Arnold Koslow & Arthur Buchsbaum (eds.), The Road to Universal Logic: Festschrift for 50th Birthday of Jean-Yves Béziau, 2015, vol. 1, Cham, Heidelberg, etc.: Springer-Birkhäuser.

Causality and Attribution in an Aristotelian Theory

Srećko Kovač

Abstract. Aristotelian causal theories incorporate some philosophically important features of the concept of cause, including necessity and essential character. The proposed formalization is restricted to one-place predicates and a finite domain of attributes (without individuals). Semantics is based on a labeled tree structure, with truth defined by means of tree paths. A relatively simple causal prefixing mechanism is defined, by means of which causes of propositions and reasoning with causes are made explicit. The distinction of causal and factual explanation are elaborated, and examples of cyclic and convergent causation are given. Soundness and completeness proofs are sketched.

Mathematics Subject Classification (2010). Primary 03A99; Secondary 01A20.

Keywords. predication, attribute, subject, reason, cause, proof, labeled tree.

1. Introduction

The role of causality in philosophical and scientific theories, for example in the last hundred years, ranges from its role as a primitive or fundamental concept (even in logic and set theory) to its dismission at least from "austere" science, and to its confinement to informal, ordinary discourse. What is equally important is that we do not presently have a unique concept of causality to which we may refer as to the standard one.[1]

Our starting background question is whether causality is (or should be) a primitive concept of theories. This leads to the question about causality and logic: since we assume that each theory should include (beside its specific axioms) some sort of logic (i.e. language and a sort of a consequence relation on the sentences of the language), the question arises whether the concept of causality – and in which sense of causality – is connected with logic as such ("essentially"), or originates from it, or is in any other way closely related to it. For example, in a Kantian approach, the

[1]For instance, the editors of the Oxford Handbook of Causation say: "Philosophers have been interested in the nature of causation for as long as there has been philosophy. . . . Despite the attention, there is still very little agreement on the most central question concerning causation: what is it?" [7, p. 1].

concept of causality stems from formal logic (from the "logical function" of hypothetical judgment, in essential connection with the principle of sufficient reason) but in application to representations given in intuition. On the other hand, Gödel reflects on causality as a fundamental philosophical concept from which even logic and set theory should be derived [12, p. 432–435]. Be it as it may, the interrelationship of logic, causality and knowledge seems to be one of the fundamental, open questions.[2]

The approach we take here is meant to go back to historical origins of logic and of the theory of causation, and to possibly see whether and in which way these conceptions could give some orientation in current reconsiderations of the concept and the role of causality. We propose a formalization of Aristotelian theory of causality, with the aim to show that Aristotelian concept of causality reinforces the view on causality as a basic concept of our theories, and moreover, that the concept of causality is of intensional nature and essentially connected with logic. One special interest is to find, along Aristotelian lines, in which way and how much a causal chain leading to some event can be reduced in order to avoid unnecessary complexity and yet to retain the part of conditions that may still be called a cause of the event.

2. Causality and attribution in Aristotle

There are some general features of the Aristotelian concept of cause:

1. a cause is something because of what (dia ti), or why, some state of affairs (thing, pragma) obtains (see e.g. An. Post. A2 71b 10–11, A6 75a 35, A24 85b 23–24 [1, 3, 4]);
2. the causation of a state of affairs is necessary, in the sense that things cannot happen otherwise when the cause is present (e.g. An. Post. A2 71b 11–12);
3. the cause of a state of affairs is a necessary and a sufficient condition of the state of affairs (if the affirmation is the cause of ϕ, the negation is the cause of $\neg\phi$; cf. An. Post. A13 78b 17–21);
4. cause essentially ("in itself") belongs to the thing it causes, i.e. the cause and the thing should be interconnected by means of what they are (their essences), or by means of how they are defined (i.e. by means of their respective concepts) (see An. Post. A24 85b 24–25, B8 93a 4).

Feature (1) distinguishes causes from mere (non-causal) reasons explaining the fact that something obtains; for example, non-twinkling of planets is a reason that explains that planets are near, but not the reason why it is so, whereas the reason why planets do not twinkle is that they are near (An. Post. A13); feature (2) distinguishes causality from what happens only accidentally, even if it happens always (An. Post. A6 75a 32–35), or otherwise may and may not happen (cf. An. Post. A6 75a 18–21); feature (3) distinguishes a cause from a remote or too general

[2] These questions are addressed, for example, in a short programmatic paper by J.-Y. Béziau [8], where a very general formalized theory is envisaged that is motivated by da Costa's formalization of the principle of sufficient reason and Shopenhauer's philosophical views on the principle.

reason, responsible for (or merely explaining) other facts too (see below the analy-sis of Aristotle's example of why walls are not breathing); feature (4) distinguishes causality from merely equivalent (reciprocal) phenomena and properties (propria) – see examples in Section 4 below.

