

Logic and Truth in Religious Belief

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1 Introduction

Logic and religious belief are narrowly interconnected. This is so in the sense of the consistency required in religious belief, and in the sense of perseverance in belief and in its consequences. We aim to show, on the ground of biblical texts and using logical tools, that there is a gradation in implementing religious faith in logic and that it is reflected in a gradation of knowledge and the corresponding notion of truth. The role of logic in religious belief consists not only in the correctness of reasoning, but also in the semantic interplay of appearance and truth in the process of the building-up of a religious belief.

Preliminarily, let us briefly outline what is specific for religious belief with respect to the concepts of belief and knowledge as they are usually understood in epistemic (and doxastic) logic. In general, “*i* believes that ϕ ” means that *i* holds that ϕ is true. Here we can, first, distinguish the objective side, the reference to truth. In the case of religious belief (in some fully realized sense), this reference to truth is in fact a sort of knowledge. This may be seen in the example of the Roman centurion from the Gospels, who, coming to Jesus, obviously knows (according to Mt 8:8) that his belief will be realized: “say the word and my servant will be healed” (cf. a slightly different formulation in Lk 7:7). However, it seems that the reason why we usually do not identify religious belief with knowledge is precisely the strong subjective side of religious belief, as can be seen, for instance, from Jesus’ words to his disciples in a storm: “Why are you terrified? Do you not yet have faith?” (Mt 8:26, Mk 4:40). Religious belief essentially depends not only on truth, but also on subjective trust, confidence, i.e. on religious faith.

2 Logic and religious belief¹

We aim to show the interconnectedness of logic and religious belief in two ways: first, religious belief includes reasoning; secondly, religious belief is a pragmatic function applied to logical forms, and hence is a part of logic in a wider sense.

As for the first aspect, we indicate with two well-known examples in which sense logical reasoning is included in religious belief.

In the first example (Lk 10:30-37), Jesus leaves his disciples to judge for themselves which of three people, encountering a robbed half-dead man on the way, really fulfils the law, i.e. the commandment of neighborly love (“Love your neighbor as yourself”). As we know from the story, a priest, “when he saw [the man], . . . passed by on the opposite side”. A Levite did the same. A Samaritan traveler, however, “was moved with compassion at the sight” and took care of the man. Logically formulated, Jesus’ question is about who instantiated the general law, or who was consistent, comparing their knowledge of the law and their behavior in the situation?

The second example is the “Golden Rule”, to which Jesus refers and, in a general premise, wants to be applied: “Do to others whatever you would have them do to you” (Mt 7:12; Lk 6:31). This entails the commandment “Love your enemies” (Lk 6:27, 35), assuming that you would have others love you. Also: “Forgive and you will be forgiven. Give and gifts will be given to you” (Lk 6:37-38). Obviously, the converse of the Golden Rule is assumed, too (in a contrapositive formulation): “Do not do to others what you would not have them do to you”. Instantiations are, for example, “Stop judging and you will not be judged. Stop condemning and you will not be condemned” (Lk 6:37, Mt 7:1).

In the next example, in the parable of the sower, we outline in which way religious belief (precisely, religious faith) can be conceived as a function applied to logic and language (as a part of logic).

In the parable, a word (*rhēma*, *logos*) is put in connection with faith, precisely: the word of God with religious faith (Mt 13:3-23; Mk 4:3-20; Lk 8:4-15). Here, the sown seed is compared to the spoken word. The *tertium comparationis*, common to the seed and word, and enabling the comparison, is giving and receiving. The giving is sowing and saying the seed and the word, respectively. The receiving is the receiving of the seed in the ground and the receiving (accepting) of the word in the mind (“heart”). In the parable, four grades of religious faith are distinguished: (1) according to the inner quality of the reception of the word (I, in

¹This section is a further elaboration of section 1 and of a part of section 4 of [?].

