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THE NUMBER OF SENSES

ABSTRACT. Many philosophers still countenance senses or meanings in the broadly Fregean vein. However, it is difficult to posit the existence of senses without positing quite a lot of them, including at least one presenting every entity in existence. I discuss a number of Cantorian paradoxes that seem to result from an overly large metaphysics of senses, and various possible solutions. Certain more deflationary and non-traditional understandings of senses, and to what extent they fare better in solving the problems, are also discussed. In the end, it is concluded that one must divide senses into various ramified-orders in order to avoid antinomy, but that the philosophical justification of such orders is, as yet, still somewhat problematic.

1. INTRODUCTION

For Frege (1892a), a sense was thought to consist in a 'mode of presentation' of a referent. Although it is fair to say that almost no contemporary philosopher subscribes to Frege's original views on senses down to the letter, many – e.g., Graeme Forbes (1990), John Searle (1983), Jerrold Katz (1990), etc. – do still countenance senses in some form or another. Even those who are wary of making room for senses in their theories of the semantics of language sometimes make room for 'modes of presentation' or 'methods of representation' in their epistemologies or philosophies of mind. Recently, certain direct reference theorists for names, although sticking to the claim that 'Hesperus is a planet' and 'Phosphorus is a planet' are indistinguishable semantically, have nevertheless invoked 'modes of presentation' or 'ways of taking' in the belief relation as a way of solving certain belief puzzles (see, e.g., Jeshion 2000). The issues raised in this paper are a concern not just for the traditional Fregean conception of senses, but, at least potentially, for *any* theory that legitimizes talk about different senses, whether or not those senses are thought to be metaphysically fundamental or irreducible. In the end, I think this *should* include just about everyone's theory, because, surely, common sense alone makes it obvious that the same object can be grasped, gotten at, conceived of, or represented in different ways, and there must be *some* explanation for this. As the title of the paper suggests, the questions I want to raise center



around how many senses there are. All that is required to get these issues off the ground is that it makes sense to discuss senses, distinguish them or identify them, enumerate them and/or quantify over them. (It is reasonable to think that if we can do any of these things, we can do all of them.) One does not have to buy into Frege's 'third realm' for this to be possible. After all, even a reductionist about persons, about countries, or about headaches can believe it is meaningful to ask how many of these things there are (in the room, in NATO, or since the neighbors bought a new stereo, etc.)

In what follows, I shall often begin by assuming a rather traditional Fregean conception of senses, but it is never to be taken as a dogmatic assumption. Along the way, I shall pause to discuss alternative conceptions of senses and to what extent those conceptions fare better (or worse) than the traditional conception when it comes to dealing with the issues under consideration.

2. THE PROBLEMS TO BE SOLVED

The question as to how many senses there are may seem purely recondite and academic. After all, no one cares how many grains of sand there are in the Sahara. Why be any more concerned with how many senses there are? However, it would be folly to remain unconcerned. The importance of the issue is perhaps most easily seen from the standpoint of the traditional Fregean theory, in which senses are necessarily existing abstract objects. Consider the difficulties that have arisen with attempts to enumerate other abstract entities, like sets, classes and numbers: Cantor's paradox, Russell's paradox, the König-Dixon paradox, the Burali-Forti paradox, etc. These problems have had an enormous influence on logic and the philosophy of mathematics. Similar paradoxes are at least potentially a concern for the attempt to enumerate senses, and just as, e.g., avoiding Russell's paradox has had the effect of greatly constraining what sorts of theories of the nature of classes and sets can be adequate, I believe similar paradoxes greatly constrain what theories of the nature of senses can be adequate.

Theories of senses seem particularly liable to get into trouble with Cantor's power-class theorem, according to which there must be more classes of entities in a certain domain (i.e., subclasses of that domain) than there are entities in that domain. However, assuming it is intelligible to speak of classes of senses, it is hard to maintain that there are more classes of senses than there are senses. Indeed, to *guarantee* that there are as many senses presenting classes as there are classes, we need only make the following three assumptions:

Principle of Conceivability (PC): *for every entity, there is at least one sense presenting it as referent.*

Principle of Determinacy (PD): *no sense presents more than one entity as referent.*

Principle of Classes as Entities (PCE): *classes are entities.*

We shall discuss the viability of these and other principles in some detail below, but given these principles, every class of senses will have at least one sense that presents it (and only it), thereby posing a violation of Cantor's theorem.

A parallel difficulty can easily arise for *thoughts* or *propositions*, which many, from Frege on, have understood as *complex senses*, senses that would be expressed by complete sentences. If there are such things as thoughts or propositions, and they can be members of classes, Cantor's theorem requires that there be more classes of propositions than propositions. The difficulty, of course, is the apparent truth of the following principle (justified perhaps by considering, for each entity, the proposition that that entity exists, or is self-identical, etc.):

Principle of Propositions (PP): *for every entity, it is possible to generate a different proposition.*

Together with PCE above, this too yields a violation of Cantor's theorem, because for each class of propositions we can generate a different proposition, guaranteeing as many propositions as classes of propositions.¹

Applying the diagonalization recipe for generating an antinomy when Cantor's theorem is violated, we arrive at the following apparent contradictions:

Class/sense paradox: Some senses that present classes are in the classes they present; some are not. The class of all senses, e.g., contains all the senses that present it, while the null class does not contain any of the senses that present it. Let us define *C* as the class of all senses that present some class in which they are not included. Now consider any sense, *C**, presenting *C*. Is *C** in *C*? It is just in case it is not.

