

RUSSELL ON “DISAMBIGUATING WITH THE GRAIN”

KEVIN C. KLEMENT
Philosophy / U. of Massachusetts
Amherst, MA 01003, USA
KLEMENT@PHILOS.UMASS.EDU

Fregeans face the difficulty finding a notation for distinguishing statements about the sense or meaning of an expression as opposed to its reference or denotation. Famously, in “On Denoting”, Russell rejected methods that begin with an expression designating its denotation, and then alter it with a “the meaning of” operator to designate the meaning. Such methods attempt an impossible “backward road” from denotation to meaning. Contemporary neo-Fregeans, however, have suggested that we can disambiguate *with*, rather than *against*, the grain, by using a notation that begins with expressions designating senses or meanings, and then alters them with a “the denotation of” operator to designate the denotation. I show that in his manuscripts of 1903–05, Russell both considered and rejected a similar notation along with the metaphysical suppositions underlying it. This discussion sheds light on the evolution of Russell’s thought, and may yet be instructive for ongoing debates.

I. INTRODUCTION

Between 1903 and early 1905, Russell held a theory of meaning that he himself later described as “very nearly the same” (*OD, Papers* 4: 415n.) as Frege’s theory of sense (*Sinn*) and reference (*Bedeutung*). While Russell exaggerated the similarities between his views of this period and those of Frege, it is still worthwhile for those of us interested in Fregean theories of meaning to examine closely Russell’s confrontation and eventual abandonment of his own early theory of “mean-

ing and denotation”, as he called it. Even a century later, Russell’s work can provide new insight into many of the unresolved philosophical questions facing Fregean theories of meaning. In this paper, I discuss an issue concerning the disambiguation of truths that have to do with the referents/denotations of senses/meanings from truths having to do with the senses/meanings themselves.¹

This issue can be understood—and in the secondary literature on Frege often *has* been understood—as an issue dealing with the development of an appropriate notation for intensional logic. Frege’s own extant logic was entirely extensional, and he provided no more than hints at how it might be expanded. There is some disagreement among more contemporary “Fregeans” as to how to best devise a logical calculus in line with Frege’s theory of meaning. One of the most important desiderata is the inclusion of some means whereby to clearly distinguish between contexts in which we wish to talk about the senses of expressions from the more normal contexts in which we use expressions to speak about their referents. There are competing views among neo-Fregeans regarding how this can best be done. It is arguable, however, that there are problems with whatever methodology is adopted. A close inspection of Russell’s manuscripts from 1903 through 1905 shows that he was aware of these problems, and seems to have come to the conclusion that they were insoluble. Moreover, while he had an appreciation for the difficulties in developing an adequate notation, by his lights, the notational issue was merely a manifestation of a more important, and more serious, extra-linguistic (metaphysical) issue. There is good reason to think that his inability to come to grips with this issue weighed heavily on his eventual abandonment of his own quasi-Fregean theory of meaning, and may yet continue to be instructive for our ongoing evaluation of similar theories.

¹ In what follows, I usually use the words “sense” and “reference” when discussing the dualism in Frege’s views or those of neo-Fregeans, reserving “meaning” and “denotation” for the similar distinction in Russell’s early work. It is worth noting, however, that Frege and Russell used these words slightly differently. The way Frege spoke, it is the *expression* that “refers to” the reference, while the way Russell usually spoke, it is the *meaning* itself (not the expression) that denotes the denotation, but he did not stick to this usage consistently.

2. THE PROBLEM FOR THE FREGEAN

Before turning to Russell's thoughts on the issue, let us discuss the issue as it has arisen within Frege scholarship. Frege's theory of meaning² is a sort of semantic dualism: an expression not only refers to an individual or function but also *expresses* a sense, and it is in virtue of a logical relation between the sense and the entity referred to that the expression refers. Senses, then, act as intermediaries between expressions and referents. On Frege's own view, the reference of a complex expression is a function of the referents of the parts. As he holds that the referents of complete sentences are truth-values, he is committed to the principle that coreferential expressions must be substitutable *salva veritate*. The apparent exceptions to this principle are explained away by his theory of indirect reference. If we consider the sentences:

- (1) George believes that Scott is a poet
- (2) George believes that the author of *Waverley* is a poet

it seems possible that these have different truth-values. Frege would explain the failure of substitutivity of "the author of *Waverley*" for "Scott" in these examples by holding that, in certain contexts which he calls "indirect speech", such as in statements of propositional attitudes, words express different senses and have different referents than they would normally. Specifically, they have as referents that which would in normal contexts be their senses. So while the names "Scott" and "the author of *Waverley*" both standardly refer to the same person, in the above examples, they do not refer to the same thing; instead, they refer to different senses.

So on Frege's view, the senses and referents of expressions appearing in ordinary language vary depending on context. Frege views this ambiguity of ordinary language as a defect, specifically to be avoided in a logically superior language.³ In a letter to Russell he gives the following

² See especially his "On *Sinn* and *Bedeutung*", in *The Frege Reader*, ed. M. Beaney (Oxford: Blackwell, 1997), pp. 151–71.

³ See "On *Sinn* and *Bedeutung*", p. 153, and Frege, letter to Peano, 29 Sept. 1896, in *Philosophical and Mathematical Correspondence*, ed. G. Gabriel *et al.* (Chicago: U. of Chicago P., 1980), p. 115.

recommendation:

To avoid ambiguity, we ought really to have special signs in indirect speech, though their connection with the corresponding signs in direct speech should be easy to recognize.⁴

However, Frege himself never specifically included means for dealing with indirect speech in his logical notation, saying only that he had “no occasion to do so” (*ibid.*, p. 149). So it has been left to Frege’s philosophical progeny to attempt to devise some sort of notation that is able to both (a) rid itself of the ambiguity of ordinary language while at the same time, (b) keeping apparent the connection between expressions occurring in indirect speech and the corresponding signs in direct speech. However, this task has met with some considerable difficulty. Notations have been developed that perform either of the tasks (a) or (b) sufficiently well, but notations that are able to do both adequately are more difficult to devise.

Let us begin with the standard sort of function calculus, and let “*s*” be our transcription of the English “Scott”, “*a*” be our transcription of “the author of *Waverley*” and “*P*” be our transcription of “... is a poet.” We then transcribe the following English sentences:

- (3) Scott is a poet
- (4) Scott is the author of *Waverley*
- (5) The author of *Waverley* is a poet.

