Bulletin of the **S**ection of **L**ogic
Volume 39:1/2 (2010), pp. 53–55

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HALLDEN INCOMPLETE CALCULUS OF NAMES

Hallden completeness is a weaker version of "disjunction property" for logical systems, defined as follows:

if $\alpha \vee \beta \in L$ and $var(\alpha) \cap var(\beta) = \varnothing$, then $\alpha \in L$ or $\beta \in L$;

where L is a system, α and β are formulae and for any formula γ, $var(\gamma)$ denotes a set of free variables contained in γ.

The notion is usually used in the context of intermediate or modal logics. In the present paper it is applied to the systems of the calculus of names, which are axiomatisations of syllogistic. A Hallden incomplete system $Sl^{\star}$, which is placed between two known systems - classical axiomatisation of J. Łukasewicz (Luk) [1] and its minimal subsystem containing all Aristotelean laws of J. Słupecki (Sl) ([3], also in [2] as system 10.3 on page 310), is considered.

Let S, M and P be individual variables and a and i denote predicates forming respectively universal and particular affirmative sentences of syllogistic. (Thus the atomic formula SaP can be read as *every S is P* and SiP - *some S are P*.) Let further $\neg$, $\wedge$, $\vee$ and $\rightarrow$ denote classical propositional functors of respectively negation, conjunction, alternative and implication.

The systems Luk, Sl and $Sl^{\star}$ are all based on classical propositional calculus (CPC). Formally they are defined by rules modus ponens and substitution for individual variables (point substitution) and axioms, including substitutions of all theses of CPC into the language of the systems and the following specific axioms for particular systems.

Luk:

$$SaM \wedge MaP \to SaP, \tag{1}$$

$$MiS \wedge MaP \to SiP, \tag{2}$$

$$SaS, \tag{3}$$

$$SiS. \tag{4}$$

Sl: (1), (2) and

$$SaP \to SiP, \tag{5}$$

$$PiS \to SiP. \tag{6}$$

$Sl^\star$: (1), (2), (5), (6) and

$$PiP \to SiS. \tag{7}$$

It is easy to check that $Sl \subset Sl^\star \subset Luk$.

THEOREM. *System $Sl^\star$ is Hallden incomplete.*

PROOF. Axiom (7) is equivalent in CPC to the formula $\neg PiP \vee SiS$. Obviously $var(\neg PiP) \cap var(SiS) = \varnothing$. Thus it is enough to show that $\neg PiP \notin Sl^\star$ and $SiS \notin Sl^\star$. Since all of the $Sl^\star$ axioms are of the form of implication with a conjunction of atomic formulae in the predecessor and an atomic formula in the consequent, they are all true in the model in which all atomic formulae are true and also in the model in which all atomic formulae are false. In the first model the formula $\neg PiP$ is false. In the second one the formula SiS is false. Thus both formulae are not elements of $Sl^\star$. □

References

[1] J. Łukasiewicz, **Aristotle's Syllogistic from the Standpoint of Modern Formal Logic**, Oxford, 1957.

[2] A. N. Prior, **Formal Logic**, Clarendon Press, Oxford 1962.

[3] J. Słupecki, *Uwagi o sylogistyce Arystotelesa (Remarks on aristotle's syllogistic)*, **Annales UMCS** I (1946), pp. 187–191.

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