Class/proposition paradox: Consider the (false) propositions expressed by such sentences as 'every entity is in the class of humans', 'every entity is in the null class', 'every entity is in the class of propositions'. Some of these propositions are themselves in the classes they are about, such as the final example. Others, like the first two examples, are not in the classes they are about. Define *W* as the class containing every proposition of this

form that is not in the class it is about. Then consider R, the proposition that every entity is in W. Is R in W? It is just in case it is not.

These paradoxes, to be sure, not only involve classes, they are quite similar in structure to Russell's paradox. However, it should not be thought that these paradoxes would automatically be solved by whatever modifications we adopt for our class theory to solve Russell's paradox. For example, the above paradoxes, unlike Russell's paradox, *do not* violate the simple theory of types for classes, nor do they in any obvious way violate the 'axiom of foundation' used by many set theorists to avoid Russell's paradox. (I shall later take up the issue of *types* of senses to see what help such theories might have for avoiding these difficulties.)

Indeed, it is possible to generate similar antinomies without making mention of classes at all. This is especially true if a theory of sense is wedded to a liberal metaphysics of attributes, concepts or properties. The formulation of such antinomies is somewhat problematic, especially because of how widely divergent the views are of different philosophers with regard to how, and even whether or not, the sense/reference distinction applies to such things. Carnap's position, for example, is that a property is the intension (sense) of a predicate expression while a class is its extension (Carnap 1947). On Frege's own view, a concept (*Begriff*), which he understands as a special kind of *function*, is the *referent* of a predicate. Predicates also express senses, for Frege, but exactly what such senses are is a matter of some controversy among Frege scholars.² On Dummett's interpretation, however, the senses of predicate expressions are themselves objects, differing from other senses only in that the referents they present are concepts rather than objects (Dummett 1973, pp. 291–294). Using Dummett's terminology, we can state the following contradiction:

Concept/sense paradox: Some senses presenting concepts fall under the concepts they present, such as any sense presenting the concept of self-identity, or the concept of being a sense, etc. However, other senses do not fall under the concepts they present, such as any sense presenting the concept of being human, or being made of cheese. Define D as the concept of being a sense that presents a concept it does not itself fall under. Then consider any sense, D*, presenting D. Does D* fall under D? It does just in case it does not.

The concept/sense paradox can be understood as simply an intensional variant of the Grelling paradox. Given those already outlined, the following paradox is perhaps an obvious addition to the list:

Concept/proposition paradox: Consider the propositions expressed by such sentences as 'every entity is red', 'every entity is human', 'every entity is self-identical', etc. Some such propositions themselves fall under the concepts that they generalize, such as the final example. Others, such as the first two, do not fall under the concepts they generalize. Define F as the concept that applies to something just in case it is a proposition of this form that does not fall under the concept it generalizes. Now consider P, the proposition that every entity is F. Does P fall under F? It does just in case it does not.

I certainly do not mean to suggest that all of these contradictions are a problem for every theory of the nature of senses. In fact, as I discuss later, there is at least some reason to think that Frege's own views would be immune to the concept/sense paradox. Each theory of the nature of senses must be examined carefully in order to determine whether or not it is prey to these (and similar) paradoxes. Given the relation of these paradoxes to Cantor's theorem, it is natural to begin our examinations by looking at *how many* senses we can assume to exist. Just as Russell's paradox seems to provide a reason to be wary of positing too many classes too easily, these paradoxes seem to give reason for caution with regard to positing too many senses too readily.

Unfortunately, however, it is very difficult to countenance senses at all without countenancing quite a lot of them. Given PC and PD listed above, all it would take to guarantee an infinite number of senses is the assumption that there is at least one entity, A, that isn't a sense, along with the following principle:

Principle of Sense as Entities (PSE): *senses are entities.*

By PC, A would be presented by at least one sense S₁, and S₁, as an entity, would be presented by at least one sense S₂, and S₂ would be presented by S₃, and so on *ad infinitum*.³ (It is necessary to suppose that A is not a sense to ensure that this chain is not circular. If A were a sense, it might be that S₁ presents A, S₂ presents S₁, and A presents S₂. PD ensures that the chain does not loop back in on itself at any point other than A.) Assuming that there is a non-denumerably infinite number of things that aren't senses (e.g., points in space), we would be guaranteed a non-denumerable number of infinite hierarchies of senses, and *a fortiori*, a non-denumerably infinite number of senses. Principle PP, along with an assumption to the effect that propositions are entities, would generate similar results. It would certainly seem desirable to avoid these results if at all possible. In order to do so, it is necessary to deny one or more of PC, PD or PSE.

3. SENSES WITH MULTIPLE REFERENTS?

At first blush, PD perhaps seems most innocuous. According to the traditional theories of sense, the way a sense works is by setting forth a certain property or characteristic, and the referent presented by the sense is precisely that thing of which this property or characteristic is uniquely true. If nothing has this property, or if several things do, the sense is usually taken to be without referent (Frege 1892a, p. 153). On this model, it is of course impossible for a sense to present more than one referent. Even on other views of the ways senses work, something like PD is usually assumed. After all, it is the task of senses to *determine* reference. If a sense presents more than one thing as referent, reference would not be determinate (at least not by sense alone).

