Studies in History and Philosophy of Science 108 (2024) 28–37

Contents lists available at ScienceDirect

Studies in History and Philosophy of Science

journal homepage: www.elsevier.com/locate/shpsa

Mathematics and society reunited: The social aspects of Brouwer's intuitionism

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ARTICLE INFO

Keywords:
Intuitionism
Significs circle
L. E. J. Brouwer
Solipsism
Social history of mathematics
Gerrit Mannoury

ABSTRACT

Brouwer's philosophy of mathematics is usually regarded as an intra-subjective, even solipsistic approach, an approach that also underlies his mathematical intuitionism, as he strived to create a mathematics that develops out of something inner and a-linguistic. Thus, points of connection between Brouwer's mathematical views and his views about and the social world seem improbable and are rarely mentioned in the literature. The current paper aims to challenge and change that. The paper employs a socially oriented prism to examine Brouwer's views on the construction, use, and practice of mathematics. It focuses on Brouwer's views on language, his social interactions, and the importance of group context as they appear in the *significs dialogues*. It does so by exploring the establishment and dissolution of the significs movement, focusing on Gerrit Mannoury's influence and relationship with Brouwer and analyzing several fragments from the significs dialogues while emphasizing the role Brouwer ascribed to groups in forming and sharing new ideas. The paper concludes by raising two questions that challenge common historical and philosophical readings of intuitionism.

1. Introduction

Intuitionism, a mathematical school of thought championed by L.E.J Brouwer, presented an alternate mathematical framework to classical mathematics that viewed mathematical entities not as Platonic objects that exist independently of the human mind nor as formulas written on a piece of paper but as mental constructions created in the mind. Since the creators of mental constructions are human beings situated in time, space, culture, and society, this makes intuitionism an exceptionally interesting case study for understanding how social and communal aspects shape individuals' ability to perform mental constructions of mathematical entities.

While traditional and contemporary discussions of the social construction of mathematics have considered several mathematical and philosophical schools,[1] they have paid far less attention to the way controversial or non-mainstream mathematical schools such as intuitionism addressed mathematical notions. Intuitionism was seriously explored from a socially oriented point of view only once, in Herbert Mehrtens' book *Moderne Sprache Mathematik* (Mehrtens, 1990), thirty years ago. Mehrtens discusses Brouwer's intuitionism in chapter 3.4 of the book, in which he provides a close examination of Brouwer's dissertation, its deleted parts, and Brouwer's views in *Life, Art, and Mysticism* (Brouwer, 1905). However, he nowhere explores Brouwer's engagement with the *Significs Circle* - a social movement that was focused on the connection between language, mathematics, and society in the Netherlands during the 1920s. The current paper aims to begin to fill this gap by highlighting the collective creation of mathematical knowledge as Brouwer addressed it in the *significs dialogues* - documented dialogues between Brouwer and other members of the significs circle – and examine them as telling cases of the social character of Brouwer's mathematics.

By "social character," I refer to the social elements that are expressed by Brouwer himself and affected his interests. Specifically, I focus on three such elements: language, interactions, and groups. As a relatively

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[1] In general, social constructionist treatments of mathematics view mathematical structures and entities as constructed by the social activities of mathematicians. Some study the connection between mathematical developments, political and social contexts, and the individual mathematicians involved in them (Bloor, 1978, 1991; MacKenzie, 1981), while others suggest that it is the subject matter of mathematics itself that is social (Ernest, 1998; Hersh, 1997). Recent discussions about the social construction of mathematics examine social treatments of mathematics alongside different mathematical accounts such as platonism, fictionalism, and modal nominalism. For example, Julian Cole's view of "practice-dependent realism" maintains that mathematical domains are the product of social construction (Cole, 2009, 2013). Jill Dieterle, in turn, has criticized Cole's view for being unable to account for the a-temporality of mathematical existents (Dieterle, 2010), and Shay Allen Logan has introduced an alternative hybrid view of fictionalism and social constructivism (Logan, 2015).

https://doi.org/10.1016/j.shpsa.2024.09.008
Received 6 April 2023; Received in revised form 10 July 2024; Accepted 22 September 2024
Available online 7 October 2024
0039-3681/

unexplored text in the history of intuitionism, the *significs dialogues* offer a unique perspective into Brouwer's social views, and more importantly, his perspective on the role of mathematics in society and how individuals' social interactions affect the use and content of mathematical knowledge. Accordingly, my analysis rests on three pillars: 1. Brouwer's (dualistic) approach to the use of language, 2. Brouwer's social and professional interactions with Gerrit Mannoury, and 3. Brouwer's views on groups and group context.

The idea that social interactions, group context, and language are important elements in Brouwer's intuitionism does not contradict the notion that the mind is the source of knowledge. It does, however, extend it, by suggesting that the process of knowledge creation in one's mind is, to some extent, social. This means that this process is influenced by the groups a mathematician belongs to, the people he interacts with, and the language he uses for such interactions and exchanges of ideas.

The paper is structured as follows: Section 2 outlines Brouwer's twofold perspective on language: as an imprecise tool that cannot be associated with proper mathematical practice on one hand, and as a valuable tool in his academic journey that helped disseminate his ideas and was the focus of his efforts to develop a new language, on the other. Sections 3 and 4 describe the evolution of the significs movement by focusing on the close relationship between Mannoury and Brouwer and its impact on Brouwer's views on language, mathematics, and society. Section 5 points to excerpts from the significs dialogues that demonstrate the importance Brouwer saw in group context as a catalyst for mutual understanding and developing new ideas, and highlights Brouwer's theory of the creating subject as an instance of when Brouwer's views about the role of the community in mathematics paved its path into the subject matter of his intuitionistic mathematics. Building on the social reading of Brouwer's intuitionism proposed so far, Section 6 challenges previous interpretations by raising two questions: one concerning Mehrtens' portrayal of intuitionism as a counter-modernistic approach, and the other regarding the relationship between intuitionism and anti-realism.

2. The dual role of language in Brouwer's life and work

Language is arguably the most important and controversial aspect of Brouwer's intuitionism. Intuitionism fundamentally challenges the traditional role of language in mathematics, as Brouwer believed that language, by its nature, cannot fully capture the essence of mathematical thought. On the other hand, his views on language have significantly changed throughout the years, as he dedicated almost a decade of his life to diagnosing the problems in language and to creating new words and a new language that truthfully reflects true human thought. Brouwer conceived and developed his new linguistic views while being part of the *International Academy for Practical Philosophy and Sociology*, which later turned into the Significs movement. To better understand the seemingly contradictory approaches to language in Brouwer's work and life, in the following sections, I examine Brouwer's motivation to engage with questions concerning the connection between language, mathematics, and society in the first place. To set the stage for this examination, let us start by laying out the dual perspective Brouwer held regarding language.