	seed	the word of God (of the kingdom)
I	on the path	heard, not understood (n)
O	birds	devil, the evil one (d)
R	trampled, eaten by birds	taken away
I	on rocky ground, little soil	received with joy, only for a time, no root(o)
O	sun, lack of moisture	tribulation, persecution (t)
R	withered	fallen away
I	among thorns	heard, mixed with anxieties and pleasures (m)
O	full-grown thorns	worldly life, riches (w)
R	choked	choked, unfruitful
I	on the good soil	embraced in the heart, perseverance (p)
O	/	/
R	fruitful	fruitful

Table 1

analogy with the inner quality of the ground), and (2) with respect to the least outer circumstances it cannot endure (O, in analogy with the outer circumstances for the growth of a plant; see result R). These outer circumstances, which, in a sense, measure the endurance of faith according to its inner quality, are ordered in the following way:

(1) the devil (without any special efforts from him) < (2) tribulation, persecution < (3) worldly life and its riches.

The corresponding order of the inner quality of faith is the following:

(1) faith without understanding < (2) rootless faith < (3) faith mixed with anxieties and pleasures < (4) persevering faith.

Inner quality (4) can endure all outer circumstances (1)–(3).

Table ?? displays the analogy between the sown seed and the spoken word of God with respect to the grades of their reception and the respective results (outcome) in the limiting outer conditions. According to this table we can conceive religious faith as a function of the sentences of a given language, and of inner and outer circumstances. In the case where a sentence is “the word of God” we get the following values: $\text{faith}(\phi, n, d) = \text{no}$, $\text{faith}(\phi, o, t) = \text{no}$, $\text{faith}(\phi, m, w) = \text{no}$, $\text{faith}(\phi, p, x) = \text{yes}$, where ‘yes’ and ‘no’ are understood as ‘yes’ and ‘no’ to the word of God, respectively (yes is like ‘amēn’ in the Bible). In the last case, of

LOGIC	Syntax	Semantics	Pragmatics
Language	Formation: linguistic forms	Interpretation: truth conditions	Faith: actualization, execution
Reasoning	Derivation: correctness	Consequence: truth preservation	Fruitfulness: perseverance

Table 2

persevering faith, no outer circumstances x can turn the result to no. In general, inner quality x can match outer circumstances $y < x$, but cannot match outer circumstances $y \geq x$:

$$\text{faith}(\phi, x, y) = \text{yes iff } x > y, \text{ where } \phi \text{ is the word of God.}$$

Religious faith can thus be conceived as a function that applies to syntactically and semantically already determined forms (sentences). It is an additional, pragmatic function, which pertains to the actual use of syntactical and semantic forms and to their execution in a context: $\text{faith}(\phi, x, y) \in \{\text{yes}, \text{no}\}$.

Let us mention that what is understood as consistent faith is the faith that perseveres in all circumstances, so that the third argument can be ignored: $\text{faith}(\phi, x) \in \{\text{yes}, \text{no}\}$. Consistent faith behaves in a standard way (as in classical logic): $\text{faith}(\neg\phi, x) = \text{yes iff } \text{faith}(\phi, x) = \text{no}$, $\text{faith}(\phi \wedge \psi, x) = \text{yes iff } \text{faith}(\phi, x) = \text{yes and } \text{faith}(\psi, x) = \text{yes}$, etc., and should correspond to some chosen (semantic) model(s).

By including religious use (pragmatics) into logic as its part, we get a more comprehensive concept of logic, which may be sketched as in Table ??.²

3 Faith pragmatics and truth

Let us, at first informally, analyze the episode of Nicodemus from the Gospel of John 3 in order to see how different stages of truth and its knowledge correspond to different stages of religious faith. Corresponding to the gradation (see Table 1) of (1) faith without understanding (merely “hearing”), (2) temporary, rootless faith dependent on natural conditions and threats, and (3) mixed faith within the

²Compare the three-fold structure of logic in Table ?? with the self-defining words of Jesus (*Logos*, Word): “I am the way and the truth and the life” (John 14:6).

richness of worldly life, we encounter in John 3 the following stages of knowledge: (1) materialistic knowledge in the sense of being based on outer signs, (2) the bio-naturalistic conception of man,³ and (3) rich (Pharisaic) historicist knowledge. The insufficiency of each of these three kinds of knowledge is uncovered in a dialogue with Jesus. Nicodemus' initially claimed knowledge that Jesus has come from God turns out to be only apparent knowledge, and is three times in succession reduced to a contradiction, due to incompatibility with knowledge in a true, spiritual sense.

