Triviality results and the relationship between logical and natural languages*

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Abstract

Inquiry into the meaning of logical terms in natural language ('and', 'or', 'not', 'if') has generally proceeded along two dimensions. On the one hand, semantic theories aim to predict native speaker intuitions about the natural language sentences involving those logical terms. On the other hand, logical theories explore the formal properties of the translations of those terms into formal languages. Sometimes, these two lines of inquiry appear to be in tension: for instance, our best logical investigation into conditional connectives may show that there is no conditional operator that has all the properties native speaker intuitions suggest *if* has.

Indicative conditionals have famously been the source of one such tension, ever since the triviality proofs of both Lewis (1976) and Gibbard (1981) established conclusions which are in *prima facie* tension with ordinary judgments about natural language indicative conditionals. In a recent series of papers, Branden Fitelson has strengthened both triviality results (Fitelson 2013, 2015, 2016), revealing a common culprit: a logical schema known as IMPORT-EXPORT. Fitelson's results focus the tension between the logical results and ordinary judgments, since IMPORT-EXPORT seems to be supported by intuitions about natural language. In this paper, we argue that the intuitions which have been taken to support IMPORT-EXPORT are really evidence for a closely related, but subtly different, principle. We show that the two principles are independent by showing how, given a standard assumption about the conditional operator in the formal language in which IMPORT-EXPORT is stated, many existing theories of indicative conditionals validate one, but not the other. Moreover, we argue that once we clearly distinguish these principles, we can use propositional anaphora to show that IMPORT-EXPORT is

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in fact not valid for natural language indicative conditionals (given this assumption about the formal conditional operator).

This gives us a principled and independently motivated way of rejecting a crucial premise in many triviality results, while still making sense of the speaker intuitions which appeared to motivate that premise. We suggest that this strategy has broad application and an important lesson: in theorizing about the logic of natural language, we must pay careful attention to the translation between the formal languages in which logical results are typically proved, and natural languages which are the subject matter of semantic theory.

1 Introduction

Inquiry into the meaning of logical terms in natural language—expressions like 'and', 'or', 'not', and 'if' (our focus here)—has generally proceeded along two dimensions. On the one hand, *semantic* theories aim to compositionally generate meanings for natural language sentences including those terms in order to predict native speaker intuitions about their patterns of felicity, entailment, and (in concert with a pragmatic theory) what they are used to communicate. On the other hand, *logical* inquiry explores the formal properties of logical terms, investigating what constraints on the possible analysis of logical terms we can derive from a purely formal point of view.

The last half century of research has yielded a number of logical results that constrain the possible meanings of the conditional in particular. Many of these results are puzzling: they seem to show that no conditional operator could have the properties that ordinary speaker intuitions suggest 'If... then...' does have. This yields a *prima facie* tension between the semantic and logical dimensions of inquiry into conditionals.

In light of this situation, it may look as though we are forced to accept an error theory of some kind: ordinary speakers must simply have mistaken intuitions about some of the properties of natural language indicative conditionals. In this paper we will give a different perspective on the relationship between limitative logical results and semantic theories of natural language, a perspective which allows us to respect the insights of semantic theory while remaining within the bounds set by logic.

We will spell out this perspective by making a case study of two kinds of triviality results regarding the logic of indicative conditionals. We will focus in particular on Branden Fitelson's recent strengthening of those results (Fitelson 2013, 2015, 2016). The first class of results stem from a famous proof of Allan Gibbard's (Gibbard 1981), so we call them *Gibbardian triviality results*. Gibbard showed that any conditional operator validating certain intuitively plausible principles is equivalent to the material conditional. This is a startling conclusion, since there is abundant evidence that 'if... then' is *not* equivalent to the material conditional. The second class of results stems from a proof of David Lewis's (Lewis (1976)); since they concern the probabilities of conditionals, we call them *probabilistic triviality results*. These results make trouble for the thesis that the probability of an indicative conditional generally goes by way of the conditional probability of its antecedent on its consequent—an unsettling conclusion, again, given the intuitive plausibility of this thesis (at least in many cases).

Fitelson's results provide a useful point of entry for our purposes because Fitelson shows that both kinds of triviality results follow from a very weak background theory—one that is difficult to object to—plus (a very weak form of) one particular logical schema, known as IMPORT-EXPORT, which states that conditionals of the form (1-a) and (1-b) are equivalent (with '$>$' intended to stand for the indicative conditional operator):

(1) a. $p > (q > r)$
    b. $(p \wedge q) > r$

As we will discuss in detail, intuitions about English indicative conditionals suggest that IMPORT-EXPORT is valid. But how can it be valid, if (together with an unobjectionable background theory) it leads to various implausible conclusions?

We argue that this tension can be resolved in a way that respects our intuitions about natural language indicative conditionals, and that this fact has broad and important upshots for the study of the logic of natural language. In the first part of the paper, we focus on Gibbardian triviality results. In §2, we summarize Gibbard's result, the subsequent dialectic, and how Fitelson's recent result has re-opened the question of how to respond to Gibbardian triviality. In §3–5, we implement our strategy for avoiding Fitelson's strengthening of Gibbard's proof. First, we distinguish IMPORT-EXPORT from a closely related principle which we call SENTENTIAL IMPORT-EXPORT. We argue that the English sentences which are typically taken to be evidence that IMPORT-EXPORT is valid are really only evidence that SENTENTIAL IMPORT-EXPORT is valid. The relationship between these two principles depends on what we assume about the operator '$>$' and its precise relation to natural language 'if... then'. In §5, we argue that, given the widespread assumption that $\ulcorner p > q \urcorner$ always expresses the same proposition no matter what linguistic environment it is embedded within, SENTENTIAL IMPORT-EXPORT may be valid while IMPORT-EXPORT is invalid. This opens up space for us to reject IMPORT-EXPORT as invalid and thus avoid the Gibbardian triviality results, while still accounting for the natural language intuitions that suggested it was valid (as these intuitions only support SENTENTIAL IMPORT-EXPORT). In §6, we show how an analogous strategy can be used to avoid Fitelson's closely related probabilistic triviality result. In §7, we conclude by discussing the broader upshots and applications of our approach, arguing for the general importance of paying careful attention to the translation between the formal languages in which logical results are typically proved, and natural languages which are the subject matter of semantic theory.

2 Gibbardian triviality

We begin by introducing the *Gibbardian triviality results*: a class of triviality results building on Gibbard (1981)'s observation that, given certain appealing principles, English indicative conditionals are equivalent to the material conditional.[1]

Gibbard (1981) showed that, if the English indicative conditional '$>$' validates the following three schemata, then it is equivalent to the material conditional '$\supset$':

IMPORT-EXPORT (IE): $\models \ulcorner (p > (q > r)) \equiv ((p \wedge q) > r) \urcorner$

MODUS PONENS (MP): $\ulcorner p > q \urcorner \models \ulcorner p \supset q \urcorner$

INDICATIVE DEDUCTION (ID): From $p \models q$ conclude $\models \ulcorner p > q \urcorner$

'$\models$' here and throughout denotes classical entailment: for Γ a set of sentences and p a sentence, $\ulcorner \Gamma \models p \urcorner$ means that any admissible model for the language (i.e. any model which conforms to intended stipulations about the meanings of connectives in the language) on which all the sentences in Γ are true also makes p true.[2]

Here is a brief summary of the proof. MP already yields one direction, namely that the indicative conditional entails the material, so we just need to show that the material conditional entails the indicative. Suppose that $\ulcorner p \supset q \urcorner$ is true. $\ulcorner p \supset q \urcorner$ is equivalent to $\ulcorner \neg p \vee q \urcorner$. Our proof goes

[1] Much of what we say regarding Gibbardian triviality applies equally to subjunctive conditionals, which *prima facie* appear to validate Gibbard's three schemata to the same degree as indicatives.

[2] Instead of $\ulcorner \varnothing \models p \urcorner$ we write simply $\ulcorner \models p \urcorner$, meaning that every admissible model for the language makes p true; and instead of $\ulcorner \{p\} \models q \urcorner$ we write $\ulcorner p \models q \urcorner$. A reviewer for this journal rightly points out that the choice of how we construe entailment (and thus validity) is crucial for present purposes. It is only if we construe entailment in a classical 'static' manner, as we do here, that we can derive the triviality results in question. Different, 'dynamic' construals of entailment (like those given in Veltman 1996 and following) will not give rise to these results in the same way. Thus, for instance, if we adopt any of Veltman (1996)'s notions of entailment, the proof of Gibbard's result given here will fail because these notions of entailment will not validate disjunction elimination. As we discuss briefly in the conclusion, we believe that dynamic construals of entailment are very important for capturing intuitions about many of the inference patterns we will explore here. But, for reasons we discuss at greater length in Footnote 33, simply adopting a dynamic construal of entailment does not yet answer the question of how to avoid the triviality results which can be derived from the principles we discuss, principles which are formulated in terms of a static notion of entailment, and it is that question which will occupy most of our attention here.

by disjunction elimination. Suppose first that $\ulcorner \neg p \urcorner$ is true. Since $\ulcorner \neg p \wedge p \urcorner \vDash q$, by ID we have $\vDash \ulcorner (\neg p \wedge p) > q \urcorner$. By IE, we have $\vDash \ulcorner \neg p > (p > q) \urcorner$. By MP, we have $\ulcorner p > q \urcorner$. Suppose next that q is true. Since $\ulcorner q \wedge p \urcorner \vDash q$, by ID we have $\vDash \ulcorner (q \wedge p) > q \urcorner$. By IE, we have $\vDash \ulcorner q > (p > q) \urcorner$; by MP, we thus have $\ulcorner p > q \urcorner$. So $\ulcorner p \supset q \urcorner \vDash \ulcorner p > q \urcorner$.

This is a puzzling result. On the one hand, indicative conditionals intuitively seem to validate these three schemata.[3] But on the other hand, indicative conditionals are intuitively not equivalent to material conditionals. This is made vivid by many well-known contrasts; we will highlight two here.

First, unlike material conditionals, not all indicative conditionals with false antecedents are true. To see this, consider (2):

(2) If Patch is a rabbit, then she is a rodent.

(2) is false, even if Patch isn't a rabbit.[4]

Second, unlike material conditionals, an indicative conditional embedded under negation is not equivalent to the conjunction of its antecedent and the negation of its consequent. To see this, consider (3):

(3) It's not the case that if Patch is a rabbit, then she is a rodent.

(3) is true even if Patch turns out not to be a rabbit, contrary to the predictions of an analysis on which the indicative conditional is the material conditional.

Thus, reflecting on our intuitions about indicative conditionals, it seems that they validate IE, MP and ID, and are also not equivalent to material conditionals. Yet Gibbard's proof demonstrates that there is no conditional operator which has these properties.

In an influential response to this argument, Kratzer (1986) argued that Gibbard's proof rests on an unwarranted assumption about the semantics of indicative conditionals, namely, that conditionals denote two-place operators at all. Kratzer instead argued that an indicative conditional sentence involves a (possibly covert) modal taking highest scope in the conditional's consequent whose domain is restricted by the conditional's antecedent. But Khoo (2013b) showed that merely

[3] We'll discuss the case for (and against) IE below. ID is intuitively quite plausible. And, although McGee (1985) provides independent arguments against MP, as we will see below, Fitelson (2013) shows that there is a version of Gibbard's proof which does not require the assumption that indicatives validate MP.

[4] The standard response to this challenge is to distinguish assertability from truth and argue that for various reasons some true indicative conditionals are unassertable (see Lewis 1976; Jackson 1979; Grice 1989; Rieger 2006). Setting aside whether this strategy can be spelled out successfully, it won't help with the second challenge, which involves embedded conditionals and thus cannot be resolved by appeal to considerations about assertability.

denying that conditionals are two-place operators does not, *by itself*, avoid the conclusion of Gibbard's proof. Khoo shows that nothing in Gibbard's proof relies essentially on the assumption that indicative conditionals denote two-place operators—Gibbard's argument can be replicated, essentially unchanged, simply by looking at the propositions expressed by conditionals, regardless of their internal syntax. But Khoo pointed out that Kratzer's semantics in fact avoids Gibbard's conclusion because it invalidates MP (importantly, it invalidates MP in a sufficiently limited way that it can still make sense of our intuitions that MP is valid).

