Conquering Mount Everett: Branch Counting Versus the Born Rule

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Abstract: This paper develops and advocates a rule for assigning self-locating credences in quantum branching scenarios, called Indexed Branch-Counting. It is argued that Indexed Branch-Counting can be justified on both accuracy-theoretic grounds and on the grounds that it satisfies a requirement of exchangeability for probability assignments. Since Indexed Branch-Counting diverges from the Born Rule, this poses trouble for Everettian approaches to probability. The paper also addresses a common argument against branch-counting, namely that the rule is incoherent in light of putative vagueness in the number of branches. Finally, the paper addresses a recent proposal from Simon Saunders that aims to reconcile branch-counting with the Born Rule, arguing that the proposal faces challenges.

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Everettian Quantum Mechanics has many virtues: it appears to be a simple and elegant solution to the measurement problem, postulating nothing in addition to the universal wavefunction and a unitary dynamical law – the Schrodinger Equation – guiding its evolution. Yet, Everettian Quantum Mechanics faces perhaps its most serious challenge in making sense of quantum-mechanical probabilities.[1] Ac-

[1]Not to mention the other central challenge for the Everett picture, namely of explaining how a pure wave theory can be ontologically adequate. This concern has been raised by Maudlin (2010), for instance. Wallace (forthcoming) defends a 'math-first' structural realism, however, and if one adopts such an ontology, a pure wave theory arguably faces no trouble with ontic adequacy. For the sake of argument, I will grant that Everettian quantum theory is ontically adequate.

cording to the theory, every non-zero amplitude component of the wavefunction describes an actually-obtaining state of affairs, and the various outcomes occur on separate 'branches' to the extent that the interference between them is sufficiently small. But since every possible outcome will occur with certainty, on one branch or another, it is not immediately clear what quantum mechanical probabilities are supposed to be probabilities *of*, on the Everett picture. This is problematic insofar as we take ourselves to have empirical confirmation of a particular probabilistic law for quantum measurements, namely

> *Born Rule*:[2] the probability of a measurement of observable $\hat{O}$ yielding eigenstate $|\alpha\rangle$, for a quantum state $|\psi\rangle = \sum_i C_{\alpha_i} |\alpha_i\rangle$ is equal to the squared wavefunction amplitude $|C_\alpha|^2$.

The problem of understanding probability in Everettian Quantum Mechanics might come from two places.[3] First, there is the *incoherence problem*: what sense could it make to talk about non-degenerate probabilities (i.e., probabilities between 0 and 1) for events that I know with certainty will occur on one branch or another? Second, there is the *quantitative problem*: assuming that it does make sense to talk about non-degenerate probabilities in an Everettian universe, why should those probabilities equal the square amplitudes?

It has been widely suggested that the incoherence problem can be solved by appeal to self-locating uncertainty: while agents are certain that a given outcome will occur on one branch or another, they are uncertain on which branch they will end up.[4] The details of how to make sense of self-locating uncertainty – especially future-directed self-locating uncertainty – are controversial, but for present purposes, we may imagine that an agent simply closes their eyes before measurement, and observes the result only after branching has occurred.[5]

In this paper, I will not be concerned with the incoherence problem. Rather, I will be concerned with whether Everettian Quantum Mechanics can recover the Born Rule probabilities, assuming it makes sense to talk about nontrivial Everettian probability in the first place. And, following Wallace (2012), I will assume that this question will be resolved just in case Everettians can show that agents in a branching universe are rationally compelled to set their *credences* in measurement outcomes equal to the Born Rule probabilities.[6]

Attempts at justifying the Born Rule for Everettian Quantum Mechanics typically start from some prior principles of rationality – decision-theoretic or epistemic – and then try to show that the Born Rule follows from the imposed constraints.[7] But perhaps the most natural way in which one might set one's

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[2]For observables with continuous spectra, we calculate the probability that the observable will be found to be within a range of values by integrating $\psi^*\psi$ over the relevant region. For instance, in the case of position in one dimension for a time-independent wavefunction $\psi(x)$, the probability that a particle will be found between *a* and *b* is $P(a < x < b) = \int_a^b \psi^*(x)\psi(x)\,dx$.

[3]The proceeding dichotomy is due to Greaves (2007).

[4]See Vaidman (1998), Saunders and Wallace (2008), Saunders (2010), Wilson (2012), Sebens and Carroll (2018), and Vaidman and McQueen (2019).

[5]See McQueen and Vaidman (2019).

[6]I will sidestep questions about whether the square amplitudes function as objective probabilities, or merely guide credence-forming and decision-making in a manner 'similar enough' to objective probabilities. Wallace (2012) adopts a functionalist approach to objective chance, while other Everettians such as Vaidman reject the notion of objective chance.

[7]It is not obvious that *any* justification of the Born Rule must go by way of showing that rational credence conforms to the Born Rule; in the case of dynamical collapse theories, for instance, the Born Rule is more or less stipulated as part of the stochastic dynamics, and as a strategy for setting credences it is just a straightforward application of the Principal Principle. But insofar as Everettians sidestep the incoherence problem by appeal to something like self-locating uncertainty rather than objective chance, it is crucial that they demonstrate that self-locating credences ought rationally to obey the Born Rule. And

credences in branching contexts is to apply an *indifference measure* over branches: roughly, if I could be on any of *n* branches, I should assign a credence of $\frac{1}{n}$ to the proposition that I am on any particular branch.[8] Such a *branch counting rule* immediately appears to conflict with the Born Rule, for it will seldom be the case that the quantum system under consideration is in a perfectly symmetric superposition in the relevant basis. (We will return to this in the final section.) If it is not, then different branches will be assigned different wavefunction amplitudes, and so their quantum-mechanical probabilities will be distinct. Therefore, if branch counting really is a rationally permissible thing to do in a branching universe, then the rational way to set one's credences would misalign with the textbook quantum probabilities, spelling trouble for Everettian Quantum Mechanics.

The aim of this paper is to propose and defend a new branch-counting rule for setting credences in branching scenarios, arguing that this branch-counting rule disagrees with the Born Rule and thus spells trouble for Everettian Quantum Mechanics. I will proceed as follows. In Section 1, I will discuss an alleged problem with branch-counting and propose a new branch-counting rule – which I call Indexed Branch-Counting – that avoids this problem. In Section 2, I will offer two independent justifications of Indexed Branch-Counting. In Section 3, I will address another common criticism of branch-counting, namely that it is nonsensical in light of putative vagueness in the number of branches. In Section 4, I will address a recent branch-counting rule from Saunders (2021c) that claims to reconcile branch-counting with the Born Rule.

1 Branch-Counting

As noted, there is intuitive appeal to the idea that, in branching scenarios, we should count branches and adopt a credence function that is indifferent between being located on one branch or another. A first-pass formulation of this idea is what Wallace dubs

> *Naive Branch-Counting*: If an agent's evidence is consistent with their being located on any one of *n* branches, *m* of which realize an eigenstate $|\alpha\rangle$ of observable $\hat{O}$, then the agent's credences should be such that $Cr(\hat{O} = |\alpha\rangle) = \frac{m}{n}$,

where $\hat{O} = |\alpha\rangle$ is the self-locating proposition that the agent is located on an $|\alpha\rangle$-branch.

Naive Branch-Counting surely conflicts with the Born Rule: if all that I'm doing in setting credences is counting branches, and assuming the square amplitudes don't co-vary with ratios in branch count, then my credences will disobey the Born Rule. For this reason, Everettians wish to rule out Naive Branch-Counting as a rationally permissible credence-forming strategy. Luckily, Wallace (2012) has pointed out a serious problem with Naive Branch-Counting. Suppose that we have a particle in the following quantum state, as written in the z-spin basis:

$$|\psi\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle\,.$$

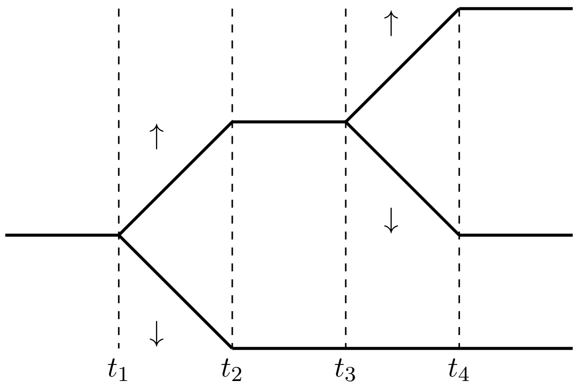
my purpose in this paper is largely not to show directly that these justifications fail, but rather to show that a rationally viable alternative to the Born Rule emerges in branching scenarios, deepening the challenge of showing that adherence to the Born Rule is the *uniquely* rational credence-forming strategy in an Everettian universe.

[8] This formulation, as with more precise formulations later in the paper, presupposes that there are finitely many branches. This, I take it, is the received view: DeWitt (1971) explicitly asserts that there are finitely many branches, and Wallace (2012) assumes as much in his decision-theoretic justification of the Born Rule. I will also make this assumption; but notice that Barrett and Goldbring (2022) have recently developed a model of Everettian Quantum Mechanics with hyperfinitely many branches, for observables with continuous spectra, that allows us to talk about ratios in cardinalities. I suspect such a model will be compatible with branch-counting, wherein certain self-locating propositions will receive infinitesimal probability.

And suppose you have a device which measures the z-spin of the particle. According to the Everett interpretation, the system undergoes unitary evolution upon measurement into a superposition of product states:

$$|\psi\rangle \otimes |R\rangle_M \to a\,|\uparrow\rangle \otimes |``\uparrow"\rangle_M + b\,|\downarrow\rangle \otimes |``\downarrow"\rangle_M.$$

Finally, suppose that at t_1, you measure the z-spin of such a particle, observing the results at t_2. Suppose further that if and only if the particle comes out spin-up, a measurement of another such particle will be performed at t_3, with results being observed at t_4. According to Everettian Quantum Mechanics, the branching structure will look as follows.