Aristotelian cause of a property is an essential attribute (of what has the property) which is the necessary and sufficient reason why the property obtains. On the other side, for each attribution (hyparchein, 'to belong') it could be asked about its "why", looking for its cause in this attribution itself (in which case it is a primitive attribution) or outside the attribution (in which case it has an external cause). Aristotelian formal logic (categorical syllogistic) gives a formal account of attribution interdependences (e.g. if A is universally attributed/belongs to B, and B is universally attributed to C, then A is universally attributed to C). Accordingly, Aristotelian formal logic is nothing but a general formal theory of attribution. Moreover, according to Aristotle's definition of a syllogism, the premises are causes of the conclusion (An. Pr. A 1, 24b 18–22 [1, 5, 6]), i.e. forms of reasoning have themselves their formal reasons why (formal causes). Aristotle explicitly states that premises are the material cause from which the conclusion necessarily follows (Metaph. Δ2 1013b 20–21 [1], see also [13][3]). Finally, special Aristotelian theories, that is special sciences, are applied logic – theories of causation in some specific area, with specific primary propositions added to a general formal logic.

3. Preliminary account of Aristotelian attribution

As far as can be seen from Aristotle's logic texts, attribution is a primitive logical relationship, and is to be distinguished from set membership in the extensional interpretation of modern P x. It is technically expressed as "A belongs to B" ("B is A", "B can be taken as an A"), where A is an attribute and B the subject of attribution. With respect to a possible expression, attribution is also called predication, attribute is a predicate, and the subject of attribution is now the subject of predication. In [13] we have analysed Aristotle's definition of the meaning of AaB proposition:

> A is predicated to all B (AaB) ⟺ no B can be taken to which A is not predicated.

Natural reading of this definition seems to imply the existential import of subject B. Namely, non-existence of anything that is B would imply that A (or whatever other predicate) is not predicated to any B, simply because there are no Bs. But this seems to be denied by AaB. So, in this reading, to verify that there is no B to which A is not predicated, there should be a B, and to each B A should not be predicated. It is not modern ¬□x(Bx □ ¬Ax), but rather something like A □ X for no X which is chosen as B (with < for 'belongs to'). In this way, we get the

[3]For the causal meaning of the expression form 'if something holds it is necessary for something (else) to hold', used in Aristotle's definition of the syllogism, see, e.g. An. Post. B11.

following propositions in the square of opposition:

(a) A ⍰ X for no X which is taken as B,
(e) A < X for no X which is taken as B,
(i) not (A < X for no X which is taken as B),
(o) not (A ⍰ X for no X which is taken as B),

which we abbreviate in the following way:

(a) A ⍰ no B
(e) A < no B
(i) not (A < no B)
(o) not (A ⍰ no B).

X that is taken (ecthesis) as a B should be meant, according to Aristotle's examples, as a lower species of B, also as a species of the species, till the individuals to which the lowest species (species infimae) are attributed. Hence, in the proposed reading "existential import" of subject terms holds for a and i propositions but not for e and o. It can be easily seen that such a reading enables all opposition in the square of propositions (for existential import in general, see [16, 17]). In our formalization we will use a modified ecthesis approach, combined with a sort of (hidden) reflexivity, and will be able to retain in logic only attributes, without individuals.

The alternative Aristotle's definition of the meaning of a proposition does not use ecthesis from the subject term but directly relates subject and predicate terms by means of the whole – part relation: AaB means "B is in A as in a whole" (this approach is used in [15]). So B (or that to which B is attributed) seems to be assumed as somehow given in order to be a part of A (or of that to which A is attributed) as a whole.[4] This manner of speaking can completely avoid any mention of individuals (no X should be taken as an example of a lowest species) and enables to consider exclusively attributes in their mutual attribution.

4. Causal language and models

In what follows we propose a formalization of Aristotle's account of a causal (scientific) theory, possibly with some simplifications, which presents in more detail the general Aristotelian logical structure of causality. Each causal theory (each special science) represents a one-rooted tree structure with a genus at the root and the low-est (unanalyzable) species as leaves, interconnected with one another and with the genus by means of a mutual causal attributions.

We define language L and the semantics for a theory that includes Aristotle's general syllogistic, accompanied with primary propositions that are specific for a given scientific field. The language includes explicit causal prefixes indicating causes in front of propositions.

[4] In this sense, Aristotle speaks, for example, of species ("man", "horse") to be a part of a genus ("animal"), Met. Γ, 26. It is the whole-part relation in the distributive sense that each of the many (parts) is one (genus), not in the collective sense of the one that consists of many.

Vocabulary: $A_0, \ldots, A_n$; operators: a, e, i, and o (subscripts and superscripts will be omitted if no ambiguity arises). P is the set $A_0, \ldots, A_n$.

Definition 4.1 (Term).

1. A_i is a term (predicate letter),
2. if Φ is a term, $\overline{\Phi}$ is a term,
3. if Φ and Ψ are terms, $(\Phi\Psi)$ is a term (usually with outer parentheses omitted),
4. if Φ and Ψ are terms, $\Phi \times \Psi$ is a causal (prefix) term.