Recent results in Fitelson 2013, 2016, however, show that this response does not suffice. Fitelson shows that MP is not the real culprit behind the Gibbardian triviality results. Fitelson's strengthening of Gibbard's proof relies on only a weak background theory for the logic of the indicative conditional and a weak 'logical' conditional '$\rightarrow$' (a conditional even weaker than the material conditional; we lay out Fitelson's assumptions in the Appendix).[5] Crucially, Fitelson does not assume that the indicative conditional validates MP (nor even that modus ponens is valid for the logical conditional). In this weak background setting, Fitelson shows that a very strong form of Gibbardian triviality—namely, the equivalence of the indicative conditional and the logical conditional—is *logically equivalent to* a very weak form of IE—namely, the restriction of IE to logical truths. In other words, Fitelson shows that, given his background assumptions, LOGICAL IMPORT-EXPORT is logically equivalent to GIBBARDIAN TRIVIALITY:[6]

LOGICAL IMPORT-EXPORT (LIE): $\models \ulcorner p > (q > r) \urcorner$ iff $\models \ulcorner (p \wedge q) > r \urcorner$

GIBBARDIAN TRIVIALITY: $\models \ulcorner p > q \equiv p \rightarrow q \urcorner$

In light of Fitelson's proof, GIBBARDIAN TRIVIALITY remains deeply problematic: contrary to Khoo's claim (and similar claims earlier in the literature, like McGee (1985)'s), we cannot avoid this conclusion simply by rejecting MP on a limited basis. Fitelson's result also helps locate the source of the problem raised by Gibbard's proof: given the weakness of Fitelson's background assumptions, we think that the most natural interpretation of Fitelson's proofs is that they show that LIE is the real culprit in GIBBARDIAN TRIVIALITY, and thus that the following are jointly inconsistent: (i) the indicative conditional validates LIE; and (ii) the indicative conditional is not equivalent to Fitelson's weak logical conditional '$\rightarrow$'.

[5] We could of course easily weaken Fitelson's result by strengthening '$\rightarrow$' to '$\supset$', bringing it into a more familiar setting.

[6] We will not rehearse Fitelson's proof, which is quite complicated; we refer interested readers to his paper for the details.

3 Apparent evidence for IE

There is, however, clear evidence that the indicative conditional is not equivalent to Fitelson's '$\rightarrow$'. In particular, the material conditional entails '$\rightarrow$',[7] and so '$\rightarrow$' shares with the material conditional the key features which led us to reject equating the material conditional and indicative conditional: in particular, $\ulcorner p \rightarrow q \urcorner$ will be true whenever p is false, and $\ulcorner \neg(p \rightarrow q) \urcorner$ will entail $\ulcorner p \wedge \neg q \urcorner$. As we saw above, the indicative conditional does not share these features; and so the indicative conditional cannot be equated with '$\rightarrow$' any more than it can with the material conditional. The natural move at this juncture is thus to question the validity of LIE.

The usual way to evaluate whether a schema like LIE is valid is to look at its instances, and see if one can find counterexamples—in this case, pairs of sentences in natural language which have a surface structure mirroring the structure of the relevant instances of LIE in our formal language, but which do not intuitively validate LIE.

Recall that LIE states that whenever an instance of the schema (4) is logically true, the corresponding instance of (5) is logically true as well—and *vice versa*.

(4) $p > (q > r)$

(5) $(p \wedge q) > r$

To evaluate this as a claim about natural language, then, it is natural to consider sentences with surface structures that mirror the structure of (4) and (5): that is, sentences of the forms of (6) and (7).

(6) If p, then if q, then r.

(7) If p and q, then r.

Thus consider, for instance, the following pair of sentences, which have the form of (6) and (7):

(8) If it's raining, then if Bob brought his umbrella, it's raining.

(9) If it's raining and Bob brought his umbrella, then it's raining.

[7] I.e. for any p and q, $\ulcorner p \supset q \urcorner \models \ulcorner p \rightarrow q \urcorner$. To see this, note that, by Axiom 2 of Fitelson's axioms (enumerated in the appendix), we know $\models \ulcorner(\neg p \wedge p) > p\urcorner$ and $\models \ulcorner(\neg q \wedge q) > q\urcorner$. By Axiom 4, it follows that $\models \ulcorner(\neg p \wedge p) > q\urcorner$. By Axiom 6, it follows that $\models \ulcorner(\neg p \wedge p) \rightarrow q\urcorner$. By Axiom 7, we have $\models \ulcorner \neg p \rightarrow (p \rightarrow q)\urcorner$. By Axiom 5, we have $\ulcorner \neg p \urcorner \models \ulcorner p \rightarrow q\urcorner$. Likewise by Axiom 1 we have $\models \ulcorner(q \wedge p) \rightarrow q\urcorner$; by Axiom 7 we have $\models \ulcorner q \rightarrow (p \rightarrow q)\urcorner$; by Axiom 5 we have $q \models \ulcorner p \rightarrow q\urcorner$. We thus have $\ulcorner p \supset q\urcorner \models \ulcorner p \rightarrow q\urcorner$.

It seems clear that (9) is necessarily true.[8] And this seems equally so for (8): under what possible circumstances would either one be false? These sentences thus seem to witness exactly the pattern predicted by LIE.

This is just one pair of sentences, not yet a pattern. But in exploring further instances of (6)/(7)—pairs like (8)/(9)—we have not been able to find any which break this mold: in other words, for all sentences with this form we have examined, if one is necessarily true, then so is the other. Perhaps the most convincing evidence that this is a uniform pattern comes from examining the best attempt we know of to construct a counterexample to this pattern. The putative counterexample, slightly modified from Kaufmann 2005 (following the presentation in Fitelson 2016), runs as follows:

> Suppose that the probability that a given match ignites if struck is low, and consider a situation in which it is very likely that the match is not struck but instead is tossed into a camp fire, where it ignites without being struck. Now, consider the following two indicative conditionals.
>
> (a) If the match will ignite, then it will ignite if struck.
>
> (b) If the match is struck and it will ignite, then it will ignite.

Fitelson (2016) claims that, in this situation, (b) is a logical truth, but (a) is not. (a) and (b) thus constitute an apparent counterexample to the pattern in question.

We agree with Fitelson's judgments here. However, we suspect the intuitive grip of this example rests on an equivocation in 'will' between a broadly dispositional meaning and a temporal meaning. We can disambiguate these readings by replacing 'will ignite' with 'is ignitable', to select for the dispositional meaning, and by replacing 'will ignite' with 'will ignite at t', to select for the temporal meaning (we also replace 'struck' with 'struck at t'', to thoroughly regiment the readings). We suspect that the reading on which (a) and (b) strike us as inequivalent is:

(a′) If the match is ignitable, then it will ignite at t if struck at t'.

(b′) If the match is struck at t' and it will ignite at t, then it will ignite at t.

(b′) does indeed strike us as a logical truth, while (a′) certainly does not. But this pair, of course, is no longer a counterexample to the pattern we are exploring; we would only get a counterexample if

[8] We will use 'necessarily true' and 'logically true' interchangeably here.

we were to disambiguate (a) and (b) in a uniform way. But no matter how we do this, the resulting sentences strike us as equivalent. Consider:

(a$''$) If the match will ignite at t, then it will ignite at t if struck at t'.

(b$''$) If the match is struck at t' and it will ignite at t, then it will ignite at t.

(a$''$) and (b$''$) strike us as obviously equivalent (and both necessarily true). We will not go through all the possible uniform substitutions into (a) and (b), but readers can verify for themselves that in no case does an intuitively inequivalent pair result.

Thus, we do not think that even the most promising extant attempt yields a pair of sentences with the form (6) and (7) which are such that one is necessarily true, and the other is not. This provides strong inductive evidence that there is no such pair.

4 Natural and formal languages

The data so far suggest that sentences with the form of (6) and (7) always have the same logical status: one is logically true just in case the other is. This appears to be strong evidence that LIE is valid. Given Fitelson's proof, accepting this conclusion would lead to the startling, and implausible, conclusion that the indicative conditional just is the weak logical conditional.

One way to avoid Fitelson's results would be to steadfastly deny the validity of LIE. But, as things stand, this would be unprincipled. What we want is a *principled* way to reject the validity of LIE: a way to reject its validity that still allows us to make sense of the intuitions discussed in the last section. In this section, we will lay out a strategy for doing just this. Our strategy is to distinguish LIE from a closely related, but distinct, principle. This opens up room to hold that corresponding instances of (6) and (7) are semantically equivalent (and thus have the same logical status), but that LIE is nonetheless invalid. In the next section, we will show that distinguishing these principles allows us to formulate a test which provides positive evidence that LIE is invalid.

In the previous section, in an attempt to evaluate LIE, we considered sentences of English whose surface form seemed to mirror the form of the substitution instances of LIE. But, although we were not explicit about this in presenting it, LIE is in fact an inference schema in a formal logical language. In particular, it is an inference schema in a standard propositional language comprising a set of atomic sentences closed under the Boolean connectives (conjunction, disjunction, negation, and the material conditional), and two binary connectives, '$>$' and '$\rightarrow$'. Thus instances of LIE are sentences of this formal language, not sentences of English. To draw conclusions about the validity of LIE from intuitions about sentences in English, we must translate the relevant sentences of English into the formal language in question (most simply, we can think of the translation as a bijection between the two languages). Without a translation which tells us how the sentences of the formal language are meant to be interpreted, LIE simply does not bear on natural language. A translation on the other hand enables us to evaluate the validity of a principle like LIE: we can do so by looking at the sentences in natural language which correspond to instantiations of LIE in the formal language and consulting speaker intuitions about whether those sentences are felt to be equivalent.

Translating between a natural language (in our case, English) and a formal language, however, is not trivial. In assuming that the intuitions elicited in the last section were evidence for LIE, we were implicitly assuming that the translation from English to the formal language would completely respect surface form, so that an instance of (10) would express the same proposition as the corresponding instance of (11):

(10) If p, then if q, then r.

(11) $p > (q > r)$

But this is a substantive assumption, and one which we may find reasons to reject: a faithful translation from a natural to a formal language may well diverge from the surface form of the natural language sentence, given facts about the meaning of 'if' and how we interpret the operator '$>$'.

To appreciate this point, consider the following schemata, which are closely related, but—we will show—independent from, IE and LIE:[9]

SENTENTIAL IMPORT-EXPORT (S-IE):

[9] Here, $[\![\ ]\!]$ is an interpretation function, which assigns extensions to expressions of English relative to a *context*, which we will generally denote with 'c', and a possible world, generally denoted with 'w', written as superscripts on the interpretation function (what a context amounts to is theory dependent, as we will see below). The extension of a sentence at a context and world is its truth value (either 1 or 0). We will sometimes omit a world superscript, writing $[\![\text{p}]\!]^c$, to indicate the *proposition* expressed by the sentence p in context c; we will assume that this is the set of possible worlds $\{w : [\![\text{p}]\!]^{c,w} = 1\}$. We use roman letters to stand for sentences in natural language, and italic letters to stand for sentences in our formal language. We have opted to use the $\vDash$ notation in stating IE and the $[\![\cdot]\!]^c$ notation in stating S-IE. This is because these are the notations most native to the two literatures we are trying to bridge here—the first is most commonly used in discussions of meaning in formal languages, the latter in discussions of meaning in natural language. However, there is nothing essential about this choice: $\ulcorner\Gamma \vDash \varphi\urcorner$ means that φ is true at every world in every intended model where all the sentences of Γ are true, and thus all of the principles we are interested in could be stated using either idiom. That is, we could state S-IE using $\vDash$ (appropriately relativized to a context parameter), and IE using $[\![\cdot]\!]$; the crucial difference between them, as we discuss at greater length in a moment, is not in this notational choice, but rather in that IE is a principle about a formal language, while S-IE is a principle about natural language, and the intended interpretation of these languages may differ in relevant ways.

