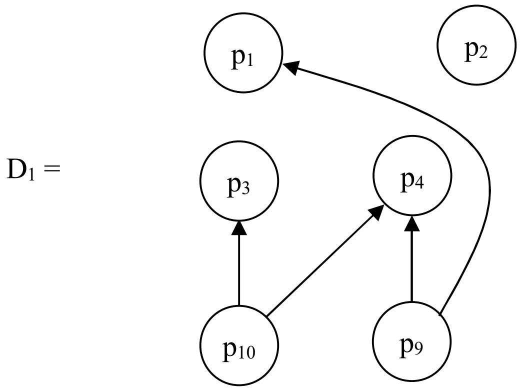


Figure 2.3.89

Clearly D_1 is a linguistic directed subgraph of Q which is not connected.

Let D_2 be the linguistic directed subgraph of Q given in the Figure 2.3.90.

$D_2 =$

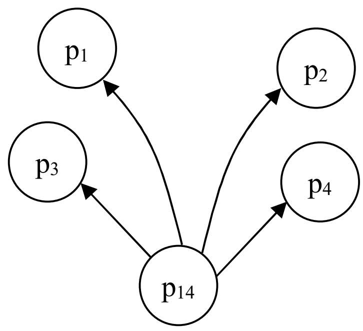


Figure 2.3.90

D_2 is a directed linguistic subgraph which is connected and is a tree.

Let E_1 be the edge linguistic directed subgraph of Q got by removing the edges $\overrightarrow{p_{14}p_1}$ and $\overrightarrow{p_{14}p_9}$ given by the Figure 2.3.91.

$E_1 =$

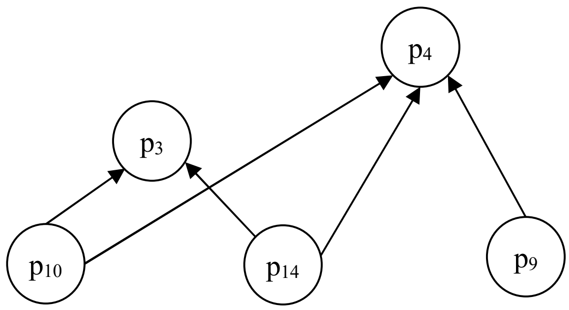


Figure 2.3.91

E_1 is a connected edge linguistic directed subgraph of Q. let E_2 be a edge linguistic directed subgraph of Q got by moving the edges $\overrightarrow{p_{14}p_1}$, $\overrightarrow{p_{14}p_2}$, $\overrightarrow{p_{14}p_3}$ and $\overrightarrow{p_{14}p_4}$ is given in the following Figure 2.3.92.

E =

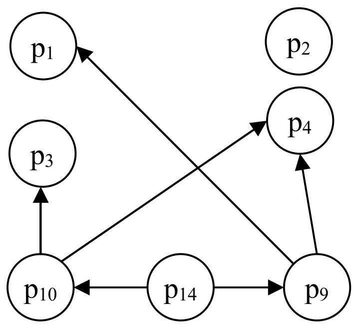


Figure 2.3.92

E_2 is a disconnected edge linguistic directed subgraph of Q and one of the components is just one vertex set and other is a proper linguistic directed subgraph of Q.

Next, we consider the linguistic directed graph W where the edge / relation is of increasing order that is $p_1 \rightarrow p_{14}$ given by the following Figure 2.3.93 using the linguistic power set P(S) of the linguistic set

S = {good, bad, fair, very good}

W =

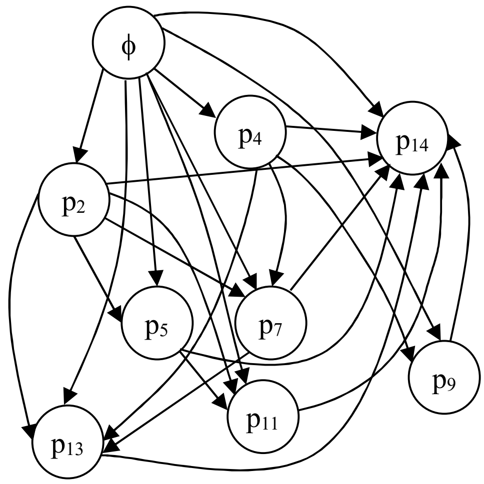


Figure 2.3.93

We find some linguistic directed subgraphs of W.

Let F_1 be a linguistic directed subgraph of W given by the following Figure 2.3.94.

F_1 =

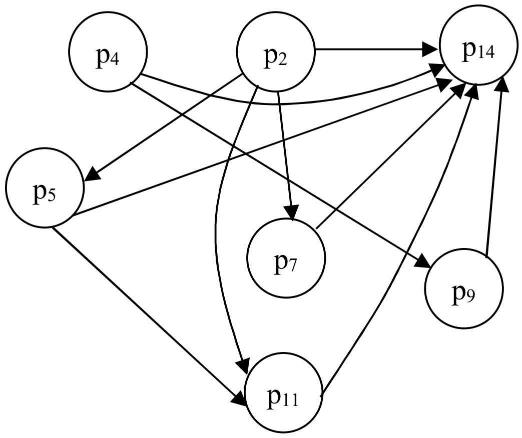


Figure 2.3.94

Let F_2 be a linguistic directed subgraph of W given by the following Figure 2.3.95.

F_2 =

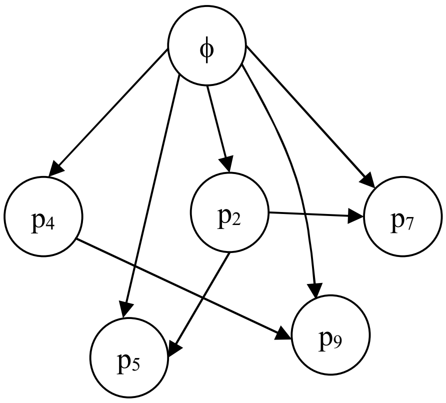


Figure 2.3.95

Let G_1 be the edge linguistic directed subgraph of W got by removing the edges

$\overrightarrow{\phi p_2}$, $\overrightarrow{\phi p_7}$, $\overrightarrow{p_2 p_7}$, $\overrightarrow{\phi p_4}$, $\overrightarrow{p_4 p_9}$, $\overrightarrow{p_2 p_5}$ $\overrightarrow{p_2 p_{13}}$, $\overrightarrow{\phi p_{11}}$, $\overrightarrow{\phi p_{13}}$ and $\overrightarrow{\phi p_{14}}$ from W.

G_1 is given in Figure 2.3.96.

$G_1 =$

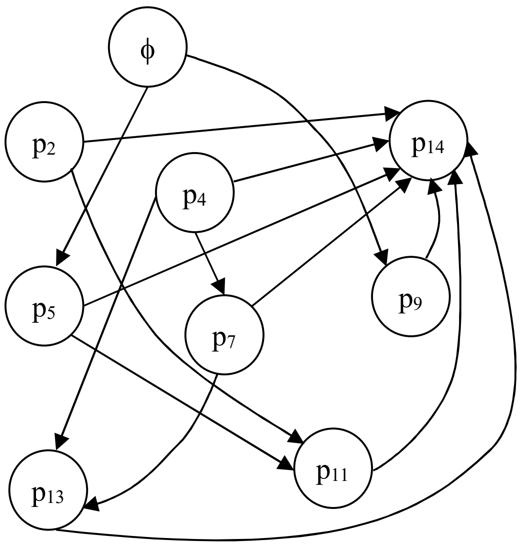


Figure 2.3.96

We see none of the nodes of W are removed by the removal of these edges from W.

G_1 is the edge linguistic directed subgraph of W with 10 edges removed from W.

Now let G_2 be a edge linguistic directed graph got by removing the following edges from W;

that is edges $\overrightarrow{\phi p_5}$, $\overrightarrow{\phi p_{14}}$, $\overrightarrow{p_2p_{14}}$, $\overrightarrow{p_4p_{14}}$, $\overrightarrow{p_5,p_{11}}$, $\overrightarrow{p_5p_{14}}$, $\overrightarrow{p_7p_{14}}$, $\overrightarrow{p_9p_{14}}$, $\overrightarrow{p_{11}p_{14}}$, $\overrightarrow{p_{13}p_{14}}$, $\overrightarrow{p_2p_{13}}$, $\overrightarrow{p_4p_{13}}$, $\overrightarrow{p_7p_{13}}$ and $\overrightarrow{p_2p_5}$, $\overrightarrow{\phi p_{13}}$.

V =

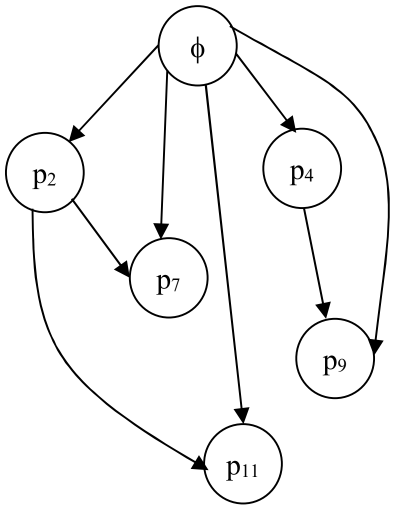


Figure 2.3.97

Now having seen linguistic directed subgraphs of both types we proceed describe define and develop the notion of linguistic adjacency matrices in case of the 3 types of matrices described in section 2.1 this chapter.

2.4 Linguistic edge weighted graphs

In this section we introduce the notion of linguistic edge weighted linguistic graphs.

The edge values also take only linguistic values relating the linguistic edges.

Before we define these concepts we describe by some examples.

Example 2.4.1. Let

S = {good, bad, best, very bad, very good, fair, just bad, just good, worst}

be the linguistic set associated with the linguistic variable, the performance of a worker in a factory.

We give the linguistic edge set which evaluates the worker is improving or not improving or remaining good or remaining bad and so on.

E = {i - improving, ni - not improving, vi - very much improved, ji - just improving, rg - remaining good, rb - remaining bad, di - drastic improvement}.

We just give a linguistic graph the values as labels. Let G be a linguistic graph with weighted linguistic edge values given by the Figure 2.4.1.

G =

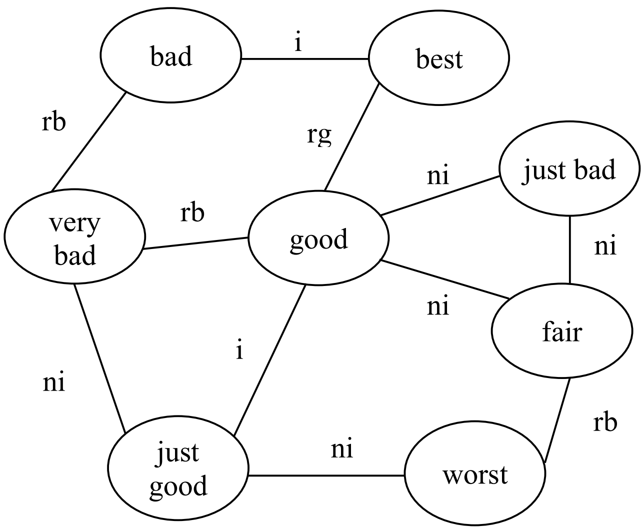


Figure 2.4.1

Let G_1 be a line linguistic edge weighted graph given by the following Figure 2.4.2.

G_2 =

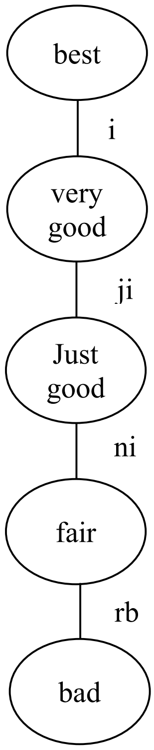


Figure 2.4.2

Clearly G_2 is a linguistic edge weight graph which is a line linguistic graph.

Let G_3 be the complete linguistic edge weighted graph which is given by the following Figure 2.4.3.

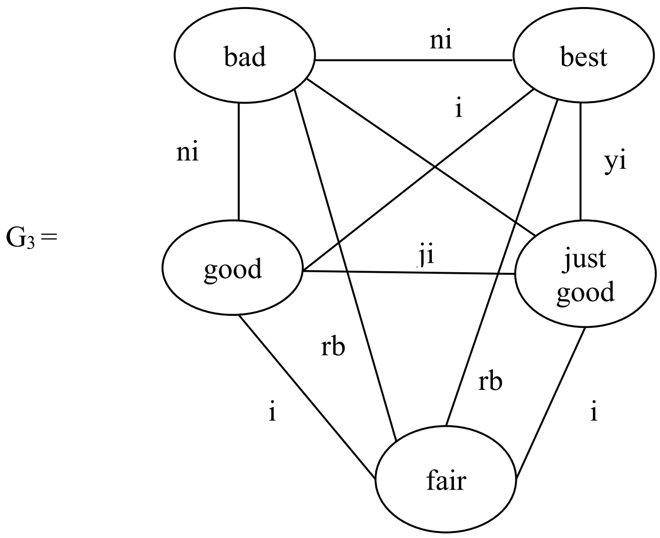


Figure 2.4.3

Let G_4 be the star linguistic edge weighted graph given by the following Figure 2.4.4.

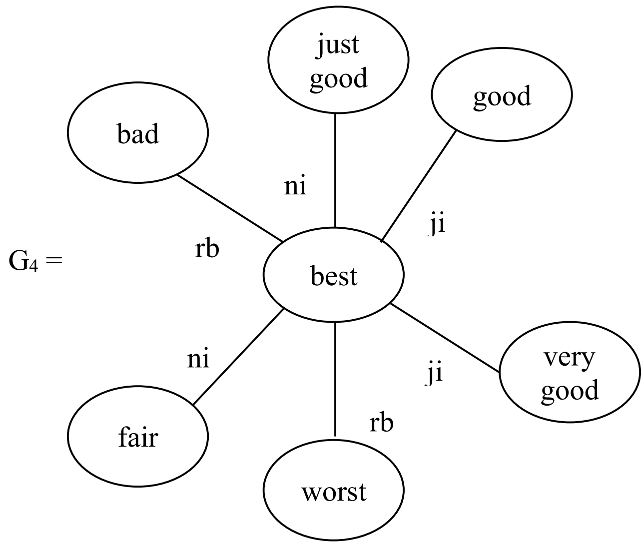


Figure 2.4.4

We now give wheel linguistic edge weighted graph in the following Figure 2.4.5.

G_5 =

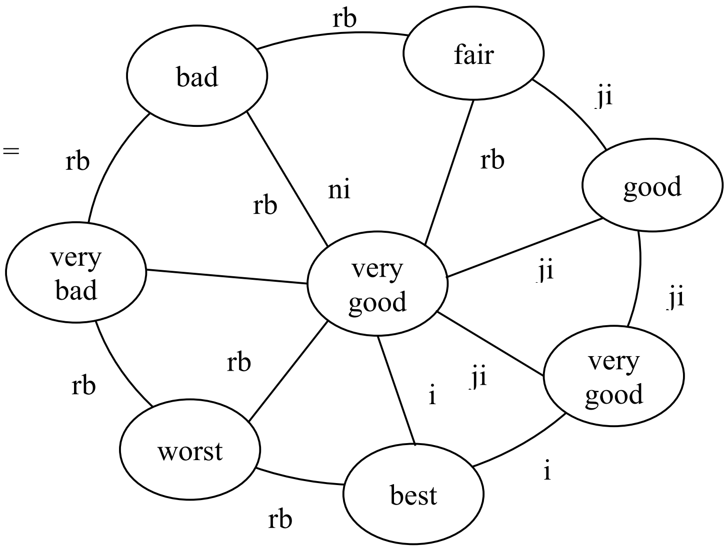


Figure 2.4.5

We wish to state though the direction is not specified in all these edge weighted linguistic graphs (dyads) we see that by seeing edge weights one can for instance say if it is

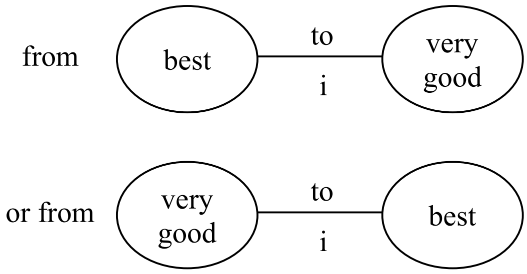


Figure 2.4.6

so the direction is the worker has been assessed from the very good worker to a best worker which implies he/she is improving. If on the other hand if the linguistic edge weight

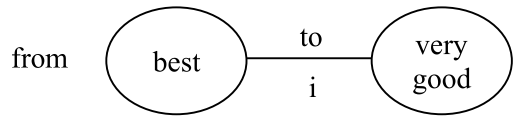


Figure 2.4.7

is ni and cannot be i.

We can have several such examples of edge weighted linguistic graphs. We leave getting different undirected edge weighted linguistic graphs as exercise to the reader.

Now we give an example of edge weighted linguistic directed graphs.

Example 2.4.2. Let

S = {tall, very tall, just tall, tallest, medium short, very medium, just medium, short, very short, just short, shortest}

be the growth of the plant as observed a scientist associated with the linguistic variable is growth of the plants in general.

Let us consider linguistic edge weights of this S given by

E = {good, bad, medium, very good, very bad, just good, just bad, worst (no proper growth), very very bad, very very good}.

That is the growth is good, very good etc etc.

Let H be the edge weighted linguistic directed graph given by the following Figure 2.4.8.

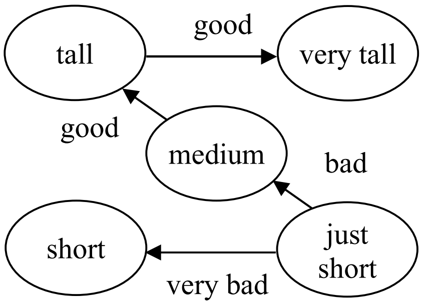
H = 

Figure 2.4.8

Let B be the edge weighted linguistic directed graph with edge weights from the linguistic set E and nodes are taken from the linguistic set S. B is given by the following Figure 2.4.9.

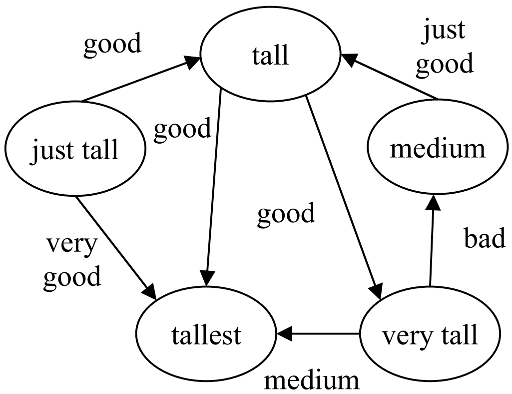
B = 

Figure 2.4.9

We can have several such examples of such linguistic directed graphs which are neither of increasing nodes or of decreasing nodes.

Now we give some examples of directed edge weighted linguistic graphs with increasing nodes and decreasing nodes.

Example 2.4.3. Let us consider the linguistic variable age of a person.

S = {old, oldest, just old, very young, adult young, youngest, just young, just middle age, middle age}

be the linguistic set associated with the linguistic variable age.

Let E be the linguistic edges which are weights which describes the functioning / ability to do work.

E = {very able - va, just able - ja, not that able - na, partially able - pa, able - a}

Let M be the linguistic directed edge weighted graph in increasing order of age.

M =

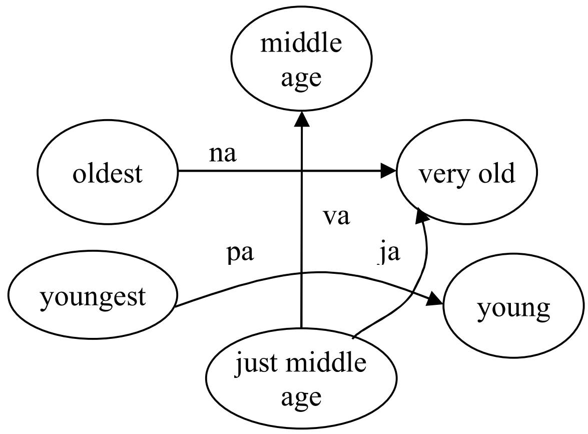


Figure 2.4.10

Let N be a edge weighted linguistic graph in the decreasing direction (old $\rightarrow$ middle age) given by the following Figure 2.4.11.

