About multidimensional spaces

Klimets AP

bure.aleksander2017@yandex.ru

Abstract

In the article, based on the philosophical analysis of the concept of "three-dimensional space", a model of multidimensional space is constructed, reflecting the properties of intersections of multidimensional spaces. The model reveals some unusual aspects of multidimensional spaces.

Philosophy of multidimensional space

Any measurement process is, in fact, an external relationship of some measured bodies or processes with other material bodies or processes acting as measuring instruments (watches, rulers, any devices, etc.).

The external character of spatial dimensions left an imprint on the formation of the corresponding natural-mathematical concepts. In particular, this was expressed in the concept of three-dimensionality of space. Real things, bodies, processes that a person encounters in practical activities are voluminous. Essentially, the bulk (or capacity) and represents the actual spatial extent. The space can not be something other than a collection of cubic meters. However, the expression of the real volume in cubic meters (centimeters, kilometers, etc.) was the result of a long development, primarily economic, but also scientific practice. The need to measure the acreage, the distances to which the herds were distilled, the migrations took place or the hunters left, strictly speaking, led to the fact that the initial basis of spatial measurements was the length and its abstract expression - the line.

Why is three-dimensional volume in Euclidean geometry? Because it is based on a line taken one-dimensionally; the lines form a two-dimensional plane, and a three-dimensional volume is constructed from the planes. Although this way is optimal and most satisfies the needs of the practice, it is still not the only possible one. Archeological data confirms that units of measurement of volume (capacity) are historically as ancient as natural units of time and length (day, month, foot, etc.). It can be assumed that if the practical needs of primitive people brought to the fore not the measurement of areas and distances, but the measurement of volumes, then the development of geometric science could take a different path than that laid out by Euclid. They say, for example: such and such a room (cave, temple, house, hall, etc.) is more than another; The new device (car) is more compact and takes up less space than the previous model. For all the approximations of the above comparisons, the real spatial volume is expressed here in one dimension: with

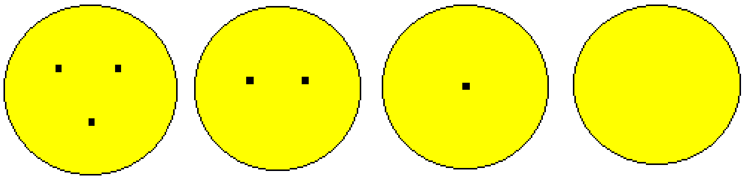


Figure 1:

respect to “more - less”. If, on the basis of similar or similar comparisons, we develop units of measurement for one-dimensional volumes and put them into the basis of some imaginary geometry, then the concept of a line in it could be completely different: for example, expressed in three dimensions, say as a third-degree root from a one-dimensional volume. Although this view seems pretentious at first glance, it’s not really unusual. The resulting line and the measured length, as well as their numerical values, are the result of a certain comparison of real volumetric objects.

From what has been said, it follows that neither two-, nor three-, or four-dimensionality, nor any other multidimensionality is identical with the real length, but reflect certain aspects of the objective relations in which it may be. The material world is the Euclidean world, the Lobachevsky world, the Riemann world, and the Minkowski world, because in terms of any of the geometries associated with the names of these eminent scientists, real spatial extent can be described and reflected as a universal attribute of material reality.[1]

Guided by the above, we construct a model of multidimensional space.

1 Model of multidimensional space

Consider three-dimensional space. Here the Pythagorean theorem is valid.

$$r^2_{m^2} = x^2_{m^2} + y^2_{m^2} + z^2_{m^2} \tag{1.1}$$

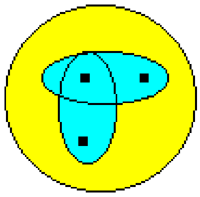
where r_m is the distance between any two points in space, in meters. It is known that the entire content of Euclidean geometry can be derived from the relation (1.1). Indeed, for example, in the geometry of Descartes, the Pythagorean theorem is an axiom.

Now consider the set consisting of three points (Fig.1). Here the points are symbols, elements of a set (instead of three points you can draw, for example, three crocodiles).

Let us assign a set of points to the set of dimensions of the 3-dimensional space. Then a 3-dimensional space corresponds to a set of three points, a 2-dimensional space is a set of two points, 1-dimensional space is a set of one points, 0-dimensional space is the empty set of points.

Consider the intersection of the subsets of points in the set of three points (Fig.2)

Recall that an intersection is a subset belonging to both intersecting subsets. In Fig.2, the subsets intersect, each of which consists of two points. As you can see, the subsets of two points can intersect at one point. In the 3-dimensional space, this corresponds to the intersection of two 2-dimensional planes intersecting along the 1-dimensional straight line.



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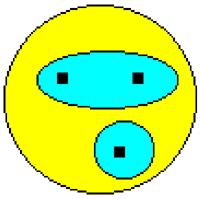


Figure 3:

Consider Fig.3. Here the intersection of two subsets of two points and one point occurs on the empty set of points.