$\forall c$: $[\![\ulcorner\text{if p, then if q, then r}\urcorner]\!]^c = [\![\ulcorner\text{if p and q, then r}\urcorner]\!]^c$.

SENTENTIAL LOGICAL IMPORT-EXPORT (S-LIE):
$\forall c : (\forall w : w \in [\![\ulcorner\text{if p, then if q, then r}\urcorner]\!]^c)$ iff $(\forall w : w \in [\![\ulcorner\text{if p and q, then r}\urcorner]\!]^c)$

S-IE says that the proposition expressed by an instance of ⌜if p, then if q, then r⌝ is always the same proposition expressed by an instance of ⌜if p and q, then r⌝. And S-LIE is a strictly weaker principle which says that such pairs always have the same logical status: one is necessarily true iff the other is. S-IE entails S-LIE, and so to streamline the discussion, we will focus on S-IE.

Recall the data we saw in the last section, which consisted of pairs of sentences which were felt to be equivalent. The most obvious interpretation of those data is that they show that, in any context, pairs of sentences with the form of ⌜if p, then if q, then r⌝ and ⌜if p and q, then r⌝ respectively, like (8) and (9), repeated here, express the same propositions:

(8) If it's raining, then if Bob brought his umbrella, it's raining.

(9) If it's raining and Bob brought his umbrella, then it's raining.

This is direct evidence for S-IE. But is it also evidence for IE? This is a more subtle question, but the answer is negative. It is possible for S-IE to be valid even though IE is invalid, and thus evidence for the former is not necessarily evidence for the latter.

We pause to clarify what we mean when we say that S-IE may be valid though IE is not. As noted earlier, '$>$' is intended to stand for the English indicative conditional operator. However, this intention may be in tension with other principles which logicians generally accept, in particular, the assumption that what proposition $\ulcorner p > q \urcorner$ expresses does not change depending on the linguistic environment in which it is embedded. Given this assumption, together with facts about natural language conditionals, it may be plausible that the best translation of right-nested conditionals in the the logician's formal language, like $\ulcorner p > (q > r) \urcorner$, is not ⌜if p, then if q, then r⌝. If that is so, then it could be that IE is not valid but S-IE is, since the instances of one would simply not be translated as the instances of the other: the two principles would end up being orthogonal.

In more detail, our idea is the following. Many plausible semantic theories for the conditional predict that what proposition is expressed by ⌜if q, then r⌝ will be sensitive to the linguistic environment in which it is embedded. Thus, on these theories, the proposition expressed by ⌜if q, then r⌝ when unembedded may be different from the proposition it expresses (in the same global context) when embedded under ⌜if p⌝. In other words, these theories hold that the interpretation of 'if' varies with a parameter (call it *the context*, without any particular commitment regarding what this amounts to), which can be shifted by sentential operators. Relative to a single admissible valuation, then, two occurrences of ⌜if p, then q⌝ may end up expressing different propositions, relative to the same global context, if one is embedded in a different linguistic environment than the other. Call this property of English 'if' *shiftiness*. This contrasts with an assumption generally made in the logical literature on triviality proofs about '$>$', namely that ⌜$p > q$⌝ always expresses the same proposition (relative to a given global context), whether embedded or unembedded. Call this latter property of '$>$' $>$-*unshiftiness*.

We will illustrate this point in more detail by sketching a semantic theory that generates these results compositionally. We will use the theory of Gillies 2009, 2010 to make this point. However, we want to emphasize that we have no special commitment to Gillies' theory; it is just a tidy compositional theory on which 'if' is shifty. Many other theories of the conditional will deliver essentially the same results in different ways (see in particular McGee 1985; von Fintel 1994; Kratzer 1981, 1986, 2012). [10] The crucial commitment shared by these theories which allows them to validate S-IE without necessarily validating IE or LIE is that conditional antecedents shift some parameter of evaluation, and thus that conditionals embedded under other conditionals express different propositions than they do when unembedded.

Gillies' theory comprises two assumptions, one somewhat familiar and the other less so. The first is that indicative conditionals are analyzed as strict conditionals over a domain of epistemic possibilities. The second is that 'if'-clauses are *shifty*: they shift both the world at which the consequent is evaluated, and the context in which it is evaluated. It then follows that ⌜if q, r⌝ expresses a different proposition when it is embedded under ⌜if p⌝ than it does when unembedded.

Formally, let E be an *epistemic domain function* from a context c and world w to a set of possible worlds (intuitively, the ones compatible with the c-relevant evidence in w). Gillies' theory assigns the following truth conditions to indicatives (as shorthand, let 'p^c' refer to $[\![\text{p}]\!]^c$):

Gillies:

[10] Other than in the case of McGee's theory, these will validate S-IE in full generality only in combination with an appropriately dynamic conjunction. Yet another way to validate S-IE is with a close variant on Stalnaker (1968); McGee (1985)'s variably strict semantics which validates S-IE as follows, with f Stalnaker's selection function:

(12) $[\![\text{If p, then q}]\!]^{f,w}$=1 iff $[\![\text{q}]\!]^{f^p,f(p,w)}$=1

For any proposition p, let f^p be the smallest function such that for all propositions r and worlds w, $f^p(r,w) = f(p \cap r, w)$. We combine this semantics for the conditional with this semantics for conjunction:

(13) $[\![\text{p and q}]\!]^{f,w}$=1 iff $[\![\text{p}]\!]^{f,w}$=1 and $[\![\text{q}]\!]^{f^p,w}$=1.

All the points that we make now using Gillies' theory could be equally well made with this variant of Stalnaker's.

$$[\![\ulcorner\text{if p, q}\urcorner]\!]^c = \{w : \forall w' \in E(c,w) \cap p^c : w' \in [\![\text{q}]\!]^{c+p^c}\}.$$

For any proposition r, we define $E(c+r,w)$ in terms of $E(c,w)$ as follows:

$$\forall c, w, r\text{: } E(c+r,w) = E(c,w) \cap r.$$

Finally, following Gillies, we assume that E is closed in the following sense:[11]

$$\forall c, w, w'\text{: if } w' \in E(c,w)\text{, then } E(c,w) = E(c,w').$$

Thus, according to the theory, $\ulcorner$if p, q$\urcorner$ is true iff every epistemically possible p-world is one in which $[\![\text{q}]\!]^{c+p}$ is true. **Gillies** predicts that simple conditionals like $\ulcorner$if p, q$\urcorner$, with p and q atomic sentences, have the same truth conditions as those assigned by standard strict conditional theories, which differ only in that the consequent is evaluated relative to an unshifted context:[12]

Strict: $[\![\ulcorner\text{if p, q}\urcorner]\!]^c = \{w : \forall w' \in E(c,w) \cap p^c : w' \in [\![\text{q}]\!]^c\}.$

However, when it comes to conditionals with conditionals embedded in their consequents, the predictions diverge. We can see this divergence by comparing the meaning **Gillies** and **Strict** assign to $\ulcorner$if p, then if q, then r$\urcorner$:

(14) **Gillies**: $[\![\ulcorner\text{if p, then if q, then r}\urcorner]\!]^c =$
$\{w : \forall w' \in E(c,w) \cap p^c : w' \in [\![\ulcorner\text{if q, then r}\urcorner]\!]^{c+p^c}\}$

(15) **Strict**: $[\![\ulcorner\text{if p, then if q, then r}\urcorner]\!]^c =$
$\{w : \forall w' \in E(c,w) \cap p^c : w' \in [\![\ulcorner\text{if q, then r}\urcorner]\!]^c\}$

Since $[\![\ulcorner\text{if q, then r}\urcorner]\!]^{c+p^c}$ need not be the same as $[\![\ulcorner\text{if q, then r}\urcorner]\!]^c$, the two theories will make different predictions about the meaning of $\ulcorner$if p, then if q, then r$\urcorner$. In particular, unlike **Strict**, **Gillies** (together with the assumption that epistemic modal bases are closed) predicts that indicatives validate S-IE.[13] We prove this in the Appendix, but here is an intuitive gloss on why this is. According

[11]This assumption, which is discussed in Gillies 2010, is basically an introspection assumption about epistemic modal bases (see Kaufmann and Kaufmann 2015, 292: they show that a slightly weaker pair of assumptions, namely shift-reflexivity plus transitivity, suffice to validate S-IE in Gillies' framework). We can achieve the same results without this assumption by removing the world-shifting component of the semantics of conditionals and locating the modal dimension of conditionals as a covert element of their logical form (as in Lewis 1975; Heim 1982; Kratzer 1981, 1986). See Khoo 2011 for some recent arguments for taking this route over Gillies'.

[12]The strict conditional analysis is ordinarily attributed to Chrysippus; see e.g. Warmbrod 1983; Lycan 2001 for modern incarnations of the standard analysis. See von Fintel 2001 for a shifty precedent to **Gillies**.

[13]This holds in full generality provided we couple **Gillies** with an appropriate asymmetric dynamic entry for conjunction, as we discuss in the Appendix. The case for that approach to conjunction is subtle, and there is a case to be made

to **Gillies**, the context in which we evaluate ⌜if q, then r⌝ when it is embedded under ⌜if p⌝ has been shifted by ⌜if p⌝. In effect, this shifting forces ⌜if q, then r⌝ to be evaluated against the epistemic domain function as updated by p^c, thus predicting that the embedded conditional will be equivalent to ⌜if p and q, then r⌝. According to **Strict**, on the other hand, the embedded conditional ⌜if q, then r⌝ is not evaluated against the epistemic domain function updated by p^c—in effect, the information contributed by p^c is lost when we get to the embedded conditional. For this reason, **Strict** yields counterexamples to S-IE (see the Appendix for a concrete counterexample).

Since **Gillies** validates S-IE, it also validates S-LIE, which follows as a special case of S-IE. But what about (L)IE? It depends on whether we assume $>$-unshiftiness. Suppose we do (we will revisit this assumption below)—this just is to assume that $\ulcorner p > q \urcorner$ always expresses the same proposition, whether embedded under further operators or not (holding fixed the global context). Given this assumption, then, we cannot translate between English and our formal language by simply following surface structure. In particular, the English sentence that corresponds to the right-nested conditional $\ulcorner q > (p > q) \urcorner$ will not be ⌜if q, then if p, then q⌝. This is because the shiftiness induced by 'if' affects what proposition is expressed by ⌜if p, then q⌝ when it is embedded under ⌜if q⌝; but given $>$-unshiftiness, this will not hold for $\ulcorner p > q \urcorner$.

Under the assumption of $>$-unshiftiness, then, to find the English sentence that translates $\ulcorner q > (p > q) \urcorner$, we need to find a sentence s which, when embedded under ⌜if q⌝ in c, expresses the proposition expressed by an unembedded occurrence of ⌜if q, then p⌝ in c. That is, we need an s such that $[\![\text{s}]\!]^{c+p^c} = [\![\ulcorner\text{if q, then p}\urcorner]\!]^c$. Then, given **Gillies** and $>$-unshiftiness, ⌜if p, s⌝ will be correctly translated into our formal language as $\ulcorner p > (q > p) \urcorner$. But then **Gillies** predicts counterexamples to LIE (see the Appendix for one). The key is to show that there are models where ⌜if p, s⌝ is false, despite the fact that there are no models which make false ⌜if p and q, then p⌝, which continues to be translated into our formal language as $\ulcorner (p \wedge q) > p \urcorner$. Therefore, **Gillies** predicts that there is a sentence which is translated into our formal language as $\ulcorner (p \wedge q) > p \urcorner$ and which is necessarily true, but that there is a sentence which is translated into our formal language (given the assumption of $>$-unshiftiness) as $\ulcorner p > (q > p) \urcorner$ which is not, and thus that LIE (and, therefore, IE) is not valid.

