# Do We Really Not Know What Toulmin's Analytic Arguments Are?

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**Abstract:** The aim of this paper is to challenge the idea that Toulmin's main focus in The Uses of Argument is to critique formal deductive logic. I first try to challenge the argument that, on the basis of what Toulmin says about analytic arguments, it is impossible to determine exactly what they are. I will then attempt to determine the basic contours of analytic arguments. Finally, I will conclude that the concept of an analytic argument involves epistemological assumptions to which formal logicians are in no way committed by the nature of their discipline.

Résumé: Le but de cet article est de remettre en question l'idée que l'objectif principal de Toulmin dans The Uses of Argument est une critique de la logique déductive formelle. J'essaie d'abord de contester l'argument selon lequel, sur la base de ce que dit Toulmin à propos des arguments analytiques, il est impossible de déterminer exactement ce qu'ils sont. Je tenterai ensuite de déterminer les contours fondamentaux des arguments analytiques. Enfin, je conclurai que le concept d'argument analytique implique des suppositions épistémologiques auxquelles les logiciens formels ne sont nullement engagés par la nature de leur discipline.

Keywords: analytic argument, deductive argument, Hamby, Toulmin

#### 1. Introduction

In what is perhaps his most famous book, *The Uses of Argument* (hereafter *Uses*), Toulmin argues that the traditional formal presentation of arguments in logic books obscures their deeper structure (2003, p. 134). It is recognition of this deeper argumenta-

tive structure that reveals some of the fundamental differences between different types of arguments, which are of immense importance for their proper evaluation. Toulmin accused logicians of promoting as the universal ideal for evaluating arguments only those criteria that are typical of one specific subtype of so-called "analytic argument," the use of which is, in fact, almost negligible in ordinary discourse (ibid., pp. 118, 133, 201-203). Toulmin thought that the claim that one set of criteria for assessing arguments should be universally applied to all other kinds of arguments was harmful. He believed that arguments should be assessed according to criteria typical of the field of knowledge or discourse to which they belong (ibid, pp. 32, 35). For example, it is unwise to use the same criteria to evaluate arguments in, say, mathematics and ethics, as some arguments would remain unappreciated as a result. Based on the structure of his microargument, Toulmin distinguishes two basic types of argument—analytic and substantial. Although the second type is disproportionately more common in argumentation than the first, Toulmin argues that logicians have done their best to promote only one specific subtype of analytic argument as the ideal for all other arguments, including substantial ones.

Following the publication of *Uses*, a number of authors wrote responses in defence of classical logic. These were often critical of both Toulmin's critique and his positive proposal. In response to Uses, for example, Castaneda concludes that "Toulmin's new logic is at best only vaguely hinted at, and that his proposals are clearly obscure or erroneous" (1960, p. 279). Cooley, who preferred the approach Toulmin had taken in The Philosophy of Science, commented on Uses as follows: "Toulmin did much better when he assumed the role of explorer rather than judge, and made up his problems step by step as he worked his way into concrete material" (1959, p. 319). Even authors who believed that there were strong reasons to revisit classical logic were not always enthusiastic about Toulmin's proposals. According to Cowan (1964), for example, the purpose of logic is to organise knowledge, for which its formal aspect plays an important role. This function of logic in organising our knowledge, Cowan believed, is somewhat hampered by the traditional assumption that it is the job of logicians to discover logical truths. Logical truths are what they are only because we have given them that role, which we have done for pragmatic reasons, to ensure the best possible organisation of knowledge. In trying to discover a universal pattern of argument and in attacking logical formalism, Toulmin makes the worst possible choice according to Cowan: "He rejects exactly that part of traditional logic which should be retained, the basic concepts and forms, and retains exactly that part which should be rejected, the main lines along which this apparatus has been misinterpreted" (1964, pp. 27–28).

The aim of this paper is not to broaden existing criticisms, which are extensive and often justified, but to describe Toulmin's account of the general features of an analytic argument. The idea that Toulmin objects to formal deductive logic is widely accepted. This is true to some extent, but I think that what Toulmin is mainly objecting to in *Uses* are certain epistemological presuppositions which, according to some, should be intertwined with the formalism of deductive logic. According to these epistemological presuppositions, true knowledge must be based on first and indisputable truths, from which derived truths are proven. When Toulmin speaks critically of "more geometrico" arguments, he is referring not only to the character of individual arguments, which it should always be possible to reconstruct in a system of formal deductive logic, but also to geometry in a broader sense, as an example of a system of truths developed through rigorous proofs from first principles.

In addition to Toulmin's texts, I will pay close attention to Ben Hamby's (2012) article, "Toulmin's 'Analytic Arguments," which is currently the most comprehensive justification of the thesis that, based on Toulmin's works, we cannot possibly know what Toulmin's analytic arguments actually *are*. A cursory glance at the above-cited works by Toulmin's critics confirms this suspicion. For example, they all seem to criticize Toulmin for his *Uses*, as if his criticism of the analytic argument were in fact a criticism of the deductive argument and deductive validity. This assumption is by no means unique to the works cited thus far. For example, the well-known *Handbook of Argumentation Theory* states that, "By calling arguments substantial, Toulmin (2003) refers to the fact

that in such arguments the conclusion is not entailed (p. 116)" (van Eemeren et al. 2014, p. 209). On the very next page, the authors of the *Handbook* give an example of a substantial argument in which an astronomer predicts the phase of the moon 100 years later. The authors say that since the astronomer's prediction is based on a substantial argument, it "will never follow necessarily from the premises" (ibid., p. 210).

However, the assumption that the distinction between substantial and analytic arguments corresponds to the distinction between deductive and non-deductive arguments is not true. Indeed, it directly contradicts Toulmin's own claim:

The division of arguments into analytic and substantial is, therefore, entirely distinct from that into conclusive (necessary) and tentative (probable) arguments. Analytic arguments can be conclusive or tentative, and conclusive ones analytic or substantial (Toulmin 2003, p. 123).

