# Is Stalnaker's Semantics Complete?\*

Alexander W. Kocurek

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#### Abstract

It is shown that one common formulation of Stalnaker's semantics for conditionals is incomplete: it has no sound and (strongly) complete proof system. At first, this seems to conflict with well-known completeness results for this semantics (e.g., Stalnaker and Thomason 1967; Stalnaker 1970 and Lewis 1973, ch. 6). As it turns out, it does not: these completeness results rely on another closely-related formulation of the semantics that is provably complete. Specifically, the difference comes down to how the Limit Assumption is stated. I close with some remarks about what this means for the logic of conditionals.

One of the most influential semantics for conditionals is Stalnaker's (1968) selection function semantics. On this theory, a conditional of the form  $\phi > \psi$  is true iff either there is no world where  $\phi$  is true or else the closest  $\phi$ -world is a  $\psi$ -world. This theory presupposes the *Limit Assumption*: if  $\phi$  is possible, there there is a closest  $\phi$ -world.

Here, I show that one common formulation of the selection semantics is incomplete: it has no sound and (strongly) complete proof system. This may seem to conflict with well-known completeness results for this semantics (e.g., Stalnaker and Thomason 1967, 1970 and Lewis 1973, ch. 6). As we'll see, it does not: these completeness results use a subtly different formulation of the semantics, which can be axiomatized. Specifically, the difference comes down to how the Limit Assumption is stated. Both formulations appear in the literature. Stalnaker himself adopts different formulations in different works. Thus, while the results proven by Stalnaker and Thomason (1967, 1970) are correct, the titular question of this paper has no simple answer: Stalnaker introduced two semantic theories, one that's complete and one that's not.

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Throughout, we'll work with a simple conditional language  $\mathcal{L}$  with an infinite set of atomics At = { $p_1, p_2, p_3, ...$ } and a conditional operator >:

$$\phi ::= p \mid \neg \phi \mid (\phi \land \phi) \mid (\phi > \phi).$$

The other booleans  $\lor$ ,  $\supset$ ,  $\equiv$  are defined as standard. As usual, we define  $\Box \phi := (\neg \phi > \bot)$  and  $\diamondsuit \phi := \neg \Box \neg \phi$ .

Here is one common formulation of the selection semantics. Models are tuples  $\mathcal{M} = \langle W, R, f, V \rangle$ , where:

- *W* is a nonempty set (of worlds)
- $R \subseteq W \times W$  is a reflexive accessibility relation; throughout, we write R(w) for  $\{v \in W \mid wRv\}^1$
- $f: \mathscr{P}(W) \times W \to \mathscr{P}(W)$  is a selection function satisfying the following constraints for all  $P, Q \subseteq W$ :
  - (i) success:  $f(P, w) \subseteq P \cap R(w)$
  - (ii) *limit*: if  $P \cap R(w) \neq \emptyset$ , then  $f(P, w) \neq \emptyset$
  - (iii) *centering*: if  $w \in P$ , then  $f(P, w) = \{w\}$
  - (iv) *rational monotonicity:* if  $P \subseteq Q$  and  $f(Q, w) \cap P \neq \emptyset$ , then  $f(P, w) = f(Q, w) \cap P$
- $V: At \rightarrow \mathscr{P} W$  is a valuation function.

This formulation allows for multiple *P*-worlds to be equally close. Many advocates of the selection semantics, including Stalnaker himself, forbid this. To ensure there is only ever at most one closest *P*-world, we can impose the following additional constraint on selection functions:

(v) uniqueness:  $|f(P, w)| \leq 1$ 

Uniqueness will not play an essential role below, so I will leave it as an optional add-on to the semantics.