Thus, **Gillies** predicts that S-IE is valid, but that LIE and IE are invalid (given $>$-unshiftiness). This shows that it is possible for S-IE and IE to come apart, and in particular for the first to be valid and the second invalid. We have already seen evidence which suggests that the former is valid.

that we should stick with a static conjunction, in which case we will validate S-IE only in the limited cases where what is imported/exported is not itself conditional. How we decide this question does not matter for present purposes; see Mandelkern 2018 for further discussion.

We suspect that it is indeed valid (though if it turns out to break down in some marginal cases, it would not matter for present purposes).[14] The crucial point is that intuitions about sentences of the form (6) and (7) constitute evidence only about the validity of S-IE, *not* the validity of LIE or IE. Thus, rejecting LIE as a way of avoiding the logical triviality results for indicative conditionals does not have any intuitive cost. In the next section, we show that not only is this move without cost, but that once we have the distinctions made here clearly in sight, we can formulate intuitive counterexamples which show that LIE is, in fact, invalid, given $>$-unshiftiness.

5 The propositional anaphora test

To have a fully satisfying case for rejecting LIE, and thus a satisfying response to Fitelson's Gibbardian triviality result, we need to do more than simply formulate an alternate explanation for the data which seemed to tell in favor of the validity of LIE; we also need evidence that LIE is, in fact, invalid (assuming $>$-unshiftiness, that is, which we'll continue to assume up to the last part of this section, where we'll revisit it). If LIE is invalid, then, provided that we can find pairs of English conditionals which correspond to instances of LIE, we expect to find intuitive counterexamples to LIE. In this section, we show that we can indeed find pairs which instantiate LIE by using propositional anaphora to force the conditional proposition expressed by an *unembedded* conditional into the consequent of another conditional. We argue that some pairs constructed this way do indeed provide intuitive counterexamples to LIE.

Suppose that Bob tends to be confused about the weather; he often brings his umbrella when it's not raining, and almost always fails to bring it when it is raining. Joe then says:

(16) [Joe]: If Bob brought his umbrella to work, then it's raining out.

Under such circumstances, (16) does not strike us as true; instead, it strikes us as being very probably false. Now suppose that Sue, overhearing Joe, says:

(17) [Sue]: If it's raining out, then what Joe said is true.

(17) strikes us as no more likely than (16). After all, (16) struck us as very probably false, *whatever* the weather was like. Assuming that it is raining out does not change our intuition that (16) was likely false; thus (17) does not strike us as true. In any case, (17) certainly does not strike us as a *necessary truth*.

[14]One place that sentences with the form (6) and (7) may come apart is in how presuppositions project out of their first argument. Another place, as briefly discussed in Footnote 13, may be when a conditional is itself imported/exported.

But now compare (17) with (18):

(18) If it's raining out and Bob brought his umbrella to work, then it's raining out.

(18), unlike (17), strikes us as true, and, indeed, as necessarily true.

We think that (17) and (18) together comprise a genuine counterexample to LIE (and thus to IE). Recall that to formulate a counterexample to LIE, we need to find a pair of sentences which would be (respectively) translated as each of a pair of sentences in our formal language that have the forms of (4) and (5), such that one is necessarily true and the other is not.

(4) $p > (q > r)$

(5) $(p \wedge q) > r$

Now note, first, that (18) is clearly equivalent to the instance of (5) where $p = r =$ 'it's raining out' and $q =$ 'Bob brought his umbrella to work'. Second, we hypothesize that (17) is equivalent to the instance of (4) with the same values for p, q, and r: that is, (4) could faithfully be translated into our formal language as (17). This is because, in (17), we use a propositional anaphor to embed the proposition expressed by (16)—an unembedded conditional—under the conditional antecedent 'If it's raining out'. On at least one reading, the referent of propositional anaphors like 'What Joe said' is the proposition expressed by Joe. By using a propositional anaphor to embed the conditional expressed in (16), we thus circumvent the shift in proposition of the embedded conditional induced by the antecedent of the conditional it is embedded under. Propositional anaphora thus allows us to find what we claimed above is quite elusive: namely, a natural language conditional which expresses what a sentence with the form of (4) expresses, under the assumption of $>$-unshiftiness.[15]

Let us pause to note an important methodological point here. Some discussions of propositional anaphora assume that a phrase like 'What Joe said' *always* refers to the proposition expressed by Joe's assertion (as opposed to e.g. the character of Joe's assertion). We do not need to make this strong—and perhaps unmerited—assumption here. All that is required for our point is that *there is*

[15]A reviewer for this journal points out that order effects sometimes affect the interpretation of embedded material; thus e.g. 'Bill and Sue got married and filed their taxes' and 'Bill and Sue filed their taxes and got married' will typically be interpreted in different ways. This effect, moreover, tends to persist when these constructions are embedded, which raises the possibility that order determines, not only pragmatic effects, but also what proposition a given sentence expresses. This, in turn, might problematize our assumption that the instance of 'Bob brought his umbrella to work' appearing in (16) and the instance appearing in (18) express the same proposition—a crucial assumption implicit in our translation of these sentences into our logical language. While this is an important issue to be sensitive to—and while it might indeed affect other examples similar to ours—we cannot find any evidence that these do indeed express different propositions in these different contexts: to our ears, there is no perceived contrast between 'Bob brought his umbrella to work and it's raining out', versus 'It's raining out and Bob brought his umbrella to work'.

a reading of a propositional anaphor like this which picks out the proposition expressed by Joe's assertion. Insofar as there is an interpretation of (17) on which it is not felt to be a necessary truth—as, indeed, there seems to be—we can best make sense of this by assuming that, on that reading, the propositional anaphor is indeed picking out the proposition expressed by Joe's assertion.

With that in mind, let us summarize the situation. (17) and (18) express what sentences with the form of (4) and (5), respectively, express. And we claim that (18) is necessarily true, while (17) is not—indeed, it strikes us as likely to be false. This pair thus constitutes an intuitive counterexample to LIE, and so to IE as well. Moreover, this pattern is not an isolated instance; it is easy to reproduce (we give one more example along these lines in the next section). Thus, once we have the distinction between (L)IE and S-(L)IE clearly in view, we can formulate intuitive counterexamples to the former principles, giving us positive and independent reason to reject them. This, in turn, gives us independent motivation for the strategy we are taking to avoid GIBBARDIAN TRIVIALITY, while still making sense of speaker intuitions about embedded conditionals: namely, endorse a semantics for indicative conditionals that invalidates (L)IE, but which validates S-(L)IE.[16]

At a high level, here is where things stand. Intuitions about conditionals in natural language seem to show that IE is valid. This is worrisome, since LIE, plus a few very weak further constraints, suffices to yield GIBBARDIAN TRIVIALITY. But appearances are deceiving. Those intuitions are evidence in favor of a closely related, but importantly different, principle, namely S-LIE. But they are not evidence for LIE. S-LIE and LIE can come apart, and indeed do come apart in a semantics like **Gillies**, which predicts that the first is valid but the second is not. And, once we distinguish these principles, we can use propositional anaphora to show that LIE (and hence IE) is,

[16]Arguably, we do not *need* propositional anaphora to formulate counterexamples to IE. What is key about our strategy is that we use an expression which, when embedded, expresses what a given unembedded conditional expresses. Propositional anaphora provides one way of executing this strategy, but other paraphrases might provide other avenues.

Yablo (2016) provides some interesting possibilities along these lines, turning on the felt equivalence between dispositional adjectives and conditionals. A different possibility, suggested to us by a reviewer for this journal, is with broadly quotative material. For instance, consider these variations on (17):

(19) If it's raining out, then what Joe said—'if Bob brought his umbrella to work, then it's raining out'—is true.

(20) If it's raining out, then what Joe said is true: if Bob brought his umbrella to work, then it's raining out.

In these constructions, what Joe said is essentially quoted (using explicit quotation marks in (19), and an implicitly quotative construction in (20)). It is thus very natural to interpret this material relative to the contextual parameters of *Joe's* context of assertion, rather than the local parameters made available in (19) and (20) (a little reflection on the interpretation of context sensitive terms like indexicals in quotation shows that generally we interpret quoted material relative to the contextual parameters in the context of assertion, rather than the contextual parameters in the context in which the material is repeated). This material will thus pick out the proposition that Joe asserted, achieving the same goal we achieve in (17) with propositional anaphora; our intuitions about these constructions match our intuitions about (17) (namely, that neither is necessarily true).

in fact, not valid for natural language conditionals. This allows us to avoid the unacceptable result of Gibbardian triviality results, while also respecting the natural language intuitions which seem, at first glance, to support (L)IE.

This argument has assumed $>$-unshiftiness. A natural question to raise at this point in the dialectic is whether Fitelson's whole proof would go through if we dropped $>$-unshiftiness and instead assumed that $\ulcorner p > q \urcorner$ is always interpreted exactly as $\ulcorner$if p, then q$\urcorner$ would be interpreted, even if the latter express different propositions, relative to the same global context, depending on where it is embedded (call this assumption *$>$-shiftiness*). Given $>$-shiftiness, IE would be equivalent to S-IE, and thus the discussion in this section would *not* give us intuitive reason to reject IE, which would stand or fall with S-IE (the latter of which, again, we think is quite plausible).[17] But if we adopt $>$-shiftiness, then we will no longer have reason to accept Fitelson's axioms (which seem unobjectionable given $>$-unshiftiness). In particular, consider Axioms 5 and 6 (see the Appendix for the full list), which in this shifty interpretation would be equivalent to the following (we're not sure exactly how to translate the logical conditional '$\rightarrow$' into natural language, but we can leave it untranslated for present purposes):

Axiom 5: If $\forall c : \forall w : [\![\ulcorner \text{p}\rightarrow\text{q}\urcorner]\!]^{c,w} = 1$, then $\forall c : [\![\text{p}]\!]^c \subseteq [\![\text{q}]\!]^c$

Axiom 6: If $\forall c : \forall w : [\![\ulcorner \text{if p then q}\urcorner]\!]^{c,w} = 1$, then $\forall c : \forall w : [\![\ulcorner \text{p}\rightarrow \text{q}\urcorner]\!]^{c,w} = 1$.

Substituting $\ulcorner$if q then p$\urcorner$ for q in these axiom schemata and assuming the meta-language 'if. . . then' in these axioms is the mathematician's material conditional (and thus validates modus ponens and hypothetical syllogism), we can prove:

(21) If $\forall c : \forall w : [\![\ulcorner \text{if p, then if q then p}\urcorner]\!]^{c,w} = 1$, then $\forall c : [\![\text{p}]\!]^c \subseteq [\![\text{if q, then p}]\!]^c$

Any semantics for the conditional which, like **Gillies**, validates S-IE will predict that the antecedent of (21) is true, since $\ulcorner$if p, then if q, then p$\urcorner$ will express the same proposition as the logical truth $\ulcorner$if p and q, then p$\urcorner$. But the consequent of the conditional in (21) is clearly false: in a semantics like **Gillies**, and indeed in any plausible semantics for the conditional, at any given context, the proposition expressed by p at that context is not necessarily a subset of that expressed by $\ulcorner$if q, then p$\urcorner$ at that context. If in fact we could conclude that $\forall c : [\![\text{p}]\!]^c \subseteq [\![\text{if q, then p}]\!]^c$, then we would already fall into familiar paradoxes of the material conditional: for instance, it would then follow that, from the falsity of a conditional, we can conclude that its consequent is false. But this is absurd; consider again (3):

[17] This response would roughly mirror the strategy of Khoo 2013b in response to Kratzer 1986.