Brouwer maintained that mathematical objects are created by the human mind through a process of mental construction (Brouwer, 1907, 32–33). From the "move of time" Brouwer says that we initially discern a "two-ity", and the recognition of the two-ity lies at the base of our grasp of numbers. This recognition is the same for every possible mind, and communication between mathematicians is a means to create the same mental process in different minds (Iemhoff, 2020; Shapiro, 2007). However, the communication between mathematicians serves this purpose only since, according to Brouwer, mathematics is a languageless creation of the mind:

> [...] Completely separating mathematics from mathematical language and hence from the phenomena of language described by theoretical logic, recognizing that intuitionistic mathematics is an essentially languageless activity of the mind having its origin in the perception of a move of time (Brouwer, 1981, 4–5).

In Brouwer's view, language is used to exchange mathematical ideas, but mathematical ideas exist independently of whether there exists a language to express and communicate them. For example, language is a means to communicate mathematical truths, but the truth of a mathematical statement derives from mental construction and consists in correspondence with actual constructions:

> *Truth* is only in *reality*, i.e. in the present and past experiences of consciousness. Amongst these are things, qualities of things, emotions, rules (state rules, cooperation rules, game rules) and deeds (material deeds, deeds of thought, mathematical deeds). But expected experiences, and experiences attributed to others are true only as anticipations and hypotheses; in their contents there is no truth. Truths often are conveyed by words or word complexes, generally borrowed from cooperation languages, in such a way that for the subject together with a certain word or word complex always a definite truth is evoked, and that object individuals behave accordingly. (Brouwer, 1948, 488).

It should be noted that Brouwer's linguistic approach does not coincide with other scholars of intuitionism. The most prominent intuitionist advocating a relation of interdependence between mathematics and language is Michael Dummett, who developed a philosophical basis for intuitionism that focused on the concepts of meaning and language. According to Dummett, language is essentially a social activity which transforms and creates knowledge, and the community plays a significant role in constituting and preserving knowledge (Dummett, 1978, pp. 425–28; 1993).[2]

While a similarly strong connection between language and mathematics cannot be traced to Brouwer's intuitionism, it would be misleading to address Brouwer's linguistic views as lacking any social dimension. Brouwer strived to develop a new mathematics, with its fundamental concepts originating from the human mind, free from linguistic dependencies on logical reasoning (Brouwer, 1907). However, as he pursued his mathematical theory, he discovered that language proved to be an invaluable tool in articulating and disseminating his ideas (van Stigt, 1982).

Within the international mathematical community, Brouwer's work was understood and welcomed, as he gained the recognition and respect of distinguished mathematicians such as Felix Klein and Henri Poincaré and received an invitation to join the editorial board of the *Mathematische Annalen*. At his home institution, the University of Amsterdam, his mentor Diederik Korteweg made significant efforts to secure Brouwer a prominent position within the department, eventually stepping down to allow Brouwer to succeed him in 1913. These efforts center on language and communication and would not have been possible without the community's acknowledgment of Brouwer's work and Korteweg's success in persuading the department to include Brouwer in their academic

[2] In a nutshell, Dummett argues that the job of a theory of language is to make an individual's implicit knowledge of how to use the language to be known explicitly. Following Wittgenstein, Dummett prioritizes the common language over the idiolect and builds his theory on Wittgenstein's idea of "meaning is use": to know the meaning of a word is to understand that word and to understand it is to be able to use it correctly (Murphy, 2012). When knowledge is captured in a theory of language use, that use will reflect the conventional practice of a community of language users (Dummett, 1993). In this sense, the community not only reflects the knowledge but also constitutes it and creates new knowledge that cannot be reduced to the mere collection of knowledge of its members. For an elaborated view on the social character of language in Dummett's theory see (Avramides, 2013).

team.

Around that time appears "Intuitionism and Formalism" (Brouwer, 1912), where one can spot a notable shift in Brouwer's views regarding language. Instead of condemning the flaws of language and logic, his critique is now aimed at those who argue that language is the only foundation of mathematical existence and deny that mathematics has a higher existence within the human mind. Brouwer's early solipsism evolves into a new trust in the potential of human communication. He believes that higher, spiritual concepts, including mathematical ideas, can indeed be conveyed through language. However, Brouwer also insists that language must be improved, and perhaps even a new language created, to more accurately mirror human thought processes and the concepts of the human mind (Brouwer, 1946). In Brouwer's perspective, redefining language and creating a more suitable one becomes the primary objective of the Academy and the significs circle.

The mission of the Academy and the significs was twofold: (1) to create new words with "spiritual meaning" for Western languages and make them enter into the mutual understanding of those languages, and (2) to identify and highlight words in major languages that misleadingly suggest spiritual meanings for ideas actually rooted in the desire for material comfort, and by doing so, to purify and correct the goals of democracy towards a universal common good (Brouwer, 1946, 465). Both goals emphasize, both implicitly and explicitly, the significance of a community of interacting individuals who use language to share mutual understandings, in order to create better political and social lives for themselves.

These two goals are an inseparable part of Brouwer's intuitionistic program. As Walter van Stigt claims, for Brouwer, "intuitionism and significs are almost identified" (van Stigt, 1982, 509). Brouwer refers to the significs movement as "intuitive significs," stating that "the Intuitive Significs movement concerns itself with the creation of new words forming a new code of elementary means of communication" (Brouwer, 1918). His broader vision of significs reform includes political influence and builds on collective forms of communication and scholars' ability to cooperate and think together as a group.

These social elements—collective communication and group thinking—were central to the significs agenda as Brouwer envisioned it. As such, their impact on Brouwer might extend beyond the conventional narrative of solitude and solipsism towards a more socially oriented narrative. Unlike van Stigt, who described Brouwer's relationship with language as a "double U turn," marking the year 1928 as the "end of Brouwer's creative life and temporary trust in the human ability to communicate" (van Stigt 1982, 512), I would like to offer a different perspective.

First, I wish to show that Brouwer's views on language are part of a broader perspective on the social elements in his intuitionism, alongside his views on group context as a catalyst for mutual understanding and the impact of social interactions with members of the significs circle on his thought. Second, I wish to show that Brouwer's views on the communal character of language and the significance of a community in creating new ideas did not disappear in 1928 but were still evident in his work on the creating subject in 1948. The next two sections are dedicated to supporting my first goal. Specifically, they explore how Brouwer's social interactions and the people with whom he had close relationships have led to his engagement with the significs theory and have shaped his views on language, social groups, and the practice of mathematics.

