Rigidity and factivity

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Abstract: David Chalmers argued against the claim that for all p, or even for all entertainable p, it is knowable a priori that p iff actually p. Instead of criticizing Chalmers's argument, I suggest that it can be generalized, in a sense, and in interesting ways, concerning other principles about contingent a priori truths. In particular, I will argue that the puzzle presented by Chalmers runs parallel to others that do not turn on 'actually'. Furthermore, stronger arguments can be presented that do not turn on apriority either, though they do entail the conclusion of Chalmers's argument. All such puzzles involve interactions between rigidifying sentence-forming devices with factive operators.

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§1. David Chalmers (2011) argues against the claim that for all p, or even for all entertainable p, it is knowable a priori that p iff actually p. Let 'A', 'E', 'K', '$\Box$' and '$\Diamond$' stand for 'actually', 'someone entertains', 'someone knows', 'necessarily' and 'possibly', let q be any entertainable and expressible proposition that no one actually entertains, and let r be $\neg Eq$, i.e. the proposition that no one entertains q. Chalmers's argument is as follows:

(1) Ar

(2) $Ar \rightarrow \Box Ar$

(3) $\Box(K(r \leftrightarrow Ar) \rightarrow (r \leftrightarrow Ar))$

(4) $\Box(r \rightarrow \neg K(r \leftrightarrow Ar))$

(5) $\neg\Diamond K(r \leftrightarrow Ar)$

Premise (1) requires simply that some proposition is not actually entertained, while (2) and (3) are instances of well-known principles from the logics of A and K. (4) follows from two principles about entertaining: (i) entertaining a proposition requires entertaining its constituents, and (ii) knowing a proposition requires entertaining that proposition. If r is true, $\neg E(r \leftrightarrow Ar)$ follows from (i), as q is a constituent of it, but then $\neg K(r \leftrightarrow Ar)$ follows from (ii).[1] The conclusion follows from (1)-(4) in the classical normal modal logic **K**.

After examining numerous ways in which the argument can be resisted, Chalmers concludes:

> [I]f one accepts an orthodox semantics for 'actually' and an orthodox understanding of apriority, one must reject the orthodox view that $p \leftrightarrow Ap$ is always a priori. Likewise, if one accepts the orthodox view that $p \leftrightarrow Ap$ is always a priori, one must adopt an unorthodox semantics for 'actually' or an unorthodox understanding of apriority. (2011: 419)

The 'orthodox' understanding of (propositional) apriority is the *modal understanding*, according to which p is a priori iff p is knowable a priori, and p is knowable a priori iff it is possible that someone knows p a priori. (5) is the negation of the claim that it is possible that someone knows $r \leftrightarrow Ar$, and *a fortiori* of the claim that it is possible that someone knows $r \leftrightarrow Ar$ a priori. If the argument presented by Chalmers is sound, either we accept the orthodox semantics for 'actually' *and* the orthodox (modal) understanding of apriority, or we accept the orthodox view that $p \leftrightarrow Ap$ is always a priori. We cannot have both.

I will not challenge the soundness of Chalmers's argument. As he himself notes, there are many different ways in which his argument can be resisted, but the overall plausibility of its assumptions is substantial. Nor will I offer an unconventional semantics for 'actually' or an alternative understanding of apriority. In fact, whether or not one accepts the modal understanding of apriority,

[1] In the same paper, Chalmers presents different versions of the argument, relying neither on entertaining nor on constituency. Similar variations are possible for the arguments presented here.

(5) is a curious result on its own, that is, apart from whether unknowability is implied by the impossibility of knowing.

Instead of presenting a solution to the puzzle, I will suggest that Chalmers's argument can be generalized, in a sense, and in interesting ways, concerning other principles about contingent a priori truths. In particular, I will argue that the puzzle presented by Chalmers runs parallel to others that do not turn on 'actually'. Furthermore, stronger arguments can be presented that do not turn on apriority either, though they do entail the conclusion of Chalmers's argument. All such puzzles involve interactions between rigidifying sentence-forming devices with factive operators – of which A and K are mere instances, respectively. In what follows I will first focus on the latter arguments entailing (5) and afterward on the parallel arguments which do not rely on 'actually'.

§2. There are different ways of generating arguments stronger than Chalmers's without relying on apriority. One might be suspicious of the very notion of apriority while still maintaining that for every entertainable p, p iff actually p is *truly entertainable*, where a proposition p is truly entertainable iff it is possible that someone truly entertains p. But there is a simple argument against this claim which runs parallel to Chalmers's original argument. Let 'E_T' abbreviate 'someone truly entertains'. Because one cannot truly entertain a falsehood, (6) is true:

(6) $\Box(E_T(r \leftrightarrow Ar) \rightarrow (r \leftrightarrow Ar))$

And (7) follows from (i) in conjunction with the truism that truly entertaining a proposition requires entertaining that proposition:

(7) $\Box(r \rightarrow \neg E_T(r \leftrightarrow Ar))$

From (1), (2), (6) and (7) one can derive (8) by an argument that is exactly parallel to the one establishing (5):

(8) $\neg\Diamond E_T(r \leftrightarrow Ar)$

Finally, given principle (ii) connecting knowledge and entertaining, (5) follows from (8).

