Epistemic capacities, incompatible information and incomplete beliefs

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Abstract. We investigate a specific model of knowledge and beliefs and their dynamics. The model is inspired by public announcement logic and the approach to puzzles concerning knowledge using that logic. In the model epistemic considerations are based on ontology. The main notion that constitutes a bridge between these two disciplines is the notion of epistemic capacities. Within the model we study scenarios in which agents can receive false announcements and can have incomplete or improper views about other agent's epistemic capacities. Moreover, we try to express the description of problem specification using the tools from applied ontology – RDF format for information and the Protege editor.

Key words: knowledge representation, dynamic epistemic logic, formal ontology

Introduction

In a multi-agent environment other agents' behaviour (including speech acts) can be a valuable source of information about the world. We are interested in building a formal system that models this kind of information acquisition. We want our system to be easily implementable in computer programs, so we aim to design it to be rich enough to perform the task and, at the same time, be as simple as possible.

Beliefs and their dynamics are widely studied in the AGM approach [1], its model counterpart DDL [10, 14] and dynamic epistemic logic (DEL) – cf. [8, 2, 15]. This paper uses a specific approach (defined in [7]) based on the notion of epistemic state. Intuitively, the epistemic state of an agent describes what the agent recognizes as possible given specific information. To analyze the complex problems, different possible pieces of information have to be considered. Thus, on the basis of the epistemic state, we build the notion of epistemic structure which describes what the agent would recognize as possible if it was given one of many possible packages of information.

Usually, in formal descriptions of epistemic changes and their implementation (see for example [6, 13]), agents' beliefs and their dynamics are taken as primitive and described *manually* in the form of a Kripke structure or topology with hypertheories. In contrast we generate an epistemic structure on the basis of ontology, i.e. a description of the world and the epistemic capacities of the agents. We believe that ontology is more fundamental then epistemology, and epistemic investigations can be based on, and sometimes derived from, an ontological basis. The concept of the epistemic capacity of an agent understood as a description of information that the agent has access to bridges both disciplines. At the same time we are not fundamentally interested in investigations about other agent's state of beliefs by themselves but wish to use them to capture information about the world in which the agents act.

The main concern of the paper is from the area of Knowledge Representation. Recently, applied ontology has attained great success in that field, so we believe that connecting epistemic/doxastic logic with it may contribute to the development of both disciplines. Thus we try to express the description of problem specification using the tools from applied ontology - RDF format for information, and the Protege editor. We try to improve the existing solutions in terms of readability and the length of description of a given situation. We believe that these issues are important from the point of view of possible applications of logic and logic based systems to real life situations. At the same time, despite the differences in the representation of information, we employ the same reasoning principles as in DEL, so we are not aiming for improvements in complexity issues.

Since we are interested in information acquisition from other agents' behaviour, we have to limit ourselves to situations in which agents share a significant part of ontology. In the basic version of our model of epistemic interaction, all the agents share the same information about the structure of the world and epistemic capacities of the agents. Their beliefs may differ only because the agents do not have access to the content of other agents' observations. In the more complex cases which we consider in the paper, the differences are larger; but we still assume that agents share a set of ontologically possible situations.

First, we will present our model of epistemic interaction described already in [7, 9], then we will analyse the cases of incompatible information and incomplete beliefs. Finally, we show how tools of applied ontology can be used for description of the problem environment in the epistemic context.

1 A general model of epistemic interactions

In the investigations on epistemic interactions we operate on five levels:

- ontological, which boils down to a description of the world,
- intermediary, which interfaces the world description to the epistemological levels,
- static epistemological, which concerns agents' belief at a certain time,
- dynamic epistemological, which defines the dynamics of beliefs,
- operational, at which we query the whole system.

As a working example we will use the following Hats Puzzle (see [7]):

Alice, Bob and Charlie sit in a row in such a way that Alice can see Bob and Charlie, Bob can see Charlie and Charlie can see nobody. They are shown five hats, three of which are red and two are black. While the light is off, each of them receives one of the hats to put on her/his head. When the light is switched back on, they are asked whether they know what the colours of their hats are. Alice answers that she doesn't know. Then Bob answers that he doesn't know either. Finally Charlie says that he knows the colour of his hat. What colour is Charlie's hat?

1.1 Ontology

A description of a problem environment includes basic information about objects (including agents), features of those objects, their possible values and relations between objects. Such a description forms a domain ontology in a way introduced by McCarthy [11]. In the Hats Puzzle there are three objects (being also agents: *Alice*, *Bob* and *Charlie*) and for each of them we consider only one feature – the colour of the hat, and there are two possible values of that feature – *red* or *black*. There are also further restrictions – *at least one hat is red*.

The basic unit of such a description refers to an actual or possible elementary situation. Any set of elementary situations will be called a situation. A maximal set of ontologically compatible situations[1] forms a possible world.

In our example we have six situations:

1. that Alice has a black hat,
2. that Alice has a red hat,
3. that Bob has a black hat,
4. that Bob has a red hat,
5. that Charlie has a black hat,
6. that Charlie has a red hat.