And they become:

- (3′) $P(s)$
- (4′) $s = a$
- (5′) $P(a)$

If we suppose our logical system to have the usual sort of inference rules, including, e.g., a rule for the substitution of identicals, the inference from (3′) and (4′) to (5′) is easily shown to be valid. However, how do

⁴ Gottlob Frege, letter to Russell, 28 Dec. 1902, *Philosophical and Mathematical Correspondence*, p. 153.

we transcribe (1) and (2) above? The naïve approach would be to render them (with "g" for George and "B" for the belief relation):

(1') $B(g, P(s))$

(2') $B(g, P(a))$

The problem with this rendering is that, provided we retain the usual rule for the substitution of identicals in our system, the inference from (1') and (4') to (2') is retained as valid, when it is in actuality an invalid inference. It is precisely for this reason that Frege advocates the use of "special signs" for capturing indirect speech.

Perhaps the most common step taken to avoid this difficulty is to use some way of altering our signs when they appear in indirect speech to mark the unusual mode of occurrence, such as placing them in special brackets or prefixing them with a certain symbol. Montague and Gallin use the sign " \wedge " for this purpose.⁵ When " \wedge " prefixes a certain expression, that expression is to be understood as referring to its customary sense (or its intension, as they would say), rather than what it normally refers to. So their transcriptions of (1) and (2) would be instead:

(1m) $B(g, \wedge(P(s)))$

(2m) $B(g, \wedge(P(a)))$

George Bealer uses a similar notation that employs square brackets around expressions when they appear in indirect contexts and are thus thought to have *shifted* or *oblique* reference.⁶ Thus, his transcription of (1) would be something such as " $B(g, [P(s)])$ ". Similar devices abound in the literature.⁷

These notations have the advantage that they make indirect contexts evident, i.e., they make plain exactly in what cases expressions must not

⁵ See Richard Montague, "The Proper Treatment of Quantification in Ordinary English", in *Formal Philosophy* (New Haven: Yale U. P., 1974); and Daniel Gallin, *Intensional and Higher Order Modal Logic* (Amsterdam: North Holland P., 1975).

⁶ George Bealer, *Quality and Concept* (Oxford: Clarendon P., 1984).

⁷ Frege's own use of underlining for this purpose in a letter to Russell (*Philosophical and Mathematical Correspondence*, pp. 153–4) can also be seen as falling in this category, though Frege himself seems to treat the methodology as provisional at best.

be taken as having their normal referents. Nevertheless, it can still be argued that these notations have not adequately purged our logic of ambiguity. This notation still has indirect contexts, contexts in which the normal substitution rules do not hold or expressions do not stand for what they would normally. Notice that “*s*” still occurs in (1m), but it cannot there be understood as standing for Walter Scott himself. We still are faced with the necessity of having to alter or restrict our rules of inference, because we cannot substitute “*a*” for “*s*” in (1m) any more than we can in (1’). The only effect of the new notation is, arguably, that it helps in specifying exactly in what contexts the normal rules of inference do not apply. This would not be seen as adequate to anyone such as Frege who holds that in an adequate notation, an expression must always have the same reference, and who holds that the truth-value of an expression is a function alone of the referents of the parts. Because there are multiple senses picking out any one thing, we cannot understand the sign “ \wedge ” as standing for a function that operates on the customary referents of the expressions that follow it. After all, if Frege is right that the referents of sentences are truth-values, the referents of “ $P(a)$ ” and “ $P(s)$ ” are the same, yet “ $\wedge(P(a))$ ” and “ $\wedge(P(s))$ ” surely cannot be understood as standing for the same thing. This notation, therefore, does not preserve the functionality and univocality of our language; it simply *reproduces* the ambiguities of ordinary language in a *more perspicuous* manner.

Alonzo Church, David Kaplan, and others, however, seem to be quite aware of these difficulties, and have opted for a different method,⁸ dubbed “the method of indirect discourse” by Kaplan. On this approach, in our logical notation, rather than altering the signs used when transcribing direct speech for use when transcribing indirect speech, we employ wholly different signs. That is, we might use one sign when transcribing the name “Scott” when it appears in ordinary contexts, and a completely different sign when transcribing that same name when it appears in an indirect context in ordinary language. In Church’s “Logic of Sense and Denotation”, type-distinctions are made between signs

⁸ See, e.g., Alonzo Church, “A Formulation of the Logic of Sense and Denotation”, in *Structure, Method and Meaning, Essays in Honor of H. M. Sheffer* (New York: Liberal Arts P., 1951); David Kaplan, “Foundations of Intensional Logic” (unpublished Ph.D. thesis., UCLA, 1974). I myself used a notation along similar lines in my *Frege and the Logic of Sense and Reference* (New York: Routledge, 2002).

standing for individuals (written with the type-symbol " ι_o " as a subscript) and signs standing for senses picking out individuals (written with the type-symbol " ι_1 " as a subscript). Thus, the sign " s_{ι_o} " might be used to stand for Scott himself, and " s_{ι_1} " for the sense of the name "Scott". For convenience, the letter " s " is used each time, but this is unnecessary. These are to be understood as distinct primitive constants of differing types; they are not complex in any way, and nothing prevents us from using " t_{ι_1} ", or any other sign of this type, for the sense of the name "Scott". Our new corollaries of (1)–(5) are written somewhat as follows:⁹

- (1c) $B_{o_o(\iota_o, o_1)}(g_{\iota_o}, P_{o_1(\iota_1)}(s_{\iota_1}))$
- (2c) $B_{o_o(\iota_o, o_1)}(g_{\iota_o}, P_{o_1(\iota_1)}(a_{\iota_1}))$
- (3c) $P_{o_o(\iota_o)}(s_{\iota_o})$
- (4c) $s_{\iota_o} = a_{\iota_o}$
- (5c) $P_{o_o(\iota_o)}(a_{\iota_o})$

Here we see that the sign " s_{ι_o} " is used to transcribe "Scott" whenever it occurs in direct speech, whereas " s_{ι_1} " is used to transcribe "Scott" whenever it occurs in indirect speech, and it is easy to see why the inference from (3c) and (4c) to (5c) is valid while the inference from (1c) and (4c) to (2c) is invalid without making any revisions to the standard inference rules regarding the substitution of identicals.

It is obvious that on Church's approach, there is no corresponding problem with the same sign being used in two different ways. Where " s_{ι_o} " occurs it always stands for Scott, and where " s_{ι_1} " occurs it always stands for a sense. There is no ambiguity here. While we choose the same letter for use in the cases of " s_{ι_o} " and " s_{ι_1} ", these constants are of different logical types and we could have used different letters just as easily. Recall that Frege himself suggested two criteria for an adequate

⁹ Here, I have recreated Church's type-subscripts for functions, and so on. Those unfamiliar with Church's Logic of Sense and Denotation can ignore them. I have, however, not been entirely faithful to Church, who actually employs lambda abstracts and functions having functions as value in such a way as to make unnecessary functions with multiple arguments.

notation for dealing with these contexts, that it (a) rid itself of the ambiguity of ordinary language, and (b) maintain the obvious connection between signs used to transcribe indirect speech and the corresponding signs used for direct speech. Montague's approach did (b) well enough but was deficient with regard to (a). The challenge facing Church's strategy is the reverse. Church has no difficulty with (a), but there is a question with regard to whether or not it does (b). There is, after all, a close connection between what the signs " s_{10} " and " s_{11} " stand for. The former is our transcription of the English name "Scott" when it occurs in a direct context, and the latter is our transcription of the *same* name when it occurs in a (singly embedded) indirect context. On Frege's view the name "Scott", when occurring in an indirect context, is thought to refer to the very sense the name "Scott" *expresses* in a direct context. So Church's sign " s_{11} " is supposed to refer to what " s_{10} " expresses. But, precisely because the choice of signs is arbitrary, and the coincidence in letters unnecessary, this connection is not at all obvious from the symbolism.

