The Tower of Turtles

or: the seeming 'paradox' of reductionism, which isn't

or: about objects which are not even intended to be entities

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Abstract. In this paper I will discuss the philosophical topic of emergence and reduction(ism). The motivation behind that is to try to overcome an inherent vagueness of the reductionist program which most obstinately shows up at the *primordial* emergence, namely that of the intended elementary ontological structure(s) of the universe itself. I propose that for overcoming the alleged nearly paradoxical situation of reductionism with respect to *primordial* emergence one has to put the attached problems in a rigorously emergence-theoretical perspective. Thus I will also come to some uncommon ontological results, e.g. the postulate of objects which are no intended entities (and no mere ideas or fictitious concepts either) and by this to the postulate that the most fundamental physical structure(s) have to be seen as being emergent from a rigorously non-physical, i.e. an entirely mathematical structure.

> Physica ergo ... desinit in geometriam, nec ante ullum phaenomenon penitus in corporibus intelligemus, quam ex primis figurae motusque ideis derivamus.[*]

Introduction: The Tower of Turtles or the Pitfall of Reductionism - as we knew it

In his - characteristically multifaceted and wide ranging - essay 'It from Bit' John A.Wheeler postulates four great No's as guidelines for a future foundation of physics. Citing William James he puts the first of the great No's the following way:

> " "No tower of turtles," advised William James. Existence is not a globe supported by an elephant, supported by a turtle, supported by yet another turtle, and so on. In other words, no infinite regress."[1]

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[*] G.W.Leibniz, Dissertatio exoterica de statu praesenti et incrementis novissimis deque usu geometriae, in: G.W.Leibniz, Mathematische Schriften, (ed. C.J.Gerhardt), Bd.VII p 325); (Physics namely ... fades away into geometry, and we won't know any phenomenon deeply in its corporeal construction as long as we haven't derived it from the first principles of geometrical figures and of motion.)

[1] J.A.Wheeler; It from Bit, in: J.A.Wheeler, At Home in the Universe, Woodbury 1994, 295-311, p 300; cf. also: J.A.Wheeler, Information, Physics, Quantum: The search for links, in: W.H.Zurek (ed.), Complexity, Entropy and the Physics of Information, Redwood City 1990, 3-28, p 8

This great No stated by Wheeler is of course a veto against reductionism. And it probably had been mainly motivated by the anticipation of a presumed potentially disastrous final outcome of the reductionist project carried to the extremes of a suspected inevitable shipwreck or sort of paradox. That seeming paradox of reductionism is expected to arise from the presumed fact that the reductionist project possibly could not terminate. And this - if it would turn out to be a justified presumption - would indeed look like a sort of paradox for it is undoubtedly and explicitly (by its proponents) the very idea and aim of the reductionist project to exactly come to the point of terminating, i.e. to discover the fundamentum inconcussum of physics or rather of the entire nature.

In this talk I will try to show that the real problem which leads to the presumption of such a paradoxical end of the reductionist project is not one of reductionism itself but rather one of a seemingly obvious but nevertheless unjustifiable tacit interpretation of reductionism, namely that of an ontological charge[2] of reductionism taken for granted not only by its proponents but by its opponents as well.

The following argumentation will bring me again in my usual position namely between at least two stools because I will argue fervently in favour of the reductionist project of which being against is probably the defining denominator of nearly all alternative natural philosophies but by arguing *for* reductionism I will also fervently argue *against* an essential reductionist belief namely that reductionism is about finding or uncovering increasingly lower or deeper *levels of nature*, made of *physical entities*, by means of a method of reductionist abstraction which is closely associated with a sort of practice of somehow dividing or separating the *physical entities* of higher levels of nature into physical entities of respectively lower levels. The problem with that belief is twofold. The first aspect relates to the ontological hypostasis which is implied in that belief. This will be a main topic of this paper. The other aspect, which is deeply intertwined with the first, is that that mentioned reductionist belief doesn't take the concept of emergence serious, but rather substitutes - even if referring to concepts like 'level' - emergence by ideas similar to a sequence of varying or extending configurations of fundamental entities, by that also implying that all the laws of nature found in more complex levels or systems would already be preformatively inherent in such a most elementary level of fundamental entities. That is not the case. And the erroneous nature of the reductionist belief will obviously become most apparent when the most innate nature of those fundamental entities - namely their 'being entities' itself - will come into question, and instead the attribute of objects to become intendable entities will turn out to just be an emergent property.[3]

It is exactly that reductionist belief which I regard to be the mentioned unjustifiable tacit interpretation of reductionism or its ontological charge. My argument in this respect is - as some might already expect - not specifically against reductionism but in general against the assumption that any scientific theory would be about or would refer to any existing entities. On the contrary I hold that physics in particular as well as science in general neither is nor ever was about any *actually existing entities* but always was and is about *intelligible objects*, and that the genuine scientific endeavour was and is the endeavour of *objectivation*, i.e. the

[2] In fact it is not a proper ontological charge but rather an epistemological charge which then yields an ontological hypostasis. But exactly that 'mechanism' of confusing epistemological presuppositions with ontological conclusions is notoriously unfathomable for the holders of that respective epistemological conviction, to which I refer, namely realists.