Here is a brief summary of what happens in the dialogue of John 3.

(1) Nicodemus claims to have knowledge of the kingdom of God, stating that on the ground of the signs Jesus has made, he knows that Jesus is a "teacher who has come from God". However, according to Jesus' reply, it is contradictory to claim knowledge of the kingdom of God on the ground of signs: "No one can see (know) the kingdom of God without being born from above" [re-born]. Hence, the pre-condition for knowledge of the kingdom of God is to be "re-born", or "born from above" (according to the ambiguity of the Greek *anothen*).

(2) Now, the requested pre-condition of (1) ("re-birth") is conceived by Nicodemus naturalistically, leading again to contradiction. Nicodemus wonders how an old person could re-enter his mother's womb and be born again. Jesus gives further specification of the pre-condition for "knowledge" of God: no one can enter the kingdom of God without being born of water and Spirit. He makes explicit the distinction between the required spiritual and Nicodemus' naturalistic conceptions: what is born of flesh is flesh and what is born of spirit is spirit.

(3) Nicodemus manifests the lack of knowledge about "spiritual re-birth" by asking: how can this happen? His knowledge as "a teacher of Israel" (he is a Pharisee), despite its possible entirety and richness, and although it should be knowledge about the kingdom of God, remains only a historicist knowledge of facts, lacking true understanding. There is therefore a further contradiction, consisting in an attempt to come to the knowledge of God on the ground of historicist knowledge.

(4) What remains for Jesus is to instruct Nicodemus in order to lead Nicodemus to an adequate understanding. Being himself a learned man and a teacher, Nicodemus should perhaps be receptive to such instruction.

The dialogue has thus a tree structure, where the claimed knowledge branches (a) on the left with apparent truth, which leads to contradictions, and (b) on the

³I have reclassified some aspects of knowledge as naturalistic following discussion on the occasion of the conference where the talk from which this paper originates was presented (*God, Truth and other Enigmas*, Warsaw, September 2013).

N: We know that you are a teacher who has come from God.

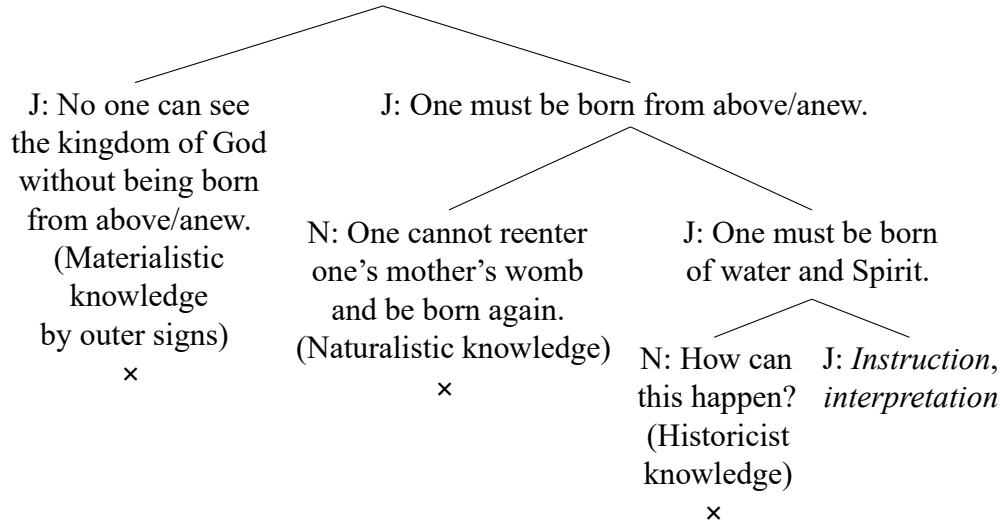


Figure 1

right with the alternative conception, within which the truth and true knowledge should be sought (see Figure ??).