By 'conclusive arguments,' Toulmin means arguments in which we derive a conclusion "unambiguously and unequivocally" (2003, p. 127). In other words, the premises of a conclusive argument rule out even the slightest possibility of the conclusion being false. Therefore, the appropriate modal qualifier of the conclusion is the qualifier 'necessarily.' According to Toulmin, we use arguments that are conclusive and substantial when we apply the methods of geometrical optics to find the length of a shadow, assuming we know the height of the wall on which the sun shines and the angle of inclination of the sun. This is a *calculation* of the conclusion from the premises (ibid.). If the premises of an argument do not rule out the possibility of the conclusion being false and merely support it to some degree, then the argument is 'tentative'—its conclusion is appropriately modified by the modal quali-

numbers are divisible by two.

<sup>1</sup> For example, the statement that Harry was born in Bermuda supports the conclusion that Harry is a British citizen, because people born in Bermuda are generally British citizens. This does not rule out the possibility that Harry is not a British citizen (he may have given up his British citizenship), but it does make it probable that he is. By contrast, the indication that 4 is an even number clearly rules out the possibility that 4 is not divisible by two, since all even

fier 'probably,' 'presumably' or 'almost certainly' (Toulmin 2003, p. 17, pp. 126–128).

Toulmin's aversion to conclusive analytic arguments as a universal ideal for all arguments and his followers' identification of conclusive analytic arguments with deductive arguments could foster unjustified resistance to deductive logic. I believe that conclusive analytic arguments cannot be fully identified with deductive arguments and thus that Toulmin's critique of conclusive analytic arguments cannot simply be identified with a critique of deductive logic.

In this paper, I formulate an answer to the question "What are analytic arguments on Toulmin's view?" I will be primarily concerned with conclusive analytic arguments, but I will also touch on quasi-syllogistic analytic arguments, whose conclusions are only tentative. This is because conclusive analytic arguments—as opposed to tentative analytic arguments—are the paradigm of valid argumentation, according to Toulmin (2003, pp. 113–114).

Hamby suggests that, given that Toulmin's distinction between analytic and substantial arguments is "problematically opaque" (2012, p. 118), we should "deemphasize" it in favour of the more familiar distinction between deductive and non-deductive arguments; after all, Toulmin's merit, which is undeniable, lies primarily in the attention he draws to non-deductive forms of inference (ibid., pp. 116-117). This suggestion may indirectly support the interpretation that Toulmin was primarily concerned with deductive rather than analytic arguments, which I would challenge.

Given the importance of analytic arguments in Toulmin's critique in *Uses*, new insights into them could contribute not only to a more adequate appreciation of that critique but also to a more precise determination of the place of Toulmin's views in the history of argumentation theory. I think there is more to be said about Toulmin's distinction between analytic and substantial arguments—but especially about analytic conclusive arguments—than Hamby's analyses would lead us to believe. In addition to these goals, I believe that this paper contributes to a deeper analysis of the structure of the microargument, which would certainly not have been possible without Hamby's work, and especially his discussion with James Freeman.

### 2. Overview of the following sections

In section 3, I briefly outline Toulmin's concept of the microargument and specify the main object of my interest, which is not all analytic arguments but only a proper subset of them.

In the next section, I present two examples of arguments that differ due to small but significant differences between some of their assumptions. My main aim here is to draw attention to a feature of analytic conclusive arguments that is often overlooked. I believe that Toulmin's well-known example of an argument involving Jack and his sisters does not have this overlooked feature either and therefore cannot be taken as a proper example of an analytic conclusive argument, as is sometimes mistakenly assumed. Furthermore, I discuss two different descriptions of the relationship between the backing and the conclusion in analytic conclusive arguments proposed by Toulmin. I then provide Toulmin's characterisations of analytic arguments.

In section 5, I present Toulmin's tests for the analyticity of arguments and Hamby's critique of these tests as a reliable tool for determining the set of analytic arguments. I will give reasons why I think that Hamby's criticism does not hold in the case of analytic conclusive arguments. I agree with Hamby that these tests are not very helpful in the case of analytic tentative arguments. However, my reasons for reaching this conclusion are different from Hamby's.

In the section that follows, I analyse an example of an analytic conclusive argument which, according to Toulmin, has all the essential properties of this type of argument. As a result of my analysis, I show that the argument is in fact an example of an analytic tentative, not a conclusive, argument.

In section 7, I try to identify all the individually necessary and jointly sufficient conditions for being an analytic conclusive argument. In doing so, I make use of Toulmin's later work, *Knowing and Doing: An Invitation to Philosophy* (1973), in which Toulmin describes the origin of formal or geometrical validity. I also give an example of an argument that satisfies these conditions. Finally, I will show which epistemological assumptions are involved in the concept of analytic conclusive arguments and then discuss them in more detail.

# 3. Microargument, working classification, and Toulmin's two characterisations of analytic arguments in *Uses*

Image 1 shows the general scheme of Toulmin's microargument:

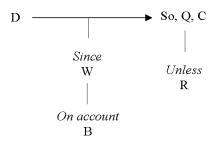


Image 1: Microargument

According to this model, we always proceed from some *datum* D (e.g., "Harry was born in Bermuda") to a *conclusion* or *claim* C (e.g., "Harry is a British citizen") via a *warrant* W that bridges the datum and the claim of the argument (e.g., "A man born in Bermuda will be a British subject") on account of *backing* B, which establishes the warrant (e.g., the concrete wording of the laws that govern the nationality of persons born in British colonies at the time of the argument). Q is a *modal qualifier* of the conclusion (e.g., 'probably,' 'inevitably,' etc.) and R is a *rebuttal* (e.g., "Harry's parents were foreigners").

Whether an argument is conclusive or not depends *only* on the relation between the data and the warrant for the conclusion of the argument (p. 126). This relationship is reflected by the modal qualifier (p. 94). An argument is conclusive when it is not possible to accept its datum and the warrant while rejecting its conclusion. This formulation is strikingly reminiscent of the definition of a deductive inference, with the difference that we are talking not about the truth-dependence relation between propositions but about the relation of acceptance of those elements that constitute particular instances of the microargument. The reason for this is that Toulmin does not speak about warrant unequivocally.<sup>2</sup> First,

<sup>2</sup> In *The Philosophy of Science* (1953, p. 93), Toulmin says that a warrant is used not as a major premise in a syllogistic argument but as an "inference-

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he claims that warrants are not propositions but rather permissions to act in a certain way<sup>3</sup>—that is, to move through or infer from some data to some conclusions. At the same time, however, he says that warrants are hypothetical in form and that they *bridge* a datum and a conclusion. If this is so, then warrants do not say what agents can do but rather resemble major premises in a syllogistic argument. Be that as it may, Toulmin himself did not hesitate to apply the term 'deductive' to substantial arguments with a "deductive form"—those arguments that share with "all other formal arguments" the following feature:

[...] that is, if the conclusion proves false, something must be wrong either with the "grounds" or with the "linking generalization" that connects those grounds to the predictive conclusion<sup>4</sup> (1976, p. 104).

