Generalized Identity, Zero-Ground, and Necessity

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March 23, 2022

1 Introduction

According to *essentialist theories of modality* (ETM), modality can be accounted for in terms of essence (cf. Fine (1994) and Hale (2002)). While the details may differ, the basic idea is that necessity can be *defined* in terms of essence, that the former can be *reduced* to the latter, or that facts about necessity can be *grounded* in facts about essence. Recently, Jessica Leech (2020: 2) has challenged proponents of ETM to show why essence "should generate necessity".

Basically, she assumes that for this challenge to be met, essence facts (e.g. facts of the form "It is part of the essence of ... that ...") themselves must be necessary and then argues that this spells trouble for ETM.[1] In particular, she considers the theory of essence (and ground) in terms of generalized identity in Correia and Skiles (2017) (short: EGI) and argues that while it initially appears to be well suited to address her challenge, it falters when it comes to accounting for the necessity of generalized *self*-identity facts themselves.

This is my plan for this paper: Section 2 introduces EGI and develops an explanatory challenge for ETM in terms of EGI that Leech might have had in mind. Section 3 considers some ways to address the challenge (including Correia's and Skiles' (forthcoming) recent definition of necessity in terms of generalized identity and logical consequence), but finds them wanting. Section 4 extends Correia's and Skiles' definition of ground in terms of generalized identity to apply to *zero-ground* and uses this to reply to the explanatory challenge.

[1] For some discussion of the former assumption see Wildman (2019).

2 An Explanatory Challenge for EGI-based ETM

To introduce the explanatory challenge for an essentialist theory of modality based on EGI, we first need a basic understanding of the latter. According to EGI, essence and grounding can be reduced to *generalized identity*. Whereas ordinary, objectual identity is expressed by a relational predicate and holds between an object and itself, generalized identity – well – generalizes this idea to other grammatical categories. Following Rayo (2013), Correia and Skiles (2017: 3) express generalized identity using the operator $\equiv$, indexed by zero or more variables, which takes two open or closed sentences and yields another. Where P and Q are open or closed sentences, they read $P \equiv Q$ as "For it to be the case that P is for it to be the case that Q" and a statement of the form $P \equiv_{x,y,...} Q$ as "For some things x, y, .. to be such that P is for them to be such that Q". For now it suffices to note moreover that according to Correia and Skiles (2017: section 2), generalized identity statements *just are* (factual) essence statements.

Now, let us first take a look at what Leech says about EGI and then develop the explanatory challenge. As pointed out above, Leech assumes that ETM requires essence facts to be necessary. Hence, ETM based on EGI would require generalized identities to be necessary as well. After having offered an argument for the necessity of generalized identity on the basis of a Leibnizian principle and the necessity of generalized *self*-identity, Leech (2020: 16) argues that "one should not appeal to the essence of anything to account for [the necessity of generalized self-identity]". She then considers that one might take (as Correia and Skiles do) the generalized self-identities like $P \equiv P$ to be logical theorems and hence to be logically necessary and asks what the source of the semantic constraints on $\equiv$ responsible for these logical theorems, as well as the link between logical theoremhood and necessity is. According to her, these questions give rise to a dilemma that arises *even if we accept the correctness of the logic*: Either essence is not involved, but then not all necessities have their source in essence, or essence is involved, but then the account given is objectionably circular.

What the exact problem is that Leech is trying to get at is not completely clear, but I will now develop an explanatory challenge for ETM based on EGI that she might have had in mind. While doing so, I will understand ETM in terms of grounding (rather than reduction or definition), but the problem plausibly generalizes. Its core is this: Since generalized identities such as $P \equiv P$ are necessary, EGI-based ETM needs to supply grounds for the corresponding necessities – e.g. $\Box(P \equiv P)$ – but it is not straightforward to see what generalized identities could ground these necessities.