However, the point has been challenged, even by those who acknowledge senses as entities that have a role in determining reference. Consider the views of Jerrold Katz (2001), who believes that senses work *along with* features of the context of use to determine reference. According to his theory of proper names, for example, the name 'George Bush', expresses the same sense when used in 1991 to speak of the person who was president then, as it does in 2002 to speak of the current president. By itself, the same sense presents both father and son, but on any given use, features of the context work to make reference determinate. Technically, then, on his view, PD is false. This same sense presents *everyone* who bears the name 'George Bush'.

This may seem to help us find a way out of many of the problems discussed so far. Both the Cantorian paradoxes and the infinite hierarchies just considered *seem* to come from PC, positing a sense for every entity, be it a class or a point in space. However, PC *by itself*, does not guarantee that there is a *different* sense presenting every class. If it were possible, for example, for the same sense to present *many different* classes, PC would not in itself violate Cantor's theorem. Similarly, if it were possible for the same sense to present *many different* points in space, then there would not necessarily need to be non-denumerably many senses to present non-denumerably many points. Consider also, e.g., the effect denying PD would have on the class/sense paradox. Normally, the antinomy would result from showing that the conclusion that C^* is not in C follows by assuming that it is, and vice-versa. Suppose that C is in C^* . This means that it presents some class that it is not in, but this class *need not necessarily be* C . It could be some *other* class that it *also* presents. This assumption is not contradictory, and the antinomy is avoided.

However, I think this response works too fast. While on Katz's theory, *some* senses present more than one thing, it is not the case that *all* senses do. The sense of the name 'George Bush' may present more than one person but the senses of the expressions 'the 41st president of the United States' and 'the 43rd president of the United States' are unambiguous in whom they present. It is not as though there are *no* senses that uniquely single out either of these men. Let us call those senses that do present no more than one thing 'strong senses'. On most theories of senses, *all* senses are strong senses. For some theories, such as that of Katz, this is not so; however, I cannot imagine any theory of senses in which *no* senses are strong senses. It is true that without PD, PC does not guarantee as many senses as classes or points in space, but the need for PD would be obviated if PC were strengthened to the following:

Strong Principle of Conceivability (SPC): *for every entity, there is at least one strong sense presenting it as referent.*

If SPC holds, then the denial of PD in its general form provides us with no help for the problems under discussion. To resurrect the class/sense paradox, e.g., we need only assume that C^* is a strong sense. This leads us to our next question: is SPC true?

4. A SENSE FOR EVERY REFERENT?

PC and SPC, I think, stand or fall together. SPC *entails* PC, and most good arguments for PC would also be arguments for SPC. Indeed, on traditional theories of senses, in which all senses are strong senses, PC and SPC are equivalent. The issue requires most careful scrutiny, because there is, I think, a natural temptation to see these principles, as bold existential hypotheses positing the existence of senses, as doing most of the work in generating the paradoxes.

There may, however, be no knock-down argument for or against taking every entity to have at least one (strong) sense presenting it that is independent of a more detailed theory of senses. However, I suspect there is good reason to think that on the most natural theories, SPC will hold. Senses are modes of presentation of entities; they are that 'through which' we conceive or grasp objects. If a certain object is not presented by any sense, it would seem that it must be impossible to conceive of or name that object. Hence the name 'principle of conceivability': this principle suggests that there is a way of conceiving of anything. While certainly there are particular entities that no one ever has conceived of or named

or will (despite Berkeley's 'master argument'),⁴ there is, I think, a strong intuition that there are no objects that are *incapable* of being named or grasped.

Indeed, in his discussion of Church's Logic of Sense and Denotation, in which, *unexpectedly*, the principle that every entity is presented by at least one sense follows as a theorem, C. A. Anderson (1980, p. 224) writes:

On the Fregean theory of meaning, an entity is *nameable* only if there is a concept [i.e., *sense*] of it (for the name, or description, to express), or so it might be argued. And is it not obvious that every entity whatever *is* nameable?

The argument is not cogent. From the fact (if indeed it is a fact) that some entities do not fall under any concepts – as things actually are – it does not follow that there are entities which *could not* fall under any concept. This reply is not affected by the suggestion that concepts have necessary existence.

His point, I take it, is this: just because no senses do present a certain object, B, this does not mean that no senses *could*. He grants that senses have necessary existence, i.e., that the same set of senses exists in every possible world. Because the relationship between senses and the referents they present is (at least in most cases)⁵ non-rigid, it is possible that in some other possible world the object B is presented by some sense that presents something *else* (or nothing at all) in the actual world. In that case, it is still *possible* to conceive of or name B, although B is not presented by any sense in the actual world.

Again, I think the argument moves too fast. If we consider senses to present their referents in virtue of the properties of the referents, a sense that presents B in some other possible world but not in the actual world could only do so if B had properties in the other world it does not have in the actual world.⁶ However, I think the intuition that we have that every entity is graspable and nameable is that every entity – *precisely as it is* – is capable of being grasped and named. That is, if B is not named or grasped in the actual world, it is still true that there is a world that differs from the actual world *only* in terms of what namers name and what conceivers conceive in which B is named or conceived of. If B is precisely the same in that world as it is in the actual world, then it is presented by the same senses in the two worlds. Hence, whatever sense through which B is conceived of or named in that world also presents B in the actual world. Therefore, if B is capable of being grasped or named – *precisely as it is* – then, even if it is not actually named or grasped, there are nevertheless senses that present it in the actual world.