In the context of our RDF framework - see section 4 below - they will be represented as the following triples:

1. $(Alice, hasHatOfColour, black)$,
2. $(Alice, hasHatOfColour, red)$,
3. $(Bob, hasHatOfColour, black)$,
4. $(Bob, hasHatOfColour, red)$,
5. $(Charlie, hasHatOfColour, black)$,
6. $(Charlie, hasHatOfColour, red)$.

The possible worlds composed out of those elementary situations, given the restriction that at least one hat is red, are as follows:

[1] We do not consider the precise meanning of the notion of compatibility in this paper. Intuitively two situations: that Bob has a red hat and that he has a black one, are incompatible.

- $\{(Alice, hasHatOfColour, black), (Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, red)\}$,
- $\{(Alice, hasHatOfColour, black), (Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, black)\}$,
- $\{(Alice, hasHatOfColour, black), (Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, red)\}$,
- $\{(Alice, hasHatOfColour, red), (Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, black)\}$,
- $\{(Alice, hasHatOfColour, red), (Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, red)\}$,
- $\{(Alice, hasHatOfColour, red), (Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, black)\}$,
- $\{(Alice, hasHatOfColour, red), (Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, red)\}$.

For the sake of brevity we will also refer to the possible worlds by giving them labels, respectively: $bbr, brb, brr, rbb, rbr, rrb, rrr$.

1.2 Ontology-to-epistemology interface

The description of the puzzle implies information about the agents' epistemic capacities – the kind of information that the agents have access to. We assume that *Alice knows the colours of Bob's and Charlie's hats* (because she can see the hats) and *Bob knows the colour of Charlie's hat*.

Thus, the interface between ontology and epistemology is constituted by the following information:

1. Alice knows what colour hat Bob has,
2. Alice knows what colour hat Charlie has,
3. Bob knows what colour hat Bob has.

In the context of our RDF framework this information will be represented as the following triples:

1. for "Alice knows what colour of hat Bob has":
   - $(Alice, knowsWhether, (Bob, hasHatOfColour, red))$,
   - $(Alice, knowsWhether, (Bob, hasHatOfColour, black))$,
2. for "Alice knows what colour of hat Charlie has":
   - $(Alice, knowsWhether, (Charlie, hasHatOfColour, red))$,
   - $(Alice, knowsWhether, (Charlie, hasHatOfColour, black))$,
3. for "Bob knows what colour of hat Charlie has":
   - $(Bob, knowsWhether, (Charlie, hasHatOfColour, red))$,
   - $(Bob, knowsWhether, (Charlie, hasHatOfColour, black))$.

Nevertheless, for the sake of brevity, we shall write $(Alice, knowsWhether, (Bob, hasHatOfColour, \dots))$ instead of the pair:

- $(Alice, knowsWhether, (Bob, hasHatOfColour, red))$,
- $(Alice, knowsWhether, (Bob, hasHatOfColour, black))$.

1.3 Static epistemology

The epistemic state of an agent is a subset of a set of ontologically possible worlds which the agent recognises as possible, taking into account the available information. We write $epist(i, t_n)$, where i is an agent and t_n is the time characteristics.

If an observer does not know what the actual epistemic state of an agent is but knows the agent's epistemic capacities, he can determine a number of possible epistemic states. We write $epist_k(i, t_n)$, where $k \in \mathbb{N}$ indexes the set of possible epistemic states, i is an agent and t_n is the time characteristics. Obviously, for some k we have $epist_k(i, t_n) = epist(i, t_n)$.

The set of all possible epistemic states of an agent at a certain time t_n – $Epist(i, t_n)$ – defines the agent's epistemic structure at that time. The overall description of the epistemic structures of all agents at a certain time t_n is represented by a set of epistemic structures of all agents ($Epist(t_n)$).

Let t_0 be the initial state of the Hats Puzzle – before any of the agents announces the information about his/her knowledge. At this point we know the actual epistemic state of Charlie (and do not know Alice's and Bob's actual epistemic states):

$$epist(Charlie, t_0) = \{bbr, brb, brr, rbb, rbr, rrb, rrr\}.$$

We can, however, recognise Alice's and Bob's possible epistemic states which form their epistemic structure:

$$Epist(Alice, t_0) =$$

$$= \{epist_1(Alice, t_0), epist_2(Alice, t_0), epist_3(Alice, t_0), epist_4(Alice, t_0)\} =$$

$$= \{\{bbr, rbr\}, \{brb, rrb\}, \{brr, rrr\}, \{rbb\}\};$$

$$Epist(Bob, t_0) = \{\{bbr, rbr, brr, rrr\}, \{brb, rrb, rbb\}\}.$$

Charlie's epistemic structure is a singleton as follows:

$$Epist(Charlie, t_0) = \{epist(Charlie, t_0)\}.$$

The initial state of the Hats Puzzle is represented by the set

$$Epist(t_0) = \{Epist(Alice, t_0), Epist(Bob, t_0), Epist(Charlie, t_0)\}.$$

Agent i knows, in a possible epistemic state $epist_k(i, t_n)$, that situation X holds ($K_{epist_k(i,t_n)}(X)$) iff $X \subseteq \bigcap epist_k(i, t_n)$.