It is a setup like this that motivates Everettian skepticism about Naive Branch Counting. As Wallace (2012) has noted, at t_2, the rule requires that an agent assign equal credence to each branch, so that $Cr(\uparrow) = Cr(\downarrow) = \frac{1}{2}$. Yet, at t_4, there are three branches, each of which must again be apportioned equal credence, so that $Cr(\uparrow) = \frac{2}{3}$ and $Cr(\downarrow) = \frac{1}{3}$ for the first z-spin measurement. Yet, this sensitivity of the prescribed credence function to times is putatively at odds with a constraint of probabilistic coherence which I will call Diachronic Updating:[9]

$$Cr(\uparrow \text{ at } t_4) = Cr(\uparrow \text{ at } t_4 \mid \uparrow \text{ at } t_2)Cr(\uparrow \text{ at } t_2) + Cr(\uparrow \text{ at } t_4 \mid \downarrow \text{ at } t_2)Cr(\downarrow \text{ at } t_2).$$

Diachronic Updating (in this case) follows from the rule of total probability. But clearly Naive Branch-Counting is, in this sense, diachronically inconsistent.[10] Consequently, it appears that Everettians are free to reject branch-counting and pursue independent arguments for the rationality of the Born Rule, having cast aside its most promising competitor.

Not so fast! While Wallace has offered a good argument against Naive Branch-Counting, we can formulate alternative branch-counting rules that are, in fact, diachronically consistent. The most straightforward such rule has been suggested by Saunders (2021a), which I will call

> *Local Branch-Counting*: If, upon a branching event (such as a quantum measurement), *n* branches are created, *m* of which realize an eigenstate $|\alpha\rangle$ of observable $\hat{O}$, then the agent's credences should be such that $Cr(\hat{O} = |\alpha\rangle) = \frac{m}{n}$.

[9] Saunders (2021a) has a similar rule, stated as a special case of the so-called consistency condition for a quantum history space.

[10] Carroll and Sebens (2018) as well as Peterson (2011) argue that, aside from putative incoherence, such intertemporal inconsistency renders naive branch-counters susceptible to an Everettian Dutch Book.

Local Branch-Counting is not diachronically inconsistent, because it only demands that agents be indifferent between being located on the branches that are created *upon a particular branching event*. Thus in adhering to Local Branch-Counting, an agent will assign equal credence to each outcome at t_2 and will then update in the usual way, adopting respective credences of $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$ in the propositions that they are located on the $|\uparrow\rangle_1 |\uparrow\rangle_2$ branch, the $|\uparrow\rangle_1 |\downarrow\rangle_2$ branch, and the $|\downarrow\rangle_1$ branch, respectively, at t_4.

I think that this proposal is on the right track. However, I don't see why the requirement of indifference should be indexed to the time of *branching* in particular. Consider the following pair of cases.

> DOUBLE OBSERVATION: At t_1, a measurement of the z-spin of a particle in the state $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$ will occur. You will close your eyes prior to measurement. The measurement result will be displayed on a screen before you, and you will then open your eyes. If, at t_2, you observe "↑", the z-spin of a second particle in the same initial quantum state will be measured at t_3. You will close your eyes again, and observe the results of the second measurement at t_4. If, at t_2, you observe "↓," nothing else will happen.

> DELAYED OBSERVATION: At t_1, a measurement of the z-spin of a particle in the state $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$ will occur. You will close your eyes prior to measurement. But this time, you will keep your eyes closed. A second measurement will occur at t_3 if and only if the first measurement outcome is spin up. Only after this second measurement, at t_4, will you open your eyes and observe the results of the first measurement.

The difference between DOUBLE OBSERVATION and DELAYED OBSERVATION is not in the induced branching structures, but only in the times at which you observe the relevant outcomes. I propose that rational agents should consequently have different credences in the two cases. In particular, I advocate

> *Indexed Branch-Counting*: If, at the time in which an agent observes the result of a measurement on observable $\hat{O}$, there are *n* branches consistent with the agent's evidence, *m* of which realize an eigenstate $|\alpha\rangle$ of $\hat{O}$, then the agent's credences should be such that $Cr(\alpha) = \frac{m}{n}$. Formally:
>
> $$Cr(\alpha \mid N^t(\alpha)/N^t = \frac{m}{n}) = \frac{m}{n},$$
>
> where t is the time of observation, $N^t(\alpha)$ is the number of α-branches at t, and N^t is the total number of branches at t consistent with the agent's evidence.

Indexed Branch-Counting, like Local Branch-Counting, prescribes that agents apply indifference reasoning only at a particular time. However, the time-index is shifted from the time of branching to the time of observation. In DOUBLE OBSERVATION, the two branch-counting rules make the same prescription: namely, $Cr(\uparrow) = Cr(\downarrow) = \frac{1}{2}$. For Local Branch-Counting, this is because t_2 is the time at which branching occurs, and at t_2, the numbers of branches for each outcome are equal. But for Indexed Branch-Counting, it is because t_2 is the time at which the agent observes the result of a measurement on $\hat{A}$. Yet, the branch-counting rules conflict in cases like DELAYED OBSERVATION. Local

Branch-Counting advocates the same thing in both cases, namely equal credence in both possible outcomes. Indexed Branch-Counting, by contrast, advocates credences $Cr(\uparrow) = \frac{2}{3}$ and $Cr(\downarrow) = \frac{1}{3}$ for the first spin-measurement, since the agent only observes the measurement results at t_4, when there are unequal numbers of branches instantiating each eigenstate.

Immediately, it may look like Local Branch-Counting gives more intuitive results than Indexed Branch-Counting. Instinctively, since there is only one way that the agent can observe two spin-up results, a credence assignment of $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$ looks natural. I will offer independent reasons in favor of a time-index at the time of observation rather than the time of branching in section 2.1, but for now, it bears mentioning that a possible reason that Local Branch-Counting looks more intuitive is because it clearly would give the correct prescriptions if we thought of the probabilities in question as robust, dynamical chances; after all, there is no clear reason that dynamical chances should depend on agent-centered facts such as when observations are made. But Everettians must think of these probabilities as (something like) self-locating credences, and when we move from dynamical chances to self-locating credence, probability assignments that depend on the time of observation are at least potentially better-motivated. This bears important analogies with the 'thirder' position in the famous Sleeping Beauty Problem (see Elga 2000): while the objective chances in Sleeping Beauty-style cases of self-locating probability are not impacted by these agent-centered facts, it is often thought that rational self-locating credences are impacted by these considerations.[11]

2 Justifications of IBC

In this section, I will offer two separate justifications of Indexed Branch-Counting. Both justifications, I think, are strong rivals to extant Everettian justifications of the Born Rule.

2.1 Accuracy

Epistemic Decision Theory is a longstanding tradition which aims to justify principles of rational credence on the grounds that they promote accuracy.[12] It is often thought that, just as the goal of belief is truth, the goal of credence is accuracy, or proximity to truth. Epistemic Decision Theory is a 'consequentialist' program, in that it is primarily concerned with the *outcomes* of adopting certain credence-forming strategies, cashed out in terms of whether those strategies are conducive to accuracy.

This subsection is devoted to justifying Indexed Branch-Counting from the perspective of Epistemic Decision Theory. I will argue that adherence to Indexed Branch-Counting maximizes accuracy on a natural way of measuring accuracy in branching contexts. (Though, I should note that one could replace 'accuracy' with other kinds of utility, such as pleasure, monetary payoffs from bets, etc., without much trouble; I will sometimes switch between these different kinds of utility when it is helpful for illustrative purposes.)

The *inaccuracy* of a credence function *Cr* is often defined by its Brier Loss Function, which is the sum of the squared distances between *Cr* and the truth function v_w at a world *w*, where v_w assigns 1 to

[11] I am indebted to Chris Dorst on this point.

[12] See Pettigrew (2013) for an overview.

all truths and 0 to all falsehoods:

$$I(Cr, w) = \sum_{X \in \mathcal{F}} (Cr(X) - v_w(X))^2,$$

where $\mathcal{F}$ is an algebra of propositions. Now, if we are to extend Epistemic Decision Theory to branching contexts, we require (a) a measure of accuracy which is diachronically consistent and (b) a way to sum accuracy across branches.

Regarding (a), I propose an accuracy-measure which is indexed to the time at which one observes the result of a quantum-mechanical experiment, rather than the times at which the relevant branching events occur. Define the Quantum Brier Loss Function for an agent with credence function *Cr* on branch *b* as follows.

$$I^t(Cr, b) = \sum_{\alpha} (Cr^t(\alpha) - v_b^t(\alpha))^2,$$

where α is the proposition that some eigenstate $|\alpha\rangle$ of observable $\hat{O}$ is realized, $v_b^t(\alpha) = 1$ if α is true on branch *b* and 0 if α is false on branch *b*, *Cr* is *A*'s credence function, and *t* is the time at which *A* observes the result of the measurement on $\hat{O}$.