Given a model  $\mathcal{M}$  and a world  $w \in W$ , we define truth at  $\mathcal{M}$ , w as follows (where  $\llbracket \phi \rrbracket^{\mathcal{M}} = \{ w \in W \mid \mathcal{M}, w \Vdash \phi \}$ ):

$$\begin{array}{lll} \mathcal{M}, w \Vdash p & \Leftrightarrow & w \in V(p) \\ \mathcal{M}, w \Vdash \neg \phi & \Leftrightarrow & \mathcal{M}, w \nvDash \phi \\ \mathcal{M}, w \Vdash \phi \land \psi & \Leftrightarrow & \mathcal{M}, w \Vdash \phi \text{ and } \mathcal{M}, w \Vdash \psi \end{array}$$

<sup>&</sup>lt;sup>1</sup> While Stalnaker and Thomason (1967, 1970) and Lewis (1973) include an accessibility relation in their models, some authors omit it, effectively treating *R* as the universal relation  $W \times W$ . None of the results to follow turn on this modeling choice.

$$\mathcal{M}, w \Vdash \phi > \psi \quad \Leftrightarrow \quad f(\llbracket \phi \rrbracket^{\mathcal{M}}, w) \subseteq \llbracket \psi \rrbracket^{\mathcal{M}}.$$

Observe that  $\Box$  and  $\Diamond$  have the following derived truth conditions:

$$\mathcal{M}, w \Vdash \Box \phi \quad \Leftrightarrow \quad \text{for all } v \in W \colon \text{if } wRv, \text{ then } v \in \llbracket \phi \rrbracket^{\mathcal{M}}$$
$$\mathcal{M}, w \Vdash \Diamond \phi \quad \Leftrightarrow \quad \text{for some } v \in W \colon wRv \text{ and } v \in \llbracket \phi \rrbracket^{\mathcal{M}}.$$

We write  $\mathcal{M}, w \Vdash \Gamma$  to mean  $\mathcal{M}, w \Vdash \gamma$  for all  $\gamma \in \Gamma$ . Consequence is truthpreservation:  $\Gamma \vDash \phi$  iff for every model  $\mathcal{M} = \langle W, f, V \rangle$  and world  $w \in W$ , if  $\mathcal{M}, w \Vdash \Gamma$ , then  $\mathcal{M}, w \Vdash \phi$ .

This semantics is not compact: there are unsatisfiable sets of formulas that are finitely satisfiable (meaning every finite subset is satisfiable). Consider the following set:

$$\Sigma = \{ \diamondsuit p_i \mid i \ge 1 \} \cup \{ (p_i \lor p_{i+1}) > \neg p_i \mid i \ge 1 \}$$

Intuitively,  $\Sigma$  says that, for each *i*,  $p_i$  is possible and the closest  $p_{i+1}$ -worlds are strictly closer than the closest  $p_i$ -worlds. This gives rise to an infinite descending chain of closer and closer worlds: there's a  $p_2$ -world closer than any  $p_1$ -world, a closer  $p_3$ -world, an even closer  $p_4$ -world, and so on. This would violate the Limit Assumption. Hence, in the selection semantics,  $\Sigma$  is unsatisfiable. Yet each finite subset only requires a finite descending chain. Hence,  $\Sigma$  is finitely satisfiable. Thus, the selection semantics, as formulated above, is not compact (see appendix for details).<sup>2</sup>

For familiar reasons, the failure of compactness entails that the failure of (strong) completeness: there's no finitary proof system  $\vdash$  such that  $\Gamma \vDash \phi$  iff  $\Gamma \vdash \phi$ .<sup>3</sup> Moreover, the proof does not appeal to (non-)uniqueness (or even centering): it only relies on success, limit, and rational monotonicity.<sup>4</sup>

<sup>&</sup>lt;sup>2</sup> Adams (1975, p. 52) gives a similar proof that his (quite different) probabilistic theory of conditionals lacks compactness, though, to my knowledge, he never considered applying this to Stalnaker's semantics. Fine (2012, fn. 2), who argues broadly against any intensional semantics for counterfactuals, suggests a related (but distinct) argument could be used to "demonstrate the non-compactness of semantics for counterfactuals with the 'limit assumption'", though he neither explains how the proof is supposed to go nor considers how this could be consistent with well-known completeness results. After submitting this article for review, Dorr and Mandelkern (2024) posted a paper on arXiv, which also observes that this semantics with the uniqueness assumption is incomplete (section 2.2). (Thanks to David Boylan for pointing this out to me.)

<sup>&</sup>lt;sup>3</sup> This semantics is *weakly* complete, however: there is an axiomatic proof system  $\vdash$  such that  $\models \phi$  iff  $\vdash \phi$ . See Lewis 1973, section 6.2. Dorr and Mandelkern (2024) likewise observe the conditional logic C2 is weakly complete for Stalnaker's semantics with the uniqueness assumption.

