ON THE TRIPLET FRAME FOR CONCEPT ANALYSIS

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ABSTRACT: The paper has two objectives: to introduce the fundamentals of a triplet model of a concept, and to show that the main concept models may be structurally treated as its partial cases. The triplet model considers a concept as a mental representation and characterizes it from three interrelated perspectives. The first deals with objects (and their attributes of various orders) subsumed under a concept. The second focuses on representing structures that depict objects and their attributes in some intelligent system. The third concentrates on the ways of establishing correspondences between objects with their attributes and appropriate representing structures.

Keywords: concepts, models of concepts, triplet description of concept models.

CONTENTS

1. Introduction

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[1] The draft. The final version, see: THEORIA - Segunda Época Vol. 14/1, 1999, 39-62.

For the sake of simplicity, we consider only models of so-called object concepts like *HORSE*, *STAR* and *ROBIN*. However, with minor reservations our approach may be applied to "abstract" concepts like attribute concepts (*RED*, *BLACK*, *SIZE*) and relational concepts (*LOVE*, *FORCE*, *DISTANCE*).

Concepts have been the objects of much concentrated attention in cognitive science and cognitive psychology (Barsalou and Medin 1986, Cohen and Murphy 1984, Komatsu 1992, Smith 1988, Smith and Medin 1981). It was found that concepts are not formless and simple entities and cannot be equated with the names, lists of attributes, meanings, or simple units of information. To understand concepts scientists have been elaborating models of concepts. Their common feature is that they describe different collections of empirical data about the usage of ordinary concepts in recognition and classification of objects.

It seems that a particular concept model is suitable for explaining outcomes of a certain empirical study of a definite kind of concept. From this point of view, each model has some empirical confirmation. However, changing either the empirical condition or conceptual kind has, as a rule, led to building new concept models. This strategy should be supplemented by revealing certain hypothetical structure that may serve as a basis for the unified treatment of concept models. This theoretical work grounds on the following circumstance. All concept models assume that in order to specify a concept we should specify attributes (properties, relations, functions) of objects subsumed under a concept. The models proposed differ in the assumptions about the nature and descriptions of these constitutive specifying attributes.

In any case, every model associates with concepts certain structures defined on constitutive attributes. We interpret these structures as substructures of some hypothetical and general underlying structure of concepts. With regard to its maturity, nature, and context of application, each kind of concept reveals or actualizes specific substructures of this underlying structure. We do not pretend that this underlying structure exists in mental or other reality. It is only a hypothetical construct effective under the unified treatment of available concept models.

The first objective of the paper is to introduce the general underlying structure of concepts and to describe semi-formally its substructures. We use the name "triplet" for calling this structure and the corresponding concept model. The second objective is to show which substructures of the triplet structure are being actualized in several concept models in the current cognitive psychology.

2. The Extended Notion of a Representation

Many cognitive scientists and psychologists have treated a concept as a mental representation. In turn, the notion of a representation is taken from the formal mathematical theories of representation (Krantz, Luce, Suppes, and Tversky 1971). According to this notion, we should distinguish the domain being represented, D, from a representational model, R of D. Often, D is a part of the actual world, and R is some scientific theory about D. ... R *represents* D, if every object in D has a corresponding name in R, and every relation in D has a corresponding relation symbol in R. Normally, R uses numeric relations (greater than, equal to, etc.), whereas D's relations are based on actual properties of the objects (mass, velocity, intelligence) (Cohen and Murphy 1984, p. 31).

This notion fits well with the context of measurement when the main task is to assign definite numeric values to attributes of objects. However, it appears that the notion of a representation associated with the expression "mental representation" is more complicated. Indeed, the mental apparatus in which a mental representation takes place operates not only with names of objects and symbols of relations, but also with other representing structures. Their examples are images, descriptions (in particular, definitions) and mental models of objects and their attributes. By the way, numbers and numerical structures are the simplest mathematical structures used in science for the reality representation. Additionally, the information about the domain D concerns both objects from D and their attributes. Thus, we need a more realistic notion of a representation. Without going into details, we will keep its following version.

Under this version, first, any representation takes place in certain conditions K (it is built by a particular subject and by specific tools; it fulfills a definite task, etc.) Second, in any case a representation does not represent completely the domain. It makes this only partially with some degree of precision. Third, to study the domain we usually split it into objects. This splitting (partition) is not an absolute, but a relative procedure in respect to both objects and their attributes. Depending on conditions, available and accessible knowledge, we separate in D different sets of objects and various collections of their attributes. Fourth, there are many representing structures. The most important of them are informal and formal models of attributes and finally models of objects as some bundles of attributes. Fifth, one consequence of allowing for many representing structures (not only names) is introducing relationships between D and representing structures other than naming relations. Examples are modeling relations between entities being modeled and their models.