Why should the measure of accuracy be indexed to the time of observation, in particular? The answer comes down, in part, to issues of theory confirmation: the only times at which we are in a position to experimentally confirm or disconfirm a scientific theory is when we actually observe the outcomes. Suppose, for instance, that someone proposed a quantum theory according to which whatever physical quantities determine the quantum probabilities may oscillate wildly in the moments leading up to observation, but the probabilities at the time of observation always, for whatever reason, conform to the Born Rule.[13] Now, this would be a strange theory, and we may have all sorts of reasons to find it distasteful. But such a theory does not seem to run into trouble with empirical confirmation *per se*, in the sense that all of the empirical evidence we have of textbook quantum theory – obtained by making experimental observations – are predicted by such a theory, and the precise relative frequencies that we observe in the laboratory are all as to be expected, according to such a theory. By contrast, a theory which gave the wrong probabilities at the time of observation – even if it gave the 'correct' probability-determining magnitudes at other times – would be straightforwardly empirically disconfirmed. Thus, from the perspective of theory-confirmation, what is crucial is that one's credences be well-calibrated to the truth at the time of observation. Insofar as we must pick a time-index – and in case different choices of time-index result in different probability assignments – it should be the time of observation. Further, any post-observation branching should be irrelevant, from an accuracy standpoint, since agents are in a position to diachronically update their credences in the measurement outcomes once they have observed them. In DOUBLE OBSERVATION, then, once the agent has observed the first measurement results, their updated credences will be maximally accurate, so it doesn't matter that they will branch in

[13] What of the probabilities our theory assigns to *records*, which are surely also part of our evidence? Even post-observation, once we have updated on the results of measurements, our records of past measurement outcomes are presumably also to be treated as evidence with which we can confirm or disconfirm a theory. My argument here should carry over to the case of records in a manner similar to the case of one-world objective chance; after updating on the actual outcome of a chance process, we can nevertheless compare what credences various chance theories instructed us (via the Principal Principle and related norms) to adopt at the time of observation. Similarly, records are part of our evidence in the Everett case insofar as there are facts about what credences we rationally ought to have had when we made the relevant observations, and how far those credences were from what we now know to be the truth. Thanks to an anonymous referee for pushing me to clarify.

the future. It is as though their epistemic utility obtained from observing the first measurement result is 'exhausted' as soon as the observation occurs.[14] In DELAYED OBSERVATION, the agent's credences should be such that they are well-calibrated to the truth at t_4, since this is the time at which the first measurement outcomes are to be observed, and so the agent is in a position to confirm or disconfirm the relevant probabilistic hypotheses.

More contentious, I suspect, is (b): the function by which we sum inaccuracy across branches. The question is whether inaccuracy should be summed via a weighted or non-weighted function. I will advocate a simply additive cross-branch accuracy measure. In particular, let $\{A_i\}$ be the set of an agent *A*'s successors and $\{b_i\}$ be the set of branches on which *A*'s successors find themselves. Letting *Cr* be *A*'s credence function, we can define the *total successor-inaccuracy* for *A* as:

$$I(\{A_i\}, \{b_i\}) = \sum_{b_i} I^t(Cr, b_i).$$

I advocate the following accuracy norm for branching agents:

> *Maximize Accuracy for Successors*: in setting their credences in the outcomes of quantum-mechanical experiments, agents should minimize total successor-inaccuracy.

As noted, what will be controversial in *Maximize Accuracy for Successors* is the use of a simply additive function of inaccuracy, as opposed to an inaccuracy function that is weighted by the square-amplitudes of each branch. At first glance, it looks like the use of a simply additive function begs the question against Everettians. So, what could motivate the simply additive approach?

Everettian justifications of the Born Rule often start out with some antecedent principles of decision-theoretic or epistemic rationality that are supposed to constrain an agent's credences. In the decision-theoretic approach, these credences are taken to be what guides an agent's calculation of expected utility, and are hence taken to determine an agent's preferences between bets. Expected Utility is calculated as:

$$EU_{\mathcal{B}} = \sum_i Cr(b_i) u(\mathcal{B}, b_i),$$

where $\mathcal{B}$ is a possible bet, $Cr(b_i)$ is the agent's credence that they will end up on branch b_i, and $u(\mathcal{B}, b_i)$ is the utility of $\mathcal{B}$ on b_i, which in the context of Epistemic Decision Theory is accuracy. Decision-theoretic justifications of the Born Rule start from 'axiomatic' constraints on $Cr(b_i)$ which are supposed to guarantee that $Cr(b_i)$ is equal to b_i's square amplitude. Here is a small sampling of such principles:

> *Measurement Neutrality*: Given a measurement of observable $\hat{O}$ for state $|\psi\rangle$, and a payoff function $\mathcal{P}$, a rational agent must be indifferent between two quantum games that agree on $\langle |\psi\rangle, \hat{O}, \mathcal{P}\rangle$ but disagree on additional physical details about *how* the measurement is performed on $\hat{O}$. (Deutsch 1999, Wallace 2002)

[14] The accuracy case (and this is why it is useful to work with Epistemic Decision Theory in particular), I take it, is a special case of a more general principle, namely: when we are trying to determine the utility of some course of action, we should index our measure to the times at which payoffs are received and exhausted. The accuracy case is special because (1) the time of payoff (if anything) is just the time of observation, and (2) the relevant 'payoffs' are almost immediately exhausted by a conditionalizing agent, since updating on observed measurement results gives a posterior probability function whose inaccuracy contribution for the event in question is always zero.

State Supervenience: Given unitary evolution operators *U*, *U'*, *V*, *V'*, if $U\psi = U'\psi'$ and $V\psi = V'\psi'$, then an agent who prefers *U* to *V* given initial state ψ should also prefer *U'* to *V'* given initial state ψ'. (Wallace 2012)

Branching Indifference: A rational agent shouldn't care about branching *per se*: if a certain operation leaves their future selves in N different macrostates but doesn't change any of their rewards, the agent should be indifferent as to whether or not the operation is performed.[15] (Wallace 2012)

These principles look intuitively plausible. But at the same time, the considerations that motivate them seem defeasible, at least under certain conditions. For one thing, they arguably trade on intuitions that come from the fact that some features (such as the details about how measurements are performed) are obviously irrelevant to the outcomes in standard one-world cases, which is far less obvious in the Everettian case, where measurement details have a dramatic influence on the 'numbers' of branches corresponding to each outcome of the measurement on $\hat{O}$.[16] And it surely isn't *obvious*, a priori, that such considerations should be ignored.[17] Furthermore, independent doubts have been raised against each of these principles. (For doubts about Measurement Neutrality, see Greaves (2004); about Branching Indifference, see Dizadji-Bahmani (2015) and Wilson (2013); about State Supervenience, see Jansson (2016).)

But my purpose here is not to offer further independent arguments against these principles. Rather, I want to point out that the program of justifying the Born Rule via axiomatic constraints on an agent's EU-guiding credences arguably gets the process backwards. Let us suppose that an agent knows all of the possible outcomes of an experiment, the utility profile of every possible bet, and the numbers of successors for each outcome. (The third of these assumptions is controversial, and will be discussed in Section 3.) Such an agent is in a position to maximize utility *outright*, given certainty about what states their post-branching successors will be in. So why should the agent be concerned to maximize *expected* utility, when (once we have picked a way to sum utility across branches) they are in a position to just maximize *utility*? To be sure, in a branching context, the agent's preferences will be such that they can be represented as though they follow from expected utility calculations via a non-trivial credence function.[18] But why should the agent be concerned that their credences obey principles like Measurement Neutrality, State Supervenience, Branching Indifference, etc., except insofar as adherence to those principles will assist them in maximizing utility outright, on their preferred way of measuring utility across branches? At minimum, the normative force of obeying these principles seems to be *defeasible*, if an alternative course of action is dominant from the perspective of utility-maximization.

Here is a way to put the point. Suppose (for the moment) that an agent has a definite number of successors about whom they indeed care equally. It seems perfectly rational that the agent should adhere to Maximize Accuracy for Successors, *even* if this means that they will fail to obey what seem, initially, to be other plausible rationality constraints on their credences. Since the agent knows how inaccurate

[15]It should be noted that Measurement Neutrality is given up, e.g., in Wallace and Greaves (2004) in favor of principles like State Supervenience and Branching Indifference. So, the three principles here are just given as examples of axioms that have assumed in Everettian justifications of the Born Rule, though they have not all appeared together, as far as I am aware.

[16]Excepting, of course, contextual hidden-variable theories, where measurement details are relevant in a different sense.

[17]One reason that Wallace thinks they *should* be ignored – and this serves primarily as a defense of Branching Indifference – is because decoherence theory only approximately defines a branching structure. The merits of this rationale will be explored in Section 3.

[18]This comes from standard Savage-style representation theorems; see Savage (1954).

each of their successors' credences will be given a particular action, they will be in a position to adopt the unique credence function that maximizes accuracy. If adhering to any of these other rationality principles will result in an accuracy-dominated credence function, come what may, then from the perspective of Epistemic Decision Theory, adherence to such principles need not take priority, when an absolute utility-maximizing strategy is available. The same argument applies to recent 'epistemic' Everettian justifications of the Born Rule, such as those of Sebens and Carroll (2018) and McQueen and Vaidman (2019). Such theories posit 'internal' constraints on rational credence functions (so-called 'Epistemic Separability' in the former case, and certain symmetry constraints in the latter) from which the Born Rule is derived. But an analogous question arises: why should agents obey these constraints, if adherence to such constraints is a strictly accuracy-dominated strategy for setting credences? We grant that agents are ignorant of centered (i.e. self-locating) propositions; but given that they are certain of all the uncentered facts, it seems that they are in a position to know exactly what course of action would maximize accuracy or utility (on a particular measure) across branches.[19] My broad point is this: appeal to 'axiomatic' principles like Measurement Neutrality, Branching Indifference, and the like, can't alone justify any particular way of measuring *utility* (or accuracy) across branches. Since these principles are meant to be 'internal' constraints on credences/preferences, they are irrelevant from the perspective of outright utility-maximization.

But it is a common line of thought that one ought never adopt a strategy that is strictly utility- or accuracy- dominated by some alternative strategy. This is the idea that underlies classic dutch-book arguments in epistemology, e.g. for the requirement that rational credences obey the probability axioms.[20] If Everettians are not working with a weighted measure of utility across branches, then all of these rationality 'axioms' lead agents to consistently adopt accuracy-dominated credence functions. (This is to be proved at the end of this subsection, where I show that Indexed Branch-Counting uniquely minimizes cross-branch inaccuracy, on the non-weighted measure.) Hence Everettians should endeavor to show that the non-weighted measure of cross-branch accuracy is somehow defective, in favor of a measure of cross-branch accuracy that is weighted in accordance with square amplitudes, lest they prescribe a credence-forming strategy that is strictly accuracy-dominated. And an account of why agents should use a weighted measure of cross-branch accuracy/utility ought to explain why the amplitudes are relevantly 'value-making' for agents.