[3] My attitude towards reductionism then can be summarised as follows: 'Reductionism is essentially right as a scientific method, but reductionism is seriously wrong by not taking emergence (and *its* ontological implications) into account.'

creation of the intelligible objects of scientific theories.[4] (A creation which then of course has to differ essentially from the creations of such artists which happen to be no scientists.) Yet the deep belief that the opposite of my argument would be the case is of course much more common amongst the rather standard reductionists than the ontologically less correct alternatives.

1 Triumph and imminent fall of reductionism

In his book 'Dreams of a Final Theory'[5] Steven Weinberg fervently praised reductionism.[6] But he did so for explicitly the opposite reason for which I would chime in his 'Two Cheers for Reductionism'.[7] This becomes quite clear when he stresses

> "the distinction ... between reductionism as a general prescription for progress in science, which is not my view, and reductionism as a statement of the order of nature, which I think is simply true."[8]

So Weinberg - as many other physicists - is a reductionist for a somewhat peculiar reason. Namely the very reason that reductionism seems to perfectly blend with epistemological realism et vice versa. Such convictions then sometimes lead to questionable conceptions, one of which is certainly the idea of reductionism being a statement of the order of nature.
It is quite clear what Weinberg means by his statement, namely that he sees nature as build up on increasingly more complex levels being based on the most universal structure of the least complex one. But it is hard to understand what, if any, significance such an insight might have beyond the context of scientific explanations. Yet fortunately Weinberg actually knows better than he said, and sometimes he also says it better

> "Our scientific discoveries are not independent isolated facts; one scientific generalization finds it explanation in another, which is itself explained by yet another. By tracing these arrows of explanation back toward their source we have discovered a striking convergent pattern – perhaps the deepest thing we have yet learned about the universe."[9]

Here the recursive structure underlying the progress of scientific explanation - which is for a good part just reductionism at work - is clearly pointed out. But again Weinberg's realist convictions are also mingled in not only when he speaks of 'discoveries' but - more significantly - when he speaks of 'tracing *back* the arrows of explanation toward their *source*' instead of thinking of them as *being directed toward their goal of ultimate convergence (or ultimate explanation)*.[10] But the most significant amalgamation of his methodological and his epistemological convictions occurs nearly invisible in that quotation, namely when he speaks about 'scientific generalization'. 'Scientific generalization' is just another word for 'abstraction'. And it is the method of abstraction - most closely related with reductionism - from which the realist connotations of reductionism

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[4] I discussed this topic of the difference of 'actually existing entities' and 'intelligible objects' and the relevance of this difference for the history and philosophy of science extensively in: D.Kurth, Actual Existence and Factual Objectivation, in: Movements, Philosophical Aspects of ANPA 23 (Proceedings of ANPA 23), Arleta D. Ford (ed.), London 2002
[5] St.Weinberg, Dreams of a Final Theory, New York 1992
[6] Ibid. pp 51-64
[7] Ibid.
[8] Ibid. p 54
[9] Ibid. p 19
[10] Since hardly any yet undiscovered fundamental physical *structure* can be seen as being the source of scientific explanations but instead the original *source* of scientific explanations had been a most elementary proper scientific *theory* (perhaps the Theory of Planetary Motion of Hipparchos). A fundamental physical structure however might have always tacitly been the ultimate goal of subsequently improving explanations in the history of scientific theories.

mainly stem. Therefore abstraction is recognised - by most advocates of reductionism - as being the true essence of their approach. My question here is not if this view is justified but if it grasps the whole truth of the endeavour of scientific imagination and its progress.
Weinberg then deserves thanks for making unmistakably clear what in his view is *not* the essence of reductionism

> " ... by elementary particle physics being more fundamental I do not mean that it is more mathematically profound or that it is more needed for progress in other fields or anything else but only that it is *closer to the point of convergence of all our arrows of explanation* (italics by me, D.K.). "[11]

> "Mathematics itself is never the explanation of anything – it is only the means by which we use one set of facts to explain another, and the language in which we express our explanations." [12]