From this it follows, that the extended notion of a representation $\mathfrak{R}$ may be described as the triple $(D(D_K, P_K(D_K), E_K(D_K)), r_K, \quad Rep_K^T(N, M))$. Here K is conditions in which a representation takes place. $D(D_K, P_K(D_K), E_K(D_K))$ symbolizes the partition of D in the set of objects D_K with the collection $P_K(D_K)$ of properties and the ensemble $E_K(D_K)$ of relations among objects. $Rep_K^T(N, M)$ symbolizes the system of names N and models M that is built and analyzed by means of certain cognitive system (language, knowledge system, theory) T. The correspondence r_K is such that (a) every object d from D_K, its every property from $P_K(D_K)$ and every relation from $E_K(D_K)$ have specific names from N; (b) some objects and some of their attributes have their appropriate models from M that contain and organize the certain relevant information about them. The models are constructed and analyzed by means of T.

It should be noted that in analyzing the extended notion of a representation, we deal directly not with its components (objects, representing structures and correspondence between them), but with the information about them. It may have different degrees of completeness, formality, precision, justification, certainty, confidence, etc.

3. The Fundamentals of the Triplet Modeling of Concepts

Considering a concept as a special mental representation, we characterize it from three interrelated perspectives. The first one deals with objects (and their attributes of various orders) subsumed under a concept. The second one operates with structures that represent objects and their attributes in some intelligent system. The third perspective deals with the ways of establishing correspondences between objects (and their attributes) and their representing structures.

Thus, we have the three structured and differently ordered kinds of information associated with these perspectives. The first information is organized around the ontological hypotheses about the concept domain (concept extension), i.e., on what a concept is about. The second information is organized according to the rules, resources and history of representative and communicative systems, primarily natural and sentative and communicative systems, primarily natural and artificial languages and knowledge systems. The third information has centered on the relationships between the concept domain and the available representative and communicative systems. These relationships are not simple one-to-one correspondences between the elements of the former and structures of the latter. Many peculiar operations and processes have contributed to generating these correspondences.

According to the triplet approach, the hypothetical triplet structure and its substructures connected with these kinds of information are modeled by means of sets of different kinds, set scales, and abstract properties. Using these constructions, the triplet model has uniformly described most of the various concept characteristics introduced by other models.

4. Ontological Structuring and Its Explication

Ontologically, we assume that the concept domain consists of objects and their attributes. We also take for granted that an object and its attributes determine each other. How these assumptions may be expressed on the level of modeling objects and their attributes? We will consider names (in a broad sense not only "ordinary" names may function as names, but also descriptions and definitions) of objects and their attributes as their "primary" models.

In a first approximation, the "secondary" models of objects and attributes may be described as follows. Let us begin from secondary models of attributes. We suppose that these models contain necessarily names of objects that may possess the attribute in question. Additional requirements to the secondary models of attributes will be given below. If we have the secondary models of object attributes, then the secondary models of an object itself may be considered as a specific composition of secondary models of their attributes.

From this it follows, that we may treat a model of the concept domain as an interrelated system of primary and secondary models of objects and their attributes. Is it possible to describe this model in the uniform and manageable way?

Usually, the treatment of a concept starts from the naming and description of objects subsumed under it. These objects may have real, ideal, or mental nature. However, naming is only the first step in the object study. Since the origin of modern science the leading strategy of investigating objects is the finding and describing of their attributes and also the establishing and describing of relationships between them. This means that the reasonable description of any object should contain information about, at least several, its attributes of the first order. Occasionally, this description contains also information about attributes of higher orders like attributes of attributes of objects, attributes of relations between objects, relations between attributes of objects, etc. Again, this information is expressed by means of primary and secondary models of these entities.

Traditionally, in concept studies objects, their sets and attributes of various orders have been taken intuitively and informally without corresponding them to some precise and easy analyzable models. One possible way of explicating this situation is to use the set-theoretical constructions of abstract property (Burgin 1985; Burgin and Kuznetsov 1993) and some generalization of Bourbaki's set scale (Bourbaki 1968; Burgin and Kuznetsov 1986).

Previously we denoted the set of objects subsumed (in the conditions K) under a concept as D_K. Let us take also into account that (i) an attribute is an attribute of some object(s); (ii) an attribute, as a rule, has some set of its values and (iii) there are usually some procedures of assigning a specific value of an attribute to a particular object. This minimal list of constraints on the notion of an attribute may be taken as the foundation for constructing its model as an abstract property.

An abstract property $P(D_K)$ as a model of an attribute of objects D_K is a triple (D_K, p, Sc). Here D_K is a set of objects c that may possess the attribute in question. A scale Sc is a set of possible values of an attribute. A partial function p symbolizes a procedure(s) of assigning the definite attribute value(s) (the element(s) $p(c) = sc \in Sc$) to the object $c \in D_K$.

The construction of an abstract property may also function for modeling relations between objects from D_K. Here we take instead of D_K the direct product $D_K \otimes D_K$ and speak of so-called abstract relational two-placed property $P^2(D_K \otimes D_K)$ of D_K. In such a manner, it is possible to model as abstract properties attributes of any order and arity connected with D_K.

