Abstract: Humean theories of chance hold that objective chances reduce to patterns in the history of occurrent events, such as frequencies. Non-Humean accounts of chance hold that objective chances are metaphysically fundamental, existing independently of the "Humean Mosaic" of perfectly natural properties and relations instantiated at spacetime points (or whatever underlies a potentially emergent spacetime in a fundamental physics inclusive of quantum gravity). It is therefore possible, by the lights of non-Humeanism, for the chances and the frequencies to diverge wildly. Humeans often allege that this undermines the ability of non-Humean accounts of chance to rationalize adherence to David Lewis' Principal Principle (PP), which states that an agent's degrees of belief should match (what they take to be) the objective chances. In this paper, I propose two novel approaches to justifying (PP) for non-Humean chance, hence defusing the Humean objection. The first approach justifies (PP) via the role it plays in informing outright beliefs about long-run frequencies. The second approach justifies (PP) by showing that adherence to (PP), even for non-Humean chance, maximizes expected epistemic utility according to the chance function that in fact obtains in any particular world. I then address two different circularity objections to this approach, one concerning our epistemic access to non-Humean chance, and another concerning the justificatory status of the antecedent rationality principles.

Rationalizing the Principal Principle for Non-Humean Chance

The major divide in the metaphysics of chance is between Humean accounts, which hold that the objective chances reduce to patterns in the history of occurrent events, such as frequencies, and non-Humean accounts, which hold that chances are somehow irreducibly modal features of the world, such as brute propensities, chancemaking relations between universals, or constituents of fundamentally stochastic dynamical laws.[1] But whatever chances turn out to be – and whatever the direction of *metaphysical* explanation between chances and occurrent events – they play an important role in explaining statistical regularities and licensing scientific explanations. At the same time, it is widely believed that chances should somehow constrain our credences: on pain of irrationality, agents ought to match their credences in

[1] I will hereby refer to propensity theories, and, following Gillies (2000), I will consider propensity theories, broadly construed, as any objective, non-frequency, non-reductive theory of probability. More specifically, propensities are thought of as *intrinsic dispositions*, logically distinct from the frequencies, to generate events with a particular probability. These probabilities are taken to *explain* the observed relative frequencies.

certain propositions to what they believe to be the chance of those propositions. This idea is captured by David Lewis' Principal Principle (PP_i)[2]:

$$(PP_i) \quad Cr(A \mid X \wedge E) = x.$$

Here, *Cr* is a rational credence function, X is a proposition to the effect that the chance of A is x, where $x \in [0,1]$, and E is an agent's total evidence, provided that evidence is *admissible*. Admissibility is difficult to define, and Lewis offers no precise definition. However, he does offer a characterization of admissibility: admissible information tells us about a proposition only by way of the *chance* of that proposition. On this characterization, for example, the reading of a crystal ball that carried future information about the outcome of a chancy event would be inadmissible. Moreover, Lewis offers two sufficient conditions for admissibility. Firstly, *historical* information up to a time *t* is admissible at *t*. Secondly, the general theory of chance at a world – namely a set of history-to-chance conditionals which give an account of which antecedent conditions give rise to which chance distributions – is always admissible. Therefore, (PP_i) can be refined to include these sufficient conditions for admissibility, where H_{tw} is the history of world *w* up to time *t*, T_w is the theory of chance that holds at *w*, and P_{tw} is the probability function for *w* at *t* generated by *w*'s theory of chance:

$$(PP) \quad Cr(A \mid H_{tw} \wedge T_w) = P_{tw}(A).$$

Informally, (PP) says that agents ought to match their degrees of belief to what they take to be the objective chances.

Despite their intuitive force, chance-credence norms like (PP) turn out to be somewhat difficult to derive. The issue becomes especially pressing insofar as it encroaches on the aforementioned metaphysical debates about chance. The issue is something like this: that *some* chance-credence norm holds seems to be an indispensable aspect of the chance-role, and the

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[2] See Lewis 1980.

proponents of any metaphysical account of chance had better be able to explain why their candidate filler of the chance-role is up to the task of constraining rational credence. Lewis (1994, 484) famously quipped that it was utterly mysterious how the "unHumean whatnots" posited by his opponents could constrain rational credence. That non-Humean accounts of chance are unable to rationalize adherence to (PP) has since become a common argument in the literature that such accounts are unsatisfactory.

This paper attempts to defuse this argument, proceeding as follows. First, I will review the informal statements of the credal argument and offer a more substantive formulation (sec. 1). Second, I will offer two separate proofs of (PP) for non-Humean chance, attempting to defuse the argument (sec. 2), arguing first that adherence to (PP) can be rationalized based on its implications for an agent's outright beliefs about frequencies (sec. 2.1) and second that adherence to (PP) can be rationalized in terms of its implications for the accuracy of an agent's credence function (sec. 2.2). Finally, I will consider two objections and offer replies (sec. 3).

1. The Credal Argument Against Non-Humean Chance

Lewis never clearly formulated his argument that non-Humean accounts of chance failed to rationalize adherence to (PP). His complaint is raised only in the following passage:

> Be my guest – posit all the primitive unHumean whatnots you like. (I only ask that your alleged truths should supervene on being.) But play fair in naming your whatnots. Don't call any alleged feature of reality "chance" unless you've already shown that you have something, knowledge of which could constrain rational credence. I think I see, dimly but well enough, how knowledge of frequencies and symmetries and best systems could constrain rational credence. I don't begin to see, for instance, how knowledge that two universals stand in a certain special relation N* could constrain rational credence about the future coinstantiation of those universals (1994, 484).

Lewis' complaints here have intuitive pull, but he gives little by way of further argumentation. Luckily, his objection has been rendered much more precise by Loewer (2004), Eagle (2004), and Hall (2004).[3] Loewer offers a compelling statement of Lewis' objection to non-Humean chance:

> Without [relying] on the PP there is no non-question begging reason to think that setting one's degrees of belief by propensity chances will result in having high degrees of belief in truths and low degrees of belief in falsehoods. And since propositions about propensity chances are facts logically completely distinct from the propositions they assign chances to it is utterly mysterious why they should tell us anything about what degrees of belief to have in those propositions (2004, 1123).

Eagle formulates the objection in a similar fashion:

> Severing the constitutive link between frequencies and chances means that we have no logical connection between the concepts of probability and rational expectation. Since, as we have seen, the events that occur in a world and the chances of those events are not logically related, why should knowledge of the chances tell us anything about which events to expect to occur? There seems no way that these single case propensities can rationalise adherence to Lewis' Principal Principle or anything like it; but without the Principal Principle we have no link between the two major uses of probability (2004, 401).