This much (at least up to the preceding paragraph's penultimate sentence) has been acknowledged in some Everettian justifications of the Born Rule. For instance, Papineau (2010) has argued that agents should weight the utility of their successors according to the square-amplitude of the branches on which their successors will live. One can then act in such a way as to *guarantee* that utility – on the square-amplitude measure – be maximized. Greaves (2004) takes a similar approach. Both authors seem to take it as primitive that agents should weight utility in this manner. I agree with Papineau and Greaves that

[19] I note that this argument sits more naturally with fission and overlap views of personal identity, and less naturally with divergence views. It is clearer why a person prior to splitting into multiple successors, or a person-stage prior to branching into multiple future person-stages, would care about maximizing utility across branches than in the divergence case, where there are many numerically distinct pre-measurement observers, each naturally concerned only with their *own* accuracy or utility, in the face of *de se* uncertainty about which of many qualitatively indistinguishable branches they happen to be on. In the fission and overlap cases, the pre-measurement observer bears the same relation to their successors as I bear to my future self (or that my present time-slice bears to my future time-slices) in one-world cases, so it makes sense that the pre-measurement observer should care about each of their successors (equally or otherwise). Not so in the divergence case. The argument of the next subsection, however, should apply regardless of your views about personal identity in branching worlds.

[20] Most famously from Joyce (1998).

the only viable way to justify the Born Rule in decision-theoretic terms is to show that adhering to it maximizes utility outright. Once this much has been assumed – that agents who can maximize utility should do so (or are rationally permitted to do so) – then the debate about probability in Everett comes down to one question: must agents weigh the utility of their successors in proportion to their square amplitudes, or are they rationally permitted to measure cross-branch accuracy/utility via a non-weighted function?

I think we can give at least a prima facie positive argument against requiring agents to weight utility in accordance with branch amplitude. Given that the various branches of the wavefunction are approximately structurally isomorphic to each other, it is very natural that I would weigh each branch equally in my quest to maximize accuracy (or utility, more broadly). After all, whatever value-making features there are in virtue of which I might care about the epistemic success of my successors seem to be equally present in all branches, since the functional/structural properties on each branch do not depend on the amplitudes.[21] For rhetorical illustration, suppose that instead of minimizing inaccuracy we are trying to minimize pain. Each of the relevant branches will correspond to my possessing (e.g.) perfectly rich neural architecture, all equally capable of firing C-fibers in response to certain sensory inputs, and the like. Shouldn't I expect, then, that each of my successors would be equally capable of experiencing pain, and to the same degree? And insofar as I had considered pain to be a negative value-making feature of a situation – a feature in virtue of which I would have wanted to *avoid* a given situation – shouldn't I then be just as averse to bringing about pain-states for my low-amplitude as my high-amplitude successors? The projects of maximizing accuracy, utility, etc., are 'consequentialist' projects insofar as they are aimed at promoting *outcomes* that we deem valuable; and states of affairs are valuable to the extent that they instantiate certain value-making descriptive properties. So if one is to care about the accuracy/utility of their high-amplitude successors more than that of their low-amplitude successors, it should be because the high-amplitude branches somehow instantiate these descriptive properties to a greater 'degree.'

Everettians could respond to this prima facie argument in one of two ways. First, they could explain how it is that whatever value-making descriptive features we are interested in (inaccuracy, pain, etc.) are indeed instantiated to a lesser degree on low-amplitude branches than on high-amplitude branches. This would serve as a satisfying explanation of why agents should weight the inaccuracy of their successors in proportion to their square amplitudes. Second, they could stipulate that, even though low-amplitude branches do instantiate value-making descriptive properties to the same degree as high-amplitude branches, agents are still rationally compelled to weight the utility of their successors in proportion to their square amplitudes.

The closest I have seen Everettians come to the first strategy is Vaidman (1998), who suggests that square amplitudes represent 'measures of existence', such that low-amplitude branches exist to a lesser degree than high-amplitude branches. Perhaps each branch, then, has a certain 'thickness', in proportion to its square amplitude, that may somehow explain why the states of affairs on low-amplitude branches should matter less to me than the states of affairs on high-amplitude branches.

Unfortunately, it is not clear why Vaidman's concept of a branch's 'measure of existence' should bear on the 'value' we assign to our successors. As Vaidman acknowledges (1998, p. 255):

[21] See Maudlin (2010) and McQueen (2015) for very similar arguments regarding the tails problem in GRW: Maudlin and McQueen both convincingly argue that we should not be comforted by the fact that the 'tails' of GRW are of low-amplitude or low mass-density, precisely because the relevant functional/structural properties are preserved.

> All physical parameters, such as mass, spin, magnetic moment etc., are independent of the measure of existence. A neutron with a tiny measure of existence moves (feels) exactly as one with measure 1.

But if all physical parameters are independent of the measure of existence, what could justify our weighting the inaccuracy of our successors in accordance with their amplitudes? Our low-amplitude successors, for instance, will not be any less disappointed that their predictions were falsified, nor will their pain be duller, nor (it seems) will they experience any relevant value-making states of affairs in a manner that is qualitatively distinguishable from their high-amplitude counterparts. What Vaidman's measure of existence quantifies is something like an 'ability to interfere' (1998, p. 255):

> When the measure of existence is less than or equal to 1/2, the [experimenter] can change probabilities of further splitting completely; when it is greater than 1/2, only partially, and when it is equal to 1 the [experimenter] cannot change the probabilities at all.

And as important as measures of existence may be when it comes to their dispositions to interfere or be interfered with, there is no clear sense in which they justify caring unequally about our high- and low-amplitude successors. That low-amplitude branches can be more easily interfered with does not justify caring about them less in any transparent way. If, for instance, we are interested in minimizing pain, then it seems that we should care more about high-amplitude pain only if it is qualitatively distinguishable from low-amplitude pain (else how could it be true that we are genuinely minimizing *pain*, which is a qualitative property?), yet Vaidman's 'measure of existence' does not give us reason to think that this is the case. Moreover, insofar as explaining why the amplitudes are value-making requires explaining how the amplitudes influence qualitative features of states of affairs, this strategy would arguably be self-defeating. For, if Everettian Quantum Mechanics is to be consistent with our experience, it had better be true that being in one element of a superposition is subjectively indistinguishable from not being in a superposition at all, else observers would be able to notice when branching occurs![22] Hence, it seems vital that being in a high amplitude branch should be subjectively indistinguishable from being in a low-amplitude branch. But this makes it exceedingly difficult to see why rational agents are obligated to value high-amplitude payoffs over low-amplitude payoffs.[23]

The second response to the argument is to just stipulate that we should weight utility in accordance with the square amplitudes, despite the fact that the relevant value-making states of affairs are equally present on all branches. This is certainly an option for Everettians, but I think the weight of independent reason is heavily against it. It is hard to overstate how extreme such a proposal would be: it would imply, for instance, that we rationally ought to take an action that we know will bring about unbearable pain for millions of our low-amplitude successors if the action would bring about positive well-being for just a handful of our high-amplitude successors, despite there being *no* qualitative difference between high- and low- amplitude pain, pleasure, etc. Consequently, this approach implies that agents are rationally required to have their 'caring measure' completely fail to supervene on the qualitative, structural, or functional features of the relevant states of affairs, even if all the agent is interested in minimizing/maximizing are functional, structural, or qualitative properties! To say the least, this is an unpleasant consequence of the 'stipulation' response.

[22] And, for what it is worth, I have had no such experience.

[23] Thanks again to Chris Dorst on this point.

One may worry that this line of argument simply begs the question against the Everettian: after all, attributing some physical significance to branch-amplitude is a core commitment of Everettian Quantum Mechanics. Once one is committed to Everettian theory, one is *already* committed to taking amplitude seriously, and to thinking that it bears some intimate connection to concepts like value and probability. So, demanding a *justification* for the idea that agents are rationally required to weight the utility of their successors in proportion to their square amplitudes simply assumes that the core commitment of Everettian Quantum Mechanics lacks justification. But I see no such core commitment. Nothing in the fundamental theory forces us to think that branch amplitude *directly correlates* with how much we value certain outcomes, or what probability these outcomes receive. As far as I can tell, one who adopts Everettian theory is committed to only two roles for branch amplitude: first, the amplitudes determine what states of affairs obtain in the first place, in the sense that only nonzero amplitude components of the wavefunction describe things that actually happen. And second, the amplitudes figure in the time-evolution of the universe and its subsystems, via the standard unitary dynamics. This seems to be the only role that the theory truly postulates for the amplitudes; so, in what sense does a commitment to the theory carry over into a commitment that the amplitudes bear any special relation to the notions of value, utility, or probability? This in no way flows from antecedent commitments of the physical theory; and if it is to be a primitive postulate, then we are forced to say revisionary and counter-intuitive things about what makes certain descriptive properties value-making for agents.