In their original (2017) Correia and Skiles do not provide grounds for (or define) necessity in terms of generalized identity, but in response to Leech's paper, they have later (forthcoming) provided a definition of necessity in terms of generalized identity and logical consequence, to which we will turn below. They do however in their first paper offer two principles that link generalized identity and modality:

(8) If $F_x \equiv_x G_x$, then $\Box\forall x\Box(Fx \text{ iff } Gx)$

(9) If $p \equiv q$, then $\Box(p \text{ iff } q)$

In trying to assign generalized identities as grounds to necessities, a plausible idea is to mirror these principles as follows:

(8<) If $F_x \equiv_x G_x$, then $(F_x \equiv_x G_x) < \Box\forall x\Box(Fx \text{ iff } Gx)$

(9<) If $p \equiv q$, then $(p \equiv q) < \Box(p \text{ iff } q)$

The intuition behind this move is that similar to how the essence operator ... can be conceived of as a more determinate and specific relative of the necessity operator (and correspondingly truth like $\Box P$ be grounded in truths like ...), generalized identity can be conceived of as a more determinate and specific relative of the strict (possibly quantified) biconditional (for similar motivated ideas concerning the relation between generalized identity and modality see Rayo (2013)).

But there is an issue that casts doubt on whether this idea could be worked into a full account of necessity: The latter two principles do at least not obviously supply us with grounds for $\Box(P \equiv P)$! Thus, the question arises what truth about generalized identities could ground $\Box(P \equiv P)$. More generally, there is a question of how to account for brute necessities that are not of conditional form (or in an appropriately intimate sense equivalent to such propositionsS, as $\Box(P \vee \neg P)$ might be).[2]

3 Some Options Considered

Let us consider some ideas. One is that generalized identities ground *their own* necessity (and analogous for essentiality):

($\equiv$ grounds $\Box\equiv$) If $P \equiv Q$, then $(P \equiv Q) < \Box(P \equiv Q)$

But there may be reason to believe that $P \equiv Q$ cannot fully ground $\Box(P \text{ iff } Q)$ because the resulting grounding claim can appear to be explanatory deficient: The grounding claim cannot be an instance of a general explanatory schema of the form "If ϕ, then $(\phi < \Box\phi)$" – lest every proposition

2 The issue also arises for the essentialist analogue of $\Box(P \equiv P)$: How can generalized identities prefixed with an essence operator such as $\Box_P(P \equiv P)$ be understood in terms of generalized identity?

be necessary, this schema has false instances - yet, it is plausible that claims of full ground must correspond to such general schemata.

This is a plausible related case: If Fa fully grounds Ga, then for any x, if Fx, then $Fx < Gx$. For if there were an entity b such that Fb but not Gb, then it appears that Fa cannot fully ground/explain a's being G, after all, b's being F does not suffice for its being G.[3] It follows that if [P]'s being the case grounds [P]'s being necessarily the case, then for every proposition, its being the case grounds its necessarily being the case. While it is doubtful that a "semantic descent" like this is in general permissible (this would at least require substantial argument), it may seem plausible in this case.

But there is a problem for this train of thought: There are instances of the schema "If ϕ, then $(\phi < \Box\phi)$" which prima facie seem explanatorily acceptable, for example: "If $\Box P$, then $(\Box P < \Box\Box P)$" (an analogous instance arises involving essence operators). This instance corresponds to a further, apparently unproblematic, general grounding schema, namely "If $\Box\phi$, then $(\Box\phi < \Box\Box\phi)$". Moreover, it's easy enough to come up with analogously restricted (albeit perhaps less principled) schemata for the case above, for example "If $\phi \equiv \phi$, then $((\phi \equiv \phi) < \Box(\phi \equiv \phi))$".

Still, the proposal remains mysterious in a way that the grounding of iterated box-claims perhaps is not: *How* exactly does the identity ground its own necessity? Without something of an answer, it is unclear what would differentiate the identity-claim from other necessary propositions with respect to their propensity to ground their own necessity. But then why not let *all* necessary propositions fully ground their own necessity as well? We can arguably characterize at least some of them (e.g. the identities and the logical necessities) without modal material, so general grounding principles that do not presuppose their necessity seem possible. Yet, it seems that such an approach would abandon the spirit of ETM. This may be attractive to some, or perhaps a rationale can be found for why the generalized approach is impossible, but there is a worry here.

In their (forthcoming) response to Leech, Correia and Skiles propose to solve the problem by defining necessity in terms of generalized identity and logical consequence. They offer three candidate definitions, a strong, a weak, and an intermediate account, which differ in whether the definiens only involves true identities or (also) their extensional correlates (e.g. the extensional correlate of $P \equiv Q$ is $P \leftrightarrow Q$), since my discussion does not turn on this difference, I will only consider the strong account here:

(Strong account) A proposition is necessary iff it is a logical consequence of the true identities.