Since Toulmin's classification of arguments into substantial, analytic, conclusive, and tentative has not taken hold, it will be useful to translate it into existing classifications where possible. Rather than 'tentative,' I will use the term 'non-deductive.' There are, of course, many types of non-deductive arguments, for example, inductive, abductive, argument by expert, etc. For our purposes here, however, a non-deductive argument will simply be an argument whose premises do not support its conclusion deductively, such that it is possible to accept the premises of the argument but not its conclusion. Non-deductive arguments are open to rebuttal (R) even if we accept their premises.

Toulmin first approximates analytic arguments by saying that if we have a formally valid argument of the form "D; W; hence C," then, in the case of analytic arguments, it follows that an argument of the form "D; B; hence C" will also be formally valid (p. 114).

ticket." In *Uses* (2003, p. 92), he distinguishes between *warrants* and *backings* by saying that backings can be categorical in form, whereas warrants are always hypothetical in form. A warrant is a *moral* based on a backing that tells us how we can argue in certain cases (ibid.). Later, Toulmin adds that under certain conditions, warrants can also be categorical in form (1976, pp. 105–6).

<sup>3</sup> See (Castaneda 1960, p. 291).

<sup>4</sup> Toulmin prefers the word 'grounds' to 'data' in later works. These words have the same meaning however (compare with Toulmin et al. 1984, p. 26).

But what exactly does this mean? On what basis can we substitute 'B' for 'W'? According to Toulmin, the backing of an analytic argument "includes, explicitly or implicitly, the information conveyed in the conclusion itself" (p. 116). According to another characterisation, the conclusion of an analytic argument "is a mere restatement in other words of something already stated implicitly in the datum and the backing" (ibid.). These characterisations are not only slightly different but also the only ones Toulmin suggests.<sup>5</sup> According to one of them, the conclusion of an analytic argument repeats the information in the backing explicitly or implicitly, while according to the another, the conclusion is in the backing implicitly and "in other words" (p. 116). I will further assume that these characteristics are disjunctive and that the conclusion of an analytic argument is contained implicitly or explicitly or "in other words" in its backing. I will also leave open for the moment what 'implicitly' means. A more serious distinction concerns the fact that in the one case the conclusion is supposed to be included in the backing, whereas according to the other characterisation, the conclusion is supposed to be included in the backing and the data. I will return to this distinction later. Toulmin goes on to say that, in the case of analytic arguments, "D; B and also C" is a tautology, but he adds in turn that even this rule has some exceptions.

The key point for us is that while the deductive character of a deductive argument depends on the relation between the data, the warrant, and the conclusion, the analytic character of an analytic argument depends on the relation between the data, the backing, and the conclusion or between the backing and the conclusion. In the first case, I described this relationship by saying that it is not possible to accept the data and the warrant but reject the conclusion; in the second case, I described the relationship between the data, the backing, and the conclusion as *being contained* (explicitly or implicitly included). According to Toulmin, these two rela-

<sup>5</sup> In addition to these descriptions, Toulmin also provides three alternative tests of analyticity: the tautology test, the verification test, and the self-evidence test. I shall discuss these tests further below.

<sup>6</sup> What is apparently being discussed here is not a tautology as a statement that is true on every interpretation but a repetition.

tions are independent, and thus there are four basic types of arguments:

- 1. Deductive analytic arguments
- 2. Non-deductive analytic arguments
- 3. Deductive substantial arguments
- 4. Non-deductive substantial arguments

Within this classification, I will focus primarily on the first category. There are two reasons for this. First, Toulmin was the first to suggest that there is a class of arguments that are both non-deductive and analytic. So, when Toulmin criticized logicians for promoting analytic arguments as the ideal for all other arguments, he could not have had non-deductive analytic arguments in mind. Second, analytic arguments do not constitute a well-defined set of arguments because their notion is vague. I will demonstrate this vagueness in section 5.

# 4. On an overlooked feature of deductive analytic arguments and on the backing and its relation to the conclusion

According to Toulmin (2003), some logicians claimed we should use the word 'deduction' only to denote arguments that meet extremely rigid criteria; that is, in the case of a deductive argument, it must be true that "the data and the support positively entail the conclusion" (p. 113). What does 'positively entail' mean? It means that asserting the data and the backing while rejecting the conclusion "would land one in a positive inconsistency, or contradiction" (ibid.). Given Toulmin's characterisation of analytic arguments, I understand a positive contradiction as being a conjunction consisting of a proposition and its negation: A & ~A.

Consider first an example of a deductive substantial argument—that is, one whose conclusion can be rejected even if one accepts the data and the backing. Toulmin describes Newton's procedure for forming a general proposition from particular re-

<sup>7</sup> Cooley rightly remarks that Toulmin here accepts "something not admitted by conventional logic" (1959, p. 304).

peated experiences. The motivation for forming a general proposition is methodological insofar as a 'statistical report' of what has happened to some number of individual objects of a particular kind does not constitute a 'principle of computation' that is useful for formulating predictions. Newton, in generalising, *went beyond* what is stated in the backing, that is, the report of some objects that we have observed.<sup>8</sup>