And yet the clearest formulation of the objection emerges out of a question posed by Ned Hall: can we show that a chance-credence norm like the Principal Principle follows from a set of normative constraints on our beliefs and credences over the purely categorical features of the world? Hall answer this question in the negative:

[3] 2004 was a rough year for propensity theorists!

> If the correct account of the metaphysics of objective chance is a thoroughgoing non-reductionist account – that is, an account according to which the categorical facts about a world place virtually no constraints on the ur-chance function for that world – then the answer is clearly `no'. For that is a metaphysics of objective chance that gives the categorical constraints no purchase. Commit yourself to such a metaphysics, and it appears that you must introduce the Principal Principle as a *sui generis* normative principle governing rational credence (2004, 107).

It will be especially fruitful to adapt Hall's formulation of the problem as I address this putative challenge for non-Humean views, as it is the sharpest. What makes Hall's formulation so compelling is that it really gets at the heart of why the connection between non-Humean chance and rational expectation is, at first glance, so mysterious. The sorts of propositions about which our beliefs and credences can be vindicated are just those propositions that concern categorical features of the world. What (PP) aims at – alignment of credence with objective chance – is not the sort of thing that can be vindicated in the ordinary, alethic sense. Our beliefs about the categorical features of the world, however, are true or false, and our credences turn out to be some precise distance from the outputs of our world's truth function. Thus, if we want to bring (PP) into the fold of our other rationality principles, we want to show that it follows from categorical constraints on our credences.

With this in mind, we can more clearly formulate the argument against non-Humean chance in terms of the putative inability of non-Humean accounts to derive (PP) from constraints on rational belief that range only over the categorical, nonmodal features of the world.[4] The argument goes as follows:

[4] It should be noted that Hall does not himself necessarily subscribe to this argument, because it is not clear that he subscribes to (P2). Nevertheless, he clearly does subscribe to (P1) which is what I will be arguing against.

Credal Argument against Non-Humean Chance:

(P1) Non-Humean accounts of chance are unable to vindicate (PP) given only categorical prior constraints on rational belief and credence.

(P2) A satisfactory account of chance should be able to show how (PP) is vindicated by such categorical constraints.

(∴) Non-Humean accounts are unsatisfactory.

I would like to concede (P2), which I consider to be well-motivated: (PP) doesn't seem like a basic constraint on rational belief. The reason we are interested in obeying (PP) is supposed to be because we think that it will aid us in forming rational expectations about the everyday events that we actually come to observe, and about which our beliefs and credences can achieve alethic vindication. Therefore, (PP) really should follow from other principles that we take to inform us as to what we should believe about these categorical features of the world.

The sticking point is (P1). Here, non-Humean accounts of chance are supposed to compare unfavorably with Humean accounts, on which it is possible to derive (PP) from other principles of rationality.[5]

2. The Argument Defused

The argument can be defused by showing that (PP) is vindicated, on a non-Humean account of chance, from independently plausible constraints on rational belief or credence which range over categorical features of the world.

2.1. Outright beliefs about frequencies

My first attempt at deriving (PP) in a propensity-friendly manner will rely on three assumptions, each of which carries independent plausibility. The first assumption is what I shall call Chance Reliabilism:

[5] Hoefer (2019), for instance, argues that (PP) follows from a consistency requirement on an agent's credence function, for Humean accounts of chance.

Chance Reliabilism: All else equal, agents are (epistemically) obligated to adopt the belief-formation processes which have the highest objective chance of producing belief sets with high ratios of true to false belief.[6]

Call any belief set with a high ratio of true to false belief an *optimal* belief set. Chance Reliabilism states that what makes a given belief rational is that it was formed by a process which has a high chance of producing such an optimal belief set. What might motivate the adoption of such a principle?

Reliability is often understood, in the literature on epistemic justification, as high objective probability of producing an optimal belief set.[7] The informal statements of the credal argument seem to suggest that the issue with non-Humean chance is that, since the *logical* connection between chance and frequency has been severed, we similarly cannot draw any logical connection between chance and rational expectation. On a Humean view, chances supervene on the actual history of occurrent events, and so facts about the chances just are, in

[6] An important thing to note at the outset is that Chance Reliabilism may appear to rationalize alignment of our credences with the *actual* objective chances, rather than our beliefs about the objective chances, as (PP) requires. This is significant for the ensuing discussion, because I will argue that the rationality principles presupposed in the Credal Argument are implausibly strong and exceedingly difficult to operationalize; but so too is the requirement that our credences match the actual objective chances! However, my basic reply is that Chance Reliabilism can rationalize adherence to (PP) conditional on our having prior beliefs about the chances which are, according to Chance Reliabilism, themselves rational. As will be discussed in the third section, such beliefs are typically informed by updating on evidence obtained from the relative frequencies. And, while I won't be able to give a comprehensive account here, it seems straightforward that such updating practices are both operationalizable and objectively reliable. After all, the objective chances say that you are very likely to observe certain approximate relative frequencies. An updating rule (given reasonable priors) will subsequently return a very high credence in the corresponding chance function.

[7] See Alston (1988), Pettigrew (2021), and Comesaña (2018), for example.

some way, facts about the actual history of occurrent events.[8] Thus, an agent who has a grasp on what the chances are, and thereby adjusts their credences to the chances, has a kind of *guarantee* that their credences will be accurate and their outright beliefs will be true. If their expectations about the frequencies turn out to be incorrect, it was because they had false beliefs about the chances. For Humeans, (PP) appears iron-clad.

But the underlying account of epistemic rationality here is too strong, because it assumes that what counts as a rational expectation is one that guarantees epistemic success. But such an emphasis on guarantees of truth or accuracy can only capture half of the story behind (PP). This is because (PP) is itself a conditional principle of rationality: *given* your belief that a particular chance function obtains, you should match your credences to that chance function. But, as Schroeder (2021) has emphasized, part of what gives us subjective reasons for belief is that our own internal doxastic circumstances lead us to believe the world to be such that we have objective, external reasons to believe certain propositions. This, at any rate, is what seems to be going on with (PP): *given* our belief that a particular probability function accurately specifies the objective chances, we believe that we have a compelling objective reason to adopt a particular credence function. Consequently, the principles of rationality that we employ in vindicating (PP) must also be able to secure our rational access to the objective chances, if they are to bridge the gap between the internally relevant facts (i.e., concerning our beliefs about the chances) and externally relevant facts (i.e., concerning the chances themsevles). But there is no logical or constitutive guarantee of epistemic success when it comes to our beliefs about the chances.