In the realist reductionists view mathematics cannot ever be more than a means for scientific explanations and their progress because in their view scientific explanations are not - in a very strong sense of that word - creative.[13] And conclusively abstraction cannot ever be - in that very strong sense - a genuinely creative act but in the end just another way of reduction. And who would deny that reducing complexity is after all a method of leaving particular secondary qualities out of account and instead of pointing out respectively more general underlying properties of whatsoever objects in question, i.e. such a reduction is a method which seems to nearly perfectly translate into - abstraction.
Now, what should be so horribly wrong with all that realist reductionism? Besides that it is based on indefensible epistemological assumptions and besides that it is inconsistent with the actual history of science which can definitely not be described as a process in which some originally coarse depiction asymptotically comes ever closer to a perfect representation of something out there realist reductionism leads - as ever more inescapably as its dreams of a final theory would come nearer to fulfilment - into the seeming paradox so graphically described by J.A. Wheeler as the *tower of turtles*.
And it does so not for being reductionist but for being *realist* reductionist. It does so for re-straining scientific explanation and imagination to a way of detection or discovery, it does so for dismissing the very essence of abstraction, namely the mathematical theories, as mere means, which as such never could play a role in constituting genuine objects of their own right. In short it does so for linking the objects of science to physical existence, i.e. it does so for treating these objects entirely and exclusively as entities.
And - as Wheeler hinted at - there cannot ever be anything as a first entity, a first appearance of physical existence which didn't emerge from something preceding. As long as by dogmatic presupposition that something preceding only has to be conceived as another physically existing entity one inevitably gets a seemingly paradoxical infinite regress, the tower of turtles. An infinite regress as such is obviously *not* a paradox, but in the special case of realist reductionism it very well becomes somewhat paradoxical because the final dream or envisioned end of realist reductionism is just to terminate, and by its very presuppositions realist reductionism ensures that it will not and cannot terminate.

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[11] Ibid. p 55, cf. also p 32
[12] Ibid. p 56
[13] Of course also realist reductionists do not say that scientist are not creative people, but that creativity is just a feature of their respective personal intellectual capacity. Yet realist reductionist probably would not see the objectivations of scientific theories as being genuinely creations of scientific imagination but rather as the result of a detectional endeavour, i.e. as discoveries. After all a realist reductionist is expected to be - a realist.

2 Aspects of theoretical imagination: reductionist abstraction versus creative objectivation

A so-called scientific realism like the one which St. Weinberg propagates must not necessarily solely or even primarily stem from ordinary epistemological convictions but is in many cases rather founded upon a certain view of how science - or in particular scientific progress - allegedly works, namely the view that it works by ever increasing abstraction.[14]
Abstraction after all is the very essence and method of reductionism. Therefore rather sophisticated reductionists are convinced that it was by the means of abstraction that their predecessors came by starting from the most common appearances and phenomena step by step closer to the increasingly deeper and less obvious underlying levels of increasingly more universal laws of nature and by that conviction ourdays reductionists also get - even if only as a by-product - a comfortable epistemological position, namely that of scientific realism.
The question is not if this reductionist view of abstraction as the predominant scientific method is correct or not because it is certainly correct in a sense. The question - in my view - is if it is sufficient. Does scientific imagination really works by abstraction - more or less - exclusively?
The question of abstraction as a method of reduction and a certain uneasiness related to a purely methodological or formal understanding of that notion of reduction were central topics in the philosophical endeavours of Edmund Husserl.
Husserl in particular emphasised a distinction between an epistemic attitude which he called 'Wesensschau' (which could be translated as 'view (or vision) of the essence') in contrast to ordinary (formal) abstraction. One of many paraphrases he used to explain what he meant by 'Wesensschau' was 'ideation'[15].
Yet then one can also see in the work of Husserl how to eventually miss the point of which really makes the difference between abstraction on the one hand and what I will call 'objectivation' on the other. In the end Husserl explained the way how his 'Wesensschau' would work as 'eidetische Reduktion (eidetic reduction)'[16]. And thus he missed the point since as any form of reduction also eidetische Reduktion inevitably comes dangerously close to what is essentially meant by the very notion of abstraction. In my view the way how Husserl explained eidetische Reduktion just shows that - despite explicit denials by Husserl - it has to be understood as just a somewhat refined and modified version of proper abstraction. The reason for that dangerously close relation of reduction and abstraction is that any abstraction simply is a kind of reduction.
It is probably most common in the form of reducing a more complex phenomenon to a more elementary underlying structure or in the form of reducing special or particular appearances to comparably more general or universal structures (or laws). These are typical examples of abstraction as carried out by the method of reduction.
Yet it should also be noted that on the other hand not any reduction must necessarily be rated as abstraction as for example in such cases when a complex system is described by a few critical parameters which are themselves just ordinary elements of the description of that very system and not elements of an underlying structure.

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[14] A comparably far less sophisticated realism had in the past been founded on the rather ridiculous assumption that science would work based on so called induction.
[15] Cf. E.Husserl, Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, Tübingen 1980 p 42/43
[16] Cf. E.Husserl, Ideen ... p 108 ff

Another serious shortcoming of Husserl's phenomenological approach then was that the intentional act which is at the core of his concept of Wesensschau was meant to be directed on something *given* to that act. By this idea the phenomenological project finally derailed and ended in the usual antinomies of epistemology. The entire presumption of something *given* to the mind as a precondition of thought then had later been impressively refuted by a famous critic of Wilfried Sellars.[17]

And now we can make the proper distinction between abstraction on the one hand and objectivation on the other which matters for our task of overcoming the fatal inconsistencies of the reductionist project which are partly caused by the epistemological preoccupation of most of its protagonists.