4 Appearance and truth in religious belief (semantics)

Since different stages of religious faith have their own truth (or apparent truth), it follows that faith pragmatics (associated to a logic) should itself be conceived and presented as some comprehensive semantics. We define such semantics (and appropriate logical language) in order to prepare the ground for a formal semantic analysis of some characteristic aspects of truth and appearance in religious belief according to the Gospel passage about Nicodemus.

4.1 Logic QB modified

We give a formal description of the language and semantics of a variant of first-order modal logic QB according to [?]. QB is a general logic of belief by means of which we can formalize contradictions and non-completeness of beliefs, and

which we want to show to be suitable to give a formal account of different stages of religious belief (from apparent to true belief) mentioned above. In [?], QB was primarily applied to model *de re* contradictions resulting from using different names of the same object as if they refer to different objects. Here we focus on the ambiguity of names: one name can be used to refer to different, apparent or real, objects. In accordance with the episode about Nicodemus, the objects concerned will be belief and knowledge agents themselves. Hence, we specify QB so as to identify domain objects with belief agents and call this logic QBA. Therein, belief agents with the empty accessibility relation can be understood as non-rational beings (where logic and belief in fact collapse: there are no epistemic possibilities, and hence everything is a vacuously epistemic necessity).

The vocabulary of language L_{QBA} contains individual constants c_1, c_2, c_3, \dots , (set C , we will also informally use c, d, j), individual variables x, y, z, x_1, \dots (set V), descriptive predicate letters P_i^n (informally, G^1 as well), logical predicates $=$ and E^1 , connectives \neg and \Box , the quantification symbol \forall , the predicate abstractor λ , belief operators B_t and parentheses. Symbols \Box, \rightarrow and \forall are defined in a usual way.

The formulas of L_{QBA} are $\Phi t_1 \dots t_n, t_1 = t_2, Et, \neg\phi, \phi \Box \psi, B_t\phi, \Box x\phi$, and $(\lambda x.\phi)(k)$, where ϕ and ψ are formulas, Φ^n is an n -place description predicate letter, t_i a term (a constant or variable), k an individual constant, and $(\lambda x.\phi)$ a predicate abstract. A subscript occurrence of t in B_t of $B_t\phi$ is an occurrence of t in the formula $B_t\phi$ (and if t is a variable, it can be bound by a quantifier).

The λ -abstraction renders unambiguous the sense in which an individual constant should be understood. For example, in $B_c(\lambda x.Px)(d)$, constant d is dependent on λ and should be understood in the sense in which agent c understands d (*de dicto*); in $B_cP d$, constant d is taken objectively and independently of agent c as well as of any other agent (*de re*); in $B_x(\lambda y.BzPy)(c)$, constant c is taken in the sense in which it is understood by agent x (not necessarily also by agent z) (see [?]).

Definition 4.1 (Frame). *Frame* $F = \langle W, S, U, Q, \{\Box_w\}_{w \in W}, \{R_u\}_{u \in U} \rangle$, where

1. W is a non-empty set of possible worlds ($w \in W$),
2. $S \subseteq W \times W$ (reflexive),
3. $U = D \subseteq A$ (a set of objects), where
 - (a) D is a non-empty set (of actual and possible things),

(b) $A \subseteq D \times C$ (a set of appearances)

(d $\in D$, $a = hd, ki \in A$, $u \in U$, in addition, other bold letters instead of d will be used),

4. $Q : W \rightarrow \mathcal{P}U \setminus \{\emptyset\}$,
5. for each w , \mathcal{Q}_w is an equivalence relation (reflexive, symmetric, transitive) on set U ,
6. $R_u \subseteq W \times W$ (serial, transitive, euclidean).

We will use abbreviations: $D_w = Q(w) \cap D$, $A_w = Q(w) \cap A$, $[u]_w = \{u' \mid u' \mathcal{Q}_w u\}$.