Of course, Church is not entirely unaware of this problem. He does introduce a sign, " Δ ", to capture the relation between a sense and the referent it picks out or presents. So Church can formalize the relation between the sense s_{11} and the person s_{10} by writing:

$$(6c) \Delta_{o_0(t_1, t_0)}(s_{11}, s_{10})$$

At the very least then, Church can assert that the sense picks out the person as referent. However, the sufficiency of this move to capture the close relation intended between the signs " s_{11} " and " s_{10} " has been challenged, particularly by Pavel Tichý.¹⁰ The first thing to note that is that, in virtue of (6c), and (4c) above, and the normal principles of substitution of identicals (which Church's system is especially designed to preserve), we conclude:

$$(7c) \Delta_{o_0(t_1, t_0)}(s_{11}, a_{t_0})$$

¹⁰ See Pavel Tichý, *The Foundations of Frege's Logic* (Berlin: de Gruyter, 1988), pp. 138–40.

This captures the thought that the sense of the name "Scott" picks out or presents the author of *Waverley* as referent. This is *true*—there is no problem with the substitution in *that* regard. The difficulty is rather one of the relative triviality of (6c) and (7c). Recall that what we wanted to capture with (6c) is the close connection between the signs " s_{11} " and " s_{10} "—one refers to the very sense the other expresses. However, it is clear that (6c) does not say *that*, it says something weaker, that what the one stands for is a sense that picks out what the other stands for. But that much holds also for " s_{11} " and " a_{10} "—though less trivially so. We want the connection between signs used to transcribe direct speech and signs used to transcribe indirect speech to be direct and obvious, apparent from the symbolism alone. But Church has no way of making this connection obvious.¹¹ (6c) above may seem like a triviality, but that is only because one is prone to forget that " s_{11} " and " s_{10} " are simply two different *primitive* constants and that the coincidence in letters is just that. Indeed, in the context of a systematized logic, (6c) would have to be added as an axiom, and would not follow from more general principles for the symbolism, and/or the laws of logic alone.

The dilemma would seem to be this. If we choose a notation that makes use of the same sign to transcribe an ordinary language phrase that occurs in indirect speech as we use to transcribe the same phrase in direct speech, then we *reproduce* the ambiguity and problems with substitutivity present in ordinary language. However, if we choose a notation that makes use of wholly different signs for these tasks, then the close, even *trivial*, relation that must hold between them is lost.

3. TICHÝ'S PROPOSED SOLUTION

Later on, I will suggest that Russell seems to have anticipated much of this debate in his struggle with his own dualism between meaning and denotation. But I want first to discuss a solution that has been proposed

¹¹ As Tichý points out, Church is not helped by attempting to get at the differences in triviality by introducing modal operators (or, more directly, a "triviality" operator), because such operators typically create oblique contexts, and he would be forced to use different signs in place of " s_{11} " and " s_{10} " which would only bring us further away from a method of showing the close relation between *these* signs.

by one neo-Fregean, viz., Pavel Tichý (*op. cit.*, Chap. 10). Tichý suggests that the difficulties found in the previous attempts at an adequate notation derive from coming at the problem the wrong way around. Given that words in ordinary language are usually thought of as *standing for* or representing their referents rather than their senses, it is natural to think that the same must be true for our logical language. If we add the additional assumptions that the same sign must always be used to stand for the same thing, and that what a complex expression stands for is a function of what the parts stand for, we are driven inevitably towards the dilemma just described. On Montague's approach, for example, the attempt is made to add to or prefix signs that, by themselves, would stand for the referents, and through prefixing or altering them, arrive at a name for the sense. However, since the relation between senses and corresponding referents is many-one, trying to get at the senses *through* the referents creates a difficulty. As Russell himself famously put it, "there is no backward road from denotations [referents] to meanings [senses]" (*OD, Papers 4*: 422). Montague meant to have " $\wedge(P(a))$ " and " $\wedge(P(s))$ " stand for different things, but since "*a*" and "*s*" (by themselves) stand for the same thing, this is impossible if what a complex expression stands for is determined entirely by what its parts stand for. Tichý describes the problem with this approach as one that attempts to disambiguate *against* the grain, as it were.

Tichý's own approach is to reject the starting place. Imagine a notation that, rather than beginning with signs that, *alone and unmodified*, stand for the referents of names, begins instead with signs that, *alone and unmodified*, are thought to stand for senses.¹² For example, take "*s*", by itself, to stand for the sense expressed by the name "Scott", and take "*a*", by itself, to stand for the sense expressed by the phrase "the author of *Waverley*". Then rather than trying to create a name for the sense by prefixing or altering a name for the referent, *pace* Montague, we create a name for the referent by prefixing what would otherwise be a name for the sense. Montague himself used a sign " \vee " for a functor carrying a sense to its referent. He intended it to be used *in conjunction* with the

¹² Tichý actually makes a distinction between senses and what he calls "presentations", and strictly speaking, he takes his simple signs to stand for presentations, not senses. But this distinction is not crucially relevant for the present discussion and I overlook this complication here.

sign " \wedge ", so that, for every expression A , " $\vee \wedge A$ " stands for the same thing as " A ". Tichý, for reasons already discussed, rejects Montague's " \wedge ", but finds no similar problems with " \vee ". Thus, on his approach, " s " and " a " are regarded as names for two *different* entities, two different senses, but " $\vee a$ " and " $\vee s$ " are taken as two different names of the same person. Tichý's transcriptions of (1)–(5) from above are then as follows:

- (1t) $\vee B(\vee g, P(s))$
- (2t) $\vee B(\vee g, P(a))$
- (3t) $\vee P(\vee s)$
- (4t) $\vee a \vee = \vee s$
- (5t) $\vee P(\vee a)$

Here we see that Tichý uses " s " *by itself* to transcribe the name "Scott" when it occurs in an *indirect* context in English, but uses " $\vee s$ " to transcribe "Scott" when it occurs in an ordinary context.¹³