(3) It's not the case that if Patch is a rabbit, then she is a rodent.

Given the truth of (3), this principle would let us conclude that Patch is not a rodent. But (3) is true no matter what, *whether or not Patch turns out to be a rodent* (i.e. (3) would be true even if Patch turned out to be a rat). So this principle is clearly absurd.

Therefore, if we reinterpret Fitelson's proof assuming $>$-shiftiness, then we will have principled reason to reject Fitelson's axioms. (Note, though, that this argument does *not* give us reason to reject these axioms given $>$-unshiftiness. For, assuming $>$-unshiftiness, $\ulcorner p > (q > p)\urcorner$ will *not* be logically valid given **Gillies**, and so this argument will not go through.)

In sum: S-(L)IE is a principle that concerns sentences in English, whereas (L)IE concerns sentences in a formal language. Given that English 'if' is shifty (as many theories of the conditional, including **Gillies**, predict), we face a choice point in how we interpret the '$>$' connective that is intended to stand for the English indicative conditional operator in our formal language. If we assume $>$-unshiftiness, then the cases we constructed using propositional anaphora show that (L)IE is invalid. If instead we assume $>$-shiftiness, then (L)IE and S-(L)IE are equivalent, and both arguably valid; but, given this assumption, other axioms which Fitelson assumes turn out to be invalid. Either way, we avoid Fitelson's triviality results in a principled manner.

6 Probabilistic triviality

We think that our strategy for responding to Fitelson's proof has general application: in addition to Gibbardian triviality results, it promises to be helpful in addressing a wide range of further puzzles in the study of the logic of natural language. In this section, we will show that distinguishing S-IE from IE, and adopting a semantics which validates the former but not necessarily the latter, helps us resolve not only Gibbardian triviality results, but also a related result of Fitelson's regarding the probability of conditionals. In the conclusion, we will briefly discuss other applications of our strategy.

Fitelson's second result concerns a principle known as THE THESIS, which says that the probability of a conditional is always equal to the probability of its consequent conditional on its antecedent:[18]

THE THESIS: $Pr(\ulcorner p > q\urcorner) = Pr(q|p)$.

[18] 'Pr' denotes a probability operator that maps propositions, or sentences in context (we move freely between the two here, suppressing reference to context for now), to values in $[0, 1]$ and obeys the Kolmogorov axioms. '$Pr(q|p)$' denotes the conditional probability of q given p, defined in the usual way: $Pr(q|p) = \frac{Pr(\ulcorner p \wedge q\urcorner)}{Pr(p)}$, when $Pr(p) > 0$.

It is *prima facie* very intuitive that THE THESIS holds in general, i.e. for all conditionals and probability distributions. For instance, suppose that John has just rolled a fair six-sided die and kept the result hidden. Now consider the following conditional:

(22) If John rolled an even number, then he rolled a prime.

The probability that (22) is true is low; intuitively, the reason for this is that the corresponding conditional probability that John rolled a prime, given that he rolled an even number, is low (1/3), in conformity with the predictions of THE THESIS.[19]

Despite its plausibility, however, a series of famous triviality proofs establish that THE THESIS cannot hold in full generality. The first of these proofs is due to Lewis (1976, 1986), and goes as follows. Assume $Pr(q) > 0$ and $Pr(\neg q) > 0$, and consider the following instance of the Law of Total Probability:

(LTP) $$Pr(\ulcorner p > q \urcorner) = Pr(\ulcorner p > q \urcorner | q) \cdot Pr(q) + Pr(\ulcorner p > q \urcorner | \neg q) \cdot Pr(\neg q)$$

Then, we have two lemmas:

(L1) $$Pr(\ulcorner p > q \urcorner | q) = 1$$

(L2) $$Pr(\ulcorner p > q \urcorner | \neg q) = 0$$

Substituting into (LTP) yields:

(C) $$Pr(\ulcorner p > q \urcorner) = 1 \cdot Pr(q) + 0 \cdot Pr(\neg q) = Pr(q)$$

and thus, given THE THESIS,

(I) $$Pr(q|p) = Pr(q)$$

This proof shows that, for any conditional that validates THE THESIS and is such that its consequent and its negation have non-zero probability, if (L1) and (L2) hold for that conditional, then the conditional's antecedent and consequent are probabilistically independent.

It is obviously not true in general that any two propositions p and q which can figure as the antecedent and consequent of a conditional are probabilistically independent whenever the probability of q is non-maximal. However, assuming that probability functions are well-defined over

[19] The plausibility of THE THESIS was first observed in Ramsey 1931 (see also especially Stalnaker 1970; Adams 1975; Edgington 1995; see Douven 2013 for recent empirical discussion). It is an open question whether intuitions invariably conform to THE THESIS: see McGee 2000; Kaufmann 2004; Khoo 2016 for some apparent counterexamples.

conditionals, the Law of Total Probability seems unassailable.[20] Thus if we are to preserve any non-trivial instances of THE THESIS, we must reject the lemmas (L1) and (L2), at least for some conditionals. Lewis showed that these lemmas follow if THE THESIS holds for any conditional across a class of probability functions closed under conditionalization (see the Appendix for his reasoning). Thus, Lewis's proofs establish:

> LEWISIAN TRIVIALITY: For any sentences p, q such that $Pr(\ulcorner p \wedge q \urcorner) > 0$, $Pr(\ulcorner p \wedge \neg q \urcorner) > 0$, and $Pr(q|p) \neq Pr(q)$: THE THESIS does not hold for $\ulcorner p > q \urcorner$ across the class of probability functions which includes Pr and is closed under conditionalization.

One response to Lewis's triviality proof is to reject its significance. For instance, Rothschild (2013b) and Bacon (2015) show that there are intuitive reasons why THE THESIS will not always be preserved across conditionalization—and hence that we might want to preserve many instances of THE THESIS, but deny that it always holds across probability functions related by conditionalization.

A response like this provides a reasonable way to temper the upshot of Lewis's proof. However, it is not so clear that this strategy can be extended to a recent strengthening of Lewis's results, discovered by Fitelson (2015). Fitelson (building on a long tradition of probabilistic triviality proofs)[21] shows that we can derive worrisome results in a way which does not involve conditionalization, and instead relies only on a few limited applications of THE THESIS, and two instances of the following principle:

> PROBABILISTIC IMPORT-EXPORT (PIE): $Pr(\ulcorner p > (q > r) \urcorner) = Pr(\ulcorner (p \wedge q) > r \urcorner)$.

Fitelson's full proof establishes a strong and worrisome triviality result, which we will not review here. Instead, to bring out the connection to Lewis's proof, we will show how Fitelson's strategy can be adapted into a novel proof of (L1) and (L2), and spell out our strategy for responding to this adaptation of Fitelson's proof (our strategy would work equally as a response to Fitelson's full triviality result).

This novel proof of (L1) and (L2), crucially, does not rely on the assumption that THE THESIS holds across a class of probability functions closed under conditionalization. Since this proof does

[20] There is a long history of denying that conditionals express propositions in response to this style of triviality result, which might open up room to deny that probability functions are well-defined over them; see Adams 1975; Gibbard 1981; Edgington 1995. We will not discuss this strategy here.

[21] For other triviality proofs, and some responses, see: van Fraassen 1976; Stalnaker 1976; Carlstrom and Hill 1978; Ellis 1978; Gibbard 1981; Blackburn 1986; Kratzer 1986; McGee 1989; Hájek 1989; Jeffrey 1991; Eells and Skyrms 1994; Hájek 1994; Edgington 1995; Adams 1998; Bradley 2000; Bennett 2003; Milne 2003; Bradley 2007; Kaufmann 2009; Hájek 2011; Bradley 2012; Kratzer 2012; Khoo 2013a; Rothschild 2013b,a; Korzukhin 2014; Charlow 2015; Bacon 2015.

not rely on conditionalization, strategies like the one pursued by Rothschild (2013b) and Bacon (2015), discussed briefly above, will not straightforwardly help us avoid its conclusion. Here is the reasoning establishing (L1):

1. $Pr(\ulcorner q > (p > q)\urcorner) = Pr(\ulcorner (q \wedge p) > q\urcorner)$ PIE
2. $Pr(\ulcorner (q \wedge p) > q\urcorner) = Pr(q|\ulcorner q \wedge p\urcorner) = 1$

   THE THESIS, Probability calculus (assp $Pr(\ulcorner q \wedge p\urcorner) > 0$)
3. $Pr(\ulcorner q > (p > q)\urcorner) = 1$ From 1,2
4. $Pr(\ulcorner q > (p > q)\urcorner) = Pr(\ulcorner p > q\urcorner|q)$ THE THESIS
5. $Pr(\ulcorner p > q\urcorner|q) = 1$ From 3,4

Given this reasoning (and analogous reasoning to (L2)),[22] we thus establish the following triviality result, given PIE:

> PROBABILISTIC TRIVIALITY: For every pair of sentences p, q such that $Pr(\ulcorner p \wedge q\urcorner) > 0$, $Pr(\ulcorner p \wedge \neg q\urcorner) > 0$, and $Pr(q|p) \neq Pr(q)$, one of the following instances of THE THESIS must be false:
>
> (a) $Pr(\ulcorner p > q\urcorner) = Pr(q|p)$
>
> (b) i. $Pr(\ulcorner (q \wedge p) > q\urcorner) = Pr(q|\ulcorner q \wedge p\urcorner)$
>
> ii. $Pr(\ulcorner (\neg q \wedge p) > q\urcorner) = Pr(q|\ulcorner \neg q \wedge p\urcorner)$
>
> (c) i. $Pr(\ulcorner q > (p > q)\urcorner) = Pr(\ulcorner p > q\urcorner|q)$
>
> ii. $Pr(\ulcorner \neg q > (p > q)\urcorner) = Pr(\ulcorner p > q\urcorner|\neg q)$

[22] Namely:

1′. $Pr(\ulcorner \neg q > (p > q)\urcorner) = Pr(\ulcorner (\neg q \wedge p) > q\urcorner)$ PIE

2′. $Pr(\ulcorner (\neg q \wedge p) > q\urcorner) = Pr(q|\ulcorner \neg q \wedge p\urcorner) = 0$

THE THESIS, Probability calculus (assp $Pr(\ulcorner \neg q \wedge p\urcorner) > 0$)

3′. $Pr(\ulcorner \neg q > (p > q)\urcorner) = 0$ From 1′,2′

4′. $Pr(\ulcorner \neg q > (p > q)\urcorner) = Pr(\ulcorner p > q\urcorner|\neg q)$ THE THESIS

5′. $Pr(\ulcorner p > q\urcorner|\neg q) = 0$ From 3′,4′

This is a troubling result. We see no independently plausible reason to think that, in every case, we should be able to reject one of these instances of THE THESIS. The (a), (b-i), and (b-ii) instances of THE THESIS are all quite plausible, since they all involve simple conditionals (conditionals whose antecedents and consequents are not themselves conditionals). What about (c-i) and (c-ii)? Assuming that natural language 'if' is shifty, it depends on what we assume about '$>$'. For now, we will assume $>$-unshiftiness; we will revisit this assumption in a moment. Assuming $>$-unshiftiness, for the reasons we discussed above, a sentence with the form $\ulcorner q > (p > q) \urcorner$ can be translated into natural language using propositional anaphora. Consider the example from the previous section:

(16) [Joe]: If Bob brought his umbrella to work, then it's raining out.

(17) [Sue]: If it's raining out, then what Joe said is true.