For example, the relation between physical bodies called "*distance*" may be modeled as a two-placed abstract property in the following manner. Here $D_K \otimes D_K$ is the direct product of the set of all physical bodies by itself, Sc is the some numerical 3-dimensional coordinate system, and *p* is realized by some procedures for determining the values of *distance* for any pair of bodies from D_K.

The construction of an abstract property allows us to distinguish systematically between the attribute and its values. Informally, all objects possessing an attribute are identical in respect to it. However, we usually differentiate these objects by values of this attribute. Sometimes values of an attribute may have structure of another attribute. We will not touch further this point here. Moreover, this construction allows us to distinguish between various orders attributes of objects under consideration. Often we need not only attributes of the first order, but also the attributes of the former attributes. For example, the attribute *weight* has such an attribute of the second order as *to be positive*.

Let us consider the construction of the set scale.

We build the set scale *S*(*X*) by the stepwise application of operations of set union, set product and constructing the power-set to its basis *X*. The basis is a collection of sets X_1, X_2,..., X_n. We obtain the definite level of the set scale on each step. Any such level consists of some sets. The set scale *S*(*X*) is the union of all its levels.

We use levels of a set scale for distinguishing attributes of various orders. For instance, the abstract property *distance* may be associated with one level of the set scale $S(X = \{D_K \otimes D_K, Sc\})$. The *positiveness of distance* is associated with other level of this scale.

The collection of object attributes has a systemic nature in the following sense. Changing the attributes of the n-th order (for example, introducing new scales), we change also and the attributes of higher than n-th orders.

Let us take for example such a property of material bodies as *weight*. We may use the set {*light*, *middle*, *heavy*} as its scale and consider such a relation between bodies as *to be heavier than.* Evidently, we may take as the scale for this relation the set {(*middle*, *light*), (*heavy*, *middle*), (*heavy*, *light*)}. This means that the relation *to be heavier than* is hold between all *middle bodies* and all *light bodies*, etc. However, taking the set of all rational numbers as the scale for the property *weight*, we obtain another scale for the relation in question and therefore, in a sense, change it.

Thus, speaking about an object *c*, we consider also its attributes, at least that which we supposed to be relevant in a concrete situation. It means that we should associate with the concept *C* of objects {...,*c*,...} the knowledge not only about *c*, but also about *c*'s attributes.

To avoid undesirable associations from here on we shall use, if necessary, capital bold symbols, letters, words, word combinations for denoting concepts. Instances are *C*, *ROBIN* and *SMALL BIRD*. We will name objects falling under a concept as *c*, *robin* and *small bird*. Correspondingly, the names of a concept might be *"C"*, *"ROBIN"*, and *"SMALL BIRD"*. The names of the objects subsumed under a concept might be *"c"*, *"robin"* and *"small bird"*. Generally, as names or terms of a concept may function not only lexically simple names, but also compound names, sentences, and even texts.

A concept *C* has, as a rule, many names of the kind $N(C)$. The same is true for the objects falling under a concept. The names differ in their exactness, effectiveness, simplicity and other characteristics. There are many relationships between the various names of the "same" concept and between the various names of the "same" object falling under a concept.

Moreover, in many contexts we do not systematically and explicitly differentiate between an object and its names. We frequently identify such an "entity" as a concept not only with some of its names, but also with some names of objects falling under it. In our notation it means that, metaphorically, *C* = *"C"* = *"c"* = *c*.

A given concept *C* informs us only about specific elements or subsets of the universe of discourse *U*. Any such informing takes place in some conditions K. Aside from describing these conditions in details, we mention only that they have been associated with the individual's mental and interpretative abilities, skills and tools, available knowledge, purposes, and even psychic state.

5. The Base of a Concept

Bearing previously mentioned distinctions and conventions in mind, we introduce the following definition.

Definition 1. Under the conditions K the potential ground set $G_K(C) \subseteq U$ of the concept *C* is the set of all elements $g \in U$ reasonably denoted by the name $N_K(C)$ of the concept *C*.

We usually say that element $g \in G_K(C)$ falls or *subsumes* under the concept *C*.

Under the traditional logical treatment, the terms "extension" and "volume" have been frequently used for labeling the ground set of a concept. The term "category" is in use in cognitive science and psychology. Here we also call elements from $G_K(C)$ "instances" or "exemplars" of a concept.

Associating the ground set with a concept is only a first step in its triplet modeling. On the one hand, the specifying of the concept C presupposes also the possibility of indicating and describing, at least, qualitatively some constitutive attributes of elements from $G_K(C)$. This means that the information about such attributes is an important characterization of a concept. As a rule, the set of these constitutive attributes is called "intension" or "content" of a concept. On the other hand, not every object labeled by the name $N_K(C)$ relates to the concept C. In particular, we may apply an inappropriate name. An object should possess some specific attributes to be counted as an instance of the particular concept.

Moreover, there are experimental and theoretical findings connected with so-called explanation-, or knowledge-, based approach to concepts. These findings have suggested that for specifying a particular concept C we need the information not only about the corresponding ground set and its members, but also about relations of these members to the members of a set $O(C)$ of other relevant objects. The set $O(C)$ is specific for the concept C.