According to this proposal, a generalized identity's being necessary is defined in that identity's

[3] This is an instance of deRosset's (2013) *determination constraint*.

following logically from itself. The account thus responds to the challenge to provide a ground or a definition of generalized identities' being necessary. Moreover, Correia and Skiles offer a candidate for a proper *definition* of necessity in terms of identity and logical consequence, as opposed to (e.g.) Rayo's suggestion.

While I cannot offer a decisive problem for Correia's and Skiles' account, it has some aspects that motivate looking for different options. First, it relies on the notion of logical consequence to define necessity and is thus less ambitious than accounts that attempt to make do with essence (or identity) alone. Second, it can capture non-conditional necessities only insofar as they logically follow from the identities (or their extensional correlates or both together in the case of the weak and intermediate account, respectively).

The third issue requires a little setup: On Correia's and Skiles' account, a true self-identity's necessity consists in that self-identity being a logical consequence of the true identities. Setting aside the possibility that true self-identities might be logical consequences of zero premises, this may seem to amount to a true self-identity's necessity consisting in its being a logical consequence of itself. In trying to identify the grounds of a true self-identity $P \equiv P$ given Correia's and Skiles' definition like this, it seems helpful to differentiate two proposals (let $\Rightarrow$ express logical consequence):

(1) $((P \equiv P) \Rightarrow (P \equiv P)) < \Box(P \equiv P)$

(2) $(([P \equiv P] \text{ is a true identity}) \wedge ((P \equiv P) \Rightarrow (P \equiv P))) < \Box(P \equiv P)$

Now, it seems to me that (1) is confronted with something like Fine's (2002: 266) trivialization worry: Since every proposition is a logical consequence of itself, the question arises why the fact that a generalized self-identity's being a logical consequence should be able to ground that self-identity's being necessary, while other (possibly contingent or even false) propositions' being logical consequence's of themselves cannot. Indeed, Correia and Skiles Correia and Skiles (2017: fn12) address Fine's trivialization worry by remarking that "[even] if every truth is a logical consequence of itself, not every truth is a logical consequence of (the extensional correlates of) the true identities. And by our lights, that makes all the difference, since we take it to be a non-trivial and substantial matter what the true identities are". This suggests that on Correia's and Skiles' account, the ground of $Box(P \equiv P)$ looks more like the one specified by (2) than by (1). But if so, then it appears that the proposal defines necessity not merely in terms of $\equiv$ and logical consequence, but moreover requires a resource to state that certain propositions are true identities.

4 Addressing the Challenge: Zero-Ground in Terms of Generalized Identity

In any case, let us now see how these issues might be avoided by combining the following ideas: Correia and Skiles do not only account for essences in terms of generalized identity, but also for grounding. Additionally, De Rizzo (2020) has proposed that necessities might be grounded in grounding claims and in particular that some necessities might be grounded in *zero-grounding* claims. Lastly, Fine (2012: 48) has suggested that identities might be zero-grounded.

Let us begin with a quick note on zero-grounding: Normally, grounding is taken to be (at least something approximately like) a relation between a plurality of propositions or facts, the *grounds*, and a single proposition or fact, the *grounded* proposition/fact or *groundee*. Zero-grounding is a limiting case of grounding in which the set of grounds is empty. A zero-grounded proposition or fact is grounded and not ungrounded, but it does not require any propositions or facts to ground it – it is grounded in *zero* propositions/facts. More precisely, if we assume grounding statements to have the form '$\Gamma < P$', then since in the case of zero-grounding statements, the 'Γ' stands for an empty plurality of grounds, statements of zero-grounding have the form '$< P$'.[4]

Plausibly, if $< P$, then $\Box P$. The first part of the present idea is that zero-grounding grounds necessity: If $< P$, then $(< P) < \Box P$ (De Rizzo (2020) suggests that *all* necessities are zero-grounded, here we will merely use the idea that a proposition's being zero-grounded grounds its being necessary). Note that this idea fits well with the general idea (sketched above) of necessities being grounded in truths involving more determinate or specific operators. Next, taking on board Fine's suggestion that identities are zero-grounded, we have $< (P \equiv P)$, which would then ground $\Box(P \equiv P)$. According to this proposal, all necessities may be grounded in generalized identities, but some, e.g. $\Box(P \equiv P)$, will be so grounded via zero-grounding propositions (which in turn are defined or grounded in generalized identities).