Definition 4.2 (Model). Model $M = \langle F, I, I' \rangle$, where

1. $I(k) \subseteq D$, $I(\Phi^n, w) \subseteq \mathcal{P}U^n$ closed under \mathcal{Q}_w , $I(E^1, w) = Q(w)$, informally, we will denote $I(k)$ by ' k ',
2. $I'(k, w)$ is the smallest subset of U such that (a) there is $hd, ki \in I'(k, w)$, and (b) for each $hd, ki \in I'(k, w)$, $d \in I'(k, w)$.

Definition 4.3 (Variable assignment). Variable assignment is mapping $v : V \rightarrow U$. An x -variant of the variable assignment v is variable assignment $v[u/x]$, differing from v at most in assigning u to x .

Definition 4.4 (Denotation of a term).

$$JtK_V^{M,w} = I(k) \text{ if } t = k, \quad JtK_V^{M,w} = v(x) \text{ if } t = x,$$

where $JtK_V^{M,w}$ is the denotation of term t in model M (at world w) for variable assignment v , and k is an individual constant.

Definition 4.5 (Satisfaction).

1. $M, w \models_v t_1 = t_2$
iff $(\mathcal{Q}_w' wSw') Jt_1K_V^{M,w} \mathcal{Q}_w' Jt_2K_V^{M,w}$ if $Jt_1K_V^{M,w} \in A$ and $Jt_2K_V^{M,w} \in A$,
 $(\mathcal{Q}_w' wSw') Jt_1K_V^{M,w} \mathcal{Q}_w' Jt_2K_V^{M,w}$ otherwise,
- $$M, w \models_v t_1 = t_2$$
- iff $(\mathcal{Q}_w' wSw') Jt_1K_V^{M,w} \mathcal{Q}_w' Jt_2K_V^{M,w}$ if $Jt_1K_V^{M,w} \in A$ and $Jt_2K_V^{M,w} \in A$,
 $(\mathcal{Q}_w' wSw') (\mathcal{Q}_w' wSw'') (\mathcal{Q}_w' u_1' \mathcal{Q}_w [u_1]_w)$ otherwise, where $Jt_1K_V^{M,w} = u_1$
 $(\mathcal{Q}_w' u_2' \mathcal{Q}_w [u_2]_{w''}) u_1' \mathcal{Q}_w u_2'$ and $Jt_2K_V^{M,w} = u_2$,

2. $M, w \models_v^T \Phi t_1 \dots t_n$
 $(\exists w' w S w') \bigwedge_{i=1}^n J t_i K_v^{M,w} \models I(\Phi, w')$
iff $(\exists w' w S w') \bigwedge_{i=1}^n J t_i K_v^{M,w} \models I(\Phi, w')$ *if* $J t_i K_v^{M,w} \models A(1 \leq i \leq n)$,
otherwise,
- $M, w \models_v^E \Phi t_1 \dots t_n$
 $(\exists w' w S w') \bigwedge_{i=1}^n J t_i K_v^{M,w} \models I(\Phi, w')$ *iff* *if* $J t_i K_v^{M,w} \models A(1 \leq i \leq n)$,
 $(\exists w^1 w S w^1) \dots (\exists w^n w S w^n) (\exists u'_1 \models [u_1]_{w^1}) \dots$ *otherwise, where*
 $(\exists u'_n \models [u_n]_{w^n}) \bigwedge_{i=1}^n J t_i K_v^{M,w} \models I(\Phi, w)$ $J t_i K_v^{M,w} = u_i,$
3. $M, w \models_v^E E t$ *iff* $J t K_v^{M,w} \models I(E^1, w)$, $M, w \models_v^E E t$ *iff* $J t K_v^{M,w} \not\models I(E^1, w)$,
4. $M, w \models_v^T \neg \phi$ *iff* $M, w \not\models_v^E \phi$, $M, w \models_v^E \neg \phi$ *iff* $M, w \not\models_v^T \phi$,
5. $M, w \models_v^T \phi \boxplus \psi$ *iff* $M, w \models_v^T \phi$ *and* $M, w \models_v^T \psi$,
 $M, w \models_v^E \phi \boxplus \psi$ *iff* $M, w \models_v^E \phi$ *or* $M, w \models_v^E \psi$,
6. $M, w \models_v^E B_t \phi$ *iff* $(\exists w' w R_u w') M, w' \models_v^E \phi$,
 $M, w \models_v^E B_t \phi$ *iff* $(\exists w' w R_u w') M, w' \models_v^E \phi$,
where $u = J t K_v^{M,w}$,
7. $M, w \models_v^T \exists x \phi$ *iff* $(\exists u \models U_w) M, w \models_v^T [u/x] \phi$,
 $M, w \models_v^E \exists x \phi$ *iff* $(\exists u \models U_w) M, w \models_v^E [u/x] \phi$,
8. $M, w \models_v^T (\lambda x. \phi)(k)$ *iff* $(\exists u \models I'(k, w)) M, w \models_v^T [u/x] \phi$,
 $M, w \models_v^E (\lambda x. \phi)(k)$ *iff* $(\exists u \models I'(k, w)) M, w \models_v^E [u/x] \phi$.