Agent i knows that situation X holds at fixed time t_n ($K(i, t_n)(X)$), when i knows that X holds in the actual epistemic state at that time ($K_{epist(i,t_n)}(X)$)

As an example let us consider one of the possible epistemic states of Alice in t_0 – $epist_1(Alice, t_0) = \{bbr, rbr\}$. The full description is as follows:

$$\{\{(Alice, hasHatOfColour, black), (Bob, hasHatOfColour, black),$$

$$(Charlie, hasHatOfColour, red)\},$$
$$\{(Alice, hasHatOfColour, red), (Bob, hasHatOfColour, black),$$
$$(Charlie, hasHatOfColour, red)\}\}$$

Thus, we have X to be the situation known by Alice ($K_{epist_1(Alice,t_0)}(Y)$) if

$$X = \bigcap epist_1(Alice, t_0) =$$
$$= \{(Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, red)\}.$$

This means that in the epistemic state $epist_1(Alice, t_0)$ Alice knows two facts: $(Bob, hasHatOfColour, black)$ and $(Charlie, hasHatOfColour, red)$, i.e. that Bob's hat is black and Charlie's hat is red.

1.4 Dynamic epistemology

Some actions taken by agents may cause changes in the epistemic structures of other agents. An obvious example of such an action is a public announcement (cf. e.g. [15]). In the Hats Puzzle Alice can announce that *Bob's hat is red* or, more interestingly, that *she does not know the colour of her own hat.* After such an action all agents should remove from their epistemic structure all possible worlds containing elementary situations incompatible with the action.

The results of such actions can be represented in the form of epistemic rules. As examples, let us consider the rules connected with the above announcements[2].

In both cases the agents remove from their epistemic structures all possible worlds that are incompatible with the information they obtain. Formally, it can be stated in the following way.

Rule 1 *If (i says that) situation X takes place, then for every agent i' $Epist(i', t_{n+1}) = \{e' : e \in Epist(i', t_n)$ and $e' = \delta_1(e, X) \neq \emptyset\}$, where*

Definition 1.

$$\delta_1(e, X) = \{w \in e : X \subseteq w\}.$$

Rule 2 *If (i says that) $\neg K_{i,t_n}(X)$ and $e = \bigcup e_k$, where $e_k \in Epist(i, t_n)$ and $X \subseteq \bigcap e_k$, then for every agent i', $Epist(i', t_{n+1}) = \delta_2(Epist(i', t_n), e)$, where*

Definition 2.

$$\delta_2(E, e) = \begin{cases} E \backslash \{e'\}, & \text{if } e' \in E \text{ and } e' \subseteq e \\ (E \backslash \{e'\}) \cup \{e' \backslash e\}, & \text{if } e' \in E \text{ and } e \cap e' \neq \emptyset \text{ and } e' \backslash e \neq \emptyset \\ E & \text{otherwise} \end{cases}$$

Let us analyse the result of Alice's announcement that she does not know the colour of her hat for Bob's epistemic structure. First, we have to find out what Alice knows in her possible epistemic states. As mentioned above:

[2] More rules are defined in [7].

$$\bigcap epist_1(Alice, t_0) = \\ = \{(Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, red)\}.$$

Moreover,

$$\bigcap epist_2(Alice, t_0) = \\ = \{(Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, black)\};$$

$$\bigcap epist_3(Alice, t_0) = \\ = \{(Bob, hasHatOfColour, red), (Charlie, hasHatOfColour, red)\};$$

$$\bigcap epist_4(Alice, t_0) = \\ = \{(Alice, hasHatOfColour, red), \\ (Bob, hasHatOfColour, black), (Charlie, hasHatOfColour, black)\}.$$

The fact that Alice knows the colour of her hat

$$(Alice, hasHatOfColour, black)$$

is an element only of $epist_4(Alice, t_0)$.

Now we can consider Bob's epistemic structure after Alice's announcement – $Epist(Bob, t_1)$. Since $epist_4(Alice, t_0)$ is the only possible epistemic state of Alice which contradicts her announcement, the set e from the definition equals $epist_4(Alice, t_0) = \{rbb\}$. By Rule 2 we have

$$Epist(Bob, t_1) = \delta_2(Epist(Bob, t_0), \{rbb\}).$$

With

$$Epist(Bob, t_0) = \{\{bbr, rbr, brr, rrr\}, \{brb, rrb, rbb\}\}$$

we have

$$Epist(Bob, t_1) = \{\{bbr, rbr, brr, rrr\}, \{brb, rrb\}\}.$$

1.5 Common knowledge

To apply and justify Rule 2 we have to accept the assumption that agents share knowledge about the state of the world and epistemic capacities. Not only do all agents share the information stated as a description of the situation (in the Hats Puzzle the information about possible hat colours and about visibility relation) but all of them know that each of them has that knowledge and know that they know that they know etc. Such an assumption is captured in the notion of common knowledge understood as in [3].

The question arises concerning what level of introspection is really relevant to the reasoning mechanisms. The answer depends on the reasoning mechanisms considered. In Rule 1 there is no reference to information about knowledge at all, in Rule 2 there is a reference to other agent's knowledge about the world.