Definition 4.2 (Sentence). If Φ, Φ', and Γ, are terms, then

1. $\Phi a \Phi'$, $\Phi i \Phi'$ are sentences,
2. $\Gamma : \Phi a \Phi'$, $\Gamma : \Phi i \Phi'$, $\Gamma : \Phi e \Phi'$ and $\Gamma : \Phi o \Phi'$, where Γ can also be a causal term, are sentences. Also
3. if ϕ is a sentence, then $\neg\phi$ is a sentence.

$\Phi e \Phi'$ and $\Phi o \Phi'$ abbreviate $\neg \Phi i \Phi'$ and $\neg \Phi a \Phi'$, respectively.

Definition 4.3 (Theory tree, T). Theory tree T is a set $\langle W, w_0, < \rangle$, which is a finite ternary tree, i.e. a finite set W that is partially ordered by $<$ with at most three different immediate successors, where for each $w \in W$, $\{w' \mid w' < w\}$ is well-ordered, and with w_0 as a least element.

We call members of W nodes, and $<$ a basic predication relation. Further, we call each maximal totally ordered subset b of W a T-branch, and each initial segment p of a branch a T-path. The height of a w, $h(w)$, is the order-type of $\{w' \mid w' < w\}$. Since p is a sequence we write $\langle w_i, \ldots, w_k \rangle \subseteq p$ for $\{\langle j, w_i \rangle, \ldots, \langle l, w_k \rangle\} \subseteq p$, where j and l are ordinals.

Definition 4.4 (Frame, F). Frame F is a set $\langle T, A \rangle$, where T is a theory tree, and A is a finite set of basic attributes such that there is a bijection from P to A.

Definition 4.5. (Attributive equivalence class, $[A]$)

$[A] = \{A\} \cup X \square \square A$ such that

1. $A \notin X$,
2. for each A and B with $A = B$, $([A] \setminus \{A\}) \cap ([B] \setminus \{B\}) = \emptyset$.

Definition 4.6 (Labeling). V is a labeling function such that $V(w \in W) \in \{[A_i]\}_{i \leq n}$ and

1. $h(w) = h(w')$ if $A \in V(w)$ and $A \in V(w')$,
2. w and w' do not have the same immediate predecessor if $V(w) = V(w')$.

Instead of $\langle w_i, \ldots, w_k \rangle \subseteq p$ we will usually write $\langle [A_i], \ldots, [A_k] \rangle \subseteq p$ if $V(w_i) = [A_i]$ and $V(w_k) = [A_k]$.

We can now introduce the concepts of species and genus.

Definition 4.7 (Species). Species is an attribute $A_0 \ldots \ldots A_k$ such that the sequence $\langle [A_0], \ldots, \ldots, [A_k] \rangle$ is by V in 1-1 correspondence with a T-path p of F.

A lowest species (infima species, atomōn eidos) is a species defined by a 1-1 correspondence with a branch b of F. Each non-lowest species is a genus.

As an example, the first genus (the general subject) of arithmetic is for Aristotle "number", with "odd" and "even" as the first pair of basic essential attributes. Further, odd numbers as well as even numbers are distinguished with respect to "non-measurable by a number" (prime) and "measurable by a number". The further distinction, applied to both previous attributes, is the distinction with respect to "non-compounded of numbers" (prime in the second sense), and "compounded of numbers" (let us have in mind that for Aristotle "one" is not a number, and that a number is not a measure of itself). For example, the lowest species that we obtain by successively determining numbers by attributes "odd", "non-measurable by numbers", "non-compounded from numbers" is "triad" (see An. Post. B13 96a35–96b1).

According to Aristotle, a causal theory should be primarily concerned with essential attributes, but sometimes equivalent peculiar properties (like "non-twinkling" of planets) occur that are dependent on essential attributes (like "near" of planets). Only essential attributes give a scientific, causal proofs (demonstratio propter quid, apodeixis tou dia ti), whereas peculiar properties could give only factual explanation (demonstratio quia, apodeixis tou hoti). Another Aristotle's example of essential and peculiar attributes is "spherical", as an essential attribute of the Moon, and "waxing", as a dependent equivalent property: the Moon is waxing in its specific way because it is spherical, not vice-versa (Moon should be here conceived as a lowest species in astronomy, that applies to only one object).

In the definition of a model the actualization function is included in order to model causal interrelationships. The idea is that the causal interrelationship between immediately connected (or non-connected) attributes in a tree is actualized internally (by means of the essences of the respective attributes themselves), and that otherwise external actualization (through intermediate attributes) is presupposed. This is a simplification of the prefixing mechanism used in causally interpreted justification logic, where a proposition prefix reproduces the whole causal structure that leads (in a system) to the proposition (see [10, 14]).

