**The principle of evolution translated into philosophy of nature**

**Abstract:** In this paper we will translate the theory of evolution from a scientific framework to a philosophy of nature framework using formal logic. The purpose of this is to allow philosopher for philosophers that because of their view of science can not use the principle of evolution in their system. To translate, we use predicate logic with some logical vocabulary coming form interval base temporal logic, and we formalize a principle of recursion in one of our premise to simulate the passing of generations.

**Keywords:** Philosophy of Nature

**1.Introduction:**

Please note that the timeline in this demonstration is limited to mankind existence, it start when the first human that existed was born and end with the death of the last human that existed.The current interval start when the first human that existed was born and end when the last human of the first generation of human die.

Let O be the predicate "being an organism from the species w"

Let P be the predicate "being a physical characteristics that is useful for survival"

Let A₁ be the sort such thatA₁= {x|O(x, w)}

with

x ∈ A₁

Let A₂ be the sort such thatA₂ = {z|P(z)}

with

z ∈ A₂

Let A₃ be the sort such that it contains all the possible combinaison of subsets of A₁ and A₂ made using the predicates that can be applied to elements of A₁ and/or A₂.

A₃ = {{x|Q(x, z}, {x|R(x)}, {x|Q(x) ∧ R(x)}, ...}

Let D be a domain of discourse such that:

D = {A₁, A₂, A₃}

Let Q(x, z) be the binary predicate "x have z"Let R(x) be the x predicate "survive in the long run"Let S(x) be the x predicate "reproduce more"Note that it means reproducing more compared to the average reproductive rate of other organism that lived at the same era as themLet T({x}) be the predicate "subset {x} increase in percentage"Note that T only apply on interval of timesSo by increase in percentage we mean that the percentage it was on the first instant of the given interval is lower than the percentage it was on the last instant of that given interval.It be computed as following (For example with the subset {x|Q(x, z)} : (|{x|Q(x, z)}| / |{x|O(x, w)}|) x 100

**2.Premises:**

P1: For all z, for every interval that start at the same instant as the current interval, for all x, if for all x, x have z and it is not the case that for all x, x does not have z, then x reproduce more.

∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))))

P2: For all z, if for every interval that start at the same time as the current interval, for all x, x have z imply x reproduce more and it is not the case that for all x, if it is not the case that x have z then x reproduce more then there exist an interval after the current one for which the subset of x that have z and reproduce more is bigger than the subset of x that does not have z and reproduce more.

∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|))

In simpler term:

∀z(P1 → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)))

P3: For all z, for every interval that start at the same instant as the current interval, if the subset of x that satisfy having z and reproducing more is bigger than the subset of x that satisfy not having z and reproducing more then then there exist an interaval after the interval that start at the same time as the current interval for which the subset of x that satisfy having z increase in percentage.

∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))))

In simpler term:

∀z([B̅](P2's consequent → ⟨L⟩(T({x|Q(x, z)}))))

P4: For all z, for every interval that start at the same instant as the current interval if the subset of x that satisfy having z and reproducing more is highter than the subset of x that satisfy not having z and reproducing more imply that there exist some interval after this one for which the subset of x that satisfy having z increase in percentage and there exist some interval after the current one for which the subset of x that satisfy having z and reproducing more is highter than the subset of x that satisfy not having z and reproducing more then there exist an interval after the current one for which every interval after this one, for all x, x have z.

∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x,z))

In simpler term:

∀z(P3 ∧ P2's consequent) → ⟨L⟩[B̅]∀xQ(x,z))

φ: For all z, there exist an interval after the current one, for which every interval that start at the same instant as this one, for all x, x have z.

∀z(⟨L⟩[B̅]∀xQ(x, z))

P1: ∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))))

P2: ∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|))

P3: ∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))))

P4: ∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, z))

φ: ∀z(⟨L⟩[B̅]∀xQ(x, z))

Note that for P4 it is a simplication of a recursion that can not be expressed with this logic, to sum it up the recursion increase the percentage at each iteration until it gets to 100%, each iteration consist of an interval that is after the current one (which is the current iteration).

