Truth, Modality, and Paradox: Critical Review of Scharp, *Replacing Truth*

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Abstract

This paper targets a series of potential issues for the discussion of, and modal resolution to, the alethic paradoxes advanced by Scharp (2013). I proffer four novel extensions of the theory, and detail five issues that the theory faces.

1 Introduction

This essay targets a series of potential issues for the discussion of, and resolution to, the alethic paradoxes advanced by Scharp, in his *Replacing Truth* (2013).

In Section **2**, Scharp's replacement strategy is outlined, and his semantic model is described in detail.

In Section **3**, novel extensions of Scharp's theory to the preface paradox; to the property version of Russell's paradox in the setting of unrestricted quantification; to probabilistic self-reference; and to the sorites paradox are examined.

Section **4** examines six crucial issues for the approach and the semantic model that Scharp proffers. The six issues target the following points of contention:

(i) The status of revenge paradoxes in Scharp's theory;

(ii) Whether a positive theory of validity might be forthcoming on Scharp's approach, given that Scharp expresses sympathy with treatments on which – in virtue of Curry's paradox – validity is not identical with necessary truth-preservation;

(iii) The failure of compositionality in Scharp's Theory of Ascending and Descending Truth (ADT) and whether the theory is not, then, in tension with natural language semantics. The foregoing might be pernicious, given Scharp's use of consistency with natural language semantics as a condition for the success of approaches to the paradoxes. A related issue concerns whether it is sufficient to redress the failure of compositionality by availing of hybrid conditions which satisfy both Ascending and Descending Truth;

(iv) Whether ADT can generalize, in order to account for other philosophical issues that concern indeterminacy;

(v) Whether Descending Truth and Ascending Truth can countenance the manner in which truth interacts with objectivity. It is unclear, e.g., how the theorems unique to each of Descending Truth and Ascending Truth – respectively, T-Elimination and T-Introduction – can capture distinctions between the reality of the propositions mapping to 1 in mathematical inquiry, by contrast to propositions – about humor, e.g. – whose mapping to 1 might be satisfied by more deflationary conditions; and

(vi) Whether the replacement strategy in general and ADT in particular can be circumvented, in virtue of approaches to the alethic paradoxes which endeavor to resolve them by targeting constraints on the contents of propositions and the values that they signify.

Section **5** provides concluding remarks.

2 Scharp's Replacement Theory

Scharp avers that two main alethic principles target the use of the predicate as a device of endorsement and as a device of rejection. When the truth predicate is governed by (T-Out), then it can be deployed in the guise of a device of endorsement. When the truth predicate is governed by (T-In), then it can be deployed in the guise of a device of rejection.

Scharp's theory aims to replace truth with two distinct concepts. His explicit maneuver is to delineate the two, smallest inconsistent subsets of alethic principles; and then to pair one of the subsets with one of the replacement concepts, and the other subset with the second replacement concept.

Thus, one replacement concept will be governed by (T-In) and not by (T-Out); and the second replacement concept will be governed by (T-Out) and not by (T-In).

Scharp refers to one of his two, preferred replacement concepts as *Descending Truth* (henceforth DT). DT is governed by (T-Out).

Scharp refers to the second of his two, preferred replacement concepts as *Ascending Truth* (henceforth AT). AT is governed by (T-In).

In his "Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability" (1963), Montague proved that, for any predicate $H(x)$, the following conditions on the predicate are inconsistent.

Montague's (1963) Lemma 3:

(i) All instances of $H(\phi) \rightarrow \phi$ are theorems.

(ii) All instances of $H[H(\phi) \rightarrow \phi]$ are theorems.

(iii) All instances of $H(\phi)$, where ϕ is a logical axiom, are theorems.

(iv) All instances of $H(\phi \rightarrow \psi) \rightarrow [H(\phi) \rightarrow H(\psi)]$ are theorems.

(v) Q – i.e., Robinson Arithmetic – is a subtheory.[1]

Scharp notes that Montague's conditions target only Predicate-Elimination, and are thus apt for governing DT.

Scharp argues that (v) is necessary, in order for languages that express the theory to refer to their own sentences. Condition (i) is necessary, because it

[1]For contemporary developments of predicate treatments of modality and knowledge, see Halbach et al (2003); Halbach and Welch (2009); Stern (2016); and Koellner (2016; 2018a,b).

captures (T-Out). Condition (ii) is necessary, because denying iterations of DT entrains a version of the revenge paradox.

Thus, either Condition (iii) or Condition (iv) must be rejected. Condition (iii) states that all tautologies are Descending True. Condition (iv) is an instance of closure. In virtue of considerations pertaining to validity (see Section **3**), Scharp is impelled to reject (iv), s.t. DT cannot satisfy closure (151).

2.1 Properties of DT and AT

Scharp argues that the alethic principles, DT and AT, ought to include the following.

DT ought to satisfy:

[$\neg$-Exc: $\mathrm{D}(\neg\phi) \rightarrow \neg\mathrm{D}(\phi)$];

[$\wedge$-Exc: $\mathrm{D}(\phi \wedge \psi) \rightarrow \mathrm{D}(\phi) \wedge \mathrm{D}(\psi)$; and

[$\vee$-Imb: $\mathrm{D}(\phi) \vee \mathrm{D}(\psi) \rightarrow \mathrm{D}(\phi \vee \psi)$.

However, DT is not governed by:

[$\neg$-Imb: $\neg\mathrm{T}(\phi) \rightarrow \mathrm{T}(\neg\phi)$; nor by

[$\vee$-Exc: $\mathrm{T}(\phi \vee \psi) \rightarrow \mathrm{T}(\phi) \vee \mathrm{T}(\psi)$.