N =

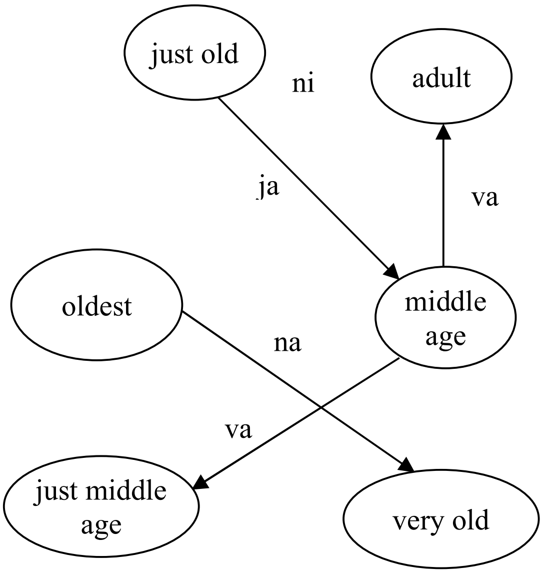


Figure 2.4.11

Here we wish to keep on record the following:

1. The linguistic edge weights given for any linguistic set S is in the hands of the expert. The expert can give any suitable / appropriate values for the same.

2. Even the linguistic set associated with any linguistic variable is in the hands of expert to give appropriately.

The flexibility at large helps the expert to choose with ease the linguistic sets which largely depends on the way he / she thinks is appropriate.

Example 2.4.4. Let S and E be as in example 2.4.2.

Let W be the linguistic edge weighted graphs given by the following Figure 2.4.12.

W =

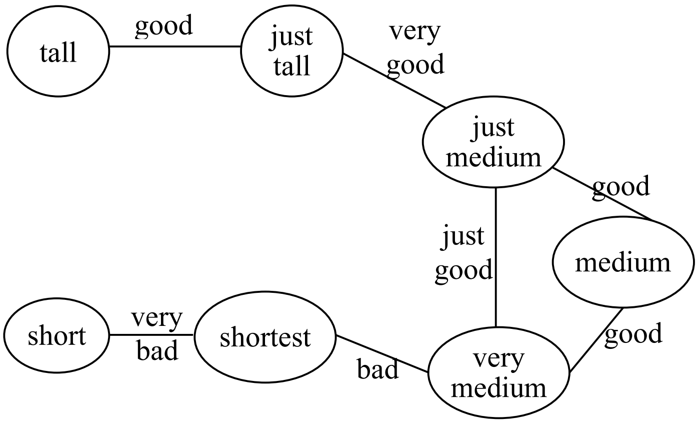


Figure 2.4.12

We give a few of its edge weighted linguistic subgraph H of W.

H_1 =

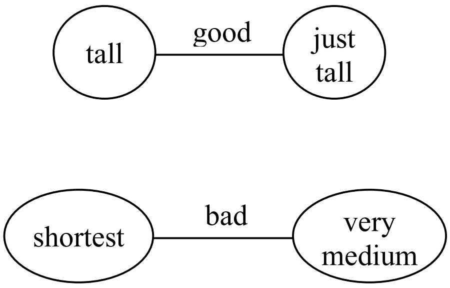


Figure 2.4.13

Let H_2 be the edge weighted linguistic subgraph H_2 of W.

$H_2 =$

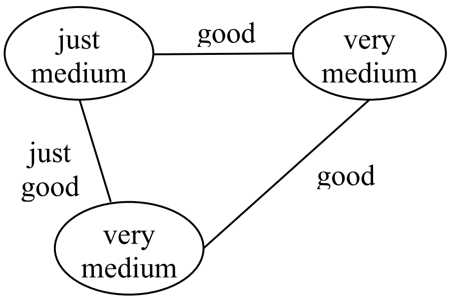


Figure 2.4.14

Let H_3 be a edge weighted linguistic subgraph of W given by the following Figure 2.4.15.

$H_3 =$

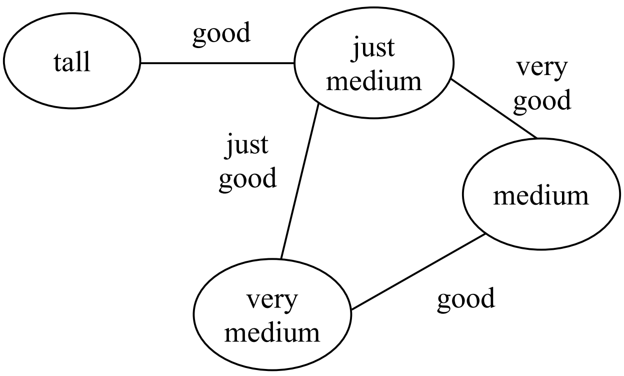


Figure 2.4.15

We see the linguistic edge weighted subgraph H_1 is disconnected it has two components both of which are linguistic edge weighted dyads.

The linguistic edge weighted subgraph H_2 of W is a linguistic edge weighted triad.

The linguistic edge weighted subgraph H_3 of W is a linguistic edge weighted connected subgraph.

Now having seen examples of linguistic edge weight subgraphs.

We now proceed onto give examples of linguistic edge weighted directed graph.

We use the same linguistic set S and the linguistic edge set E given in example 2.4.2. and obtain a directed edge weighted linguistic graph P.

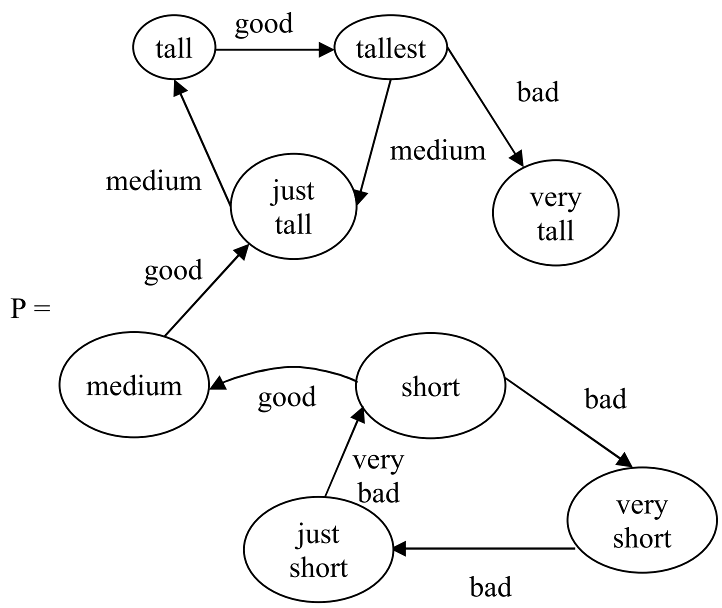


Figure 2.4.16

Let Q_1 be the linguistic edge weighted directed subgraph of P given by the following Figure 2.4.17.

Q_1 =

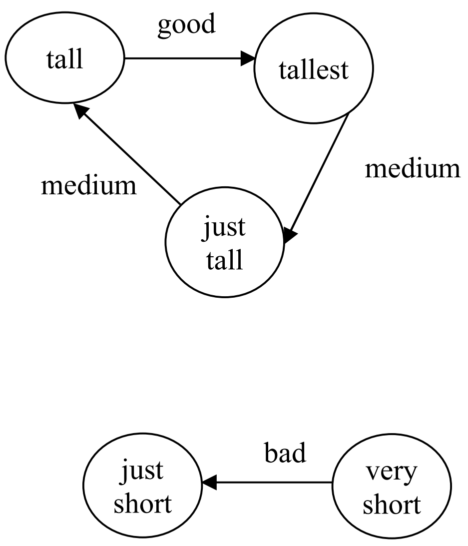


Figure 2.4.17

Clearly Q_1 is a disconnected edge weighted linguistic directed subgraph of P.

The two components of Q_1 is a linguistic edge weighted directed triad and a linguistic edge weighted directed dyad.

Let Q_2 be a edge weighted linguistic directed subgraph of P given by the following Figure 2.4.18.

Q_2 =

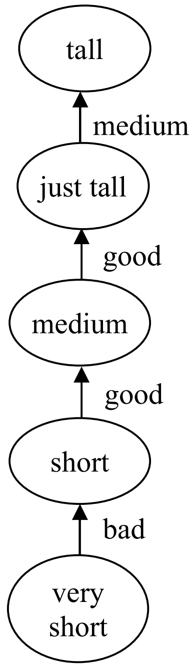


Figure 2.4.18

Now certain factors are important, when we put

Q_3 =

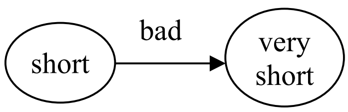


Figure 2.4.19

We mean that the plant was short after a time period it is not grown so for; its time of growth it is termed very short so we are forced to indicate it by “bad”.

Q_4 is a connected edge weighted directed linguistic subgraph of P.

Consider Q_4 a edge weighted linguistic subgraph of P given by the Figure 2.4.20.

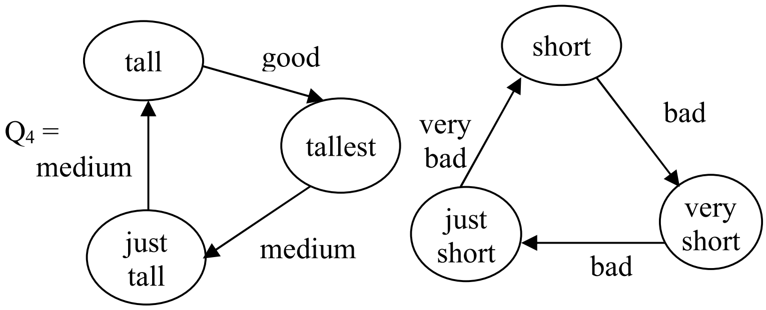


Figure 2.4.20

Q_4 is a disconnected edge weighted linguistic directed subgraph of P which are two components both are edge weighted linguistic directed triads of P.

Now the interesting difference between the usual directed linguistic edge weighted graphs and the directed linguistic edge weighted graphs

(increasing tall $\xrightarrow{\text{good}}$ very tall) are as follows.

We can in case of the directed linguistic edge weighted graphs we can also mark

very tall $\xrightarrow{\text{bad}}$ tall

this implies at one particular phase of time the plants was very tall for its age (growth was good) but as they become older after a stipulated time they were only tall that is no growth so we cannot say the growth is good so only we mark it by bad.

This way of mapping is not possible in increasing or decreasing edge values. In fact, these are unique linguistic edge weighted directed graphs unlike the edge weighted linguistic directed graph which can be many and has flexibility.

Now in most case we get only a complete edge weighted linguistic graph if the linguistic set associated with the linguistic variable can be totally orderable.

This is entirely different if the linguistic set is only a partially ordered set. We have surplus number of such partially ordered set by considering the linguistic power set P(S) of the linguistic set S.

We provide example of linguistic edge weighted directed graph (increasing order and find its subgraph).

For this also we use the linguistic set S and the linguistic edges E given in example 2.4.2.

Let R be the linguistic edge weighted directed graph given by the following Figure 2.4.21.

R =

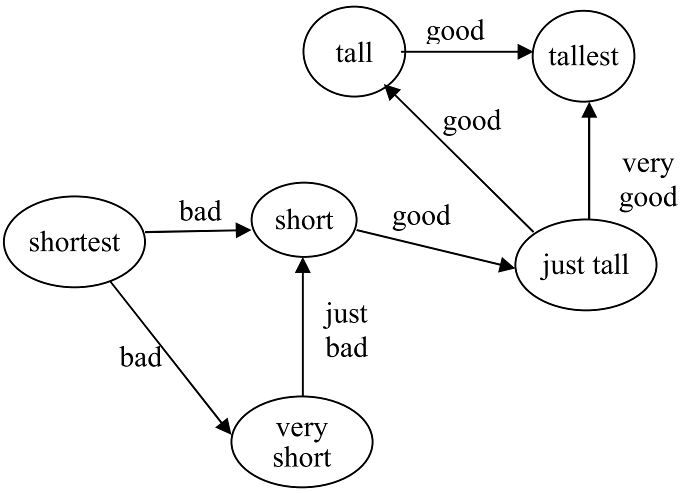


Figure 2.4.21

Let F_1 be the linguistic edge weighted directed subgraph of R given by the following Figure 2.4.22.

F_1 =

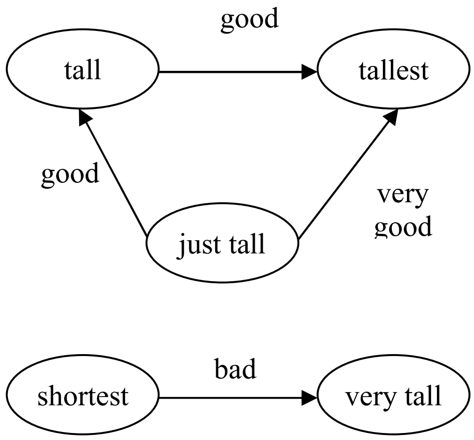


Figure 2.4.22

F_1 is a disconnected edge weighted directed subgraph of R which has two distinct components one is a edge weighted linguistic directed triad whereas the other one is a edge weighted linguistic directed dyad.

Let F_2 be a linguistic edge weighted directed subgraph of R given by the following Figure 2.4.23.

F_2 =

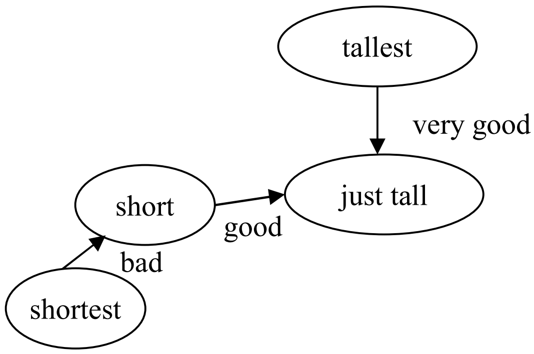


Figure 2.4.23

F_2 is a connected edge weighted linguistic directed subgraph which is a line graph.

Now using the same set of linguistic set S and the edge linguistic set E given in example 2.4.2.

We give an example of a linguistic edge weighted directed graph (decreasing direction relation or edge) Z given by the following Figure 2.4.24.

Z =

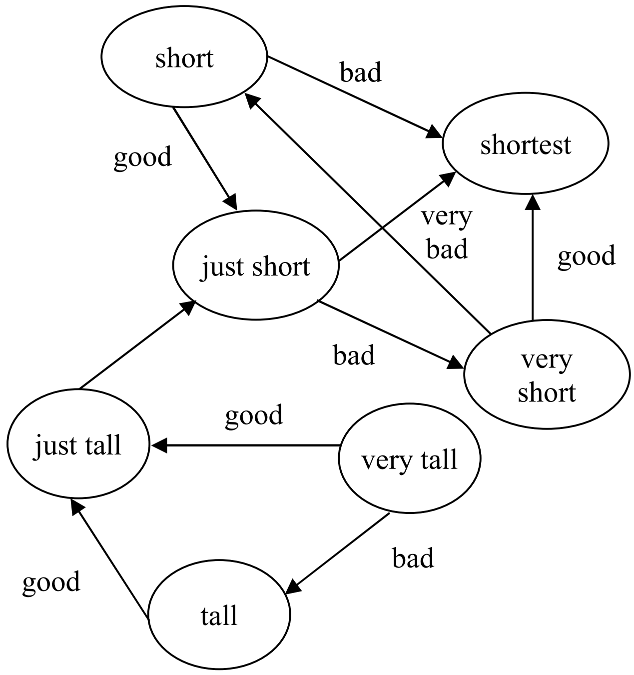


Figure 2.4.24

Let X_1 be the linguistic directed edge weighted subgraph of E given in Figure 2.4.25.

X_1 =

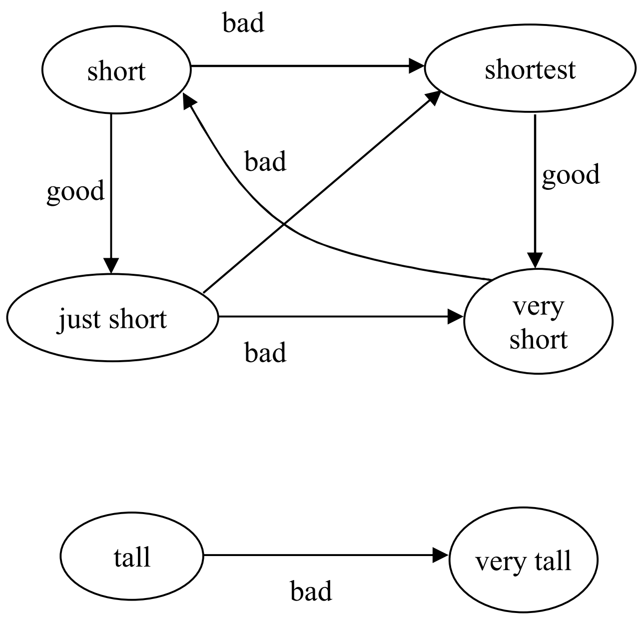


Figure 2.4.25

X_1 is a disconnected edge weighted linguistic directed subgraph of E.

It has two components one is complete edge weight linguistic directed graph of order 4 and other component is just a linguistic directed edge weighted dyad.

Let X_2 be the edge weighted directed linguistic subgraph of E given by the following Figure 2.4.26.

$X_2 =$

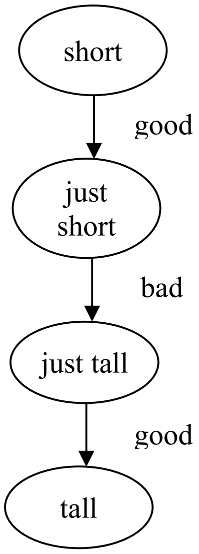


Figure 2.4.26

This is a line linguistic edge weighted directed subgraph of E.

Now we find edge linguistic edge weighted directed subgraphs of linguistic edge weighted directed graph E in the following.

Let Y_1 be a edge-linguistic edge weighted directed subgraph of E got by removing the edges.

$\overrightarrow{(\text{just tall})(\text{just short})}$, $\overrightarrow{(\text{just short})(\text{very short})}$,

$\overrightarrow{(\text{short})(\text{just short})}$ and $\overrightarrow{(\text{just short})(\text{shortest})}$

is given in Figure 2.4.27.

$Y_1 =$

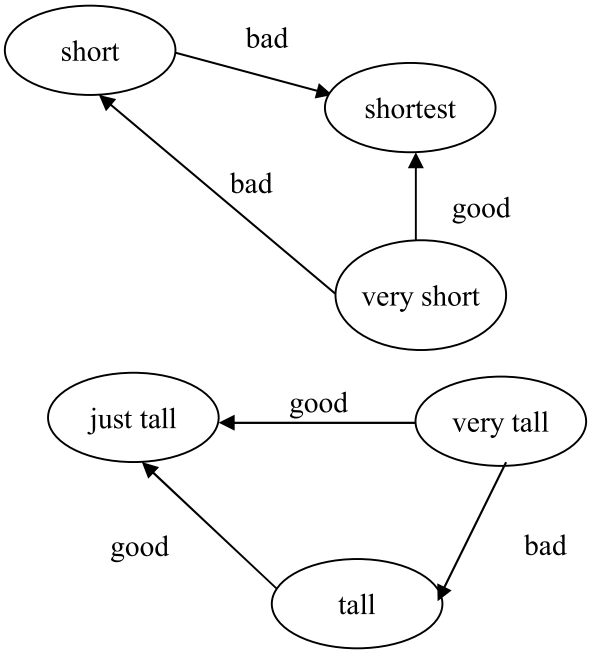


Figure 2.4.27

Y_1 is a disconnected edge linguistic edge weighted directed subgraph of E. There are two components for Y_1 both are linguistic edge weighted directed triads.