Definition 4.8 (Model). Model M is a quadruple $\langle \mathsf{F}, \mathsf{V}, \mathsf{I}, \mathsf{A} \rangle$ where

1. F is a frame,
2. V is a labeling function,
3. I is an interpretation function such that $\mathsf{I}(A_i) = \mathsf{A}_i$, $\mathsf{I}(\overline{\Phi}) = \overline{\Phi}$, $\mathsf{I}(\Phi\Psi) = \Phi\Psi$, $\mathsf{I}(\Phi \times \Psi) = \Phi \times \Psi$,
4. let p be a path, and $[\mathsf{A}]^{\mathsf{k}}$ a node of height k to which $[\mathsf{A}]$ is assigned; A is an actualization function from the set of formulas to the set of attributes:
   (a) $\mathsf{B}, \mathsf{C} \in [\mathsf{A}] \implies \mathsf{A}(BaC) = \mathsf{A}$,
   (b) (i) $\exists \mathsf{p}\, \langle [\mathsf{A}]^{\mathsf{k}}, [\mathsf{B}]^{\mathsf{k}+1} \rangle \in \mathsf{p}\ \&\ \neg \exists \mathsf{p}'\, \langle [\mathsf{A}']^{\mathsf{k}}, [\mathsf{B}]^{\mathsf{k}+1} \rangle \in \mathsf{p}' \implies \mathsf{A}(AaB) = \mathsf{A} \times \mathsf{B}$, where $\mathsf{A}' = \mathsf{A}$,
   (ii) $\exists \mathsf{p}\, \langle [\mathsf{A}]^{\mathsf{k}}, [\mathsf{B}]^{\mathsf{k}+1} \rangle \in \mathsf{p} \implies \mathsf{A}(AiB) = \mathsf{A} \times \mathsf{B}$,
   (iii) $\exists \mathsf{p}\, \langle [\mathsf{A}]^{\mathsf{k}}, [\mathsf{B}]^{\mathsf{k}+1} \rangle \in \mathsf{p}\ \&\ \exists \mathsf{p}' \langle [\mathsf{A}]^{\mathsf{k}}, [\mathsf{C}]^{\mathsf{k}+1} \rangle \in \mathsf{p}' \implies$ $\mathsf{A}(BeC) = \mathsf{A}(BoC) = \mathsf{B} \times \mathsf{C}$, $\mathsf{A}(BoA) = \mathsf{B} \times \mathsf{A}$, $\mathsf{A}(CoA) = \mathsf{C} \times \mathsf{A}$,

(c) $A(\Phi \square \Psi) = \Gamma \Longleftrightarrow A(\Psi \# \Phi) = \Gamma$,
$\square, \#$ are the following possible pairs of operators (according to the possible conversions): a, i; i, i; e, e.

(d) (i) $A(\Sigma \square \Phi) = \Gamma \; \& \; A(\Phi \# \Psi) = \Phi \times \Psi$
$\Longrightarrow A(\Sigma \S \Psi) = \Phi$,
(ii) $A(\Sigma \square \Phi) = \Gamma \; \& \; A(\Phi \# \Psi) = \Delta$
$\Longrightarrow A(\Sigma \S \Psi) = \Delta$, where $\Delta = \Phi \times \Psi$,
$\square, \#, \S$ are the following possible sequences of operators (according to the first syllogistic figure): a, a, a; e, a, e; a, i, i; e, i, o.

We defined the actualization function so as to have one attribute as a value, following Aristotle's view that cause should be a sufficient and necessary condition for its effect (An. Post. A 13, 78b 15–21; B 15-16). Other causes are only causes in a non-strict sense (e.g. propria of the real cause). However, one and the same effect can sometimes be causally explained, in different approaches, by different sort of causes, e.g. by an efficient and a final cause (light shines through a lantern because of its consisting of small particles as well as in order to save one from stumbling, see An. Post. B 11, 94b 27–37). Often it seems that Aristotle takes that one of the explanations is the primary one.

If the causation actualization of a proposition is internal, i.e. $A(\Phi \square \Psi) = \Phi \times \Psi$, the actualization is directly due to the related attributes, which in accordance with Aristotle's theory should be recognized by means of direct knowledge of essences if all attributes are essential, or by means of induction and perception if any of the attributes is a proper attribute. In the first case, the knowledge of the causal relationship is according to Aristotle essential, in the second case only factual.

Let us extend the notion of a basic predication path to the notion of attribution path, r. If $B \square [A]$ at w and $C \square [B]$ at w', then the attribution path extends from w to w', although w and w' might not be connected by any tree path.

Definition 4.9. Attribution path r satisfies term Φ ($r \models \Phi$) iff,

1. $[A] \square r$ for $\Phi = A$,
2. $r \models A$ for $\Phi \square [A] \backslash \{A\}$,
3. $r \models \Psi$ for $\Phi = \overline{\Psi}$,
4. $r \models \Psi \; \& \; r \models \Psi'$ for $\Phi = \Psi\Psi'$.

In the definition of truth, only necessary truth is considered. We have excluded accidental, contingent truth, since it does not pertain to knowledge in Aristotelian sense.