Note that w -equivalent objects (objects in relation \boxplus_w) need not behave in the same way with respect to the belief operator B_t , since w -equivalent objects need not have the same accessibility relation R_u .

As remarked in [?], the idea of modally relativized formulas (here in relation to S -accessible worlds) is well-known in inconsistency logics (“paraconsistent” logics), as, for example, in Jaśkowski’s discussive logic [?, ?] (see also [?, ?] as well as a comprehensive overview and discussion in [?]). The idea of universal quantification under the “mode of presentation” (here, formulas $(\lambda x. \phi)(k)$) in Definition ??, case ??, originates from Ruili Ye (see [?] and [?]).

Definition 4.6 (Satisfiability). *A set Γ of formulas is satisfiable iff there is a model M , world w and variable assignment v such that for each formula $\phi \in \Gamma$, $M, w \models_v \phi$.*

Definition 4.7 (Consequence).

$\Gamma \models \phi$ iff, if $M, w \models_v \psi$ for each $\psi \in \Gamma$, then $M, w \models_v \phi$.

4.2 Formal analysis of John 3

We now describe a concrete model M for section John 3. Operator B_t covers belief (*pistis*) as well as knowledge (seeing; *idein, ginōskein*). Religious belief is the belief at w_1 , which includes the belief that Jesus (j) has come from God (Gj). But this Gospel section describes Nicodemus (c) in a state of unrealized religious belief, comparable to the stage 3 of the religious faith in the synoptic Gospels (see above). That is, we encounter Nicodemus' belief as almost "choked" by his naturalistic knowledge (in paraphrase: "should one re-enter one's mother's womb to be born again?") as well as his historicist knowledge (Jesus says to him: "You are a teacher of Israel and you do not understand this?"), preventing him from coming to the belief and knowledge proposed by Jesus.⁴

In M there are two worlds:

1. world w_1 : all of j, j', hj, ji, hj', ji are mutually equivalent (\approx_{w_1}), they are all and the only members of $I'(j, w_1)$, and they are members of $I(G, w_1)$.
2. world w_2 :
 - $j \approx_{w_2} j', j \approx_{w_2} hj', ji, hj, ji \approx_{w_2} j', hj, ji \approx_{w_2} hj', ji$,
 - $I'(j, w_2) = \{j', hj', ji\}$,
 - $j' \in I(G, w_2), hj', ji \in I(G, w_2), j \notin I(G, w_2), hj, ji \notin I(G, w_2)$.

Relations R_j, R_s, R_f, R_c for j, s, f and c (Jesus, Spirit, "flesh" and Nicodemus, respectively) as members of U , and S are presented in Figure ?? (S is dashed).

Let us analyze a few statements that characterize Nicodemus' (partly ambivalent) religious belief.

$$M, w_1 \models B_c(\lambda x. Gx)(j) \tag{1}$$

⁴As a distinction, in the episode about a Samaritan woman (John 4), the starting point is stage 2 of religious belief (see above), the rudimentary belief of the Samaritan woman, who step by step comes towards the Christian belief (see [?]).