Abstraction is essentially a method of generalisation or of explaining a class of phenomena by reducing it to a set of essential properties which then can be explained by a theory of another class of phenomena of an underlying structure related to the original class of phenomena in question in short: abstraction works by reducing a class of phenomena to another class of phenomena the later being respectively invariant to the various manifestations and appearances of the original one. Therefore abstraction is essentially reductionist and directed on something presumed to be *given* to the act of abstraction (this later condition of course implies that the *given* in question also exists independently from that very act of abstraction).

In contrast to that *objectivation* is first of all meant to be *creative*. Yet with a certain difference to that notion of creativity as it is used in the cases of artists or inventors. Objectivation is creative but only under the very strict rules of rigorous theoretical justifiability.[18] This justifiability has to do as well with corroboration by evidence as with coherence with respect to the possible embedding of the results of objectivation in the entirety of accepted theoretical knowledge, i.e. with intertheoretical relations. In the case of the seeming paradox of reductionism or the foundation of an ultimate theory there will hardly be any evidence available for a long time which could serve to select between competing candidates. So the work of selection will probably have to be done mainly by considerations concerning coherence.

Yet notwithstanding the importance of such aspects of justification there are also the even more important aspects of what has traditionally be called 'the logic of discovery'. Since I do not believe in the common understanding of the notion of discovery as such I will rather speak of a 'logic of conceptual or theoretical creation'.[19]

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[17] Cf. W.Sellars, Empiricism and the Philosophy of Mind, in: W.Sellars, Science, Perception and Reality, London 1963, pp. 127-196; Sellars' critic of 'the myth of the given' - so stolidly uphold by the sense-data theorists of his time - culminates when he comes to the conclusion that the proponent of that myth "confuses his own creative enrichment of the framework of empirical knowledge, with an analysis of knowledge as it was. He construes as *data* the particulars and arrays of particulars which he has come to be able to observe, and believes them to be antecedent objects of knowledge which have somehow been in the framework from the beginning. It is in the very act of *taking* that he speaks of the *given*.", ibid. p 195.

In our case of the theoretical imagination or objectivation the mentioned 'creative enrichment' plays an even bigger role than in the case of ordinary perception and cognition which are the focus of Sellars' consideration in 'Empiricism and the Philosophy of Mind'.

[18] For a deeper look into what is meant here by 'objectivation' and 'justifiability' cf. D.Kurth, Actual Existence and Factual Objectivation, in: Movements, Philosophical Aspects of ANPA 23 (Proceedings of ANPA 23), Arleta D. Ford (ed.), London 2002

19 Using a rather traditional 'continental' terminology one could call this concept of creative objectivation as being about bringing something immanent transcendent into appearance. This 'something immanent transcendent' being the results of objectivation i.e. the objects of theories seen as being intelligibly yet not independently (of the acts of objectivating) existing.

Creative objectivation then is based like Husserl's Wesensschau on intentionality. But objectivation also essentially transcends intentionality, since intentionality had been thought to be directed on something *given in advance* to the respective act of intending. Yet instead objectivation also partly *creates* the intended object at first. That difference has to do with the fact that objectivation is a genuinely theoretical attitude and not a practical one. By 'creation' I do not in any sense mean something like 'invention' but rather something similar to 'bringing into appearance', with a connotation changing somewhat between 'unveiling' and 'revealing'. The best metaphor - to my knowledge - for that aspect of creativity of theoretical objectivation is the description of a sculptors work (which had been in particular related to Michelangelo) as a work of liberating the already inward statue of the veiling marble.

The seeming paradox of reductionism illustrated by J.A.Wheeler as a tower of turtles or infinite regress will only as long seem to be paradoxical as one assumes that the objects allegedly discovered by iterated reductionist abstractions are taken to be something *given* in advance or to be (intended) entities, i.e. as long as these objects are thought of to possibly exist physically.

Thus it should become obvious that this seeming paradox of reductionism isn't necessarily a paradox of reductionism as such in the first row but it certainly is a paradox of a reductionism with a certain epistemological flavour, i.e. a realist reductionism, a reductionism which pretends to uncover its objects by iterated reductionist abstraction till it finally would come to that abstractively anticipated "point of convergence of all our arrows of explanation" and then would come to an end, would come to rest, would come to terminate.