Definition 4.10 (Truth).

1. $M \models \Phi a \Phi' \Longleftrightarrow \square r (r \models \Phi') \; \& \; \square r (r \models \Phi' \rightarrow r \models \Phi)$,
2. $M \models \Phi i \Phi' \Longleftrightarrow \square r (r \models \Phi' \; \& \; r \models \Phi)$,
3. $M \models \Psi : \Phi \square \Phi' \Longleftrightarrow M \models \Phi \square \Phi' \; \& \; A(\Phi \square \Phi') = \Psi$,
4. $M \models \neg \phi \Longleftrightarrow M \models \phi$,

Note that self-predication is also included as essential and internal case of causation. Let us give two examples of how the definition of truth functions in case of compound terms.

1. M |= AaBC iff □rhB, Ci □ r & □r (hB, Ci □ r → A □ r),
2. M |= ABaCD iff □r hC, Di □ r & □r (hC, Di □ r → (A □ r □ B □ r)).

Remark 4.11 (Paraconsistency). Genus is according to Aristotle, a distributive whole of its species, for example, “number” is a distributive whole of “odd” and “even”. Since “odd” and “even” are mutally exclusive, their genus has, in a sense, “para-consistent” character – it contains a contrariety, without implying triviality. Both of genus’ (distributive) parts can be attributed to the genus (“number is odd”, “num-ber is even”), and the genus is an element in their essential natures (odd, as well as even, are essentially numbers) (cf. An. Post. A4 73b 20–21, A6, A22). This seems to be essentially connected with paraconsistency of Aristotelian logic as desribed by Gomes and D’Ottaviano [11] on the ground of Aristotle’s example with contradictory concepts (“Callias and non-Callias”, “man and non-man”) occuring under the non-contradictory major term of a syllogism (“animal”).

The definition of satisfiability is in its formulation quite usual:

Definition 4.12 (Satisfiability). A set Γ of sentences is satisfiable iff there is model M such that for each ϕ □ Γ, M |= ϕ.

5. System

The system of an Aristotelian causal theory contains a finite number of primitive propositions and rules of inference.

a) Finite number of primitive propositions (archai).

We first define predicative equivalence class in the same way as an attributive equivalence class by replacing in the definition attributes A, B, . . . with predicate letters A. B, . . . , respectively. The system may be called hybrid because we use the same tree structure (theory tree) as in semantic frame, and a labeling function U, which is isomorphic to V, the only difference being that each attribute A_i in an equivalence class that is associated to a node is replaced by a predicate letter A_i.

Now we build a finite ternary tree by means of immediate relation $[A_i] < [A_j]$ (where A_i has height h, and A_j has height $h + 1$) with predicative equivalence classes as nodes. We describe the tree by primitive propositions (archai):

1. For each pair $[A_i] < [A_j]$ where A_i has height h, and A_j has height $h + 1$, there is primitive proposition A_i □ A_j, where □ = a if A_j does not occur more than once at $h + 1$ (and in the tree at all), and otherwise □ = i.
2. If there is A_i such that A_i □ A_j and A_i □ A_k are primitive propositions, then $A_j e A_k$ is a primitive proposition.
3. If A_j, A_k □ $[A_i]$, then $A_j a A_k$ and $A_k a A_j$ are primitive propositions.

b) Conversion rules:

$$\Phi a \Psi / \Psi i \Phi; \qquad \Phi i \Psi / \Psi i \Phi; \qquad \Phi e \Psi / \Psi e \Phi.$$

c) Indirect inference rule (S: a set of propositions, Contrd: negation of a member of S):

$$S, \neg\chi \,/\, \text{Contrd} \quad \Longrightarrow \quad S \,/\, \chi.$$

d) Categorical syllogism rules:

$$\Sigma a\Phi, \Phi a\Psi \,/\, \Sigma a\Psi; \quad \Sigma e\Phi, \Phi a\Psi \,/\, \Sigma e\Psi;$$
$$\Sigma a\Phi, \Phi i\Psi \,/\, \Sigma i\Psi; \quad \Sigma e\Phi, \Phi i\Psi \,/\, \Sigma o\Psi.$$

e) Causal rules:

$A \square B \,/\, A\times B : A \square B$ if $A \square B$ is a primitive proposition on the ground of $<$;
$A \square B \,/\, C : A \square B$ if $A \square B$ is a primitive proposition on the ground of the membership in $[C]$;
$\Gamma : \Phi \square \Psi \,/\, \Gamma : \Psi \# \Phi$, for $\square$ and $\#$ as in the conversion rules above;
$S, \Gamma : \neg\chi \,/\,$ Contrd $\Longrightarrow$ $S \,/\, \Gamma : \chi$;
$\Gamma : \Sigma \square \Phi,\ \Phi\times\Psi : \Phi\#\Psi \,/\, \Phi : \Sigma\S\Psi$,
$\Gamma : \Sigma \square \Phi,\ \Delta : \Phi\#\Psi \,/\, \Delta : \Sigma\S\Psi$, for $\Delta = \Phi\times\Psi$,
$\Gamma : \phi \,/\, \phi$.