3. The significs – from Lady Welby to the significs circle

The theory of significs is an analysis of communicative acts starting from the ideas laid down by Victoria Lady Welby, a self-educated philosopher of language. In her paper "Sense, Meaning and Interpretation" (Welby, 1896), Welby coined the term "Significs" for her theory of meaning and special approach to studies on signs, language, and communication.[3] Her approach concentrated on signs used by a speaker or author and the interpretation of such signs by a listener or a reader where both perspectives were given equal importance, thereby distancing herself from other philological-historical approaches to meaning (Petrilli, 2009). One of her leading assumptions was that an advanced sign theory could improve interpersonal communication processes, and she was convinced that a deep understanding of such processes would eventually prevent or solve most social problems (Schmitz 1990).

Welby's research gave rise to the Significs Movement in the Netherlands. In 1908, the poet, psychiatrist, and social reformer Frederik Willem van Eeden published the first significs study in the Netherlands (van Eeden, 1908), but his study did not specifically cite or mention significs or its founder, Lady Welby. A few years later, in 1912, the Dutch poet and lawyer Jacob Israel de Haan was the first to introduce Lady Welby's communication-oriented theory of signs in his article "New Philosophy of Legal Language" (de Haan, 1912). During World War I, a group of Dutch scholars became interested in significs research (each due to his own very different reasons), and in 1917 they founded the "International Academy for Practical Philosophy and Sociology" (to use Brouwer's words (Brouwer, 1946, 465)) in Amsterdam. The Academy director and chairman was the mathematician and philosopher Gerrit Mannoury, one of Brouwer's teachers and mentors; de Haan was the secretary, and the other participating members besides Brouwer were van Eeden, Henri Borel, L. S. Ornstein, and H. P. J. Bloemers. One of the primary goals of the significs movement and the new Academy, as Brouwer saw it, was to better understand how individuals' ideas can find their way into the commons and thus magnify their social impact. As Brouwer himself quotes from the prospectus of the significs movement:

> The undersigned are well aware of the fact, that such a task as assigned by them to the International Academy for Practical Philosophy and Sociology, has been taken up several times by philosophers individually. However they are convinced that precisely in consequence of the individual character of the work of those philosophers their words could be efficient only for memorizing the expressed thoughts in the minds of the writer and his isolated readers, but never could find a place in the mutual understanding of the multitude and therefore had only a slight social influence.
>
> They hold the opinion that when the same task could be undertaken in common by a group of independent thinkers with subtle and pure human feeling, their thoughts *formed in the mutual understanding of their circle*, would necessarily find a corresponding language, *allowing them to enter into the mutual understanding of the multitude*.
>
> Finally, as regards the realization of the proposals of the Academy, it must he kept in mind, that a thought, in its quality of embryonic deed, has a far greater possibility of development when it is the common intimate conviction of a group of human beings, than in the case of its belonging to one individual only, however courageous that individual may be and however numerous the company of half understanding followers who surround him. (Brouwer, 1946, 466)

[3] Victoria Lady Welby's contribution is interdisciplinary and widespread, and she presented ground-breaking ideas spanning different fields of study. Her significs is at the origin of important 20th-century philosophical trends, including linguistic philosophy, ethics, pragmatism, and modern semiotics (Petrilli & Ponzio, 2005; Petrilli & Sebeok, 2001). It has been suggested that Welby's research on cultural and linguistic philosophy is related to the intellectual environment in which the Vienna Circle was functioning (Pietarinen, 2009). Beyond that, she introduced the concept of translation into the study of signs and meaning, anticipating developments in translation studies in the 20th and 21st centuries, and her innovative concepts such as "mother sense" and "father sense" translate to what we recognize today as "women's studies."

Brouwer's specific choice of words, especially in the second paragraph where he highlights that scientists' thoughts are "formed in the mutual understanding" of their groups, suggests that, at the very least, Brouwer noticed a difference between individualistic work and collective work, acknowledging the benefits of the latter. When a group of individuals forms an idea, it enables them to enter the "mutual understanding" of the commons in a much more effective way than individualistic thinking does. Put differently, since thoughts are shaped by their thinker (or thinkers), group thinking makes thoughts sharable because they are the product of a collective mind. According to the quoted paragraphs above, an idea can evolve best and prosper when cultivated in a group mode rather than in the mind of one individual. Hence the group, community, or collective holds a key to creating and spreading knowledge. This is a very atypical statement for someone like Brouwer, whose philosophical views were often charged with solipsism (Blum, 2005). I return to discuss this issue in section 5.

The International Academy did not achieve its social and reformative goals, mainly due to financial issues and the divergent positions each member held. Nevertheless, the short-lived Dutch Academy was considered by significs scholars as a major milestone in the development of Lady Welby's theory of signs, and led to a cooperation of scientists from widely varying disciplines for the purpose of joint significs research (Schmitz 1990, p. 222).

In May 1922, after de Haan left for Palestine and the linguist and theologian Jacques van Ginneken had joined the group, the group changed its name to the "Significs Circle" ("Signifische Kring"). With the establishment of the *Significs Circle,* the group had redefined its goals, which now went beyond criticism and synthesis of language. It aimed to gain a better understanding of "the connections between words and the needs and tendencies of the soul," thereby affecting "the future social and mental conditions of man" (Brouwer, 1946, 468). The circle met regularly for several years, but due to van Eeden's deteriorating health condition and the retirement of van Ginneken, the circle dissolved in 1926. While de Haan and van Eeden brought significs into the Netherlands, Gerrit Mannoury was the driving force behind the International Academy and the significs circle. To understand Brouwer's engagement with the significs circle, we must take a brief detour to explore the relationship between Brouwer and Mannoury.

4. On the importance of social interactions: Gerrit Mannoury

Among scholars of Brouwer's intuitionism, Gerrit Mannoury is considered one of the most influential people in the evolution of Brouwer's philosophical and mathematical ideas (van Atten, 2020; van Dalen, 2013; Hesseling, 2003; van Stigt, 1990). According to Dirk van Dalen, Mannoury "meant more to him and his work in foundations than any other man" (van Dalen, 1978, 303). Brouwer himself described Mannoury's profound impact on him in a formal address he delivered in 1946, at the occasion of awarding Mannoury a honorary doctorate:

> […] I began to attend the meetings of the Amsterdam Mathematical Society. There I saw a man apparently not much older than myself, who after lectures of the most diverse character debated with unselfconscious mastery and well-nigh playful repartee, sometimes elucidating the subject concerned in such a special way of his own that straight away I was captivated. I had the sensation that, for his mathematical thinking, this man had access to sources still concealed to me or had a deeper consciousness of the significance of mathematical thought than the majority of mathematicians. At first I only met him casually, but I at least knew his tuneful name, which guided me to some papers he had recently published […] They had the same easy and sparkling style which was characteristic of his speech, and, when I had succeeded, not without difficulty, in understanding them, an unknown mood of joyful satisfaction possessed me, gradually passing into the realization that mathematics had acquired a new character for me. For the undertone of Mannoury's argument had not whispered: "Behold, some new acquisitions for our museum of immovable truths", but something like this: "Look what I have built for you out of the structural elements of our thinking" […] (Brouwer, 1946, 192–93)