Imagine asking a Humean how an agent is to rationally form beliefs about the objective chances. This agent, after all, does not have epistemic access to the entire actual history of occurrent events taken to subvene on the Humean chances. The agent's present total evidence *indicates*, but does not *guarantee*, that the global frequencies, and so the objective chances, have certain values. Thus, while a Humean could say that adherence to (PP) guarantees success

[8] See, again, Hoefer (2019).

conditional on one's knowing what the chances, the underlying account of epistemic justification must not forbid this antecedent condition from coming to bear.[9]

This point can be strengthened by considering some plausible constraints on normative theories. Consider, for instance,

> *Transparency*: A normative theory is adequately guiding only if, whenever it requires you to φ, you are in a position to know that it requires you to φ.

The idea behind Transparency is that a normative theory can only be adequately action-guiding if its requirements are epistemically transparent or *luminous* (in Williamson's (2000) phrase). Transparency is itself controversial – see Hughes (2022) for criticism – and may be too strong. This is because, arguably, we are rarely in a position to know that *any* non-trivial condition obtains, including constraints imposed by a normative theory.[10] However, we can consider a modified version of the principle:

> *Transparency**: A normative theory is adequately guiding only if, whenever it requires you to φ, you are in a position to permissibly believe that it requires you to φ.

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[9] Fernandes (forthcoming) makes this point vividly. First of all, locally speaking, agents have only a high chance of doing well given that they match their credences to what they believe to be the objective Humean chances, even when an agent's beliefs about the chances are rationally-formed and evidenced by observed relative frequencies. This is for the simple reason that, ordinarily, there is some non-trivial chance that the local frequencies at a given spacetime region will diverge from the global frequencies. Secondly, there is even a small chance that the global frequencies will diverge from the Humean chances in so-called "undermining worlds."

[10] See Srinivasan (2015).

Something in the ballpark of this constraint has been advocated, inter alia, by Kiesewetter (2016).[11] If the antecedent rationality principles we employ in justifying (PP) require that our belief-formation processes guarantee truth or accuracy then we will have to say that agents are required to adopt such-and-such credence function without being in a position to permissibly believe that they ought to do so. This is because their being in a position to know which credence function they should adopt requires that they know which credence function is the chance function. Yet, the observed relative frequencies accessible to ordinary agents never actually guarantee what the global frequencies are, or even what the region-specific frequencies for an extra-frequency-tolerant Best Systems view of Humean chance are, just as the observed relative frequencies could never guarantee what the propensity chances are.

Humeans should lower their standards. In particular, they should de-emphasize the supposed guarantee of success that the chances are supposed to bring about, and accept that our justification of (PP) may reasonably make reference to which belief- and credence- forming processes have the highest objective chance of "performing well" over the long run.[12] Chance Reliabilism, therefore, is a more plausible starting point than any norm which demands a guarantee of epistemic success. Prominent contemporary theories of rationality often have it that objective chance holds the world of epistemic normativity together at a fundamental level. I suspect there may be no satisfactory justification of (PP) which does not appeal to more basic

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[11] Schroeder (2021) argues against *Transparency** for normative theories of action, but agrees that it obtains in the case of epistemic normativity.

[12] There may, in fairness, be other ways to go for the Humean. For instance, one could apply self-locating indifference reasoning. One imagines that the global frequencies, and so the Humean chances, have certain values. Then, one applies a restricted indifference principle so as to obtain a uniform self-locating credence function over possible regions of the Humean Mosaic. One would thereby have high credence in the proposition that they find themselves in a typical region where the frequencies match the Humean chances. Yet, as Ismael (2009) points out, it is somewhat puzzling why one would be indifferent between what they did not already think were equiprobable outcomes. And, in the end, a justification of (PP) which treats (say) an indifference norm as bedrock is not obviously going to be more satisfying or less controversial than one which starts from reliabilist truth- or accuracy- norms.

principles about what is *likely* to produce true belief or accurate credence. The hope, then, is to offer an account of how chances provide external reasons – via principles like Chance Reliabilism – which allows us to derive (PP) without circularity. My goal here is to offer a sketch of such an account.

One may worry, instead, not that non-Humeans are unable to offer a guarantee of truth or accuracy, but rather that the rationality principles behind any non-Humean justification of (PP) will be forced to sneak in some non-categorical content, in violation of (P2) in the Credal Argument.[13] Chance Reliabilism appears clearly to smuggle in non-categorical facts, namely facts about which belief-formation processes are objectively likely to produce optimal belief sets.[14]

However, I take it that the Credal Argument's second premise does not forbid the invocation of principles which in *any* way refer to non-categorical properties, but rather that it forbids the invocation of principles whose epistemic prescriptions *range* over the non-categorical features of the world. By this, I mean that categorical constraints (i) tell us what to believe *about* categorical propositions – e.g. that a certain coin will come up heads, or that repeated coin tosses will have certain approximate relative frequencies – and (ii) aims at promoting some non-dispositional features of the target doxastic states, such as truth or accuracy or consistency, rather than alignment with the objective chances. (PP), then, is a non-categorical constraint insofar as it essentially aims at alignment of your doxastic states with the objective chances. (PP), thus, says nothing about accurate credence or true belief in categorical propositions. Yet, our reason for obeying (PP) is supposed to be that we think it will assist us in forming rational expectations about the categorical facts we will actually encounter – the kinds of facts to which we can assign zeros and ones! If we are going to rationalize adherence to (PP), therefore, we need some principle(s) which will bridge the manifest divide between the

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[13] One may also worry that these appeals to what belief-formation processes are likely to yield true belief and accurate credence are *circular*, when applied to discussions of chance. I will address this worry in the next section.

[14] Thanks to [OMITTED FOR BLIND REVIEW] for pushing me to clarify this point.

categorical events that we actually observe, on the one hand, and the chances that underlie them, on the other. But to insist that our most fundamental rationality principles can have no dispositional content whatsoever is to rule out some of our best contemporary theories of rationality. Though Chance Reliabilism is ultimately cashed out in dispositional terms, it ranges over propositions about the categorical features of the world: it tells you how to form beliefs about the categorical features of the world, such as frequencies.