If we apply the usual rules for the reduction to the first syllogistic figure, we obtain the following causal syllogisms for the second figure:

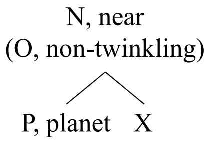
$\Gamma : \Phi \square \Sigma,\ \Delta : \Phi\#\Psi \,/\, \Delta : \Sigma\S\Psi$ ($\Gamma : \Sigma\S\Psi$ in Camestres),
for $\Delta = \Phi\times\Psi$ (for $\Gamma = \Phi\times\Sigma$ in Camestres),
$\Gamma : \Phi \square \Sigma,\ \Delta : \Phi\#\Psi \,/\, \Phi : \Sigma\S\Psi$,
for $\Delta = \Phi\times\Psi$ (for $\Gamma = \Phi\times\Sigma$ in Camestres),

and the following causal syllogisms for the third figure:

$\Gamma : \Sigma \square \Phi,\ \Delta : \Psi\#\Phi \,/\, \Delta : \Sigma\S\Psi$ ($\Gamma : \Sigma\S\Psi$ in Disamis),
for $\Delta = \Phi\times\Psi$ (for $\Gamma = \Phi\times\Sigma$ in Disamis),
$\Gamma : \Sigma \square \Phi,\ \Delta : \Psi\#\Phi \,/\, \Phi : \Sigma\S\Psi$,
for $\Delta = \Phi\times\Psi$ (for $\Gamma = \Phi\times\Sigma$ in Disamis),

Let us analyze some characteristic situations that can appear within an Aristotelian causal theory T, containing all primitive propositions and closed under consequences.

Example (Cause and fact, see An. Post. A 13, 78a30–78b4). We show how “non-twinkling” may be used as a middle term in a syllogism to prove the nearness of planets, although it is not the real cause of the nearness of planets, “non-twinkling” being only a peculiar property corresponding to the nearness of planets. Such non-causal middle terms serve for Aristotle only to demonstrate a fact (demonstratio quia), not the reason why (demonstratio propter quid). The proposition that planets are near is taken to be a primitive (astronomical) proposition. In the right side proof below, we start from causal prefixes according to the causal theory of the left-side proof – middle term O of the right-side proof does not have a causal role.



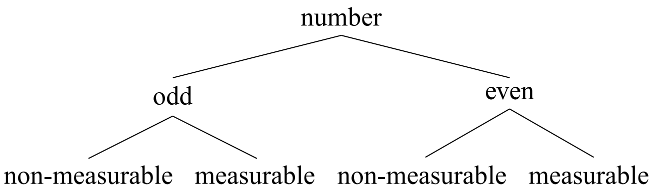
1	N : OaN	prim. prop.	1	N : NaO	prim. prop.
2	N × P : NaP	prim. prop.	2	N : OaP	left. syll, line 3
3	N : OaP	from 1 and 2, caus.	3	N : NaP	from 1 and 2, caus.

Example (A too remote "cause", An. Post. A 13, 78b13–31). It seems that in the second syllogistic figure a proof may be given through a non-adequate, too remote cause. In Aristotle's example, "not being animal" is not an adequate reason for why wall does not breathe, since wall is also not an animal that does not breathe (there are non-breathing animals) - although it is a reason of the sole fact that wall does not breath. However, the syllogism seems to suggest "not being an animal" as the reason due to the minor, negative, premise. The real cause for why wall does not breathe seems to be, in the analysis below, "being an animal" (that what breathes is an animal). Let 'A' stand for 'animal', 'B' for 'breathing', and 'W' for 'wall'. Second-figure proof (Camestres) is to the left, and its reduction to the first figure (with an improvement of causal explanation) is to the right.

1	A × B : AaB	primitive prop.
2	A × W : AeW	primitive prop.
3	$\overline{A}$: BeW	⍰from 1 and 2

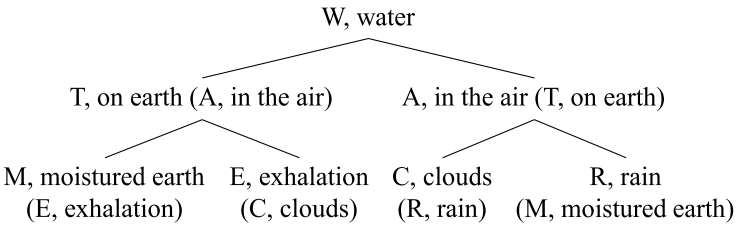
1	A × W : WeA	caus. conv.
2	A × B : AaB	primitive prop.
3	A : WeB	from 1 and 2, caus.
4	A : BeW	from 3, caus. conv.