Thus when Rule 2 is used once, only second order knowledge of agents, and our (observer's, reasoning system) knowledge about it is sufficient. However if agents reason on the basis of other agents' reasoning (e.g. results of application of Rule 2), higher level knowledge should be considered.

1.6 Querying the system

A description of the world, agents' beliefs and their dynamics, is sufficient for detailed investigations about possible scenarios. In the example we may be interested in Charlie's epistemic structure after Alice and Bob announce one after another that they do not know the colours of their own hats. Applying the rule will lead us to the conclusion that at the end Charlie knows that his hat is red. To formulate such queries standard DEL languagecan be used.

Something is true after action (public announcement) – $[\Phi]\Psi$ means that after Φ is (publicly and truthworthy) announced Ψ holds.

The main question of the Hats Puzzle is:

$$\text{find } X\text{, such that}$$
$$[\neg K_{(Alice,t_0)}\Phi_1][\neg K_{(Bob,t_1)}\Phi_2](Charlie, hasHatOfColour, X),$$

where $\neg K_{(Alice,t_0)}\Phi_1$ states that Alice does not know the colour of her own hat and $\neg K_{(Bob,t_1)}\Phi_2$ states that Bob does not know the colour of his own hat.

It is important to observe that in order to analyse the situation and answer such a query one does not have to know what is the real situation from the beginning. Thus we can extract information about the reality from the announcements (or, more generally, behaviour) of the agents. The precondition here is that agents are reliable.

1.7 Relation to Kripke structures

The description of the problem environment, defined in the presented way can be transformed into a Kripke structure that can be a subject of further investigations (see [9]).

Let Agt be a set of agents, Π a set of possible actions and V a set of propositions defining elementary situations. We consider the structure $\mathcal{M} = (S, RB, I, v)$ where

- S is a non-empty set of states (the universe of the structure),
- RB is a doxastic function which assigns a binary relation to every agent, $RB : Agt \longrightarrow 2^{S\times S}$,
- I is an interpretation of actions performed by agents, $I : \Pi \longrightarrow (Agt \longrightarrow 2^{S\times S})$,
- v is a valuation function, $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^V$.

We have to define such a structure on the basis of the set of all possible epistemic states $Epist$ and the rules describing epistemic changes after the agent's actions. Since we want to include in the model all the possible scenarios, we have to drop the time layer of the model and consider the elements of $Epist$ as valid after performing a sequence of triggers. For this reason we will use for them symbols $Epist(x)$, $x \in \mathbb{N}$. Since we consider a finite number of agents, features and possible values, the number of states is also finite. Let $Epist(1)$ be an initial epistemic state. Let further $Epist^* \subseteq Epist$ be the smallest set containing $Epist(1)$ closed under the operation of applying an epistemic rule. Intuitively, $Epist^*$ is a set of epistemic states reachable from an initial state by the actions of agents.

Not all ontologically possible worlds are epistemically possible in every epistemic state, because epistemic rules exclude some of them. $\bigcup\bigcup Epist(x)$ is the set of possible worlds occurring in epistemic state $Epist(x)$. Let $S_x = \{\langle w, x\rangle : w \in \bigcup\bigcup Epist(x)\}$. Now, we can define $\mathcal{M} = (S, RB, I, v)$ in the following way:

$$S = \bigcup\{S_x : Epist(x) \in Epist^*\};$$

$$RB(i) =$$
$$= \{\langle\langle s, x\rangle, \langle s', x'\rangle\rangle : x = x' \textit{ and there exists } e \in Epist(i, x), \textit{ such that } s, s' \in e\};$$

$$I(P)(i) = \{\langle\langle s, x\rangle, \langle s', x'\rangle\rangle :$$
$$s = s' \textit{ and action } P \textit{ performed by agent } i \textit{ leads from } Epist(k) \textit{ to } Epist(k')\};$$

v is a valuation such that in state $\langle s, x\rangle$ proposition Φ is assigned to value $\mathbf{1}$ iff a respective elementary situation $es \in s$ and to $\mathbf{0}$ otherwise.

2 Errors, cheating and incompatible information

Our approach can be applied to situations that are more complex than simple knowledge acquisition. In the case considered in the previous section we assumed that the ontology is shared by all agents – it belongs to common knowledge and the only possible actions are reliable announcements. Now we drop those assumptions. To deal with such cases we need to extend the language of the description of epistemic capacities and modify the rules.

2.1 Empty epistemic structure

The first extension covers beliefs that can be false. This may be caused by errors in the reasoning of other agents or their behaviour different from the one described in the scenario (such as cheating). This situation may eventually lead to contradictory beliefs. Otherwise, the agent that received false information does not know it. The incompatible information makes the set of possible epistemic states empty.

Let us look at the Hats Puzzle example again. Let us analyse Bob's beliefs when Alice first (falsely) announces that she does not know the colour of her hat and then that Bob's hat is black and that Charlie's hat is black.