Thus, modeling a concept, we should depict in model terms three kinds of information. The first one is on the concept ground set. The second one is on some properties and attributes of elements and subsets from the ground set. The third one is on certain relations of members from $G(C)$ to members of $O(C)$. One way to do this is to use the construction of a set scale $S(X)$.

Under the triplet concept modeling, the basis X of the corresponding concept set scale necessarily includes the ground set G ($G = X_1$). To allow for the findings connected with the explanation-based view we include also the set $O(C)$ in the basis X of a concept. It should be noted that the other contemporary views of concepts specify a concept without reference to the set $O(C)$.

We may express any attribute associated with the ground set $G(C)$ and the set $O(C)$ by resources of the set scale with the appropriate basis. For this purpose the basis X should include auxiliary sets that are scales of attributes of elements from G and O. Examples of auxiliary sets are qualitative scales, real numbers, vector spaces, truth values, etc.

Definition 2. The potential base $B^*_{\mathrm{K}}(C)$ (in relation to the conditions K) of the concept C is the collection of elements of $G_{\mathrm{K}}(C)$ and their attributes that are necessary for the usage of C in conditions K.

Structures from finite number of levels of the set scale $S(G^*)$ represent these attributes. Here the basis G^* is equal to $\{G_{\mathrm{K}}, X_2 = O, X_3,..., X_n\}$, where $X_3,..., X_n$ are auxiliary sets.

Eventually, we have centered the information about the concept base around the general ontological structuring (objects-attributes) of the reality under study. The hypotheses about specific nature of concrete objects and their attributes concretize this structuring. Informally, the base includes the information about the world knowledge of a subject, that is his or her knowledge about the attributes and interaction of objects.

In other words, as viewed from a base, a concept is modeled ontologically not only by all instances that exemplify it, but also by (relevant in the conditions K) attributes of its instances.

The available concept views vary also in constraints on attributes associated with the concept potential base. To allow for these constraints we introduce the notion of a real concept base or simply a concept base. For example, a particular attribute may belong to all concept instances or only to some subset of instances.

Definition 3. The (real) base $B_{\mathrm{K}}(C)$ is generated by imposing specific constraints on the potential base $B^*_{\mathrm{K}}(C)$.

For simplicity's sake, in what follows we will consider constraints as a part of the conditions K.

6. The Representing Part of a Concept

Apparently, components from the concept ground set and their attributes do not themselves bear their names, descriptions, definitions, statements about them, etc. Such structures are human creations. Thus, any realistic concept model should allow for this fundamental fact. Without the loss of generality, we may speak of only about linguistic structures. Here we understand language in a very broad sense. The second triplet characteristic of a concept -its representing part- deals with these linguistic structures.

Let us assume that we use some language L with the alphabet A, the vocabulary V, the set P of word combinations, the set E of expressions (sentences), and the set T of texts. The language L may include sublanguages (sign, pictorial, natural, artificial, common, scientific, mathematical, etc.). The basis L^* of set scale $S(L^*)$ of language L is $\{A, V, P, E, T\}$. The set scale $S(L^*)$ contains everything expressible in the language L.

Definition 4. The representing part $R_K(C) \subseteq S(L^*)$ of the concept C is a set of linguistic structures by which the base $B_K(C)$ of a concept C is depicted (mapped, represented) under conditions K in some intelligent system.

For example, the representing part of pre-scientific concept *PLANET* contains some descriptions of images of huge pieces of matter moving round a star. The representing part of its scientific counterpart includes material points of classical mechanics and various theoretical models of *planets*.

Structures from the concept representing part differ in their representative and expressive capacity. Some of them only denote the ground set as a whole or its selected subsets or its individual elements. Other structures designate attributes from the base. The third group of structures provides the relatively complete and/or exact description of elements from the ground set or even their attributes. The fourth group models attributes in question.

In this paper, we consider only structural aspects of the concept representing part. It should be mentioned in this connection that any "formal" structure may be interpreted in various ways. For example, T is possible to interpret as a description, an explanation, a definition, a theory.

7. The Linkage of a Concept

Various (conscious and unconscious, mental and physical) processes and operations associate the base components with the appropriate representing structures. In this sense, such associations are outcomes of human activity. They depend on developmental levels of civilization, culture, language, science, the person's knowledge, maturity, purposes and mental capacities. These associations are conditional and ephemeral, but necessary for building (forming) and functioning concepts. Thus, we should provide a careful characterization of links between base components and representing structures.

Let we point out only some aspects of these links. There are many ways to establish them: by custom, by training, by language acquisition, by convention, by analogy, by procedure, etc. The almost commonly accepted approach treats these links as simple naming relations. Representing structures play the role of names and base components play the role of entities named by the appropriate former structures. However, these links are not all reducible to naming relations that assign the names to the entities. For example, if the representing part contains mathematical model of an attribute, then this model usually not only names the attribute, but also conveys the knowledge about its values.