Having said all that, Maximize Accuracy for Successors – which relies on a non-weighted accuracy function – looks like a perfectly sensible rule for setting credences in a branching universe. And it is straightforward to show that Indexed Branch-Counting satisfies Maximize Accuracy for Successors. Suppose that $\frac{m}{n}$ branches at the time of observation realize an eigenstate $|\alpha\rangle$ of observable $\hat{O}$. Now suppose that an agent *A* adopts credence function *Cr* and that $Cr(\alpha) = \delta$. Notice first that the Brier Loss Function is additive over propositions, such that $I(Cr, w) = \sum_X \mathfrak{s}(X)$, with $\mathfrak{s}(X) = (Cr(X) - v_w(X))^2$. Now, given that $\frac{m}{n}$ branches host $|\alpha\rangle$, it will be the case that on $\frac{m}{n}$ branches, $\mathfrak{s}(X) = (1-\delta)^2$, and on $1 - \frac{m}{n}$ branches, $\mathfrak{s}(X) = \delta^2$. Thus:

$$\mathfrak{s}_X(\{A_i\}, \{b_i\}) = \left((1 - \frac{m}{n})\delta^2 + \frac{m}{n}(1-\delta)^2\right) n.$$

Maximize Accuracy for Successors demands that agents select a credence function that minimizes $I(\{A_i\}, \{b_i\})$, which would be satisfied by a credence function that minimizes $\mathfrak{s}_X(\{A_i\}, \{b_i\})$ for all X. We can easily minimize the function by setting its first derivative to zero and confirming that at the relevant δ-value, the second derivative is positive.

$$\frac{d\mathfrak{s}}{d\delta} = 2n\delta - 2m,$$

as can easily be verified. To minimize, we set:

$$2n\delta - 2m = 0,$$

which sets $\delta = \frac{m}{n}$. Finally, we notice that

$$\frac{d^2\mathfrak{s}}{d\delta^2} = 2n,$$

which is clearly positive if we assume (as we surely do!) that the number of branches is positive. This establishes that setting $Cr(\alpha) = \frac{m}{n}$ minimizes successor-inaccuracy. This is precisely what Indexed Branch-Counting recommends, so Indexed Branch-Counting follows from Maximize Accuracy for Successors.

2.2 Exchangeability

If one is unconvinced by the 'consequentialist' accuracy-first arguments of the previous subsection, there is a very plausible internal constraint on rational credences, central to probability and decision theory, the extension of which to branching scenarios justifies adherence to Indexed Branch Counting, rather than the Born Rule.

Suppose that at the time of observation, there are *n* branches consistent with the agent's evidence, *m* of which host some outcome $|\alpha\rangle$. We hold fixed a set of branches $\{b_1, ..., b_n\}$ and the outcomes on each branch, and so define $b_\alpha = \{b_1, ..., b_m\}$ and $b_\alpha^C = \{b_{m+1}, ..., b_n\}$.

The outcome space Ω over partitions of which the agent's self-locating probabilities are to be defined is the set of all permutations of observers across the branches, i.e. Ω is a set of $n!$ ordered *n*-tuples: $\Omega = \{\omega_1, ..., \omega_{n!}\}$, with a given $\omega = (A_{\pi_1}b_1, ..., A_{\pi_n}b_n)$ being such a possible permutation of observers. '$A_{\pi_i}b_i$' is to be read 'the ${\pi_i}^{\text{th}}$ observer is on the i^{th} branch,' and the $(\pi_1, ..., \pi_n)$ represent a possible permutation of the indices $\{1, ..., n\}$.

An agent's credences are to be distributed over a partition $\{X_1, ..., X_n\}$ of Ω, where X_i corresponds to the proposition that the agent is located on the i^{th} branch. Hence, in determining their self-locating credences over possible results of an experiment, the agent distributes credences over subsets of this partition. For instance, the agent's credence that they are on an α-branch is equal to their credence in $\bigcup_{i=1}^{m} X_i$, which corresponds to the proposition that they are in one of the *m* branches that instantiates outcome $|\alpha\rangle$.

A very popular constraint on rational credence functions in probability and decision theory is known as *exchangeability*, which states roughly that an agent's credences ought to be indifferent between outcomes that agree on the frequencies with which the relevant events occur.[24] The intuitive idea is that two sequences of events do not differ in their joint probability distributions just because they differ in the order in which the events are distributed. In the present context, we can define an exchangeability constraint on rational credences in branching scenarios.

> *Quantum Exchangeability*: for any $\omega, \omega' \in \Omega$ that agree on the frequency of observers who see outcome $|\alpha\rangle$, for any α, the agent should assign credences $Cr(\omega) = Cr(\omega')$.

Now, since each of the $n!$ outcomes agree on the number of branches hosting each measurement result, it follows that each ω receives the same self-locating probability. And for any X_i the identity of the observer on the i^{th} branch (namely one's own identity!) is held fixed, leaving $(n-1)!$ possible permutations of observers on the other branches remaining, each of which (again) agree on the frequencies. Thus,

$$P(X_i) = \frac{(n-1)!}{n!} = \frac{1}{n}.$$

[24] Exchangeability is famously central to de Finetti's theorem. See Gillies (2000) for an accessible overview.

Finally, for an outcome $|\alpha\rangle$ realized on branches $\{b_1, ..., b_m\}$, there are m propositions $X_1, ..., X_m$ each with probability $\frac{1}{n}$. Thus we get:

$$Cr(\hat{O} = |\alpha\rangle) = Cr(\bigcup_{i=1}^{m} X_i) = \frac{m(n-1)!}{n!} = \frac{m}{n}.$$

This is precisely the prescription given by Indexed Branch-Counting. Hence, agents with exchangeable credences in branching scenarios must obey Indexed Branch-Counting rather than the Born Rule. Given the importance of exchangeability in decision theory, this strikes me as a strong point in favor of Indexed Branch-Counting. Exchangeability has a much more robust history in decision theory, and is much more widely and independently defended, than the central assumptions made in extant Everettian justifications of the Born Rule – such as Measurement Neutrality, Branching Indifference, Epistemic Separability, symmetry considerations, etc.[25]

It is worth pointing out, too, that Quantum Exchangeability is a much weaker and more plausible constraint than any *general* principle of indifference. Indifference reasoning, in general, has a number of well-known problems.[26] And this makes it much easier to privilege other intuitively plausible principles of rationality, insofar as they conflict with indifference reasoning.[27] But Indexed Branch-Counting, which prescribes indifference in branching contexts, can be motivated by something much weaker, less problematic, and more widely-defended than a general principle of indifference. In particular, Indexed Branch-Counting is motivated by the plausible idea that agents should have exchangeable credences – i.e., that they should be indifferent between sequences that differ merely in the ordering, and not in the frequency, of the relevant events. It is not necessary that agents in general be indifferent between any two outcomes compatible with their evidence *whatsoever*, because in an Everettian universe, the possible outcomes of an experiment all agree on the cross-branch frequencies of observers who see each result. Because of this, a much weaker constraint that the general principle of indifference can justify indifference reasoning in this particular case.

2.3 Taking Stock

There is a guiding idea behind both the accuracy-based and exchangeability-based justifications of Indexed Branch-Counting that I have offered, namely that knowledge of frequencies constrains our credences more strongly than most principles of rationality that apply when agents are otherwise ignorant of the frequencies. In the accuracy case, this boils down to the fact that our antecedent knowledge of cross-branch frequencies allows us to adopt credences that maximize epistemic utility *outright*. And this means that our ability to simply maximize accuracy overrides any initial considerations in favor of certain constraints on the ur-credences that guide our calculation of *expected* accuracy, such as those assumed in extant Everettian justifications of the Born Rule. In the exchangeability case, this amounts to the fact that antecedent knowledge of frequencies allows agents to readily implement a widely defended link between

[25] Exchangeability is central, for instance, to de Finetti's theorem. For a recent philosophical application, see Schwarz (2014) for a use of exchangeability to justify the Principal Principle.

[26] Though see Eva (2019) for an attempt to rehabilitate the principle of indifference via the notion of comparative confidence judgments.

[27] For instance, Carroll and Sebens (2018) acknowledge that indifference reasoning conflicts with the Born Rule, and yet assert that we should abandon indifference in favor of Epistemic Separability, in part because of the general problems with indifference reasoning.

frequencies and single-case credences.

And this seems like a very plausible idea. If, for instance, I ask you about your credence that some die will land on a 3, it is perfectly natural for you to respond with $\frac{1}{6}$, on the basis (let's say) of considerations relating to symmetries in the die. But if I then tell you that the frequency of '3' in the next ten trials will be $\frac{1}{2}$, it seems that you now have information that clearly overrides the initial appeal of a symmetry-based argument in favor of credence $\frac{1}{6}$: your credence that the next die-roll will land on a 3 should be $\frac{1}{2}$. Such information (i.e., information about future frequencies), in single-world cases, is extremely rare – sufficiently rare that it is often regarded as 'inadmissible' in the Principal Principle. When agents have knowledge of frequencies, all bets are off, and often, principles that would otherwise constrain rational credence no longer apply. In an Everettian universe such information is readily available: since agents know what the future branching structure looks like – and are merely ignorant about self-locating propositions – they have perpetual access to information about frequencies that very plausibly override whatever considerations have thus far motivated justifications of the Born Rule.[28] And it is *this* fact, I think, that presents the greatest challenge for Everettian accounts of probability: the access agents have to frequency-information means that there is a ready competitor to the Born Rule, licensed by principles that quite plausibly override those that justify the Born Rule.

Finally, it bears mentioning that one does not have to fully buy Maximize Accuracy for Successors or Quantum Exchangeability to think that these arguments present a challenge for Everettian approaches to probability. For anyone aiming to justify some precise probabilistic rule in branching scenarios will have to make *some* substantial addition to classical decision theory or standard Bayesian epistemology. Everettians must always postulate numerous 'axioms' of rationality that are nowhere to be found in classical decision theory or Bayesian epistemology. But postulates are cheap; why should we adopt the rationality axioms that figure in Everettian justifications of the Born Rule – like Measurement Neutrality, Branching Indifference, Epistemic Separability, etc. – rather than those – like Maximize Accuracy for Successors and Quantum Exchangeability – that justify Indexed Branch-Counting instead? Even if one is not convinced that all rational agents must obey these principles (e.g., if they favor some more minimal, less demanding constraints on rationality), I think I have given reasons to think that these principles have at least as good of a claim, if not better, to be requirements of rationality as do the axioms that figure in Everettian justification of the Born Rule.