Mannoury's lectures revealed to Brouwer that mathematical elements are created by construction and that mathematical truths are not simply out there, existing independently of human beings, but rather a human creation, a view that was in accordance with Brouwer's pre-existing philosophical ideas. The relationship between the teacher and the student soon converted into a "dialectic partnership," making Mannoury one of the few friends Brouwer had throughout his life. The strong friendship and the continued dialectics between the two led to discussions about their mutual interest in language, philosophy, symbolic logic, conceptual criticism, and of course – significs (Kirkels, 2013).

Gerrit Mannoury was a self-taught mathematician, philosopher, psychoanalyst, and political activist.[4] He worked as a school teacher while pursuing a mathematics degree at the University of Amsterdam, but he never completed his studies or received a formal degree (van Dalen, 2013). Diederik Korteweg, a mathematics professor at the University of Amsterdam (and official supervisor of Brouwer's doctorate), noticed the talented young man and tutored him for a while. The private lessons led to Mannoury's appointment in 1903 as a *privaat docent* (unpaid lecturer) in logical foundations of mathematics at the University of Amsterdam and later to become a professor at the same university, succeeding Korteweg (van Atten, 2020). In 1917 Mannoury became a professor *extraordinarius*, and in the same year, he delivered his inaugural speech entitled 'The Social Significance of the Mathematical Form of Thought.'

In the speech, Mannoury claims that mathematics is a practical science and describes the task of the mathematician as a task of great social value: to free human thought from the dogmas it has given itself up to, such as determinism or "scientific" reason (Meertens, 1956). Real scientific progress can only be achieved by setting ourselves free from these dogmas and acknowledging that even in mathematics, "rock-solid convictions" are changeable (Mannoury, 1917). Such changes can be hard to accommodate at first, but eventually, they are the forces that move the discipline forward and advance science. To demonstrate his argument, he gives the example of "pan geometry" (non-Euclidian geometry):

> The oldest rights, but also the most limited ones, belong to the so-called pan geometry, whose genesis runs almost from Lobatchefsky to Riemann, and thus came to a certain conclusion about half a century ago […] which had remained untouched since Euclid's days: the axiom of parallels and the three-dimensionality of space. And if we try to summarize the results of these investigations, we say to them that they have taught us that a geometry which departs from these fundamental principles bears no contradiction in itself, and that, therefore, the rock-solid conviction which it always had cherished with regard to the necessity of these basic principles, at any rate does not lie in the field of mathematics. Of course this did not mean that this necessity itself does not exist, but the thinking person nevertheless had learned a precious lesson in modesty: propositions that for centuries had been taken for granted, the denial of which had been called impossible and unreasonable, […] turned out to require support for their validity […] Precious lesson indeed! How long had the "axioms" and the "postulates" been revered and respected as sacred inviolability in all fields of human knowledge and belief, how

[4] It should be noted that this section does not aim to provide a comprehensive overview of Mannoury's biography; for an extensive biographical account, see (Kirkels, 2019). The following paragraphs merely present a brief and incomplete sketch of Mannoury's scientific and personal background, mainly focusing on his relationship with Brouwer.

long had it been thought that the human spirit of inquiry had to respect certain limits, under pain of being plunged into an abyss of bewilderment, and lo[ok], there one of those unassailable prayers, upon closer examination, proved to stand only on a clay foot, and the perilous limit could be crossed with impunity at least at one point. […] this experience was bound to deal a serious blow to the authority which had hitherto been accorded to other more far-reaching "fundamental truths"; and it seems to me that between this advance in geometry and the so much more far-reaching discoveries in that infinite point-sets must have existed if not any direct, at least some causal relationship. (Mannoury, 1917, 18–19)

The "blow to the authority" was, to some extent, a blow to the dogma of how mathematics and geometry should be properly done (Meertens, 1956, p. 455). Mannoury's specific choice of words is not coincidental; it fits the general atmosphere in the mathematical community during the first decades of the 20th century. The beginning of the 20th century was a period of uncertainty and "blows to the authority" that started with the publication of Russell's antinomies in set theory and continued in the culmination of the foundational crisis of mathematics in the 1920s - the *Grundlagenkrise* (Ferreirós, 2008; Hesseling, 2003). The debate about the proper foundations of mathematics was mainly between three approaches: logicism, formalism (whose leading representative was Hilbert) and intuitionism, advocated by Brouwer. While in the late 1920s, the tone of the debate abated somewhat, the conflict between Brouwer and Hilbert turned personal and eventually resulted in Brouwer's dismissal from the *Mathematische Annalen.*[5]

Interpersonal conflicts and political turbulence were two fundamental human-centered problems Mannoury wished to address in his work. He placed considerable emphasis on the role of language in comprehending human behavior. His interest in the theory of significs derived from his belief that mutual understanding is the key to a better community. As Mannoury saw it, misunderstandings lead to the inability to communicate with each other, which leads to ethical problems, political conflicts, and scientific disagreements. To attend to the problem of misunderstandings, Mannoury intended to improve the mechanism he thought was responsible for it: language.

Brouwer shared Mannoury's views of language as the source of misunderstandings. Brouwer restricted the role of mathematical language to be only "an instrument for keeping in memory mathematical constructions or for suggesting them to other people" since "in spite of its efficiency [mathematical language] can never completely safeguard us against misunderstanding" (Brouwer, 1947, 477). Yet unlike Mannoury, Brouwer did not regard language as important or significant for practicing mathematics (van Dalen, 1999), and for him, mathematics was a "mental construction essentially independent of language" (Brouwer, 1947, 477). Hence, his participation in the significs circle raised some eyebrows, and, to quote Dirk van Dalen, "it is a small miracle that a man who had such a low opinion of language and communication joined this circle" (van Dalen 1978, 303).

However, Brouwer's participation in the significs circle can be seen as somewhat less surprising once shifting the focus to his social interactions. Brouwer's close friendship with Mannoury and other significs members such as Frederik van Eeden,[6] highlight the importance of social interactions in narratives about the development of intuitionism. Interactions among individuals or between an individual and the groups he belongs to are sometimes overlooked in historical narratives, mainly because such interactions are basic and common elements of social life. Precisely because interactions exist everywhere, we should not ignore them when we explore the trajectory of mathematical theories. The people a mathematician talks to, the groups he is part of, and the institutions where he conducts his research shape his thinking process in explicit and implicit ways. Social interactions play a critical role in understanding how ideas evolve, specifically for a recluse person like Brouwer.