Ideally, however, our acceptance of SPC should be based on an actual argument and not on mere intuition. On the sort of theory of the nature of senses we have been considering, the bare bones of such an argument is not difficult to construct. On such a theory, whenever there is a set of

conditions or attributes uniquely held by some entity, there exists a (strong) sense that presents that entity (and that entity alone) in virtue of this fact. Given the identity of indiscernibles, no two entities share all of their properties. For every entity, therefore, there is a set of conditions or attributes held by it alone, such as the logical conjunction of all its properties. Therefore – or so it might be argued – at least one strong sense exists presenting every entity, and SPC is validated.

There are, I suppose, a number of different ways in which this argument might be challenged. The identity of indiscernibles, for example, is by no means uncontroversial. If it is false, then we actually have very good reason to think that SPC is false as well. (After all, if two entities are exactly alike, how could it be at all possible to conceive or name one as opposed to the other?) Another potential failing with the argument is that, although the identity of indiscernibles guarantees that any two objects differ in at least one way, it does not guarantee that there is one specific property or *finite* combination of properties that by itself differentiates a given object from *all* other objects. It might be argued that senses, due to the role they play in cognition and understanding, can only be finitely complex. It may be true that the logical conjunction of all an object's properties is unique to that object, but if this conjunction is infinitely large, perhaps it does not correspond to a sense.

This is a difficult issue. I take it that complexity in the nature of senses is not itself problematic. After all, the standard examples given of senses in the literature (e.g., that which presents Venus in virtue of its shining the brightest in the morning sky) are by no means *simple*. If senses are abstract entities, independent of mind and language, as Frege himself thought, I see no principled reason why senses of *infinite* complexity would be excluded. However, other views on the nature of senses may have more trouble allowing infinitely complex senses. Still, it is not clear that they are required for the argument. On many metaphysical theories, properties divide into *primitive* properties (and relations) and derived or *defined* properties (and relations), and the number of primitive properties entities are thought to have is limited. If all other properties 'supervene' upon the primitives, then the logical conjunction of all its primitive properties alone will be unique to an entity. If these are finite in number, then the sense that presents it in virtue of this conjunction will only be finitely complex.

Of course, however, a full answer to this question would require settling some of the most fundamental questions about the metaphysics of properties and their relation to senses, which we cannot hope to do here. Still, I think there is good reason to think that on most traditional theories of the nature of senses, SPC (and hence PC) will likely hold (at least if the

identity of indiscernibles does). Indeed, in many metaphysical theories, the denial of PC or SPC would have very damaging consequences.⁷ For example, a *fact* is often equated with a true proposition (e.g., in Frege 1918, p. 342). If propositions are complex senses, and are about what their constituent senses present, the denial of PC would lead to the consequence that there are entities about which *there are no facts* – a very curious result indeed! Even on a more Russellian view of propositions, on which the constituents of propositions are not senses, but the very objects the proposition is about, if facts are true propositions, then in order to avoid the curious result just mentioned, PP would have to be assumed. While PP does not lead to precisely the same hierarchies and Cantorian paradoxes as PC and SPC, it does generate similar hierarchies and paradoxes, and hence, difficulties every bit as troubling.

5. CONSERVATIVE ONTOLOGIES OF SENSES

Frege himself thought that senses were abstract entities existing in a 'third realm', neither created by our mental or linguistic practices nor destroyed by their cessation. Surely, with this robust understanding of the reality of senses, it is not difficult to imagine that Frege's 'third realm' must be rather heavily populated. What, however, of more conservative ontologies of senses, such as those that make the existence of senses depend in some way on the mind or on language? Whatever else might be said for or against such theories, at first blush, they do seem to offer quite a lot of hope for reducing the number of senses posited. However, the appearance may be illusory.

On a psychological understanding of senses, it might be argued that there are no senses that are never 'used' by any mind. Therefore, only those entities about which people or other thinkers have thought, are thinking, or will think, have senses that present them. Surely, on this view, PC and SPC are likely to be false. Many, perhaps most, entities in existence, never come into contact with any mind.⁸ On this view, there is likely no violation of Cantor's theorem, because not every subclass of senses has been thought of, and hence there may be fewer senses than classes of senses. Similarly, not every concept applying to senses has been grasped by a mind. And the infinite hierarchies are not generated, because not every sense that has been *used* by a mind has also been *thought about* by a mind.

This sort of view of the nature of senses, to be sure, is not initially very attractive on other grounds, as Frege himself was prone to argue. That Venus can be represented both as that object that shines brightest in the morning sky, and also as that object that shines brightest in the evening sky,

seems to be entirely a feature of Venus itself; it does not seem contingent upon anyone's actually conceiving of Venus in these ways. The same sort of view, when applied to propositions as complex senses, at least in its crude form, would seem to yield the result that when a person realizes a new truth that no one has entertained before, he or she is not so much discovering a truth that was already there, he or she is instead *bringing* what it is that is true (the proposition) into existence. There also seems to be a difficulty on this view with explaining how it is that multiple people can seemingly make use of the same sense or grasp the same proposition. If senses and propositions are mental entities, they would seem, like mental states, to be private to a single mind. If they are capable of being shared, then it seems they must be independent of any one particular mind, and in principle, capable of existing independently of any minds. But perhaps the advantages of such a conservative ontology of senses in solving the difficulties here under discussion may provide sufficient motivation for attempting to work out these problems with a psychological theory of senses.