The advantages of Tichý's approach are many. In fact, he likely would claim that it is the only approach that succeeds with regard to both of Frege's desiderata when it comes to an adequate notation. Firstly, it is not ambiguous. The same simple expression always stands for the same thing, a sense. In both (1t) and (3t), the sign " s " is used to refer to a sense; it is only that, in (3), the *functor* \vee is used to map this sense onto the referent it picks out. There is no problem with substitutivity of identicals. (4t) states not that " a " and " s " stand for the same thing, but rather that " $\vee a$ " and " $\vee s$ " stand for the same thing. So wherever one of *these* expressions occurs, the other can replace it, as in the move from (3t) to (5t). No similar move is possible with regard to (1t) and (2t), since these expressions do not appear there at all. Yet at the same time, Tichý has no trouble in explaining the close connection between " s " and " $\vee s$ ". Let us imagine that Tichý introduced a sign much like Church's " Δ "—though, since sense and reference can be distinguished even here, and given Tichý's general approach of having simple signs stand for senses, we write this instead as " $\vee \Delta$ ". Given that the functor \vee maps a

¹³ There is a complication to Tichý's approach as regards *asserted* sentences, because Tichý believes that when we make an assertion, *what* we assert is the sense of the proposition, and so these renderings would not technically be his renderings if these were taken as *asserted*. See Tichý, pp. 162–6.

sense onto its referent, the following is an obvious triviality:

$$(6t) \quad \forall \Delta(s, \forall s)$$

However, the following, while true, does not share the same triviality:

$$(7t) \quad \forall \Delta(s, \forall a)$$

The difference is that (6t) is obvious just from the intended way in which the signs “ Δ ” and “ \forall ” are to be understood, whereas (7t) cannot be inferred from this alone. The close relation between “ $\forall s$ ” and “ s ” is obvious from the symbolism; indeed the latter sign is thought to occur *within* the former sign. Unlike the other approaches, Tichý views his approach as disambiguating *with* the grain. Russell may have been right that there is no backward road from referents to senses, but if we fashion our logical notation correctly, perhaps all we need is the *forward* road from senses to referents.

4. RUSSELL ON DISAMBIGUATING AGAINST THE GRAIN

Let us then turn to Russell. It is well known that in “On Denoting”, Russell objected to theories that distinguish between meaning and denotation (or sense and reference) because of the lack of any means to distinguish between occasions in which we want to talk about the one side as opposed to another. The examples he gave there (*OD, Papers* 4: 421) were the following pairs of sentences:

[(8)] The centre of mass of the Solar System is a point, not a denoting complex.

[(9)] “The centre of mass of the Solar System” is a denoting complex, not a point.

Or again,

[(10)] The first line of Gray’s *Elegy* states a proposition.

[(11)] “The first line of Gray’s *Elegy*” does not state a proposition.

For each sentence pair, the first seems to be about what a certain phrase *denotes* or *refers* to, while the second seems to be about a certain complex

meaning or sense. It is not difficult to see here that the problem Russell is discussing is akin to the problem for the Fregean we have just discussed. The examples used above concerned indirect discourse, and specifically, reports of propositional attitudes. Frege's theory of indirect discourse is in effect the view that, in statements about beliefs, for example, we are not really talking about the (customary) referents of the words used to describe the content of the belief, but rather the senses. With the examples of (8)–(11), Russell has chosen cases that *more directly* show the need for some means whereby to distinguish between contexts in which we wish to make assertions about meanings or senses from the more normal contexts in which we merely use meaningful phrases to make assertions about their denotations or referents. The problem posed by these examples is essentially the same.

In "On Denoting", Russell primarily focuses on the device of using inverted commas, which he describes as the "natural mode" (*OD, Papers* 4: 421) that one would invoke to mark off contexts in which one means to talk about the meanings, as opposed to the denotations, of expressions.¹⁴ In the end, he finds the device to be unworkable. The argument he offers for this conclusion, however, is notoriously difficult to interpret fully and has been the subject of much controversy. Especially because my primary interest in this paper is some of Russell's less well-known works, I shall not attempt to offer my own worked-out interpretation of Russell's infamous Gray's Elegy Argument. Moreover, I believe there have been a number of breakthroughs in understanding Russell's argument in recent years, and I do not think I have much to add to the existing literature. But for present purposes, it is worth briefly discussing certain aspects of these developments here.

First, I believe that the recent breakthroughs stem largely from taking Russell's true target not to be Frege's theory of sense and reference, but his own earlier theory of meaning and denotation and the theory of

¹⁴ So far throughout the paper (except for the quotation on the previous page) I have been using double quotation marks, "...", to mention linguistic phrases rather than use them, in the normal American fashion. Because I still need such a device for the purposes of this paper, and because this obviously is not the way Russell means his inverted commas, in what follows I will use straight (or typewriter-style) quotation marks ("") for the purpose of reproducing the methodology of inverted commas Russell dislikes, while continuing to use curly quotation marks, ("") and (""), to mention expressions. Cf. note 17.

“denoting concepts” given in *The Principles of Mathematics*.¹⁵ There are significant differences between Russell’s earlier theories and Frege’s views on meaning that are important to bear in mind. The differences are especially important, given that my discussion will draw heavily on Russell’s manuscripts of 1903–05. The earlier works in this period represent Russell’s attempts to work out and state clearly the details of his own theory of meaning and denotation, while the later works in this period represent Russell’s last-ditch efforts to make this theory work along with increasing focus on the difficulties from which he could not find escape, and finally, in “On Fundamentals” (*Papers* 4: 359–413), discussion of what he later called “the reasons for the new theory of denoting [i.e., the theory of descriptions]” (p. 359). It is also worth bearing in mind, however, that although it is true that the arguments of “On Denoting” were more directly targeted against the views of the early Russell than those of Frege, this does not mean that Russell’s discussion there can have no relevance to the difficulties facing the Fregean discussed earlier.

What are the differences? The differences reveal themselves nicely in the correspondence between Russell and Frege between 1902 and 1904.¹⁶ On Frege’s theory of meaning, the distinction between sense and reference applies to every meaningful expression, simple or complex. A complete declarative sentence has a thought for its sense, and a truth-value (the True or the False) as its referent. The phrases that make up a sentence divide into two kinds of components, proper names and function names; the former referring to objects and the latter referring to functions. Each name also expresses a sense distinct from its referent, and in fact, Frege regards the thought expressed by the whole sentence as composed or built up from the senses of its parts. Thoughts are com-

¹⁵ This suggestion was first made years ago by Peter Geach (“Russell on Meaning and Denotation”, *Analysis*, 19 (1958): 69–72), and has been followed up more recently, with some success, by others. See Peter Hylton, *Russell, Idealism and the Emergence of Analytic Philosophy* (Oxford: Clarendon P., 1990); Russell Wahl, “Russell’s Theory of Meaning and Denotation and ‘On Denoting’”, *Journal of the History of Philosophy*, 31 (1993): 71–94; Gregory Landini, “‘On Denoting’ against Denoting”, *Russell*, n.s. 18 (1998): 43–80; Michael Kremer, “The Argument of ‘On Denoting’”, *The Philosophical Review*, 103 (1994): 249–97; Gideon Makin, *The Metaphysicians of Meaning: Russell and Frege on Sense and Denotation* (London: Routledge, 2000); William Demopoulos, “On the Theory of Meaning of ‘On Denoting’”, *Noûs*, 33 (1999): 439–58.