We argued above that, given $>$-unshiftiness, (17) expresses what a sentence with the form $\ulcorner q > (p > q) \urcorner$ expresses (where $\ulcorner p > q \urcorner$ expresses the proposition Joe asserted). What is the probability of (17)? Intuitively, the probability of (17) will equal the ratio of the probability that it's raining and what Joe said is true to the probability that it's raining. So, it seems plausible that the probability of a conditional which expresses what $\ulcorner q > (p > q) \urcorner$ does indeed equal the probability of the proposition expressed by $\ulcorner p > q \urcorner$ conditional on the probability of what q expresses. Similar considerations motivate thinking the same for conditionals with the form of (c-ii). It thus seems to us that, in many cases, all of these substitution instances of THE THESIS will be true. This makes one line of response to PROBABILISTIC TRIVIALITY look less attractive to us: namely, one which attempts to argue that there are independent intuitive grounds to think that one of the above instances of THE THESIS must fail (for any p and q). This kind of response would generalize Rothschild and Bacon's response to LEWISIAN TRIVIALITY; however, in light of the present considerations, it does not look promising.

Rather, it seems more promising to avoid PROBABILISTIC TRIVIALITY by rejecting PIE, the crucial background premise used to establish it. Indeed, counterexamples to PIE are not hard to come by; we can find one, again, by using the method from the last section, namely using propositional anaphora to generate counterexamples (we continue to assume $>$-unshiftiness). Suppose that John has just rolled a fair six-sided die and kept the result hidden. Now suppose that Smith says:

(23) [Smith]: If John rolled an even number, he rolled a prime.

In this case, we do not yet have enough information to conclude either that (23) is true or that it is false. The best we might do is to say that the probability that (23) is true is about 1/3.[23] If John in fact rolled a 2, then what Smith said seems true. If John in fact rolled a 4 or a 6, then what Smith said seems false. But suppose for a moment that John rolled a 5. In that case, (23) has a false antecedent and it is hard to evaluate what Smith said as clearly true or clearly false. But notice that having a false antecedent does not automatically result in what Smith said being true—if it did, (23) would be equivalent to the material conditional 'John rolled an even number $\supset$ John rolled a prime', but, as we discussed above, there is abundant evidence that indicatives are not equivalent to material conditionals. Thus, it seems plausible that rolling a 5 does not (even epistemically) entail that what Smith said is true.[24] So—the crucial observation here—learning that John rolled a 5 would not give us reason to conclude decisively that (23) is true.

Now suppose that Jane, overhearing Smith, says in response:

(25) [Jane]: If John rolled a prime, then what Smith said is true.

We submit that (25) is also not certainly true. There are three prime possibilities: 2, 3, 5. Suppose John rolled a 5. Then the antecedent of (25) is true, in which case, plausibly, the entire conditional is true only if its consequent is true.[25] But we have already argued above that we do not know that its consequent ('what Smith said is true') is true if John rolled a 5. Thus, since it's an open possibility that John rolled a 5, for all we know, (25)'s antecedent is true and its consequent not true, and hence, for all we know, (25) is not true. Thus we should not be certain that (25) is true.

By contrast, we should be certain that (26) is true:

[23] Some claim that (23) is automatically false because it is epistemically possible that John rolled an even non-prime number. Moss (2013) argues persuasively against this claim. Adapting one of her arguments here, consider the contrast between:

(24) a. It is unlikely that if John rolled an even number then he rolled a prime.
b. It's not the case that if John rolled an even number then he rolled a prime.

(24-a) sounds true given the information in our setup. By contrast, (24-b) strikes us as not clearly true—the information given in the setup is not enough to conclude (24-b). This is evidence that (23) is not false merely because it is epistemically possible that John rolled an even non-prime number.

[24] We do not here take a stand on the status of (23) if John rolled an odd number. There are a variety of ways we can go, compatible with our main interests here. For instance, Stalnaker (1980) suggests that it is neither true nor false but rather indeterminate; Edgington (1995) suggests that it has a verity rather than a truth value (where a verity is some number in the unit value); and Hawthorne (2005) suggests that it is either true or false but we cannot know which (Hawthorne's suggestion is about chancy counterfactuals—we here are extending his suggestion to indicatives). See also Stalnaker and Jeffrey 1994; Kaufmann 2009; Bradley 2012; Bacon 2015; Khoo 2016.

[25] While it is not clear that MP is valid in full generality for indicative conditionals—and, indeed, theories like **Gillies** do not validate it—it is uncontroversial that it is valid for conditionals whose antecedents and consequents are both non-modal, non-conditional sentences, as is the case for (25).

(26) If John rolled a prime and John rolled an even number, he rolled a prime.

Suppose John rolled a prime. Then, whatever else is true, he rolled a prime! It is thus hard to see how (26) could be anything short of necessarily true, and so we should have maximal credence in (26). As before, (26) would uncontroversially be translated into our formal language as having the form of (5). And, for the same reasons appealed to in our counterexample to LIE above, we claim that (25) should be translated as the corresponding instance of (4). But, since we should have credence 1 in (26) and credence of less than 1 in (25), (25) and (26) together form an intuitive counterexample to PIE, and thus give us independent reason to reject it.[26]

Thus we think that propositional anaphora furnishes intuitive counterexamples to PIE, just as it does to LIE.[27] However, notice that, just as for LIE, denying PIE seems to fly in the face of intuition: all of the intuitions appealed to above as *prima facie* support for IE can equally be used to provide support for PIE, since IE entails PIE. As above, we can resolve this tension by distinguishing PIE from its sentential cousin SENTENTIAL PROBABILISTIC IMPORT-EXPORT (S-PIE), and endorsing a theory that validates S-PIE but not PIE:

SENTENTIAL PROBABILISTIC IMPORT-EXPORT (S-PIE):
$\forall c : Pr(\llbracket \ulcorner \text{if p, then if q then r} \urcorner \rrbracket^c) = Pr(\llbracket \ulcorner \text{if p and q, then r} \urcorner \rrbracket^c)$.

S-PIE says that the probability of the proposition expressed by $\ulcorner$if p, then if q then r$\urcorner$ is always the same as the probability of the proposition expressed by $\ulcorner$if p and q, then r$\urcorner$. S-PIE follows from S-IE. But, just as S-IE is independent of IE, likewise S-PIE is independent of PIE; the former is a claim about sentences in a natural language, whereas the latter is a claim about sentences in a formal language. Given $>$-unshiftiness, one of these can be true, while the other is false. For instance, a theory like **Gillies** will validate S-PIE, in virtue of validating S-IE, but will invalidate PIE, in virtue of invalidating IE (it is straightforward to construct a countermodel based on our countermodel to IE in the Appendix). Crucially, validating S-PIE, but not PIE, allows us to reject PIE in a way which still makes sense of the intuitions which seemed to support it in the first place. That is, by validating S-PIE, we can make sense of intuitions that conditionals with the form $\ulcorner$if p, then if q, then r$\urcorner$ and $\ulcorner$if p and q, then r$\urcorner$ are generally felt to be equivalent, and thus to have the same probability. By contrast, if we adopted a theory which invalidates both S-PIE and PIE (like a standard strict or variably strict analysis), we could not appeal to this kind of move, and thus we would fail to make sense of the intuitions that seemed to motivate adopting PIE in the first place.

[26] Since IE entails PIE, this counterexample to PIE is also another counterexample to IE.

[27] Others have made similar moves; for instance, the theories of Rothschild (2013b) and Bacon (2015) both invalidate IE, and thus allow for counterexamples to PIE.

Thus, just as above, distinguishing S-PIE from PIE allows us to avoid triviality results, while still respecting ordinary intuitions about natural language conditionals.[28]

Just as with GIBBARDIAN TRIVIALITY, as we discussed in the end of §5, we might wonder what happens if we drop the assumption of $>$-unshiftiness and instead assume $>$-shiftiness. Given $>$-shiftiness, PIE will end up being equivalent to S-PIE, and thus both will arguably be valid. Does PROBABILISTIC TRIVIALITY then follow, given this translation schema? We think it does not. If we interpret the proof in these terms, it would rely on the following premise (the analog of step [4]):

4″ $\quad Pr(\llbracket \ulcorner\text{if q, then if p, q}\urcorner \rrbracket^c) = Pr(\llbracket \ulcorner\text{if p, q}\urcorner \rrbracket^c \mid \llbracket \text{q} \rrbracket^c)$.

But (4″) is false. The reason is that $Pr(\llbracket \ulcorner\text{if q, then if p, q}\urcorner \rrbracket^c) = 1$ (by S-PIE), but $Pr(\llbracket \ulcorner\text{if p, q}\urcorner \rrbracket^c \mid \llbracket \text{q} \rrbracket^c)$ can be less than 1. To see this, compare (27) with (23):

(27) [Mabel]: If John rolled a prime, then if John rolled an even number, he rolled a prime.

(23) [Smith]: If John rolled an even number, he rolled a prime.

What is the probability that what Mabel said is true? We think it is clearly 1. But what is the probability that what Smith says is true, *given that John rolled a prime*? There are three prime possibilities: 2, 3, 5. In one of them (the 2-possibility) Smith's claim is true. But in two of them (the 3- and 5-possibilities) Smith's claim is not necessarily true (since it would then have a false

[28] Kratzer (1986, 2012) spells out a different way to respond to related triviality results which, like our approach, relies on distinguishing the logical form of a conditional from its surface form. The basic idea is that, when we are asked to judge the probability of a conditional, we are not judging the probability of the proposition expressed by that conditional, but are rather judging the probability of its consequent relative to a probability measure restricted (at a distance) by its antecedent (see Kratzer 2012: 107-8). We agree with criticism in e.g. Rothschild 2013b; Charlow 2015 that this approach faces serious challenges. One problem is that we seem able to *think* about the probabilities of the propositions expressed by conditional propositions, and when we do, those probabilities are generally equal to the corresponding conditional probabilities; yet positing some kind of restricting operation at the level of thought is much less plausible than at a linguistic level. A similar point can be made using propositional anaphors like 'What X said' to target the content of an utterance of a conditional—since to utter a conditional is to assert the proposition it expresses, *not* to assert its consequent, 'What X said' should pick out that proposition and allow us to evaluate its likelihood. Yet when we elicit intuitions about the probability of conditionals using constructions like this, the probability of the conditional proposition generally seems to go by the corresponding conditional probability of its consequent on its antecedent. Thus, we think Kratzer's strategy for responding to probabilistic triviality results is unsuccessful. Though this is not to say that Kratzer's restrictor theory is incorrect: on the contrary, as we noted above, Kratzer's restrictor theory provides an elegant semantic strategy for validating S-IE without necessarily validating IE, and thus avoiding both GIBBARDIAN TRIVIALITY and PROBABILISTIC TRIVIALITY in the way we've advocated here, and it seems entirely open to us that her theory (or something very close to it) is the correct theory of the conditional. But the crucial point about Kratzer's theory is not that it allows us to reconstrue apparent intuitions about the probabilities of conditionals as intuitions about the truth of probabilistic conditionals, but rather that her theory validates S-IE but not IE.

antecedent, we cannot be sure that it is true). So, it seems you should not be certain that what Smith said is true given that John rolled a prime. Thus it seems to us that, contrary to the predictions of (4″), the probability that what Smith said is true, given that John rolled a prime, is *not* equal to 1 (the probability that what Mabel said is true). Thus, given $>$-shiftiness, PIE is indeed much more plausible, since it ends up being equivalent to S-PIE; but, then other premises in Fitelson's proof (namely, relevant instances of THE THESIS) are straightforward to reject in a principled manner.

Here is a brief summary of the dialectic. Assuming $>$-shiftiness, we argued that PIE is not equivalent to S-PIE, and that the former (but not the latter) is invalid. This is sufficient to avoid Fitelson's PROBABILISTIC TRIVIALITY result. Alternately, given $>$-shiftiness, PIE and S-PIE are equivalent, and both are arguably valid. But then we have reason to reject another premise of Fitelson's proof (a premise which was plausible assuming $>$-unshiftiness, but not plausible assuming $>$-shiftiness). These probabilistic triviality results thus illustrate, once again, the importance of paying careful attention to how we are translating between formal and natural languages.