In 3-dimensional space, this corresponds to the intersection of a 1-dimensional straight line and a 2-dimensional plane at a 0-dimensional point.

Similarly, we can consider intersections in the 2-dimensional space and the 1-dimensional space. The correspondence between the set of points and the set of dimensions of the spaces will be the same.

Now consider a set of four points, which corresponds to a 4-dimensional space (Fig.4)

As can be seen, in a 4-dimensional space, two 2-dimensional planes can intersect along a 0-dimensional point. This is impossible to do in 3-dimensional space.

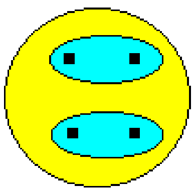
In general, if we consider a set of n points, which corresponds to a n-dimensional space, it is easy to find that the following relation holds

$$l \geq m + k - n \tag{1.2}$$

where l is the subset of points at the intersection of the subsets of m and k; n is the entire set of points.

In the theory of finite-dimensional vector spaces there is a similar relationship

$$\dim l \geq \dim m + \dim k - \dim n \tag{1.3}$$



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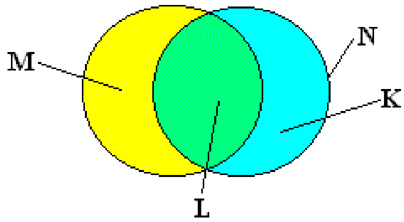

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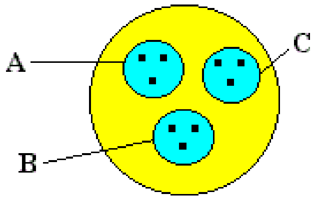

Figure 6:

where dim is the dimension; $\dim l$ is the dimension of the subspace resulting from the intersection of the subspaces m and k; $\dim n$ is the dimension of the enclosing space.[2]

Suppose we have infinite-dimensional spaces M and K. Then in our model, their intersection will be displayed by a subset of L of an infinite number of points (Fig.5)

The equations (1.2) and (1.3) will be here

$$L \geq M + K - N \tag{1.4}$$

Now consider a set of 9 points, which corresponds to 9-dimensional space (Fig.6)

If this set is divided into subsets of three points - A, B, C, then it is easy to see that the intersections of the subsets of A, B, C is similar to the intersection of subsets of three points. In the 9-dimensional space, this means that its three 3-dimensional subspaces can intersect at one point and be mutually orthogonal. Thus, the 3-dimensional subspace in this case can play the role of a coordinate "axis". Then what corresponds to the 2-dimensional planes in the 3-dimensional space, here is the 6-dimensional subspace. We took three points in A, B, C as an example only. Let A, B and C have n points in each subset. Then we get an analogue of the $3\,n$-dimensional space. "Cube", for example, in such a space should look like this (Fig.7)

Here each edge is n-dimensionally, each face is $2\,n$-dimensionlly, the cube itself is $3\,n$ -dimensional, but the vertices will be eight. If we take its n-dimensional subspace as a "line" in the $3\,n$-dimensional space, then we get with this definition usual 3-dimensional geometry, where each point can be characterized by three numbers with respect to the n-dimensional coordinate "axes". The only difference from the 3-dimensional space will be that the "length" of this "line" will be measured in meters to the degree n (cm, km, etc.). The Pythagorean theorem in this case will have the form

$$r^2_{m^{2n}} = x^2_{m^{2n}} + y^2_{m^{2n}} + z^2_{m^{2n}} \tag{1.5}$$

With this definition, such "3-dimensional" geometry will not be formally any different from the 3-dimensional geometry of Euclidean with all its content.

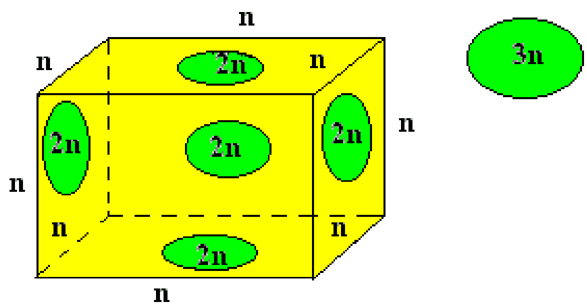

Figure 7:

In principle, n can be equated to infinity and we get a "3-dimensional" geometry with an infinite number of internal degrees of freedom. "Points" in such a space (that is, very small volumes) will be infinite-dimensional.

We conclude that even if observers use the 3-dimensional geometry formalism, space itself may be in the above sense multi- and even infinite-dimensional, but additional dimensions of space are unobservable. At what level this multidimensionality manifests itself - this is already a matter of physics.

References

[1] Demin V.N. The basic principle of materialism: The principle of materiality and its role in scientific knowledge. - M .: Politizdat, 1983. - 239 p. - (What they are working on, what philosophers are arguing about)

[2] Arkhangelsky A.V. Finite-dimensional vector spaces, Moscow University Press, 1982, p.32