The seeming paradox then is that there is no end, no rest and no termination. And that there cannot and will not be any such thing. At least not if we interpret 'termination' generously as 'final reference'. Because that seeming paradox works only as a paradox as long as we take the objects (which become infinitely subdivided in a sense) as (intended) entities. Thus it is not the infinite regress as such which makes that restless shrinking so unsympathetic but the fact that we interpret the infinite regress as something which seems somehow comparable to a 'shrinking of size' i.e. as physical process involving physical entities.

What I really want to stress by this is that (possible) *physical existence itself* is - as temperature, colour, life and thought and many other properties of complex entities - just an *emergent property*.

I.e. an emergent property of objects which together with this property (of being possibly physically existent) emerge up from an underlying level of pre-physical objects which nevertheless should be as clearly presentable as theoretical objects always should be but which should not even possibly be intended to be entities. (Potentially ascribable) *physical existence itself should therefore be thought as being emergent* together with some certain kind of objects, namely as a new property of these particular probably rather primordial objects which themselves then would emerge from logically preceding objects which simply would lack this property (of bearing potentially ascribable physical existence).

I.e. I will try to make a clear distinction between the ordinary objects of physical theories which I would regard to be objects which are intended to be entities or at least to stand in a particular established relationship to such intended entities and - in contrast to those - another kind of objects which are not (and could not even) be intended to be entities at all.

Concerning the mentioned ordinary objects which are intended to be entities a realist would say they simply *are* entities whereas I would say they are objects the theoretical significance of which is corroborated by evidence. That is obviously an epistemological distinction which somewhat matters but it is not the kind of distinction I try to point out here specifically.[20]

What matters here is that by restricting our concept of objects to such ordinary objects which could be intended to be entities (or which could be characterised by properties which are - in the broadest possible sense - related to physical existence) one cannot - in my view - overcome the seeming paradox of reductionism.

The reason for this is that the - in my view - overwhelmingly successful project of reductionism anyway cannot come to an end, cannot come to rest and cannot terminate in any ordinary sense of these words. In particular it cannot terminate by discovering a particular kind of fundamental physical entities which may be describable as a particle or as any other kind of physical entity like for example a quantized discrete part of space, since any of such fundamental entities will only be a demarcation of our respective actual theoretical capabilities and not the unveiling of a unshakeable fundament of nature.

Now let me just make a cautionary remark: such unshakeable fundament of nature, even build of proper entities is not *logically* excluded, but it would need an additional ironclad corroboration to prove such claim of just being the latest atomon. Such a corroboration would have to be - in my view - at first a proof of rigid selfconsistency. But - and now comes the really hard nut to crack - it would also have to prove an effective exclusion of any possible alternative with comparable selfconsistency. That of course is not a very fair burden for such a proof but that lack of fairness is directly due to the fact that (physical) existence simply doesn't follow from logical or mathematical consistency as such.[21]

Therefore it is no wonder that since now we had and have a nice supply of alternative selfconsistent candidates (e.g. holistic approaches, the bootstrap program etc.) to choose between.

The alternative I will suggest is different: The reductionist project can only be successfully completed if it doesn't come to such an expected proper end, if it doesn't come to rest and if it doesn't terminate - at least not in the usual way, i.e. by presenting the latest fundamental entity. Instead it would have to show that fundamental entities might very well be fundamental entities but that there is something more fundamental than any entity, i.e. that *there is something more fundamental than physical existence and that physical existence itself emerges from that level beneath*.

Trying to be consistent with my previous terminology and my epistemological or rather anti-epistemological convictions I will call these pre-physical objects 'objects which are not intended to be entities'. The reason for that longwinded name is that I'm convinced that no theoretical object represents an extra-theoretical entity anyway. But I would agree that these comparably normal objects of theories are in many cases undoubtedly *intended* to somehow

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[20] Cf. D.Kurth, Actual Existence and Factual Objectivation, loc.cit.

[21] To make that point clear in advance I have to stress that the proposal I will make in the following and in particular in the corresponding more formal paper "The Topos of Emergence" (see the "The Scientific Aspects of ANPA 24") differs essentially from such approaches which rely primarily on selfconsistency and are opposed to the reductionist approach. Just in the opposite to the convictions of the proponents of such approaches I am convinced that one has to follow the lead shown by the reductionist program and just at the very end of it to go a step further. Simply because otherwise there never would be such 'very end'. But then I have to admit that of course my proposal can also justifiably be confronted with similar questions of selfconsistency as well as exclusivity, as it will be the case for any theoretical construct which can not effectively be corroborated by evidence anymore.

represent or even be equivalent to existing entities. The new pre-physical objects here in question however couldn't even possibly be intended to be or to represent entities.
And thus I come to the following statements or postulates:

> (physical) existence as such is emergent from and supervenient on a deeper (mê)ontological level

at which this level has to be seen not only as pre-physical but also as pre-natal, i.e. not exactly trans-cending nature but rather de-scending it. From this I tend to draw the conclusion that

> (physical) existence itself is rather an accidental property related to such an underlying level, i.e. that (physical) existence is contingency (in the same sense in which live is a contingent phenomenon in the perspective of ordinary physics).