We should note in concluding that our discussion in this section has been entirely negative: we have only shown that our general strategy provides a principled way to respond to Fitelson's probabilistic triviality result. This is not meant to resolve more broadly the many subtleties regarding the relation between conditional probabilities and probabilities of conditionals. Without further elaboration, **Gillies** does not answer to the intuitions which provide support for THE THESIS; specifically, and problematically, **Gillies** does not predict any instances of THE THESIS in which the probability of the conditional is non-extreme.[29] One plausible way to bring a theory like **Gillies**, or any other theory we end up adopting for 'if', broadly in line with THE THESIS would be to follow the strategy of Rothschild 2013b, assuming that, at least as a default, our theory of 'if' validates two principles concerning the logic and probabilities of conditionals, namely STRONG CENTERING and PROBABILISTIC INDEPENDENCE, allowing for failures of PROBABILISTIC INDEPENDENCE to predict cases where THE THESIS seems to fail, as in McGee 2000; Kaufmann 2004.[30] We will not spell out this strategy here; our present goal is not to settle on a particular the-

[29]Here is a sketch of why. Suppose that $Pr(\llbracket \text{q} \rrbracket^c \mid \llbracket \text{p} \rrbracket^c) = x$, for some $x : 0 < x < 1$; this means that there are some epistemically accessible $p \wedge q$-worlds and some epistemically accessible $p \wedge \neg q$-worlds. Given the assumption from §4 that epistemic modal domain functions are Closed, so that at each $w' \in E(c,w)$, $E(c,w) = E(c,w')$, **Gillies** predicts that $Pr(\llbracket \ulcorner \text{if p, then q} \urcorner \rrbracket^c) = 1$ if $\forall w' \in E(c,w) \cap p^c : w' \in \llbracket \text{q} \rrbracket^{c+p^c}$, and $Pr(\llbracket \ulcorner \text{if p, then q} \urcorner \rrbracket^c) = 0$ otherwise. By hypothesis, there are $p \wedge \neg q$-worlds in $E(c,w)$, so it follows that $Pr(\llbracket \ulcorner \text{if p, then q} \urcorner \rrbracket^c) = 0$. But given our assumptions, $Pr(\llbracket \text{q} \rrbracket^c \mid \llbracket \text{p} \rrbracket^c) > 0$.

[30]STRONG CENTERING says that $\models \ulcorner p \supset (q \equiv p > q) \urcorner$. PROBABILISTIC INDEPENDENCE says that $Pr(p > q|p) = Pr(p > q)$. Rothschild, building on Ellis 1978, shows that these are together equivalent to THE THESIS. Adding these both to a semantics like **Gillies** as non-defeasible defaults would lead to triviality, but the hope would be that adding both as defeasible defaults would predict THE THESIS in the cases where it is intuitively valid, but not in cases where it is not, and avoid triviality.

ory that can handle the various complexities regarding the probabilities of conditionals, but rather to show that the strategy we took to resolve Gibbardian triviality has broad application as a means of avoiding the implausible conclusions of various proofs—in this case, regarding the probabilities of conditionals—while simultaneously making sense of intuitions about natural language.

7 Conclusion

IMPORT-EXPORT and its corollary PROBABILISTIC IMPORT-EXPORT seem to lead to disaster: given a very weak background theory, each entails triviality results which we have strong evidence are false. Yet, at first glance, natural language conditionals seem to validate IE. Can we make sense of this fact, without landing in triviality?

We have argued that we can. Doing so depends on distinguishing IE from a subtly different principle, namely S-IE. S-IE and IE are equivalent only under the assumption that the instances of the former are best translated into our formal language as instances of the latter. Given that natural language 'if' is shifty, if we assume $>$-unshiftiness (as is common in the logical literature on triviality proofs) then we can distinguish S-IE from IE. We illustrated this point using the theory of indicative conditionals in Gillies 2009, 2010, but that choice was simply for concreteness; as we have emphasized, there are many semantic theories which make (or can make, given minor adjustments) the same prediction. We have argued that the intuitions which are often elicited to show that IE is valid in fact only tell in favor of S-IE, vitiating the motivation for adopting IE in the first place. Furthermore, we have formulated a new test, using propositional anaphora, which shows that IE is not valid for natural language indicative conditionals, assuming $>$-unshiftiness. This gives us a principled way to reject IE (and its probabilistic corollary) given that translation, and thus avoid the conclusion of Fitelson's recent strengthenings of the Gibbardian and probabilistic triviality results, while still respecting speaker intuitions about natural language indicative conditionals. If instead we assume $>$-shiftiness—the assumption that '$>$' just means whatever 'If. . . then' means, and the proposition expressed by $\ulcorner p > q \urcorner$ can vary depending on its embedding environment—then S-(L)IE and (L)IE (and likewise S-PIE and PIE) end up being equivalent, and we think these principles are then much more plausible. However, given the assumption of $>$-shiftiness, we have reason to reject other principles in the triviality proofs, so we can still avoid the proofs in a principled way.

This conclusion advances the dialectic in a number of ways. Most specifically, it provides a principled response to Fitelson's new logical results, which have threatened to revive the spectres of Gibbardian and probabilistic triviality. More generally, it provides a new perspective on the relationship between semantics and logic, and illustrates the importance of paying careful attention to the translation between the formal languages in which logical results are typically proved and natural languages which are the subject matter of semantic theory. Logical rules like IE make predictions about inferences in a formal language. Translating from natural language to the formal language—which is crucial for sorting out the import of logical results for natural language semantics—must be taken deliberately. Given certain assumptions about '$>$', we can distinguish formal language inference schemata (like IE) from closely related natural language schemata (like S-IE), and use propositional anaphora to give principled reasons for rejecting the formal language inference schema, while validating the natural language one; on other assumptions about '$>$', we can find principled reasons to reject other, apparently innocuous, premises in the triviality proofs.

In concluding, we would like to suggest that the moves we have made here are quite general, and can be brought to bear across the board in thinking about the logic of natural language. We have focused on IE, a crucial principle for understanding the logic of natural language conditionals, but similar distinctions can be made for other central inference schemata. Thus, for instance, consider again THE THESIS:

THE THESIS: $Pr(\ulcorner p > q \urcorner) = Pr(q|p)$.

Here are two ways we might bring THE THESIS to bear on the propositions expressed by sentences of natural language:

THE UNSHIFTY THESIS: $Pr([\![\ulcorner \text{if p, then q} \urcorner]\!]^c) = Pr([\![\text{q}]\!]^c | [\![\text{p}]\!]^c)$.

THE SHIFTY THESIS: $Pr([\![\ulcorner \text{If p then q} \urcorner]\!]^c) = Pr([\![\text{q}]\!]^{c+p^c} | [\![\text{p}]\!]^c)$.

The crucial difference between these involves the shifted context parameter on the right-hand side of the latter, which matches the shifting effect induced by $\ulcorner$if p$\urcorner$ on the proposition expressed by q as it appears on the left-hand side. For reasons similar to those we discussed with respect to IE, we believe that THE SHIFTY THESIS better captures the intuitions behind THE THESIS than does the THE UNSHIFTY THESIS. In particular, there will be counterexamples to the latter which the former avoid. One class of counterexamples will come from sentences with the following form:

(28) $Pr([\![\ulcorner \text{if q, then if p, q} \urcorner]\!]^c) \neq Pr([\![\ulcorner \text{if p, q} \urcorner]\!]^c | [\![\text{q}]\!]^c)$.

By S-PIE, we predict that the value on the left side should be 1, but the value on the right need not be 1 (as we saw above in our rejection of PROBABILISTIC TRIVIALITY). Similarly, and more concretely, consider the following case, due to Paolo Santorio (p.c.). Suppose that Mary rolled a fair six-sided die. Then the probability of (29) is, intuitively, 1:

(29) If the die landed on an odd number, then if it landed on a number greater than three, it landed on five.

But the probability of (30) is, intuitively, close to or equal to 1/2:

(30) The die landed on an odd number.

And the probability of (31) is, intuitively, close to or equal to 1/3:

(31) If the die landed on a number greater than three, it landed on five.

Given these probability judgments, however, there is no way that the probability of (29) can be equivalent to the probability of (31) conditional on the probability of (30) (the highest possible value for that conditional probability would be obtained if the probability mass of (31) was entirely contained in that of (30); but even then, the conditional probability of the latter on the former would only be 2/3).[31] These examples are counterexamples to the UNSHIFTY THESIS. But they are not counterexamples to the SHIFTY THESIS. For the SHIFTY THESIS says that the probability of the proposition expressed by (29) is the probability, not of the proposition expressed by (31), but rather of the proposition that would be expressed by a sentence with the form of (29) (which will be 1), conditional on the probability of the proposition expressed by (30) (which gets us 1 again).[32]

Likewise, in thinking about whether natural language conditionals validate modus ponens, we should be careful to distinguish two versions of such a principle, a shifty and unshifty version—the key difference again being in how the context parameter gets shifted:[33]

[31]That is, given Bayes' Theorem, we have:

$$Pr(\llbracket\text{if greater than 3, then 5}\rrbracket^c \mid \llbracket\text{odd}\rrbracket^c) = \frac{Pr(\llbracket\text{odd}\rrbracket^c \mid \llbracket\text{if }{>}3\text{, then 5}\rrbracket^c) \cdot Pr(\llbracket\text{if }{>}3\text{, then 5}\rrbracket^c)}{Pr(\llbracket\text{odd}\rrbracket^c)}$$

Even supposing that $Pr(\llbracket\text{odd}\rrbracket^c \mid \llbracket\text{if greater than 3, then 5}\rrbracket^c) = 1$, the maximum value of $Pr(\llbracket\text{if greater than 3 then 5}\rrbracket^c \mid \llbracket\text{odd}\rrbracket^c)$ is 2/3, since $Pr(\llbracket\text{if greater than 3 then five}\rrbracket^c) = 1/3$ and $Pr(\llbracket\text{odd}\rrbracket^c) = 1/2$.

[32]You might wonder whether we can recreate our Fitelson-inspired proof of PROBABILISTIC TRIVIALITY by appealing only to S-PIE and instances of THE SHIFTY THESIS. Doing so is sufficient to reach the conclusions (L1*) and (L2*):

(L1*) $Pr(\llbracket\text{if p, q}\rrbracket^{c+q^c} | \llbracket\text{q}\rrbracket^c) = 1$

(L2*) $Pr(\llbracket\text{if p, q}\rrbracket^{c+\neg q^c} | \llbracket\text{not-q}\rrbracket^c) = 0$

But these lemmas cannot be substituted into the Law of Total Probability (rather, what's needed is the non-shifty versions, $Pr(\llbracket\text{if p, q}\rrbracket^c|\llbracket\text{q}\rrbracket^c) = 1$ and $Pr(\llbracket\text{if p, q}\rrbracket^c|\llbracket\text{not q}\rrbracket^c) = 0$), and hence no similar result is threatened by (L1*)/(L2*).

[33]The shifty reformulation of modus ponens is closely related to 'dynamic' or 'reasonable inference' versions of modus ponens (see e.g. Stalnaker 1975; Gillies 2004), and more generally to dynamic notions of entailment and validity like those given in Veltman 1996. One may wonder whether we could similarly appeal to dynamic entailment relations to

UNSHIFTY MODUS PONENS: $(\llbracket \text{p} \rrbracket^c \cap \llbracket \ulcorner \text{if p, then q} \urcorner \rrbracket^c) \subseteq \llbracket \text{q} \rrbracket^c$.

SHIFTY MODUS PONENS: $(\llbracket \text{p} \rrbracket^c \cap \llbracket \ulcorner \text{if p then q} \urcorner \rrbracket^c) \subseteq \llbracket \text{q} \rrbracket^{c+p^c}$.