AT ought to satisfy:

[$\neg$-Imb: $\neg\mathrm{A}(\phi) \rightarrow \mathrm{A}(\neg\phi)$;

[$\wedge$-Exc: $\mathrm{A}(\phi \wedge \psi) \rightarrow \mathrm{A}(\phi) \wedge \mathrm{A}(\psi)$; and

[$\vee$-Imb: $\mathrm{A}(\phi) \vee \mathrm{A}(\psi) \rightarrow \mathrm{A}(\phi \vee \psi)$.

However, AT is not governed by:

[$\neg$-Exc: $\mathrm{T}(\neg\phi) \rightarrow \neg\mathrm{T}(\phi)$]; nor by

[$\wedge$-Imb: $\mathrm{T}(\phi) \wedge \mathrm{T}(\psi) \rightarrow \mathrm{T}(\phi \wedge \psi)$.

Scharp argues, further:

–that classical tautologies are Descending True;

–that the axioms governing the syntax of the theory are Descending True;

–that the axioms of PA are Descending True, in order to induce self-reference via Gödel-numbering; and

–that the axioms of the theories for both AT and DT are themselves Descending True (152).

DT takes classical values, and, in Scharp's theory, there are no restrictions on the language's expressive resources. This is problematic, because 'a' := '$\neg\mathrm{A}(x)$' and 'd' := '$\neg\mathrm{D}(x)$' can be countenanced in the language, and thereby yield contradictions:

Because $\mathrm{A}(x)$ is governed by (T-In), 'a' entails that A(a), although a states of itself that $\neg\mathrm{A}(x)$. Contradiction.

Because $\mathrm{D}(x)$ is governed by (T-Out), 'd' entails that replacing 'd' for x in 'd' is not a descending truth, i.e., $\neg$D(d)]. So – by condition (ii) – 'D[D(x)] $\rightarrow$ x' entails that it is not a descending truth that replacing 'd' for x in 'd' is not a descending truth [i.e., $\neg$D($\neg$d)].

Thus, Scharp concedes that there must be problematic sentences in the language for his theory, s.t. both the sentences and their negations are Ascending True, and s.t. the sentences and their negations are not Descending True (op. cit.).

Scharp endeavors to block the foregoing, by suggesting that DT can be governed by both unrestricted (T-Out), as well as a restricted version of (T-In). Similarly, AT can be governed by both unrestricted (T-In), as well as a restricted version of (T-Out).

To induce the foregoing, Scharp introduces a 'Safety' predicate, S(x). A sentence ϕ is *safe* if and only if ϕ is either (DT and AT) or not AT.

Thus,

$\mathrm{S}(\phi) \wedge \phi \rightarrow \mathrm{D}(\phi)$; and

$\mathrm{S}(\phi) \wedge \mathrm{A}(\phi) \rightarrow \phi$.

A sentence ϕ is *unsafe* if and only if ϕ is AT and not DT:

$\mathrm{S}(\phi) \iff \mathrm{D}(\phi) \vee \neg\mathrm{A}(\phi)$.

From which it follows that:

$\neg\mathrm{S}(\phi) \iff \neg\mathrm{D}(\phi) \wedge \mathrm{A}(\phi)$, s.t.

$\mathrm{A}(\phi) \rightarrow \neg\mathrm{D}(\phi)$;

$\mathrm{D}(\phi) \rightarrow \mathrm{A}(\phi)$;

$\neg\mathrm{A}(\phi) \rightarrow \neg\mathrm{D}(\phi)$;

$\neg\exists\phi[\mathrm{D}(\phi)\wedge \neg\mathrm{A}(\phi)$ (153).

Scharp avers too that AT and DT are duals. Thus,

$\mathrm{D}(\phi) \iff \neg\mathrm{A}\neg(\phi)$; and

$\mathrm{A}(\phi) \iff \neg\mathrm{D}\neg(\phi)$ (152).

2.2 Scharp's Theory: ADT

Scharp's Theory is referred to as ADT. The necessary principles comprising ADT are as follows (cf. 154):

- Descending Truth

(D1): $\mathrm{D}(\phi) \rightarrow \phi$
(D2): $\mathrm{D}(\neg\phi) \rightarrow \neg\mathrm{D}(\phi)$
(D3): $\mathrm{D}(\phi \wedge \psi) \rightarrow [\mathrm{D}(\phi) \wedge \mathrm{D}(\psi)]$
(D4): $[\mathrm{D}(\phi) \vee \mathrm{D}(\psi)] \rightarrow \mathrm{D}(\phi \vee \psi)$
(D5): If ϕ is a classical tautology, then $\mathrm{D}(\phi)$
(D6): If ϕ is a theorem of PA, then $\mathrm{D}(\phi)$
(D7): If ϕ is an axiom of ADT, then $\mathrm{D}(\phi)$.

- Ascending Truth

(A1): $\phi \rightarrow \mathrm{A}(\phi)$
(A2): $\neg\mathrm{A}(\phi) \rightarrow \mathrm{A}(\neg\phi)$
(A3): $[\mathrm{A}(\phi) \vee \mathrm{A}(\psi)] \rightarrow \mathrm{A}(\phi \vee \psi)$
(A4): $\mathrm{A}(\phi \wedge \psi) \rightarrow [\mathrm{A}(\phi) \wedge \mathrm{A}(\psi)]$
(A5): If ϕ maps to the falsum constant, then $\neg\mathrm{A}(\phi)$
(A6): If ϕ negates an axiom of PA, then $\neg\mathrm{A}(\phi)$

- Transformation Rules

(M1): $\mathrm{D}(\phi) \iff \neg\mathrm{A}\neg(\phi)$
(M2): $\mathrm{S}(\phi) \iff \mathrm{D}(\phi) \lor \neg\mathrm{A}(\phi)$
(M3): $\mathrm{S}(\phi) \land \phi \rightarrow \mathrm{D}(\phi)$
(M4): $\mathrm{A}(\phi) \land \mathrm{S}(\phi) \rightarrow \phi$

(E1): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $\mathrm{D}(\mathrm{p}) \iff \mathrm{D}(\mathrm{q})$

(E2): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $\mathrm{A}(\mathrm{p}) \iff \mathrm{A}(\mathrm{q})$

(E3): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $\mathrm{S}(\mathrm{p}) \iff \mathrm{S}(\mathrm{q})$.