<sup>&</sup>lt;sup>4</sup> The proof likewise does not rely on the ordering of worlds satisfying comparability. Thus, the proof equally applies to the premise semantics defended by Kratzer (1979, 1981, 1986).

This incompleteness result might seem to conflict with the completeness results given by Stalnaker and Thomason (1967, 1970) and Lewis (1973). In fact, it does not. To see why, let's look at two different formulations of the Limit Assumption. Here is how Stalnaker (1980, p. 89) states the Limit Assumption:

for every possible world *i* and non-empty proposition *A*, there is at least one *A*-world minimally different from *i*.

Translated into the selection semantics above, this condition says the following: for any set  $P \subseteq W$  and any world w, if  $P \cap R(w) \neq \emptyset$ , then  $f(P, w) \neq \emptyset$ . In other words, for any *proposition* P, if P is possibly true, then there is always a closest P-world. This is how we stated the limit constraint above.

Here is how Lewis (1973, p. 120) states the Limit Assumption:

for any  $\phi$ , if  $\llbracket \phi \rrbracket$  overlaps  $\bigcup \$_i$  there is some smallest member of  $\$_i$  that overlaps with  $\llbracket \phi \rrbracket$ .

Translated into the selection semantics above, this condition says the following: for any  $\phi$  and any world w, if  $[\![\phi]\!]^{\mathcal{M}} \cap R(w) \neq \emptyset$ , then  $f([\![\phi]\!]^{\mathcal{M}}, w) \neq \emptyset$ . In other words, for any *formula*  $\phi$ , if  $\phi$  is possibly true, then there is always a closest  $\phi$ -world.

Notice the subtle difference between these formulations. The first is stated in terms of *propositions*; the second in terms of *formulas*. Every formula expresses a proposition, but not every proposition is, in general, expressed by a formula. Indeed, if there are infinitely many worlds in our model, there will be uncountably many propositions but only countably many formulas. Lewis's sentential formulation of the Limit Assumption only quantifies over *definable* subsets, i.e., propositions *expressed* by some formula. Stalnaker's propositional formulation above quantifies over *all* subsets.

This difference is crucial, as the proof of non-compactness relies on the assumption that  $f(P, w) \neq \emptyset$  where, intuitively, P is the proposition that would be expressed by the infinite disjunction  $(p_1 \lor p_2 \lor p_3 \lor \cdots)$  in an infinitary version of our language (formally,  $P = \bigcup_{i \ge 1} V(p_i)$ ). Such a P isn't guaranteed to be expressed by a finite formula, however: there may be no  $\phi$  in our finitary language such that  $\llbracket \phi \rrbracket^{\mathcal{M}} = P$ .

Both formulations of the Limit Assumption appear in the literature. For example, Pollock (1976); Herzberger (1979); Nute (1980); Warmbröd (1982) adopt the sentential formulation. By contrast, Swanson (2012); Schulz (2014); Kaufmann (2017); Cariani and Santorio (2018); Mandelkern (2020); Khoo (2022) adopt the propositional formulation, as do many linguists (see the references in Kaufmann 2017, pp. 9–10).<sup>5</sup>

Interestingly, Stalnaker himself seems to use different formulations in different places. Stalnaker and Thomason (1967, 1970) clearly use the sentential version to prove completeness. By contrast, Stalnaker (1980, p. 88) and Stalnaker (1984, p. 120) use the propositional formulation (see also Stalnaker 2014, p. 120). Stalnaker's (1968) own notation and terminology are unclear as to which formulation is intended: he defines selection functions as taking propositions as inputs, but then uses the same notation (capitalized italics) for both the inputs of selection functions (writing "f(A, w)") and for sentences of the object language (writing "A > B" and " $\Box A$ "), suggesting he has in mind the sentential formulation.