Let us note that because of possible re-occurring of attributes in a tree, a convergence can appear in a labeled tree. For example, in the mentioned example of Aristotle's arithmetic, an odd number (triad), as well as an even number (dyad), can be prime in the sense of being "non-measurable by numbers":



However, there are no two different essences of prime numbers (odd, even) but of different species of prime numbers.

Example (Causal cycle, see An. Post. B 12, 95b38–96a7). Sometimes, a specific causal substructure has to be added in a model, more precisely, into an equivalence class [A] at a tree node. This should be done, for instance, in case of water cycle (see An. Post.B12 95b38–96a7). We describe such causal cycles by transitivity, assigning to properties in [A] a double relative role of cause attributes as well as of peculiar properties (propria). A peculiar property corresponding to an attribute and relatively caused by the attribute is given in our scheme in parentheses. E.g. there is moistured earth iff there is exhalation (exhalation belongs to moistured earth as its peculiar property) – at least on some regular but essential basis. However, moistured earth causes exhalation (under some general conditions, like Sun), not vice versa.



Here are the first syllogistic proof of the syllogistic chain (left side) and the whole circular chain of the causal inference in a form of sorites (right side). We have put causal prefixes in parentheses to indicate that they have only relative causal role (water being a real cause):

1	(M) : MaE	prim. prop.
2	(E) : EaC	prim. prop.
3	(E) : MaC	from 1 and 2, caus.

1	(M) : MaE	prim. prop.
2	(E) : EaC	prim. prop.
3	(C) : CaR	prim. prop.
4	(R) : RaM	prim. prop.
5	(R) : MaM	from 1 to 5, caus.

6. Soundness and completeness

We outline main features of the soundness and completeness proofs.

6.1. Soundness

According to the construction of the set of primitive propositions of a system, it is immediate that due to an interpretation function (in a model) there is a set of attributive equivalence classes that corresponds to a chosen set of predicative equivalence classes. It is also obvious that to each tree of primitive propositions there corresponds some theory tree that which is a part of a frame. Finally, it is straightforward to prove that each inference rule of the system preserves truth. For example, for conversion from $\Phi \mathsf{a} \Phi'$ to $\Phi' \mathsf{i} \Phi$, if $\Phi \mathsf{a} \Phi'$ is true in a model, it is obvious from Definition 4.10 that there is a an attribution path r in the theory tree of the model such that r satisfies both Φ and Φ', i.e. makes $\Phi' \mathsf{i} \Phi$ true. As a further example, categorical syllogism Barbara is obviously semantically confirmed by means of the transitivity of $<$ in a theory tree, i.e. if each attribution path r that satisfies Ψ also satisfies Φ, and each r that satisfies Φ also satisfies Σ, then each r that satisfies Ψ also satisfies Σ. In addition, explicit causal conditions in a syllogism strictly correspond to the definition of the actualization function within Definition 4.8 of a model.

6.2. Completeness

We sketch a proof that for each consistent Aristotelian causal theory there is a corresponding model confirming precisely the sentences that are members of the theory (completeness).

We say that a set S of sentences is inconsistent iff it contains contradictories (ϕ and $\neg\phi$) as members, or contradictories are syllogistically deducible from S. Let a theory T contain all sentences deducible from it. Then T is inconsistent iff T contains contradictories as members.

Let us start from a consistent set S of sentences of an Aristotelian causal theory T. We extend S to a maximal consistent set U adding each φ (of T) that can be added without a contradiction. It can be easily seen that U obeys the square of opposition conditions for SP (subject-predicate) sentences: U contains one and only one sentence of each contradictory pair of SP sentences, at least one of subcontraries and at most one of contraries (cf. [9] in a different Aristotelian formal system).

Proposition 6.1.

1. ΦaΨ ∈ U iff for each X such that ΨaX ∈ U, ΦaX ∈ U,
2. ΦiΨ ∈ U iff there is X such that ΨaX ∈ U, and ΦaX ∈ U,
3. Φ □ Ψ ∈ U iff for some Γ, Γ : Φ □ Ψ ∈ U,
4. φ ∈ U iff ¬φ ∉ U.

Proof.

1. Suppose ΦaΨ ∈ U as well as ΨaX ∈ U but ΦaX ∉ U. Then ΦoΨ ∈ U. Contradiction.
2. Similarly for ΦiΨ.
3. Suppose that Φ □ Ψ ∈ U. The proof is based on the causal rules of the system. If (a) Φ□Ψ ∈ U is a primitive proposition, then for some Γ, Φ□Ψ / Γ : Φ□Ψ, and thus Γ : Φ □ Ψ ∈ U. If (b) Φ □ Ψ is derived by conversion and Γ is the causal prefix of the starting proposition, then Γ is the causal prefix of Φ □ Ψ. If (c) Φ □ Ψ is derived by indirect proof, then it gets the same causal prefix under which the negation of Φ□Ψ was supposed. If (d) Φ□Ψ is obtained by a syllogism, after establishing the causal prefixes of the premises, we can derive the causal prefix of the conclusion (according to the causal inference rules).
4. Obvious from the definition of U.