2.3 Semantics for ADT

Scharp advances a combination of relational semantics for a non-normal modal logic, as augmented by a neighborhood semantics. (A modal logic is normal if and only if it includes axiom K and the rule of Necessitation; respectively '$\Box[\phi \rightarrow \psi] \rightarrow [\Box\phi \rightarrow \Box\psi]$' and '$\vdash\phi \rightarrow \vdash\Box\phi$'.) He refers to this as *xeno semantics*.

A model, M, of ADT is a tuple, $\langle$D, W, R, I$\rangle$, where D is a non-empty domain of entities constant across worlds, W denotes the space of worlds, R denotes a relation of accessibility on W, and I is an interpretation-function mapping subsets of D to W. The clauses for defining truth in a world in the model are familiar:

$\langle$M,w$\rangle \Vdash \phi$ iff w$\in V(\phi)$
$\langle$M,w$\rangle \Vdash \neg\phi$ iff it is not the case that $\langle$M,w$\rangle \Vdash \phi$
$\langle$M,w$\rangle \Vdash \phi \land \psi$ iff $\langle$M,w$\rangle \Vdash \phi$ and $\langle$M,w$\rangle \Vdash \psi$
$\langle$M,w$\rangle \Vdash \phi \lor \psi$ iff $\langle$M,w$\rangle \Vdash \phi$ or $\langle$M,w$\rangle \Vdash \psi$
$\langle$M,w$\rangle \Vdash \phi \rightarrow \psi$ iff, if $\langle$M,w$\rangle \Vdash \phi$, then $\langle$M,w$\rangle \Vdash \psi$
$\langle$M,w$\rangle \Vdash \phi \iff \psi$ iff [$\langle$M,w$\rangle \Vdash \phi$ iff $\langle$M,w$\rangle \Vdash \psi$]
$\langle$M,w$\rangle \Vdash \Box\phi$ iff $\forall$w'[If R(w,w'), then $\langle$M,w'$\rangle \Vdash \phi$]
$\langle$M,w$\rangle \Vdash \Diamond\phi$ iff $\exists$w'[R(w,w') and $\langle$M,w'$\rangle \Vdash \phi$]

Scharp augments his relational semantics with a neighborhood semantics. M = $\langle$D, W, R, I$\rangle$ is thus enriched with a neighborhood function, N, which maps sets of subsets of W to each world in W.

Necessity takes then the revised clause:
$\langle$M,w$\rangle \Vdash \Box\phi$ iff $\exists X\in$N(w)$\forall$w'[$\langle$M,w'$\rangle \Vdash \phi \iff$ w'$\in X$]

Possibility takes the revised clause:
$\langle$M,w$\rangle \Vdash \Diamond\phi$ iff $\neg$[$\exists X\in$N(w)$\forall$w'[$\langle$M,w'$\rangle \Vdash \neg\phi \iff$ w'$\in X$]]

Let L be a language with Boolean connectives, and the operators $\Box$, $\Diamond$, and Σ. $\Box$ is the Descending Truth operator. $\Diamond$ is the Ascending Truth operator. Σ is the Safety operator. A *xeno model* M = $\langle$F,R,N,V$\rangle$ where F denotes a xeno frame, R is an accessibility relation on wff in L, N is a function from W to 2^{2^w}, and V is an assignment-function from wff in L to the values [0,1].

Truth in a world is defined inductively as above.

The operators take the following clauses:

Descending Truth:
$\langle$M,w$\rangle \Vdash \Box\phi$ iff $\forall$w'$\in$W[R_ϕ(w,w') $\rightarrow \exists X\in$N(w')$\forall$v$\in$W[$\langle$M,v$\rangle \Vdash \phi \iff$ v$\in X$]

Ascending Truth:

$P(\phi)$ denotes the neighborhood structure – i.e., the set of subsets of worlds – at which ϕ is true.

$\langle M,w\rangle \Vdash \Diamond\phi$ iff $\neg[\forall w'\in W[R_{\neg\phi}(w,w') \rightarrow P(\neg\phi)\in N(w')]$

Safety:

$\langle M,w\rangle \Vdash \Sigma\phi$ iff $\forall w'\in W[R_{\phi}(w,w') \rightarrow P(\phi)\in N(w')] \vee \exists w'\in W[R_{\neg\phi}(w,w') \wedge P(\neg\phi)\neg\in N(w')]$

A reflexive and co-reflexive xeno frame is equivalent to a neighborhood frame:

(Reflexivity) $\forall\phi\forall w\in W[R_{\phi}(w,w)] \wedge$

(Co-reflexivity) $\forall\phi\forall w\in W\forall w'\in W[R_{\phi}(w,w') \rightarrow w = w']$

A sentential xeno frame is *acceptable* iff

(i) $\forall w\in W$ $N(w) \neq \emptyset$

(ii) $\forall w\in W$ $\forall X\in N(w)$ $X \neq \emptyset$

(iii) $\forall w\in W$ $\forall X\in N(w)$ $w\in X$

(iv) $\forall\phi\in L\forall w\in W[R_{\phi}(w,w)]$

(v) if ϕ and ψ are of the same syntactic type, then $R_{\phi} = R_{\psi}$

A constant-domain xeno frame is a tuple, $F = \langle W,N,R_f,D\rangle$. A constant-domain xeno model adds an interpretation-function I to F, s.t. I maps pairs of F and worlds w to subsets of D, s.t. $M = \langle F,R_M,I\rangle$.

A substitution is a function from a set of variables to elements of D. A substitution v' is *x-variant* of v, if v(y) = v'(y) for all variables y.