Concerning the tower of turtles (or the seeming paradox of reductionism) an infinite regress then might turn out to be rather a part of the solution than the problem.

3 The Mêontology of primordial emergence: objects which are not intended entities

After all these epistemological preludes the question may arise how such an object which doesn't have a physical existence and which shouldn't even be intended to be an entity then should be thought of.
At first let me distinguish between some different kinds of objects which are no entities.
Undoubtedly not an entity is something pretended to be a correlate of a logical contradiction like for example a square circle but it also should better not called an object but instead rather regarded to be a misuse of language. Anything like this is of course of no relevance for our question.
Also of no interest for us are fictitious concepts which are not correlates of logical contradictions but are trivially excluded by simple theoretical premises like for example an iron cage which contains the whole universe. Such a nonsense thing would anyway rather be imagined as an pseudo-entity than as an object.
The same holds for merely fictional things or figures of literature like centaurs, Cyclopes, unicorns etc. such things are fictional entities which do not and can not exist in the context of the biological evolution on the earth but which very well could exist in accordance to physical principles. Again such things are fictitious or fictional entities but not objects in the sense we are interested in.
Of more interest than all the previous things are objects which had been postulated by definitely given up ('empirical') theories like for example the phlogiston and the ether. Now we know that nothing exists which could be regarded to be a correlate of these objects. But this doesn't mean that these objects had not been intended to be entities, they obviously had so just like so many other conceptual objects of other theories. So this is also nothing what we are looking for.
And then there is another class of objects which are neither fancy nor fictional nor given up and which - at least according to the majority of philosophers concerned with that subject - are also explicitly not intended to be possible physical existing entities. These objects are

sometimes called mathematical objects, sometimes they are also called mathematical entities, but then there is rather something meant by 'entity' what we would call an object.[22]
But on the other hand most mathematical objects like groups, algebras, topologies or sets are rather that what sober realists like Weinberg take them to be, namely just elements of 'the language in which we express our explanations'. They are deeply amalgamated with the objects which couldn't sometimes even become characterised without the means of such mathematics. But the mathematical objects themselves are nevertheless not the same as the physical objects explained with their help and the latter couldn't simply be substituted by their mathematical counterparts. Agreeing to such conventional assessment of the role of mathematics in physical theories brings us in clear opposition to a sort of neohyperpythagoreanism which had been proposed by Max Tegmark in his paper 'Is "the theory of everything" merely the ultimate ensemble theory?'[23] This paper has undeniably the merit of directly addressing the peculiar question of the relation of mathematical and physical existence and insisting that the former is somehow prior to the later. The main thesis he tries to defend is

> "Everything that exists mathematically exists physically."[24]

In the end the apparent weakness of this argument is that it doesn't really try to overcome the peculiarity of the relation of mathematical and physical existence but instead proclaims that the problem, namely that mathematical existence is seen (by some) as being prior to physical existence (and that for this reason it seems that physical existence somehow would have to come from mathematical existence) would itself already be the solution.
But there are mathematical objects which do not exist, will not exist and in no possible universe ever can exist. One of these objects (and also the one which is the most relevant for our question of how to overcome the seeming paradox of reductionism by presenting an object which underlies the objects of intended physical existence but can itself not be intended to exist) is the continuum and the Euclidean point as described by continuum mathematics.[25]
But then there is also a further insurmountable deficiency of the continuum namely that it not only doesn't exist and cannot exist but that nothing what could ever be intended to exist or to be an entity could ever *arise* from the continuum. The continuum is - related to the question of any even most indirect physical or pre-physical significance - nothing but a fruitless abstraction and not an objectivation of an at least intelligibly existing object.
Yet to overcome the seeming paradox of reductionism one needs an object of at least such an indirect pre-physical significance, i.e. an object which itself cannot be intended to be an entity but from which objects can emerge which then could be intended to be possible primordial entities. And for such objects one would have to look - so to speak - in the closest neighbourhood of the points of the continuum, since that is the neighbourhood of the least complexity attachable to such pre-geometrical as well as pre-physical objects.

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[22] Speaking of 'mathematical entities' could somehow suggest at least tacitly an inclination towards the metamathematical position of Platonism. Then this would imply something like an intelligible existence of mathematical objects. In my view such a position might be defendable even without the obligation to become a sort of Platonist or dualist.
[23] M.Tegmark, Is "the theory of everything" merely the ultimate ensemble theory?, Annals of Physics, 270, 1-51 (1998)
[24] Ibid. p 1
[25] By saying that the continuum does not exist I of course do not mean that there do not exist things which have properties which can (and even must) be described by means of continuum mathematics for example such simple things like balls or cups. What I do mean is that there doesn't even possibly exist a physical structure which is in its most fine grained resolution equivalent to the intrinsic 'structure' of the continuum, namely the structure of the Real Numbers. I.e. actual infinity doesn't exist physically.