As we stated above, Bob's epistemic structure after Alice's first announcement ($Epist(Bob, t_1)$)is as follows:

$$Epist(Bob, t_1) = \{\{bbr, rbr, brr, rrr\}, \{brb, rrb\}\}.$$

After the second announcement (that Bob has a black hat) we eliminate, according to Rule 1, possible worlds that do not contain that fact. Thus we have

$$Epist(Bob, t_2) = \{\{bbr, rbr\}\}.$$

Up to that moment Bob cannot recognise that some of the information he receives is indeed false. Now if we apply Rule 1 to incorporate information from Alice's third announcement we obtain

$$Epist(Bob, t_3) = \emptyset.$$

The empty epistemic structure represents the state of beliefs in which no situation is perceived as a possible world. This means that the information the agent has obtained is incompatible and at least some of it is false. However, there is no hint as to which announcement is false.

In this case there is no bridge between the announcements and states of beliefs on the one side and the real situation on the other. We cannot perform the reasoning as we did in the original puzzle; however, we can still analyse beliefs. The analyses may be richer when we know the real situation.

2.2 Kripke models accepting empty epistemic structures

Let us now construct a Kripke model that takes into account an empty epistemic structure. As in Section 1.7, we use S for a set of states, RB for a doxastic relation, I for an interpretation of actions and v for a valuation function. Let $\mathcal{M} = (S, RB, I, v)$ be a model defined on the basic of the epistemic structures of agents and their possible actions with respective epistemic rules as defined in section 1.7. Let us now define a new structure, $\mathcal{M}_e = (S_e, RB_e, I_e, v_e)$ in the following way:

$$S_e = S \cup \{\langle \emptyset, 0\rangle : w \in \bigcup\bigcup Epist(1)\};$$

$$RB_e(i) = RB(i) \cup \{\langle\langle \emptyset, 0\rangle\langle \emptyset, 0\rangle\rangle\};$$

$$I_e(P)(i) = I(P)(i) \cup \{\langle\langle s, x\rangle, \langle s, 0\rangle\rangle : \neg\exists r(\langle\langle s, x\rangle, r\rangle \in I(P)(i))\};$$

v_e for non-epistemic sentences is any global valuation, i.e. a valuation which is the same for every possible world.

Let Φ be a proposition describing elementary situstion es. Agent i believes that Φ in a state $r \in S_e$ iff for every $r' = \langle s', x'\rangle \in S_e$ such that $\langle r, \langle s', x'\rangle\rangle \in RB_e(i)$ $es \in s'$. In the state of the model corresponding to an empty epistemic state – $\langle \emptyset, 0\rangle$ agent believes in nothing.

We cannot get from beliefs to reality. Thus we have two possibilities of using Kripke structures defined in such a way, corresponding to an observer knowing the real situation or not. In the former case we consider just one valuation, v, representing the real world. In the latter, we have to consider all possible valuations and create a suitable logicfal system.

2.3 Ontology driven belief revision

The important question, discussed within belief revision theory and dynamic epistemic and doxastic logic, is how can we resolve the inconsistency. Many possibilities have been discussed here. All strategies of revision can be expressed in terms of modal logic and models [4], [5].

As we aim to connect epistemic investigations with ontological background we are interested in revision strategies based on factors coming from ontology. The investigations on the subject exceed the scope of the present paper, so we only list some possible directions of ontologically based methods of ordering beliefs.

- One type of information is more reliable then others. (e.g. An agent relies more on simple facts than the results of other agents' reasoning.)
- One informant is more reliable than another.
- Newer information is more reliable than older.

2.4 Ontological representation of different beliefs about epistemic structures

Now let us consider yet another variation on the Hats Puzzle – false recognition of other agents' epistemic capacities. The children sit as in the original puzzle but Charlie believes falsely that Alice and Bob sit in a different order, such that Bob sees Alice and Alice does not see Bob. To encode this new situation we just need to state it explicitly and assume that it becomes common knowledge. We use the following statements to express Charlie's beliefs about the epistemic capabilities of our agents:

- $(Charlie, believes, (Bob, knowsWhether, (Alice, hasHatOfColour, ...)))$;
- $(Charlie, believes, (Bob, knowsWhether, (Charlie, hasHatOfColour, ...)))$;
- $(Charlie, believes, (Alice, knowsWhether, (Charlie, hasHatOfColour, ...)))$.

To interpret the whole situation we assume that, unless explicitly stated, the agents' beliefs about plain facts, epistemic capabilities and other agents' beliefs concerning them are correct.

To capture reasoning in such a situation we have to extend our model with the explicit information about agents' beliefs about other agents' epistemic structures. Thus instead of a simple epistemic structure of agent i in time t_n ($Epist(i, t_n)$) we have to consider the structure as seen by an other agent, i'. To do this we add another argument, and obtain:

$$Epist(i', i, t_n).$$

For real epistemic structures of agent i in t_n we still use $Epist(i, t_n)$.

We can now formulate the new rule.

Rule 3 *If (i says that)* $\neg K_{i,t_n}(X)$ *and* $e = \bigcup e_k$, *where* $e_k \in Epist(i', i, t_n)$ *and* $X \subseteq \bigcap e_k$ *then for every agent* i'', $Epist(i'', i', t_{n+1}) = \delta_3(Epist(i'', i', t_n), e)$, *where* $\delta_3(Epist(i', i, t_n), e) = \delta_2(Epist(i, t_n), e)$.