Suppose we have had n cases thus far in which a steel sphere with property A (the sun shines on the sphere) also had property B (the sphere heats up). Suppose we see the sun shining on the sphere again. Will it heat up? According to the warrant (principle of calculation) Whenever the sun shines on this steel ball, it heats up obtained by generalisation from the backing (which is the sum of experience with the steel ball, which has thus far always heated up when we let the sun shine on it), unequivocally yes—if it does not warm up, we will have to reject the warrant. But it seems entirely consistent with the actual data and backing that the sphere will not heat up. If the backing is a report of what has happened to the sphere thus far when the sun has shone on it, then the backing refers to a prediction that the sphere will heat up at most only indirectly, through its warrant, but in fact says nothing about the sphere or its further heating up. What is the logical structure of the backing? I suggest that, in accordance with Toulmin's description of Newton's procedure for constructing the general hypothesis, it is best to represent it as a *list of* conjuncts that express the observed connections between the sun shining on the sphere and its heating. If, for example, each item in the list confirms this connection, then we have clear justification for expecting it to be repeated in this way in the future. Let pn+1 be the datum for prediction (the sun shines on the sphere for the nth+1 time), and let qn+1 be the conclusion we predict (the sphere has been heated for the nth+1time). Let On be a backing that is a list of conjuncts that jointly express our actual experience. I wrote above that Toulmin is ambiguous about the analyticity condition we have stated thus far. In the first case, an argument is analytic if and only if we cannot

<sup>8</sup> This is actually an inductive argument, which Toulmin also refers to as a "warrant-establishing" argument. See Toulmin (2003, pp. 112–113).

accept the datum and the backing and reject the conclusion. In the second case, an argument is analytic if and only if we cannot accept the backing and reject the conclusion. If we understand the analyticity of an argument to mean that the datum or backing *already contains* information expressed in the conclusion, which merely repeats it, then with respect to our example of the steel sphere, we can state the conditions for analyticity for time t, which is *after*, or *just when*, we have obtained datum pn+1 but before we could verify the prediction qn+1 as follows:

i) 
$$t: q_{n+1} \in \{p_{n+1}; O_n\}$$

ii) 
$$t: q_{n+1} \in \{O_n\}$$

According to (i), at time t, the conclusion of the argument belongs to the set consisting of the datum and the statements that constitute the backing of the argument. According to (ii) at time t, the conclusion of the argument belongs to the set that consists of the statements that constitute the backing of the argument. Of these two options, the first hardly seems acceptable. If the connection between the events or states of affairs expressed by the datum and the conclusion of the argument is truly empirical, then there can be no relation based on meaning between the statement datum and the statement conclusion: there is no logical connection between the statements "The sun is shining on this sphere" and "This sphere is heating up."

Let us now proceed to the deductive analytic argument. Before considering Toulmin's examples of this type of argument, I will provide my own example. I will obtain it by changing certain assumptions that characterised the immediately preceding situation.

Let us imagine the world as a history—a series of events ordered according to time relations such as 'earlier than' and 'later than,' which we will leave undefined. And suppose for the sake of example that we have a divine perspective that is not relative to this or that moment in history. Let us further suppose that from this perspective we "see" that in every single instance in which the sun has shone on the steel sphere, it has also heated up. Now imagine that, with this knowledge, we "enter" the particular point in the history of the world at which the sun has just begun to shine on the sphere and make the following prediction:

Whenever the sun shines on the steel sphere, it will heat up; The sun shines on the steel sphere; Thus, necessarily, the steel ball will heat up.

Now, there is one feature of deductive analytic arguments that to my knowledge has been overlooked. Toulmin claims that the rejection of the warrant in a deductive analytic argument leads to a contradiction and that such warrants are backed not "by experience but by entailment" (2003, p. 128). I suggest that Toulmin's reasoning may have been as follows: experience will never provide enough data on which to base a general statement that has (i) a negation that is logically incompatible with the data and (ii) can be used to derive *new* predictions. This seems trivially true—if there are any new predictions, we couldn't possibly have all the data. But we would have to have all the data in order to be unable to reject the warrant without contradicting the data. In our somewhat artificial example, we used the absolute perspective to get all the data. We then used this data from the relative perspective to formulate the prediction. It is true that the conclusion of the argument is contained in the backing of the argument, and it is also true that rejecting the warrant of the argument leads to a contradiction. But it is certainly not true that warrant is not acquired by experience. A and B objects have been observed, albeit from a very unique perspective. Moreover, it is not clear why we would even try to justify the prediction with the argument. After all, we already had to assume the conclusion in order for our data to logically entail the warrant we needed to justify it. It is precisely for these reasons that I have doubts about whether Toulmin's first example of a deductive analytic argument in *Uses* is appropriate—the example of Jack's sisters on p. 117, which is analogous to the example just given and bears its problems as well.

### 5. Analyticity tests—are they unreliable?

Toulmin introduced three tests for identifying analytic arguments: the tautology test, the verification test, and the self-evidence test. According to the tautology test, it must be true that by stating the conclusion of an analytic argument, we simply repeat something that was already in the backing, and thus we can say "D, B, or in other words C." I have already mentioned this condition, and I have also pointed out that the property that this test identifies is not universal among analytic arguments. According to the verification test, it is not possible to verify the backing of an analytic argument without checking the truth of its conclusion. Note that the verification test does not mention the verification of the data and the backing of an analytic argument but only the verification of the backing, which should be sufficient to check the truth of the conclusion. Finally, according to the self-evidence test, the conclusion of an analytic argument should be evident to anyone presented with the data and the backing of the argument. According to Toulmin, (a) in some cases these tests give different results, and (b) the self-evidence test corresponds "more nearly" to the verification test than to the tautology test (2003, p. 121).

Let us proceed to Hamby's (2012) critical observations, which can be summarised in the following points:

- 1. It is not obvious which test is the main one. Toulmin claims it is the verification test but then uses the tautology test as the main authority (p. 120).
- 2. The tautology test does not give the expected result in the example of Jack's sisters (pp. 121–124).
- 3. The verification test also identifies arguments as analytic whose premises actually constitute support for the negation of their conclusions (p. 126).
- 4. There is an example of an argument that passes the tautology test but not the verification test (pp. 127–128).

# Ad 1)

I introduced our classification of arguments in part to clearly articulate the boundaries beyond which the tautology test is no

longer a reliable identifier of analytic arguments. The tautology test is valid in the case of deductive analytic arguments. In fact, all three tests are equivalent in the case of this category—that is, they always give the same result. Hamby notes that Toulmin first states the limited validity of the tautology test compared to the verification test in the case of so-called analytic "quasi-syllogisms"—that is, what we have identified in our classification as non-deductive analytic arguments (2012, p. 120). According to Hamby, the suggested difference in the scope of these tests is not compatible with what Toulmin says only 15 pages prior: "A valid analytic syllogism cannot in its conclusion tell us anything not already included in the data and warrant-backing" (Toulmin 2003, p. 139).