Let Y_2 be the edge linguistic edge weighted directed subgraph of E got by removing the edge

$\overrightarrow{(\text{just short})(\text{very short})}$, $\overrightarrow{(\text{very short})(\text{short})}$ and

$\overrightarrow{(\text{very short})(\text{shortest})}$

given in the following Figure 2.4.28.

Clearly Y_2 is a connected edge linguistic edge weighted directed subgraph of E.

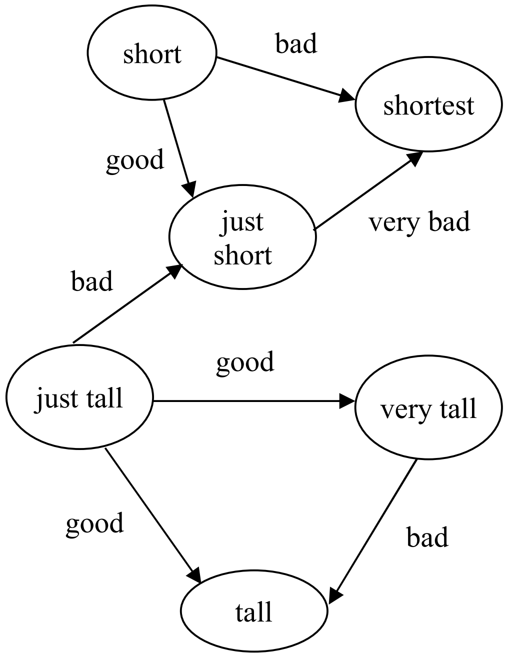


Figure 2.4.28

Now we find the edge linguistic edge weighted directed subgraph of the linguistic edge weighted directed graph R given in Figure 2.4.21.

Let H_1 be the edge linguistic edge weighted directed subgraph of R got by removing the edge

$\overrightarrow{(\text{short})(\text{just tall})}$ given by Figure 2.4.29.

H_1 =

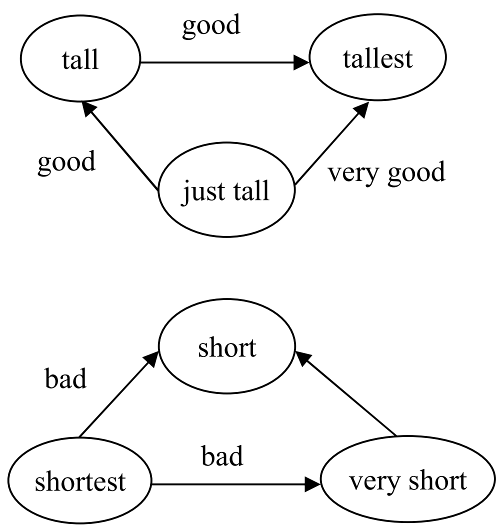


Figure 2.4.29

Clearly H_1 is a disconnected edge linguistic edge weighted directed subgraph of R and both the components are linguistic edge weighted directed triads.

Let H_2 be a edge linguistic edge weighted directed subgraph of R given by the following Figure 2.4.30.

This linguistic subgraph H_2 is got by removing the edges

$\overrightarrow{(\text{tall})(\text{good})}$, $\overrightarrow{(\text{just tall})(\text{tall})}$, $\overrightarrow{(\text{shortest})(\text{short})}$ and

$\overrightarrow{(\text{shortest})(\text{very short})}$.

H_2 =

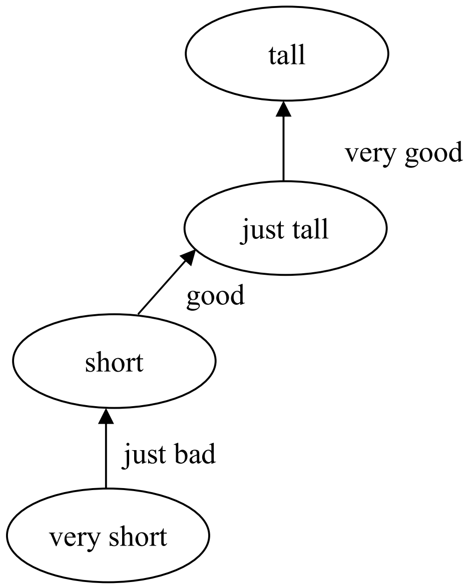


Figure 2.4.30

Clearly H_2 is a connected edge-linguistic edge weighted directed subgraph of R.

Now we find the edge linguistic edge weighted directed subgraphs of P given in Figure 2.4.18.

Let N_1 be a edge linguistic edge weighted directed subgraph of P obtained by removing the edges

$$\overrightarrow{(\text{medium})(\text{just tall})} \text{ and } \overrightarrow{(\text{short})(\text{medium})}$$

from P given by the following in Figure 2.4.31.

$N_1 =$

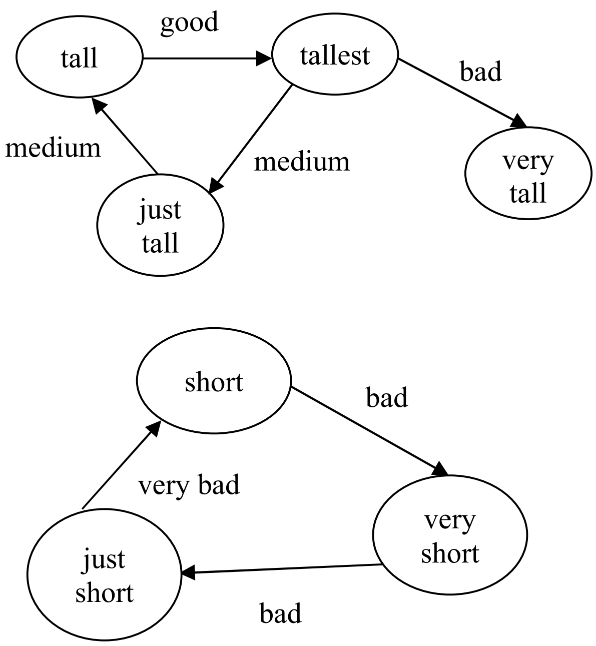


Figure 2.4.31

Let N_2 be a edge linguistic edge weighted directed subgraph of P given by the following Figure 2.4.32 got by removing the edges

$$\overrightarrow{(\text{tall})(\text{tallest})},\ \overrightarrow{(\text{just tall})(\text{tall})},\ \overrightarrow{(\text{short})(\text{very short})}$$

given by the following

N_2 =

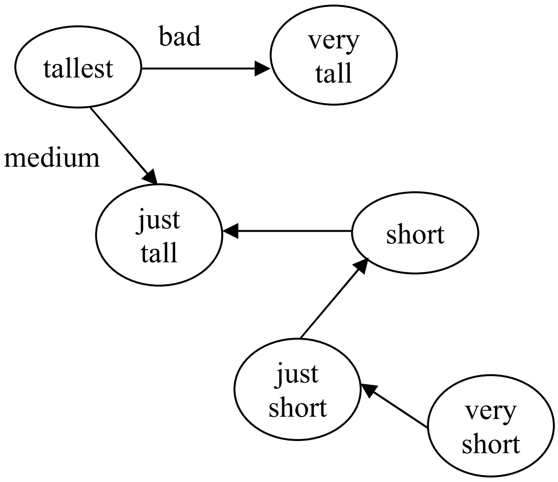


Figure 2.4.32

Clearly N_2 is a connected subgraph of P.

2.5 Suggested Problems

In this section we propose some problems which are mostly exercises for the reader to carefully attempt them in order to build a complete knowledge about this newly developing linguistic graphs.

This expertise will help the reader to build linguistic networks and linguistic models using these linguistic graphs.

2.5.1 Find all linguistic graphs using the linguistic set S where S = {tall, very tall, just tall, short, very short, medium, just medium, very very short}.

i) How many of these linguistic graphs using S are complete linguistic triads?

ii) How many linguistic graphs using S are complete graphs of order 5?

iii) Find the biggest linguistic circle graph using S.

iv) Find the largest linguistic wheel graph using S.

iv) Give the largest linguistic star graph using the linguistic S.

2.5.2 Let G be a linguistic graph given in the following using the linguistic set S in problem.

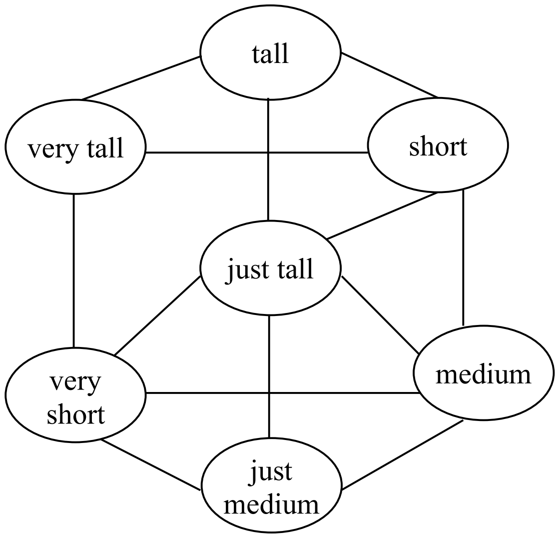


Figure 2.5.1

i) Find all linguistic complete subgraphs of G of order four.

ii) Find all linguistic subgraphs of G of order four.

iii) Find all complete linguistic triads of G.

iv) Find all linguistic subgraphs of G of order 5.

v) What is the biggest linguistic subgraph of G?

vi) How many such linguistic subgraphs of G exist mentioned in problem 5?

vii)

H_1 =

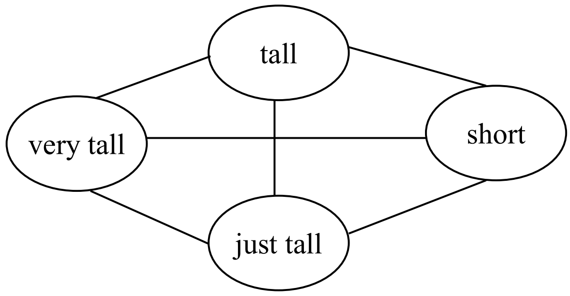


Figure 2.5.2

Is H_1 given in the above Figure a linguistic subgraph of G? Justify your claim?

viii) H_2 =

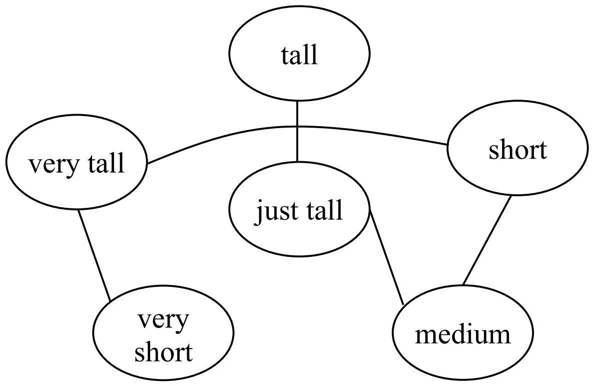


Figure 2.5.3

Is H_2 a linguistic subgraph of G or a linguistic edge subgraph of G? Justify your claim.

2.5.3 Find some interesting properties enjoyed by linguistic graphs in general.

2.5.4 Let S = {fastest, fast, just fast, very fast, slow, just slow, slowest, very slow, medium speed, very very slow, ϕ = (no speed just in static state which usually occurs in signals)}

be the linguistic set associated with linguistic variable moving vehicle on road.

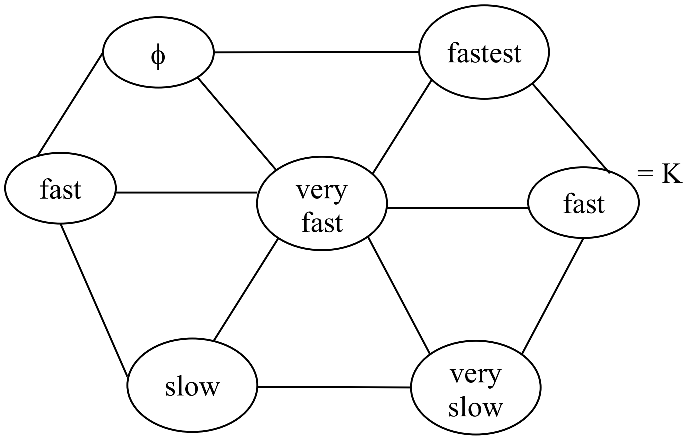


Figure 2.5.4

be the linguistic wheel.

i) Find all linguistic triads associated with the linguistic graph K.

ii) Find all linguistic subgraphs of K of order four.

iii) Find all linguistic subgraphs of K of order 5.

iv) How many linguistic circles can be got using the linguistic graph K?

v) Find all linguistic subgraphs of order 67.

vi) Obtain any other special property associated with K.

2.5.5 Let S be the linguistic set associated with the linguistic variable growth of paddy plants from the time it is sown till they are harvested.

S = {average, not up to mark, no good yield good, stunted, very stunted, not good, good in h eight but was not green, infested by insects, very good, average, yield good yield, just stunted, just good, just average}.

Since it is over all growth S cannot be ordered for we cannot find any relation with colour.

G =

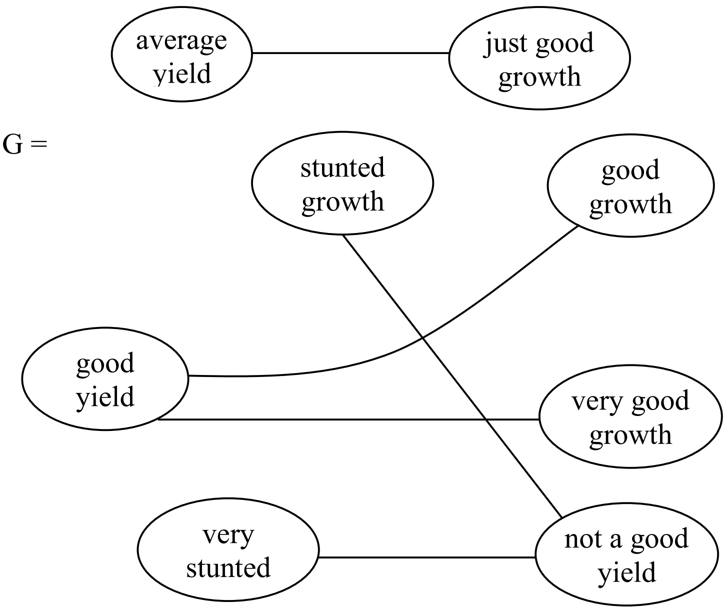


Figure 2.5.5

Let G be the linguistic graph.

i) Find all linguistic subgraphs of G.

ii) Find all connected linguistic subgraphs of G.

iii) Find all edge linguistic subgraphs of G.

iv) Does G contain a linguistic triad?

v) Obtain any other special features associated with this G.

vi) Using the linguistic set S as the nodes obtain at least 3 linguistic graphs of order 12, 10 and 9.

vii) Study questions (i) to (iv) in case of these 3 linguistic graphs given in (vi).

viii) Can S be a partially ordered set?

ix) How many such type of partial orders can be defined on the linguistic set S?

x) Show linguistic sets can give many types of partial ordering on it.

xi) Can we separate S into two classes (say) S_1 and S_2 so that both S_1 and S_2 are totally ordered?

xii) Can you supply a linguistic edge set for this linguistic set S?

xiii) Can you give directed edges to the linguistic nodes used in the linguistic graph G given in Figure 2.5.5?

xiv) Can you give directed increasing edges using the linguistic nodes of the linguistic graph G given in Figure 2.5.5?

xv) Draw the linguistic directed graph with decreasing edges using nodes of G given in the linguistic graph 2.5.5.

xvi) Now can we give edge weights to the linguistic graph G given in Figure 2.5.5?

xvii) Obtain a linguistic edge weighted subgraphs of the linguistic edge weighted graph given in problem (xvi).

xviii) Using the nodes of the linguistic graph G given in Figure 2.5.5 draw.

a) The linguistic edge weighted directed graph (increasing order).

b) The linguistic edge weight directed graph (decreasing order).

xix) Find all linguistic edge weighted directed subgraphs in xviii (a) and (b).

xx) Find all edge linguistic edge weighted subgraphs in xviii of (a) and (b).

xxi) Does these linguistic edge directed weighted subgraphs in xviii (a) have linguistic directed edge weighted triads?

xxii) How many edge linguistic edge weighted directed subgraph in xviii(b) are linguistic directed edge weighted triads?

2.5.6 Let S = {good, bad, very good, best, lazy, very fair, medium, very bad, fair, just fair, just good, just bad, very medium}

be the linguistic set associated with the linguistic variable performance of the worker at a factory.

i) Is the linguistic set S totally orderable?

ii) Give a linguistic edge set E for the linguistic set S.

iii) Let H be the linguistic graph with linguistic set S given by the following Figure 2.5.6.

H =

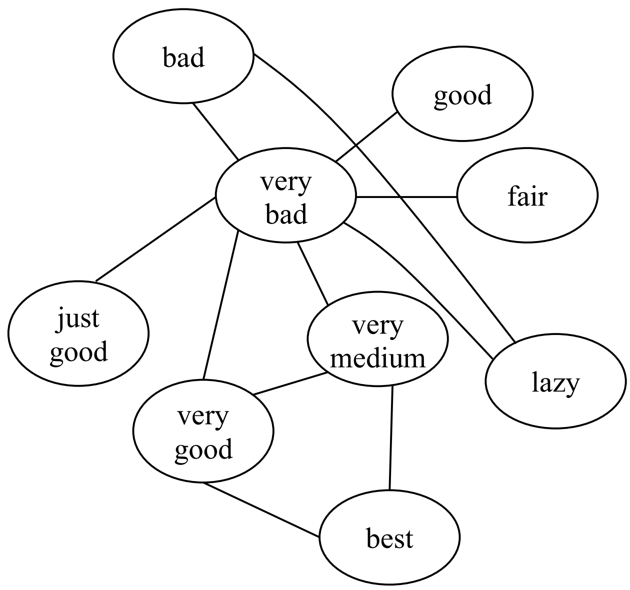


Figure 2.5.6

iv) Study all the problems (i) to (xxii) of 2.5.5 for this H given in linguistic graph in figure 2.5.6.

2.5.7 Let S = {good, bad, best, very good, just bad, fair, worst, very bad}

be the linguistic set. P(S) be the linguistic power set of S.

Let K =

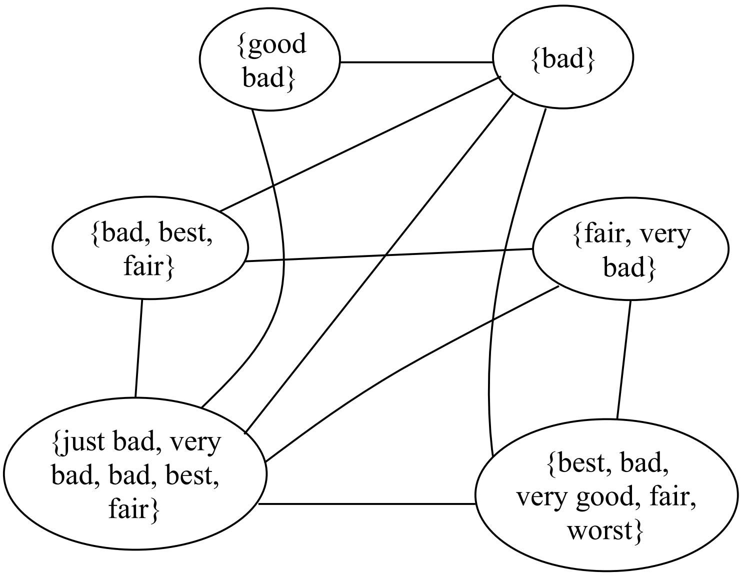


Figure 2.5.7

a) Study all the problem 2.5.5 (i) to (xxii) using the linguistic graph K given in Figure 2.5.7.
b) What is the difference between the linguistic set associated with the graph K (Figure 2.5.7) and graphs G (Figure 2.5.5) and H (Figure 2.5.6)?