A complete description of the problem state at a certain time t_n consists of the real epistemic structure of every agent at t_n and beliefs about epistemic structure of agents among agents ($Epist(i', i, t_n)$ for every pair of agents i and i'). Rules 2 and 3 can be applied to get from one state to another.

Now we can express the initial epistemic structures for Alice, Bob and Charlie derived from the descriptions of capacities. As we have stated before, the real epistemic structures for agents at t_0 are as follows:

$$Epist(Alice, t_0) = \{\{bbr, rbr\}, \{brb, rrb\}, \{brr, rrr\}, \{rbb\}\};$$

$$Epist(Bob, t_0) = \{\{bbr, rbr, brr, rrr\}, \{brb, rrb, rbb\}\};$$

$$Epist(Charlie, t_0) = \{\{bbr, brb, brr, rbb, rbr, rrb, rrr\}\}.$$

Moreover

$$Epist(Alice, Alice, t_0) = Epist(Bob, Alice, t_0) = Epist(Alice, t_0);$$

$$Epist(Charlie, Alice, t_0) = Epist(Bob, t_0);$$

$$Epist(Alice, Bob, t_0) = Epist(Bob, Bob, t_0) = Epist(Alice, t_0);$$

$$Epist(Charlie, Bob, t_0) = Epist(Alice, t_0);$$

$$Epist(Alice, Charlie, t_0) = Epist(Bob, Charlie, t_0) = \\ = Epist(Charlie, Charlie, t_0) = Epist(Alice, t_0).$$

Let us now analyse Charlie's epistemic structure after Alice and Bob state consecutively that they do not know the colours of their hats (at time t_1 and t_2).

$Epist(Charlie, t_n)$ represents what Charlie would believe if he knew the real eistemic capacities of Alice and Bob while $Epist(Charlie, Charlie, t_n)$ represents what Charlie's actual beliefs are. Thus

$$Epist(Charlie, Charlie, t_0) = Epist(Charlie, t_0) = \\ = \{\{bbr, rbr, brr, rrr, brb, rrb, rbb\}\};$$

$$Epist(Charlie, t_1) = \{\{bbr, rbr, brr, rrr, brb, rrb\}\};$$

$$Epist(Charlie, Charlie, t_1) = \{\{bbr, rbr, brr, rrr, brb, rrb, rbb\}\};$$

$$Epist(Charlie, t_2) = \{\{bbr, rbr, brr, rrr\}\};$$

$$Epist(Charlie, Charlie, t_2) = \{\{bbr, rbr, brr, rrr, brb, rrb\}\}.$$

We can build on this basis the corresponding Kripke structure. The basic difference between the new structure and the model defined for simple knowledge acquisition is that in every state we have all possible worlds. Some of them (the ones present in model $\mathcal{M}$ from section 1) are really possible, while others are possible from the point of view of agents wrongly recognising other agent's epistemic capabilities. The rest of the construction is a straightforward extension of the definition of model $\mathcal{M}$. Let $States \subset \mathbb{N}$ be the set of numbers of states.

Now, we can formally define model $\mathcal{M}_c = (S_c, RB_c, I_c, v_c)$ in the following way:

$$S_c = \{\langle w, x\rangle : w \in \bigcup\bigcup Epist(1), x \in States\};$$

$$RB_c(i) =$$
$$= \{\langle\langle s, x\rangle, \langle s', x'\rangle\rangle : x = x' \textit{ and there exists } e \in Epist(i, i, x), \textit{ such that } s, s' \in e\};$$

$$I(P)(i) = \{\langle\langle s, x\rangle, \langle s', x'\rangle\rangle :$$
$$s = s' \textit{ and action } P \textit{ performed by agent } i \textit{ leads from } Epist(k) \textit{ to } Epist(k')\};$$

v is a valuation such that in state $\langle s, x\rangle$ proposition Φ is assigned to value **1** iff a respective elementary situation $es \in s$ and to **0** otherwise.

More complicated cases can also be studied, but the description and reasoning are becoming very complex with the introduction of higher level beliefs[3].

3 Incomplete beliefs as alternatives

3.1 Representation using epistemic structures of agents

Another extension of the basic model covers a situation in which an agent has an incomplete picture of the other agents' epistemic capacities. In the hats puzzle this would take place if Charlie did not know in which order Alice and Bob sat. Thus, in Charlie's opinion, either Alice sees Bob or Bob sees Alice.

This situation can be expressed by stating alternative possibilities. Every element of such an alternative is a complete description of the situation.

1. alternative 1
   - $(Alice, knowsWhether, (Bob, hasHatOfColour, \dots))$, $(Alice, knowsWhether, (Charlie, hasHatOfColour, \dots))$, $(Bob, knowsWhether, (Charlie, hasHatOfColour, \dots))$.
2. alternative 2
   - $(Bob, knowsWhether, (Alice, hasHatOfColour, \dots))$, $(Alice, knowsWhether, (Charlie, hasHatOfColour, \dots))$, $(Bob, knowsWhether, (Charlie, hasHatOfColour, \dots))$.