Lewis (1973) seemed aware of these different formulations of the Limit Assumption, though he only comments on it briefly near the beginning of the book (p. 19; emphasis added):<sup>6</sup>

If there are sequences of smaller and smaller spheres without end, then there are sets of spheres with no smallest member. . . Yet it might still happen that for every entertainable antecedent in our language, there is a smallest antecedent-permitting sphere. For our language may be limited in expressive power so that not just any set of worlds is the set of  $\phi$ -worlds for some sentence; and, in that case, it may never happen that the set of  $\phi$ -permitting spheres is one of the sets that lacks a member, for any antecedent  $\phi$ .

Lewis goes on to state the Limit Assumption in terms of formulas, rather than arbitrary propositions, without further explanation for why it's preferable to state the Limit Assumption in such a restricted way. Perhaps it did not matter to Lewis, since he famously rejects even the weaker, sentential formulation. But as we'll see, it does matter for the purposes of the logic of conditionals.

Let's consider formulating the selection semantics with the sentential Limit Assumption, as Stalnaker and Thomason (1967, 1970) do. A *premodel* is a tuple of the form  $\mathcal{M} = \langle W, R, f, V \rangle$  where W, R, and V are as before and  $f: \mathcal{L} \times W \to \mathscr{P} W$  is a *preselection function*, mapping each formula  $\phi$  and world w to a set of worlds  $f(\phi, w)$ . No further conditions are placed on preselection functions. The truth conditions relative to worlds in premodels

<sup>&</sup>lt;sup>5</sup> Thanks to Willow Starr and Seth Yalcin for pointing this out to me.

<sup>&</sup>lt;sup>6</sup> He is careful in other places in the book to state the Limit Assumption so it only applies to formulas (pp. 57–58). He does briefly consider allowing selection functions to take propositions, but still restricts attention to expressible propositions (e.g., p. 60).

are as before, replacing the conditional clause with:

$$\mathcal{M}, w \Vdash \phi > \psi \quad \Leftrightarrow \quad f(\phi, w) \subseteq \llbracket \psi \rrbracket^{\mathcal{M}}$$

We now redefine a model to be a premodel M whose preselection function f satisfies the following conditions for all  $\phi$  and  $\psi$ :

- (i') success:  $f(\phi, w) \subseteq \llbracket \phi \rrbracket^{\mathcal{M}} \cap R(w)$
- (ii') *limit*: if  $\llbracket \phi \rrbracket^{\mathcal{M}} \cap R(w) \neq \emptyset$ , then  $f(\phi, w) \neq \emptyset$
- (iii') centering: if  $w \in \llbracket \phi \rrbracket^{\mathcal{M}}$ , then  $f(\phi, w) = \{w\}$
- (iv') rational monotonicity: if  $\llbracket \phi \rrbracket^{\mathcal{M}} \subseteq \llbracket \psi \rrbracket^{\mathcal{M}}$  and  $f(\psi, w) \cap \llbracket \phi \rrbracket^{\mathcal{M}} \neq \emptyset$ , then  $f(\phi, w) = f(\psi, w) \cap \llbracket \phi \rrbracket^{\mathcal{M}}$

Stalnaker's selection semantics, on its sentential formulation, is obtained by adding the following:

(v) uniqueness:  $|f(\phi, w)| \leq 1$ 

Consequence is defined, like before, as truth-preservation over *models* (not premodels). So defined, this semantics is axiomatizable. Table 1 presents standard axioms and rules for this semantics. Completeness can be proven via the familiar canonical model strategy (Lewis 1973; Stalnaker and Thomason 1967, 1970; the latter extends completeness to the first-order case).

Lewis (1980) drew the following "moral" from his completeness results: "the Limit Assumption is irrelevant to the logical properties of the counterfactual" (p. 83). In a technical sense, he was right, though largely thanks to his subtle restriction on the Limit Assumption. In spirit, however, the moral is misleading: how one formulates the Limit Assumption can quite dramatically affect the underlying logic for conditionals.<sup>7</sup>

To be clear, the fact that the selection semantics on its propositional formulation is incomplete isn't necessarily a reason to reject it. It is well-known that while the propositional Limit Assumption does not correspond to any finitary principle, it does correspond to a principle in an infinitary language (with infinitary conjunction) known as "infinite agglomeration" (Pollock, 1976; Herzberger, 1979; Stalnaker, 1980):

<sup>&</sup>lt;sup>7</sup> Kaufmann (2017) draws a related, but distinct, moral. He observes that different formulations of the Limit Assumption yield different verdicts about whether certain kinds of counterfactual necessity are equivalent. The different formulations, however, concern whether we quantify over all subsets or just downward-closed subsets, which is separate from whether we quantify over only definable subsets (as definable sets need not be downward-closed on the world-ordering).