A linguistic understanding of senses, i.e., one in which the existence of senses is tied to their being expressed (or being expressible) in a language, may seem to have similar merits. Though most philosophers that countenance senses see them as playing a role in their philosophies of language, few take senses themselves to be linguistic entities. Indeed, the language-independence, and hence, language-neutrality, of senses plays a large role in their philosophical motivation, as they are often invoked in explaining what makes correct translations between languages synonymous. But it is perhaps not overly implausible to suppose that there is some connection between language and the existence of senses. There are several degrees of strength we might imagine the connection to have. The strongest, I suppose, would be to suggest that the only senses that exist are those that have actually been (or will be) expressed by actual uses of expressions in some existent natural language. This seems overly strong. Languages have a somewhat curious ontological status. They themselves arguably are abstract objects (as argued, e.g., in Katz 1981), and there certainly *seem* to be well-formed sentences and phrases of English that never have been (or will be) spoken or written by anyone. It would be difficult, I think, to maintain that these expressions are, as things actually are, without sense.

A more modest claim would be to suggest that there are no more senses than could *ideally* be expressed in *an ideally expressive* natural language. This more modest claim may itself greatly reduce the number of senses posited. While there are an infinite number of grammatically well-formed and meaningful sentences one could put together in English (or any other

natural language we have managed to invent), the number is only a countable infinity. Assuming that an 'ideally expressive language' would share this feature, there would, on such a view, be at most a countable infinity of senses. We know that if there are an uncountably infinite number of entities in the universe then not all of them can be named in any given language. On this view of senses, a similar result seems to follow: not every entity is presented by a (different) sense, and hence, SPC is false. Without SPC, the infinite hierarchies do not get underway. Similarly, the Cantorian paradoxes might be thought to be solved. If there are \aleph_0 senses, then the number of classes of senses is 2^{\aleph_0} , hence it is impossible that every such class is presented by a sense. This is as it should be, considering that some such classes of senses might have an infinite number of members with no common characteristics, and it would be impossible to describe or delimit them with a finitely long expression in language. If the class cannot be described or named in language, on this view, no sense could present it. Something similar might be argued to provide a response to the other violations of Cantor's theorem.

Though they seem to have their unattractive features, relatively conservative ontologies of senses, such as the psychologistic or linguistic varieties, do seem to be in a somewhat better shape vis-à-vis traditional theories when it comes to keeping the number of senses at manageable levels. However, I think the advantages are not quite so large as they may seem. It is true that these sorts of views do greatly restrict the number of senses we are moved to posit, and they can, to an extent, reconcile the existence of senses with Cantor's theorem in the sense that they can explain why we were too hasty in concluding that there must be as many senses as classes (or as many senses as properties or concepts, etc.) However, when it comes to the actual *paradoxes* discussed in Section 2, it is not at all clear that they are in any better shape. This requires careful scrutiny.

The paradoxes listed in Section 2 are Cantorian paradoxes in the sense that one can generate them by applying Cantorian diagonal methods. Cantor's own argument for the theorem that every domain must contain more subclasses than members was *entirely* based on the claim that whenever this condition is violated, such paradoxes result. Therefore, the only reason why a violation of Cantor's theorem should bother us at all is that it leads to such paradoxes. Now it may be true that the more conservative ontologies do not end with the result that there are as many or more entities in a certain domain as classes of entities in that domain, but it is not at all clear that they fare any better in solving the paradoxes. The paradoxes may have been originally inspired by Cantor's theorem, but they can take on a life of their own. If the more conservative ontologies of senses also succumb to

the paradoxes, then it is not clear that they offer *any* particular advantages over the more traditional theories.

Do the more conservative ontologies of senses help us at all with the paradoxes? It seems clear that they do not. Consider first the class/sense paradox. Even if it is not true for whatever reason that *every* class has at least one sense that presents it, so long as some classes have senses that present them, it still makes sense to speak of those senses that present classes that they are not in. So long as there is a class, C, containing all such senses, and any sense, C*, presenting C, there remains the question as to whether or not C* is in C. I do not see how the more conservative ontologies of senses would help us at all here. The only conceivable help such ontologies could give would be to deny that there is any such sense C* presenting C. Such ontologies, after all, make room for entities that have no senses presenting them. Could C be one such entity? This is very doubtful. The psychologistic and linguistic understanding of senses seems only to deny senses to those entities that no one has ever thought about or referred to using language. However, while writing this paper, I have conceived of C, and, I certainly *seem* to have just denoted it. Provided that the English expression 'the class of all senses presenting classes that they are not in' expresses a sense, then *that* sense presents C, and the question remains.

Similar considerations apply in the cases of the other paradoxes. On a more conservative ontology of propositions, propositions might only be posited to exist when someone has a certain thought or crafts a certain sentence. Only those properties or concepts that someone has thought about or named in language might be thought to have senses presenting them. But it is not at all clear how this will help us in getting rid of, e.g., D* from the concept/sense paradox, a sense presenting the concept of being a sense that presents a concept it does not itself fall under, or P from the concept/proposition paradox, the proposition that every entity falls under the concept of being a proposition asserting the generalization of a concept it doesn't fall under. Unluckily for the proponents of the conservative ontologies of sense, I seem to have made use of these senses in my own mental states and verbal behavior. I hope they will not blame me too much.