The close mentorship-turned-friendship between Mannoury and Brouwer led to Brouwer's engagement with language-centered ideas, such as the role of language in life and the difference between everyday language and scientific language. This engagement would not have occurred without this kind of social interaction, starting from a professional interaction and moving to a personal one. As a mentor and a friend, Mannoury's continuous interest in language and communication has affected Brouwer's views on these topics, but Brouwer's interest in the connection between mathematics, language, and society was genuine. Mannoury's influence brought Brouwer to the significs circle in the first place, but his active participation and the documented dialogues between him and other circle members imply that Brouwer had formed additional social interactions with other group members. These interactions contributed to him giving these topics serious consideration and feeling that they were important enough to discuss with linguists and philosophers.

Through conversations, deliberations, and exchange of ideas, the interactions between Brouwer, Mannoury, van Eeden, and other members of the Significs circle have shaped the development of Brouwer's thought. All these social elements – conversations, dialogues, idea exchanges, interactions – are practiced using the tool of language *and* within a group context (involving two or more people). Building on Brouwer's own conviction that people understand each other better when they come together to form an idea rather than forming it apart (Brouwer, 1946, 466), it seems that Brouwer acknowledged the importance of group context for understanding and promoting new ideas. In the following section I trace this line of thought to several fragments from the significs dialogues, demonstrating that for Brouwer himself, group thinking and interactions between people and their communities are critical for practicing mathematics.

5. The social character of Brouwer's intuitionism

5.1. Group context and mathematical truth

The fragments from the significs dialogues I refer to below derive from three main sources. The first is a book version of the dialogues titled "Signifische dialogen" published in 1939 (Brouwer et al., 1939).

[5] The *Mathematische Annalen* incident is an interesting moment in the history of intuitionism, on which van Dalen elaborates in his paper "The war of the frogs and the mice" (van Dalen 1990). The *Annalen* was the most prestigious mathematics journal at the time, and being one of its editors was considered the highest recognition for a mathematician. Brouwer was part of the *Mathematische Annalen*'s editorial board (one of its chief editors being Hilbert) from 1914, but despite his diligent editorial work, in 1928 Hilbert sent a request to the other chief editors to remove Brouwer due to personal conflicts and mathematical differences. The editorial board (comprised of Blumenthal, Carathéodory, and Einstein) did not comply at first, but after an extensive exchange of letters between all parties, including the publisher Ferdinand Springer and his legal advisors, the editorial board was dissolved and immediately reassembled without Brouwer. The incident had a tremendous effect on Brouwer, who, from that point onwards, refrained from publishing in the *Annalen* and convinced his student, Arend Heyting, to do the same. It has been claimed that 1928 marks the beginning of the end for Brouwer's intuitionistic program as he never regained the energy to promote intuitionism as he did before (Posy, 1998).

[6] Van Eeden and Brouwer had a close friendship, dating back to 1915. Van Eeden recognized Brouwer as a man of exceptional intellect who could provide the intellectual strength necessary for realizing his social and significs ideals. Brouwer was flattered by the admiration of the renowned poet and shared van Eeden's ambition for a prominent role in a campaign for social reform, as van Eeden believed that the well-being of society and the conditions of the poor could be improved through the positive influence of wise individuals like themselves (van Stigt 1982).

Parts of it already appeared as articles in the journal *Synthese* in 1937 under the same title, written by Brouwer, Mannoury, van Eeden and van Ginneken (Brouwer et al., 1937). These two sources are in Dutch; an English translation of some parts was published in Brouwer's *Collected Works* in 1975, edited and translated by Arend Heyting (Brouwer, 1975).

The English translation of the dialogues begins with a statement of the circle's basic program on its foundation date made by all four members, followed by Brouwer's statement on the purpose of the movement, and then a discussion on the premises of a group ("Groep-spraemissen") where two out of four of Brouwer's statements are translated. Following that appears the discussion on the formalistic method in significs ("De Formalistische Methode In De Signifika"). The English translation lacks the first comments made by Mannoury, van Eeden, and van Ginneken, as well as the first sentence of Brouwer's comment, where he claims that Mannoury goes way too far when he says that mathematical language is "dead" language, which can become "alive" by losing its mathematical character (Brouwer et al., 1937, p. 42, my translation). And then Brouwer continues to exchange the following words with Mannoury:

> *Brouwer*: True mathematics never lacks significance, because it is never without a social cause. Man came to think mathematically because only by applying this method of thinking was he able to prevail in the struggle for life which became more and more complicated and difficult. And in its turn this method of thinking gave rise to the need for a corresponding language which carries a will-impulse not word for word but only after longer and more complicated periods.
>
> *Mannoury*: Yet mathematics is cultivated to a large extent as 'l'art pour l'art' and then the formal structure does grow quite independent of social aims?
>
> *Brouwer*: I am not so sure of that. When mathematical calculations are made simply by way of a game or a recreation, or as long as they serve only as a subject of instruction, it is difficult to point out a direct connection with social causes. But this does not prove that such a connection does not exist at all. For that matter I have not meant to say that any isolated mathematical formula represents a 'jump from the end to the means' but only that mathematical thought, or rather mathematical sentiment, is a sort of network connecting the data which are necessary or valuable for our life […] Still I feel that also a formula, and perhaps even a calculating-rule, as long as it is manipulated, really has an asserting character in many more cases than would follow from Mannoury's ideas, and I think even that the formula borrows its only importance from that asserting character (Brouwer, 1975, 450–51).

This is an interesting and undiscussed quote of Brouwer, implying that Brouwer did not think that mathematics is practiced independently of social circumstances, in the sense that it depends on other individuals to manipulate it. According to Brouwer, the significance of a mathematical formula derives from its "asserting character," which is constituted by the formula's being manipulated or operated by members of the community (Brouwer, 1975, 451; Placek, 1999). This quotation refers back to Brouwer's remarks on group thinking in the prospectus of the significs movement discussed in section 3, where he explicitly emphasizes the importance of groups or communities in fostering mutual understanding among individuals (Brouwer, 1946, 466). By integrating these two quotations, we gain a clearer understanding of the significance Brouwer attributed to interactions between individuals and groups within a community: it is through these social interactions within a group structure that mathematical formulas become meaningful and are utilized by the community.