Chapter Three

ADJACENCY LINGUISTIC MATRICES OF LINGUISTIC GRAPHS AND THEIR APPLICATIONS

3.1 Introduction

In this chapter, we introduce adjacency matrices, which are linguistic in nature and related to linguistic graphs.

Here we also introduce the notion of linguistic bigraphs, and finally, we describe and discuss their applications. This chapter has four sections. Section 1 is introductory in nature; section two introduces the adjacency matrices in the case of linguistic graphs and linguistic graphs, which are directed generally directed in increasing order and decreasing order.

The third section describes the adjacency matrices of edge-weighted linguistic matrices. Section four suggests problems based on these studies.

3.2 Adjacency linguistic matrices of linguistic graphs

Here we obtain adjacency matrices of four types of graphs and obtain some interesting properties about them.

We will first illustrate this by some examples.

Example 3.2.1. Let

S = {good, bad, best, just good, fair, just bad, very good, very bad}

be a linguistic set associated with the linguistic variable performance aspects of a school student in the classroom.

Let G be the linguistic graph given by the following Figure 3.2.1.

G =

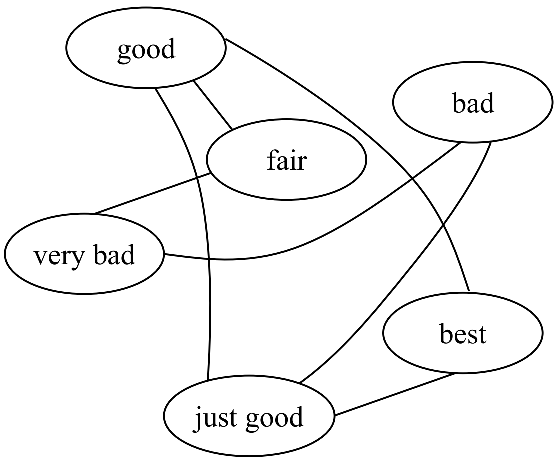


Figure 3.2.1

Recall we have given in Chapter two of this book that if we have an edge between two linguistic terms, it is denoted by e, which means edge exists, and if the linguistic terms say good and fair, we say good and fair are adjacent and denote it by, good fair = e.

If they have no edge connecting two linguistic terms, we say they are not adjacent and denote it by f, which is no edge or empty edge. This is similar to the classical graphs where 1 denotes the edge between any two vertices and 0 no edge between two vertices.

Further, we assume that there is no edge between a vertex with itself, that is, there is no loop.

Now with this similarity in mind, we can get the linguistic adjacency matrices in the case of linguistic graphs. The edge set in this case would be {f, e} analogous to {0, 1}, 0 replaced by empty term f and 1 replaced by e.

The linguistic adjacency matrix associated with the linguistic graph G given in Figure 3.2.1 is as follows.

Let M be the linguistic adjacency of the linguistic graph G.

$$M = \begin{array}{c} \\ \text{good} \\ \text{bad} \\ \text{very bad} \\ \text{fair} \\ \text{best} \\ \text{just good} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{good} & \text{bad} & \text{very bad} & \text{fair} & \text{best} & \text{just good} \end{array} \\ \left[\begin{array}{cccccc} \phi & \phi & \phi & e & e & e \\ \phi & \phi & e & \phi & \phi & e \\ \phi & e & \phi & e & \phi & \phi \\ e & \phi & e & \phi & \phi & \phi \\ e & \phi & \phi & \phi & \phi & e \\ e & e & \phi & \phi & e & \phi \end{array} \right] \end{array}.$$

The following observations are mandatory.

i) Always the adjacency matrix of a linguistic graph (which is not a bipartite graph) is a square linguistic matrix (we call M the linguistic adjacency matrix of the linguistic graph G).

ii) They are always symmetric linguistic matrices unless the given linguistic graph is a directed linguistic graph.

iii) Always the values or entries in a linguistic matrix is from the set {ϕ, e} unless we take edge-weighted linguistic graphs and the main diagonal is always ϕ.

With these concepts in mind, we now define the linguistic adjacency matrix of a linguistic graph G.

Definition 3.2.1*. Let S = {a_1, …, a_n} be the linguistic set associated with the linguistic variable L. Let G = (V, E) be a linguistic graph with V a linguistic vertex subset of S where edges set E = {ϕ, e} and V = {a_1, a_2, …, a_m}.*

The linguistic adjacency matrix M of G is given by

$$M = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & \dots & a_m \end{array} \\ \begin{bmatrix} \phi & e & \phi & \dots & e \\ e & \phi & e & \dots & \phi \\ \phi & e & \phi & \dots & e \\ \vdots & \vdots & \vdots & \dots & \vdots \\ e & \phi & e & \dots & \phi \end{bmatrix} \end{array}$$

Clearly, M is an m × m linguistic square matrix which is symmetric about the main diagonal.

We provide more examples of adjacency linguistic matrices.

Example 3.2.2. Let

S = {tall, just tall, very tall, tallest, short, very short, medium, just short, shortest, just medium}

be the linguistic variables associated with the linguistic variable height.

Let H be the linguistic graph given by Figure 3.2.2.

H =

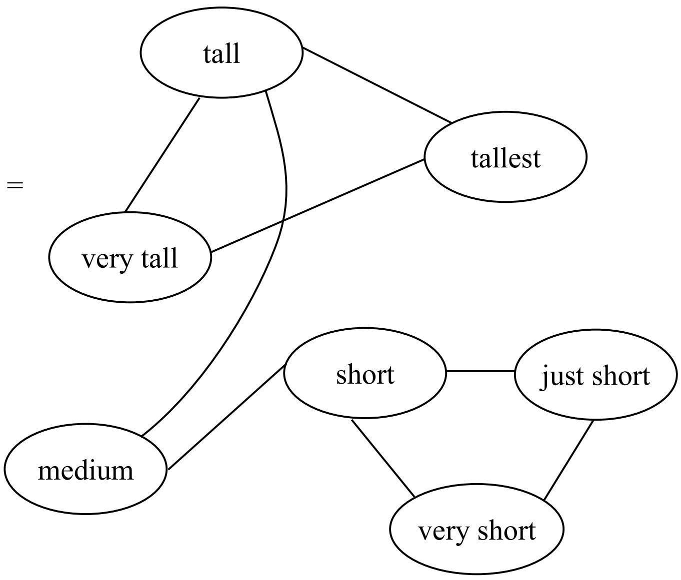


Figure 3.2.2

Let W be the linguistic adjacency matrix associated with H.

W =

	tall	very tall	medium	short	tallest	just short	very short
tall	ϕ	e	e	ϕ	e	ϕ	ϕ
very tall	e	ϕ	ϕ	ϕ	e	ϕ	ϕ
medium	e	ϕ	ϕ	ϕ	e	ϕ	ϕ
short	ϕ	ϕ	ϕ	ϕ	ϕ	e	e
tallest	e	e	e	ϕ	ϕ	ϕ	ϕ
just short	ϕ	ϕ	ϕ	e	ϕ	ϕ	e
very short	ϕ	ϕ	ϕ	e	ϕ	e	ϕ

W is a symmetric 7 × 7 square linguistic matrix.

Here we make the following observation the nodes

{tall, very tall, …, very short} is not ordered by us.

When getting the linguistic adjacency matrices for a totally ordered set, it is not mandatory, we need not mention the naming of rows or columns in the increasing or in decreasing order they can be anyway. It need not be in a form given by the adjacency matrix X, the adjacency matrix in W is also equally allowed.

Now if we order this linguistic subset, that is the vertex subset of H we see

very short < short < just short < medium < tall < very tall < tallest

So we can denote the vertex set V of H by

V = {very short, short, just short, medium, tall, very tall, tallest}.

Let X denote the linguistic matrix of H associated with V.

X =

	very short	short	just short	medium	tall	very tall	tallest
very short	ϕ	e	e	ϕ	ϕ	ϕ	ϕ
short	e	ϕ	e	e	ϕ	ϕ	ϕ
just short	e	e	ϕ	ϕ	ϕ	ϕ	ϕ
medium	ϕ	e	ϕ	ϕ	e	ϕ	ϕ
tall	ϕ	ϕ	ϕ	e	ϕ	e	e
very tall	ϕ	ϕ	ϕ	ϕ	e	ϕ	e
tallest	ϕ	ϕ	ϕ	ϕ	e	e	ϕ

Clearly W and X look different however one can be got from the other by shift of rows.

In W if we shift last row to first row, 4^{th} row to 2^{nd} row, 6^{th} row to 3^{rd} row and so on we will get the linguistic matrix X.

Now having seen example of adjacency linguistic matrix of a linguistic graph we now give an example of the adjacency linguistic matrix of linguistic directed graph.

Example 3.2.3. Let S the linguistic set be as in example 3.2.1.

Let P = {V, E}; V ⊆ S be the linguistic directed graph with entries from S given by the following Figure 3.2.3.

P =

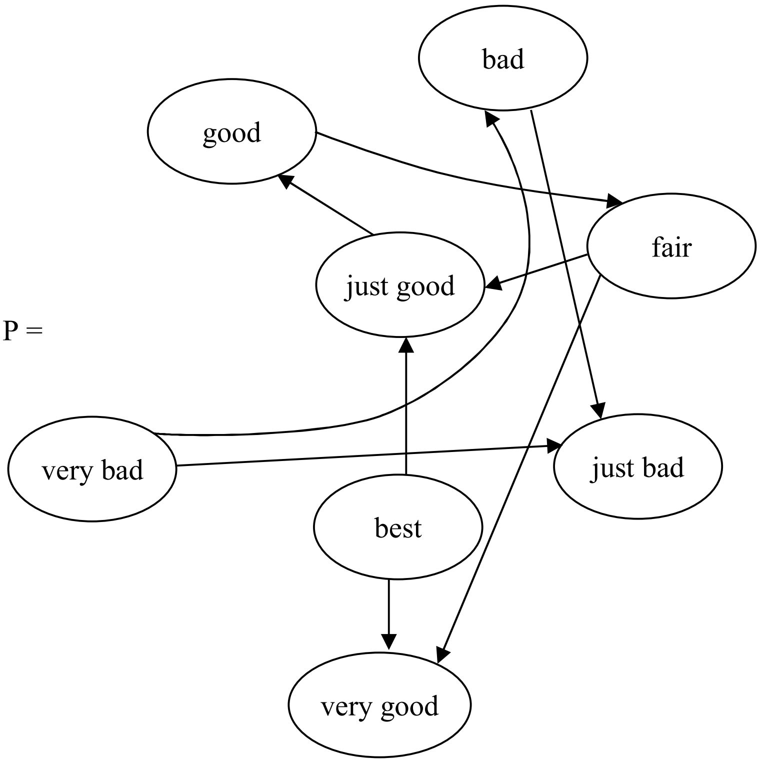


Figure 3.2.3

Let K be the linguistic adjacency matrix of the linguistic directed graph P given in the following.

K =

	good	bad	very bad	best	just bad	very good	fair	just good
good	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ
bad	ϕ	ϕ	ϕ	ϕ	e	ϕ	ϕ	ϕ
very bad	ϕ	e	ϕ	ϕ	e	ϕ	ϕ	ϕ
best	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ	e
just bad	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
very good	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
fair	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ	e
just good	e	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

Clearly K is also a square linguistic matrix with the main diagonal entries to be ϕ.

Clearly K is not a linguistic symmetric matrix.

Now if totally order the vertex set of K as

very bad < bad < just bad < fair < just good

< good < very good < best

We get the following adjacency matrix with the order set of vertices for P;

V = {very bad, bad, just bad, fair, just good, good, very good, best} $\subseteq$ S.

Let R be the linguistic adjacency matrix of K using the totally ordered set V.

R =

	very bad	bad	just bad	fair	just good	good	very good	best
very bad	ϕ	e	e	ϕ	ϕ	ϕ	ϕ	ϕ
bad	ϕ	ϕ	e	ϕ	ϕ	ϕ	ϕ	ϕ
just bad	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
fair	ϕ	ϕ	e	ϕ	ϕ	ϕ	ϕ	ϕ
just good	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ	ϕ
good	ϕ	ϕ	ϕ	e	ϕ	ϕ	ϕ	ϕ
very good	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
best	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ

We see the linguistic matrix R can be got from K by row echelon method by shift of rows.

Now having seen examples of adjacency linguistic matrices related with linguistic directed graphs we now proceed onto find linguistic adjacency matrices of linguistic directed graphs (increasing order of nodes) by some examples. The above linguistic graph 3.2.3 is not in the increasing order of nodes but only arbitrary.

Example 3.2.4. Let the linguistic set S be as in example 3.2.2 associated with the linguistic variable height.

Let B = (V, E), V ≤ S; be the linguistic directed graph (increasing order of nodes) given in Figure 3.2.4.

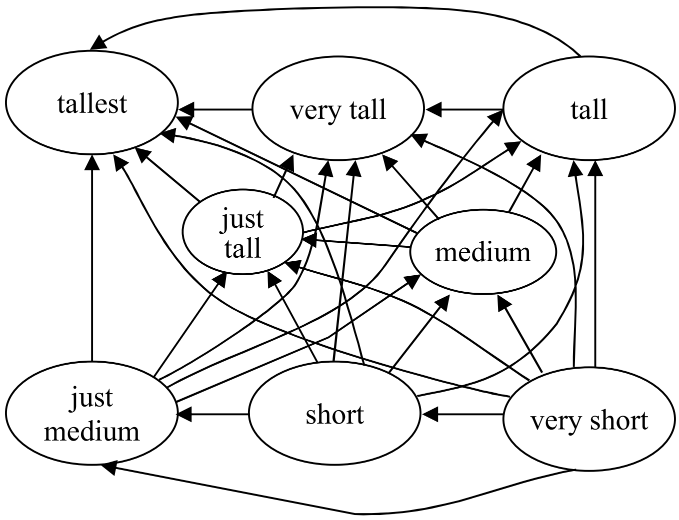


Figure 3.2.4

Clearly

very short < short < just medium < medium

< just tall < tall < very tall < tallest

is a totally ordered linguistic set

V = {very short, short, just medium, medium, just tall, tall, very tall, tallest} $\subseteq$ S

be the totally ordered linguistic set associated with the linguistic nodes of the linguistic directed graph B where the direction as in the case of performance of students, cannot be changed as it is height we can only get the line in increasing order.

The adjacency linguistic matrix Q of B is as follows.

Q =

	very short	short	just medium	medium	just tall	tall	very tall	tallest
very short	ϕ	e	e	e	e	e	e	e
short	ϕ	ϕ	e	e	e	e	e	e
just medium	ϕ	ϕ	ϕ	e	e	e	e	e
medium	ϕ	ϕ	ϕ	ϕ	e	e	e	e
just tall	ϕ	ϕ	ϕ	ϕ	ϕ	e	e	e
tall	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e	e
very tall	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
tallest	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

Clearly, Q is a square linguistic matrix but it is not symmetric.

In fact the main diagonal elements are ϕ.

Q is upper triangular linguistic matrix with empty main diagonal entries.

For any linguistic directed graph with (increasing order of nodes (or direction)) we see the resultant adjacency linguistic matrix is not symmetric but upper triangular matrix whose main diagonal is also empty.

This will be the case only in case of graphs where totally ordered linguistic sets are used. We give an example of a linguistic directed graph (decreasing order of nodes) using the same set S.

Let C = {V, E}; V $\subseteq$ S be the linguistic subset of S for the linguistic directed graph.

We give V in the decreasing order, V = {tallest ≤ very tall ≤ tall ≤ just tall ≤ medium ≤ short ≤ just short ≤ shortest} and E = {0, e}

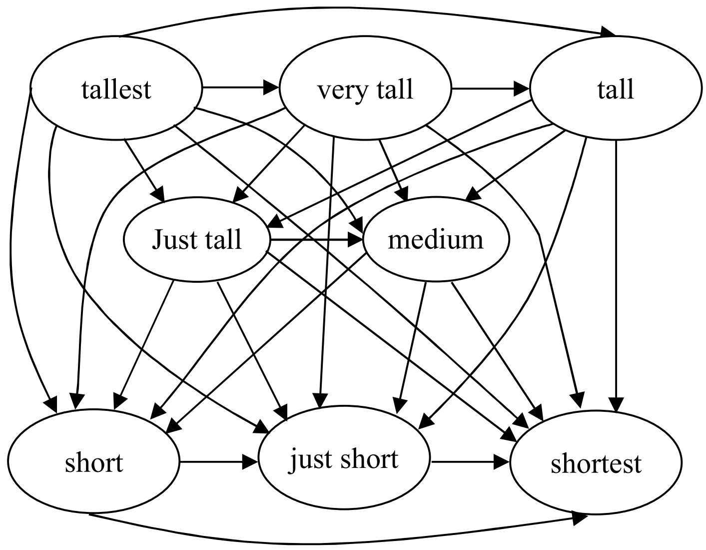


Figure 3.2.5

Now we give the linguistic adjacency matrix T associated with the linguistic directed graph C (decreasing order) is given by T.

T =

	shortest	just short	short	medium	just tall	tall	very tall	tallest
shortest	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
just short	e	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
short	e	e	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
medium	e	e	e	ϕ	ϕ	ϕ	ϕ	ϕ
just tall	e	e	e	e	ϕ	ϕ	ϕ	ϕ
tall	e	e	e	e	e	ϕ	ϕ	ϕ
very tall	e	e	e	e	e	e	ϕ	ϕ
tallest	e	e	e	e	e	e	e	ϕ

T is a 8 × 8 linguistic square matrix whose main diagonal is empty. But it is only a linguistic lower triangular matrix whose main diagonal entries are empty.

Now we proceed on to get linguistic directed graphs using partially ordered sets.

Example 3.2.5. Let

S = {good, bad, best, very good, very bad, fair}

be a linguistic set associated with the performance of teacher in a class room. The linguistic power set P(S) of S is as follows:

P(S) = {ϕ, t_1 = {good}, t_2 = {bad}, t_3 = {best}, t_4 = {very good}, t_5 = {very bad}, t_6 = {fair}, t_7 = {good, bad}, t_8 = {good, best}, t_9 = {good, very good}, t_{10} = {good, very bad}, t_{11} = {good, fair}, t_{12} = {bad, best}, t_{13} = {bad, very good}, t_{14} = {bad, very bad}, …, t_{31} = {S}}.

Let L = {V, e} be a linguistic directed graph where V is a subset of P(S) and E = {ϕ. E}.

V = {ϕ, t_2, t_4, t_7, t_{12}, t_9, t_{18} = {good, bad, best},

t_{20} = {very bad, fair, best},

t_{26} = {very bad, good, best, fair, very good},

t_{27} = {bad, good, very good},

t_{31} = {S}}

L =

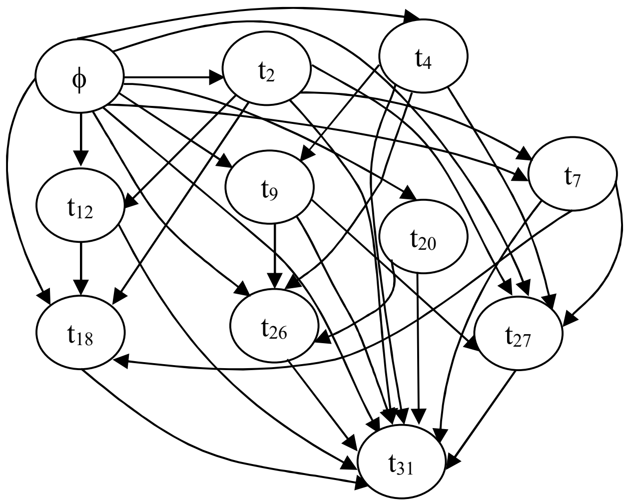


Figure 3.2.6

We have Z to be the adjacency linguistic matrix.