In such a situation, in order to model Charlie's belief state after the epistemic action of Alice or Bob, one has to consider parallelly both alternatives and accept only those sentences that are true in both of them. Eventually, one of the alternatives may lead to the empty epistemic structure and then the other one

[3] Implementation of such cases is the subject of ongoing work.

remains the only one to be considered. If something is true in both alternatives, then it is true in general.

Let us analyse Charlie's epistemic structure after Alice and Bob, and again Alice state consecutively that they do not know the colours of their hats (at time t_1, t_2 and t_3). We use $Epist(i, t_n, k)$ for the epistemic state of agent i at time t_n in alternative possibility k and define the alternatives as follows:

1. alternative 1
   - $Epist(Charlie, t_1, 1) = \{\{bbr, rbr, brr, rrr, brb, rrb\}\}$;
   - $Epist(Charlie, t_2, 1) = \{\{bbr, rbr, brr, rrr\}\}$;
   - $Epist(Charlie, t_3, 1) = \{\{bbr, rbr, brr, rrr\}\}$;
2. alternative 2
   - $Epist(Charlie, t_1, 2) = \{\{bbr, rbr, brr, rrr, brb, rrb, brr\}\}$;
   - $Epist(Charlie, t_2, 2) = \{\{bbr, rbr, brr, rrr, brb, rrb\}\}$,
   - $Epist(Charlie, t_3, 2) = \{\{bbr, rbr, brr, rrr\}\}$.

Since $Epist(Charlie, t_3, 1) = Epist(Charlie, t_3, 2)$ we can treat that set as $Epist(Charlie, t_3)$ and derive that Charlie knows that he has a red hat.

3.2 Corresponding Kripke structures

Let $\mathcal{M}_1 = (S_1, RB_1, I_1, v_1)$ and $\mathcal{M}_2 = (S_2, RB_2, I_2, v_2)$ be Kripke structures defined as the structure in section 1, corresponding to two alternative epistemic capacities of the agents, as for example in those described above. To obtain a structure corresponding to such an alternative we define structure $\mathcal{M}_a = (S_a, RB_a, I_a, v_a)$ as follows:

$$S_a = S_1 \cup S_2$$
$$RB_a(i) = RB_1(i) \cup RB_2(i)$$
$$I_a(P)(i) = I_1(P)(i) \cup I_2(P)(i)$$
$$v_a = v_1 = v_2$$

4 RDF description of ontology and epistemic capacities

We believe that expressing problems from epistemic logic in the framework of Knowledge Representation will be a step towards a wider application of both. To close the conceptual gap between those two methodologies we utilize RDF (*R*esource *D*escription *F*ramework) language to encode the information from the ontological and intermediary level. RDF may be seen as a language for representing information about various information resources, including those available in the World Wide Web (see, e.g., [12]). The main rationale is to facilitate it for application agents to exchange information without a loss of meaning. The characteristic feature of RDF is the way it represents information. Namely, the basic RDF pattern is a triple: (subject s, property p, value v), whose intuitive meaning may be captured by the expression "subject s has property p of value v".

Currently, there is a number of RDF stores that contain a substantial amount of data. We use RDF to represent elementary situations and epistemic capacities.

An RDF description of our elementary ontological situation is effectively given by a triple: $(agent, \text{hasHatOfColour}, colour)$. For instance, the fact that Alice has a red hat is encoded by the triple: $(Alice, hasHatOfColour, red)$. On the other hand, we suggest that an RDF description of an epistemic capacity should be given by a set of complex triples:
$(agent, \text{knowsWhether}, reified\_elementary\_situation\_triple)$, where
an $reified\_elementary\_situation\_triple$ is a special kind of RDF construct that allows one to represent within an RDF document its own RDF triples.

Given these assumptions, we created for each possible world a single RDF document that contains the description of this world and the specification of the ontology-to-epistemology interface using the Protege editor - see figure 1.