Now, for this to work, EGI has to be able to capture zero-grounding claims. In general, EGI (Correia and Skiles (2017: 14)) defines grounding as follows:

> "a collection of facts $p_1, p_2, ...$ grounds another fact q iff conjoining $p_1, p_2, ...$ gives you a disjunctive part of q (thus each of $p_1, p_2, ...$ is a conjunctive part of a disjunctive part of q), yet there's *no* way of conjoining facts with q that gives you a disjunctive part of *any* of $p_1, p_2, ...$ (thus q is *not* a conjunctive part of *any* disjunctive part of the facts in that

[4] Zero-grounding has been introduced by Fine (2012: 47f.), who argues for it by applying principles of the logic of ground to certain edge cases. Further applications include Litland (2017)), Muñoz (2020), De Rizzo (2021), and Kappes (2020).

collection). In symbols:

FACTUAL-GROUNDING $p_1, p_2, ... < q$ iff: (i) $p_1 \wedge p_2 \wedge ... \sqsubseteq^{\vee} q$; and (ii) neither $q \sqsubseteq^{\wedge\vee} p_1$, nor $q \sqsubseteq^{\wedge\vee} p_2$, nor ..."

To capture zero-grounding, we allow that the plurality of grounding facts $p_1, p_2, ...$ be empty (i.e. we allow the grounding facts to be none). Furthermore, allow for the conjunction in (i) to conjoin the empty set of facts. Do do this, we use '$\bigwedge\{...\}$' to express the conjunction of an arbitrary set of facts. This leaves us with the task of extending condition (ii), which we achieve by quantifying over the facts in the set $\{p_1, p_2, ...\}$:

EXTENDED FACTUAL-GROUNDING $p_1, p_2, ... < q$ iff: (i*) $\bigwedge\{p_1, p_2, ...\} \sqsubseteq^{\vee} q$; and (ii*) for no p in $\{p_1, p_2, ...\}$: $q \sqsubseteq^{\wedge\vee} p$

Alternatively, we could use 'for no p among $p_1, p_2, ...$, $q \sqsubseteq^{\wedge\vee} p$' for (ii*), but then we would need to allow for the plurality $p_1, p_2, ...$ to be empty here as well.

Now, for the ordinary cases in which $p_1, p_2, ...$ are not none, i.e. in which $\{p_1, p_2, ...\}$ is non-empty, **EXTENDED FACTUAL-GROUNDING** amounts to **FACTUAL-GROUNDING**: Both (i) and (i*) demand that the conjunction of $p_1, p_2, ...$ be a disjunctive part of q, while (ii*) holds iff (ii) holds. On the other hand, if the $p_1, p_2, ...$ are none, we obtain the following instance of **EXTENDED FACTUAL-GROUNDING**:

FACTUAL-ZERO-GROUNDING $< q$ iff: (i*) $\bigwedge\emptyset \sqsubseteq^{\vee} q$; and (ii*) for no p in $\emptyset$: $q \sqsubseteq^{\wedge\vee} p$

Here, (ii*) is trivially satisfied (there are no facts in the empty set of facts). Therefore, according to this definition, a fact q's being zero grounded amounts to the empty conjunction being a disjunctive part of q. According to Correia and Skiles, we can understand the notion of disjunctive part in terms of generalized identity as follows: p is a disjunctive part of q iff for some r, for q to hold is for $p \vee r$ to hold. Given this, a fact q is zero-grounded iff there is an r such that $q \equiv (\bigwedge\emptyset \vee r)$.

Accordingly, for the proposal to work, for any generalized self-identity $p \equiv p$ an r has to be found such that $(p \equiv p) \equiv (\bigwedge\emptyset \vee r)$. One idea here is to choose the empty conjunction itself for r: According to Correia's and Skiles' (2017, p. 6) assumptions about generalized identity, for q to hold is for $q \vee q$ to hold (i.e. $q \equiv (q \vee q)$), and from $p \equiv q$ we can infer $q \equiv p$, so if r is the empty conjunction, then what it is for $p \equiv p$ to be the case is for the empty conjunction to be the case.

Here it might be objected that the identity-claims required by the proposal are perhaps not immediately intuitive: It is not completely straightforward to see what what it is for a generalized self-identity $P \equiv P$ to be the case has to do with what it is for the empty-conjunction to be the case. Note that this problem does not depend on a particular choice for r, it already arises given that according to the proposal, there is an r such that $(P \equiv P) \equiv (\bigwedge \emptyset \vee r)$.