To illustrate the idea of a formula's communal character and the connection between language, truth, and social context, Brouwer gives the example of a bookkeeper at a bank:

> For the clerks who enter the items in the books, these items have no sense or importance whatever; they do their job to gain a living, but the fact that the board of the bank pays them for book-keeping is accounted for by the interests that are served by the transactions. The banker sends an important telegram: this is living, asserting language; the codewords in which it is written form what I call a sequence of entities which represents another sequence of perceptions and unattained aims, in other words they are as many links between the ultimate aims and the nearest means. If now the content of that telegram is reproduced in the bookkeeping, it does not lose that character, but it becomes only more difficult to perceive the connection between means and aims: the sequence of entities is transformed. For the bookkeeper this connection is then dissolved into a general feeling that he has conscientiously performed his duty, a feeling that is positively akin to the sentiment that constitutes for the mathematician the notion of 'truth'. (Brouwer, 1975, 451)

The "codewords" in the banker's telegram represent what Brouwer calls "asserting language" in the sense that these words carry connections to other aims and means of the bank. When these words are reproduced in different circumstances, tracing the connections becomes more challenging since the context of these words has changed. The analogy to mathematics is that mathematical expressions and formulas are part of our everyday language, and individuals acquire them in certain social circumstances. The process of acquisition and the social context differs between individuals, thereby affecting the way individuals use and manipulate these formulas.

The idea that group context affects the way individuals can convey, understand, and exchange ideas with members from other groups[7] is explicitly acknowledged by Brouwer:

> […] I see two kinds of premisses (which facilitate mutual understanding within the group, but hamper it between groups): partly they consist in certain judgements that are endorsed or accepted by every member of the group (in most cases tacitly) and partly they consist only in the paths of associations, which are analogously (of course not identically) interlaced for different members of the same group. One might speak of nets of associations which confer their particular character to the different special languages of groups and races, whilst however the structure of these nets is never mentioned. I feel this for instance quite clearly when I think of the difference between Eastern and Western languages and when I try to understand why it is so difficult for the average Oriental and the average modern European to convey their thoughts to each other, no matter how well they know each other's language-in-the-narrow-sense. (Brouwer, 1975, 449)

According to Brouwer, if two individuals do not belong to the same group and share the same group context, it will be difficult for them to engage and understand each other's ideas properly, even if language is not a barrier.[8] Hence the "group context" is more than the language spoken between group members; it is also the norms, standards, ideas, and judgments that are endorsed or accepted by the group. Moreover,

[7] The difference between conveying and exchanging an idea lies in the in the direction and interaction involved in the communication process. Conveying an idea is typically one-way communication process while exchanging an idea involves a two-way interaction that involves dialogue, discussion, and mutual sharing. Hence, conveying an idea is about transmitting information from one person to another, while exchanging an idea involves a reciprocal process where multiple parties share and discuss their thoughts and concepts. Both conveying and exchanging ideas are processes that occur *between* individuals, thus require the need of a group to happened.

[8] The notion that a shared group context is essential for effective communication is also articulated in Brouwer's earlier writings. In *Life, Art, Mysticism*, he contends that "language can only be the accompaniment of an already existing mutual understanding" (Brouwer, 1905, 401).

the "paths of associations" differ between individuals even if they are from the same group since Brouwer mentions that these paths or nets are similar but not identical. This suggests that there are differences among individuals' contexts, which can affect the way individuals acquire mathematical expressions and formulas.

Philosopher Tomasz Placek presents an interestingly relevant analysis of these passages. According to Placek, even though "mathematical constructions are assumed to be mental, there is no essential difference between a way mathematical vocabulary and other expressions are learned," (Placek, 1999, pp. 99–100) in the sense that both are learned within societal contexts. Such an analysis suggests that Brouwer's views of language do not coincide with the doctrine of private language and that for Brouwer, the ability to communicate in mathematics rests on "invoking appropriate mental constructions in the minds of other people" (Placek, 1999, p. 100). This analysis rephrases the problem of language by reinstating a communal aspect as one of its basic assumptions. The inability of words to evoke the same constructions in other minds is not because language is private or inaccurate, but due to the fact that language is a network of intentions and links that transform upon transmission. This happens because every expression, including mathematical expressions, is learned, understood, and practiced within a social context.

A similar approach can be seen in the rest of Brouwer's argument concerning the idea of mathematical truth:

> For it is also in this respect that I differ fundamentally from Mannoury's conception, that mathematics, when it is made less formal, will pay for it by a loss of 'exactness', i.e. of mathematical 'truth'. For me 'truth' is a general emotional phenomenon, which by way of 'Begleiterscheinung' [accompanying phenomenon] can be coupled or not with the formalistic study of mathematics. And therefore I do not recognize as true, hence as mathematics, everything that can be written down in symbols according to certain rules, and conversely I can conceive mathematical truth which can never be fixed down in any system of formulas. Again just as in the administration of the bank: On the one hand it is quite possible to falsify the books though scrupulously heeding all the rules of the art of book-keeping, on the other hand it is impossible to enter into the books all the cons iderations of the banker and all the other factors that influence the financial power of the bank. (Brouwer, 1975, 451–2).

Just as mathematical formulas cannot be reduced to linguistic symbols without losing some of their essence, so does mathematical truth. The essence of mathematical expressions and formulas is rooted in the accompanying elements of the entity, such as the social context in which it was used or learned. The same is true for mathematical truth: the phenomenon of truth is too general to be accurately articulated in linguistic symbols since, resorting again to the illustration of the banker, it is impossible to account for all the considerations and all the other factors that influence the concept of mathematical truth.

In other words, Brouwer's views on the practice of mathematics, as reflected in the significs dialogues, emphasize that (1) mathematical truth is influenced by external factors, some of which are related to the social context and environment of the individual, and (2) the 'asserting character' of mathematical formulas is shaped by external manipulation occurring in various social contexts. The quotes from the significs dialogues highlight a communal trend in Brouwer's views—a trend that sharply contradicts his solipsistic perspectives. This trend, however, extends beyond the significs dialogues and into the core of Brouwer's intuitionism. A critical point where this trend emerges is in the concept of the creating subject.

5.2. *The creating subject and solipsism*

The creating subject is an idea Brouwer introduced in 1948,[9] and it has been argued that it is the closest thing to a psychological interpretation of mathematics (Brouwer, 1948; van Atten, 2017). The creating subject is an idealized subject, who can, by itself, do whatever mathematics can in principle be done, and whose activity is structured as an ω-sequence (finite sequence of the ordinal ω). It is ideal in the sense that is does not have time or space limitation, nor does it make mistakes. It has the ability to look back at its earlier activity (to reflect), and to project an initial segment of an ω-sequence (such as the natural numbers) onto its earlier acts. Otherwise stated, the creating subject does not forget anything, and it basically holds the accumulation of knowledge that exists. The creating subject can generate choice sequences, which are some of the most important and complicated mathematical entities of Brouwer's intuitionism. This might be the motivation behind Brouwer's introduction of the creating subject, which is a natural successor of Brouwer's counter-examples and yields much stronger results (van Atten, 2018).