Thus,

$\langle M,w\rangle \Vdash_v F[(a_1), \ldots, F(a_m)]$, where

a_i is either an individual constant or variable iff $\langle f(a_1), \ldots, f(a_m)\rangle\in I(F,w)$, s.t. if a_i is a variable x_i, then $f(a_i) = v(x_i)$, and if a_i is an individual constant c_i, then $f(a_i) = I(c_i)$

$\langle M,w\rangle \Vdash_v \neg\phi$ iff it is not the case that $\langle M,w\rangle \Vdash_v \phi$

$\langle M,w\rangle \Vdash_v \phi\wedge\psi$ iff $\langle M,w\rangle \Vdash_v \phi$ and $\langle M,w\rangle \Vdash_v \psi$

$\langle M,w\rangle \Vdash_v \phi\vee\psi$ iff $\langle M,w\rangle \Vdash_v \phi$ or $\langle M,w\rangle \Vdash_v \psi$

$\langle M,w\rangle \Vdash_v \phi\rightarrow\psi$ iff, if $\langle M,w\rangle \Vdash_v \phi$ then $\langle M,w\rangle \Vdash_v \psi$

$\langle M,w\rangle \Vdash_v \phi \iff \psi$ iff $[\langle M,w\rangle \Vdash_v \phi$ iff $\langle M,w\rangle \Vdash_v \psi]$

$\langle M,w\rangle \Vdash_v \forall x[\phi(x)]$ iff for all x-variant v' $\langle M,w\rangle \Vdash_{v'} \phi(x)$

$\langle M,w\rangle \Vdash_v \exists x[\phi(x)]$ iff for some x-variant v' $\langle M,w\rangle \Vdash_{v'} \phi(x)$

$\langle M,w\rangle \Vdash \Box\phi$ iff $\forall w'\in W[R_{\phi}(w,w') \rightarrow \exists X\in N(w')\forall v\in W[\langle M,w\rangle \Vdash [\phi \iff v\in X]$

$\langle M,w\rangle \Vdash \Diamond\phi$ iff $\exists w'\in W[R_{\neg\phi}(w,w') \wedge P(\neg\phi)$ is not in $N(w')$.

3 New Extensions of ADT

In his discussion of Priest's (2006) inclosure schema, Scharp disavows of a unified solution to the gamut of paradoxical phenomena (Scharp, 2013: 288). Despite the foregoing, I believe that there are at least four positive extensions of Scharp's theory of Ascending and Descending Truth that he does not discuss, and yet that might merit examination.

3.1 First Extension: The Preface Paradox

The first extension of the theory of ADT might be to the preface paradox. A set of credence functions is *Easwaran-Fitelson-coherent* if and only if (i) the credences are governed by the Kolmogorov axioms; and it is not the case both (ii) that one's credence is dominated by a distinct credence, s.t. the distinct credence is closer to the ideal, vindicated world, while (iii) one's credence is assigned the same value as the remaining credences, s.t. they are tied for closeness (cf. Easwaran and Fitelson, 2015).[2] Rather than eschew consistency in favor of a weaker epistemic norm such as Easwaran-Fitelson coherence, the ADT theorist might argue that consistency can be preserved, because the preface sentence, 'All of the beliefs in my belief set are true, and one of them is false' might be Ascending True rather than Descending True. Because the models in Scharp's replacement theory can preserve consistency in response to the Preface, ADT might, then, provide a compelling alternative to the Easwaran-Fitelson proposal.

3.2 Second Extension: Absolute Generality

A second extension of Scharp's ADT theory might be to a central issue in the philosophy of mathematics; namely unrestricted quantification. A response to the latter might further enable the development of the property versions of AT and DT: i.e., being Ascending-True-of and being Descending-True-of. For example, Fine (2005) and Linnebo (2006) advance a distinction between sets and interpretations, where the latter are properties; and suggest that inconsistency might be avoided via a suitable restriction of the property comprehension scheme.[3] A proponent of Scharp's ADT theory might be able: (i) to adopt the distinction between extensional and intensional groups (sets and properties, respectively); yet (ii) circumvent restriction of the property comprehension scheme, if they argue that the Russell property, R, is Ascending True-of yet not Descending True-of. The foregoing maneuver would parallel Scharp's treatment of the derivation, within ADT, of the Ascending and Descending Liars and their revenge analogues (see Section **2.1** above).

3.3 Third Extension: Probabilistic Self-reference

A third extension of ADT might be to a self-referential paradox in the probabilistic setting. Caie (2013) outlines a puzzle, according to which:

(1) '*' := $\neg$CrT(*) $\geq$ .5

[2]A credence function is here assumed to be a real variable, interpreted as a subjective probability density. The real variable is a function to the [0,1] interval, and is further governed by the Kolmogorov axioms: normality, 'Cr(T) = 1'; non-negativity, 'Cr(ϕ) $\geq$ 0'; finite additivity, 'for disjoint ϕ and ψ, Cr($\phi \cup \psi$) = Cr(ϕ) + Cr(ψ)'; and conditionalization, 'Cr($\phi|\psi$) = Cr($\phi \cap \psi$) / Cr(ψ)'.

[3]See Field (2004; 2008) for a derivation of the Russell property, R, given the 'naive comprehension scheme: $\forall u_1 \ldots u_n \exists y$[Property($y$) $\wedge$ $\forall x$(x instantiates $y \iff \Theta(x, u_1 \ldots u_n)$)]' (2008: 294). R denotes 'does not instantiate itself', i.e. $\forall x[x \in \mathrm{R} \iff \neg(x \in x)]$, s.t. R$\in$R $\iff$ $\neg$(R$\in$R) (2004: 78).

that is, (*) says of itself that it is not the case that an agent has credence in the truth of (*) greater than or equal to .5. As an instance of the T-scheme, (1) yields: ‘T(*) $\iff$ ¬CrT(*) $\geq$ .5’. However, CrT(*) ought to map to the interval between .5 and 1. Then, ‘Cr(ϕ) + Cr(¬ϕ) $\neq$ 1’, violating the normality condition which states that one’s credences ought to sum to 1.