Name	Axiom/Rule
Taut	$\vdash \phi$ where $\phi$ is any instance of a propositional tautology
Id	$\vdash \phi > \phi$
NC	$\vdash \Box  \psi \supset (\phi > \psi)$
Cen	$\vdash \phi \supset ((\phi > \psi) \equiv \psi)$
RM	$\vdash \neg(\phi > \neg \psi) \supset (((\phi \land \psi) > \chi) \equiv (\phi > (\psi \supset \chi)))$
MP	if $\vdash \phi$ and $\vdash \phi \supset \psi$ , then $\vdash \psi$
CNec	if $\vdash (\psi_1 \land \cdots \land \psi_n) \supset \chi$ , then $\vdash ((\phi > \psi_1) \land \cdots \land (\phi > \psi_n)) \supset (\phi > \chi)$
For Stalnaker, add:	
CEM	$\vdash (\phi > \psi) \lor (\phi > \neg \psi)$

Table 1: Axioms for the Lewis-Stalnaker semantics with the sentential Limit Assumption

### **Infinite Agglomeration.** $\phi > \psi_1, \phi > \psi_2, \phi > \psi_3, \ldots \models \phi > (\psi_1 \land \psi_2 \land \psi_3 \land \cdots).$

This principle seems highly plausible: it is just the infinitary version of conjunction-introduction in the consequent of conditionals. Indeed, this is precisely one of the main reasons advocates cite in defense of the Limit Assumption. Incompleteness might therefore be a cost worth accepting if it ensures the validity of such principles. Advocates may try to soften the blow further, e.g., by pointing out the semantics is weakly complete (footnote 3) or suggesting that such non-compact sets do not matter for practical purposes.

Nevertheless, incompleteness is a theoretical cost. After all, recall the example of a non-compact set from before:

$$\Sigma = \{ \diamondsuit p_i \mid i \ge 1 \} \cup \{ (p_i \lor p_{i+1}) > \neg p_i \mid i \ge 1 \}$$

This set does not contain any infinitary connectives: all the premises in  $\Sigma$  are stated in the *finitary* language used by Lewis and Stalnaker (and allies) to regiment conditional claims. Thus, with the propositional Limit Assumption, even the logic of conditionals stated in this finitary language a language that is meant to reflect conditionals from ordinary language—is incomplete.

Could one avoid this cost by endorsing the sentential Limit Assumption while rejecting the propositional one? On this view, there are violations of the propositional Limit Assumption: we just can't express any of them in our language.

This is a theoretical possibility, of course. To my knowledge, however, no one (except Lewis in the brief passage above) has even commented on this possibility, let alone defended it. Why think the sentential Limit Assumption holds while the propositional one fails? What would prevent limit violations from being expressible in our language?

Stalnaker (1980, p. 97) seems to suggest an answer in response to Lewis's "counterexample" to the Limit Assumption.<sup>8</sup> Take a line that is exactly one inch long and consider the antecedent 'if the line were more than one inch long'. Lewis argues there is no closest world where that antecedent is true. Stalnaker replies:

If relative to the issue under discussion, every difference in length is important, then it is just inappropriate to use the antecedent, *if the line were more than an inch long*. This would, in such a context, be like using the definite description, *the shortest line longer than one inch*. The selection function will be undefined for such antecedents in such contexts.

At first, Stalnaker seems to be *conceding* that the Limit Assumption may fail in some contexts even on its sentential formulation. In a context where every difference in length matters, Stalnaker says the selection function is undefined (or, in our notation, returns the empty set) on the proposition *that the line is more than one inch long*, even though that proposition is possible and even expressible in our language. This is precisely what the sentential Limit Assumption rules out. So it looks as though Stalnaker is acknowledging that Lewis was right that the Limit Assumption has its limits.