Brouwer's theory of the creating subject has been a rich area of exploration for philosophers, mathematicians, and historians (see for example: Troelstra & van Dalen, 1988; Kreisel, 1967; Troelstra, 1969; van Atten, 2004). However, within the context of this paper, the crucial question is precisely what the term "creating subject" alludes to. According to Joop Niekus, there is no idealized subject in Brouwer's notion of the creating subject; there is only us, human beings, and Brouwer refers to "I" or "we":

> According to Brouwer's view, mathematics is a creation of the human mind and by using the expression creating subject Brouwer only made explicit his idealistic position; it can be replaced by we or I. Interpreted in this way, an idealized mathematician is not needed at all for the reconstruction […] We interpreted the expression creating subject as we, and anybody else can interpret it as himself. Brouwer's definition is a description of a construction, as any intuitionistic definition. But the construction is not completely determined. The values of the sequence under consideration depend on the mathematical experience of the maker of the sequence, the creating subject. (Niekus, 2010, 32, 39)

Niekus asserts the existence of an actual, human mathematician or a group of mathematicians rather than an "idealized mathematician." Philosopher Carl Posy concurs with this stance and extends it further in relation to communal dimensions:

> I'm assuming an 'intersubjective' notion of the creating subject: distinct mathematicians addressing distinct issues. CrS3 (and certainly CrS3+) show that this differs inessentially from so-called 'solipsistic' versions. In practice, creating subject arguments assume only that we can track the outputs of research on particular problems or the status of knowledge about particular objects. We might, in fact, be tracking the entire mathematical community. (Posy, 2020, 71)

Posy's approach not only distances the creating subject from a solipsistic standpoint but also emphasizes the concepts of intersubjectivity and the indispensable role of the mathematical community. In light of Brouwer's own statements from the Significs dialogues regarding the community's role in asserting the character of mathematical formulas and shaping knowledge, such an interpretation implies another connection between Brouwer's social engagements and the subject matter of his intuitionism. The knowledge held by the creating subject encompasses the cumulative knowledge of the entire community,

[9] It has been argued that the origin of Brouwer's theory of the idealized mathematician or 'creative subject' dates back to 1927 (Troelstra & van Dalen, 1988; van Dalen 1999).

signifying that the community not only reflects existing knowledge but also utilizes it, shapes it, and actively contributes to its creation.

Later developments of intuitionism support the detachment of the creating subject from solipsism. Mathematician Anne Troelstra contended that a "reconstruction which would be based on the 'solipsistic explanation' of the creative subject seems to us to be undoubtedly anachronistic" (Troelstra, 1982, 479). Arend Heyting, a prominent student and successor of Brouwer, claimed that Brouwer had sometimes described mathematics as an activity of the mathematical community as a whole (Heyting, 1974). Heyting himself acknowledged the community's role in reinforcing individuals' beliefs, emphasizing that "the individual cannot be separated from the culture where he lives" (Heyting, 1978, F8.3; Franchella, 2007, p. 6). He attributed a substantial role to intersubjectivity and posited that our beliefs are justified by the social community around us: "Why am I convinced of the existence of Japan? Well, because I have been taught so at school and imagine that some men there perceive things as I myself perceive my environment. Here intersubjectivity is going to play a big role." (Heyting, 1978, F7.11-12; Franchella, 2007, p. 6).

These passages highlight that prominent intuitionism scholars, including Heyting, Troelstra, Niekus, and Posy, actively contemplated the intersubjective dimension in Brouwer's thought, however scattered their contemplations might be. By compiling these scattered interpretations into a more coherent standpoint, we observe, on one hand, the consistency of a communal trend in Brouwer's thought, and on the other hand, its clash with his solipsism.

The extent to which Brouwer's views amount to solipsism is a subject of debate among intuitionism scholars (Blum, 2005; Dummett, 1975; Franchella, 1995; Hesseling, 2003). While some argue that solipsism is a cornerstone of Brouwer's intuitionism, suggesting that without it, mental constructions lose their power, others consider this interpretation too simplistic and advocate for the possibility of intersubjectivity in Brouwer's philosophy (Pambuccian, 2022; Placek, 1999). Brouwer's own writings about the mind do not provide much clarity on the matter. In his paper "Consciousness, Philosophy, and Mathematics," Brouwer presents somewhat enigmatic views on solipsism. He argues that there can be no proof of the existence of other minds, yet hints at the possibility of other people's autonomous existence, which suggests a way out of solipsism (Brouwer, 1948, 485).

Reinforced by several intuitionism scholars, the analysis thus far on the role of language, groups, and social interactions in Brouwer's life and work presents a picture that is less solipsistic than one might initially think. While it may be difficult to determine exactly how solipsistic Brouwer was at different stages of his life, his quotes from the dialogues suggests that he attached at least some significance to group context and interactions between individuals as means of exchanging ideas and promoting mutual understanding. Similar to the evolution of his thought regarding the tool of language, Brouwer may have changed his views about solipsism in his mathematics over the years. Given the complexity of his character, it would not be surprising to find that he held a dualistic view not only about language, but also regarding solipsism.

6. Two questions: on modernism and realism

Embracing a position that fleshes out the intuitionistic view of the role played by social factors in mathematics might have a broader impact on discussions about the social history of mathematics, and on philosophical debates concerning questions of existence and truth. In this section, I briefly address two questions that challenge common historical and philosophical readings of intuitionism based on the social analysis presented in this paper.

On a historical level, the tension between Brouwer's communal and solipsistic views complicates Mehrtens' casting of Brouwer's intuitionism with counter-modernism. Mehrtens proposed to view developments in mathematics as part of a dialectic process between two interwoven currents, modernism and counter-modernism[10]. He opted for the term 'modernism' in mathematics to incorporate the history of science within its cultural context (Mehrtens, 1996, p. 521). Modernism is marked by the autonomy of cultural production, and its independence from the influence of any authority or common sense. Counter modernism, although existing within the modern world, stands as a counterpart to modernism. It posits the existence of a "natural substance to the truth and meaning of mathematics" referred to as intuition (Mehrtens, 1996, p. 522).