In ADT, the probabilist self-referential paradox might be blocked as follows. Axiom (A2) states that $\neg A(\phi) \rightarrow A(\neg\phi)$; so if it is not an Ascending Truth that ϕ, then it is an Ascending Truth that not ϕ. However, (A2) does not hold for Descending Truth. Thus, in the instance of the T-scheme which states that ‘T(*) $\iff$ ¬CrT(*) $\geq$ .5’, the move from ‘¬CrT(*) $\geq$ .5’ to ‘Cr(¬T(*) $\geq$ .5’ is Ascending True, but not Descending True. So, if the move from ‘¬CrT(*)’ to ‘Cr(¬T(*))’ is not Descending True, then the transition from ‘Cr(ϕ) + ¬Cr(ϕ) = 1’ to ‘Cr(ϕ) + Cr(¬ϕ) = 1’ is not Descending True. Similarly, then, to the status of the Descending Liar in ADT, the derivation of probabilistic incoherence from the probabilist self-referential sentence, (1), is Ascending True, but not Descending True.

3.4 Fourth Extension: The Sorites Paradox

A fourth extension of ADT might, finally, be to the sorites paradox. Scharp’s xeno semantics is non-normal, such that the accessibility relation is governed by the axioms T (reflexivity) and 4 (transitivity), although not by axiom K. Suppose that there is a bounded, phenomenal continuum from orange to red, beginning with a color hue, c_i, and such that – by transitivity – if c_i is orange, then c_{i+1} is orange. The terminal color hue, in the continuum, would thereby be orange and not red. The transitivity of xeno semantics explains the generation of the sorites paradox. However, xeno semantics appears to be perfectly designed in order to block the paradox, as well: The neighborhood function in Scharp’s xeno semantics for ADT is such that one can construct a model according to which transitivity does not hold. Let M_k be a neighborhood model, s.t. W_k = {a,b,c}; N_k(a) = {a,b}; N_k(b) ={a,b,c}; N_k(c) = {b,c}; $V_k(\phi)$ = {a,b}. Thus, $\langle M_k, a\rangle \Vdash \Box\phi$; but not $\langle M_k, b\rangle \Vdash \Box\phi$. So, it is not the case that $\langle M_k, a\rangle \Vdash \Box\Box\phi$; so transitivity does not hold in the model. Scharp’s semantics for his ADT theory would thus appear to have the resources both to generate, and to solve, the sorites paradox.[4]

In the remainder of the essay, I examine six issues for ADT.

4 Issues for ADT

4.1 Issue 1: Revenge Paradoxes

- Descent

[4] Scharp suggests that the truth predicate is contextually invariant, although assessment-sensitive (9.4). A second means by which the proposal could be extended in order to account for vagueness is via its convergence with the interest-relative approach advanced by Fara (2000; 2008).

$\delta \iff$ '$\neg$D(δ) $\vee$ $\neg$S(δ)'
(i) Suppose that D(δ).
Then, D['$\neg$D(δ) $\vee$ $\neg$S(δ)'].
So, [$\neg$D(δ) $\vee$ $\neg$S(δ)], contrary to the supposition.
(ii) Suppose that [$\neg$D(δ) $\vee$ $\neg$S(δ)].
So, D['$\neg$D(δ) $\vee$ $\neg$S(δ)']. So, D(δ), contrary to the supposition.

- Ascent

$\alpha \iff$ '$\neg$A(α) $\vee$ $\neg$S(α)'
(i) Suppose that A(α).
Then, A[$\neg$A(α) $\vee$ $\neg$S(α)]
So, [$\neg$A(α) $\vee$ $\neg$S(α)], contrary to the supposition.
(ii) Suppose that A[$\neg$A(α) $\vee$ $\neg$S(α)]
Then A(α), contrary to the supposition.

Similarly to the response to the alethic paradoxes, Scharp avers that α and δ are unsafe, and so they are Ascending True although not Descending True.

However, Scharp concedes that ADT does not invalidate all unsafe sentences, because some theorems of ADT are not Descending True (cf. 154). Crucially, then, Scharp's response, both to the alethic paradoxes and to the revenge sentences which are generated using only the resources of his own theory, fails to generalize. Because some theorems of ADT are not Descending True, some theorems of ADT are unsafe, and therefore Scharp's proposed restriction to safe predicates in order to avoid paradox serves only, as it were, to temper the flames on one side of the room, while they flare throughout the remainder.

A second maneuver exploits the fact that some unsafe sentences are derivable in ADT.

A sentence, γ, comprising the singleton U$^+$ is *positively unsafe* iff it is derivable in ADT.

A sentence, γ, comprising the singleton U$^-$ is *negatively unsafe* iff it is not derivable in ADT.

$\gamma \iff \neg$D(γ) and γ is not U$^+$.

To show that γ is unsafe, suppose for reductio that D(γ). Then, D[$\neg$D(γ) and γ is not in U$^+$], so $\neg$D(γ) and γ is not in U$^+$. So, $\neg$(γ), contrary to the supposition. So, by reductio, $\neg$D(γ). So γ is unsafe.

Suppose for reductio that $\neg$A(γ). Then, $\neg$A[$\neg$D(γ) and γ is not in U$^+$]. So, $\neg\gamma$, i.e. either D(γ) (by the definition of γ), or D is in U$^+$. If D(γ), then A(γ) (from the definition of Safety). Thus, by reductio, A(γ). Thus, γ is unsafe.