6. UNDERMINING THE INTELLIGIBILITY OF THE PARADOXES

We are left in need of a new solution. In addition to the notion of senses, the paradoxes themselves make heavy use of notions such as *classes*, *concepts* and/or *properties*. It is perhaps somewhat tempting to think that the paradoxes really arise from illegitimate use of these notions, and may not

have to do with how many or what sorts of senses are posited. There is perhaps something to this. Classes (or sets) have produced many other paradoxes and problems. The nature of concepts and properties is not all uncontroversial, and I have more or less been assuming that any grammatically well-formed predicate (regardless of complexity) corresponds to a concept or property of the same sort. None of the paradoxes or issues under discussion here can be given a complete diagnosis or analysis until the nature and existence (or otherwise) of such entities as classes and concepts have been thoroughly explored. However, I think it would be a mistake to think that we could expect the problems under discussion to be solved straightforwardly by examining the nature of these entities independently of discussing the nature of senses and how many and under what conditions they can be posited to exist. Firstly, as I suggested in passing above, it is not at all clear that the problems here under discussion can be solved by those theories of the nature of classes (or sets) and concepts that have been invoked to solve the paradoxes not relating to senses. The simple theory of types in class-theory, for example, divides classes into (i) those whose elements are urelements, (ii) those whose elements are classes of type (i), (iii) those whose elements are classes of type (ii), and so on. Senses and propositions are (presumably!) all urelements, and the classes defined in the class/sense and class/proposition paradoxes are classes of urelements, so there is no violation of simple type-theory. The axiom of foundation in many modern set theories rules out a set containing itself (or a different set in which it is included, etc.) Again, the paradoxes do not in any straightforward way require a violation of this axiom. Similar considerations apply for the concept/sense and concept/proposition paradoxes: neither involves something as dubious as asking whether or not a property applies to itself (or even something in the same 'ontological category' as itself.)

Secondly, even if the paradoxes could be solved by a theory regarding the nature of classes or concepts that was adopted for reasons independent of the particular sorts of worries under discussion here, it is not all obvious that such a theory could be motivated completely independently of any consideration of the nature of senses. Theories of *types* for example, are usually understood as a theories of *meaningfulness*. The various versions of Russell's paradox are sometimes said to be avoided because it is not even *meaningful* to ask whether or not a class is in itself, nor is it even meaningful to ask whether or not a certain property applies to itself. But if meaning is to be understood in terms of senses, how they work and how they fit together, e.g., to form propositions, such a claim could not be justified independently of considerations of the nature of senses. Frege (1891, 1892c), for example, himself argued – even before he was aware of the

paradoxes plaguing the foundations of logic – that it is never meaningful to ask whether or not a concept applies to itself. He based this largely on his understanding of the composition of thoughts: thoughts consist of senses; the sense of a (first-level) concept is incomplete or 'unsaturated', and thus coheres together with a sense presenting an individual to form a singular thought (proposition); however, one could not replace the sense presenting an individual with a sense presenting another first-level concept: the senses simply wouldn't 'fit together'. This conception of fitting together is largely metaphorical, but it has had a powerful influence on the imagination of many philosophers.⁹

Can a conception of the nature of senses similar to Frege's, involving the varieties they come in, and so on, provide help solving the paradoxes? Perhaps. I think Frege's views are probably *not* subject to the concept/sense paradox precisely as stated in the second section.¹⁰ For Frege, concepts come in varying levels. For Frege, both concepts and senses presenting concepts come in different varieties or 'levels'. A sense presenting a concept would probably not be of the right 'level' for the question to even arise as to whether or not that sense would fall under the concept. Frege dubs concepts that apply to individuals or objects 'first-level concepts'. However, senses presenting such concepts, as 'incomplete' or 'unsaturated' themselves, would not be objects. However, I fail to see how Frege's views regarding varieties of senses would provide any help for the other three paradoxes discussed in Section 2. The class C from the class/sense paradox is defined as a class of senses presenting classes. Assuming that all senses presenting classes are of the same 'type', then C*, as one such sense, would be of the right type for the question to arise as to whether or not it falls within C, a class containing such senses. Assuming all propositions fall into the same type, so that the same questions arise as to under what concepts they fall and in which classes they are included, no solution would be forthcoming for the class/proposition and concept/proposition paradoxes. As far as I can tell, for Frege himself, propositions (*Gedanken*) as 'complete' would all be considered objects of the same sort.

In his original formulation of the Logic of Sense and Denotation, Alonzo Church (1951) adopted a somewhat complicated theory of types of senses. In addition to treating senses presenting individuals and functions of different levels as themselves in different types, he also divided senses into different types depending on whether the referents they present are themselves senses or not. Thus, in addition to individuals, his type structure included the following hierarchy of senses: (i) senses presenting individuals, (ii) senses presenting senses of type (i), (iii) senses presenting senses of type (ii), etc. This hierarchy has interesting consequences for the

infinite hierarchy of senses discussed at the end of Section 2; while it does not block the result that every entity, including every sense, has another sense that presents it, each member of the infinite chain is of a different logical type, so one is not able to prove an infinity of entities of the same type simply by assuming a single entity that is not a sense. However, when it comes to the paradoxes themselves, these complications to the theory of types provide little help. Indeed, Church's first formulation of the Logic of Sense and Denotation was found to be formally inconsistent due, in essence, to the class/proposition paradox (see Myhill 1958, Anderson 1980). In later reformulations (e.g., Church 1993), we find Church advocating various forms of ramified type-theory.