Z =

	ϕ	t_2	t_4	t_7	t_9	t_{12}	t_{18}	t_{20}	t_{26}	t_{27}	t_{31}
ϕ	ϕ	e	e	e	e	e	e	e	e	e	e
t_2	ϕ	ϕ	ϕ	e	ϕ	e	e	ϕ	ϕ	ϕ	e
t_4	ϕ	ϕ	ϕ	ϕ	e	ϕ	ϕ	ϕ	e	e	e
t_7	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e	ϕ	ϕ	e	e
t_9	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e	e	e
t_{12}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
t_{18}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
t_{20}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
t_{26}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
t_{27}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e
t_{31}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	e

Clearly Z is not a symmetric linguistic matrix however it a 11 × 11 square matrix whose diagonal entries are ϕ; empty linguistic term.

Let W = {V, E} be the linguistic directed graph (decreasing order) with

V = {ϕ, t_3, t_5, t_6, t_8, t_9, t_{11} = {fair, good},

t_{10} = {fair, good, bad, best}, t_{31}, t_{22} = {fair, good, fair, bad, best}}

and E = {ϕ, e} given in the following Figure 3.2.7.

$$
X = \begin{array}{c} \\ \phi \\ t_3 \\ t_5 \\ t_6 \\ t_8 \\ t_9 \\ t_{11} \\ t_{19} \\ t_{22} \\ t_{31} \end{array}
\begin{array}{c}
\begin{array}{cccccccccc} \phi & t_3 & t_5 & t_6 & t_8 & t_9 & t_{11} & t_{19} & t_{22} & t_{31} \end{array} \\
\left[\begin{array}{cccccccccc}
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & e & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
e & \phi & \phi & e & \phi & \phi & \phi & \phi & \phi & \phi \\
e & e & \phi & e & e & \phi & \phi & \phi & \phi & \phi \\
e & e & \phi & e & e & \phi & \phi & e & \phi & \phi \\
e & e & e & e & e & e & e & e & e & e
\end{array}\right]
\end{array}
$$

X is not symmetric linguistic matrix only a lower triangular matrix filled with linguistic empty words / terms along the main diagonal.

W=

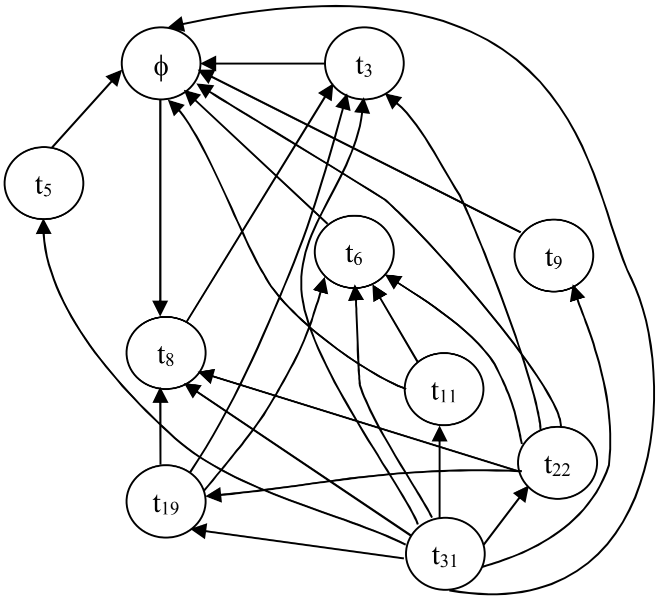


Figure 3.2.7

Next, we proceed on to describe and develop the notion of adjacency linguistic matrix, which are edge weighted by linguistic terms/words by some examples.

Example 3.2.6. Let

S = {bad, very bad, very good, good, just good, just bad, fair, best, worst}

be the linguistic set associated with the linguistic performance of workers in a factory.

Let us define the linguistic edge set

E = {sincere, very sincere, careless, careful, loyal, trust worthy, not lazy, lazy, active, medium, industries, indifferent, efficient, inefficient}

for the linguistic set S.

Now we construct the edge weightage linguistic graph G = {V, G} using the subset of S for its nodes and edge set E for the edges. G is given by the following Figure 3.2.8.

G =

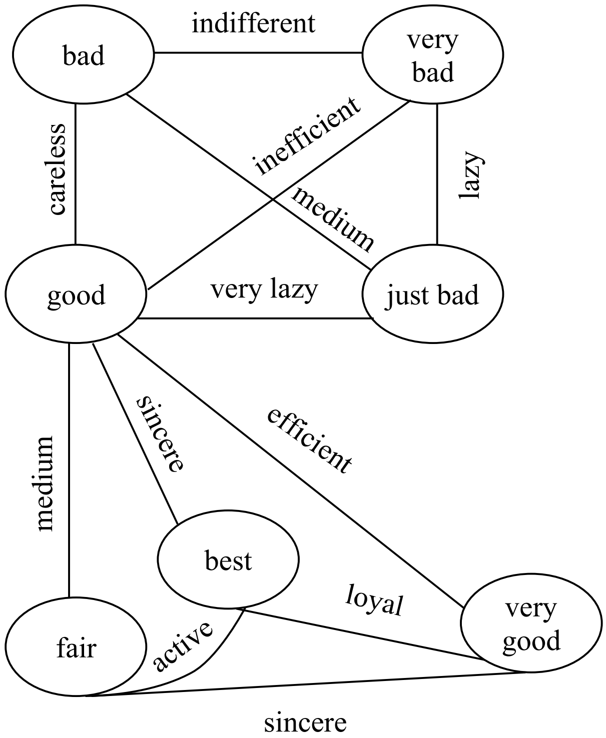


Figure 3.2.8

Let B denote the linguistic adjacency matrix associated with the linguistic graph G.

B =

	bad	very bad	good	just bad	fair	best	very good
bad	ϕ	indifferent	careless	medium	ϕ	ϕ	ϕ
very bad	indifferent	ϕ	inefficient	lazy	ϕ	ϕ	ϕ
good	careless	inefficient	ϕ	ϕ	ϕ	sincere	efficient
just bad	medium	lazy	very lazy	ϕ	ϕ	ϕ	ϕ
fair	ϕ	ϕ	medium	ϕ	ϕ	active	sincere
best	ϕ	ϕ	sincere	ϕ	active	ϕ	loyal
very good	ϕ	ϕ	efficient	ϕ	sincere	loyal	ϕ

We see B is a linguistic edge-weighted matrix, which is a square matrix. It is symmetric. However, we have not ordered the linguistic set S.

Next, we proceed onto give the linguistic matrix for the same linguistic graph G but now we order the vertex set

very bad ≤ bad ≤ just bad ≤ fair ≤ good ≤ very good ≤ best

V = {very bad, bad, …, best}.

Let W be the linguistic (adjacency) matrix.

W =

	very bad	bad	just bad	fair	good	very good	best
very bad	ϕ	indifferent	lazy	ϕ	inefficient	ϕ	ϕ
bad	indifferent	ϕ	medium	ϕ	careless	ϕ	ϕ
just bad	lazy	medium	ϕ	ϕ	very lazy	ϕ	ϕ
fair	ϕ	ϕ	ϕ	ϕ	medium	ϕ	ϕ
good	inefficient	careless	very lazy	medium	ϕ	efficient	sincere
very good	ϕ	ϕ	ϕ	ϕ	efficient	ϕ	loyal
best	ϕ	ϕ	ϕ	ϕ	sincere	loyal	ϕ

This is also a symmetric linguistic matrix.

One can easily get from the linguistic matrix B to W and vice versa by interchanging the rows.

Next, we proceed onto describe directed edge weighted linguistic graphs by an example.

Example 3.2.7. Let S be a linguistic set associated with the linguistic variable, nature or character of people,

S = {loyal, disloyal, active, inactive, sincere, not sincere, efficient, not efficient, very efficient, just efficient, industrious, traitor, lazy, careless, very lazy, trustworthy, not trustworthy, very careful}

and let E be the linguistic edge set associated with S.

E = {good, bad, very good, best, just good, just bad, very bad, worst, very fair, fair}

be the linguistic edge set associated with S.

Let H be the linguistic edge weighted directed graph given by the following Figure 3.2.9.

H =

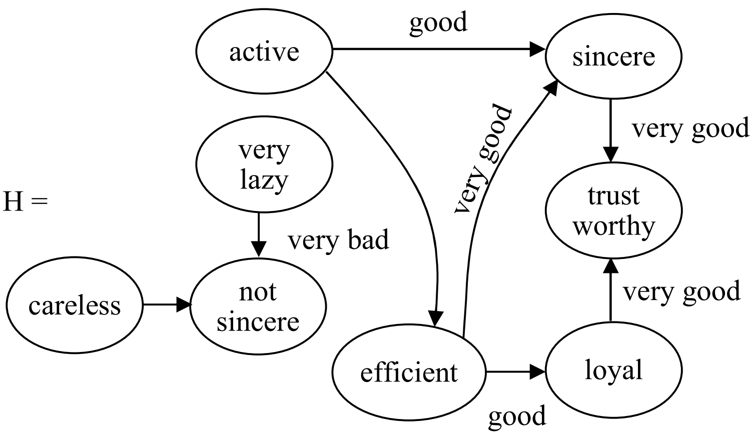


Figure 3.2.9

Let M be the linguistic adjacency matrix associated with the edge weighted linguistic directed graph H given in Figure 3.2.9.

M =

	very lazy	careless	not sincere	active	sincere	trustworthy	loyal	efficient
very lazy	φ	bad	very bad	φ	φ	φ	φ	φ
careless	φ	φ	worst	φ	φ	φ	φ	φ
not sincere	φ	φ	φ	φ	φ	φ	φ	φ
active	φ	φ	φ	φ	good	φ	φ	best
sincere	φ	φ	φ	φ	φ	φ	φ	φ
trustworthy	φ	φ	φ	φ	φ	φ	φ	φ
loyal	φ	φ	φ	φ	φ	very good	φ	φ
efficient	φ	φ	φ	φ	very good	φ	good	φ

We see the linguistic directed graphs adjacency matrix is a 8×8 square matrix with all the main diagonal elements are empty term ϕ and is not symmetric.

It is pertinent to keep on record that the linguistic set S is only a partially ordered set so if we try to label the edge weight only which falls under order $\leq$ (or $\geq$) for otherwise the two linguistic nodes are not adjacent they only yield a empty linguistic term edge ϕ.

To give a concrete example of both edge-weighted linguistic directed graph (in increasing order of nodes) and linguistic edge-weighted directed graphs (with decreasing order of linguistic nodes) we have to have basically a totally ordered linguistic set for its nodes. So, we take a totally ordered linguistic set and provide an example of the same.

Example 3.2.8. Let us consider linguistic set S associated with age of people who have specific all worked in a factory and their productive capacity in those age groups.

S = {just old, middle aged, just middle age, youth close to middle age, just youth (just very away from middle age) youth (at the prime age), old}.

Let us define the linguistic set E associated with their performance related to their age which is given as weight of the edges.

E = {good, bad, very good, just good, very bad, very fair, best}.

Now we see the linguistic set S is a totally orderable set S has the following linguistic chain

just youth ≤ youth (prime) ≤ youth

≤ just middle age ≤ middle age ≤ just old ≤ old.

Let F = (V, E) be the directed linguistic edge weighted graph (increasing order) where V ≤ S and E_1 ≤ E given by the following Figure 3.2.10.

F =

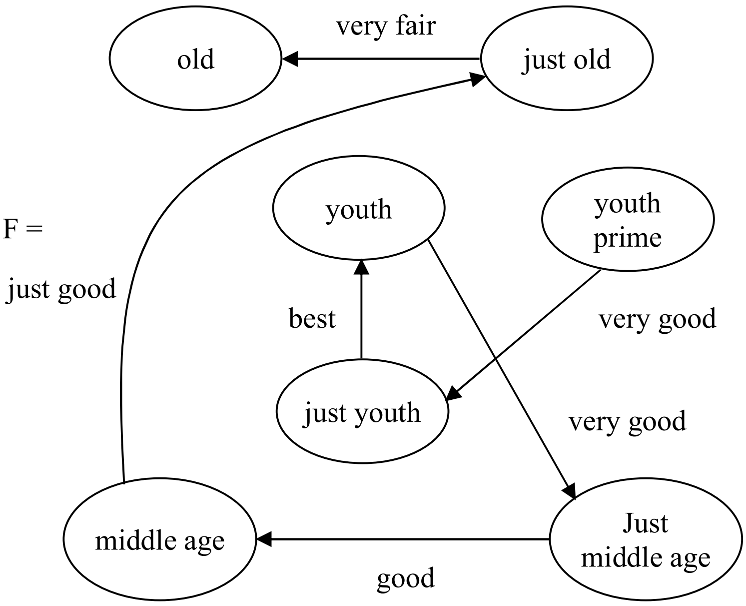


Figure 3.2.10

Let R be the adjacency linguistic matrix associated with the linguistic graph F.

R =

	old	just old	middle age	just middle age
old	ϕ	ϕ	ϕ	ϕ
just old	very fair	ϕ	ϕ	ϕ
middleage	ϕ	just good	ϕ	ϕ
just middleage	ϕ	ϕ	good	ϕ
youth	ϕ	ϕ	ϕ	very good
just youth	ϕ	ϕ	ϕ	ϕ
youth prime	ϕ	ϕ	ϕ	ϕ

youth	just youth	youth prime
ϕ	ϕ	ϕ
ϕ	ϕ	ϕ
ϕ	ϕ	ϕ
ϕ	ϕ	ϕ
ϕ	ϕ	ϕ
best	ϕ	ϕ
ϕ	very good	ϕ

Infact the linguistic adjacency matrix has only the values just below the main diagonal and all other values are linguistic empty word ϕ.

Further it is to be noted the rows or columns are formed by decreasing order only the edge weights are given in the increasing order.

How does the adjacency linguistic matrix look like if the row elements and column element are marked of the decreasing order.

Let Q be the linguistic adjacency matrix of the linguistic graph F where the row and column nodes are given in increasing order Q is as follows.

Q =

	youth prime	just youth	youth	just middle age	middle age	just old age	old age
youth prime	φ	very good	φ	φ	φ	φ	φ
just youth	φ	φ	best	φ	φ	φ	φ
youth	φ	φ	φ	very good	φ	φ	φ
just middle age	φ	φ	φ	φ	good	φ	φ
middle age	φ	φ	φ	φ	φ	just good	φ
just old age	φ	φ	φ	φ	φ	φ	just good
oldage	φ	φ	φ	φ	φ	φ	φ

We see Q is a linguistic square matrix which has only nonempty linguistic just above the diagonal when the linguistic nodes are ordered in increasing order.

Let K denote the linguistic edge weighted directed graph with (decreasing order of nodes) given by the following Figure 3.2.11. Decreasing order of nodes can never exist in the case of the linguistic variable age. So the linguistic graph K is empty.

K =

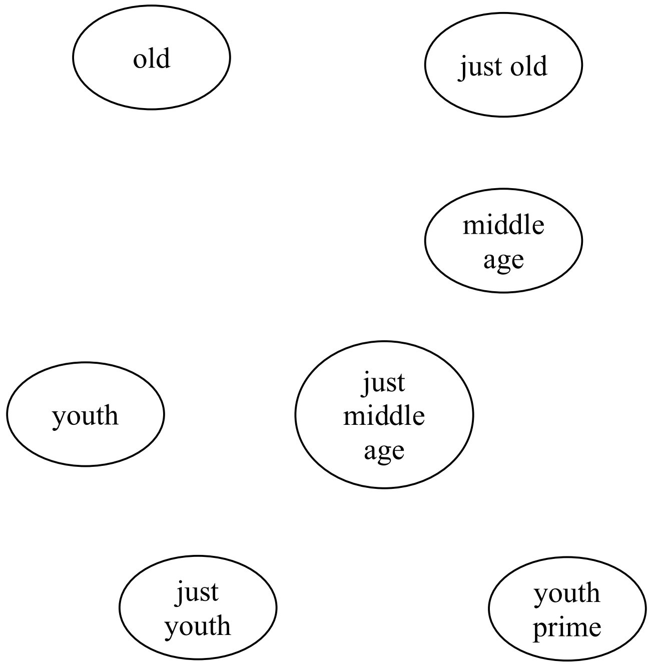


Figure 3.2.11

We see we cannot measure practically from old age to just old age for age is always an increasing factor.

So the linguistic directed graph is only an empty directed graph.

We just proceed onto describe an example where this sort of linguistic graph is possible.

Example 3.2.9. Let S = {good, bad, best, very good, just good, very bad, fair, just fair, just bad}

be the linguistic variable associated with the performance of a student in a classroom.

very bad ≤ bad ≤ just bad ≤ fair ≤ just fair ≤ just good

≤ good ≤ very good ≤ best.

Now let E be the linguistic set giving the edge weights set for linguistic graphs using the linguistic set S.

E = {hard working, lazy, just work, very hard worker, indifferent, does not work, does work, not alternative, very lazy}

be the linguistic set associated with S.

Let G be the linguistic edge weighted directed graph (decreasing order of nodes) given by the following Figure 3.2.12.

G =

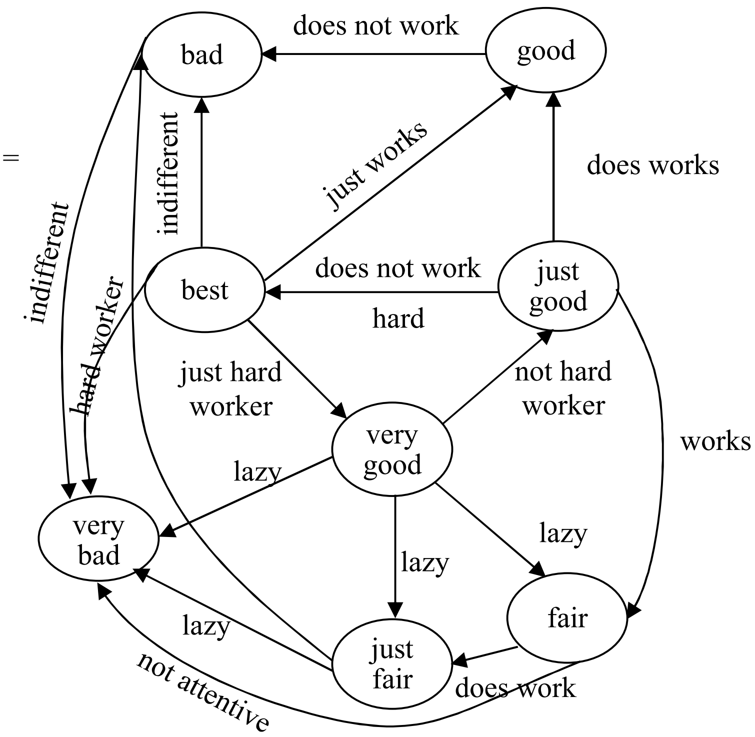


Figure 3.2.12

We provide the linguistic adjacency matrix M related with G.

M =

	very bad	bad	just fair	fair	just good	good	very good	best
very bad	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
bad	indifferent	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
just fair	not attentive	lazy	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
fair	not attentive	ϕ	does work	ϕ	ϕ	ϕ	ϕ	ϕ
just good	ϕ	ϕ	ϕ	works	ϕ	ϕ	ϕ	ϕ
good	ϕ	does not work	ϕ	ϕ	does not work	ϕ	ϕ	ϕ
very good	lazy	ϕ	lazy	lazy	not a hard worker	ϕ	ϕ	ϕ
best	inefficient	in different	ϕ	ϕ	not a hard worker	just work	just hard worker	ϕ

We see M is a linguistic lower diagonal 8 × 8 square matrix which is lower diagonal and the main diagonal elements are just the empty term.