Canonical model M_U is a model where, instead of labeling attributes, corresponding labeling predicates are associated with the nodes of a theory tree. Hence, in M_U attribution path r satisfies Φ, in the basis case where Φ = A, iff [A] ∈ r. Hence, truth in a canonical model is based on predicates and on their association with nodes of a theory tree (attribution collapses to predication).

Proposition 6.2. φ ∈ U iff M_U |= φ.

Proof.

- Basis (φ = A_i □ A_j). The proposition is immediate on the ground of the identity of the canonical attribution tree with the tree of primitive propositions for theory T.
- We take ΦaΨ as an example of φ = Φ □ Ψ. ΦaΨ ∈ U means that for each X such that ΨaX ∈ U, also ΦaX ∈ U (X = Ψ for Ψ being a primitive predi-cate with the greatest height in the theory tree). According to the hypothesis, M_U |= ΦaX for every X such that M_U |= ΨaX. This means that each path satisfying Ψ also satisfies Φ, that is, M_U |= ΦaΨ.
- Let φ = Γ : Φ □ Ψ. This means that Φ □ Ψ ∈ U. According to the hypothesis, M_U |= Φ □ Ψ. But in a model, actualization function A assigns to each formula some causal prefix Γ (Definition 4.8). It can be checked that this

prefixing mechanism corresponds to the prefixing mechanism of the system. Accordingly, $M_U \models \Gamma : \Phi$ [illegible] Ψ.

- Let $\phi = \neg\psi$. Sentence $\neg\phi \in U$ iff $\phi \notin U$. That is, in accordance with the hypothesis, $M_U \models \psi$, and equivalently, $M_U \models \neg\psi$.

Lemma 6.3. Each consistent set of T is satisfiable.

Proof. Follows from Proposition 6.2.

7. To sum up

Aristotle's account of causality is deeply interconnected with his accounts of logic and attribution. Although an Aristotelian causal theory, as formalized in this paper, is restricted to one-place predicates and a finite domain (of attributes) it can, as an example, offer some hints on how to unify some philosophical features of the concept of causality (including necessity and essential character). At the same time, and in comparison with causally interpreted jusftification logic, the proposed formalization indicates a possible way how to simplify (although not without a loss in expressivity) a causal prefixing mechanism .

References

[1] Aristotle, Analytica Priora et Posteriora. D. Ross and L. Minio-Paluello (eds.). Oxford University Press, 1964.

[2] Aristotle, Metaphysica. W. Jaeger (ed.). Oxford University Press, 1973.

[3] Aristotle, Posterior Analytics. Transl. by J. Barnes. 2nd. ed. Oxford University Press, 2002.

[4] Aristotle, Posterior Analytics. Transl. by G. R. G. Mure. The Internet Classics Archive, 1994-2000. http://classics.mit.edu//Aristotle/posterior.html

[5] Aristotle, Prior Analytics, Book 1. Transl. by G. Striker. Oxford University Press, 2010.

[6] Aristotle, Prior Analytics. Transl. by A. J. Jenkins. The Internet Classics Archive, 1994-2000. http://classics.mit.edu//Aristotle/prior.html

[7] H. Beebee, C. Hitchcock, P. Menzies, The Oxford Handbook of Causation, Oxford University Press, 2012.

[8] J.-Y. Béziau, On the formalization of the principium rationis sufficientis, Bulletin of the Section of Logic, 22 (1993), 2–3.

[9] J. Corcoran, Completeness of an ancient logic, Journal of Symbolic Logic, 37 (1972), 696-702.

[10] M. Fitting, Possible world semantics for first order LP, Annals of Pure and Applied Logic, 165 (2014), 225–240.

[11] E. L. Gomes, I. M. L. D'Ottaviano, Aristotle's Theory of Deduction and Paraconsistency, Principia : An International Journal of Epistemology, 14 (2010), 71–97.

[12] K. Gödel, Texts relating to the ontological proof, Collected Writings, S. Feferman et al. (eds.), vol. 3, Oxford University Press, 1995, pp. 429–437.

[13] S. Kovač, Causation and intensionality in Aristotelian logic, Studia Philosophiae Christianae 49 (2013), 117–136.

[14] S. Kovač, Modal collapse in Gödel's ontological proof. Ontological Proofs Today, M. Szatkowski (ed.), Frankfurt: Ontos [de Gruyter], 2012.

[15] M. Malink, Aristotle's Modal Syllogistic, Harvard University Press, 2013.

[16] T. Parsons, Things that are right with the traditional square of opposition, Logica Universalis, 2 (2008), 3–11.

[17] S. Read, Aristotle and Łukasiewicz on Existential Import, http://www.st-andrews.ac.uk/~slr/Existential_import.pdf, 2013.

Srećko Kovač
Institute of Philosophy
Ul. grada Vukovara 54
HR-10000 Zagreb
Croatia
e-mail: skovac@ifzg.hr