Suppose for reductio that γ is U$^+$, s.t. it is an unsafe theorem of ADT. Some sentences of ADT are not Descending True, e.g. β. So, assume that $\beta \rightarrow \gamma$. So, (a) $\neg$D(γ) and (b) γ isn't U$^+$. Thus, by reductio, γ is not U$^+$. Thus, γ is U$^-$, i.e. unsafe and not derivable from ADT. (In order to make this proof work, Scharp needs to assume (c), i.e. that β is itself not in U$^+$. No argument is advanced for this. In some cases it could so be, and then the proof would be blocked.)

Scharp endeavors to minimize the crucial lacuna in his proposal to the effect that ADT validates sentences that are not Descending True. He argues:

–that a valid argument cannot take one from a D(ϕ) to a $\neg$A(ϕ);

–that – while D(ϕ) can still entail $\neg$S(ϕ) (by the construction of the paradoxes in ADT) – $\neg$S(ϕ) entails A(ϕ);

–that the Descending Liar is unsafe (caveat: the Descending Liar is provable in ADT);

–that the conjunction of the Ascending Liar and its negation is not Ascending True (caveat: the Asending Liar is unsafe, and unsafe sentences are derivable from ADT); and

–that the axioms of ADT are at least Descending True.

4.2 Issue 2: Validity

Scharp mentions Field's (2008) argument against identifying validity with necessary truth-preservation, although does not reconstruct the argument.

In order to argue against identifying validity with necessary truth-preservation, Field draws, inter alia, on Curry's Paradox.

The argument from Curry's Paradox is such that – by (T-In) and (T-Out) – one can derive the following. If ϕ is a false sentence then,

1. $\phi \iff [\mathrm{T}(\phi) \rightarrow \bot]$
2. $\mathrm{T}(\phi) \iff [\mathrm{T}(\phi) \rightarrow \bot]$
3. $\mathrm{T}(\phi) \rightarrow [\mathrm{T}(\phi) \rightarrow \bot]$
4. $[\mathrm{T}(\phi) \wedge \mathrm{T}(\phi)] \rightarrow \bot$ (by importation)
5. $\mathrm{T}(\phi) \rightarrow \bot$
6. $[\mathrm{T}(\phi) \rightarrow \bot] \rightarrow \mathrm{T}(\phi)$
7. $\mathrm{T}(\phi)$
8. $\bot$

So, necessary truth-preservation entails contradiction.

However, the argument need not be valid, if one preserves (T-In) and (T-Out) yet weakens the logic. One can avail of the strong Kleene (1952) valuation scheme, such that $|\phi|$ is ungrounded, i.e. maps to 1/2. One can then add a Determinacy operator, such that it is not determinately true that ϕ and it is not determinately true that not ϕ; so, it is indeterminate whether ϕ.

Field argues, in virtue of the foregoing, that validity ought to be a primitive. In more recent work, Field (2015) argues that validity is primitive if and only if it is 'genuine', such that the notion cannot be identical with either its model-theoretic or proof-theoretic analyses. As an elucidation of the genuine concept, he writes that 'to regard an inference or argument as valid is to accept a constraint on belief [...; s.t.] (in the objective sense of 'shouldn't') we shouldn't fully believe the premises without fully believing the conclusion' (op. cit.). (The primitivist notion is intended to hold, as well, for partial belief.)

Scharp is persuaded by Field's argument, and endorses, in turn, a primitivist notion of validity, as a primitive canon of reasoning without necessary truth-preservation. Scharp takes this to be sufficient for the retention of Condition (iii), in Montague's Lemma (151). Scharp does not provide any further account of the nature of validity in the book. In later sections of the book, he reiterates his sympathy with Field's analysis, and also avails of Kreisel's 'squeezing' argument (section 8.8), to the effect that the primitive notion of validity extensionally coincides with a formal notion of validity (i.e., derivation in a first-order axiomatizable quantified logic with identity). However, one potential issue is that, in a subsequent passage, Scharp writes that: 'an argument whose premises are the members of the set G and whose conclusion is p is valid iff for every point of evaluation e [i.e., index], if all members of G are assigned tM-value [i.e., an AT- or DT-value of] 1 at e, then p is assigned tM-value 1 at e' (240); and this would appear to be a definition of validity as necessary truth-preservation.

The primitivist approach to validity is the primary consideration that Scharp explicitly avails of, when arguing that closure ought to be rejected (Condition iv, in Montague's Lemma), rather than rejecting logical tautologies as candidates for the axioms of ADT (Condition iii, in Montague's Lemma) (151). So, further remarks about the nature of validity would have been welcome. An objection to prescinding from more substantial remarks about the nature of validity might also be that Scharp exploits claims with regard to its uses. So, e.g., he writes that 'a valid argument will never take one from descending truths to something not ascending true' (177). However, that claim is itself neither a consequence of either Kreisel's squeezing argument, nor the primitivist approach to validity.

4.3 Issue 3: Hybrid Principles and Compositionality

- $\wedge$-T-Imb.

  $\mathrm{D}(\phi) \wedge \mathrm{D}(\psi) \to \mathrm{A}(\phi \wedge \psi)$

  v.

  $\mathrm{T}(\phi) \wedge \mathrm{T}(\psi) \to \mathrm{T}(\phi \wedge \psi)$

- $\vee$-T-Exc.

  $\mathrm{D}(\phi \vee \psi) \to \mathrm{A}(\phi) \vee \mathrm{A}(\psi)$

  v.

  $\mathrm{T}(\phi \vee \psi) \to \mathrm{T}(\phi) \vee \mathrm{T}(\psi)$

  (cf. 147, 171)

Feferman's (1984) theory countenances a primitive truth predicate (Feferman-true, in what follows); a primitive falsity predicate; as well as a Determinacy operator (op. cit.). This is by salient contrast to Scharp's approach, on which truth is replaced with DT and AT. Scharp argues that Feferman overemphasizes the significance of the compositionality of his Determinacy operator, at the cost of not having either logical truths or the axioms of his own theory satisfy Feferman-truth. By contrast, Scharp believes that he can avail of hybrid principles, such that it is not a requirement of ADT that Descending Truth and Ascending Truth obey compositionality (157).