Indeed, there is good reason to suppose that in order for a type-theory to provide solutions to all of the paradoxes, ramification will be necessary. Specifically, we must divide propositions into various orders. Propositions that are about other propositions or involve quantification over other propositions would necessarily be of a higher order than that of which they are about or over which they quantify. Properties of propositions would also be limited to those a certain order. For example, the property *F* from the concept/proposition would have to be refined to something like: being a general proposition of order *n* that does not fall under the concept it generalizes. Any proposition such as *P* that invokes *F* would not be of order *n*, and the question giving rise to the antinomy would disappear. The system of ramification would have to be rather complex if it were to be able to deal adequately with the class/sense paradox, as it would need to explain why a sense presenting a class of senses would need to be of a higher order than the senses included in that class. The details of such a system of ramification would perhaps be best worked out in the context of a formal system of intensional logic; then, one could explore in some detail whether or not the sorts of paradoxes discussed in Section 2 would be formulable in the system. We cannot make this attempt here.

What impact would ramification have on the question with which we began this paper, that as to how many senses there are? Ramification would seem to make it impossible to make *any* statement about the *totality* of senses or propositions. Because no proposition or sense can be about a range that includes itself, it would be impossible to think or talk about *all* senses or propositions irrespective of type or order. The only questions that could be meaningfully asked would be about the number of senses within a given type and order. Here, it is not quite so clear that one would be able to construct an argument for multiplying senses in objectionable ways. This is not to say that no arguments could be found for quite a lot of senses, perhaps even an infinite number. For example, given a sufficiently

fine-grained understanding of the identity conditions of propositions, one might argue that the propositions expressed by the members in the series of statements, '*p*', ' $\sim p$ ', ' $\sim\sim p$ ', ' $\sim\sim\sim p$ ', ..., etc., are all distinct propositions of the same type and order, or perhaps, better yet, the same might be argued about those in the series, 'Sam has fewer than 1 brother', 'Sam has fewer than 2 brothers', 'Sam has fewer than 3 brothers', etc. However, these arguments do not seem to result in contradictions or paradoxes.¹¹

This is not to say that there are not problems with this sort of ramification. Firstly, the very theory of ramification seems self-contradictory. Consider the statement, 'propositions must be divided into various orders'. This statement, if meaningful, expresses a proposition, and *seems* to express a proposition about *all* propositions, which is precisely what ramification is supposed to rule out. Similar problems, of course, arise in any type-theory. Russell himself, the first to suggest that propositions form 'an illegitimate totality' so that it is impossible to discuss all of them or enumerate them at once (see, e.g., Russell 1908), was himself very aware of the difficulties, and tried for years to avoid ramification. Indeed, at first, he found the suggestion 'harsh and highly artificial' (Russell 1903, p. 528). There may be reason to think that ramification would be even more difficult to justify within the context of a broadly Fregean theory of senses, in which propositions do not *contain* the entities they are about, but only senses that present those entities non-rigidly.¹² I do not pretend to have a solution to these difficulties.

7. CONCLUSION

Any theory that countenances senses, and allows that they can be identified and individuated, whether or not the theory in question takes senses to be ontologically basic, is left with a number of very difficult questions to answer regarding how many senses there are and under what conditions they can be posited to exist. We have not here presented a fully-worked out theory of senses, but it seems clear that providing an adequate solution to the paradoxes involving senses, classes and concepts, greatly constrains what sorts of theories of senses might be acceptable. Indeed, there may be reason to think that the theory must provide some grounds for dividing senses into a ramified hierarchy. There are many difficulties even on this strategy that remain to be worked out.

It might be thought that the whole mess can be avoided by eschewing senses altogether. But it is not at all obvious that this can be done successfully. It is true that there are strategies dating back to Russell's 'On Denoting' for denying that proper names and descriptive phrases like 'the

author of *Waverly*' have any meaning (sense) on their own, and instead suggesting that such phrases are only meaningful in the context of a complete sentence. However, this only pushes back the problem, because we are still left with the paradoxes dealing with propositions. It is also true that many philosophers have attempted to do without propositions, or proxy a theory of propositions in some way. For example, there is the common suggestion that a theory of propositions can be proxied in terms of sets of possible worlds. But even the *proxies* for propositions are often enough to generate paradoxes. Recently, a number of paradoxes similar to those discussed here have been found within the dominant theories of possible worlds (see, e.g., Kaplan 1995). The difficulties that arise with regard to the number of senses and number of propositions are *quite* difficult to escape from. My aim here has been the modest one of summarizing, and perhaps clarifying, the problem. No 'mode of presentation' has, unfortunately, presented the complete solution yet.

NOTES

¹ Russell (1903, Appendix B) was the first to appreciate this problem, alleging that it posed a great difficulty for establishing the number of propositions. He even wrote to Frege about the problem, suggesting that a similar problem might face Frege's theory of *Gedanken* (see Frege 1980, pp. 147–166). Frege himself seems to have missed Russell's point, as I have argued elsewhere (Klement 2002b).

² See Frege (1892b). For discussion of the interpretative issue, see Klement (2002a, pp. 65–76).