Another similar argument can be given which turns on propositions being *truly believable*. Let 'B_T' abbreviate 'someone truly believes'. By notational variants of the pairs (3)/(6) and (4)/(7), assumption (i) and the claim that truly believing a proposition requires entertaining that proposition, (9) can be derived in a parallel way:

(9) $\neg\Diamond B_T(r \leftrightarrow Ar)$

Under the widely held assumption that knowing a proposition requires truly believing that proposition, (9) entails (5). More generally, we can now claim that where 'F' stands for any (modally) factive propositional attitude requiring entertaining and required by apriority, the following will be derivable given (1) and (2), and it will entail (5):

(10) $\neg\Diamond F(r \leftrightarrow Ar)$

Being truly entertainable, truly thinkable, or even truly graspable are more gerrymandered (or less 'natural') than their corresponding non-factive states. Likewise for being truly believable. Still, less natural properties are often serviceable. In the case at hand, for instance, they clearly illustrate how a different understanding of apriority alone would not solve the puzzle (whatever that is, exactly) in its full generality.

§3. Suppose *S* introduces the name 'George' to denote the unique set that actually contains all and only true propositions – or all truths, for short. 'George' will then rigidly designate the set of all truths, and so *S* will be able to know a priori that George is the set of all truths.[2] To be sure, it is not terribly important whether *S* attains *a priori* knowledge in this case, as there will be many other propositions involving George that can plausibly be known a priori by *S* or anyone else, at least in principle. For it seems natural to suppose, given the way 'George' was introduced, that for all propositions *p*, or simply for all entertainable *p*, it is knowable a priori that *p* iff *that p* is a member of George. It is knowable a priori, for instance, that the number of stars is odd iff *that the*

[2]Typically, such claims of apriority are conditional on the existence of the baptized object, so that *S* will be able to know a priori that George is the set of all truths if anything is. See, for instance, Ray (1994) for discussion. See also §4 for more details.

number of stars is odd is a member of George. But there is a simple argument against this general principle.

Substitute every occurrence of 'Ar' throughout Chalmers's original argument with '*that r* is a member of George'. The first premise then becomes:

(1′) *that r* is a member of George

This premise is true given that George is the set of all truths and q is a proposition no one in fact entertains.

The second premise becomes an instance of the widely held thesis that set membership is rigid, so that it is not contingent whether this is a member of that:

(2′) *that r* is a member of George $\rightarrow \Box$ *that r* is a member of George

The third premise remains an instance of the factivity of knowledge:

(3′) $\Box(K(r \leftrightarrow$ *that r* is a member of George$) \rightarrow (r \leftrightarrow$ *that r* is a member of George$))$

And, finally, the fourth premise is motivated by the same considerations about entertaining a proposition motivating (4), that is, (i) and (ii):

(4′) $\Box(r \rightarrow \neg K(r \leftrightarrow$ *that r* is a member of George$))$

By an argument which is exactly parallel to Chalmers's original argument, one can derive:

(5′) $\neg\Diamond K(r \leftrightarrow$ *that r* is a member of George$)$

Another argument can be constructed with pluralities instead of sets, if we suppose S introduces 'Georges' to stand rigidly for the plurality of all truths, instead of the set of all truths. Then it is just as natural to expect that for all entertainable p, it is knowable a priori that p iff *that p* is one of the Georges. But substitute every occurrence of '*that r* is a member of George' in the previous argument with '*that r* is one of the Georges'. The second premise now becomes an instance of the widely held thesis that it is not contingent whether this is one of them, while the other premises become simple variants of those figuring in the set-theoretic argument. A parallel argument then establishes $\neg\Diamond K(r \leftrightarrow$ *that* r is one of the Georges$)$.

Other arguments can be constructed via the introduction of other singular terms. For example, if 'T' stands for the (infinitary) conjunction of all truths, then for all entertainable p, p iff *that* p is a conjunct of T would seem to be knowable a priori, though a parallel argument will once more establish otherwise.

§4. There are multiple ways of replying to the present arguments. In what follows I will mention only some responses which are not variants of possible responses to Chalmers's original argument, many of which are already considered in his paper.