This seems to be the analyticity condition determined by the tautology test. But note that Toulmin is talking not about "quasi-syllogisms" in this quotation but about deductive analytic arguments. Toulmin himself said that the tautology test does not apply beyond this class of arguments. Therefore, it should not be surprising if, within the boundaries of this class of arguments, he agrees with the applicability of this test. Moreover, the above quotation is found in a section in which Toulmin is critical of logicians' alleged claim to the universal applicability of their standards of argumentation, in essence, the notion that all arguments must be deductive, analytic, and unambiguous (ibid.). Thus, I do not think that the textual evidence put forward by Hamby constitutes a challenge to the hierarchization of the analyticity tests.

# Ad 2)

Hamby (2012) then notes that the tautology test does not give the expected result in the case of the argument regarding Jack's sisters:

Anne is one of Jack's sisters; All of Jack's sisters have red hair; So, Anne has red hair. (Toulmin 2003, p. 115)

<sup>&</sup>lt;sup>9</sup> Compare Toulmin (2003, p. 123)

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The first line of this argument contains the datum, the second contains the warrant, and the third the conclusion. Toulmin also gives a version of the argument in which the warrant is replaced by a backing:

Anne is one of Jack's sisters; Each one of Jack's sisters has (been checked individually to have) red hair; So, Anne has red hair. (2003, p. 115)

Toulmin points out that if Jack's sisters are observed prior to the moment at which the conclusion is stated, the argument will not be analytic insofar as Jack's sisters could (for example) have dyed their hair in the meantime. In that case, we could produce at most a non-deductive substantial argument with a moderate modal modification of the conclusion. Accordingly, Hamby proposes the following formulation of the analytic argument regarding Jack's sisters:

Anne is one of Jack's sisters; Each one of Jack's sisters (it is now being observed, i.e., it now appears) has red hair; So, Anne has red hair. (2012, p. 121)

According to Hamby, however, these arguments are not formally valid—for this to be so, the phrase in brackets would have to be in the conclusion as well:

Anne is one of Jack's sisters; Each one of Jack's sisters (it is now being observed, i.e., it now appears) has red hair; So, Anne (it is now being observed, i.e., it now appears) has red hair. (p. 122)

According to this observation, Toulmin's characterisations of the deductive analytic argument fail. It is not true that if we substitute backing for warrant, we get a formally valid argument in the sense

that the conclusion is merely a repetition of something stated in the premises of the argument thus modified.

Hamby's approach to interpreting Toulmin's example is not very accommodating. After all, in his example of Newton, Toulmin described a backing as a report about a repeated experience with *particular* objects. We have interpreted this "report" as a list of conjuncts. A similar interpretation of the backing was brought to Hamby's attention by Freeman in private correspondence, and Hamby mentions Freeman's suggestion in the concluding sections of his article:

Anne is one of Jack's sisters; Anne has red hair, & Sister #2 has red hair, & ..., & Sister #n has red hair; So, Anne has red hair (2012, p. 129).

The backing of the argument is in the second line. The argument as presented meets the tautology test, but it also meets the verification test because its backing cannot be verified without also checking the truth of the conclusion. Hamby was not entirely convinced by Freemen's interpretation. He conceded that the Jack example, thus interpreted, passes the tautology test, but he failed to notice that it also passes the verification test. And as for the tautology test, Hamby, in response to Freeman's criticism, returns to the objection that, on the basis of Toulmin's description of the tests, it is impossible to say which test has what authority (Hamby 2012, pp. 129-130).

But given Toulmin's characterisation of deductive analytic arguments, according to which negation of the warrant leads to a positive contradiction, Freeman's reconstruction of the backing is not entirely correct. The backing is what is invoked to justify the warrant, and the warrant must rely on the backing in a way that does not allow the warrant to be denied so long as one accepts the backing. My suggestion for the backing is as follows (assuming that Jack has three sisters, for example Anna, Eve, and Beata):

Anna is Jack's sister & Anna has red hair; Eva is Jack's sister & Eva has red hair; Beata is Jack's sister & Beata has red hair & Jack has no other sisters except Anna, Eva & Beata.

Even this proposal is not entirely acceptable, however. The statement "Jack has no sisters other than Anna, Eva, and Beata" is not verifiable simply by staring at Anna, Eva, and Beata.

We have already mentioned that Toulmin has not offered an adequate illustration of a deductive analytic argument. The example of Jack's sisters is similar to the example of the steel sphere—the backing summarizes information about *all* objects, including the object mentioned in the conclusion. Moreover, it is also true in this example that arguing in favour of the conclusion seems futile—after all, the conclusion must first be assumed in the backing so that the warrant of the argument can be used to derive the conclusion in the desired way, in essence, analytically and unambiguously. As Toulmin writes, "[t]he thing to do now is use one's eyes, not hunt up a chain of reasoning" (2003, p. 117). And just as in the case of the steel sphere, even here it is not true that experience does not play a role.

# Ad 3)

Since non-deductive analytic arguments (analytic "quasi-syllogisms") are not our main concern, I will touch on this category of arguments and objections to them only briefly. Toulmin introduces this type of argument on p. 122:

Petersen is a Swede; Scarcely any Swedes are Roman Catholics; So, almost certainly, Petersen is not a Roman Catholic.

This argument is obviously not deductive since the warrant is compatible with a state of affairs in which some Swedes are Roman Catholics. The datum and the warrant do not exclude the possibility of Petersen being a Roman Catholic. The argument's backing is as follows: "The proportion of Roman Catholic Swedes was (say) less than 5%" (p. 123).