3 Imprecisifying IBC

Indexed Branch-Counting, now, looks like an attractive strategy for setting credences. At minimum, it is thus far a viable competitor to the Born Rule, and thus threatens to undermine the requirement that rational credences obey the Born Rule, and hence that square amplitudes behave as probabilities. For this reason, however, Everettians have sometimes not denied that some version of branch-counting/indifference reasoning *would* be rational, were it possible in an Everettian Universe. Instead, many Everettians argue that branch-counting is *impossible* or *incoherent* in light of the ontology of Everettian quantum mechanics.[29] This, in fact, is by far the most popular argument against branch-counting, as far as I can tell.

[28]This is, of course, complicated by the fact that the branch numbers are typically regarded as vague; I will address this complication in the following section.

[29]This argument has been especially advanced in Wallace (2012), but see also Greaves (2004), section 5.3, and Saunders (2005), section 5.4.

The argument runs as follows: in an Everettian universe, there is no precise number of branches, and hence it is impossible to 'count' them. This argument flows from the idea that the branching structure is an emergent feature of reality, defined in terms of decoherence theory. The criterion for when something gets to count as an independent 'branch,' however, is not – and allegedly need not – be precisely defined in decoherence theory nor in the account of higher-order ontology with which decoherence theory is supplemented.

Here is a simple example, from Wallace (2012), of how decoherence works. Suppose you have a system in the state

$$|\psi\rangle = \alpha\,|\psi_{q_1}\rangle + \beta\,|\psi_{q_2}\rangle$$

with $|\psi_{q_1}\rangle$ and $|\psi_{q_2}\rangle$ being localized in position space around q_1 and q_2, respectively. And imagine a scattering interaction on a second particle in the initial state $|\phi_0\rangle$, leading to post-scattering states $|\phi_1^+\rangle$ and $|\phi_2^+\rangle$, corresponding to a scattering interaction with the respective wavepackets of the first particle. The dynamics are:

$$|\psi_{q_i}\rangle \otimes |\phi_0\rangle \to |\psi_{q_i}\rangle \otimes |\phi_i^+\rangle\,.$$

So unitary evolution gives:

$$|\psi\rangle \otimes |\phi_0\rangle \to \alpha\,|\psi_{q_1}\rangle \otimes |\phi_1^+\rangle + \beta\,|\psi_{q_2}\rangle \otimes |\phi_2^+\rangle\,,$$

so that the two systems become entangled. The first system's density operator evolves, in matrix form, as:

$$\rho_0 = \begin{pmatrix} |\alpha|^2 \alpha\beta^* \\ \alpha^*\beta|\beta|^2 \end{pmatrix} \to \rho_+ = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\,\langle\phi_2^+|\phi_1^+\rangle \\ \alpha^*\beta\,\langle\phi_1^+|\phi_2^+\rangle & |\beta|^2 \end{pmatrix}.$$

When the two systems are highly entangled, the post-scattering states are approximately orthogonal and so $\langle\phi_2^+|\phi_1^+\rangle$ and $\langle\phi_1^+|\phi_2^+\rangle$ are small. Hence the off-diagonal terms in the first system's time-evolved density matrix – which quantify the degree of interference between the two components of the first particle's wavefunction – are negligible. The point generalizes: the more a system gets entangled with its environment (as happens, e.g., in the context of standard quantum measurements), the less its wavefunction components interfere with each other. This is decoherence.

Supplement decoherence with a suitable theory of macro-ontology, and you get an 'emergent' branching structure. Wallace (2003, 2012) uses what he calls:

> *Dennett's Criterion*: A macro-object is a pattern, and the existence of a pattern as a real thing depends on the usefulness—in particular, the explanatory power and predictive reliability—of theories which admit that pattern in their ontology.

Wallace and other Everettians think that the approximate non-interference between decohering wavefunction components, brought about by interaction with the environment, justifies our describing the situation in terms of an emergent structure of separate branches.

Treating branches as part of the emergent ontology leads naturally to treating the branching structure as *vague*. If branches are defined in terms of the existence of sufficiently negligible interference terms, how small must the interference terms be? The received view is that the matter is vague. As Wallace says (2012, p. 100):

> To be sure, by choosing a certain discretization of (configuration-)space and time, a discrete branching structure will emerge, but a finer or coarser choice would also give branching. And there is no 'finest' choice of branching structure: as we fine-grain our decoherent history space, we will eventually reach a point where interference between branches ceases to be negligible, but there is no precise point where this occurs. As such, the question 'How many branches are there?' does not, ultimately, make sense.

So, there is no precise number of branches. And for Wallace this means that any branch-counting rule will be untenable.[30] If there is no such thing as the number of branches, what is there for us to count? Greaves (2004), using similar reasoning, asserts that what she calls 'egalitarianism' – the notion that I ought to care equally about all of my future successors – is 'incoherent.' This, indeed, tends to be the dialectic around branch-counting: arguments that branch-counting would be a tenable alternative to the Born Rule (such as those of the last section) are not so much denied as they are dismissed as irrelevant in light of the putative vagueness in the number of branches.

Alas, this is too quick. I don't think that vagueness in branch count truly undermines Indexed Branch-Counting as a rational credence-forming strategy. For starters, this would seem to be a highly questionable inference for analogous credence norms. Consider the archetypal chance-credence norm,

> *The Principal Principle*: agents ought to set their credences in chancy propositions equal to their subjective estimates of the objective chances. I.e.,
>
> $$Cr(\phi \mid X\&E) = x,$$
>
> where X is the proposition that $ch(\phi) = x$ and E is any admissible proposition.[31]

The Principal Principle is a conditional norm of credence which instructs agents to defer to (what they take to be) the chances in setting their credences.[32] Indexed Branch-Counting similarly instructs agents to defer to ratios in branch count.

The objection against Indexed Branch-Counting seems to be that it is untenable because it requires agents to conditionalize on something, about which there are no precise facts. Is this a fair objection? Consider an analogous argument for the Principal Principle; i.e., suppose the objective chances are vague. This may well be true: for instance, take the statistical-mechanical case, where objective chance arguably comes from (1) the Past Hypothesis: a law that the initial state of the universe was one of very low entropy, and (2) a uniform probabilty distribution over initial microstates compatible with the Past Hypothesis. As Chen (2022) has convincingly argued, the Past Hypothesis is a plausible instance of 'fundamental nomic vagueness,' namely a case in which the fundamental laws of nature are vague. Vagueness in the Past Hypothesis may come from a number of places: vagueness in the precisification of 'low entropy,'

[30]This is Wallace's second objection to Naive Branch-Counting, in addition to the diachronic inconsistency worries addressed earlier in this paper. Unlike the diachronic inconsistency worry, the vagueness objection seems to go through for Indexed Branch-Counting as well, insofar as the objection works at all.

[31]Admissibility is famously difficult to characterize, but very roughly: E is admissible with respect to ϕ iff $ch(\phi \mid E) = ch(\phi)$.

[32]More precisely: the Principal Principle instructs agents to adopt a whole bunch of prior conditional credences, each conditional on any possible chance proposition. When supplemented with the law of total probability – which says that $P(\phi) = \sum_i P(\phi \mid B_i)P(B_i)$, where $\{B_i\}$ is a partition of the sample space – the Principal Principle instructs agents to set their credences equal to their *expectation* of the chances: $Cr(\phi) = \sum_T Cr(T)ch_T(\phi)$, where the T are candidate chance theories.

vagueness in the definition of other thermodynamic variables such as temperature, vagueness in the partitioning of initial phase space into macrostates, etc. In any such case, the initial probability distribution will similarly be vague. Thus, insofar as we obtain statistical mechanical probabilities by conditionalizing on the present macro-state of a system, the Past Hypothesis, and the initial probability distribution, we will end up with vague objective probabilities. Furthermore, as Hall (2020) argues, imprecise chance fits naturally in a Humean framework, where precise chances might substantially impede simplicity, computability, and the like. And Isaacs, Hájek, and Hawthorne (2022) argue that imprecise chance is motivated by considerations of non-measurable sets (see below).

Should we abandon the Principal Principle, in light of vagueness in the objective chances? This doesn't seem to follow. Just because the chances are vague does not mean that they should no longer be treated as epistemic guides for rational credence. If this were a plausible inference, it would (for instance) completely undermine the predictive power of statistical mechanics, to which it is absolutely essential that we treat the objective chances (vague or otherwise) as epistemic guides.[33] It would, to say the least, be an extraordinary result; and I think we ought to treat it with skepticism. Why, then, should vagueness in branch count mean that Indexed Branch-Counting is not a viable norm of rationality? As a rough moral: if a strategy of credence-formation is rationally compelling (or permissible) in *non-vague* contexts, we should not abandon these strategies just because the relevant ontology is vague, unless there really is no way to make sense of such strategies in vague contexts. But, contrary to what some Everettians claim, there are a number of ways that one might think about credence norms in light of this kind of vagueness. To fully explore all such options and assess them would require its own paper, but I will survey two possibilities. At minimum, this should make clear that Indexed Branch-Counting is not rendered nonsensical in light of vagueness in branch numbers, and shouldn't be dismissed out of hand for this reason.

First, one might think that the Principal Principle and Indexed Branch-Counting recommend that agents adopt imprecise credences.[34] Imprecise credences are motivated, for instance, by the consideration of non-measurable sets: the probability of a proposition associated with a non-measurable subset of outcome space can be considered vague on the interval defined by the least upper bound of the measures of its measurable subsets and the greatest lower bound of the measures of its measurable supersets.[35] Similarly, following Isaacs, Hájek, and Hawthorne (2022), the Principal Principle might require agents faced with imprecise chances to adopt credences that are imprecise on the interval defined by the range of precise chance ascriptions consistent with the vague chance law:[36]

Imprecise Principal Principle: $Cr(\phi \mid ch(\phi) = [x_j, x_k]) = [x_j, x_k]$,

which is to be read: an agent's credence in ϕ, given that the chance of ϕ is imprecise on the interval $[x_j, x_k]$, should itself be imprecise on the interval $[x_j, x_k]$.