Now having seen examples of totally ordered linguistic sets we proceed onto work with partially ordered set; in particular the linguistic power set P(S) of S.

Example 3.2.10. Let S = {bad, good, fair, very fair, best, very good, very bad, just bad, just good}

be a linguistic set associated with the linguistic variable performance aspects of a teacher.

Let P(S) be the power set of S;

P(S) = {ϕ, t_1 = {bad}, {good} = t_2 {fair}, = t_3 {very fair} = t_4, t_5 = {best}, $_6$ = {very good}, t_7 = {very bad}, t_8 = {just bad}, t_9 = {just good}, t_{10} = {bad, good}, t_{11} = {bad, fair}, t_{12} = {bad, very fair}, t_{13} = {bad, best}, t_{14} = {bad, very good}, t_{15} = {bad, very bad}, t_{16} = {bad, just bad}, t_{17} = {bad, very bad}, …, t_{51} = {S}},

which is a partially ordered set.

Let G = {V, E} be a linguistic directed graph with linguistic edges given by

E = {very weak, weak, just weak, medium, just medium, strong, very strong, just strong}.

V = {ϕ, t_1, t_2, t_3, t_6, t_9, t_{12}, t_{16}, t_{26} = {just bad, just good}, t_{28} = {just good, very good}, t_{33} = {bad, good, very good}}, t_{51} = {S}}.

We see V $\subseteq$ P(S) and o(V) = 11.

We say very weak when the relation is with ϕ

just weak if we have a singleton set related with higher order sets weak if we have a pair of elements in S contained in any set and so on}

very strong if the set $t_i \leq t_j$ with $|t_i| = 2^8$ and $|t_j| > 2^8$.

The graph of G is given in Figure 3.2.13.

G =

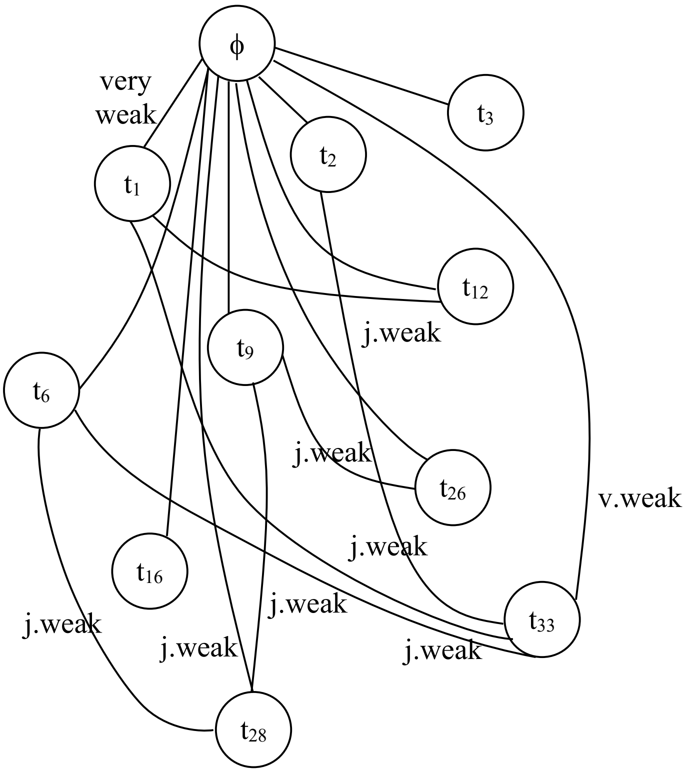


Figure 3.2.13

G is just a linguistic graph with edge weights.

Let D be the linguistic adjacency matrix associated with G.

D =

	ϕ	t_1	t_2	t_3	t_6	t_9	t_{12}	t_{16}	t_{26}	t_{28}	t_{33}
ϕ	ϕ	v.weak	v.weak	v.weak	v.weak	v.weak	v.weak	v.weak	v.weak	v.weak	v.weak
t_1	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	ϕ	ϕ	ϕ	j.weak
t_2	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak
t_3	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
t_6	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak
t_9	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	j.weak	ϕ
t_{12}	v.weak	j.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
t_{16}	v.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
t_{26}	v.weak	ϕ	ϕ	ϕ	ϕ	j.weak	ϕ	ϕ	ϕ	ϕ	ϕ
t_{28}	v.weak	ϕ	ϕ	ϕ	j.weak	j.weak	ϕ	ϕ	ϕ	ϕ	ϕ
t_{33}	v.weak	j.weak	j.weak	ϕ	j.weak	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

v.weak – very weak; j.weak – just weak.

We see D the linguistic adjacency matrix of G. D is a 11 × 11 square symmetric linguistic matrix.

Let J = (W, E_2) be a linguistic edge weighted directed graph where W ≤ P(S) and $E_2 \subseteq E$ given in the following Figure 3.2.14.

Here W = {t_1, t_3, t_5, t_7, t_9, t_{11}, t_{15}, t_{13}, t_{18} = {bad, just good},

t_{19} = {bad, fair, very bad}, t_{20} = {bad, best, just good}

t_{21} = {bad, fair best, very bad, just good, very good}}.

Let J be the linguistic edge-weighted directed graph given by the following figure.

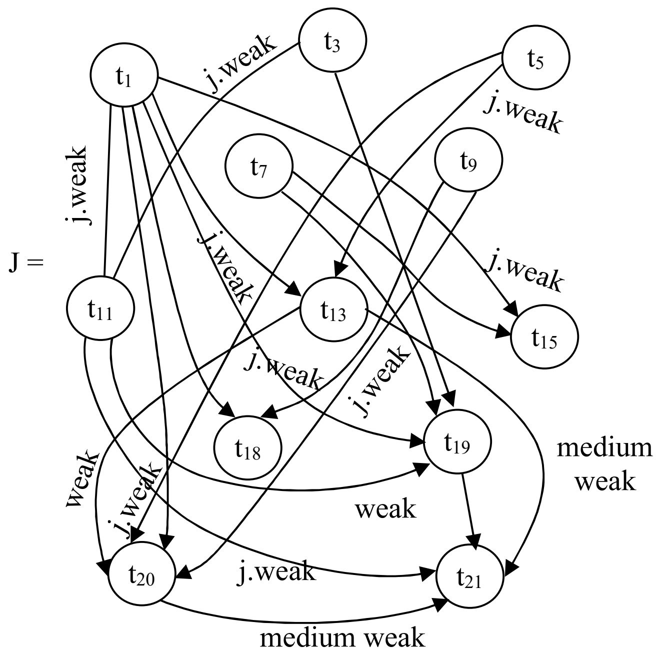


Figure 3.2.14

Let T be the linguistic adjacency matrix related with linguistic edge weighted directed graph J:

T =

	t_1	t_3	t_5	t_7	t_9	t_{11}	t_{13}	t_{15}	t_{18}	t_{19}	t_{20}	t_{21}
t_1	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	j.weak	j.weak	j.weak	j.weak	j.wek	j.weak
t_3	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak
t_5	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	j.weak	ϕ	ϕ	j.weak	j.weak
t_7	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	ϕ	ϕ	ϕ	j.weak
t_9	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	j.weak	j.weak	j.weak
t_{11}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	j.weak	j.weak	j.weak	j.weak
t_{13}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	weak
t_{15}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	weak	weak
t_{18}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	weak
t_{19}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	weak	weak
t_{20}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	medium
t_{21}	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	medium

j.weak – just weak

Now we just show how the linguistic subgraph H of a linguistic graph G relates the adjacency linguistic matrix.

If K is the linguistic adjacency matrix of the linguistic graph G then the linguistic adjacency matrix of H is a linguistic matrix K. This linguistic matrix K of the subgraph H is defined as linguistic submatrix of the linguistic matrix M.

We will illustrate this situation by an example or two.

Example 3.2.11. Let Q be the linguistic subgraph of G given in Figure 3.2.1.

A =

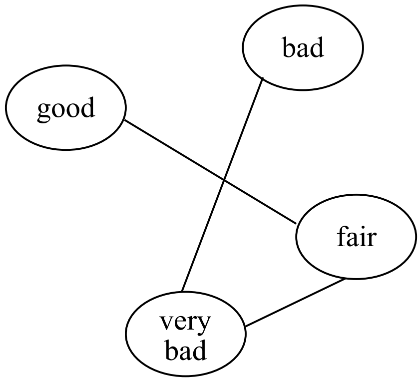


Figure 3.2.15

Let X be the linguistic adjacency matrix associated with the linguistic subgraph A of the linguistic graph G.

X =

	good	bad	very bad	fair
good	ϕ	ϕ	ϕ	e
bad	ϕ	ϕ	e	ϕ
very bad	ϕ	e	ϕ	e
fair	e	ϕ	e	ϕ

Clearly X is a linguistic submatrix of the linguistic matrix M. Thus, the subgraph of G corresponds to a submatrix of M.

Example 3.2.12. Consider the linguistic subgraph K of the linguistic graph P given in Figure 3.2.8.

K =

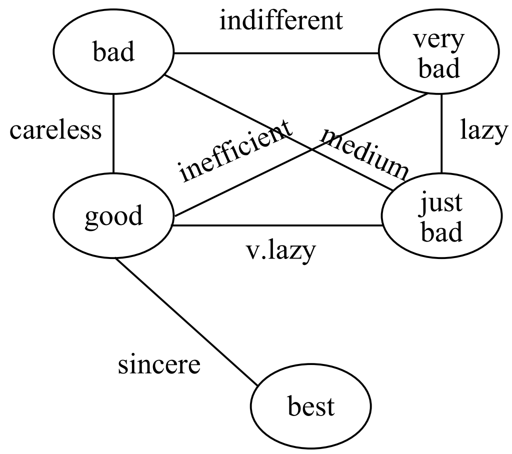


Figure 3.2.16

The adjacency linguistic matrix Y associated with K is as follows.

$$Y = \begin{array}{c} \\ \text{bad} \\ \text{very bad} \\ \text{good} \\ \text{just bad} \\ \text{best} \end{array} \begin{array}{c} \begin{array}{ccc} \text{bad} & \text{very bad} & \text{good} \end{array} \\ \left[\begin{array}{ccc} \phi & \text{indifferent} & \text{careless} \\ \text{indifferent} & \phi & \text{inefficient} \\ \text{careless} & \text{inefficient} & \phi \\ \text{medium} & \text{lazy} & \text{v.lazy} \\ \phi & \phi & \text{sin cere} \end{array} \right. \end{array}$$

just bad	best
medium	ϕ
lazy	ϕ
v. lazy	sincere
ϕ	ϕ
ϕ	ϕ

Clearly Y is a linguistic submatrix of the linguistic matrix B of the linguistic edge weighted graph G for which K is a linguistic subgraph of G.

Example 3.2.13. Let us consider a linguistic edge-weighted directed subgraph L of the linguistic edge weighted linguistic directed graph H given in Figure 3.2.9.

L =

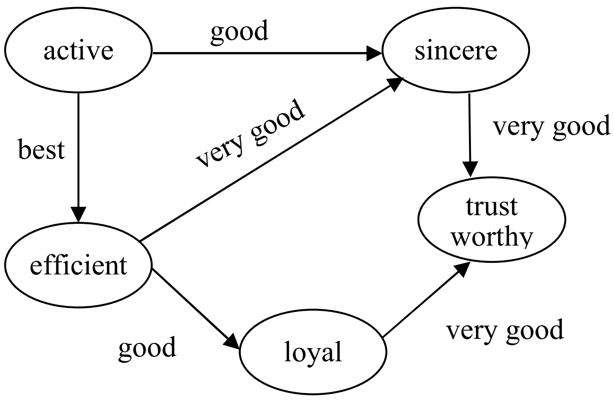


Figure 3.2.17

Let N be the adjacent linguistic matrix associated with linguistic edge weighted directed subgraph L of H.

N =

	active	sincere	trust worthy	loyal	efficient best
active	ϕ	good	ϕ	ϕ	ϕ
sincere	ϕ	ϕ	very good	ϕ	ϕ
trust worthy	ϕ	ϕ	ϕ	ϕ	ϕ
loyal	ϕ	ϕ	very good	ϕ	ϕ
efficient	ϕ	very good	ϕ	ϕ	ϕ

Clearly N is a linguistic submatrix of M got by moving the first three rows and first three columns.

Example 3.2.14. Consider the linguistic edge weighted directed subgraph Q of the linguistic edge weighted directed graph F given in Figure 3.2.10.

Q =

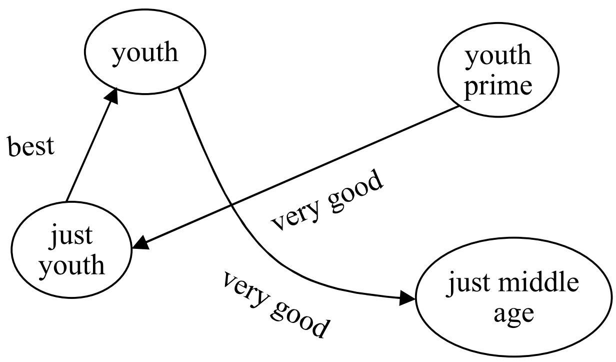


Figure 3.2.18

Let U be the linguistic adjacency matrix of Q

U =

	just middle age	youth	just youth	youth prime
just middle age	ϕ	ϕ	ϕ	ϕ
youth	very good	ϕ	ϕ	ϕ
just youth	ϕ	best	ϕ	ϕ
youth prime	ϕ	ϕ	very good	ϕ

Clearly U is a submatrix of the linguistic matrix R got by removing the first three rows and first three columns.

Example 3.2.15: Let W be the linguistic edge weighted directed subgraph of the linguistic edge weighted directed graph G given in Figure 3.2.12.

W =

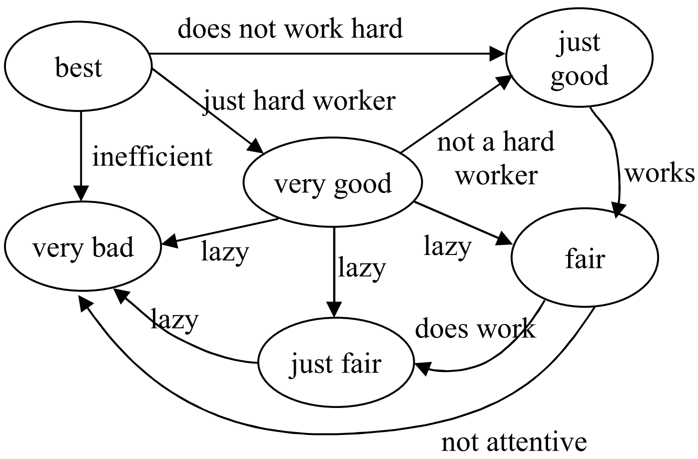


Figure 3.2.19

Let Q be the linguistic adjacency matrix associated with the linguistic subgraph W of G.

Q =

	very bad	just fair	fair	just good	very good	best
very bad	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
just fair	lazy	ϕ	ϕ	ϕ	ϕ	ϕ
fair	not attentive	does work	ϕ	ϕ	ϕ	ϕ
just good	ϕ	ϕ	works	ϕ	ϕ	ϕ
very good	lazy	lazy	lazy	not hard worker	ϕ	ϕ
best	in efficient	ϕ	ϕ	does not work hard	just hard worker	ϕ

We see Q is a linguistic submatrix of the linguistic matrix M got by removing the 2^{nd} row and 2^{nd} column 6^{th} row and 6^{th} column of the matrix M.

Thus we have a correspondence from subgraph to the submatrix of the adjacency matrix and vice versa.

For instance take the submatrix T of M where

$$T = \begin{array}{c} \\ \text{bad} \\ \text{just fair} \\ \text{fair} \\ \text{good} \\ \text{best} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{bad} & \text{just fair} & \text{fair} & \text{good} & \text{best} \end{array} \\ \left[\begin{array}{ccccc} \phi & \phi & \phi & \phi & \phi \\ \text{lazy} & \phi & \phi & \phi & \phi \\ \phi & \text{does work} & \phi & \phi & \phi \\ \text{does not work} & \phi & \phi & \phi & \phi \\ \text{in different} & \text{does not work hard} & \phi & \text{just work} & \phi \end{array}\right] \end{array}$$

The linguistic directed edge weighted subgraph A associated with the submatrix T of the linguistic adjacency matrix M is as follows.

A =

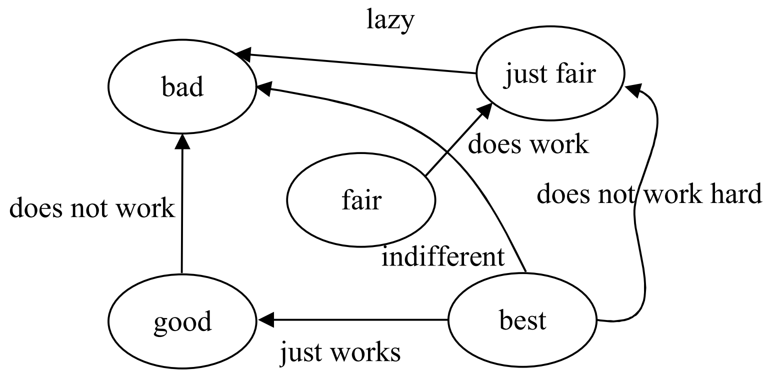


Figure 3.2.20

Now in view of all these notions we have the following theorem which we leave it is an exercise to the reader.

Theorem 3.2.1. *Let S be a linguistic set associated with some linguistic variable. E be the edge set either {ϕ, e} or {linguistic* *weighted edges connecting nodes of S}. Let G be a linguistic edge weighted directed graph or otherwise.*

Let M be the linguistic adjacency matrix of G with entries from E.

Any linguistic submatrix of M has an associated linguistic subgraph with it and vice versa.

Next we proceed on to describe linguistic bipartite graphs in the following section.

3.3 Linguistic Bipartite Graphs and their Properties

In this section we for the first time define the concept of linguistic bipartite graphs. The main criteria for building these graphs is we must have two linguistic sets D and R they should be such that D is disjoint in relations with itself and similarly R should also be disjoint with itself, so that we have linguistic relations are edges only connecting linguistic nodes from D to linguistic nodes from R.

Before we define it we illustrate this situation by some examples.

Example 3.3.1. Suppose we want to relate the industry's profit or loss and the employee's work with the type of payment he gets. We relate them using linguistic bipartite graphs.

Let D be the linguistic variables related to the employee working in an industry is taken as the linguistic domain space, and R is the linguistic range space which describes the attributes related to the industry (like loss or gain).

D = { W_1 = Pay with bonus and allowances,

W_2 = Only pay no bonus or allowances,

W_3 = Pay with either bonus (or allowance)

W_4 = Employee performance is at best

W_5 = Average performance by the employee

W_6 = Employee performance poorly

W_7 = Employee works more hours and

W_8 = Employee works for only less number of hours}

Let the range space R be the linguistic attributes/terms associated with the industry.

The linguistically associated variable with industries are loss, gain, profit, no loss, less gain, good profit, no loss or gain and so on.

R = { I_1 = Maximum profit made by the industry,

I_2 = only just profit to the industry,

I_3 = Neither profit nor loss to the industry,

I_4 = loss to the industry,

I_5 = heavy loss to the industry}.

Now we get the linguistic directed bigraph B_1 given in Figure 3.3.1.