One objection to this maneuver is that AT and DT are separated, in the hybrid principles, between the antecedent and consequent of the conditional.[5] So, it is unclear whether Scharp's hybrid principles are sufficient to redress the failure of compositionality in ADT; i.e., there being truth-conditions for sentences whose component semantic values are, alternatively, DT and AT.

A further objection is that the foregoing might be in tension with Scharp's repeated mention of natural-language semantics, in order to argue against competing proposals. If natural-language semantics were to vindicate principles of compositionality, then this would provide a challenge to the empirical adequacy of Scharp's ADT theory, and thereby the viability of his replacement concepts for the traditional alethic predicate.

4.4 Issue 4: ADT and Indeterminacy

This issue concerns whether ADT might generalize, in order to account for other philosophical issues that concern indeterminacy. Whether ADT can be so extended to other issues, such as vagueness and types of indeterminacy, is not a necessary condition on the success of the theory. However, it might be a theoretical virtue of other accounts – e.g., classical, paracomplete, intuitionist, and supervaluational approaches – that they do so generalize; and the extensions of logic and semantics to issues in metaphysics are both familiar and legion.[6]

E.g., McGee (1991) suggests replacing the truth predicate with (i) a vague truth predicate, and (ii) super-truth. The replacement predicates are not intended for deployment in inferences implicated in reasoning, such as conditional proof and arguments by reductio (155). McGee introduces a Definiteness operator, μ, in order to yield the notion of super-truth relative to a set of precisifications. There is thus a truth predicate and a super-truth predicate. Super-truth is governed by (T-In) and (T-Out). Vague truth is governed by neither.

Thus:

–If μ(p), then μ'T(p)'

(If p is definitely true, then 'p is true' is definitely true)

–If $\mu(\neg$p), then $\mu\neg$'T(p)'

(If p is definitely not true, then 'p is true' is definitely not true)

–If p is vague, then 'T(p)' is vague

(vagueness here is secured by availing of the strong Kleene valuation scheme, such that p is ungrounded, i.e. maps neither to true nor false, and rather to .5)

McGee endeavors to avoid Revenge, by arguing that

'u' := 'u is false or u is vague'

collapses to u is vague. So, u is not definitely true, and not definitely vague. Further, u is not derivable within McGee's supervaluationist theory, nor within a separate, fixed-point theory that he also advances.

[5]This objection is owing to Stephen Read.

[6]Cf. Williamson (2017), for an argument for the retention of classical logic despite the semantic paradoxes, based on the abductive strength of its generalization.

Scharp raises several issues for the supervaluational approach. One issue is that vague sentences cannot be precisified via supervaluation – i.e., rendered determinately true – on pain of Revenge (156).

Scharp argues that Descending Truth and Ascending Truth obey (T-Out) and (T-In), respectively, whereas – according to McGee – vague sentences do not. So, McGee's replacement restricts expressivity, whereas Scharp argues that there are no expressive restrictions on his proposal. Scharp notes, as well, that some of the axioms for McGee's theory are not definitely true – and are thus vague and not governed by T-Out or T-In – which would appear to be a considerable objection.

However, Field's (2008) approach – K3 plus a Determinacy operator, with a multi-valued semantics for the conditional – would appear to remain a viable proposal. Extensions of Field's proposal can be to an explanation of vagueness (Field, op. cit: ch. 5); to the logic of doxastic states (cf. Caie, 2012); and to the model-theory of metaphysics.

With regard, e.g., to the extension of Field's treatment of the paradoxes to the logic of doxastic states, Caie demonstrates that – rather than rejecting the Liar sentence – it would no longer be the case, by K3 and indeterminacy, that one could believe the Liar, and it would no longer be the case that one ought not to believe the Liar. In addition to this proposal, I provide in section **5** an epistemicist approach to Curry's paradox which is able to retain both classical logic and the normal truth rules.

With regard to the extension of Field's treatment of the paradoxes to the logic and model-theory of metaphysics, consider the following. Given Curry's paradox, the validity of an epistemic norm might depend, for its explanation, on one's choice of logic. However, one's choice of logic might depend for its explanation on considerations from metaphysics. Suppose, e.g., that one distinguishes between fundamental and derivative metaphysical states of affairs. The fundamental states of affairs might concern the entities located in $3n$-dimensional spacetime, such as whatever is represented by the wavefunction. The derivative states of affairs might concern emergent entities located in lower, 3-dimensional spacetime. In order to capture the priority of the fundamental to the derivative, fundamental states of affairs could take the classical values, [0,1]; by contrast, derivative states of affairs could take the value .5 (in K3+indeterminacy), such that – while fundamental states are always either true or false – it is not determinate that a derivative state of affairs obtains, and it is not determinate that a derivative state of affairs does not obtain.

On the supervaluational treatment of the paradoxes, the approach can more generally be extended in order to account, e.g., for the metaphysical issues surrounding fission cases and indeterminate survival. Approaches which avail of a supervaluational response to fission scenarios, and similar issues at the intersection of nonclassical logic, metaphysical indeterminacy, and decision theory, can be found, e.g., in Williams (2014).

Thus, while it is not a necessary condition on the success of treatments to the alethic paradoxes that their proposals can generalize – in order, e.g, to aid in the resolution of other philosophical issues such as epistemic and metaphysical indeterminacy – there are viable proposals which can be so extended. The competing approaches thus satisfy a theoretical virtue that might ultimately elude ADT.