But there's a less concessive way to interpret this passage: perhaps Stalnaker intends for the Limit Assumption to act as a constraint on our expressive capacities in a context. While the phrase 'the line is more than one inch long' normally expresses a contingent proposition, perhaps the phrase 'if the line were more than one inch long' does not express this proposition in a context where every difference in length matters. This may be why he draws the analogy with 'the shortest line longer than one inch': 'the line is more than one inch long' effectively goes undefined when trying to use it in the antecedent of a conditional in that context. So perhaps one could maintain, along these lines, that the only limit violations in a given

<sup>&</sup>lt;sup>8</sup> Thanks to an anonymous referee for pointing out this passage and how it could be leveraged into a defense of the sentential Limit Assumption.

context involve propositions that are not expressible in that context, even if such propositions are normally expressible in other contexts.

I am unsure whether this is what Stalnaker had in mind. At any rate, this line of defense is problematic. Even in contexts where every difference in length matters, it does not seem as though 'The line is more than one inch long' fails to express a contingent proposition: if anything, it expresses the contingently false proposition *that the line is more than one inch long*. Moreover, this approach can't explain why other conditionals involving the very same antecedent sound fine in such contexts. Thus, even when every difference in length matters, it does not sound "inappropriate" to say 'If the line were more than one inch long, the line would be too long for our purposes'—indeed, this is a natural way of expressing the idea that "every difference in length matters"!

This is not to say the view that the sentential Limit Assumption holds while the propositional one fails is indefensible. But it does seem as though a defense of this view is hard to come by. Until such a defense is provided, completeness may just be a feature that advocates of the Limit Assumption must learn to live without.

#### Appendix.

We'll now show the following set violates compactness, i.e., it is unsatisfiable yet finitely satisfiable:

$$\Sigma = \{ \diamondsuit p_i \mid i \ge 1 \} \cup \{ (p_i \lor p_{i+1}) > \neg p_i \mid i \ge 1 \}.$$

First:  $\Sigma$  is finitely satisfiable. For let  $\Sigma_0 \subseteq \Sigma$  be finite. Then there's a largest *n* such that  $p_n$  occurs in a formula in  $\Sigma_0$ . Define a model  $\mathcal{M}_n = \langle W_n, R_n, f_n, V_n \rangle$  where:

- $W_n = \{0, 1, \dots, n+1\}$
- $R_n = W_n \times W_n$
- $f_n(\emptyset, i) = \emptyset$  and for each nonempty  $P \subseteq W_n$ :

$$f_n(P,i) = \begin{cases} \{i\} & \text{if } i \in P\\ \max P & \text{if } i \notin P \end{cases}$$

where max  $P = \{j \in P \mid \neg \exists k \in P \colon j < k\}.$ 

• 
$$V_n(p_i) = \{i\} \text{ for } 1 \le i \le n+1; V_n(p_i) = \emptyset \text{ for } i > n+1.$$

#### Then $\mathcal{M}_n$ , $0 \Vdash \Sigma_0$ .

Next:  $\Sigma$  is not satisfiable. For suppose  $\mathcal{M}, w \Vdash \Sigma$ . Let  $P = \bigcup_{i \ge 1} V(p_i)$ . Since  $\mathcal{M}, w \Vdash \Diamond p_i$  for each  $i, V(p_i) \cap R(w) \ne \emptyset$ , and so  $P \cap R(w) \ne \emptyset$ . By limit,  $f(P, w) \ne \emptyset$ . Let  $v \in f(P, w)$ , then. By success,  $v \in P \cap R(w)$ , i.e., there is some n such that  $v \in V(p_n) \cap R(w)$ . Let  $Q = (V(p_n) \cup V(p_{n+1})) \cap R(w)$ . Thus,  $v \in f(P, w) \cap Q$  and  $Q \subseteq P$ . By rational monotonicity,  $f(Q, w) = f(P, w) \cap Q$ . Hence,  $v \in f(Q, w)$ . But  $Q = [p_n \lor p_{n+1}]^{\mathcal{M}}$ . Since  $\mathcal{M}, w \Vdash (p_n \lor p_{n+1}) > \neg p_n$ , that means  $f(Q, w) \cap V(p_n) = \emptyset$ . This contradicts  $v \in f(Q, w) \cap V(p_n)$ . Q.E.D.

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