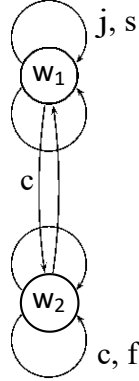


Figure 2

(‘Nicodemus believes that Jesus has come from God.’)

Proof. In each c -accessible world from w_1 , i.e. in w_2 , $(\lambda x.Gx)(j)$ should be true. That means that for each $u \models j$ in (j, w_2) , i.e. for j' and hj', ji , $M, w_2 \models_{v[u/x]} Gx$. Now, according to the definition of satisfaction, for j' it suffices that in at least one world w that is S -accessible from w_2 , $j' \models I(G, w)$, and this holds for $w = w_1$, as well as for $w = w_2$. For hj', ji , $hj', ji \models I(G, w)$ should hold for each w that is S -accessible from w_2 , which is in fact the case in our model M . Therefore, $B_c(\lambda x.Gx)(j)$ is true at w_1 . \square

$$M, w_2 \models B_c(\lambda x.Gx)(j) \quad (2)$$

(‘Possibly, Nicodemus believes that Jesus has come from God.’ ‘Possibly’ refers to possible world w_2 .)

Proof. Since the only world that is c -accessible from w_2 is w_2 itself, the rest of the argumentation is the same as in (??). \square

$$M, w_1 \models B_c B_c(\lambda x.Gx)(j) \quad (3)$$

(‘Nicodemus is aware of his belief that Jesus has come from God.’)

Proof. This easily follows from (??) since w_2 is the only world c -accessible to w_1 . \square

$$M, w_1 \models B_c \neg(\lambda x.x = j)(j) \quad (4)$$

(‘It is not the real Jesus about whom Nicodemus believes that he is Jesus.’)

Proof. This means that at each world w that is c -accessible from w_1 , the formula $(\lambda x. x = j)(j)$ is falsified. World w_2 , which is the only world c -accessible from w_1 , falsifies $(\lambda x. x = j)(j)$ since there is at least one $u \models (j, w_2)$ for x , namely j' , which does not satisfy $x = j$ at w_2 . This can be shown in the following way: w_2 is S -accessible to itself, and on the one side, $j' \models [j']_{w_2}$ since, of course, $j' \models_{w_2} j'$, while on the other side, analogously, $j \models [j]_{w_2}$; but $j \not\models_{w_2} j'$; hence, according to the definition of the falsification of identity formulas, $M, w_2 \models_{v[j'/x]} x = j$. In fact, with j' for x , formula $x = j$ is falsified in w_2 by any S -accessible w (since for each w , $j' \models [j']_w$ and $j \not\models [j]_w$). Thus, $(\lambda x. x = j)(j)$ is falsified and $\neg(\lambda x. x = j)(j)$ verified at w_2 , and therefore $B_c \neg(\lambda x. x = j)(j)$ is verified in w_1 .

Moreover, □

$$M, w_1 \models^T B_c(\lambda x. \neg x = j)(j) \quad (5)$$

(‘It is the real Jesus of whom Nicodemus believes that he is not Jesus.’)

Proof. In each c -accessible world from w_1 , i.e. in w_2 , it should be true that $(\lambda x. \neg x = j)(j)$. This means that $\neg x = j$ should be satisfied, and hence $x = j$ falsified at w_2 by each $u \models (j, w_2)$, i.e. by j' and by hj', ji as well. For j' , we have already shown this in the proof of (?). Similarly, $x = j$ is falsified at w_2 since $hj', ji \models_{w_2} j$ (this non-equivalence follows already from $hj', ji \models (G, w_2)$ and $j \not\models (G, w_2)$, because, according to the definition of model, (Φ, w) should always be closed under \models_w). This proves that at w_1 , $B_c(\lambda x. \neg x = j)(j)$ is true.

$$M, w_1 \models^T \neg B_c G j \quad (6)$$

(‘It is not so that about the real Jesus Nicodemus believes that he has come from God.’)