¹⁶ See Frege, *Philosophical and Mathematical Correspondence*, pp. 130–70.

posed of senses and of senses alone, which are a special kind of object existing in the third realm.

Russell's early views on meaning and denotation differed markedly. His views changed often during this period, but if we focus on the view he presented in the 1903 piece entitled "On the Meaning and Denotation of Phrases" (*Papers* 4: 283–96), Russell held that phrases divide into different categories. Some phrases *only* have a denotation without expressing any distinct meaning at all; this is true of simple proper names like "Plato" and "Socrates". Other phrases only have a meaning with no distinct denotation. The verb "loves" that occurs in, e.g., the sentence "Plato loves Socrates", falls in this category: it *means* the relation of love, but does not *denote* anything in this context. Lastly, certain phrases had distinct meanings and denotations. Russell's paradigm for this last category of phrases were those beginning with the definite article, "the", i.e., those he would eventually call "definite descriptions". To give one of his frequent examples, the phrase "the centre of mass of the Solar System at the beginning of the twentieth century" has, for its denotation, a certain point, something utterly devoid of complexity, whereas this expression has as its *meaning* a certain "denoting complex", which is not simple and has constituents. Yet even in this last case there are important differences from Frege. For Frege, the senses of complex expressions are also complex, but they are complex *senses* and they consist entirely of senses. For Frege, the sense of "the centre of mass of the Solar System at the beginning of the twentieth century" consists of more simple senses such as the sense of "the Solar System" and "the twentieth century". For Russell, however, the Solar System *itself* and the twentieth century *itself* are constituents of the complex meaning of "the centre of mass of the Solar System at the beginning of the twentieth century". Russell's meanings, i.e., his denoting complexes, should not be understood as abstract objects with abstract constituents in the way that Frege's senses must be.

When it comes to complete sentences, Russell's views seemed to change often during this period, and perhaps never took a definite form until the distinction between meaning and denotation was abandoned in "On Denoting". At times, Russell seems to think of sentences as falling into the category of expressions that have meanings without denotations, specifically, as having *propositions* for their meaning. On other occasions Russell seems to think that sentences have propositions as their mean-

ings, and that true propositions *denote* facts while false propositions lack denotation. At still other times, Russell seems to regard sentences as having denoting complexes for their meanings, and to regard propositions as being what these denoting complexes denote. On this view, one might regard the sentences “the author of *Ivanhoe* is a poet” and “the author of *Waverley* is a poet” as expressing different meanings that denote the same proposition. Lastly, Russell sometimes talks as though sentences stand for propositions, and that *propositions* are complexes that themselves have two sides, meaning and denotation. Unfortunately, it would take another paper to chart the different views contemplated by Russell on this matter, and the considerations that led to adopting or rejecting them. However, on his initial and usual view, denoting complexes or meanings are themselves constituents of propositions, differing only from other propositional constituents in that, when they appear in a proposition, the proposition is not *about them*, but rather *about* those things that they denote (see, e.g., *PoM*, §56). Yet he does, as we shall see, consider other views of the functioning of meanings within propositions.

To repeat, it is this theory of meaning and denotation that Russell attacks in “On Denoting” and not Frege’s theory of meaning. Indeed, interestingly, Russell was aware of many of these differences between his earlier views and those of Frege, and even in those passages in “On Denoting” in which he specifically *claims* to be addressing Frege’s views, the views he describes are actually his own prior views, making note of the important differences only in a footnote (*OD*, *Papers* 4: 418–19).

In “On Denoting”, Russell concludes that, on a dualistic theory of meaning, there are difficulties in attempting to disambiguate assertions about the meaning from assertions about the denotation by means of inverted commas. While again, I cannot here provide a full account of how he gets to this conclusion, it is interesting that Russell’s dislike of the inverted commas notation stems from considerations very similar to those that have led certain neo-Fregeans to the dislike of devices for speaking of senses or intensions with Montague’s hat-notation, “ $\wedge(\dots)$ ”, or Bealer’s bracket notation, “[...]”. Indeed, the infamous arguments Russell gave about the denoting phrase “‘*C*’”, written with inverted commas, could just as easily have been made about “ $\wedge(C)$ ” or “[*C*]”. (Let us use the letter “*C*” here, as Russell probably did, schematically.) Indeed, Tichý (*ibid.*, pp. 142–4) has made heavy appeal to Russell’s authority in his attack on methods that “disambiguate against the grain”,

as he sees it. Earlier, we saw fault with these notations insofar as they reproduce, rather than eliminate, the ambiguity of ordinary language. What Montague's " $\wedge(C)$ " stands for cannot be seen as a function of what " C " ordinarily stands for. So in the context of " $\wedge(C)$ ", the " C " cannot be seen as standing for what it normally does. This is roughly the same point Russell makes with regard to reading "' C '" as shorthand for "the meaning of C ", because in that case, "'the first line of Gray's Elegy'" would be shorthand for "the meaning of the first line of Gray's Elegy", which, as Russell puts it, "gives us the meaning (if any) of the denotation" rather than what we wanted (*OD, Papers 4: 422*). The inverted commas, Bealer's brackets or Montague's hat, cannot be understood as signs standing for a *function* that operates on the denotation of that to which it is applied.

Even during the years in which Russell himself held a meaning and denotation view, he struggled long and hard with the attempt to find an adequate notation for disambiguating. In a manuscript entitled "Dependent Variables and Denotation" (*Papers 4: 297–304*), thought to have been written in July 1903, we find him attempting various strategies. There, he writes, sadly, "it seems demonstrable that no symbolism will do what we want" (p. 299). His rejection of a style that attempts to disambiguate *against* the grain is exceedingly swift. He claims:

We want a general symbol by which we can denote a given meaning, e.g. if we wish to denote the meaning of " $p \supset q$ ", we might write $Mg(p \supset q)$. But this is not a function of what is denoted by " $p \supset q$ ". (*Papers 4: 301*)

The notation " $Mg(\dots)$ " is an obvious variant of Montague's hat-notation " $\wedge(\dots)$ " and the like; it suffers from the defects we've already seen. Here we have an early precursor to the rejection in "On Denoting" of the inverted commas notation.