I suggest we respond as follows: The present notion of generalized identity is *worldly* (Correia and Skiles (2017: 4)) and thus insensitive to mere representational differences that are responsible for this intuitive problem. Specifically, zero-grounded claims are insubstantial as to their worldly content – they demand nothing of the world. The empty conjunction is like this, but it is plausible that generalized self-identities such as $P \equiv P$ are like this as well. Hence, their worldly demand is the same – the worldly aspect of what it is for them to be the case is the same. While they are representationally different, both $P \equiv P$ and $\bigwedge \emptyset$ require nothing of the world, just like all other zero-grounded propositions.

If the proposal succeeds, then the idea that a proposition's being zero-grounded grounds that proposition's being necessary can be employed in an account of necessity based on generalized identity. The necessities of generalized self-identities, but also generalized identities and other broadly non-conditional necessities may then be grounded in the corresponding propositions' being zero-grounded, which in turn is defined in terms of generalized identity as above. At least for these cases, the proposal would allow to avoid the issues mentioned above that confront Correia's and Skile's proposal. For example, the proposed grounds for the necessity of generalized self-identity do not involve a notion of generalized self-identity or a device to state that certain propositions are generalized identities. Moreover, at least in principle, the proposal applies to any proposition which can plausibly be said to be zero-grounded. Finally, the proposal promises to retain the intuitive idea of grounding necessity in terms of more determinate or specific operators.

Now, in contrast to Correia and Skiles, I have not offered a proper *account* (i.e. a definition) of necessity in terms of generalized identity here that harnesses the above idea. So, it could still turn out for different reasons that necessity can only be defined in terms of generalized identity plus further material, such as logical consequence. Nevertheless, I have argued that given the above suggestion, accounting for the necessity of generalized identities need not require any such further material.

There are various conceivable ways a definition of necessity in terms of generalized identity that makes use of the above definition of zero-ground could look like. For example, one could plug the above definition of grounding into De Rizzo's account of necessity in terms of grounding. Alternatively, one could attempt to give an inductive definition of necessity that takes as its base the zero-grounded propositions as well as the extensional correlates of the identities. Lastly, the above idea could conceivably be employed to give a variant of Correia's and Skiles' definitions, for example, one could consider a variant of their weak account, according to which a proposition would be necessary iff it is a logical consequence of the extensional correlates of the true identities and the zero-grounded propopositions. I leave developing these ideas further as a task for another occasion.

Bibliography

Correia, Fabrice and Alexander Skiles (2017), "Grounding, essence, and identity." *Philosophy and Phenomenological Research*.

Correia, Fabrice and Alexander Skiles (forthcoming), "Essence, modality, and identity." *Mind*, fzab017.

De Rizzo, Julio (2020), "Grounding grounds necessity." *Analysis*, URL `https://doi.org/10.1093/analys/anz083`.

deRossett, Louis (2013), "No free lunch." In *Varieties of Dependence* (Hoeltje, Schnieder, and Steinberg, eds.), 243–270, Philosophia.

Fine, Kit (1994), "Essence and modality." *Philosophical Perspectives*, 8, 1–16.

Fine, Kit (2002), "Varieties of necessity." In *Conceivability and Possibility* (Tamar Szabo Gendler and John Hawthorne, eds.), 253–281, Oxford Up.

Fine, Kit (2012), "Guide to Ground." In *Metaphysical Grounding* (Correia and Schnieder, eds.), 37–80, Cambridge University Press.

Hale, Bob (2002), "The source of necessity." *Noûs*, 36, 299–319.

Kappes, Yannic (2020), "The explanation of logical theorems and reductive truthmakers." *Philosophical Studies*, 1–18.

Leech, Jessica (2020), "From essence to necessity via identity." *Mind*.

Litland, Jon Erling (2017), "Grounding Grounding." *Oxford Studies in Metaphysics*, 10.

Muñoz, Daniel (2020), "Grounding nonexistence." *Inquiry: An Interdisciplinary Journal of Philosophy*, 63, 209–229.

Rayo, Augustín (2013), *The Construction of Logical Space*. Oxford University Press.

Wildman, Nathan (2019), "A Note on Lange on Contingent Necessity-Makers." *Erkenntnis*, 1–9.