We may separate various kinds of links under consideration. Among them are reference links (naming, denoting, describing, visualizing, imaging), truth and modeling links.

Definition 5. The linkage $Lin_K(C)$ of a concept C is a system of links between the base components from $B_K(C)$ and the representing structures from $R_K(C)$.

It should be mentioned that enormously complex (sensual, perceptual, mental, scientific, etc.) activity generates this linkage for any concept. For example, the linkage of the common concept *ANIMAL* has been partially established by sensual perception (for *animals* that really were observed by a subject), pictures, photos (for "rare" *animals*). For the synonymous scientific concept we construct such a linkage in the framework of the available scientific knowledge, observational and measurement data.

For many scientific concepts, we may "control" some constituents of the linkage. In particular, with the help of measurement and calculation procedures scientists attach quite specific linguistic and mathematical (numeric, vector, etc.) values to some attributes of concept instances. The concept linkage is changing with the changes in scientific equipment, methods of its use, and available scientific theories.

8. The Triplet Model of a Concept

From stated above we obtain

Definition 6. Under conditions K the triplet model $Tr_K(C)$ of the concept C is the triple ($B_K(C)$, $Lin_K(C)$, $R_K(C)$), where $B_K(C)$ is a base of C, $R_K(C)$ is a representing part of C, and $Lin_K(C)$ is a linkage between $B_K(C)$ and $R_K(C)$.

Informally, to characterize a concept C we need to describe, at least, three kinds of information. The first one is the information about $B_K(C)$, that is about classes or sets of "naked" objects ("what C represents") and also about their attributes relevant in the conditions K. The second kind is the information about $R_K(C)$, that is about structures of representation of $B_K(C)$ in some intelligent system. The third kind is the information about $Lin_K(C)$, i.e. about the ways, operations and procedures of matching (corresponding, juxtaposing) components from $B_K(C)$ and structures from $R_K(C)$.

It should be noted that the triplet model is an abstract model of general structure of a concept. To apply this model to a particular concept we should concretize or, metaphorically speaking, to fill its formal structures with a content. Only few triplet substructures characterize any particular concept in the conditions K. We would like also to stress that the specific treatment of a concept depends not only on a concept itself, but also on subject's approach to it.

Let us mention briefly the problem of concept identity. If we have two particular concepts each of which is characterized by the same triplet structure *Str*, then we say that they are *Str*-identical. The concepts may have a specific list $List = \{...,Str_m,...\}$ of the common structures Str_m. In such a case, we speak of *List*-identical concepts.

Introducing different types of concept identity allows us to be more conscious about the situation of concept identity. For example, concepts may have the same ground set (G or the extension), that is to be G-identical. However, from this does not follow that G-identical concepts are *Str*-identical relative to an arbitrary triplet structure *Str*. Thus, the triplet analysis avoids the well-known problem of identifying the concept with its characteristics (extension, names, attributes and so forth). The concepts may have the same triplet structure, but differ due to other non-identical triplet structures.

From the point of view on concepts as special carriers of specifically ordered and organized information, it means that in various conditions we may associate context-relative knowledge with concepts. Practically, in a particular situation of concept usage we need only the limited amount of specific and situation relative conceptual information.

9. Structures Associated with a Concept in the Triplet Model

In the subsequent discussion we will consider some main current models of a concept and concept structures introduced by them. We will show that all these structures have been also separated in the triplet model.

We may speak about several principles of the triplet analysis of concepts:

- the principle of limitation of the subject's knowledge, according to which a subject associates with a concept only limited number of triplet structures;
- the principle of relevant choice, according to which in the conditions K only specific triplet structures become to be actualized;
- the principle of graded deepness, according to which there are various degrees of completeness and precision in representing chosen structures.

From here on we limit ourselves only to the demonstrations that structures introduced by other models are among structures introduced by the triplet model. It means that our main concern will be structural aspects of concept modeling. In this paper, we will leave aside the consideration of pragmatic, descriptive, psychological and other concept aspects.

In the Table 1, we list some triplet structures of two concrete concepts.

The name of a triplet structure	Examples for the concept	
	ROBIN	*PLANET*
Structures of the concept ground set		
The ground set	The set of all *robins*	The set of all *planets*
The cardinality of the ground set	Indefinite finite number	
A subset of the ground set	The set of all *garden variety robins*	The set of all *planets of the Solar system*
The cardinality of a subset of the ground set	Indefinite finite number	9