With that, the second assumption that I will make concerns the probabilistic connection between chances and frequencies, as observed in the well-known Weak Law of Large Numbers (WLLN):

$$\lim_{n \to \infty} \mathrm{P}(|f_\phi - ch(\phi)| < \varepsilon) = 1$$

Here, ϕ is a repeatable proposition-type (e.g. that a fair coin will land heads), f_ϕ is the frequency of ϕ in a sequence of n trials, $ch(\phi)$ is the single-case objective chance of ϕ, and ε is an arbitrary constant. Intuitively, WLLN says that the objective probability that the frequencies and the chances will diverge approaches zero as the number of trials gets infinitely large.[15]

One may understandably wonder whether WLLN holds for propensity accounts. I take it that the notion of single-case propensity, and the way in which propensities "govern" or "constrain" chancy events, is brute for the non-Humean, and not subject to any further metaphysical explanation. Single-case propensities, then, metaphysically ground propensities for aggregations of events, in accordance with WLLN. It is here that I can see legitimate worries about the potentially mysterious character of single-case propensities, but this is not an objection concerning the ability of propensity theories to rationalize adherence to (PP). It is rather an expression of skepticism about their foundational metaphysical plausibility. It is,

[15] Standardly, WLLN is formulated in terms of the probability that the sample mean $\bar{x}$ will diverge from the population mean μ. The formulation above adapts the standard formulation for the ensuing discussion of chances and frequencies. See Ballentine (2016) for an application of the Law of Large Numbers for propensity chance.

therefore, a story for another day: I am concerned, here, with whether we can rationalize adherence to (PP) *given* that a propensity account – or any other non-Humean interpretation of chance – happens to be true. And if one cannot, this would of course count against propensity analyses' overall plausibility, as it would undermine their ability to satisfy a manifest aspect of the chance-role.

WLLN, as stated above, is a statement about the objective probabilistic connection between chances and frequencies. However, there is a related aspect of WLLN of which I will also make use: WLLN itself functions as a kind of normative constraint on our credences. Conditional on an agent's having a particular degree of belief that some event will occur, they ought to expect that the frequency of that event in the relevant reference class is approximately equal to their single-case credence. Call this epistemic version of WLLN the Subjective Law of Large Numbers (SLLN):

$$\lim_{n\to\infty} Cr\left(\left|f_{\phi} - \mathrm{x}\right| < \varepsilon \;\middle|\; Cr(\phi) = \mathrm{x}\right) = 1$$

SLLN merely demands of agents that their credence function display the kind of internal coherence that any probability function is expected to display. Both WLLN and SLLN play important roles in the first vindication of (PP) for propensity chances that I will offer.

The last assumption I will make is known as the Lockean Thesis:

> *Lockean Thesis*: If one's credence in p is sufficiently high, then one should take up the outright belief that p.

There are a number of different versions of Lockean Thesis. It is sometimes taken that sufficiently high credence is necessary and sufficient, or just necessary, for outright belief. For my purposes, suitably high credence will need to be sufficient, but not necessary, for rational outright belief. How to understand "sufficiently high" is also a point of contention in the literature. Some, for instance, take it that there is a fixed threshold for sufficiently high credence, while others take it that the relevant threshold is context- and proposition- dependent.[16] For whatever level of credence $1 - \varepsilon$ one thinks is suitably high to justify application of Lockean Thesis, one simply needs to consider a sample size sufficiently large to generate a divergence of single-case and long-run credence of less than ε via SLLN.

Given these three assumptions, we can see how a non-Humean about chance would be able to justify (PP). First, as an example, consider a sufficiently long sequence of chancy events: tosses of a fair coin, where $ch(H) = 0.5$, let's say. If an agent S obeys (PP), and thereby generalizes their credence function $Cr(\text{-})$ via SLLN from single events to long sequences, we will have:

$$Cr(f_H \approx 0.5) \approx 1$$

This is because, given an arbitrarily small constant ε, as the number of trials approaches infinity, SLLN instructs agents to set their credence that the frequency of heads differs from their single-case credence $Cr(H)$ by a factor of ε or greater equal to zero. Thus, for any ε, there is a finite – but perhaps very long – sequence, such that an SLLN-obeying agent will assign a credence of approximately 1 to the proposition that the frequency of heads will be within ε of 0.5.

It follows from WLLN that for any ε, there is a long enough sequence that there is a chance of approximately 1 that the frequency of heads will be within ε of 0.5. Consequently, given a sufficiently small ε-value, and a correspondingly long sequence, we can obtain:

$$ch(f_H \approx 0.5) \approx 1.$$

Since S has a credence in $f_H \approx 0.5$ of approximately 1, application of Lockean Thesis yields the result that S believes outright that the frequency of heads will be approximately 0.5. Consequently, by substitution, it follows that *ch*(S's belief is true) is *also* approximately 1.

[16] See Jackson 2020 for an overview of the Lockean Thesis, and see Dorst 2019 for a recent argument to the effect that adherence to the Lockean Thesis maximizes expected epistemic utility.

It is similarly clear, moreover, that an agent who adopts a non-PP-obeying credence function will have a much lower chance of believing the truth about the frequency of heads over long sequences. For instance, imagine an agent S* with credence function *Cr** such that $Cr^*(\mathrm{H}) = 0.8$. Similarly, S* obeys SLLN. S* will consequently disbelieve truths and believe falsehoods with a very high objective probability. This is because S* will have a very high credence in the proposition that the frequency of heads is approximately 0.8:

$$Cr^*(f_H \approx 0.8) \approx 1,$$

This occurs for just the same reason that S, above, had a credence of approximately 1 in the proposition that the frequency of heads is approximately 0.5. As before, if S* obeys Lockean Thesis, then S* will believe outright that the frequency of heads is approximately 0.8. Yet, the objective chance that this belief is true will be incredibly low:

$$ch(f_H \approx 0.8) \approx 0.$$

S* will similarly have a credence approximating zero in the proposition that the frequency of heads is approximately 0.5, and thus disbelieve outright that the frequency of heads will be approximately 0.5. As established above, the objective chance of this proposition is very high, so it is very likely that S* will disbelieve a true proposition (and believe a false one).