Of course, to his own mind, Russell was not *merely* rejecting or criticizing a style of notation, the way in which Tichý might want to criticize the style of notation used by Montague, Bealer or even Church and Kaplan. In "On Denoting", Russell is doing something much more ambitious: he is arguing that the entire theory that adopts a distinction between meaning and denotation is misguided. Indeed, Russell himself rarely thought about notation as such. What might appear to us as notational issues would have appeared to him as metaphysical ones. For

him, the structure of a form of notation was simply a reflection of the structure of the propositions expressed. When seemingly considering notational styles that incorporate disambiguating against the grain, Russell was really considering a certain theory about how meanings or denoting complexes operate in the context of a proposition. On the notational side, for these styles, a phrase that has both meaning and denotation, *alone and unmodified*, is thought to stand for the denotation, and a modification is made to the phrase only when one wishes to speak about the meaning itself. Russell is apt to equate this notational approach with the theory that when a meaning (i.e., the denoting complex) occurs in a proposition *alone and unmodified*, the proposition is about the denotation, and some modification or elaboration must be made to this entity in order to get the proposition to be about the meaning itself. (This is more or less the view he had initially espoused about denoting concepts in *PoM*, §56.) In his eyes, this style of notation would be appropriate if and only if the corresponding theory of the constituents of propositions was correct. Consequently, much of his discussion of these issues seems to move back and forth between the linguistic and the metaphysical levels. For him, the questions were inseparable.

So when Russell criticizes the inverted commas or “Mg” notation, he rejects not merely a notational style, but a metaphysical thesis. Consider, metaphysically, what such a thesis would amount to. The phrase “the author of *Waverley*”, unmodified, would, by itself, represent the denoting complex occurring in a proposition in its normal guise, in which the proposition is about the denotation, Scott himself. However, the phrase “Mg(the author of *Waverley*)” would have to represent some modification or more complex propositional constituent that would somehow make the proposition about the meaning or denoting complex itself. But this more complex propositional constituent—if its structure is to read off the symbolism—would seemingly *contain* the original denoting complex, in exactly the same way that the complex denoting complex represented by “the paternal grandfather of the author of *The Principles of Mathematics*” contains as a constituent the denoting complex represented by “the author of *The Principles of Mathematics*”. However, in this embedded spot, the denoting complex would have to operate as normal, thus shifting the “aboutness” to the denotation. Insofar as the remainder of the more complex meaning operates as a function, the function would have to operate on the denotation. For this reason, the

denoting complex represented by "the paternal grandfather of the author of *Waverley*" denotes the same person as that represented by "the paternal grandfather of Scott". By like token then, using "*a*" to transcribe "the author of *Waverley*" and "*s*" for "Scott", " $Mg(a)$ " would have to denote the same thing as " $Mg(s)$ ". The "against the grain" mode of symbolism goes hand in hand with the "backward road" approach to meaning, and as such, is intrinsically flawed.

5. RUSSELL ON DISAMBIGUATING WITH THE GRAIN

Russell's dislike of such "backward road" approaches, and especially his intricate discussion of it in "On Denoting", has been explored at greater length by other authors. My own interests lie elsewhere, in particular in his confrontation with notational strategies closer to Tichy's approach of disambiguating *with* the grain, along with the metaphysical views he associates with this rival approach. It may be that there are some traces of this discussion in "On Denoting", but I believe the more explicit treatment of such approaches comes in some of Russell's earlier manuscripts, especially 1903's "Dependent Variables and Denotation", and 1905's "On Fundamentals".

While Russell's mature "substitutional theory", as it is typically called, was not adopted until late 1905, already in 1903 he was concerned with the notion of the substitution of one entity for another within a complex. In "Dependent Variables and Denotation", he considers a strategy in which signs were generally taken to stand for ("concern themselves with") meanings as opposed to denotation, a move that was found to be independently helpful for understanding such substitutions. This approach also made use of a new primitive function sign, "Dn", to map a meaning onto its denotation. He writes:

In $p \frac{y}{x}$, we want p to be a *meaning*. Thence we must go to $Dn(p \frac{y}{x})$, which we must define for all cases. And Dn is an indefinable function. We may put:

If p is a meaning which unambiguously denotes q , then $Dn|p$ is to be q ; if not, $Dn|p$ is to be p . (*Papers* 4: 301)

Russell's "Dn" here has obvious parallels to Tichy's " \vee "; the advantages over attempting to disambiguate using " \wedge " and "Mg" are clear enough from the previous discussion.

A similar approach taken elsewhere in the same paper employs an inverted iota, “ ι ”, familiar to readers of Russell’s later works as part of his abbreviation for definite descriptions. This is not at all coincidental. Earlier I claimed that phrases of the form “the so-and-so” represented Russell’s paradigmatic example of phrases that had *both* meaning and denotation. At times, he was convinced that *all* phrases that have both sides are of this form, or can be reduced to this form. The meaning of such a phrase involves a class concept or propositional function, and the denotation of such a phrase is precisely that individual to which that concept uniquely applies. He then considers a notation such as “ $\iota\phi$ ” to be read as “the ϕ ”; the constituents of ϕ are then the constituents of the *meaning*, while the denotation, written “ $\iota\phi$ ”, may be simple. The iota, then, works just as a denotation functor, taking the potentially complex concept doing the denoting to its denotation. He summarizes the approach this way:

We should have to use p to *denote* the complex meaning; then $p \frac{y}{x}$ would denote another complex meaning. Then ιp , $\iota p \frac{y}{x}$ would be what these meanings denote.

Put: if x denotes a meaning which denotes a single object, then ιx is to denote that object. (*Papers* 4: 298)

Immediately after the passage about “Mg” quoted earlier, Russell continues,

It is better to keep to ι . Thus ιx is a function of x , but not x of ιx ; for when the denotation is given, the meaning is not given. (*Ibid.*, p. 301)

The advantage listed here for disambiguating with “ ι ” instead of “Mg” is precisely akin to the reason Tichý would put forth for disambiguating with “ \vee ” instead of “ \wedge ”.

But interestingly, Russell does not seem content with either “Dn” or “ ι ”. His critical remarks about them are somewhat cryptic. For “ ι ”, he writes, “the above view of ι is ridiculous. For ιx still is a meaning, and must *denote* what we want” (*ibid.*, p. 299), and for “Dn”, he complains, “but this won’t quite do, because ... in $\phi|x$, we must take what is denoted as the indefinable, or else Dn| p will be the *meaning*, which is not what is wanted” (*ibid.*, p. 301). Cryptic as these remarks are, it is not

altogether impossible to piece together the problem.