We can do something similar for Indexed Branch-Counting. Given that there are a range of allowable precisifications of 'branch,' with ratios in branch count being imprecise over the interval $[r_j, r_k]$, we get:

Imprecise Branch-Counting: $Cr(\alpha \mid N(\alpha)/N = [r_j, r_k]) = [r_j, r_k]$.

[33] That is, at any rate, if one interprets the statistical-mechanical probabilities as chances.

[34] See Berger and Das (2020), Peden (forthcoming), and Isaacs, Hájek, and Hawthorne (2022) for a small sampling of recent work on imprecise credence.

[35] See, again, Isaacs, Hájek, and Hawthorne (2022).

[36] Pardon the mouthful.

Hence, we can understand Indexed Branch-Counting to prescribe imprecise credences for rational agents. The rule remains in conflict with the Born Rule, since the latter prescribes precise credences equal to the square amplitudes.

Another option is to say that the normative facts themselves are vague.[37] In other words, there are a range of precisifications of 'branch,' and on every such precisification, Indexed Branch-Counting prescribes a precise credence function. Yet, because the number of branches is vague, the prescriptions of Indexed Branch-Counting are vague in turn. And thus, there are certain propositions about what precise credences agents ought to adopt which are neither true nor false. A popular approach to vagueness in logic is Supervaluationism, according to which propositions are *supertrue* – which standardly just means 'true' when a sentence contains vague terms or is otherwise sensitive to vague states of affairs – just in case they are true on all allowable precisifications of their contents.[38] Thus:

Supervaluationism:

- ϕ is true iff ϕ is true on every precisification
- ϕ is false iff ϕ is false on every precisification
- ϕ is neither true nor false iff ϕ is true on some precisifications and false on others

On a supervaluationist approach, there will be no unique credence function *Cr* for which the sentence '*A* ought to adopt *Cr*' is supertrue. But there are (for instance) supertrue sentences about what intervals your credences should fall in, and there are also supertrue sentences about what credence-forming *strategies* are rational or irrational, given certain antecedent constraints on rationality. For instance, we may select a precisifcation of 'branch' on which you ought to have certain precise credences, according to Indexed Branch-Counting, and on any such selection, the credences that we ought to adopt fail to co-vary with the square amplitudes. This is because the interference terms do not in general co-vary with the square amplitudes, and so letting the amplitudes vary does not change the ratios in branch count.[39] It is thus supertrue that you should *not*, in general, adhere to the Born Rule: there is no precisification of branch on which adherence to the Born Rule is the rational credence-forming strategy.

Furthermore, that Indexed Branch-Counting is rationally required, *given* either Maximize Accuracy for Successors or Quantum Exchangeability, is super-true. In other words, the following biconditionals

(a) *Cr* is quantum-exchangeable iff *Cr* obeys Indexed Branch Counting; and

(b) *Cr* is accuracy-maximizing iff *Cr* obeys Indexed Branch Counting

are both super-true. This is because, on *any* precisification of 'branch,' there will be definite branch numbers which (via the arguments in Section 2) uniquely fix a credence function that adheres to Indexed Branch-Counting. So, while there is no *particular* credence function that is non-vaguely justified by the arguments for Indexed Branch-Counting, there are facts about what agents are required to do, given the accuracy or exchangeability constraints on their credences. This is analogous to the fact that 'there is *some n* such that *n* grains of sand count as a heap but $n-1$ grains do not' is (super-)true, while it is *not*

[37] For more on vagueness in branch count in connection with different accounts of vagueness, see Wilson (2013).
[38] See Williamson (1994) for more on supervaluationism.
[39] See the next section for more on this.

true of any particular n, that n grains of sand count as a heap while $n - 1$ do not. This is completely ordinary, given a metaphysics that countenances vague terms; so why should it be a special problem in the branch-counting case?

Both of these approaches – imprecise credence and supervaluations with imprecise normative facts – strike me as viable and natural extensions of Indexed Branch-Counting to contexts in which the number of branches is vague. They seem like plausible approaches to vagueness in general, and for analogous norms like the Principal Principle. Thus there is no clear reason, as far as I can see, that ontic vagueness in branch count should undermine Indexed Branch-Counting as a competitor to the Born Rule.

4 Saunders' Reconciliation

Throughout this paper, I have argued that Indexed Branch-Counting is a promising candidate for a rationally permissible (or required) credence-forming strategy in an Everettian universe. If the arguments succeed, they cause trouble for the Everettian approach to probability insofar as Indexed Branch-Counting conflicts (and so competes) with the Born Rule. In the remainder of the paper, I will consider a recent proposal from Saunders (2021c) that purports to render branch-counting consistent with the Born Rule.

Saunders' proposal is to count the numbers of branches in such a way that ratios in branch count are proportional to ratios in square amplitude. The proposal is as follows:

Equi-Amplitude Branch-Counting:

1. The ratio of branches realizing an eigenstate $|\alpha_j\rangle$ of observable $\hat{O}$ to branches realizing eigenstate $|\alpha_k\rangle$, given a state $|\psi\rangle = \sum_i C_i |\alpha_i\rangle$, is equal to

$$\frac{N(\alpha_j)}{N(\alpha_k)} = \frac{|C_j|^2}{|C_k|^2}.$$

2. A rational agent should match their credences to ratios in branch count. Hence, given a state $|\psi\rangle = \sum_i C_i |\alpha_i\rangle$, a rational agent should adopt

$$Cr(\alpha_j) = \frac{|C_j|^2}{\sum_i |C_i|^2} = |C_j|^2.$$

Part (2) of the Equi-Amplitude Branch-Counting rule delivers the Born Rule. Hence, given this method of counting branches, branch-counting (and allegedly, all of the original motivations behind it) can be made consistent with the Born Rule.[40]

I should first say that I do think that making branch-counting consistent with the Born Rule is the most promising strategy for solving the probability problem. If it can be sufficiently argued that Equi-Amplitude Branch-Counting is the only viable way to count branches, then the arguments in this paper can be read as a positive case in favor of this approach to the probability problem, against other Everettian

[40] My presentation of Equi-Amplitude Branch-Counting may unintentionally make the rule appear contrived; in particular, part (1) may look as though Saunders is nearly working backwards from the Born Rule. This would be somewhat unfair; what is really going on is that Saunders starts from the inevitable task of picking *some* or other coarse-graining of the decoherent history-space, and ends up picking one that partitions history-space into coarse-grained cells all of equal amplitude. I will soon argue that this rule does look unnatural on close inspection; but understood in these terms, the rule does not look immediately contrived or arbitrary.

approaches. If Everettians are to solve the quantitative probability problem, in my view, they should do so by resolving the tension between branch-counting and the Born Rule. Hence, I think the goal of Saunders' project is right on target.

Nevertheless, I have at least one substantial worry about Equi-Amplitude Branch-Counting. In particular, I am worried that the rule ignores the branching structure given to us by decoherence theory (plus a suitable theory of macro-ontology). This is because it seems that decoherence theory already gives us a branching structure, and one that naturally does not align with the equi-amplitude rule. The picture of branching that decoherence theory gives us is that branches arise whenever the interference terms between components of the wavefunction are sufficiently small. As discussed in the previous section, there is a fuzzy range of allowable precisifications, differing in what counts as 'sufficiently small' in order to give rise to branching structure.

But on any such precisification, the numbers of branches fail to co-vary with the state. As Saunders notes, we can let there be an apparatus measuring spin states, such that there are $n_\uparrow$ approximately orthogonal states $|\Phi^k_\uparrow\rangle$ of the detector recording spin up and $n_\downarrow$ states $|\Phi^k_\downarrow\rangle$ of the detector recording spin down. (We suppose, for simplicity, that we have selected a single coarse-graining of the decoherence basis on which any $\langle\Phi^i_\uparrow|\Phi^j_\uparrow\rangle$ is small enough, for $i \neq j$, that the branches decohere (and same for spin-down).) Then the unitary dynamics gives:

$$|\psi_\uparrow\rangle \otimes |\Phi_0\rangle \to |\psi_\uparrow\rangle \otimes |\Phi^1_\uparrow\rangle + \cdots + |\psi_\uparrow\rangle \otimes |\Phi^{n_\uparrow}_\uparrow\rangle$$

and

$$|\psi_\downarrow\rangle \otimes |\Phi_0\rangle \to |\psi_\downarrow\rangle \otimes |\Phi^1_\downarrow\rangle + \cdots + |\psi_\downarrow\rangle \otimes |\Phi^{n_\downarrow}_\downarrow\rangle\,.$$

Notice, however, that in such a case the numbers of branches depend only on the various ways that the measuring apparatus can register spin-up and spin-down, and hence the ratios $\frac{n_\uparrow}{n_\downarrow}$ are independent of the amplitudes for the possible states of the spin-system.

But this just is the branching structure that follows naturally when we pick an interference threshold below which components of the wavefunction get to count as branches. While the selection of a *single* such threshold is an idealization (see the previous section), this absence of interference is nevertheless what gives rise to a branching structure. So, in what sense could Equi-Amplitude Branch-Counting be a viable interpretation of the decoherent branching structure?

One option is to say that Equi-Amplitude Branch-Counting (in particular, its first postulate) is a sharp addition to the ontology of Everettian Quantum Mechanics. But, as I see it, there are two problems with this approach. First, the approach constitues a substantial metaphysical addition to the theory. Insofar as one is motivated to be an Everettian for reasons of parsimony – that Everettian Quantum Mechanics adds nothing in addition to the universal wavefunction, that the theory is the most direct realist approach to quantum theory, etc. – the addition of a sharp metaphysical link between square-amplitudes and branch numbers arguably undermines this motivation.