4.5 Issue 5: Descending Truth, Ascending Truth, and Objectivity

Scharp claims that considerations of space do not permit him to elaborate on the interaction between Descending Truth, Ascending Truth, and objectivity (Section 8.3). Suppose that – depending on the target domain of inquiry – the truth-conditions of sentences might be sensitive to the reality of the objects and properties that the sentences concern. So, e.g., second-order implicit definitions for the cardinals might be true only if the terms therein refer to abstract entities. By contrast, what is said in sentences about humor might be true, if and only if the sentence satisfies deflationary conditions such as the T-schema.

Another objection to the replacement strategy, and of Scharp's candidate replacements in particular, is that it is unclear how – in principle – either Descending Truth or Ascending Truth can be deployed in order to capture the foregoing distinctions.

4.6 Issue 6: Paradox, Sense, and Signification

One final objection concerns the general methodology of the book. Scharp proceeds by endeavoring to summarize all of the extant approaches to the alethic paradoxes in the literature, and to marshall at least one issue adducing against their favor. However, there is at least one approach to the paradoxes that Scharp overlooks. This approach targets the notion of what is said by an utterance, i.e. the properties of sense and signification that a sentence might express. One such proposal is inspired by Bradwardine (c.1320/2010) and pursued by Read (2009). According to the proposal, if a sentence such as the Liar does not wholly signify that it is true, then one invalidates T-Introduction for the sentence. In a similar vein, Rumfitt (2014) argues that paradoxical sentences are a type of Scheingedanken, i.e. mock thoughts that might have a sense, although take no value; so, T-Introduction is similarly restricted.

Scharp takes it to be a virtue of his account that he can retain the disquotational principles, even though they get subsequently divided among his replacement concepts. He might then reply to the foregoing proposal by suggesting that they similarly induce expressive restrictions in a manner that his approach can circumvent.

However, there are other approaches which avail of what I shall refer to as the sense and signification strategy, and which eschew neither T-Elimination nor T-Introduction. Modulo a semantics for the conditional, K3 and indeterminacy at all orders ensures not only that hyper-determinacy – and therefore an assignment of classical values to the paradoxical sentences – is circumvented; but, furthermore, that revenge sentences cannot be derived either. Against this approach, Scharp reiterates his concern with regard to restrictions on expression. He writes, e.g., that 'Field avoids revenge only by an expressive limitation on his language' (107). However, a virtue of the approach is that, as in xeno semantics for ADT, T-Elimination and T-Introduction are preserved. Against ADT theory, K3+indeterminacy does not arbitrarily select the alethic principles that the semantic theory should satisfy. Crucially, moreover, the approach does not say more than one should like it to, as witnessed, e.g., by the derivability in ADT of both the DT and AT Liars and their revenge analogues. Rather, the language of paracompleteness and indeterminacy demonstrates that – without the loss of the foundational principles governing the alethic predicate – there are propositions which can satisfy the values in an abductively robust semantic theory.

5 Concluding Remarks

In this essay, I have outlined Scharp's theory of ADT and its semantics. I then proffered four novel extensions of the theory, and detailed five issues that the theory faces.[7]

[7] Acknowledgements:

References

Beall, JC, and J. Murzi. 2012. Two Flavors of Curry's Paradox. *Journal of Philosophy*, 120.

Bradwardine, T. c.1320/2010. *Insolubilia*, tr. and ed. S. Read. Dallas Medieval Texts and Translations, 10.

Caie, M. 2012. Belief and Indeterminacy. *Philosophical Review*, 121:1.

Caie, M. 2013. Rational Probabilistic Incoherence. *Philosophical Review*, 122:4.

Easwaran, K., and B. Fitelson. 2015. Accuracy, Coherence, and Evidence. In T. Gendler and J. Hawthorne (eds.), *Oxford Studies in Epistemology, Volume 5*. Oxford University Press.

Fara, D.G. 2000. Shifting Sands. *Philosophical Topics*, 28. Originally published under the name 'Delia Graff'.

Fara, D.G. 2008. Profiling Interest Relativity. *Analysis*, 68:4.

Feferman, S. 1984. Toward Useful Type-free Theories, I. *The Journal of Symbolic Logic*, 49:1.

Field, H. 2004. The Consistency of the Naive Theory of Properties. *Philosophical Quarterly*, 54:214.

Field, H. 2008. *Saving Truth from Paradox*. Oxford University Press.

Field, H. 2015. What is Logical Validity? In C. Caret and O. Hjortland (eds.), *Foundations of Logical Consequence*. Oxford University Press.

Fine, K. 2005. Class and Membership. *The Journal of Philosophy*, 102:11.

Gödel, K. 1931. On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I. In Gödel (1986), *Collected Works, Vol. I*, ed. S. Feferman. Oxford University Press.

Linnebo, O. 2006. Sets, Properties, and Unrestricted Quantification. In A. Rayo and G. Uzquiano (eds.), *Absolute Generality*. Oxford University Press.

McGee, V. 1991. *Truth, Vagueness, and Paradox*. Hackett Publishing.

Montague, R. 1963. Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability. *Acta Philosophica Fennica*, 16.

Priest, G. 2006. *In Contradiction*, 2nd ed. Oxford University Press.

Read, S. 2009. Plural Signification and the Liar Paradox. *Philosophical Studies*, 145.

Read, S. 2010. Field's Paradox and Its Medieval Solution. *HIstory and Philosophy of Logic*, 31.

Rumfitt, I. 2014. Truth and Meaning. *Proceedings of the Aristotelian Society Supplementary Volume*, 88.

Scharp, K. 2013. *Replacing Truth*. Oxford University Press.

Williams, J.R.G. 2014. Nonclassical Minds and Indeterminate Survival. *Philosophical Review*, 123:4.

Williamson, T. 1994. *Vagueness*. Routledge.

Williamson, T. 2002. *Knowledge and Its Limits*. Oxford University Press.

Williamson, T. 2017. Semantic Paradoxes and Abductive Methodology. In B. Armour-Garb (ed.), *Reflections on the Liar*. Oxford University Press.