$B_1 =$

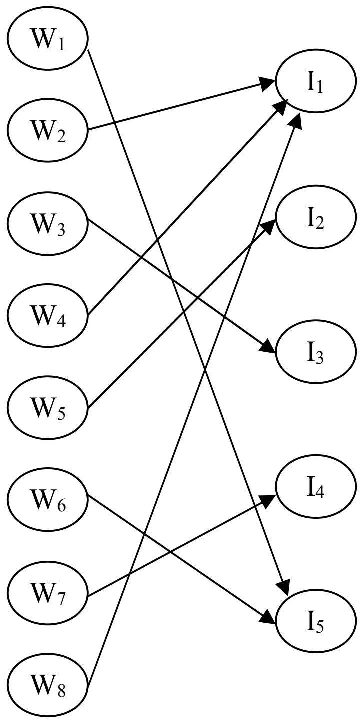


Figure 3.3.1

Now we provide the related linguistic adjacency matrix N_1 of the linguistic bipartite graph B_1 is as follows.

$$N_1 = \begin{array}{c} \\ W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \end{array} \begin{array}{c} \begin{array}{ccccc} I_1 & I_2 & I_3 & I_4 & I_5 \end{array} \\ \left[\begin{array}{ccccc} \phi & \phi & \phi & \phi & e \\ e & \phi & \phi & \phi & \phi \\ \phi & \phi & e & \phi & \phi \\ e & \phi & \phi & \phi & \phi \\ \phi & e & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi & e \\ \phi & \phi & \phi & e & \phi \\ e & \phi & \phi & \phi & \phi \end{array}\right] \end{array}$$

Thus the linguistic adjacency matrix of B_1 is a rectangular 8×5 matrix with edge set from E = {ϕ, e}.

Suppose B_1 is the linguistic bipartite graph given by the first expert.

We can take the second experts opinion and get the linguistic bipartite graph B_2 and the related linguistic adjacency matrix N_2 in the following Figure 3.3.2.

B_2 =

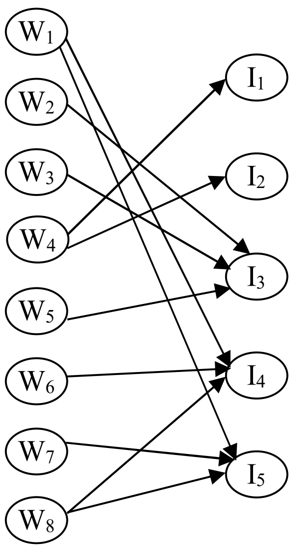


Figure 3.3.2

N_2 is the linguistic adjacency matrix associated with the linguistic bipartite graph B_2.

$$N_2 = \begin{array}{c} \\ W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \end{array} \begin{array}{c} \begin{array}{ccccc} I_1 & I_2 & I_3 & I_4 & I_5 \end{array} \\ \left[\begin{array}{ccccc} \phi & \phi & \phi & e & e \\ \phi & \phi & e & \phi & \phi \\ \phi & \phi & e & \phi & \phi \\ e & e & \phi & \phi & \phi \\ \phi & \phi & e & \phi & \phi \\ \phi & \phi & \phi & e & \phi \\ \phi & \phi & \phi & \phi & e \\ \phi & \phi & \phi & e & e \end{array}\right] \end{array}.$$

Thus depending on the expert we can have different linguistic bipartite graphs.

These are related with the real world of problem of analyzing the employees pay and work and its impact on the gain or profit or either of the industry.

Now we give some properties of linguistic directed bipartite graphs in general.

We provide some examples linguistic bipartite graphs.

Example 3.3.2. Let G be a linguistic bipartite graph with $D = \{d_1, d_2, d_3, d_4, d_5, d_6\}$ a set of linguistic sets which is neither totally ordered or partially ordered. That is no relation exists among them.

Let $R = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7\}$ be a set of linguistic terms which do not hold any relation among themselves which will serve as the linguistic set for the range space the nodes are R and D are related linguistically.

G =

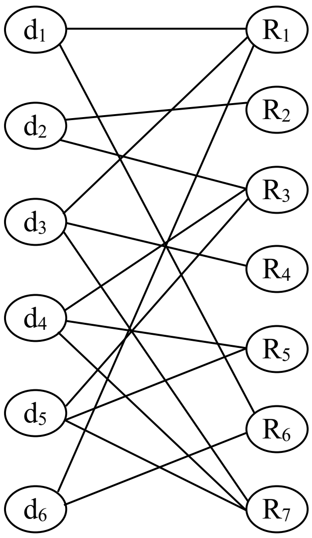


Figure 3.3.3

Let W be the linguistic adjacency matrix W associated with the bipartite G is given in the following.

$$W = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 \end{array} \\ \begin{bmatrix} e & \phi & \phi & \phi & \phi & e & \phi \\ \phi & e & e & \phi & \phi & \phi & \phi \\ e & \phi & \phi & e & \phi & \phi & e \\ \phi & \phi & e & \phi & e & \phi & e \\ \phi & \phi & e & \phi & e & \phi & e \\ e & \phi & \phi & \phi & \phi & e & \phi \end{bmatrix} \end{array}$$

W is a 6×7 linguistic matrix with entries from $E = \{\phi, e\}$.

On these we define some operations which are of a special type.

Suppose if X = {(a_1, …, a_6) where $a_i \in$ {0, e};

$1 \le i \le 6$ then we can define

max{min{X, W}} = max{min{(e, ϕ, ϕ, e, ϕ, e), W}}

(where X = (e, ϕ, ϕ, e, ϕ, e} and min {e, ϕ} = ϕ

and max {e, ϕ} = e).

$$\max\{\min\{X, W\}\} = \max\{\min X, \begin{bmatrix} e & \phi & \phi & \phi & \phi & \phi & \phi \\ \phi & e & e & \phi & \phi & \phi & \phi \\ e & \phi & \phi & e & \phi & \phi & e \\ \phi & \phi & e & \phi & e & \phi & e \\ \phi & \phi & e & \phi & e & \phi & e \\ e & \phi & \phi & \phi & \phi & e & \phi \end{bmatrix}\}$$

= (e, ϕ, e, ϕ, e, e, e} = Y.

max{min{Y, G}}= max{min{(e, ϕ, e, ϕ, e, e, e),

$$\begin{bmatrix} e & \phi & e & \phi & \phi & e \\ \phi & e & \phi & \phi & \phi & \phi \\ \phi & e & \phi & e & e & \phi \\ \phi & \phi & e & \phi & \phi & \phi \\ \phi & \phi & \phi & e & e & \phi \\ e & \phi & \phi & \phi & \phi & e \\ \phi & \phi & e & e & e & \phi \end{bmatrix}\}\} = (e, e, e, e, e, e) = X_1$$

max{min{X, G}} = (e, e, e, e, e, e, e) = Y_1

max{min{Y_1, G^t}} = (e, e, e, e, e, e) = X_2 = X_1.

Thus we get a same value how many time we repeat.

We call this as a linguistic dynamical fixed point of the linguistic system.

Next we proceed onto define linguistic graphs (bipartite) with linguistic edges different from using the set E = {ϕ, e} to {0, 1} where 0 corresponds to the empty linguistic term / word and 1 corresponds to the linguistic term e, which means an edge e exists.

Example 3.3.3. Now we study the linguistic variables ‘economic’ states and ‘female infanticide’ as a huge social problem in India for people to give birth to many female children. It is also considered a social stigma. It is an enormous burden on the parents who give birth to many female children, for the parents have to pay much dowry in marriage for the female children.

Unlike male children, female children are viewed more as a liability than an asset.

Here we study this by first defining the linguistic variables associated with riches and the problems faced by families with more female children.

We take the domain space of the linguistic bipartite group to be the economic status of the people denoted by D

e_1 – very rich

e_2 – rich

e_3 – upper middle class

e_4 – middle class

e_5 – lower middle class

e_6 – poor

e_7 – very poor

D = {e_1, e_2, e_3, e_4, e_5, e_6, e_7} is taken as the linguistic nodes of the domain space D.

Let the nodes of the range space R tests out the social / economic problems due to many female children in then families. The linguistic terms associated with the linguistic variable social or economic problem is listed in the following.

s_1 - number of female children

s_2 - social stigma of having many female children

s_3 - torture by in-laws for having many / only female children

s_4 - economic loss / burden due to female children

s_5 - insecurity due to having only female children (for in days when parents are old they have no male child / son to take care of them)

R = {s_1, s_2, s_3, s_4, s_5}

Now we give the linguistic edges connecting the possible domain and range linguistic nodes

E = { t_1 = family problems

t_2 = societal problems

t_3 = economic loss

t_4 = no one to take care of parents in their older age

t_5 = feeding them a loss to family

t_6 = social burden and responsibility }.

We have mentioned a few probable edges. We have explained each of the linguistic edges t_i; $1 \le i \le 5$.

t_1 – family problems:

In the existing scenario, in India, male children are preferred. Family problems refer to the problems faced by a young woman married into another family. It is customary to marry a female child at a young age and sometimes even to a man who is her parents' choice, not that girl's choice. It is a tradition in India that the girl must live with her husband's family and take care of her husband and his parents. Most household work and child care are done by the woman, whether she is educated or otherwise. In this state of affairs, if she gives birth to only female children, she is scorned by her husband and his parents.

They will torture her and ill-treat her. Some families will send her back to her parents or kill the female babies. In some cases, they may remarry her husband with the hope that the second marriage will provide male children.

These women face a lot of social insults and humiliations and torture by others.

t_2 : Societal problems:

When we say societal problems, we mean they are ill-treated in all family functions and sometimes are not allowed to step out of the house.

Society in India looks down upon a woman with no male children.

t_3: Economic loss:

The very unusual tradition in India of marrying a female child/adult in India to a stranger, mostly a man chosen by her parents, with a huge amount of dowry. To get her married, the parents of the female child have to spend money on performing the marriage, give cash as dowry, and jewellery in gold for the daughter, which will be demanded from the husband's side as social status and so forth. So ultimately, it entitles in a considerable loss to the family when they are not very rich.

So they consider this only as a tremendous economic loss to them.

t_4 : No one to take care of parents in their older age

It is essential to mention that it is a custom in India that parents only will live with their sons in their old age. They feel it is less prestigious to live in their daughter's house. So if a couple has no male child, they have to spend their old age alone with no help or live with their daughter. So they curse a daughter's birth and rejoice in a son's birth.

t_5: Feeding them a loss to the family:

Now because a female child will incur that significant economic loss, some families in India do not feed them with good or nutritious food. In fact, many families feed the female children what is left over after others eat. They constantly brood over that they are a burden to them.

Some uneducated families never send female children to school or give them a college education because they think it is a waste to invest in them. Only male children are educated in their families.

In most families, the female children are trained from the age of four or five to take care of their younger siblings and do all household work.

t_6: Social burden and responsibility:

In most families, they always feel female children are a social burden and their responsibility, hence a liability to them. The terms social burden and responsibility mean the following.

When they say responsibility, they have to guard her from a young age so that she is kept under their eyes, for they fear she may marry someone of another caste or religion and thereby bringing dishonour to their family.

Most honour killings are committed to harm younger women who marry into other castes and whose parents disagree with it.

That is why the term social burden and responsibility is used.

After explaining all the terms, we develop a linguistic bipartite graph using D as the linguistic domain space, R as the linguistic range space and E as its linguistic edges connecting D and R. In the existing scenario, in India, male children are preferred.

Let B be the linguistic bipartite graph given by the Figure 3.3.4.

B =

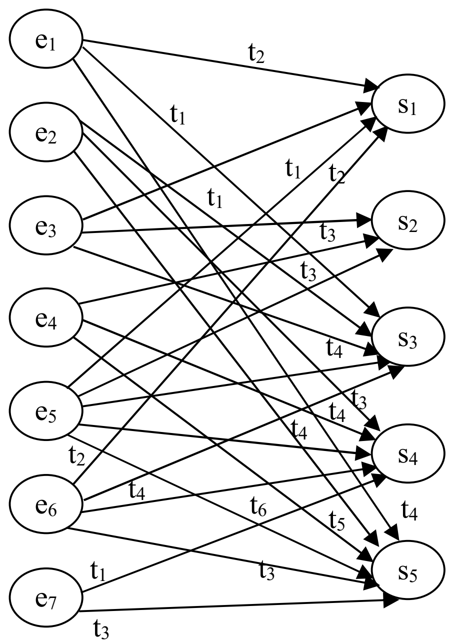


Figure 3.3.4

Let S be the linguistic adjacency matrix of the linguistic bipartite graph B.

$$S = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} t_2 & \phi & t_1 & \phi & t_4 \\ \phi & \phi & t_1 & t_3 & t_4 \\ t_6 & t_2 & t_1 & \phi & \phi \\ \phi & t_3 & \phi & t_4 & t_5 \\ t_1 & t_2 & t_3 & t_4 & t_5 \\ t_1 & \phi & t_2 & t_4 & t_5 \\ \phi & \phi & \phi & t_1 & t_3 \end{bmatrix} \end{array}.$$

The t_i's are already described.

We give a simple social problem and show how to work with them by an example.

Example 3.3.4. Let us now study social problems of child labour in India.

The linguistic terms / attributes associated with government for tackling the child labour problem are

g_1 = children do not contribute to vote bank

g_2 = the main source of vote politics are industrialists and business men

g_3 = government should provide free and compulsory education for children

g_4 = government should put stringent laws against those who practice child labour

R = {g_1, g_2, g_3, g_4} is taken as the range space of our linguistic bipartite graph.

Now for the domain space of the linguistic bipartite graph as the linguistic attributes associated with children who are forced to do child labour.

c_1 - abolition of child labor not put as law

c_2 - parents are not educated

c_3 - school dropout or children who have never attended any school

c_4 - social status of child labourers

c_5 - poverty or children a source of livelihood

c_6 - child labourers are runaways from home, orphans, parents in prison / parents are beggars.

c_7 - habits like cinema, smoking, drinking etc.

The linguistic bipartite graph associated with child labour taking the domain linguistic nodes as

$$D = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$$

and range of the linguistic nodes R = {g_1, g_2, g_3, g_4} the given by the following Figure 3.3.5.

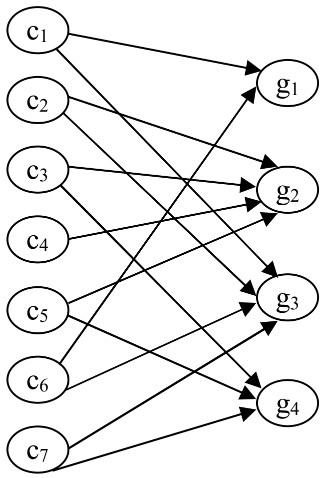


Figure 3.3.5

Let M be the linguistic adjacency matrix associated with the linguistic bipartite graph G.

$$M = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{array} \begin{array}{c} \begin{array}{cccc} g_1 & g_2 & g_3 & g_4 \end{array} \\ \begin{bmatrix} e & \phi & e & \phi \\ \phi & e & e & \phi \\ \phi & e & \phi & e \\ \phi & e & \phi & \phi \\ \phi & e & \phi & e \\ e & \phi & e & \phi \\ \phi & \phi & e & e \end{bmatrix} \end{array}$$

Suppose we take $X = (e, \phi, \phi, \phi, \phi, \phi, \phi)$ and find

$\max\{\min\{X, M\}\} = (\phi, \phi, e, \phi) = Y$

$\max\{\min\{Y, M^t\} = (e, e, \phi, \phi, \phi, e, e) = X_1$

$\max\{\min\{X_1 M\}\} = (e, e, e, e) = Y_1$

$\max\{\min\{Y, M^t\}\} = (e, e, e, e, e, e, e) = X_2$

$\max\{\max\{X_2, M\}\} = (e, e, e, e) = Y_2 (= Y_1)$

and so we see if "Abolition of child labour law not put as law", we see all the linguistic nodes come to on state in both the linguistic nodes.

Now having seen how these linguistic bipartite graphs can be used as linguistic models we now proceed onto define this concept for the sake of completeness and for researchers to use them in real world problems related to society.

Definition 3.3.1, *Let* $D = \{d_1, \ldots, d_n\}$ *the domain space where* $d_i \in D$ *are linguistic terms / attributes which serve as linguistic nodes* $(1 \leq i \leq n)$ *associated with some problem.*

Let $R = \{r_1, r_2, \ldots, r_n\}$ *be the range space where* r_j *in R are linguistic nodes / terms / attributes associated with the related problem with D;* $1 \leq j \leq m$.

Let $E = \{\phi, e\}$ *be the linguistic state*

(ϕ *- empty linguistic set, e the existence of edge).*

Let $X = \{(x_1, .., x_n) / x_i \in E, \quad 1 \leq i \leq n\}$

be the linguistic state vector space associated with D and

$Y = \{(y_1, \ldots, y_m) / y_i \in E; 1 \leq i \leq m\}$ *be the linguistic state vector space associated with R.*

So a state vector $(x_1, \ldots, x_n)$ *denotes the on-off state position of the linguistic node at an instant*

$x_i = \phi$ *off state*

$= e$ *on state.*

We have the bipartite linguistic $n \times m$ *graph M given by*

$$M = \begin{bmatrix} m_{11} & \dots & m_{1m} \\ m_{21} & & m_{2m} \\ \vdots & & \vdots \\ m_{n1} & \dots & m_{nm} \end{bmatrix}$$

where $m_{ij} \in E = \{\phi, e\}$; $1 \leq i \leq n$ *and* $1 \leq j \leq m\}$.

We find max{min{X,M}} where X is an instantaneous linguistic vector, let max min{X, M} = Y where Y is a $1 \times m$ *resultant state vector.*

We find max $\{\min\{Y, M^t\}\} = X_1$ *(say) then we find*

$\max\{\min\{X, M\}\} = Y_1$ *and so on.*

We will arrive at a fixed linguistic pair or a linguistic limit cycle which we call as the equilibrium state.

We for the present call this as Linguistic Relational Maps (LRM) is a linguistic directed graph or a map from D to R as linguistic nodes and causalities as linguistic edges. It represents a linguistic causal relations between D and R with edges weighted from $E = \{\phi, e\}$; *linguistic terms or linguistic set.*

We assume the linguistic edges form a directed cycle. That is LRM is said to be a cycle if it possesses a directed cycle.

LRM with cycles is said to be a LRM with feedback when there is feedback in the LRM that is when the causal linguistic relations flow through a cycle in a revolutionary manner; the LRM is called a linguistic dynamical system.

LRM functions analogous to Fuzzy Relational Maps (FRM) where the values of edges or edge weights are linguistic and the nodes of the linguistic bipartite graph is also linguistic nodes. These are very helpful in applications of real word problems especially social problems.

Next on similar lines we define and develop the new notion of Linguistic Cognitive Maps (LCMs) in the following.

Definition 3.3.2. *A Linguistic Cognitive Map (LCM) is a linguistic directed graph with linguistic attributes / nodes / events etc. as linguistic nodes and linguistic causalities as edges. This represents linguistic causal relationship between concepts.*

We will provide an example of the same in the following.

Example 3.3.5. In India there are several graduates and post graduates in last one decades. Several of them are unemployed or under employed. A expert who is a socioscientist spells out the five major concepts relating to unemployed graduates and post graduates.

e_1 - frustration

e_2 - unemployment

e_3 - increase of educated criminals

e_4 - under employment

e_5 - taking upto drugs, drinks etc.

The linguistic directed graph where e_1, e_2, e_3, e_4 and e_5 are taken as the linguistic nodes and causalities as linguistic edges taking values from the linguistic set E = {ϕ, e}.

The expert gives the following linguistic directed graph G.

G =

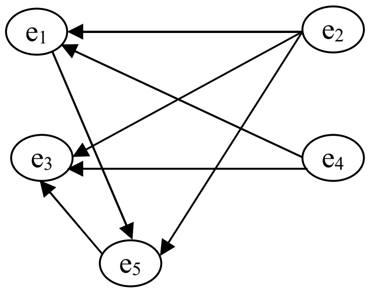


Figure 3.3.6

This is just an illustration of Linguistic Cognitive Maps (LCMs).

Definition 3.3.3. *When the nodes of the FCMs are linguistic sets then they are called as Linguistic Cognitive Maps (LCMs).*

Definition 3.3.4. *LCMs with edge weights from E = {ϕ, e} are called as simple LCMs.*

Definition 3.3.5: *Consider the linguistic nodes or concepts c_1, c_2, …, c_m of the LCM.*

Suppose the linguistic directed graph is drawn using the linguistic edge weight from $E = \{\phi, e\}$. The matrix M is defined by $M = (m_{ij})$ where $m_{ij} \in \{\phi, e\}$; $1 \leq i, j \leq n$ is the linguistic weight of the directed edge c_ic_j. M is called the linguistic adjacency matrix of the LCM also known as the linguistic connection matrix of the LCM.