According to Mehrtens, the difference between the two streams resides in their different approaches to the concept of truth in mathematics. Mathematical modernism, which originated from Dedekind and Riemann, views the truth of a mathematical object as determined from within the object itself, with no need for external representation or confirmation (Mehrtens, 1990, p. 67). As long as there are no contradictions in the mathematical system, its truth is internal to it and does not represent anything outside the system itself. Counter-modernists oppose this view and claim that the roots of mathematical truth exist outside the objects themselves. Hence, mathematicians are restricted to some extent and can never obtain complete freedom in their work (Mehrtens, 1990, pp. 76–78). Thus, for counter-modernists, there are certain ways in which the mathematician has to work, whereas modernists put themselves forward as the champions of freedom (Mehrtens, 1990, pp. 7–10).

In Mehrtens' casting, modernism is embodied in Hilbert's formalistic approach, and counter-modernism culminated with Brouwer's intuitionistic opposition to Hilbert (even though in Mehrtens' story, Hilbert's adversary is Felix Klein[11]). The question of modernity, according to Mehrtens, is how, if at all, language can create knowledge (Mehrtens, 1990, p. 286). In Mehrtens view, Brouwer rejects mathematical modernity as something peculiar that deviates from normal mathematics, and advocates a counter approach in which he speaks against language (Mehrtens, 1990, p. 258). Mehrtens reads Brouwer as someone who views language as contaminated by the will to power over people and nature and hence distances himself from the progressive or modern science, which goes the opposite way away from the individual and the truth (Mehrtens, 1990, p. 284).

However, a more nuanced interpretation of Brouwer's views on language can be found in the significs dialogues, which Mehrtens does not mention. Brouwer's views on the practice of mathematics, as reflected in these dialogues, align more closely with modernist ideas rather than counter-modernism for three central reasons. First, Brouwer's perspective on truth in the dialogues suggests that he believed mathematical expressions and formulas are shaped by the social context in which they are used or learned (Brouwer, 1947, 451). This implies that our understanding and use of mathematics are influenced by the environment and the people around us. The same applies to mathematical truth, which is not merely an abstract concept but is also impacted by the social situations in which it is taught and applied, situations that require the use of language (Brouwer, 1975, 452). Second, Brouwer views the 'asserting character' of mathematical formulas as influenced by the interactions of individuals within a group context and their mutual understanding, all formed using the tool of language. Third, the analysis of the creating subject reveals a communal aspect in the theory itself, suggesting that the community plays a crucial role in

[10] It should be noted that counter-modernism is not the same as anti-modernism; counter-modernism arises *with* modernism and is part of the modern world (Mehrtens, 1990, p. 521).

[11] While Mehrtens sees Brouwer's intuitionism as a major factor in the establishment of counter-modernism, he describes Felix Klein's *Erlangen Program* as the first move of counter-modernism. Recently it has been argued that Hilbert and Klein were not adversaries but modernist allies and that Hilbert's views on intuition are closer to Klein's views than Mehrtens is willing to allow (Bair et al., 2017).

creating and disseminating knowledge, and for this, language is essential. This threefold argument raises the question of whether it is accurate to characterize Brouwer's intuitionism as counter-modernism based on his views on language, which have proved to have a dual character.

On a philosophical level, the existence of a communal trend in Brouwer's work challenges the affiliation of intuitionism with anti-realism. Mathematical realism views mathematical entities as existing independently of the human mind. According to this view, mathematicians do not create mathematics but rather discover it, and the truths of mathematics are objective since they are independent of any human activities, beliefs, or capacities. As a result, mathematical realists believe there may be truths about mathematical reality that we can never know. Mathematical anti-realists find this particular argument unacceptable, as they insist that only what can be proved is true. According to realists, mathematical truths are objective in the sense that they do not depend on the human mind. In intuitionism, the concept of truth depends on our ability to provide proof, which is a focal point that connects intuitionism to anti-realism. As such, it has been commonly accepted that Platonism and intuitionism reside at the two ends of the realism and anti-realism spectrum, respectively.

However, the affiliation of intuitionism with anti-realism is not that explicit. It has been claimed that mental constructions fail to do justice to the communal nature of mathematical research since it makes each mathematical object accessible only to its creator (Blanchette, 1998). As the current paper suggests, mental constructions are not necessarily in conflict with the shared and collective character of mathematical work.[12] On the contrary, mathematical entities are creations of the mind, but they are also influenced by social factors, such as the community and group context in which they are practiced, and the language tools used to communicate them. Brouwer's quotes from the significs dialogues illustrate this connection by showing how the significance of a mathematical statement changes with variations in its use, users, and applications. Therefore, the presence of a communal trend in Brouwer's thought distances intuitionism from anti-realism and possibly aligns it more closely with situated, vantage point-based approaches like perspectival realism (Massimi, 2022).

7. Concluding remarks

This paper has examined Brouwer's engagement with the significs circle and his views on philosophy and mathematics through a socially oriented prism. It focused on three key social elements: Brouwer's approach to language, his social and professional interactions with members of the significs circle, and his views on groups and group context. Specifically, the paper explored the formation and dissolution of the significs movement, with a focus on Mannoury's influence and his relationship with Brouwer. It analyzed several excerpts from the significs dialogues, emphasizing the role of groups and communities in the creation of knowledge. Furthermore, it underscored Brouwer's theory of the creating subject as an area where these social elements affected his mathematics, and concluded by raising two questions that challenge contemporary readings of intuitionism.

This paper, thus, highlights a tension between Brouwer's supposedly solipsistic statements and his communal trend as evident in the significs dialogues. It builds on the idea that mathematical entities are creations of the mind, but then points to Brouwer's own words about the impact of the community on the creation, use, and practice of these entities. Given that Brouwer's thoughts on these matters are scattered among several sources, this paper consolidates some of Brouwer's insights concerning the intersection of mathematics and society into a more cohesive standpoint.

Clearly, much work remains to establish such a standpoint; this paper represents only the beginning of such an endeavor. Its goal is to set the stage for pursuing this kind of research trajectory and to demonstrate that the significs dialogues offer valuable insights for uncovering these connections. By highlighting the link between a social interpretation of Brouwer's intuitionism and debates about realism, truth, and modernism in mathematics, this paper aims to broaden the scope of questions derived from specific case studies, such as intuitionism, towards more general narratives in the history of mathematics.

Funding

This paper was supported by the Marie Skłodowska-Curie Actions fellowship, European Commission Horizon Europe. Project number 101108426.

Conflicts of interest/Competing interests

The author has no relevant financial or non-financial interests to disclose.

Availability of data and material

Not applicable.

Code availability

Not applicable.

CRediT authorship contribution statement

Kati Kish Bar-On: Writing – review & editing, Writing – original draft, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

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[12] A related concept can be found in the philosopher Jean Largeault's book *Intuition et intuitionisme*, where he argues that intuitionism is universal rather than individual, and that it refutes the legend of solipsism attributed to it (Largeault, 1993, p. 224).

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