An element of the ground set	A concrete *robin*	*Mars*
Components of the concept base		
Attributive properties of all elements from the ground set=necessary properties	*wing-bearing biped* (*two-legged*) *small size specific shape*	*mass size visual color magnetic field*
Attributive properties of subsets of the ground set=non-necessary properties	*specific features of all garden variety robins*	*to have satellite(s)*
Relational properties of all elements from the ground set	*sexual reproduction of robins*	*mutual distance between planets relative velocity of planets*
Relations among properties or interproperty relations	*correlations among properties*	*Kepler's laws*
Relational properties of attributive properties of all elements from the ground set	*similarity in shape of robins*	*density of planets*
Relational properties of subsets of properties of some elements from the ground set	*family resemblance between sets of properties of different subspecies of robins*	*family resemblance between different subgroups of planets*
Attributive properties of relational properties of all elements from the ground set	*necessity of sexual reproduction of robins*	*positiveness of the mutual distance between planets*
Combinations (sets) of properties sufficient for recognition (in conditions *K)* of some objects as instances of a concept (usually, these properties are called sufficient)	*birdhood (birdiness) smallness brown back and wings red back*	*relative to immovable stars and regular moving enough big mass revolving round a star*
Values of a property	*height of robins in centimeters*	*mass of planets in tons*
Concept representing structures		
Simple and complex names of base components	*"robin", "garden variety robin", "small bird", "color", "size"*	*"planet", "Mars", "small planet", "mass"*

Informal and formal models of base components	The model of a property *P* as (i) a linguistic predicate ("x is small") (ii) a logical predicate ($P(x) \rightarrow \{0,1\}$) (iii) an abstract property ($P(x) = (D, p, S)$, where D is a set of names of objects which may have the property in question, S is a scale of the property, and p is a partial function ascribing the value(s) of the property to an object) (iv) a mathematical function $P(x) = f(x)$, where f(x) is real twice differentiable function) (v) a composition of functions ($P(x) = f(x)g(x)$, where f(x) and g(x) are some mathematical functions) (vi)	
Definitions and descriptions of base components	"a common small European bird with a brown back and wings and a red front"	"a large body in space that moves round a star, esp. round the sun"
The linkage as a collection of constituents connecting base components and representing structures		
Naming	Ascribing the name a "*robin*" to the subset of birds through language acquisition and socialization	Ascribing the name a "*planet*" to the subset of celestial bodies through acquisition of astronomical knowledge and observation procedures
Modeling	A *robin* as a bird with specific biological and behavioral features and habits	A *planet* as a material point of classical mechanics
Determination of values of structures from the base	Visual evaluation of size of *robins*	Measurement and calculation procedures of determination of mass value of *planets*

Table 1. Certain triplet structures of concrete concepts

10. The triplet components of the main concept models in cognitive psychology

In what follows, we will shortly characterize several main concept views and concept structures introduced by them. Our objective is not to analyze the empirical validity of the proposed concept models or their theoretical consistency. We are going only to "extract" structures that those models associate with concepts and to show that these structures are included also in the triplet model.

We do not pretend to be too formal and limit ourselves only to an informal set-theoretical characterization of conceptual structures. Here we will not consider the quantitative aspects of the triplet model connected, for example, with some numerical characterization of property values, the so called weighted attributes, prediction of reaction times in sentence verification tasks, etc.

According to the experts (Smith and Medin; Komatsu) in current psychology there are five views on concepts: the classical, the family resemblance, the exemplar, the schema and the explanation-based. They vary in the nature and kind of information associated with a concept. As a rule, it is information about which constitutive attributes should possess objects to be counted as instances of a concept in question.

To illustrate the concept views, we will take again the concepts *ROBIN* and *PLANET*.

According to the classical view, these concepts represent (or consist of) information about the necessary and sufficient attributes (properties and relations) of, respectively, *robins* and *planets*. Examples of necessary attributes are *wing-bearing* for "normal" *robins* and *mass* for *planets*.
According to the family resemblance view, the concepts *ROBIN* and *PLANET* are pieces of summary information about what *robins* and *planets*, on average, are like. Examples of such pieces of information (representations) may be found in dictionaries, e.g., "a common small European bird with a brown back and wings and a red front" (see Longman 1992, p. 904). This summary information is nothing but an informal description of a certain combination of attributes that any instance of a concept should possess. Frequently, scientific concepts include also descriptions of some relations between concept instances and other objects in the world. For example, such an informal description of a *planet* as "a large body in space that moves round a star" (see Longman 1992, p. 785) contains the reference to *space* and *motion* round a star.

According to the exemplar view, the concept *ROBIN* (*PLANET*) consists of the information of past exemplars of *robins* (*planets*) that a person has experienced. It is possible to treat this information as a collection of pieces of information each of that corresponds to a particular *robin* (*planet*). In terms of attributes, the distinction between the family and exemplar views is as follows. In the former case, the description is built by means of the notion of a attribute that belongs to a set of objects. In the latter case, the description is built only by means of the notion of a attribute value. In other words, the particular exemplar information is the description of a combination of property values that a particular object possesses.

According to the schemata view, the concept *ROBIN* (*PLANET*) consists both of information about *robin* (*planet*) exemplars and of information about what *robins* (*planets*), on average, are like. Evidently, here a concept includes both family resemblance and exemplar information. Each concept instance is characterized by its specific combination of attributes and combination of its appropriate attributes values. The former combination is shared by all concept instances. The latter is supposed to be unique for a particular instance.