All this is nothing new, Bernhard Riemann and William Kingdon Clifford already tried to graft an as minimal as possible enriched structure upon the continuum to enable it to bear an elementary physical layer. Even though they didn't succeed Einstein later followed up this trail and he succeeded impressively to use a dynamical version of the continuum as a platform for GR. Yet this platform was neither a fundament nor a primordial physical structure but just a necessary mathematical abstraction, a - for that time - inevitable conceptual limitation at which the creative physical objectivation of GR eventually ceased into the absurdity of physical singularities.
Today the prevalent ideas how to overcome such absurdity are no more to try to enrich the continuum but to substitute it by a mathematical structure better apt to do the job. But such an idea had also been proposed nearly two centuries before Riemann and Clifford by Leibniz - and it might turn out that his idea was more to - so to speak - the point than the modern preoccupation with the discrete.

3.1 Punctum et Conatus: dynamical Leibniz-point objects

To be a bit more precise: Leibniz came to a slightly different point. Not to the point of Euclid and not to the point of the continuum but to a point of his own, for which reason I will call that point a Leibniz-point object. Let us see what he had to say about this point of his own in his Theoria Motus Abstracti

> „5) Punctum non est, cujus pars nulla est, nec cujus partes non consideratur; sed cujus extensio nulla est, seu cujus partes sunt indistantes ..." (Theoria Motus Abstracti, Fundamenta praedemonstrabilia, in: G.W.Leibniz, Mathematische Schriften, (ed. C.J.Gerhardt), Bd.VI p 68)
>
> *„5) A point is not something which has no part or the parts of which cannot be considered but something which has no extension or the parts of which are not distant ..."*

For a point having parts equals to having an internal structure. Even if there cannot be found any reference of Leibniz to a presumption of a physical reality of infinitesimals there can also be no doubt that exactly such a presumption is behind that quoted definition. This becomes even more obvious when one takes into account the very close relation of this definition of a point with the concept of an intrinsic minimal motion called conatus

> "6) Quietis ad motum non est ratio quae puncti ad spatium, sed quae nullius ad unum.
> 7) Motus est continuus seu nullis quietulis interruptus."(Theoria Motus Abstracti, Fundamenta praedemonstrabilia, in: G.W.Leibniz, Mathematische Schriften, (ed C.J.Gerhardt), Bd.VI p 68)
>
> *"6) The ratio of rest to motion is not like the ratio of a point to the space but like the ratio of zero to one.*
> *7) Motion is of a continuous nature, i.e. not interrupted by whatever small phases of rest."*

> "10) Conatus est ad motum ut punctum ad spatium, seu ut unum ad infinitum, est enim initium finisque motus." (Theoria Motus Abstracti, Fundamenta praedemonstrabilia, in: G.W.Leibniz, Mathematische Schriften, (ed. C.J.Gerhardt), Bd.VI p 68)
>
> *"10) A conatus is compared to motion like a point is compared to space or like (the number) one is compared to the infinite, for a conatus is the beginning and the end of a motion."*

Thus it is obvious that for Leibniz that infimum spatii which is the point as defined above and the infimum motus which is the conatus have to be seen as being rigorously copresent. This then implies a further vindication of his principle of excluded rest

> "... nulla est unquam quies vera in corporibus, nec a quiete aliud nasci potest quam quies;" (G.W.Leibniz, specimen dynamicum, pars II, in: Mathematische Schriften, (Hrsg. C.J.Gerhardt), Bd.VI p 252)
>
> *"... there is never actual rest in material bodies, and nothing else can emerge from a state of rest than just merely rest [i.e. from a state of rest merely nothing can emerge, therefore it must be conceived as impossible that there ever will be or ever had been such a state of rest, D.K.]."*

After having heard these definitions and considerations of Leibniz one might get second thoughts, as for example:

a) how can anything what has no extension then have parts? And

b) if one would assume that there would be such postulated parts of a point, and if there is also no state of rest whatsoever, are then these parts of a point themselves in some state of motion?