***Definition 3.3.6**. Let $c_1, c_2, \ldots, c_n$ be the linguistic nodes of a LCM.*

$X = (x_1, x_2, \ldots, x_n)$ where $x_i \in \{\phi, e\} = E$. X is called as the linguistic instantaneous state vector and it denotes the on-off position of the linguistic node at an instant.

$$x_i = \begin{cases} \phi \text{ if } a_i \text{ is off and} \\ e \text{ if } a_i \text{ is on} \qquad \text{for } i = 1, 2, \ldots, n \end{cases}$$

***Definition 3.3.7**. Let $c_1, c_2, \ldots, c_n$ be the linguistic nodes of a LCM. Let $\overrightarrow{c_1,c_2}$, $\overrightarrow{c_2,c_3}$ …, c_ic_j be the linguistic edges of the LCM $(i \neq j)$. Then the linguistic edges form a directed cycle. A LCM is said to be cyclic if it possesses a linguistic directed cycle. A LCM is said to be acyclic if it does not possesses a linguistic directed cycle.*

We define a LCM with cycles is said to have a feedback. When there is a feedback in an LCM, i.e., when the causal relations flow through a cycle in a revolutionary way, the LCM is called linguistic dynamical system.

Let $\overrightarrow{c_1,c_2}$, $\overrightarrow{c_1,c_2}$ …, $\overrightarrow{c_{n-1},c_n}$ *be a linguistic cycle. When* c_i *is switched on and if the causality flows through the linguistic edges of a cycle and if it again causes* c_i, *we say that the linguistic dynamical system goes round and round. This is true for any node* c_i, *for* $i = 1, 2, \ldots, n$.

The equilibrium state for this linguistic dynamical system is called the linguistic hidden pattern.

If the equilibrium state of the linguistic dynamical system is a unique linguistic state vector then it is called a linguistic fixed point.

If the LCM settles down with a linguistic state vector repeating in the form

$$X_1 \to X_2 \to \ldots X_i \to X_1$$

Then this equilibrium is called a linguistic limit cycle.

Now for the socio economic problem of the unemployed graduates and post graduates in India let the linguistic causal connection matrix M of the linguistic directed graph in Figure 3.3.6 is as follows.

$$M = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \left[\begin{array}{ccccc} \phi & \phi & \phi & \phi & e \\ e & \phi & e & \phi & e \\ \phi & \phi & \phi & \phi & \phi \\ e & \phi & e & \phi & \phi \\ \phi & \phi & e & \phi & \phi \end{array}\right] \end{array}$$

Now we find using the linguistic dynamical system M we work with different linguistic instantaneous state vectors.

Let us consider the linguistic state vector

X = (e, ϕ, ϕ, ϕ, ϕ) where only the linguistic node frustration is in the on state all other nodes are in the off state.

We find max{min{X, M}}

$$= \max\{\min\{(e, \phi, \phi, \phi, \phi), \begin{bmatrix} \phi & \phi & \phi & \phi & e \\ e & \phi & e & \phi & e \\ \phi & \phi & \phi & \phi & \phi \\ e & \phi & e & \phi & \phi \\ \phi & \phi & e & \phi & \phi \end{bmatrix}\}\} = (\phi, \phi, \phi, \phi, e).$$

Now we update the resultant linguistic vector

(ϕ, ϕ, ϕ, ϕ, e) ↪ (e, ϕ, ϕ, ϕ, e) = X_1 (say)

(↪ symbol denotes the linguistic vector has be updated).

max{min{X_1, M}} = (ϕ, ϕ, e, ϕ, e) ↪ (e, ϕ, e, ϕ, e)

= X_2 (say)

max{min{X_2, M}} = (ϕ, ϕ, e, ϕ, e) ↪ (e, ϕ, e, ϕ, e)

= X_3 (say) = X_2.

Thus the resultant is a linguistic fixed point given by

(e, ϕ, e, ϕ, e).

We see when the linguistic node frustration alone is in the on state, then it leads a fixed linguistic point which makes the linguistic nodes e_3 which is "increase of educated criminals" and e_5 which is "taking up to drug, drinks etc" comes to on state their by making one understand the frustration due to unemployment or underemployment of the educated (graduate and post graduates) leads to increase in educated criminals and they taking drugs and drinks to cover up their frustration.

Now let us consider the on state of the linguistic node unemployment of the postgraduate and undergraduates, that is $Y = (\phi, e, \phi, \phi, \phi)$ be the linguistic state vector.

We study the effect of Y on M;

$\max\{\min\{Y, M\}\} = (e, \phi, e, \phi, e) \hookrightarrow (e, e, e, \phi, e) = Y_1$ (say)

$\max\{\min\{Y_1, M\}\} = (e, \phi, e, \phi, e) \hookrightarrow (e, e, e, \phi, e) = Y_2$ (say).

Now we find out

$\max\{\min\{Y_2, M\}\} = (e, \phi, e, \phi, e) \hookrightarrow (e, e, e, \phi, e)$

$= Y_3 \ (= Y_2)$.

We see the on state of the linguistic node unemployment makes the linguistic nodes frustration, increased educated criminals and taking up drugs as the resultant linguistic state vector.

Now let us consider the linguistic state vector in which only the node increase of educated criminals, e_3 is in the on state.

We find the effect of $Z = (\phi, \phi, e, \phi, \phi)$ on the linguistic dynamical system M.

$\max\{\min\{Z, M\}\} = (\phi, \phi, \phi, \phi, \phi) \hookrightarrow (\phi, \phi, e, \phi, \phi) = Z_1$ (=Z).

We see according to this socio scientist increase of educated criminals have no effect on the dynamical system M.

Now consider the on state of the linguistic node under employment e_4.

Let $A = (\phi, \phi, \phi, \phi, e, \phi)$ be the linguistic state vector, we study the effect of A on M.

$\max\{\min\{a, M\}\} = (e, \phi, e, \phi, \phi) \hookrightarrow (e, \phi, e, e, \phi) = A_1$ (say)

Now we find the effect of A_1 on the dynamical system M.

$\max\{\min\{A, M\}\} = (e, \phi, e, \phi, e) \hookrightarrow (e, \phi, e, e, e) = A_2$ (say).

Now we find the effect of A_2 on the dynamical system M.

$$\max\{\min\{A_2, M\}\} = (e, \phi, e, \phi, e) \to (e, \phi, e, e, e)$$

$$= A_3 \ (= A_2).$$

Thus the linguistic resultant of A on M gives a linguistic fixed point given by

$$(e, \phi, e, e, e).$$

That is on state of the linguistic node under employment leads to the on state of the linguistic nodes, frustration, increased educated criminals and to taking upto drugs etc.

The only linguistic nodes which remains in the off state is

e_2 = unemployment.

Finally we study the effect of the linguistic node taking upto drugs etc. that is e_5 in the on state.

Let B = (ϕ, ϕ, ϕ, ϕ, e) be the linguistic state vector in which only the linguistic node e_5, taking up to drugs etc., is in the on state and all other nodes variable, taking upto drugs, drinks etc. leads to the on state of the increase of educated criminals.

Thus we have provided as example of how we can use the concept of LCM to model social problems / issues.

Now we have given some simple basic applications of linguistic graphs to linguistic models LCM and LRM.

3.4 Suggested Problems

In the following section we provide some problems for the reader to solve so that by working out these problems they become well versed in linguistic graphs and their properties and applications to some simple linguistic models.

The problems with * or starred problems are different to be solved.

3.4.1. Let G be a linguistic graph associated with the linguistic set

S = {tall, very tall, short, very short, just short, just tall, very medium, tallest medium and just medium}

given by the following Figure 3.4.1.

G =

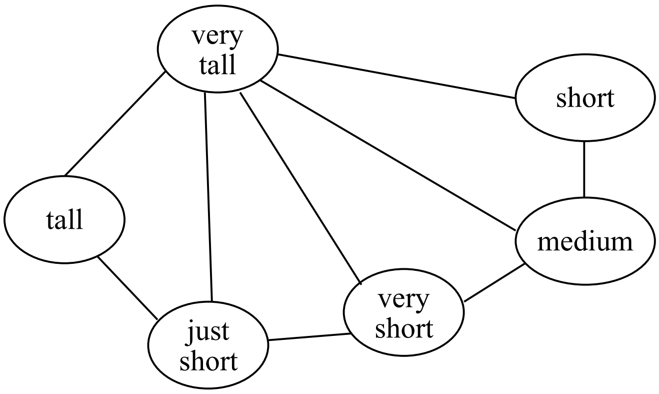


Figure 3.4.1

i) Give the adjacency linguistic matrix M associated with G.

ii) Is M a square symmetric matrix?

iii) Find all the linguistic subgraphs of G of order four and find their linguistic adjacency matrices.

iv) Is the linguistic adjacency matrices in problem (iii) linguistic submatrices of M? Justify your claim!

3.4.2 Let S be as in problem (1)

Let H be a linguistic directed graph with linguistic nodes from S given by the following Figure 3.4.2

H =

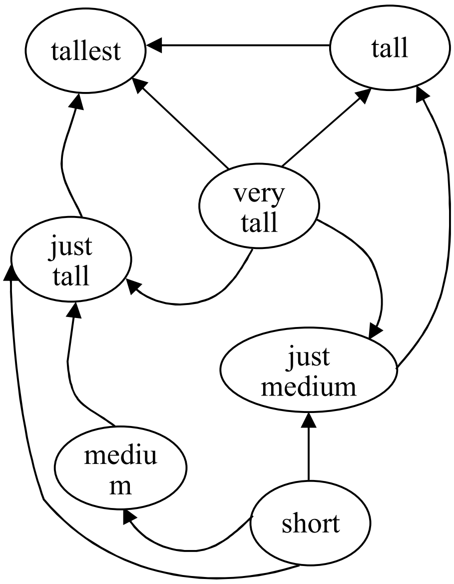


Figure 3.4.2

i) Study questions (i) to (iv) of problem (1) for this H.

ii) Let P =

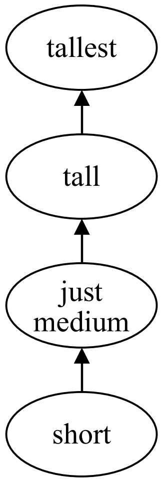


Figure 3.4.3

be the linguistic directed subgraph of H.

a) Find the linguistic adjacency matrix T associated with P.

b) Is T a linguistic submatrix of the linguistic matrix associated with H?

c) What are the special features enjoyed by this linguistic graph P and its adjacency linguistic matrix?

3.4.3 Enumerate some special features enjoyed by linguistic directed graphs in general.

3.4.4 Let S be a linguistic set associated with the linguistic variable speed of a vehicle on the road;

S = {fastest, just fast, very fast, fast, moderate speed, just moderate speed, slow, very slow, ϕ, just slow}.

Let G be the directed linguistic graph (increasing order of linguistic nodes) given by the following Figure 3.4.4.

G =

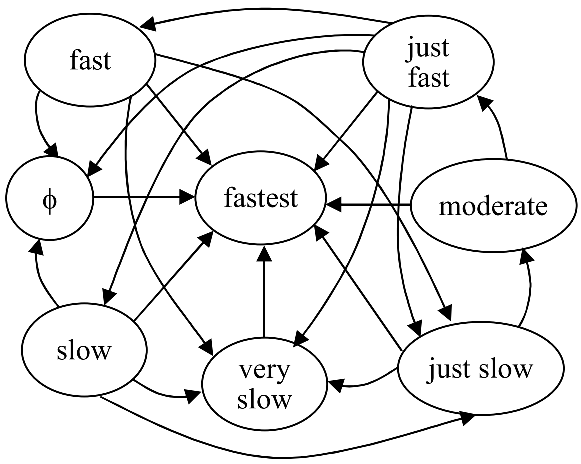


Figure 3.4.4

i) Is the linguistic set S or totally ordered set?

ii) Find the adjacency linguistic matrix M associated with G.

iii) Is M a symmetric linguistic matrix?

iv) Is M a upper triangular linguistic matrix or a lower triangular linguistic matrix?

v) How many linguistic directed subgraphs in G are complete linguistic subgraphs?

vi) Find at least four linguistic submatrices of M and find the corresponding linguistic directed subgraphs associated with them.

vii) Find five distinct linguistic direct subgraphs of orders 3, 4, 5, 6 and 7 and find the corresponding linguistic matrices and show they are also linguistic submatrices of M.

viii) Let K be an edge directed linguistic subgraph of G given in Figure 3.4.5.

K =

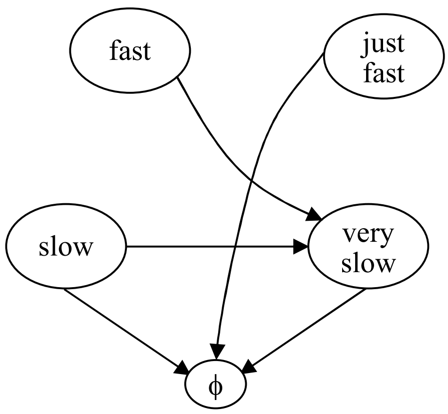


Figure 3.4.5

Find the related linguistic adjacency matrix of K. Is K a linguistic submatrix of M?

ix) Can we prove every edge linguistic directed submatrix of G contribute to a linguistic submatrix of M? Prove or disprove the same!

3.4.5 Let G be a linguistic bipartite graph associated with some linguistic domain and range sets D and R given by the Figure 3.4.6

(D = $\{x_1, x_2, x_3, x_4, x_5\}$ and R = $\{y_1, y_2, y_3, y_4\}$)

G =

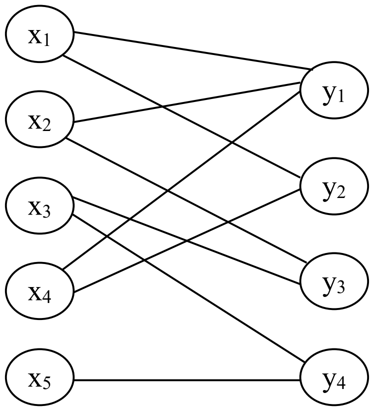


Figure 3.4.6

i) Find the linguistic adjacency matrix N of G.

ii) Prove all linguistic bipartite subgraphs of G is associated with the linguistic submatrix of N and vice versa.

iii) Is (ii) true if linguistic bipartite subgraph is replaced by edge linguistic subgraph? Justify your claim.

3.4.6* Let S = {dense, very dense, not that dense, scattered, moderate, very moderate, very scattered, just dense, just scattered, sparse, just sparse, very sparse}

be the linguistic terms / set associated with the linguistic variable growth of paddy sown in hand in a field.

Let the linguistic edge set E which measures the yield using the linguistic set S.

E = {good yield, moderate yield, poor yield, very poor yield, just good yield, just moderate},

i) Construct linguistic edge weighted graph using the linguistic set S and edge weight from E.

ii) Construct the linguistic adjacency matrix for the graph constructed in (i)

iii) Obtain a linguistic edge weighted directed graph H using the linguistic nodes S and the linguistic edges from E.

iv) Is the linguistic adjacency matrix L associated with graph H given in (iii) a upper triangular matrix or a lower triangular matrix or neither?

v) Is S a totally ordered linguistic set?

vi) Is the linguistic edge weighted directed graph W(in the increasing order of nodes) yield a upper triangular linguistic adjacency matrix? Justify your claim.

3.4.7 Obtain any interesting property associated with linguistic edge weighted directed graphs.

3.4.8* Construct a LCMs which involves indeterminacy as linguistic nodes.

3.4.9 Apply linguistic directed graph with edge weights from E = {ϕ, e} using LCM for any social problem (real world) and draw conclusions based on it.

Compare it with classical FCM.

3.4.10 Apply LCM model to the problem of symptom-disease model in children. The 8 linguistic nodes are supplied in the following

d_1 - cold with fever

d_2 - vomiting, loose motion with fever

d_3 - fever with loss of appetite

d_4 - cough and fever

d_5 - respiratory diseases

d_6 - gastroenteritis

d_7 - jaundice

d_8 - tuberculosis

The linguistic directed graph G given by an expert who is a doctor is as follows

G =

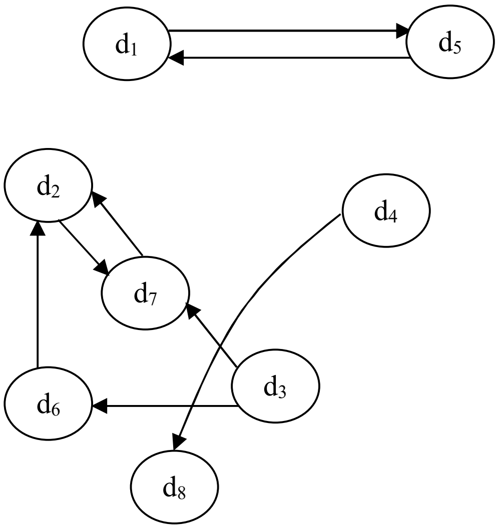


Figure 3.4.7

i) Obtain the adjacency linguistic matrix/connection linguistic matrix M of the linguistic graph G with edge weight from E = {ϕ, e}.

ii) Find the linguistic fixed point / limit cycle for the linguistic point / limit cycle for the linguistic state vector X = (e, ϕ, ϕ, ϕ, e, ϕ, ϕ, ϕ)

iii) Suppose Y = (ϕ, ϕ, ϕ, e, e, ϕ, ϕ, ϕ) be a linguistic state vector using linguistic dynamical system M of LCM find the linguistic hidden pattern of Y.

(The equilibrium state of the linguistic dynamical system of the LCM is called as the linguistic hidden pattern).

iv) Suppose A = (ϕ, ϕ, ϕ, ϕ, ϕ, e, e, e) is the linguistic state vector find the linguistic hidden pattern of A of the LCM.

v) Find the maximum number of linguistic fixed points got using LCM of all 2^8 state vectors.

vi) How many of the linguistic hidden patterns are linguistic fixed points?

vii) How many of the linguistic hidden patterns are linguistic limit cycles?

viii) Obtain any other special feature associated with the problem.

3.4.11 Let us stud the relation between the teacher and the student. Let D denote the linguistic nodes associated with teacher

D = {(t_1, t_2, t_3, t_4, t_5, t_6 t_7}

Where t_1 = good teacher

t_2 = poor teacher

t_3 = teacher is kind

t_4 = teacher is harsh (rude)

t_5 = teacher is indifferent

t_6 = teacher is mediocre

t_7 = teacher is devoted

Let R be the linguistic nodes associated with the student R = {s_1, s_2, s_3, s_4, s_5} where

s_1 = good student

s_2 = bad student

s_3 = average student

s_4 = student indifferent to studies

s_5 = lazy student in class

i) Using the linguistic nodes D and R construct a LRM; that is a linguistic bipartite graph B.

ii) If P_1 is the 7 × 5 linguistic connection matrix of B with edge weights from E = {ϕ, e}; determine the following.

a) The linguistic hidden pattern of X = (e, ϕ, ϕ, ϕ, ϕ, ϕ, ϕ) using the LRM.

b) Using the LRM the dynamical system P_1^t find the linguistic hidden pattern of Y = (ϕ, e, ϕ, ϕ, ϕ).

Will Y lead to a linguistic fixed point pair or a limit cycle pair?

c) Find out at least 2 linguistic state vector from X = {(a_1, a_2, a_3, a_4, a_5, a_6, a_7) / a_i ∈ E = {ϕ, e}; $1 \leq i \leq 7$} whose linguistic hidden pattern is a linguistic limit cycle pair.

d) Find out all linguistic hidden patterns which are linguistic fixed points pair of the LRM, P_1.

iii) Can E = {ϕ, e} be replaced by linguistic edge sets for this D and R?

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