Finally, according to the explanation-based view, the concept *ROBIN* (*PLANET*) includes two kinds of information. The first one is the information about the relationships that hold among different attributes of *robins* (*planets*). The second one is the information about how *robins* (*planets*) interact with the other (relevant) objects in the world. As a rule, these kinds of information are not perceptually given, but are outcomes of processing some knowledge systems.

This informal description of the main concept views has shown that they constructed mainly for the concepts of everyday objects taken in the situations of their recognition and classification.

After this general characterization of concept views, let us single out structures they assign to concepts.

The Classical View

Figuratively speaking, this view states that a concept is defined by (information of) individually necessary and collectively sufficient attributes of its instances. If we use the triplet model, then it means that this view associates with a concept a certain set of the first order attributes of concept instances and also some combination of the second order attributes of instances (namely, attributes like the individual necessity and collective sufficiency of instance properties). From the stated above, it follows that all these structures are among structures of the concept triplet base. These structures appear under the appropriate choice of auxiliary sets and levels of the concept base.

An attribute may be an ordinary (sharp) as well as fuzzy attribute (cf. Burgin and Kuznetsov 1993). The latter case opens a way for considering the fuzzy membership of objects in the ground set of a concept.

Next, researchers usually suppose that the information about classical view attributes of concept instances has the form of definition. It means that this view uses such a triplet structure as a definition from the representing part of a concept. It seems also that this view also uses names of objects and their attributes.

The classical view says nothing about the concept linkage. The same is true for other concept models.

The Family Resemblance View

The notion of "family resemblance" is rather vague. It is possible to give various interpretations. Generally, the family resemblance view rejects the idea that all instances of a particular concept possess the same set of individually necessary and collectively sufficient attributes. Instead, this view states that the extension (the ground set or the category) of a concept is constituted due to so-called family resemblance relationship. It may be described as follows.

Let us suppose that G is decomposed in subsets $G_1, \ldots, G_q$. Members of each subset G_j have a collection P_j of attributes $\{P^1(G_j), \ldots, P^m(G_j)\}$, and none of the attributes is common to all members of G. We say that there is family resemblance among these subsets if the intersection of collections of attributes for any two subsets is non-void. Members of the set G_{pro} come to be viewed as prototypical of G in proportion to the extent to which they share some common (specific for any other subset) attributes with all other subsets.

From this description of the family resemblance view, we may conclude that all its structures have triplet analogues. Indeed, in this case we should assume that G is decomposed in subsets with members characterized by a specific collection of attributes. The relations between these collections are constrained by the demand of family resemblance. This may be done by an appropriate choice of decomposition of G and relations among its subsets.

Concerning the representing structures using by the family resemblance view, we mention that this view prefers not definitions, but descriptions of attributes of subsets of *G* and relationships among them.

The Exemplar View

Both the classical and family resemblance views start from the notion of an attribute. Here, it is usually supposed that an attribute belongs to more than one object. However, these views vary in the ways of the distribution of attributes constitutive for a concept among objects from the concept extension. In the former case, a particular attribute characterizes every instance, while in the latter case it characterizes typically only some instances, that is a certain subset of *G*. The exemplar view speaks about the so-called unique attributes which are attributes that are unique to specific instances of a concept. Attributes that are constitutive for a particular concept need not hold true for more than one instance.

It seems that using the model of an attribute as an abstract property, we may explicate the situation as follows. Let us remind that according to this model, an attribute is characterized through three components (D_K, *p*, *Sc*). From the point of view of this model, the above considered concept views have used in a sense different reduced notions of an attribute. The classical and family resemblance views have emphasized on the first component stressing that an attribute is shared among members of some non-void set. Other two attribute components are beyond of using by these views. The exemplar view accented the possibility of unique individualization of a concept instance by means of assigning the unique attribute value (or unique combinations of values of various attribute) to it. However, this view refuses to consider an attribute as a whole entity consisting of three components. In the light of this, the exemplar view associates with a concept only information about attribute values and leaves aside information about the set D_K of objects that may possess this attribute. In contrast to this, the first two concept views associate with a concept only information about the set D_K and leave aside information about attribute values (from *Sc*) by means of which it is possible to individualize different instances. These views do not care about the procedures (*p*) of assigning specific attribute values to particular concept instances.

The Schemata View

At first glance (Komatsu 1992, p. 510), the schemata may be treated as a single structure that captures characteristics of both the family resemblance view (by storing information that is abstracted across sets of instances from *G*) and the exemplar view (by retaining unique information about actual instances). It means that the schemata information about a concept combines the family resemblance information and exemplar information. Above we characterize the family information as information about the set D_K, i.e. only about the first component of attribute. We describe also the exemplar information as information about the members of the scale *Sc,* i.e. only about the third component of an attribute. Thus, the schemata information is information both about the set D_K and the members of the scale *Sc*. This information characterizes more completely attributes constitutive for a concept. Indeed, what go under the names of slots, and slot values are nothing but attributes and attribute values.