These questions even make some sense in the context of Leibniz' own considerations especially insofar as they relate to the principle of continuity of which Leibniz is known to have been a fervent proponent. Let us have a look at the principle of continuity as it has been put by Leibniz

> „Le moindre corpuscule est actuellement subdivisé à l'infini, et contient un monde de nouvelles creatures, dont l'Univers manqueroit, si ce corpuscule étoit un Atome, c'est à dire un corps tout d'une piece sans subdivison."
>
> *"The smallest corpuscle is actual infinitely divided; and it contains a world of new creatures, of which the universe would be devoid, if this corpuscle would be an atom, i.e. an entity consisting in one entirely indivisible piece."*
>
> (Streitschriften zwischen Leibniz und Clarke, Postscript zu Leibniz' viertem Schreiben, in: G.W.Leibniz, Die Philosophischen Schriften (ed. G.J.Gerhardt), Bd.VII, p 377/378)

Now it is on the one hand obvious that a point - let alone a part of a point - should - whatever it may be - not be taken for to be the same as a smallest corpuscle but then on the other hand points (as well as the conatus) as defined by Leibniz in his *Theoria Motus Abstracti* are at the infinitesimal level to which they are confined explicitly of an at least semi-physical nature and by that it would be probably impossible to tell them apart from such infinitesimal physical objects as the smallest corpuscles mentioned in the quoted version of the principle of continuity.

All his ideas about the concept of a not Euclidean point, the conatus, the principle of excluded rest and the principle of continuity show Leibniz as a radical dynamist who tries to found physics in an intrinsically dynamical structure, which would have again be indiscernible from the mathematical structure into which physics had to fade away at the level of the infinitesimal.

> „Physica ergo ... desinit in geometriam, nec ante ullum phaenomenon penitus in corporibus intelligemus, quam ex primis figurae motusque ideis derivamus."
>
> *"Physics namely ... fades away into geometry, and we won't know any phenomenon deeply in its corporeal construction as long as we haven't derived it from the first principles of geometrical figures and of motion."*
>
> (G.W.Leibniz, Dissertatio exoterica de statu praesenti et incrementis novissimis deque usu geometriae, in: G.W.Leibniz, Mathematische Schriften, (ed. C.J.Gerhardt), Bd.VII p 325)

Before we will have a look at if and how one might make some use of the hints we got from Leibniz for the purpose of putting the tower of turtles on a profound basis I would like to point to a puzzling difficulty concerning Leibniz' philosophy. In the quotations above Leibniz appears to be an uncompromising champion of the continuum principle as well as

of a radical dynamism. That seems to stand in a striking contrast if not contradiction to his stand for relationalism for which he is also famous. An example of this relationalism is the definition of space he gave directed against Newton's definition of space as being an absolute sensorium dei.

> „Spatium est ordo rerum quae sunt simul."
>
> „Et hoc ... modo spatium fit ordo coexistentium phaenomenorum, ut tempus successivorum;" (Brief von Leibniz an des Bosses vom 16.06.1712, in: G.W.Leibniz, Die Philosophischen Schriften (ed. G.J.Gerhardt), Bd.II, p 450)
>
> *„In this way ... space becomes the order of the coexisting things as time becomes the order of the successively existing things"*

I wondered for a long time why that seeming contradiction didn't draw more attention to it because it obviously affects the very fundaments of Leibniz philosophy. Yet maybe it will turn out that just such a peculiar amalgamation of dynamism and relationalism might open up the way to putting the tower of turtles on a more profound basis by using modern mathematical means but still inspired by Leibniz' struggle for understanding of how the realm of mathematical objects had been in infinitely close contact with a world of emerging physical entities. This will be the topic of a related paper about 'The Topos of Emergence'.[26]
Here I just will summarise what might be called Leibniz' postulates for mathematical objects which could perhaps turn out to be a matrix for an emerging primordial physical layer. Such mathematical objects should have
a) an intrinsical structure (by Leibniz referred to as 'having parts'), but nevertheless it should
b) strictly be confined to an infinitesimal level (by Leibniz referred to as 'having no extension'). But then it also should be co-present with
c) some kind of an infinitesimal motion (by Leibniz called 'conatus'). This motion obviously cannot be thought as very small physical motion, because there cannot be a proper physical motion without extension. But there exist mathematical analogues of motion like morphisms, transformations and mappings which perhaps could serve as modern incarnations of 'conatus'. The mathematical structure in question should
d) also be able to be consistent with a non-vicious infinite regress (by Leibniz implied by the principle of continuity). And notwithstanding all the dynamism and continuity it should
e) also be apt to bear a relational structure as well.

A mathematical structure which would as well satisfy these conditions as also minutely connect to the most elementary physical level of the reductionist program then might be fit for providing a more profound basis for the tower of turtles. Such a mathematical structure then will - in my view - not be something obvious, it will not follow from such shallow wisdom as 'all physical existence is genuinely mathematical existence' or similar insights. And I'm also convinced that just these mathematical means which are used as *applied mathematics* in the respective areas of high energy physics are *not* the candidates for a mathematical structure from which a primordial physical structure first of all emerged.
But who has ever said that building a fundament for the tower of turtles would make no work?[27]

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[26] Cf. D.Kurth, The Topos of Emergence, in: The Scientific Aspects of ANPA 24, Proceedings of ANPA 24
[27] A sketch of how I think that work should be started one can find in (as you probably already guessed): D.Kurth, The Topos of Emergence, loc.cit.