Now again, I believe that the backing thus expressed is merely shorthand for the "real" backing, which consists of a list of conjuncts such as *Olaf is a Swede & Olaf is a Protestant; Ebba is Swede & ...* Toulmin argues that this argument fails the tautology test, which is obvious since the conclusion of the argument, "Petersen is not a Roman Catholic," need not (explicitly or implicitly) be contained in the backing. But he also believes that the example meets the verification test since one cannot verify the backing without verifying the conclusion of the argument (Toulmin 2003, pp. 122–123). It seems that, according to Toulmin, verifying the truth of the backing would mean checking the religious beliefs of every single Swede, including Petersen (p. 123).

Hamby formulates the following counterexample to the verification test:

Petersen is a Swede;

The proportion of Roman Catholic Swedes is less than 5%; So, almost certainly, Petersen *is* a Roman Catholic. (2012, p. 126)

The same criticism was made earlier by Cooley (1959, pp. 303–304). The crux of it is that the premises of this "analytic" argument do not support the conclusion but rather its negation. A conception that puts forward the concept of an analytic argument with these implications has serious problems. What both authors overlook, however, is that in addition to the tautology and verification test, Toulmin also introduces the self-evidence test, which might play an important role with respect to this counterexample. In fact, it seems obvious that Toulmin applies precisely this test to the counterexample of the argument given above (2003, p. 122). After all, both Cooley and Hamby put forward this counterexample precisely because they find it to be *obviously* unacceptable.

However, both authors' comments point out that the verification test cannot always be applied *without the* simultaneous use of the self-evidence test—the verification test defines a set of potentially analytic arguments. This set is then further "narrowed" by the self-evidence test. If we were to accept this answer as a solution, we would still face the problem of the *vagueness* of the self-

evidence test. For example, would we agree that an argument satisfies the self-evidence test if 6% of Swedes were Catholic? Obviously, we would have strong doubts about the analyticity of the argument if the ratio of Catholics to non-Catholics in Sweden were 40:60. At what ratio would our attitude change?

#### Ad4)

Hamby formulated an example of an analytic argument that supposedly fails the verification test but meets the tautology test. Now, if Toulmin has determined the set of analytic arguments using a verification test that is supposed to be essential to this type of argument, and if Hamby claims to have an analytic argument that fails this test, then it is questionable whether this is a counterexample or a consequence of adopting a different conception of analytic arguments. Moreover, Toulmin nowhere says (as far as I know) that the set of arguments that pass the tautology test is a subset of the set of arguments that pass the verification test. So, it's quite possible that he would admit the existence of an argument that passes the tautology test but not the verification test. Based on what we know, however, it seems certain that he would disagree that such an argument is analytic. Hamby gives the following example of the argument (after substituting the backing for a warrant):<sup>10</sup>

#### Petersen has a beard;

Every person named Petersen I've met so far has been Swedish, and every person with a beard I've met has been male; That is why Peterson is a Swedish man. (Hamby 2012, p. 127).

Let us leave (without comment) the observation that in terms of structure this argument has little to do with a syllogism. Hamby (2012) correctly notes that the argument fails the verification test if the one who utters it has never encountered Petersen. On the other hand, Hamby claims that the argument passes the tautology

<sup>10</sup> I haven't presented a version of the argument with a warrant because the warrant is irrelevant when testing the analyticity of the argument.

test and adds that this is true unless we are operating with the "strong notion" of the conclusion *being implicit*. Hamby clarifies what it means for a conclusion to be "not strongly" implicit in its backing in a footnote:

Toulmin gives no indication as to what exactly counts as an implicit reference. But, given the clear object in his book of showing the inadequacy of deductive standards, it is reasonable to think that he had a loose idea of the kind of implicitness that was meant to satisfy the tautology test. (2012, p. 128)

I would disagree—Toulmin said that backing is supposed to repeat (explicitly or implicitly) what the conclusion says, and as a result of this repetition, we cannot reject the conclusion and accept the backing. Hamby himself accepted the premise that the person who utters the argument has never met Petersen. It follows that rejecting the backing (or datum) and accepting the conclusion does not lead to a contradiction. Thus, this example of an argument fails the tautology test and is not a counterexample to Toulmin's conception of an analytic argument.

# 6. An example of a "real" deductive analytic argument

Immediately after Toulmin gives the example of the analytic argument about Jack's sisters, he presents another example of an analytic argument, this time from the field of mathematics. Unlike the argument with Jack's sisters in which the person who utters the argument must stare at the sisters at the very moment they are arguing, in the case of this next argument, as Toulmin points out, we are "perfectly safe" given that solutions to mathematical problems are "above time," and they have no "expiration":

Given the assurance that every sequence of six or more integers between 1 and 100 contains at least one prime number, and also the information that none of the numbers from 62 up to 66 is a prime, I can thankfully conclude that the number 67 is a prime (2003, p. 118).

Let us reconstruct this argument according to the microargument model:

D: None of the numbers from 62 up to 66 is a prime.

W: Every sequence of six or more integers between 1 and 100 contains at least one prime number

C: The number 67 is a prime number.

R: -

B:

- 1, 2, 3, 4, 5, 6 is a sequence of six or more integers between 1 and 100 & 1 belongs to this sequence & 1 is not a prime number; 2 belongs to this sequence & 2 is a prime number;
- 2, 3, 4, 5, 6, 7 is a sequence of six or more integers between 1 and 100 & 2 belongs to this sequence & 2 is a prime number;
- 3, 4, 5, 6, 7, 8 is a sequence...

D is the datum of the argument. Together with the warrant, this entry should lead us to the conclusion that 67 is prime. The presence of the connective 'or,' along with the part of the warrant that follows it, is problematic, although there is a reason why the warrant has been extended in this way—there is not a single prime number in the seven-digit sequence from 90 to 96.<sup>11</sup> The fact that Toulmin (2003) used the quantifier 'all' in the warrant suggests that his intention was to produce a *deductive* analytic argument. But this argument is not deductive. The warrant might in fact read as follows: *Almost every sequence of six numbers between 1 and 100 contains at least one prime number*. With this warrant, we can

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<sup>&</sup>lt;sup>11</sup> If there were at least one prime number in every six-digit sequence of numbers from 1 to 100, then the assurance would be true without the need to addition of 'or more.' However, since it is not the case that there is at least one prime number in every six-digit sequence of numbers between 1 to 100, Toulmin had to state the assurance exactly as he did. Otherwise, the assurance would be false. Therefore, I believe that the statement "Every sequence of six or more integers between 1 and 100 contains at least one prime number" is, in this case, equivalent to the statement "Almost every sequence of six integers between 1 and 100 contains at least one prime number." Warrants containing qualifications like 'almost' are typical of tentative arguments.

qualify the conclusion only moderately, for example, with the modal qualifier 'almost certainly.' We have only hinted at the backing of the argument, but it should be obvious that the statement "The number 67 is a prime number" belongs to it. Thus, it is true that the backing contains the information that is given in the conclusion: the argument would pass all three tests—the verification test, the tautology test, and the self-evidence test—if it were correct.