The above example, I hope, begins to make lucid why agents who fail to obey (PP) are objectively unlikely to form true beliefs about frequencies, while those who succeed in obeying (PP) are objectively likely to do so. It can also be shown more generally that *any* credence function which differs non-trivially from the objective chances is in this sense unreliable. Suppose $Cr(\mathrm{H}) = ch(\mathrm{H})$ and $Cr^*(\mathrm{H}) = ch(\mathrm{H}) + \varepsilon$, for an arbitrary ε. Then, by WLLN and uniform substitution of identicals, as the number of sequences gets sufficiently large:

$$ch(|f_H - Cr(\mathrm{H})| < \varepsilon) \approx 1.$$

However, in order that $|f_H - Cr^*(H)| < \varepsilon$, it must be the case that $|f_H - ch(H)| > \varepsilon$, since *ch*(H) and *Cr**(H) differ by a factor of ε. Therefore, by WLLN, as the number of sequences gets sufficiently large:

$$ch(|f_H - Cr^*(H)| < \varepsilon) \approx 0.$$

Cr obeys (PP) by assigning a credence of 0.5 to the proposition that the coin will land heads. *Cr**, on the other hand, fails to obey (PP). Consequently, *Cr* produces credences which are almost certain to converge with the frequencies in a long enough sequence of trials. *Cr**, by contrast, has the opposite result. Therefore, an agent who obeys (PP) by setting their credence function equal to the objective chance function will, in the long run, have an objective chance approximately equal to 1 of having their credences fall within some arbitrarily small interval from the frequencies. Consequently, they will be very likely to possess accurate outright expectations about the frequencies, via adherence to both SLLN and Lockean Thesis. An agent who adopts a credence function such as *Cr**, on the other hand, has a very low objective chance of yielding accurate expectations about the frequencies.

Therefore, an agent who fails to adhere to (PP) is much less likely to yield optimal-ratio belief sets than an otherwise similarly situated agent who does.[17] Moreover, Chance Reliabilism is only concerned with outright belief – that is, the probability that one will believe truths or falsehoods outright – rather than credence. Therefore, when it comes to the sorts of credences that we should have, Chance Reliabilism instructs us only with regard to the *outright* beliefs that can be generated from our credences, via adherence to Lockean Thesis. In the chancy cases,

[17] It is important to note that this will only apply when beliefs about the observed relative frequencies are defined only over the same reference class as the chance function.

these are (ordinarily) just the cases that involve long sequences.[18] Consequently, what has just been established is sufficient to rationalize adherence to (PP) via Chance Reliabilism.[19]

2.2. *Accuracy*

Before getting into the crux of my case about accuracy, and the assumptions I will make in giving the argument, I will give some background regarding the notions of accuracy and vindication.

In the literature on accuracy measures for credence functions, we start out with a *fully vindicated*, i.e. maximally accurate, credence function. In particular, for a world w, the fully vindicated credence function is just the truth-function, v_w, which assigns 1 to all truths and 0 to all falsehoods.[20] With this, we can define a particular credence function's accuracy in terms of its

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[18] There are a few exceptions, such as single cases involving statistical-mechanical probabilities that approximate 1 (e.g., that the gas will spread throughout the box when a divider is removed). In such cases, again, application of Lockean Thesis in conjunction with adherence to (PP) will generate outright beliefs which are virtually certain to be true.

[19] The scope of this justification of (PP) can be extended to non-repeatable, non-i.i.d., and non-long-run cases, given a few plausible additional assumptions. Take a proposition like P = <Democrats will hold the Senate in 2024>. If a propensity account of chance is correct, then P's obtaining will presumably supervene on chancy events that occur at a more fundamental level, such as wavefunction collapses for many of the universe's elementary degrees of freedom. These events are very plausibly repeatable, unlike some of the macroscopic events that they together realize. As a result, (PP) picks out a rational credence function for them. We then only need to add the assumption that, if Cr is your credence function and φ analytically entails χ, then it should be the case that $Cr(\chi) = Cr(\varphi)$ to obtain the result that you credences in non-repeatable propositions ought to obey (PP) as well.

[20] Hicks (2017) derives (PP) for Humean accounts of chance on accuracy grounds. Hicks defines Humean chance as the maximally accurate credence function which respects a particular constraint which he calls *Evidential Equivalence*, which says that if no evidence can distinguish E from E*, then $ch(A|E) = ch(A|E^*)$. So, the chance function is the "most accurate credence function that obeys the same evidential constraints that we do" (942). If an agent fails to satisfy (PP), then, they are either failing to obey Evidential Equivalence, in which case their credences depend on more than their present total evidence, or they

distance from vindication, where the distance between two credence functions is standardly defined as the sum of the squared distances between the credences in each function:

$$D\left(Cr_j, Cr_k\right) = \sum_{\gamma \in \Gamma} (Cr_j(\gamma) - Cr_k(\gamma))^2$$

where Γ is a finite set of propositions. We can then define the inaccuracy of any given credence function, *Cr*, at a world *w*, in terms of its distance from the vindicated credence function v_w, called its Brier Score:

$$I(Cr, w) = \sum_{\gamma \in \Gamma} (Cr(\gamma) - v_w(\gamma))^2$$

I can now present the central assumption in my second attempt to justify (PP). It is a modification of Chance Reliabilism to accommodate accurate credence rather than true belief:

> *Accuracy Reliabilism*: All else equal, agents are (epistemically) obligated to adopt the credence-forming processes which have the highest objective chance of producing accurate credences, provided they have access only to admissible evidence.

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have a credence function which is accuracy-dominated. In either case, they are believing irrationally. Since chance is defined in terms of accuracy, which plays perfectly well by the Humean rules, one can clearly show why adherence to (PP) is rational, by the lights of any sensible accuracy norm of belief. This derivation is unobjectionable, as far as I can tell, save for the fact that just as with the other Humean derivations of (PP), it only applies to the standard internal formulation. On Hicks' view, an agent who obeys (PP) is guaranteed to have the most accurate credence function, *conditional* on their beliefs about the objective chances being true. But how they are to have justified beliefs about the chances themselves is, again, left unanswered, and it is unclear that a reliabilist answer would be any less satisfying than the few conceivable alternatives.

The most straightforward way to cash out Accuracy Reliabilism in more formal terms is that it instructs agents to minimize *ch*-expected inaccuracy.[21] Where *Cr* is a credence function defined over a set of propositions $\wp$, *ch*-expected inaccuracy is the sum of the products of the inaccuracy of *Cr* at *w* and the actual objective probability that *w* is the actual world.

$$EI_{ch}(Cr, \wp) = \sum_{w} ch(w) I(Cr, w)$$

Accuracy Reliabilism follows a rich tradition known as *epistemic utility theory*, a program which justifies principles of rationality via the constraint that they promote accuracy. Accuracy Reliabilism, essentially, instructs agents to minimize expected inaccuracy according to the chance function that in fact obtains in their world.