Once again, we believe that the shifty version of modus ponens better tracks the intuitions that have motivated the logical principle than does the unshifty version. One payoff of drawing this distinction is that it allows us to make sense of McGee (1985)'s counterexamples to modus ponens (which are counterexamples to UNSHIFTY MODUS PONENS, but not SHIFTY MODUS PONENS), while still making sense of the intuitions that support modus ponens in the first place. We leave further discussion of this for future work.[34]

These remarks are promissory, but we hope they show that the strategy pursued in this paper will have broad application in clarifying the significance of results in logic for the semantics of expressions of natural language.

Appendix

Fitelson's logical assumptions

Fitelson (2016)'s background assumptions about the logic of $>$, his logical conditional $\rightarrow$, and $\wedge$:

(1) $\models \ulcorner (p \wedge q) \rightarrow p \urcorner$.

(2) $\models \ulcorner (p \wedge q) > q \urcorner$.

(3) If $p \models q$ and $p \models r$, then $p \models \ulcorner q \wedge r \urcorner$.

(4) If $p \models q$ and $q \models p$ then $\models \ulcorner p > r \urcorner$ iff $\models \ulcorner q > r \urcorner$.

avoid the triviality results discussed above, and do so in a way that vindicates ordinary intuitions, providing an alternate resolution to the one we have suggested here. However, we think the answer here is negative. Both of the triviality results we discuss center on a schema which is a theorem schema, not an inference pattern—namely IMPORT EXPORT and its sentential cousin, SENTENTIAL IMPORT EXPORT (and their probabilistic analogues). Static and dynamic notions of entailment differ in very substantial ways when it comes to entailment, but, as far as we can tell, they do not differ in relevant ways when it comes to theoremhood: on the standard static notion, a theorem is a sentence true in every model; on the standard dynamic notion, a theorem is a sentence accepted at every context. We do not see how this difference could help us avoid these results. A key part of predicting this, again, on our view, is positing shiftiness within sentences (i.e., from the if-clause to its consequent). If we shift to a dynamic notion of entailment, however, nothing follows about shiftiness within sentences. Thus we think that, while using a dynamic notion of entailment may play an important role in explaining phenomena in the neighborhood of those we discuss, we do not think it can solve the particular puzzles we discus centering on IMPORT EXPORT and its kin. Thanks to an anonymous referee for this journal for pushing us to discuss this option.

[34] Although we do not argue for this here, we believe that this kind of response has advantages over other existing responses to McGee's counterexamples (cf. Sinnott-Armstrong et al. 1986; Lowe 1987; Over 1987; Lycan 2001; Bennett 2003; Gillies 2004; Weatherson 2009).

(5) If $\models \ulcorner p \to q \urcorner$, then $p \models q$.

(6) If $\models \ulcorner p > q \urcorner$ then $\models \ulcorner p \to q \urcorner$.

(7) $\models \ulcorner p \to (q \to r) \urcorner$ iff $\models \ulcorner (p \wedge q) \to r \urcorner$.

Crucially, (1)-(7) do not imply modus ponens for either conditional (i.e. we have neither $\ulcorner p > q \urcorner \models \ulcorner p \supset q \urcorner$ nor $\ulcorner p \to q \urcorner \models \ulcorner p \supset q \urcorner$ as derived rules).

Gillies validates S-IE

Gillies validates S-IE provided we adopt this semantics for 'and':

$$[\![\ulcorner \text{p and q} \urcorner]\!]^{c,w} = 1 \text{ iff } [\![\text{p}]\!]^{c,w}{=}1 \text{ and } [\![\text{q}]\!]^{c+p,w}{=}1$$

Proof: For arbitrary p, q, r, c, and w:

- $w \in [\![\ulcorner \text{if p, then if q, r} \urcorner]\!]^c$ iff
- $\forall w' \in E(c,w) \cap p^c$: $w' \in [\![\ulcorner \text{if q, r} \urcorner]\!]^{c+p^c}$ iff (by **Gillies**)
- $\forall w' \in E(c,w) \cap p^c : \forall w'' \in E(c+p^c, w') \cap q^{c+p} : w'' \in [\![\text{r}]\!]^{c+p^c+q^{c+p^c}}$ iff (by **Gillies**)
- $\forall w' \in E(c,w) \cap p^c : \forall w'' \in E(c,w') \cap p^c \cap q^{c+p} : w'' \in [\![\text{r}]\!]^{c+p^c+q^{c+p^c}}$ iff

  (definition of $E(c+p)$)
- $\forall w' \in E(c,w) \cap p^c : \forall w'' \in E(c,w) \cap p^c \cap q^{c+p} : w'' \in [\![\text{r}]\!]^{c+p^c+q^{c+p^c}}$ iff

  (since E is closed)
- $\forall w' \in E(c,w) \cap p^c \cap q^{c+p} : w' \in [\![\text{r}]\!]^{c+p^c+q^{c+p^c}}$ iff

  (by vacuous quantification)
- $\forall w' \in E(c,w) \cap [\![\text{p and q}]\!]^c : w' \in [\![\text{r}]\!]^{c+[\![\text{p and q}]\!]^c}$ iff

  (by semantics for 'and')
- $w \in [\![\text{if p and q, then r}]\!]^c$. (by **Gillies**)

Thus **Gillies** validates S-IE (and hence S-LIE and S-PIE, since both follow from S-IE).

Gillies invalidates LIE

Here, we state a formal counterexample to LIE (and hence to IE, which entails LIE), given **Gillies** and the assumption of $>$-unshiftiness (that '$>$' is to be interpreted so that $\ulcorner p > q \urcorner$ expresses the same proposition whether embedded or not):

- Choose p to be atomic; then $[\![\ulcorner$if p and q, then p$\urcorner]\!]^c$ is necessarily true.

  (Proof sketch: this says that for any world w, $E(c,w) \cap p^c \cap q^{c+p^c} \subseteq p^{c+p^c+q^{c+p^c}}$; but since p is atomic, we have $p^c = p^{c+p^c+q^{c+p^c}}$, and thus this says that for any world w, $E(c,w) \cap p^c \cap q^{c+p^c} \subseteq p^c$, which is necessarily true.)

- Consider a context c. Now let s be an expression that denotes, in any context, the proposition expressed by $\ulcorner$if q, then p$\urcorner$ in c, i.e. $\forall c' : [\![\text{s}]\!]^{c'} = [\![\ulcorner \text{if q, then p} \urcorner]\!]^c$.[35]

- Thus, $\ulcorner$if p, then s$\urcorner$ can be faithfully translated into our formal language as $p > (q > p)$.

- However, in some worlds in some models, $[\![\ulcorner$if p, then s$\urcorner]\!]^c$ is false. Here is one such model:

- Let the set of worlds be $\{w_1, w_2\}$, and let $[\![\text{p}]\!]^c = \{w_1\}$, and $[\![\text{q}]\!]^c = \{w_1, w_2\}$.

- Let $E(c,w_1) = E(c,w_2) = \{w_1, w_2\}$. Then:
  - $w_1 \in [\![\ulcorner \text{if p, then s} \urcorner]\!]^c$ iff $\forall w \in E(c,w_1) \cap p^c$: $w \in [\![\text{s}]\!]^{c+p^c}$.
  - Since $E(c,w_1) \cap p^c = \{w_1\}$, we only need to check to see if $w_1 \in [\![\text{s}]\!]^{c+p^c}$
  - By construction, $[\![\text{s}]\!]^{c+p^c} = [\![\ulcorner \text{if q, then p} \urcorner]\!]^c$.
  - Thus, $w_1 \in [\![\text{s}]\!]^{c+p^c}$ iff $w_1 \in [\![\ulcorner \text{if q, then p} \urcorner]\!]^c$ iff $\forall w \in E(c,w_1) \cap q^c$: $w \in [\![\text{p}]\!]^{c+q^c}$.
  - But $E(c,w_1) \cap q^c = \{w_1, w_2\}$, and $[\![\text{p}]\!]^{c+q^c, w_2} = 0$.
  - Therefore, $w_1 \notin [\![\text{s}]\!]^{c+p^c}$, and so $w_1 \notin [\![\ulcorner \text{if p, then s} \urcorner]\!]^c$.

Strict invalidates S-LIE (and thus S-IE)

Suppose **Strict** is true. Then, $[\![\ulcorner$if p and not p, then p$\urcorner]\!]^c$ will be necessarily true, but $[\![\ulcorner$if p, then if not p, p$\urcorner]\!]^c$ not necessarily true. $\ulcorner$if p and not p, then p$\urcorner$ will be true at every world in any model, since quantification will always be trivial, as there are no worlds in any model which make the antecedent true. But not so $\ulcorner$if p, then if not p, then p$\urcorner$. Here is a countermodel:

- The set of worlds is $\{w_1, w_2\}$. Let $[\![\text{p}]\!]^c = \{w_1\}$.

- Let $E(c,w_1) = E(c,w_2) = \{w_1, w_2\}$.

[35] In §5, we appealed to propositional anaphors like 'What X said', targeting X's utterance of a conditional, to play the role of s. This proof does not rely on the assumption that there always will be a sentence which plays this role, just that there sometimes is.

- Among the p-worlds, there is one, w_1, which can access a world (both w_1 and w_2) where ⌜if not p, then p⌝ is false (since both w_1 and w_2 can access $\neg p$-worlds which are not p-worlds, namely w_2). So ⌜if p, then if not p, then p⌝ is false at both worlds.

Lewis's reasoning supporting (L1)/(L2)

Recall (L1):

(L1) $Pr(\ulcorner p > q \urcorner | q) = 1$

Lewis's reasoning to support (L1) relies on assuming that THE THESIS applies to conditionalized probability functions $Pr^{|x}$, as follows. For all x, r, p, q:

1. $Pr^{|x}(r) = Pr(r|x)$ — Definition
2. $Pr^{|x}(\ulcorner p \to q \urcorner) = Pr^{|x}(q|p)$

   THE THESIS holds across a class of probability measures closed under conditionalization
3. $Pr^{|q}(\ulcorner p \to q \urcorner) = Pr(\ulcorner p \to q \urcorner | q)$ — From 1
4. $Pr^{|q}(\ulcorner p \to q \urcorner) = Pr^{|q}(q|p)$ — From 2
5. $Pr^{|q}(q|p) = \dfrac{Pr^{|q}(\ulcorner q \wedge p \urcorner)}{Pr^{|q}(p)}$ — Defn Conditional Probability
6. $\dfrac{Pr^{|q}(\ulcorner q \wedge p \urcorner)}{Pr^{|q}(p)} = \dfrac{Pr(\ulcorner q \wedge p \urcorner | q)}{Pr(p|q)}$ — From 1
7. $\dfrac{Pr(\ulcorner q \wedge p \urcorner | q)}{Pr(p|q)} = \dfrac{\dfrac{Pr(\ulcorner q \wedge p \wedge q \urcorner)}{Pr(q)}}{\dfrac{Pr(\ulcorner p \wedge q \urcorner)}{Pr(q)}}$ — Defn Conditional Probability
8. $\dfrac{\dfrac{Pr(\ulcorner q \wedge p \wedge q \urcorner)}{Pr(q)}}{\dfrac{Pr(\ulcorner p \wedge q \urcorner}{Pr(q)}} = Pr(q | \ulcorner p \wedge q \urcorner)$ — Defn Conditional Probability, Algebra
9. $Pr^{|q}(q|p) = Pr(q | \ulcorner p \wedge q \urcorner)$ — From 5–8
10. $Pr(\ulcorner p \to q \urcorner | q) = Pr(q | \ulcorner p \wedge q \urcorner)$ — From 3, 4, 9
11. $Pr(q | \ulcorner p \wedge q \urcorner) = 1$, assuming $Pr(\ulcorner p \wedge q \urcorner) > 0$ — Probability Calculus
12. $Pr(\ulcorner p \to q \urcorner | q) = 1$, assuming $Pr(\ulcorner p \wedge q \urcorner) > 0$ — From 10,11

The reasoning for (L2) is analogous.

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