After the problematic example of Jack's sisters, we were presented with an example from "pure mathematics," which, according to Toulmin, is the true domain of deductive analytic arguments. For the reasons I have given, the methodological purpose of using the example (i.e., clarification) was not fulfilled in this case either.

### 7. What is a deductive analytic argument?

The purpose of this section is to define the class of deductive analytic arguments. I have found Toulmin's later book, Knowing and Acting: An Invitation to Philosophy (1976), to be an excellent interpretive aid for selected parts of *Uses*. Relevant for our purposes is the chapter "The Philosopher as Geometer" and its two subchapters "The Claims of Logic" and "Philosophy as the City of Truth." In these, Toulmin describes how Plato inspired philosophers by taking "formal geometry" as a model for the type of knowledge that was to be the ideal for other fields of inquiry (1976, p. 62). In this tradition it is thought that, like geometry, other fields of knowledge should form a system based on first "self-evident" principles, or axioms, from which all other truths are derived through deductive networks of arguments, rendering those truths as irreducible as the first principles (p. 70). Logic has established itself as a discipline that criticizes deductive arguments in all fields by the standards of geometry—on this model, if the truths of a field are not deductive consequences of the first selfevident, or necessary, principles, then they are not "well-founded." "Formally valid" arguments are supposed to be those that lead from more fundamental truths to derived truths (pp. 71–72). The result of knowledge, or its goal, is the so-called "Eternal City of well-founded truths" (p. 82). Its main characteristics are that it has a firm foundation (axioms) and a "rigid structure" provided by deductive or formal arguments that connect axioms to derived truths (p. 82–84). As Toulmin writes:

From the standpoint of this ancient tradition, "giving reasons" is the same as *proving*, "having good reasons" is the same as *being able to prove*, and the "rationality" of our thoughts and beliefs is measured by the strength and solidity of the *formal proofs* and *grounds* on which they rest (1976, pp. 84–85).

This is how Toulmin describes the origin and development of the formal or geometrical validity of an argument, which supposedly was then to become the standard for all other areas of knowledge. I have already mentioned the conditions which, according to Toulmin, must hold with regard to the warrants of deductive analytic arguments— the rejection of the warrant of a deductive analytic argument leads to a contradiction, and that warrant must be based not on experience but on entailment (p. 128). Later, Toulmin describes the warrants of deductive analytic arguments as being "connected by formally valid arguments back to self-evident axioms and principles" and their truth as therefore having "an absolute guarantee" (1976, p. 105).

All this brings us to the proposal of two principles that together capture how, in the Eternal City of Truths, derived truths "inherit" the necessity of first principles:

$$A 
ightharpoonup B \rightarrow (\Box A \rightarrow \Box \beta)$$
 $((A 
ightharpoonup B) \land (B 
ightharpoonup C) \rightarrow A 
ightharpoonup C$ 

According to the first principle, if B is derivable from A, then if A is necessary, B is necessary. By the second principle, if B is derivable from A and C is derivable from B, then C is derivable from A. Both principles hold in the Eternal City of Truths, and together they express the fact that every proposition belonging to it is necessary in the same way that its foundations are necessary. All arguments in such a system are deductive, and every proposition that constitutes them is necessary. If knowledge came with such

requirements, then the proposition that is supposed to express knowledge would have to be necessary. In other words, for an utterance to express knowledge, it would have to be shown to follow logically from first truths.

I will now give a very simple example of a deductive analytic argument from the field of arithmetic:

Suppose that every even number is divisible by 2 and that 6 is an even number. Then we can conclude that 6 is divisible by 2.

If we project this argument onto the structure of the microargument, we get the following.