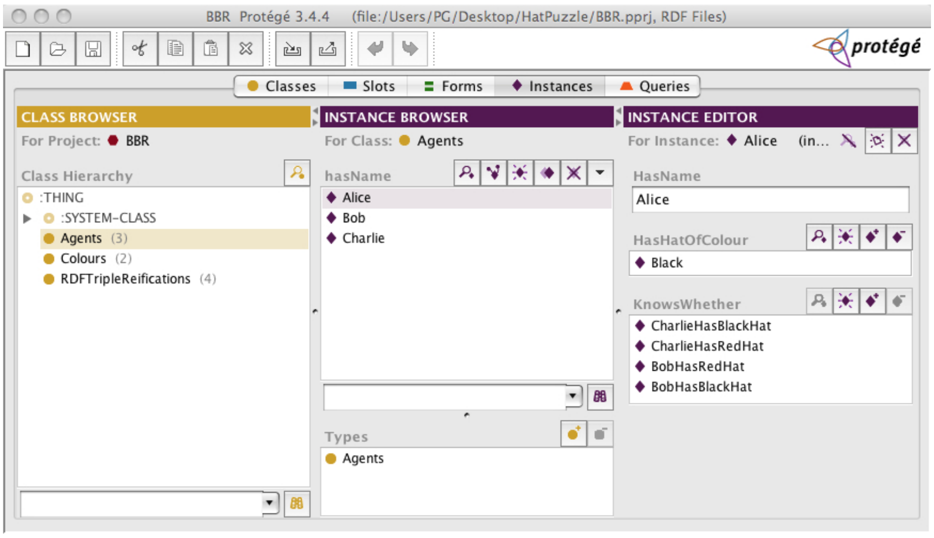

Fig. 1. Hats Puzzle representation in Protege

The whole content of one RDF document is given below. All the documents may be found at: `www.l3g.pl/hatpuzzle`.

```
<?xml version='1.0' encoding='UTF-8'?>
<!DOCTYPE rdf:RDF [
 <!ENTITY rdf 'http://www.w3.org/1999/02/22-rdf-syntax-ns#'>
 <!ENTITY rdfs 'http://www.w3.org/2000/01/rdf-schema#'>
 <!ENTITY hatpuzzle 'http://www.l3g.pl/hatpuzzle#'>
]>
<rdf:RDF xmlns:rdf="&rdf;"
 xmlns:hatpuzzle="&hatpuzzle;"
 xmlns:rdfs="&rdfs;">
<hatpuzzle:RDFTripleReifications rdf:about="&hatpuzzle;HatPuzzleWithReification_Class0"
```

```
 hatpuzzle:hasName="BobHasBlackHat"
 rdfs:label="BobHasBlackHat">
<hatpuzzle:isSubjectOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class11"/>
<hatpuzzle:isValueOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class13"/>
<hatpuzzle:isPropertyOf rdf:resource="&hatpuzzle;hasHatOfColour"/>
</hatpuzzle:RDFTripleReifications>
<hatpuzzle:RDFTripleReifications rdf:about="&hatpuzzle;HatPuzzleWithReification_Class1"
 hatpuzzle:hasName="CharlieHasRedHat"
 rdfs:label="CharlieHasRedHat">
<hatpuzzle:isSubjectOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class12"/>
<hatpuzzle:isValueOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class14"/>
<hatpuzzle:isPropertyOf rdf:resource="&hatpuzzle;hasHatOfColour"/>
</hatpuzzle:RDFTripleReifications>
<hatpuzzle:Agents rdf:about="&hatpuzzle;HatPuzzleWithReification_Class10"
 hatpuzzle:hasName="Alice"
 rdfs:label="Alice">
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class0"/>
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class1"/>
<hatpuzzle:hasHatOfColour rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class13"/>
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class16"/>
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class2"/>
</hatpuzzle:Agents>
<hatpuzzle:Agents rdf:about="&hatpuzzle;HatPuzzleWithReification_Class11"
 hatpuzzle:hasName="Bob"
 rdfs:label="Bob">
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class1"/>
<hatpuzzle:hasHatOfColour rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class14"/>
<hatpuzzle:knowsWhether rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class2"/>
</hatpuzzle:Agents>
<hatpuzzle:Agents rdf:about="&hatpuzzle;HatPuzzleWithReification_Class12"
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 rdfs:label="Charlie">
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 rdfs:label="Black"/>
<hatpuzzle:Colours rdf:about="&hatpuzzle;HatPuzzleWithReification_Class14"
 hatpuzzle:hasName="Red"
 rdfs:label="Red"/>
<hatpuzzle:RDFTripleReifications rdf:about="&hatpuzzle;HatPuzzleWithReification_Class16"
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 rdfs:label="BobHasRedHat">
<hatpuzzle:isSubjectOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class11"/>
<hatpuzzle:isValueOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class14"/>
<hatpuzzle:isPropertyOf rdf:resource="&hatpuzzle;hasHatOfColour"/>
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<hatpuzzle:RDFTripleReifications rdf:about="&hatpuzzle;HatPuzzleWithReification_Class2"
 hatpuzzle:hasName="CharlieHasBlackHat"
 rdfs:label="CharlieHasBlackHat">
<hatpuzzle:isSubjectOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class12"/>
<hatpuzzle:isValueOf rdf:resource="&hatpuzzle;HatPuzzleWithReification_Class13"/>
<hatpuzzle:isPropertyOf rdf:resource="&hatpuzzle;hasHatOfColour"/>
</hatpuzzle:RDFTripleReifications>
</rdf:RDF>
```


Conclusion

In the paper the interactions between rational agents are studied using the approach in which these phenomena are seen from the point of view of formal ontology. We try to derive the agent's states of beliefs and their dynamics from the information about the structure of the world in which agents act and the agents' epistemic capacities.

There are two points in which this paper contributes to the model of epistemic interactions defined earlier. One is an extension of the model for situations in which agents can receive false announcements and can have incomplete or incorrect view about the other agent's epistemic capacities. The other is a representation of a problem situation concerning knowledge and beliefs in the framework of RDF triples – a leading standard within Knowledge Representation.

Some questions that remain open for us are as follows.

- Is the ontological description of epistemic capacities rich enough for real life situations? Does one need, for example, higher levels of introspection for reasoning?

- Building Kripke structures on ontological descriptions determines a specific class of models. Does it form an interesting logic?

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