Closer examination of the schemata view shows that it operates with additional conditions on attributes as well. Among them are specifications of values that can and cannot fill each slot/role (attribute) and the probability distribution of values that the slot may be filled with. In the terms of an abstract property, the first specification deals with the scale of values specific for a given attribute, while the second specification deals with relations among unique values of various attributes. The latter specification naturally extends to the specification of relationships among slots (attributes) and the description of schemata as networks. In triplet terms, this means that we should consider not only different combinations of attributes of concept instances, but also relations among these combinations. Under the appropriate choice of the concept base, all these structures may find the proper place in it.

Additionally, according to Rumelhart (1980), the schemata information about a concept includes information how conceptual information is to be manipulated. This condition correlates with using in the schemata view some formalization of the notion of an attribute in terms of slot/role and slot values. The concept views discussed above use the informal notion of an attribute and take for granted that this notion is manipulated by resources and tools of common knowledge and natural language. Thus, the schemata view uses explicitly such a structure as some formal model of a property.

The Explanation-Based View

According to this concept view a concept includes information about how that concept is related to other concepts (or how its instances relate to other objects) and about the relationships -especially the functional, causal, or explanatory- that hold among the attributes associated with its instances (Komatsu 1992, p. 515).

This view also holds that the information constitutive for a concept is the information about attributes of concept instances. We can interpret this quotation as follows. First, the information about relations of a given concept to other concepts may be treated as the information about relations of constitutive attributes of the former concept to constitutive attributes of other concepts. Second, the information about relationships that hold among attributes of concept instances is the information comprised in a model of instances.

Let us consider a possible triplet interpretation of structures associated with this description of the explanation-based view. According to it, the concept base includes not only attributes of elements from the ground set *G*, but also their relations to elements from the set *O* of other relevant objects. Thus, for the explanation-based specification of a particular concept we also should consider other objects and their relations to the instances of this concept. Additionally, the base contains also relations among attributes of concept instances. All these structures are generated by an appropriate choice of the basis $\{G_K, X_2 = O, X_3, ..., X_n\}$ of the set scale and constraints.

Moreover, the explanation-based view supposes that the information about attributes constitutive for a given concept has stored not only in a form of directly accessible names and descriptions of components from the concept base. Typically, to extract this information a subject should make some processing of the concept representing part. It means that the information vital for the decision making about the membership of an object to the concept extension is an outcome of specific processing available components of the concept representing part.

The triplet model may be modified to accommodate this. Before proceeding to a modification, we should change a mode of the triplet description of the concepts representing part. *Definition 4* characterized it from the point of view of expressing its structures by means of the forms of some language *L* with the alphabet *A*, the vocabulary *V*, the set *P* of word combinations, the set *E* of expressions (sentences), and the set *T* of texts. Any of these language structures may be a carrier of information constitutive for a concept.

It is an assumption of the classical, family resemblance, exemplar and schemata views of concepts that this information is ready for use and it is actualized during the access to it. The explanation-based view rejects this assumption and states that, at least in some cases of concept usage in recognition, this information is obtained by means of some inference processes over relevant and pre-stored information.

From this point of view, the representative structures of the concept representing part may be divided into two classes. The first class consists of structures supposed to be directly accessible. These structures are subjects of inference processes and their processing creates structures from the second class. The latter structures are accessible only through some transforming of the former structures.

Let us consider briefly two illustrative examples. For common concepts, we may speak about such structures from the first class as informal descriptions, beliefs, pieces of intuitive knowledge, etc. They have been operated on processes associated with ordinary discourse. For scientific concepts structures from the first class may be definitions, initial propositions, postulates, axioms, theorems that have been operated on processes of scientific argumentation (deduction, induction, etc.). In either case, the second class consists of derived structures that used in decision making about membership of presented objects to the concept extension.

There are eight main ways of doing the modification of the triplet model of concepts. In either case, the concept representing part should be divided into classes of initial (directly accessible) and derived (indirectly accessible) structures. These ways may be described by two criteria each of that has two values. The first criterion deals with an inclusion or non-inclusion of (deductive, abductive, inductive, explanatory and others) processes and structures by means of which the derivation takes place in the concept representing part. The second criterion deals with particularity or generality of derivation processes and structures. In the former case, the derivation processes are supposed to be unique for a given concept, while in the latter case they are supposed to be shared by many concepts of the same kind.

Thus, compared to other views, the explanation-based view elaborates more carefully the idea that the concept representing part is "immersed" in the available knowledge systems. Figuratively speaking, to make a decision about the membership of objects to the extension of a given concept, we need not only description-like information about object attributes and corresponding attribute values. We should realize some processes that involve transformations of this information and lead to relevant and non-evident consequences. According to this view, the concept representing part is not a simple disjoint collection of homogeneous names and descriptions of components from the concept base. This part is characterized also by relevant relationships between different kinds of names and descriptions, which correspond to the different base components.

11. Perspectives

The triplet model of concepts may be applied in the classification of concepts and concept relations, in determining the maturity degrees of different concepts, in study of types and trends of concept developments, in analysis of knowledge organization at the level of concepts, etc.

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