One may worry about the very existence of George in light of Patrick Grim's (1984) argument to the effect that there cannot be a set of all truths. Yet, there are ways out of Grim's argument that allow for George to exist. For instance, one might take some members of George to be *sufficiently* coarse-grained so as to fail to discriminate between some different classes (Uzquiano 2015), thereby avoiding one of the key premises in Grim's argument. Alternatively, there are sets the existence of which would be less controversial to assume and that would do just as well for the present arguments, as one could replace George with, say, Georgina, which is the set of all truths of the form $\ulcorner \neg Eq \urcorner$, where q is an entertainable proposition of a sufficiently restricted type avoiding cardinality issues.[3] A parallel argument can then be developed starring Georgina instead of George.

Still, one may insist that premises $(3')$ and $(4')$ are problematic on the grounds that knowing $(r \leftrightarrow$ *that* r is a member of George$)$ requires knowing that George exists. But on the proposed ways of avoiding Grim's argument, there is no pressing reason to believe one cannot know there is a set like George or Georgina. After all, it is more or less common in presenting ZFCU to

[3]Perhaps, for example, singular propositions about some x. It bears mentioning that, under particular assumptions, especially concerning the fine-grainedness of truths, unentertained truths generate their own class of cardinality paradoxes. For let T be the set of all unentertained truths. If for all subsets R of T there is a truth p_R, say, *that no one entertains every truth in* R, which is itself never entertained, and for every $R, R' \subseteq T$, $p_R = p_{R'}$ only if $R = R'$, all the ingredients are in place for a cardinality argument to the effect that T does not exist.

simply assume there is a set of all ur-elements, and one might naturally take propositions to be ur-elements. Moreover, there are alternatives to ZFCU on which the existence of a set very much like George follows from typical separation axioms.[4]

A more obvious objection would challenge (2′) – and its plural version – on the grounds that George exists only in possible worlds wherein its members likewise exist, and that objects have properties only in worlds wherein they exist. Moreover, if propositions exist only contingently, worlds in which some actual truths fail to exist are worlds in which George likewise fails to exist, and therefore fails to have any members; consequently, the proposition that *that r* is a member of George may not even be true in some such worlds.[5] This objection is especially salient when applied to arguments involving sets like Georgina, as many think singular propositions are ontologically dependent upon the individuals that they are about, and those may exist only contingently. In light of this one might naturally propose that set membership be rigid though contingent on the set's existence – likewise for pluralities. But there are coherent views that would challenge this. Elegant modal set theories have been defended according to which set membership is unconditionally rigid.[6] Moreover, the present cases of unconditional rigidity might be welcomed by those who think truths, propositions, and even singular propositions, exist necessarily, perhaps like numbers and other abstract entities. Some such view would be especially appealing for those who think proper names refer to *individual essences* (Plantinga 1978) which necessarily exist, or to objects which are, in some instances, *concrete* in some worlds and non-concrete in other worlds, wherein they exist nevertheless (Williamson 2001).

Given the fact that the present arguments make use of baptisms in much the same way typical examples of the contingent a priori do, one might also think they are implausible on the grounds that they would inherit much of the same difficulties faced by some of those typical examples. For instance, one may suspect that many purported cases of the contingent a priori fail to involve a *real* act of naming, despite existential questions. Perhaps all such cases do is to create the *appearance*

[4]See Menzel (2014) for the latter and Uzquiano (2015: 330-331) for discussion.

[5]These and related issues are discussed by Prior (1969), Adams (1981), Plantinga (1983), Williamson (2001), and many others.

[6]See Fine (1981) and Parsons (1983: chapter 11).

that a name was introduced into a public language, but whether a name was really introduced is a question that depends on success conditions for genuine performatives – conditions which are presumably violated here.[7] This type of objection is, of course, much more general, for it would plausibly apply in many other examples of the contingent a priori in which somewhat contrived baptisms are essentially involved – such as Evans's (1979) 'Julius', for instance.

Nevertheless, this sort of objection misses a more general point. First, it is important to emphasize that the present arguments do involve assumptions that are also shared by many. A number of philosophers accept a wide variety of instances of contingent a priori knowledge involving reference-fixing in a more unconstrained fashion, which would likely include the present examples.[8] This type of widespread acceptance is noteworthy, and it provides some motivation for the cases at hand. But, more importantly, baptisms and names are not essential for generating similar puzzles that are still distinct from what is presented by Chalmers. Rigid designation is not a property that is exclusive of names. For example, in the puzzle figuring 'George', this name could be replaced by a *rigid description* of the set of all truths. This would of course involve talking about the *actual* set of all truths, of which *that p* is a member iff *p*. And this seems a priori for all *p*, or all entertainable *p*, in which case a similar puzzle can be generated by substituting throughout in the argument the proper name in question with the appropriate rigidified description, providing a counterinstance to the just mentioned general principle, just like in the other puzzles. 'Actual', in this case, does not call for an *operator*. Whatever the views one might have about that operator, rigidified descriptions are simply a different piece of technology.

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[7] Jeshion (2002: 63-66), for instance, suggests a number of constraints for genuine acts of naming.

[8] See, for example, Chalmers (2011: 414).

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