An admissibility constraint is built into Accuracy Reliabilism for the following reason. There is a sense in which the truth function at *w* is the "likeliest" credence function to have a low inaccuracy measure, since that inaccuracy measure is certain to be zero. But there is no credence function which is certain, *de re*, to be the truth function, nor is there any operationalizable credence-forming process which reliably latches onto the truth function. As such, so long as there are non-degenerate objective chances in *w*, *ch* will not assign probability 1 to any particular credence function being maximally vindicated. For an agent to know which

[21] This is somewhat similar to the second argument for (PP) given by Pettigrew (2016). Pettigrew suggests that, for any two credence functions *Cr* and *Cr**, where *Cr* obeys (PP) and *Cr** does not, $\mathrm{Exp}_{\mathfrak{U}}(Cr|ch(-|E)) > \mathrm{Exp}_{\mathfrak{U}}(Cr^*|\ ch(-|E))$ for any possible ur-chance function *ch*, where $\mathfrak{U}$ is an epistemic utility function. The vindication of (PP) I am offering is in the same spirit as Pettigrew's, but is distinct insofar as I don't rely on the constraint that an agent's credence function need $\mathfrak{U}$-dominate alternative credence functions by the lights of every possible ur-chance function. (PP)-obeying credences will rather be justified conditional on any particular chance function, and unconditionally justified only insofar as one's credences over candidate chance functions obey Chance Reliabilism or Accuracy Reliabilism for the chance function that in fact obtains in that agent's world. I also pursue details of how the derivation is supposed to go, which seem to be assumed in Pettigrew's argument. Moreover, I spell out and offer a limited defense of the underlying rationality principles, including in a non-Humean context.

credence function actually is the truth function, they would need to have loads of inadmissible information (time-travelling testimonials, crystal balls, and the like). Inadmissible information is difficult, if not impossible, to come by. Accuracy Reliabilism applies to doxastic circumstances in which the only evidence that is available is the kind to which we ordinarily have access. Hence, the same considerations regarding the practical utility of a chance-backed reliability norm over a prescriptive truth norm that I discussed in the context of Chance Reliabilism should carry over, *mutatis mutandis*, to Accuracy Reliabilism. And, for both principles, the sorts of circumstances in which they do *not* apply are precisely those in which (PP) does not apply, namely those in which inadmissible evidence is available to an agent.

With this in mind, I will attempt to show that (PP)-obeying credence functions are the likeliest credence functions to have a low measure of inaccuracy, and are thus vindicated by Accuracy Reliabilism. For vividness, I will start by making use of the simple example of coin flips with a binary and symmetrical sample space, and then generalize the proof.

Assume, then, that $ch(\text{H}) = 0.5$. Assume, also, that Cr obeys (PP) so that $Cr(\text{H}) = 0.5$, while Cr^* fails to obey (PP), so that $Cr^*(\text{H}) = ch(\text{H}) + \varepsilon$. Then, for a sufficiently long sequence of n trials:

$$I(Cr, w) = 0.25\text{n}$$

This is because $v_w(\text{H})$ is always equal to 0 or 1, so the squared distance between Cr and v_w for any particular coin flip will always be $|\pm 0.5|^2 = 0.25$. Consider, on the other hand, Cr^*. Note that, in the long-run, it is almost certain that $v_w(\text{H}) = 1$ roughly half of the time and $v_w(\text{H}) = 0$ roughly half of the time. In the former case, the distance between Cr^* and v_w is $1 - (0.5 + \varepsilon) = 0.5 - \varepsilon$. In the latter case, the distance between Cr^* and v_w is $1 - (0.5 - \varepsilon) = 0.5 + \varepsilon$. Therefore, with an objective probability of approximately 1:

$$I(Cr^*, w) \approx \frac{(0.5 - \varepsilon)^2 + (0.5 + \varepsilon)^2}{2}\text{n}$$

In order that $I(Cr, w) < I(Cr^*, w)$, the following inequality must obtain:

$$\frac{(0.5-\varepsilon)^2+(0.5+\varepsilon)^2}{2}>0.25$$
$$\text{for all } |\varepsilon|>0.$$

The inequality simplifies to ε > 0 or ε < 0, and is thus true for all $|\varepsilon| > 0$. Therefore, with an objective probability of approximately 1, *Cr** will be accuracy-dominated by *Cr*. In the long run, then, $I(Cr^*, w) > I(Cr, w)$, with very high objective probability, considering credences about single events as well as credences about the long-run frequencies.

We can now generalize the proof. Suppose, for a repeatable proposition-type ϕ, $ch_w(\phi) = \delta$. Now suppose $Cr(\phi) = \delta$ and $Cr^*(\phi) = \delta + \varepsilon$. Consequently $|v_w(\phi) - Cr(\phi)|$ is δ with a chance of approximately 1 – δ, and 1 – δ with a chance of approximately δ. Similarly, $|v_w(\phi) - Cr^*(\phi)|$ is δ + ε with a chance of approximately 1 – δ, and 1 – (δ + ε) with a chance of approximately δ. Therefore:

$$EI_{ch}(Cr,w)\approx\big((1-\delta)\delta^2+\delta(1-\delta)^2\big)\mathrm{n}$$
$$EI_{ch}(Cr^*,w)\approx\Big((1-\delta)(\delta+\varepsilon)^2+\delta\big((1-(\delta+\varepsilon)\big)^2\Big)\mathrm{n}$$

Hence, to insure that $EI_{ch}(Cr, w) < EI_{ch}(Cr^*, w)$, we need:

$$(1-\delta)\delta^2+\delta(1-\delta)^2<(1-\delta)(\delta+\varepsilon)^2+\delta\big((1-(\delta+\varepsilon)\big)^2$$
$$\text{for all } |\varepsilon|>0.$$

Which simplifies to $\delta-\delta^2<\delta-\delta^2+\varepsilon^2$ and finally to $\varepsilon^2>0$, which is true for all ε ≠ 0. Hence, we have proved that (PP)-obeying credence functions EI_{ch}-dominate all non-(PP)-obeying credence functions.[22]

[22] There is an important qualification to make here. While this proof goes through unconditionally for Brier inaccuracy – which is certainly the most popular scoring rule – it is more complicated with alternative scoring rules, particularly scoring rules which sum non-squared distances from the truth

2.3. *The argument reassessed*

Note that, on either of the above approaches, we can restrict ourselves to the chance that agents will have of performing well over long sequences solely when it comes to their beliefs and credences about categorical features of the world, either because we are measuring the probability that an agent will have a true belief about the approximate relative frequencies, or because we are measuring a credence function's *ch*-expected distance from vindication. Thus, even if we restrict our prior principles of epistemic normativity to our beliefs about those categorical features, we find that agents are epistemically obligated to obey (PP) for single-case chancy events.