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D: 6 is an even number;
W: Every even number is divisible by 2;
C: The number 6 is divisible by 2.
R: -
B:
-...
-2 is an even number & 2 is divisible by 2.
-4 is an even number & 4 is divisible by 2.
-6 is an even number & 6 is divisible by 2.
-...
```

The datum, the warrant and the claim of this argument are not only true but also necessarily true because there is no possible world or point in time in which they are false. Furthermore, negating the warrant of this argument is incompatible with accepting the backing, which consists of a list of pairs of conjuncts where the first member of the pair is the claim that some number is even and the second member is the claim that the number is divisible by 2. The reason is that the backing contains *all* even numbers—including the one stated in the conclusion—and that *each* of them is divisible by 2. Moreover, even the elements of the pairs in the backing are necessary and non-empirical truths. Thus, I think it is true to say that the warrant of this argument is backed not "by experience but by entailment"—or at least I can think of no more appropriate circumstances in which to say this.

Notice that even the claim is contained in the backing. Thus, metaphorically speaking, the analytic argument just presented can be said to resemble a tree of knowledge whose trunk (backing) corresponds to fundamental truths from which branches (claim, warrant, conclusion) grow. It resembles the conception of knowledge according to which there are first truths (seeds) from which, through chains of deductive arguments, other truths, and ultimately the whole system of knowledge, "unfold."

Did Toulmin want the microargument to be the universal pattern for analysing and evaluating arguments? There is no simple answer to this question. For example, in the case of non-deductive substantial arguments, the microargument plays an important role not only in developing critical thinking skills but also in allowing us to recognise a deeper level of argument, to see its limitations, and to qualify conclusions appropriately. So yes, the microargument can be seen here as a model by which we can construct, reconstruct, and evaluate arguments.

The situation is different in the case of formal deductive logic, however. Take, for example, the argument with the premises *All men are mortal*, *Socrates is a man* and the conclusion *Socrates is mortal*. According to Toulmin, if the claim regarding Socrates' mortality is made in the time before his death, then the conclusion of the argument still "remains a supposition" (1976, p. 99). The reason, of course, is that the conclusion is not contained in the argument's backing. It is a substantial deductive argument. Moreover, the warrant of the argument is not "guaranteed absolutely" (ibid., p. 104). Toulmin adds, however, that in view of the logic, the argument is a "formal deduction." Similarly, in *Uses*, Toulmin says that some substantial arguments are properly called "deductive." "Micro-physiologically," he continues, these arguments

[...] may thus remain mathematical in structure", and this even though "we make genuine and even far-reaching steps [in these arguments], passing from our original data and warrant-backing to conclusions at once fresh and of quite different types (2003, p. 194).

This means that formal logic is, at least to some extent, blind to the distinction between deductive substantial arguments and deductive

analytic arguments. For these reasons, I think, Toulmin proposed the microargument as a means of capturing the differences between arguments that are obscured by the formal treatment of such arguments in deductive logic. This, in my view, is the main purpose of the microargument, which was never intended to supplement or replace the argument schemes of formal deductive logic, such as the categorical syllogism or modus ponens. It is possible that the microargument may have been introduced by Toulmin as part of his point that we can make "formal deductions" even if the arguments do not meet the criteria of deductive analytic arguments. What Toulmin was objecting to is the requirement that the warrant of the argument must be unquestionable and that its conclusion must be contained in the backing (2003, p. 104), for very few arguments would "survive" such an immense demand. The deductive analytic argument is the concept that Toulmin introduced to identify just this group of potential survivors.

There are two ways to describe deductive analytic arguments. The first discusses their "natural habitat," the second their formal aspects.

According to the first, deductive analytic arguments come from areas of knowledge where the derived truths in some sense say no more than has already been implicitly or explicitly stated in the more fundamental truths from which they logically follow—from areas of knowledge where deductive reasoning can be interpreted as an analysis or elaboration of the meaning of first truths. Toulmin's example is the Pythagorean theorem, which in a sense says nothing that is not in the axioms of Euclidean geometry (2003, p. 178). The problem with this description is that it is not very illuminating. It says nothing specific about the characteristics of deductive analytic arguments that distinguish them from all other arguments.

Let us now turn to the formal description. Let A-reasons be those reasons that are taken to be true and that are not subject to justification or challenge; they are initial assumptions. A deductive argument X is analytic if and only if all its premises are A-reasons or if its premises are deductively entailed either directly or indirectly by A-reasons. In the first case, A-reasons are premises of deductively valid arguments whose conclusions are premises of X.

In the second case, premises of X are connected to A-reasons via a finite or infinite chain of deductively valid arguments.

Now, the problem with this formal way of determining the class of deductive analytic arguments is that it is insufficient without additional epistemological assumptions. Why, for example, couldn't we take the propositions "Anna is Jack's sister" and "All of Jack's sisters have red hair" as A-reasons? Toulmin returns to the "Jack's sisters" argument later in *Uses*, and I think that this later comment is very helpful in terms of answering this question. We may next begin to feel a little shaky even about things at present in sight or within earshot. After all, if we really ask what we have to go on when we make claims to knowledge about these things too, we can point only to the way things look to us and sound to us at this moment, and all the traditional arguments leading to scepticism about the senses can immediately be brought to bear on us: no collection of data, however large, about how things seem to us now can entail the truth of a conclusion about how they in fact are (Toulmin 2003, p. 206).

The reason why Toulmin ends up doubting the analyticity of the "Jack's sisters" argument is that its premises are uncertain, or "shaky." Deductive analytic arguments, it seems, must therefore be based on premises that are unquestionable, and premises can be questionable either because they are not A-reasons or because they are conclusions of substantive arguments. Returning to our question about why the premises of the "Jack's sisters" argument are not A-reasons, the answer is that they are based on substantial and therefore shaky arguments.

Since the formal description of deductive analytic arguments is not sufficient to determine their precise class, it is necessary to add a further characterisation of A-reasons. A-reasons are not true because of any other arguments that would support them. Nor is their truth a matter of mutual agreement or convention. A-reasons are simply self-evident, and as such, by definition, they cannot be a matter of doubt or proof. A-reasons are first truths.

Toulmin, of course, tried to propose his own formal description of deductive analytic arguments, which was couched in terms of 'containment,' 'tautology,' 'entailment,' and so on. However, the description obscured the fact that deductive analytic arguments are not a purely logical concept but are imbued with epistemological presuppositions, and as such they are alien to deductive formal logic.

Given what has been said above, it is not difficult to see why examples such as the "Jack's sisters" argument and the Socrates argument, and even the steel sphere argument, are deductive but not analytic. Their premises are conclusions of "shaky" substantial arguments. In light of Toulmin's later comment on the "Jack's sisters" argument, however, it is possible to think of an even simpler criterion or test. This test is negative; in essence, it tells us when a deductive argument is not analytic. Toulmin rejected the idea that beliefs based on how things "look" or "sound" to us are ever sufficiently solid to be part of deductive analytic arguments. This means that propositions with empirical content are never part of deductive-analytic arguments.

#### 8. Conclusion

If my observations are correct, Toulmin's criticism is not really about deductive logic, for although deductive logic has tools for modelling deductive analytic arguments, these arguments are still only a very small and narrow class of arguments that fall within its scope. Toulmin's criticism seems more relevant not to deductive logic itself but to those who claim that deductive logic is the only proper tool for achieving knowledge and that knowledge must be unquestionable and thus based on necessary truths. The first claim is related to deductivism—the theoretical position that we should reconstruct every argument as if there were a deductive inferential relation between its premises and its conclusion—while the second claim corresponds to epistemological foundationalism. But these two claims are not necessarily related. It seems that if one believes that genuine knowledge must be based on first truths from which it derives its necessity, then one must consequently agree with deductivism. But the reverse is not true—it is perfectly possible to hold deductivism but not epistemological foundationalism. And it is certainly false that there is a direct conceptual relation between deductive logic and any particular view of what conditions must be satisfied for something to count as knowledge. In other words,

the logicians' tools in no way commit them to the epistemological presuppositions that are embedded in the concept of a deductive analytic argument.

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