Proof. As we have mentioned in the proof of (?), $j \not\models (G, w_2)$, and w_2 is c -accessible from w_1 . □

It even holds that $M, w_1 \models^T B_c \neg G j$, since no world other than w_2 is c -accessible to w_1 (see the proof of (?)).

$$M, w_1 \models^T B_c \neg j = j \quad M, w_1 \models^T B_c j = j \quad (7)$$

(‘About real Jesus, Nicodemus believes that he is and that he is not self-identical.’)

Proof. The left proposition follows from the facts that $hj', ji \in [j]_{w_1}$ and $j \in [j]_{w_1}$ ($j \in [j]_{w_2}$ as well), w_1 (and w_2) being S-accessible to w_2 , whereas $j \notin_{w_2} j$. The right proposition follows from the definition of the frame ($u \in_w u$ for any w). \square

$$M, w_1 \models^T B_j \neg(\lambda x. B_c Gx)(j) \quad (8)$$

(‘Jesus believes (knows) that it is not about him that Nicodemus believes that he has come from God.’)

Proof. In w_1 (the only j-accessible world to w_1), sentence $(\lambda x. B_c Gx)(j)$ is false since for some $u \in [j](j, w_1)$, i.e. for j (as well as for hj, ji), Gx is falsified at w_2 (the only c-accessible world from w_1), which is S-accessible to itself. \square

$$M, w_1 \models^T B_j (\lambda x. B_c (\lambda y. \neg x = y)(j))(j) \quad (9)$$

(‘Jesus believes (knows) that about him Nicodemus believes that he is not Jesus.’)

Proof. In each j-accessible world to w_1 , i.e. in w_1 , formula $(\lambda x. B_c (\lambda y. \neg x = y)(j))(j)$ holds, since for each $u \in [j](j, w_1)$, i.e. for j, hj, ji, j' , and hj', ji as values of x , $B_c (\lambda y. \neg x = y)(j)$ is satisfied. The reason is that in the only c-accessible world from w_1 , in w_2 , objects j, hj, ji, j' , and hj', ji for x satisfy $(\lambda y. \neg x = y)(j)$. Namely, all four objects for x satisfy $\neg x = y$, i.e. falsify $x = y$ for each $u \in [j](j, w_2)$ as a value for y . These values are j' and hj', ji . Thus, we have the following cases: (a) $M, w_2 \models_{v[j/x, j'/y]} \neg x = y$, (b) $M, w_2 \models_{v[j/x, hj', ji/y]} \neg x = y$, (c) $M, w_2 \models_{v[hj, ji/x, j'/y]} \neg x = y$, (d) $M, w_2 \models_{v[hj, ji/x, hj', ji/y]} \neg x = y$, (e) $M, w_2 \models_{v[j'/x, j'/y]} \neg x = y$, (f) $M, w_2 \models_{v[j'/x, hj', ji/y]} \neg x = y$, (g) $M, w_2 \models_{v[hj', ji/x, hj', ji/y]} \neg x = y$. The respective reasons are: (a) $j \in_{w_2} j'$, (b) $j \in_{w_2} hj', ji$, (c) $hj, ji \in_{w_2} j'$, (d) $hj, ji \in_w hj', ji$, each of them together with $u \in [u]_w$ (for each u and w) and $w_2 S w_2$; further, together with $w_2 S w_1$ and $j \in_w j'$: (e) $j \in [j']_w$ and $j' \in [j']_w$, (f) $j \in [j']_w$ and $j' \in [hj', ji]_{w_1}$, (g) $j \in [hj', ji]_{w_1}$ and $j' \in [hj', ji]_{w_1}$. \square

In John 3, Jesus’ way of seeing (knowledge, belief) is conceived as spiritual, while Nicodemus’ way is conceived as ambivalent between Spirit and flesh. Formally, we have subsumed j-accessibility under the spiritual, s-accessibility, and c-accessibility partly under f-accessibility. Hence, Nicodemus could achieve true belief, which turns out to be knowledge (with reflexive accessibility), by abandoning f-accessibility and changing his c-accessibility for s-accessibility = $\{hw_1, w_1i\}$, i.e. in the Gospel words, he should be “re-born” in Spirit.

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