Let's return, then, to the Credal Argument Against Non-Humean Chance. The argument's first premise fails to consider that, while non-Humean accounts indeed give these categorical features of reality no purchase in determining *what the chances are*, the non-Humean chances can still enter through the "backdoor," as it were, in determining what sorts of belief-formation processes count as reliable and in making certain relative frequencies objectively

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function. For instance, the logarithmic scoring rule defines inaccuracy as $-\Sigma \log(x)$ where, for a credence function *Cr* defined over a set of propositions Φ:

$$x(Cr, \varphi) = \begin{cases} Cr(\varphi) & \text{if } v_w(\varphi) = 1 \\ 1 - Cr(\varphi) & \text{if } v_w(\varphi) = 1 \end{cases}$$

An accuracy-maximization rule would therefore instruct agents to maximize x, since log(x) is a nondecreasing function. But adherence to (PP) will only maximize x (and minimize the non-squared distances) for single-case propensities on the condition that $\delta \neq 0.5$. (Consider, if $\delta = 0.5$, the chance-weighted, non-squared distances for *Cr* and *Cr** both converge to about 0.5 per trial in the long run. Indeed, this is part of why Brier Scores take the squared distances: such a measure gives greater weight to large errors.) But all that it takes to break the tie is to consider our credences about long-run frequencies: there, *Cr* will perform much better than *Cr**; hence, a modified version of this argument can be given for alternative measures of inaccuracy. Similarly, a modified proof can be given for (PP) for cases of events which are not independent and identically distributed, but if such a proof invokes a non-squared measure of inaccuracy, it requires that $\delta \neq 0.5$.

likelier than others. If one accepts Chance Reliabilism or Accuracy Reliabilism, then one accepts that at the bottom of prescriptive epistemic normativity lies an emphasis on the probabilistic disposition of certain doxastic behaviors to generate optimal belief sets. That adherence to (PP) is *likely, by the lights of the non-Humean chance distribution which accurately describes our world,* to generate optimal belief sets, or to yield accurate credences, is all that it takes to rationalize adherence to (PP).

One may understandably worry about these attempted vindications of (PP) that they are circular, or that they open the door to further problems. I will turn, in the next section, to anticipating some of these potential concerns and addressing them.

3. Some Objections

There are two potential objections to my attempt to defuse the Credal Argument which I shall wish to address. The first objection concerns the kind of epistemic access we have to propensity chances. The second concerns the status of reliability in Chance Reliabilism and Accuracy Reliabilism.

Must, on this view, any account of how agents gain epistemic access to the propensities be circular? I take it that the worry here is that since propensities are irreducible and intrinsic features of entities, which we are unable to observe directly, it is unclear how we are supposed to get an epistemic handle on what the chances are in the first place so as to inform ourselves about what our credences ought to be. Humean accounts, it might be assumed, can offer a straightforward way in which we might come to know what the chances are, since the Humean chances supervene on actual patterns in observable events. Non-Humeans, the worry goes, have no such luxury.

Suppose we consider a collection of candidate propensity functions, P^1, P^2,…, P^N, each of which satisfies the axioms of the probability calculus. The different probability functions assign different numbers within the unit interval to different propositions. There is a Principal Principle for each function, PP^1, PP^2,…, PP^N, each of which tells you to match your credences to the respective propensities. The question before agents now is, "which one should you follow?"[23]

Of course, the answer is that you should follow whichever propensity function accurately describes the objective chances in our world! The "correct" propensity function, whatever this comes to, is just what is in the background of (PP) and the principles I have used to vindicate it, such as Chance Reliabilism, Accuracy Reliabilism, and WLLN. The world has decided which propensity function is the correct one.

A more serious issue is how agents could ever come to *know* which of these propensity functions is the correct one. The most immediate response to this worry is that agents can use Bayes Theorem[24] to infer the propensities from the observed relative frequencies. Given any reasonable prior distribution over the candidate propensity functions,[25] an agent could generate a posterior credence function over the candidate propensity functions based on which renders the observed relative frequencies most likely.[26]

The obvious worry here is that one needs to assume (PP) in order to translate the various propensity distributions into Bayes-Theorem-applicable likelihood functions for the observed relative frequencies. This is certainly true: but why is it a problem? The above attempt at vindicating (PP) did not invoke any assumption about the kind of epistemic access that one has

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[23] This objection (as well as the problem with the initial response) was raised to me by [OMITTED].

[24] Bayes Theorem states that the posterior conditional probability of hypothesis A given evidence B, P(A|B), is proportional to the prior probability P(A) multiplied by the likelihood of the evidence given the hypothesis, P(B|A).

[25] Your prior over chance theories need not be perfectly symmetrical, but it can't be arbitrarily peaked, it can't assign zero credence to what turns out to be the objective chance function, etc. Thanks to [OMITTED] for pointing this out.

[26] We can also modify (PP) to instruct agents to set their credences equal to a weighted mixture of the chance assigned to A by each possible chance theory, weighted according to how high your credence is in each of those particular chance theories. I.e., $Cr(A)=\Sigma a_T Ch_T(A)$, where T is a candidate chance theory and a_T is your credence that T is the correct chance theory. This is Ismael's (2008) General Recipe, and it would prove useful to an agent who hasn't settled on one propensity function through the above method.

to the non-Humean chances. Thus, invoking (PP) to explain how agents can infer, from the observed frequencies, which propensity function is the right one, is not circular. An agent who has already determined that, in general, it is rational to obey (PP), is now free to use (PP) to figure out to which particular chance function they should match their credences. Moreover, as noted above, Humeans will have to offer the same kind of explanation for why *their* agents are justified in inferring certain propositions about the chances for a given reference class from the limited data that they observe.

Is it circular to define reliability in terms of objective chance? The worry here is that, if one is trying to rationalize adherence to (PP) by defining reliability in terms of objective chance, they will have to assume (PP) in order to explain why reliability, so understood, matters in the first place. Why, after all, should agents expect that Chance Reliabilism (or Accuracy Reliabilism) is a sensible epistemic norm, without first assuming that a certain doxastic habit's having a high propensity to generate optimal belief sets thereby entices one to have a high degree of belief that said habit will generate optimal belief sets?

Let's pause to consider an illustrative analogy with another pressing issue in the philosophy of science, namely the asymmetry between our epistemic access to the past and the future. David Albert (2000, 2015) points out that there are two different ways to infer, from the state of the world at one time, the state of the world at another time. The first mode of inference employs prediction and retrodiction. Such an inference method takes the present total macro-state of the world at some time, *t*, along with a uniform probability distribution over the possible micro-states that could realize the macro-state at *t* and uses this information to make probabilistic predictions about the future or past via the time-evolved micro-states.