More seriously, besides not *following* from Everettian ontology, the approach seems to stand in sharp *tension* with the ontology. This is because the branching structure, defined in terms of the absence of interference, was supposed to follow *naturally* from Everettian theory in conjunction with some moderate assumptions about emergent ontology. For example: decoherence theory plus Dennet's Criterion are supposed to entail that there is branching structure, and that the numbers of branches are vague between certain values that are exactly determined by the various precisifications of 'branch.' But since on any such precisification the interference terms fail to co-vary with the state, both this picture and the square-amplitude picture cannot both be true, since they expressly disagree on how many branches of various types there are. The 'standard' picture holds explicitly that there are indeterminately many branches, but that on any allowable precisification the branch numbers are *not* proportional to the square amplitudes, while the equi-amplitude picture asserts that there *are* precise (or approximately precise) branch numbers and that ratios in branch count always equal ratios in square amplitudes. And hence, despite there being precisely (or almost precisely) many branches, there is *no* consistent interference threshold that gives the branching structure![41]

These pictures, then, are incompatible. So how is the defender of Equi-Amplitude Branch-Counting to resolve this tension? Surely they will not deny decoherence theory. So (it seems) they must deny the relevant criteria for macro-ontology, according to which the branching structure is given by the absence of interference beyond a certain (perhaps vague) threshold. But this absence of interference was supposed to be something like a *definition* of 'branch.' We begin to lose our grip on what branches are, in the first place, if they are detached from the underyling assumptions about emergent ontology.

So it seems to me that Equi-Amplitude Branch-Counting is costly, because it requires us to say highly revisionary things about the branching structure. But perhaps a motivation can be given for this move, if Equi-Amplitude Branch-Counting can be shown to follow from some crucial constraints on the branching structure. Saunders purports to have such a motivation: the 'old' branch-counting rule entails that the probabilities are discontinuous functions of the state, while the square-amplitude rule has no such implication.

The argument runs as follows. First, notice that, on any branch-counting rule, we will presumably count only branches of non-zero amplitude. This seems reasonable: it is essential to Everettian Quantum Mechanics that only non-zero amplitude components of the wave function correspond to physically real states of affairs, so we should surely say that zero-amplitude components of the wavefunction get zero branches. And notice, second, that the branch numbers do not co-vary with the amplitudes: as noted above, where the amplitude for states of the measured system are non-zero, the ratios in branch count remain constant. But these facts, taken together, imply that the branch numbers are discontinuous with the state: as the amplitude of a certain component approaches zero, the ratios in branch count remain the same (or at least do not perfectly co-vary with the amplitudes), and yet when the amplitude *equals* zero, that component gets zero branches.

To take a concrete example, suppose you measure the z-component of spin for a state

$$|\psi_k\rangle = \cos\theta_k\,|\uparrow\rangle + i\sin\theta_k\,|\downarrow\rangle \qquad (\theta_k = \tfrac{\pi}{2k+1})$$

[41]The Dennetian functionalism assumed by Wallace, admittedly, may not be as attractive to Saunders, who frames the branch-counting rule in terms of coarse-grainings of a decoherent history-space. But some story will presumably need to be told about why coarse-graining was necessary in the first place. To see this, notice that on Saunders' approach (2021b) there are fine-grained cells $\alpha = \{\alpha_n, \alpha_{n-1}, ..., \alpha_1\}$ of parameter space with associated 'chain operators' $C_a = P_{\alpha_n}(t_n)\cdots P_{\alpha_1}(t_1)$ which are products of non-commuting projectors and represent Heisenberg-picture histories. But there are further coarse-grainings of the α's, and Saunders is precisely arguing that we should coarse grain into equi-amplitude cells. But the need for a coarse-graining, all along, was (precisely as Wallace argues) that without it the branches or 'histories' will interfere. But then my worry about the equi-amplitude counting rule carries over: the equi-amplitude coarse-graining 'groups together' elements of history-space without consistent regard for the degree of interaction between them. So, in what sense is it an interpretation of the emergent branching structure?

Clearly, $|\psi_k\rangle$ converges smoothly to $|\uparrow\rangle$ in the Hilbert space norm $||\psi|| = \sqrt{\langle\psi|\psi\rangle}$:

$$\lim_{k\to\infty} ||\,|\psi_k\rangle - |\uparrow\rangle\,|| = 0.$$

But the ratio in branch count, for any *k*, remains constant as:

$$r_k = \frac{n_\downarrow}{n_\uparrow},$$

which does not converge to $r_\infty = 0$. Hence, the branch ratios are discontinuous functions of the state. Evidently, however, when the branch ratios are identified with ratios in square amplitudes, they will converge smoothly, along with the state.

This is supposed to be a serious problem for the 'old' branch-counting rule, though Saunders, as far as I know, gives rather minimal argument for this conclusion. The most extensive justification of this idea that I have found in Saunders' work is in his (2021a, p. 11), which I quote below in full:

> For of all of these branches, thus defined, waiting to be counted: is it all of them, or only the ones with non-zero amplitudes? Neither choice makes any sense. If it is all of them, then the numbers are completely independent of the state; in what sense is this an interpretation of the structure of the state? But if only non-zero amplitude branches are counted, the numbers will be discontinuous functions of the amplitudes. The tiniest variation in amplitude may make for arbitrarily large variations in branch numbers. Yet continuity in the amplitudes (in the Hilbert-space norm, the norm topology) is essential to decoherence theory and the unitary dynamics; as likewise to spectral theory, and the whole edifice of quantum mechanics.

As before, I concede the problems with counting zero-amplitude branches. But I find Saunders' argument concerning discontinuity puzzling. In what sense does non-weighted branch-counting threaten 'continuity in the norm topology?' The branch-counting rule is not an interpretation of the norm topology at all: it starts with a standard interpretation of the *emergent branching structure*, and it is a rule about how to set credences in light of that structure. No doubt continuity in the amplitudes is crucial, e.g., for the unitary dynamics, since the Schrodinger Equation requires differentiability – and so continuity – in the *wavefunction*. But what could this have to do with the branch-counting rule? The dynamics proceed exactly as normal, independently of what rule we use to calculate probabilities. The dynamics, the norm topology, and decoherence theory, are (as far as I can tell) left completely untouched by our choice of branch-counting rule. If there is some reason that letting one's credences be discontinuous with the state will cause the whole edifice of quantum mechanics to come crumbling down, this reason is opaque to me. So why is continuity crucial in this case? It may be a nice formal feature of the Equi-Amplitude Branch-Counting rule given the importance of continuity in *other* aspects of quantum theory, but this alone hardly justifies such a dramatic revision of the Everettian ontology.

Surely Saunders is not suggesting that *any* discontinuous function on Hilbert space is somehow physically pathological. Consider, for example, the functional $\xi(\cdot)$, defined as:

$$\xi(\psi(\vec{r})) = \begin{cases} 1 & \text{if } \psi(\vec{r}) \text{ is an energy eigenstate} \\ 0 & \text{otherwise} \end{cases}$$

$\xi(\cdot)$ is clearly discontinuous on Hilbert space, with certain state vectors being energy eigenstates while (for instance) arbitrarily small rotations yield state vectors that are not energy eigenstates.[42] But as much as continuity figures in certain aspects of quantum theory, $\xi(\cdot)$ doesn't look remotely physically pathological. So what bearing, for instance, could continuity in the dynamics possibly have for our choice of branch-counting rule? What makes it the case that $\xi(\cdot)$ is (as it seems to be) a physically respectable discontinuous function on Hilbert space, while a discontinuous branch-counting rule would be a catastrophe? Until such an explanation is given, we should not feel pressed to abandon the standard, natural interpretation of the decoherent branching-structure.

As I have argued, Equi-Amplitude Branch-Counting appears to face the serious cost of not offering a consistent interpretation of the emergent branching structure, as defined by decoherence theory. Consequently, I don't think it is supported by the considerations that I have argued support Indexed Branch-Counting. If it can be argued that the natural way of counting branches faces some insurmountable conflict with 'the whole edifice of quantum mechanics,' this might provide reason to think that Equi-Amplitude Branch-Counting is the only viable game in town. But I am aware of no such arguments, and at present I have no sense of how such arguments would go.

5 Conclusion

In this paper, I have offered arguments in favor of a new branch-counting rule, which I have called Indexed Branch-Counting. I have argued that this rule is well-motivated by considerations of accuracy-maximization, and by a principle of exchangeability for rational credences. I have further argued against the central Everettian objection to branch-counting, namely the idea that branch-counting is incoherent in light of vagueness in the numbers of branches. Finally, I considered a recent proposal from Simon Saunders aimed at unifying branch-counting with the Born Rule, and raised concerns about this approach.

I think that Everettian Quantum Mechanics faces an especially sharp problem of understanding probability precisely because it opens the door to an extremely plausible alternative to the Born Rule for setting credences. If the arguments in this paper succeed, they will place branch-counting on firmer footing, thus deepening the challenge of understanding probability in Everettian Quantum Mechanics.

Acknowledgements

Many thanks to Diego Arana Segura, Dave Baker, Jeff Barrett, David Builes, Eddy Chen, Chris Dorst, Meir Hemmo, Tim Maudlin, Wayne Myrvold, Jill North, Ezra Rubenstein, Laura Ruetsche, Chip Sebens, and Ted Sider, and two anonymous BJPS referees, for much helpful discussion on these matters and for comments on earlier drafts of this paper. Thanks especially to Barry Loewer, David Albert, and Shelly

[42] Many thanks to Dave Baker for the example.

Goldstein for their feedback on this paper, and for their invaluable mentorship in foundations of quantum theory in general.

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