³ It should be noted that this result is independent from the question that arises within Frege's theory of indirect speech about the referents of clauses in multiply embedded intensional contexts. For Frege (1892a), in an intensional context, the referent of an expression shifts from its customary referent to its customary sense. On an orthodox reading of Frege, in a *doubly* embedded context, the referent of an expression is *doubly* shifted to a sense presenting its customary sense, and so on. This of course would guarantee an infinite hierarchy of senses, since intensional verbs can be embedded without limit. However, one could still hold an interpretation such as that of Dummett (1973) or Parsons (1980), in which the referent of an expression is the same in a singly embedded context as in a multiply embedded context, while still admitting that there are senses that present other senses, and perhaps even an infinite hierarchy of the sort described here. I see no reason why Dummett's or Parsons's theory of indirect reference would in any way call PC, PSE or PD into doubt, and with these principles in place, the result is unavoidable.

⁴ In his 'master argument', Berkeley (1713, pp. 35–36) suggested that the idea that there are entities that no one conceives of is an absurd hypothesis, because it is impossible even to conceive of something no one conceives of. This argument commits an evident scope fallacy. It is possible for me to conceive *that* there are things no one conceives of without there being something I conceive of that no one conceives of, just like I can believe that there are spies without there being someone I believe to be a spy.

⁵ In his theory of senses, Pavel Tichý (1988), e.g., countenances 'rigid presentations', which present the same object in all possible worlds in which that object exists. However, Tichý's work certainly will not help the opponent of SPC, because, on his view, every entity (including every sense) has a rigid sense that presents it. Rigid senses are strong senses, by itself guaranteeing SPC.

⁶ It should be noted that for this to be strictly correct, we must include relational properties. The sense of the expression, 'the tallest building in the United States' might denote the Empire State Building in another possible world in which all of its intrinsic properties are the same, so long as its relational properties (to, e.g., the Sears Tower) are different.

⁷ In addition to any metaphysically damaging consequences of denying SPC, there may also be rather serious logical and mathematical consequences. In logic, for example, if we admit that there is a class of entities that are not presented by any sense, we may very well be *forced* into denying or restricting the applicability of the axiom of choice or any sort of epsilon operator, because such things would allow us to select a single member of this class and speak of, e.g., *the entity that is not presented by any sense that is picked out by the selection function*, or, more loosely, the '*first*' (or '*least*') *entity not presented by any sense*. However, this seems to generate a paradox analogous to the König–Dixon paradox in mathematics, because I seem to have just provided a sense presenting this entity by singling it out as the entity picked out by the selection function. Cf. Anderson (1987, pp. 106–107).

⁸ Except perhaps, God's mind. If God exists, and God's cognition also works through 'senses', then surely SPC is undeniable, but these topics are obviously too large to be tackled here.

⁹ Of course, not all maneuvers that do the work of a theory of types are meant as restrictions on meaningfulness. Consider Quine's principle of 'stratification' in his 'New Foundations' (1937). Quine posits classes defined only for certain open expressions; an expression such as ' $x \in x$ ', as unstratified, corresponds to no class of things satisfying it. But Quine does not rule out ' $x \in x$ ' as ill-formed. Thus, he restricts his *existential posits*, not what formulae he considers to be meaningful. However, in order for a similar maneuver to solve the all of difficulties under discussion here, we would have to weaken our existential posits not only with regard to classes, but also with regard to *senses* and *propositions*. That is, rather than ruling out certain formulae as defining classes, we would have to deny *senses* or *propositions* corresponding to certain formulae. But this seems simply to collapse into a restriction on meaningfulness, because it would seem to imply that these formulae are without sense, without meaning. This is important to bear in mind when we turn to the evaluation of recent work done, e.g., by Cocchiarella (2000), who has aimed to expand a (broadly Quinean) notion of stratification to cover such paradoxes as Russell's paradox of the totality of propositions (essentially, the *class/proposition* paradox). Cocchiarella fashions his logical systems in such a way that some complete formulae, though well-formed (and thus, in a sense, meaningful) do not correspond to propositions as entities falling in the range of the quantifiers. Cocchiarella understands the meaning of expressions to consist in something like *truth conditions*, and hints that for some formulae, these truth conditions can be reified into individual objects, though not for others. This suggestion is in grave need of philosophical elucidation, for it seems to suggest that some meaningful assertions express propositions, others do not, which has an air of paradox to it.

¹⁰ Though this is not true if Dummett's interpretation of Frege is correct. I argued elsewhere against Dummett's view on independent grounds (Klement 2002a, pp. 72–74), and this is perhaps another reason to reject his interpretation.

¹¹ Citing Dedekind, Frege himself gave a similar argument for an infinity of thoughts. See Frege (1897, pp. 236–237n).

¹² It is perhaps easy to justify ramification by means of a vicious-circle principle if a general proposition ontologically depends on those entities it quantifies over, because then a proposition quantifying over a range that includes itself might be thought to depend ontologically on itself in some impossible fashion. But on a Fregean theory of meaning, senses do not ontologically depend on the existence of their referents; indeed, senses can exist with no referents at all. A general proposition would similarly exist independently of the entities in the range it quantifies over. So it is not easy to see what would be vicious about a proposition quantifying over a range that happens to include itself. I have discussed this issue in greater depth elsewhere (Klement 2002a, pp. 224–228).

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