But Albert points out that we have another type of epistemic access to the past, which we lack toward the future: we can keep records of the past. The kinds of devices which are taken to keep records of the past (measuring devices, memories, and so forth) are taken to undergo a *dynamical* transition in the time interval between their ready state and their record-bearing state, which allows us to make inferences about what happened to the measured system within that time interval. In order for our records to be reliable, however, we need another physical system (for instance, our own memories) whose present state is a record of our first device's having been in a ready state a certain time ago. This initiates a "world-devouring regress" (Albert 2015, 38) of record-keeping systems which, so says Albert, bottoms out in the Past Hypothesis, which is the stipulation that our universe began in a macro-state of very low entropy. The Past Hypothesis is something which we are in a position to infer not solely because it follows from the sum total of our finite empirical evidence, but rather because it is confirmatory of our experience.[27]

But consider the kinds of evidence that we *do* have for the low-entropy initial state of the universe, as observed in the Cosmic Microwave Background. This data serves as a kind of record of the initial state, but its reliability *as* a record depends on the initial low-entropy state itself. This is not an instance of our record-based inferences being viciously circular: the Past Hypothesis is the objective feature of the world which *makes* our record-bearing devices reliable. This is so irrespective of our own *knowledge* of the Past Hypothesis. No one has to have studied statistical mechanics to reliably recall what they ate for breakfast this morning! The world has already settled that record-based inference is a reliable process, and it has done so in virtue of this low-entropy boundary condition. Now, agents are perfectly justified in exploiting this fact in order to extract reliable information about past states of the world, *including* the Past Hypothesis itself! In this sense, the connection between the Past Hypothesis and our inferences to past times is epistemologically *self-sustaining*.[28]

[27] In particular, the Past Hypothesis is justified by its ability to reconcile the time-directed regularities of the macroscopic world (reflected in the Second Law of Thermodynamics) from the apparently time-reversal invariant dynamical laws of motion, and because it allows us to make successful statistical inferences about the future.

[28] Ultimately, of course, Albert's account of the epistemic asymmetry is controversial, and not everyone will be satisfied with it. However, I take it that no one will object that the account is circular in the sense addressed above, even if they have other reasons for rejecting it. I address this issue only for purposes of analogy with the way that chances, in a non-circular fashion, provide external reasons. Hence, while I find the above analogy to be helpful for illustrative purposes, the views are entirely separable.

The same is true, I think, for chance. The objective chances make certain belief-forming processes *objectively reliable*. That certain principles – those which are rationalized by Chance Reliabilism or Accuracy Reliabilism – are genuine requirements of rationality is made true by the objective chances, and they are made so *irrespective of anyone's particular degrees of belief*. An agent could, moreover, reason that (PP) is rationalized by Chance Reliabilism or Accuracy Reliabilism, *before* adopting any beliefs or credences over candidate chance functions. After all, that is exactly what has happened in this paper! Agents, therefore, are permitted to take advantage of the resulting, Chance Reliabilism- and Accuracy Reliabilism- backed principles of rationality, even when it comes to connecting up the objective chances with what they should rationally expect about the future occurrence of chancy events.

One may, instead, worry not that the justification is circular, but that it leaves unexplained the justificatory status of the antecedent, chance-backed rationality principles. While this is true, the issue is whether Humeans can obviously do better. As I have emphasized, Humeans can only guarantee accuracy, truth, and the like, when their agents start off with knowledge of the objective chances. As far as I can tell, an account of how agents form justified beliefs about the chances in the first place is going to have to take one of three things as bedrock: (i) self-locating indifference reasoning[29], (ii) updating, or (iii) some chance-backed principle of truth- or accuracy- maximization. And it isn't immediately obvious, absent further motivation, why we should feel more queasy taking (iii) to be a brute principle of rationality than (i) or (ii). After all, such an appeal to which belief-formation processes are likely to yield epistemic success is already at the heart of reliabilist approaches to epistemic justification. I have shown that, if one's epistemology is reliabilist in character, then non-Humeans can justify

[29] Moreover, the indifference in question is not going to be the indifference we are used to. Self-locating indifference usually pertains to agents who know what the universe looks like and are trying to locate themselves in it – for instance, in Everettian quantum mechanics. But here, an agent employs indifference to infer that their local frequencies are approximately equal to the global frequencies (and so the Humean chances). In other words, indifference reasoning aimed at inferring the global Humean chances would seem to get the process backwards, at least as we typically understand indifference reasoning. Many thanks to [OMITTED] on this point.

(PP) from more basic reliabilist constraints – the kind of constraints that range over categorical features of the world, and which (one would hope!) can rationalize adherence to all sorts of *other* principles of epistemic normativity.

4. Conclusion

I want to emphasize that I have not, in this paper, attempted to issue a global defense of non-Humean accounts of chance. Rather, I have tried to show that one can vindicate the Principal Principle for non-Humean chance given some independently plausible background assumptions. Surely, though, one could dispute these principles. What I hope to have shown, however, is that the common assumption that non-Humean accounts of chance are unable to rationalize adherence to (PP) via consideration of categorical constraints on rational credence is too hasty. I think of the justification of (PP) given in this paper as a proof of concept that the connection between the irreducibly modal domain of non-Humean chance and the kinds of expectations we ought to have about the nonmodal domain of frequencies and the like, can *in principle* be made intelligible. Non-Humean accounts of chance, then, need not take (PP) as a primitive constraint on credence. At the very least, there are other options, and it may even be that both Humean and non-Humean theories of chance are on roughly equal footing, in that they must both posit some more basic chance-backed principles of rationality in order to rationalize adherence to (PP), though I don't take myself to have demonstrated this decisively.

This all opens up a puzzle for so-called *functional* analyses of probability, in particular functional analyses which restrict the chance-role to (PP) or similar chance-credence principles, so that chance is just whatever, in the world, plays the role of constraining rational credence in the right way. Lewis, for instance, thought that (PP) captures "all we know about chance" (1980, 266). But the argument advanced in this paper shows that any theory of chance, given some deeply plausible antecedent principles of rationality, can justify (PP) *on its own terms*. And if this is true, then the chance-role, restricted to the chance-credence link, underdetermines our candidate theories of chance. It is necessary, then, to find another way to assess theories of chance than via the ability of any given account to rationalize adherence to (PP). We must search for other potential aspects of the chance-role, and see how the competing theories shape up, lest we find ourselves in a dialectical impasse.

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