By way of introduction, let me describe a difficulty—real or imagined—that occurred me when I first read Tichý's suggestions explained above, prior to my reading of Russell's manuscripts. I should note that I do not put much stock into these considerations; it is merely something that seemed puzzling to me at the time. The move to the attempt to disambiguate with the grain is made precisely to retain functionality, to allow that the signs occurring in the argument places of the functors used to disambiguate stand for the same things they would in any other context. But disambiguating with the grain requires that the signs, alone and unmodified, themselves stand for senses. But the same would then be true for function signs, and if we insisted upon this for *all function signs*, it is arguable that it would become *impossible to escape from the realm of sense*, because by adding function signs, even those supposedly used to do the disambiguating, all we get are more complex senses.

In the case of Tichý, take the examples considered earlier of " $P(a)$ " and " $P(s)$ ". We know here that " a " and " s ", as such, stand for the senses of "the author of *Waverley*" and "Scott", respectively. What of the function sign " P "? Without " \vee " prefixing it, this too stands for a sense, understood here as a function from senses to more complex senses. To get the actual function—the Fregean concept (i.e., a function from individuals onto truth-values)—picked out by this sense, one would need to prefix the " P " with " \vee ". Consequently, " $P(a)$ " and " $P(s)$ " stand for complex senses—Fregean thoughts—which is what allows them to serve their intended purpose in (1t) and (2t). Now, Tichý's " \vee " is *supposed* to work just like any other function sign; this is its entire alleged advantage over other approaches to disambiguating. But it is obvious that it *couldn't* work *exactly* like the other function signs. If " \vee " worked like other function signs, it would, *alone and unmodified*, stand for a sense, and written together with an argument, it would stand for a more complex sense. Then " $\vee s$ " and " $\vee a$ " would, like " $P(s)$ " and " $P(a)$ ", again stand for different senses with the same referent. If " \vee " really worked just like " P ", then for the same reason we need " $\vee P$ " to get to the actual function *denoted* by the ordinary language predicate "is a poet", we would need " $\vee\vee$ " to get at the actual function intended by " \vee ", which might be plausible, except that the very same difficulty arises with the first occurrence of " \vee ", and adding more isn't likely to help. So it seems that Tichý's " \vee " is really rather different from other function signs; in

fact, it must fall into its own category altogether as *the only thing that itself escapes the sense/reference dualism*. The alleged function sign “ \vee ” surely can’t be taken to have a sense. If it did, it would be the one and only one sense it would be impossible to talk about, for surely “ \vee ”, *alone and unmodified*, can’t stand for that sense, or we could get nowhere.

I think Russell disparages his “Dn” for largely similar reasons. For any other function-sign “ ϕ ”, the application of that sign to a name for an argument “ x ” yields a sign for a more complex meaning, “ $\phi|x$ ”; if Dn operates as just another function, then Dn| p is just another meaning, and still hasn’t brought us to the denotation we want.

In “On Fundamentals” two years later, we find Russell struggling with the same problems. There too Russell is tempted to make use of the tack of disambiguating with the grain, along with a denotation function, now written “ δ ”:

In such a case as the above, “ $\delta'p$ ” for “the denotation of p ” would be useful. It is useful when ... p stands for the meaning, not its denotation. Then $\delta'p$ will do for the denotation. (*Papers* 4: 363)

But of course the same difficulty re-emerges, as he says, “if we put (say) $\delta'a$ to stand for “the denotation of a ”, that still only denotes the denotation, and does not mean it” (4: 360).

To fully understand these comments of Russell’s, one must remember that, again, he would not understand these as purely notational issues. Just as styles of notation that attempt to disambiguate against the grain would be understood by Russell as coinciding with a certain theory of the functioning of meanings within propositions, notational styles that involve disambiguating with the grain would be seen by him as going hand in hand with a *rival theory*. Here the supposition would be that a meaning or denoting complex normally occurs in a proposition in much the same way as any other entity; as such, a proposition in which it occurs is *about it*. Just as at the level of notation, one must add a functor to get something that actually stands for the denotation, at the level of the proposition, the denoting complex or meaning itself must be further specified or elaborated or made more complex in order for the “aboutness” of the proposition to shift to the denotation. But exactly how are we to understand this?

Again, consider more complex denoting complexes or meanings. On

this view, when the meanings that "*a*" and "*s*" stand for occur in a proposition, as such, the propositions are about those *meanings* themselves, not what they denote. These meanings can form parts of more complex meanings or denoting complexes by adding a functional constituent to the denoting complex. Hence if we use "*f*" as our transcription of "the paternal grandfather of ...", then "*f|a*" and "*f|s*" would stand for the more complex meanings expressed by "the paternal grandfather of the author of *Waverley*" and "the paternal grandfather of Scott", respectively. Here, "*f|s*" and "*f|a*", without "Dn" involved, still stand for two *different*, albeit codenotative, meanings, with one containing the meaning that "*a*" stands for in the place in which the other contains the meaning that "*s*" stands for.

However, assuming the function Dn works just like the function *f*, it would appear that "Dn|s" and "Dn|a" should also stand for two different complex meanings, when they are supposed to stand for the same denotation. The puzzle really involves what exists at the level of the proposition over and above the meanings themselves in order to make the "aboutness" shift—what does the "Dn" add to the proposition? On this view, there can't be *nothing* there, because, in their normal mode of occurrence, the meanings do not shift aboutness. But there can't be *more complex* denoting complexes, Dn|s and Dn|a, every bit like *f|s* and *f|a*, because then the propositions would be about *these* two different more complex denoting complexes, not what those complexes denote.

The only other option, I suppose, would be to take "Dn" to stand for a special sort of propositional constituent, indeed, a unique sort of function, very different from "*f*". When it comes together in a proposition with a meaning, it forces a shift in aboutness, unlike all other functions or propositional constituents. Hence sentences in which "Dn|a" occurs will express propositions about the same person as those expressed by sentences in which "Dn|s" appears. Here, the approach would be completely akin to Tichý's approach of treating "∨" as unlike all other function signs. Russell does not seem to consider such an approach, but I do not think it would have taken him long to reject such an approach had he considered it. After all, even if corresponding sentences in which "Dn|a" appears in the place of "Dn|s" express propositions that are about the same person and are alike in truth-value, surely they must nevertheless be *different* propositions, and hence, the *actual* constituents of the propositions contributed by "Dn|a" and "Dn|p" must be different. *And*

it must be possible to disambiguate here too; it must be possible for there to be propositions about *these* entities. However, if “Dn” works in this special way that always shifts aboutness, there could be no such propositions, as there would be no way to keep the aboutness from shifting once Dn were involved.