P1: ∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))))

P2: ∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|))

P3: ∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))))

P4: ∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, z))

φ: ∀z(⟨L⟩[B̅]∀xQ(x, z))

Δ = {P1, P2, P3, P4}

**3.Demonstration:**

Let's consider the following proof system:{∀x.θ(x) ⊢ θ(a)}{{θ}, {θ → ψ}) ⊢ {ψ}}{θ}, {ψ} ⊢ (θ ∧ ψ)

Let us prove that Δ ⊢ φ

{∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))))},

{∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|))},

{∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))))},{

∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, z))}

⊢

{∀z(⟨L⟩[B̅]∀xQ(x, z))}

Δ ⊢ φ

{∀x.θ(x) ⊢ θ(a)}

on P1 with

θ: [B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x))))

[B̅](∀x(Q(x, a) → S(x)) ∧ (¬(∀x(¬Q(x, a)) → S(x))))

{∀x.θ(x) ⊢ θ(a)}

on P2 with

θ: [B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

[B̅](∀x(Q(x, a) → S(x)) ∧ (¬(∀x(¬Q(x, a)) → S(x)))) → ⟨L⟩(|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}|)

{∀x.θ(x) ⊢ θ(a)}

on P3 with

θ: [B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)})))

[B̅](|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, a)})))

{∀x.θ(x) ⊢ θ(a)}

on P4 with

θ: [B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, z)

[B̅](|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, a)}))) ∧ ⟨L⟩(|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, a)

{{θ}, {θ → ψ}) ⊢ {ψ}}

on P2 with

θ: [B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x))))ψ: [B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

Let's call the new sentence S1

S1: ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

{θ}, {ψ} ⊢ (θ ∧ ψ)

on P3 and S1 with

θ: [B̅](|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, a)})))

ψ: ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

Let's call the new sentence S2

S2: [B̅](|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, a)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

{{θ}, {θ → ψ}) ⊢ {ψ}}

on S2 and P4 with

θ: [B̅](|{x|Q(x, a) ∧ S(x)}| > |{x|¬Q(x, a) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, a)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|)

ψ: ⟨L⟩[B̅]∀xQ(x, a)

Let's call the new sentence S3

S3: ⟨L⟩[B̅]∀xQ(x, a)

Since we instantiated z with the arbitrary constant "a" we can generalize back to zLet's call the new sentence S4

S4: ∀z(⟨L⟩[B̅]∀xQ(x, z))

∴Δ ⊢ φ

Now Let us prove that Δ ⊨ φ

(Δ ⊨ φ) ≡ ((Δ ∪ {¬φ}) ⊢ ⊥)

Δ ∪ {¬φ} ⊢ ⊥

{∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))))},

{∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀x(¬Q(x, z)) → S(x)))) → ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|))},

{∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))))},

{∀z([B̅](|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}| → ⟨L⟩(T({x|Q(x, z)}))) ∧ ⟨L⟩(|{x|Q(x, z) ∧ S(x)}| > |{x|¬Q(x, z) ∧ S(x)}|) → ⟨L⟩[B̅]∀xQ(x, z))},

{¬(∀z(⟨L⟩[B̅]∀xQ(x, z)))}

⊢

⊥

Because of the structure of the reasoning, Δ ∪ {¬φ} ⊢ ⊥ can be proven only after φ is derived from Δ such that we get Δ ∪ {φ} ∪ {¬φ} ⊢ ⊥.

Let's therefore take the sentence S4 from the previous demonstration.

S4: ∀z(⟨L⟩[B̅]∀xQ(x, z))

{θ}, {ψ} ⊢ (θ ∧ ψ)

on S4 and ¬φ with

θ: ∀z(⟨L⟩[B̅]∀xQ(x, z))

ψ: ¬(∀z(⟨L⟩[B̅]∀xQ(x, z)))

Let's call the new sentence S5

S5: ∀z(⟨L⟩[B̅]∀xQ(x, z)) ∧ ¬(∀z(⟨L⟩[B̅]∀xQ(x, z)))

{θ ∧ ¬θ ⊢ ⊥}

on S5 with:

θ: ∀z(⟨L⟩[B̅]∀xQ(x, z))

∴Δ ⊨ φ

**4.Remark:**

It is true that from experience, not every organism in a species, nor every interval after the current one, necessarily satisfies the properties described in the premises.However, the reasoning here does not require absolute universality at each point. The premises are generalizations intended to hold across most organisms and most intervals.Since each premise describes properties that converge or increase in satisfaction over successive intervals, this reasoning approximates universality and thus reliably leads to the conclusion.Therefore, even though the premises are generalizations, the cumulative structure ensures the "real premises" to be syntactically and semantically entailed to the conclusion.

For example, for

P1: ∀z([B̅](∀x(Q(x, z) → S(x)) ∧ (¬(∀xQ(x, z) → S(x)))))

There may exist an interval after the current one for which ∀x(Q(x, z) → S(x)) is false, maybe because most of the organisms that satisfied Q also satisfied something else which made them not being able to satisfy S.But it would not matter, as for the reasoning to still hold all that is required is that for most interval and most organism Q(x, z) → S(x) hold.