6. CONCLUSION

Later in “On Fundamentals” Russell carries on a discussion that is echoed strongly in the Gray’s Elogy passage of “On Denoting”. He considers the inverted commas notation for taking a name of a denotation to be its meaning; he also considers a notation that takes “C” by itself to be the name of a denoting complex and adding a “the denotation of” operator; noting only that we get another, more complex denoting complex out of such a construction as “the denotation of C”, and do not actually get what C denotes as such. The point finally comes to a head in what might be taken as the most crucial turning-point of the discussion of “On Fundamentals” (and one might argue, of Russell’s career). It is worth quoting it at length:

It might be supposed that the whole matter could be simplified by introducing a relation of denoting: instead of all the complications about C and "C", we might try to put "x denotes y". But we want to be able to speak of what x denotes, and unfortunately "what x denotes" is a denoting complex. We might avoid this as follows: Let C be an unambiguously denoting complex (we may now drop the inverted commas); we then have

$$(\exists y) : C \text{ denotes } y : C \text{ denotes } z \cdot \supset_z \cdot z = y.$$

Then what is commonly expressed by $\phi' C$ will be replaced by

$$(\exists y) : C \text{ denotes } y : C \text{ denotes } z \cdot \supset_z \cdot z = y : \phi' y.$$

Thus, e.g., ϕ' (the author of *Waverley*) becomes

$$(\exists y) : \text{"the author of } Waverley\text{" denotes } y : \text{"the author of } Waverley\text{"} \\ \text{denotes } z \cdot \supset_z \cdot z = y : \phi' y$$

Thus "Scott is the author of *Waverley*" becomes

$(\exists y) :$ "the author of *Waverley*" denotes y : "the author of *Waverley*"
denotes $z \cdot \supset_z \cdot z = y$: Scott = y .

This, then, was what surprised people, as well it might. On this view, we shall not introduce $\iota' u$ at all, but put

$$\phi' \iota' u . = : (\exists y) : y \varepsilon u : z \varepsilon u \cdot \supset_z \cdot z = y : \phi' y.$$

This defines all propositions about $\iota' u$, which is all we need. But now $\phi' \iota' u$ is a bad symbol; we shall have to substitute (say)

$$(\phi 1)' u .$$

On this view, "the author of *Waverley*" has no significance at all by itself, but propositions in which it occurs have significance. Thus in regard to denoting phrases of this sort, the question of meaning and denotation ceases to exist. It remains to examine other sorts of denoting phrases. (*Papers* 4: 383–4¹⁷)

Russell begins this discussion with the problem we have been discussing: the problem of adequately defining a notation for his theory of meaning and denotation. He ends with his very first statement (however rough) of the theory of descriptions.

Interpreted in light of the discussion of the previous sections, we can look at what is going on in the following way. Suppose "C" (here taken schematically) is our transcription into our logical language of some natural language phrase that has a distinct meaning and denotation—Russell's favourite example being "the author of *Waverley*". Here we can take two tacks. If we take "C", alone and unmodified, to stand for the denotation, then we would need some other method of talking about the meaning. However, inverted commas or Montague-like hats do not work properly as "meaning of" functions operating on the denotation, since there is no backward road. If instead we take "C" as, alone and

¹⁷ In this quotation, all the double inverted commas penned by Russell have been rendered as typewriter, or "straight", quotation marks, in line with the policy discussed in note 14. It is clear that at most occurrences he is using them for the to-be-rejected method for speaking about meanings. Especially at the second and last occurrences, it is not entirely clear that they were meant in this way, but I have not seen fit to impose any distinction on Russell's punctuation that wasn't made by his own pen.

unmodified, standing for a meaning, and then if we try to disambiguate by introducing a function sign “ δ ”, or the like, the result will stand for a more complex meaning or denoting complex, and again will not get us to what we want.

Realizing this, in the above passage, Russell moves to a notation that involves a denotation *relation*. In effect, what Russell is doing can be seen as *sliding* back from either a Montague-like or a Tichý-like approach to a Church-like approach. Here, we just have completely different signs for meaning and denotation, and we use a relation sign to express the connection between the entities involved. However, the difficulty comes in that, without a Tichý-like “ \vee ” function, it becomes difficult to speak of *the* entity—whatever it is—that the meaning *C* denotes. What Russell realizes, however, is that it can be done by means of quantifiers and identity, since meanings have unique denotations. Finally, bearing in mind the relationship between a meaning or “denoting complex” and the predicate or “class-concept” it contains as constituent, Russell realizes one can make do *without* even a denotation relation and simply with the quantifiers and the class-concepts. The result, however, completely eliminates any need to treat denoting complexes or meanings as nameable entities, or even include them at all in the analysis. One can treat expressions that would seem to have the two sides as what Russell would later call “incomplete symbols”. There is more to this story that we cannot delve into here,¹⁸ but it seems Russell was not only aware of the sort of difficulties facing those who would attempt to devise an adequate notation for an intensional logic built on a Fregean-style semantic dualism, but that his dissatisfaction with the available options was largely responsible for his abandoning that dualism and his replacing it with a sophisticated form of semantic monism: the theory of descriptions.

Given the immense importance of the theory of descriptions historically, Russell’s confrontations with the issues we have been discussing are certainly of historical interest. However, I believe they may be of current interest, especially to neo-Fregeans. We have seen that a parallel issue has arisen more recently among more contemporary Frege scholars, and that a satisfactory solution has yet to be found there. Russell’s own confronta-

¹⁸ For more of the story, see, e.g., Makin, *The Metaphysicians of Meaning*, Chap. 2.

tions with the issue many years earlier may be instructive in many ways. Can a semantic dualist explain disambiguation, either against the grain or with the grain? In most discussions among neo-Fregeans, the issue has been posed largely as a notational issue. One thing that is striking about Russell's treatment of the issue is that for him, the issue was entangled with related metaphysical issues. At first blush, this may seem to be some sort of conflation, another instance of Russell's infamous sloppiness about use and mention. However, I think the approach is actually quite refreshing, especially in the context of philosophers who want their notation to reflect their metaphysical commitments. Can Tichý or other neo-Fregeans explain the metaphysical difference—at the level of the *thought* or *proposition* expressed—between some statement about a sense " $\phi(a)$ " and the supposedly different statement about its referent, " $\phi(\vee a)$ "? Indeed, can they explain the metaphysics behind disambiguation at all?

I don't mean to suggest that the challenge that Russell offers on these issues is unanswerable. Certainly, some of his arguments seem to stem from the peculiarities of the brand of meaning/denotation theory to which he was, for a time, attracted. Tichý's own brand of neo-Fregeanism actually involves a semantic "tri-alism", separating out not only reference and sense, but reference, sense and what he calls "presentation". This complication in Tichý's views may actually provide some help in providing an answer to these challenges. An exploration of whether or not various neo-Fregean approaches have the resources to respond to Russell would have to be the task of a further paper. However, there may certainly be some grounds to Russell's pessimism. He himself abandoned a fairly worked-out theory in virtue of these difficulties, and Bertrand Russell, after all, was no buffoon. He did not discard theories without due consideration. So the challenge he offers is not one to be taken lightly.
