Semantic Criteria of Correct Formalization[1]

Timm Lampert

Abstract

This paper compares several models of formalization. It articulates criteria of correct formalization and identifies their problems. All of the discussed criteria are so called "semantic" criteria, which refer to the *interpretation* of logical formulas. However, as will be shown, different versions of an implicitly applied or explicitly stated criterion of correctness depend on different understandings of "interpretation" in this context.
In particular, I will discuss the benefits and problems of the following criteria of correctness:

- *criterion of verbalization* (CRVERB): $\phi(A)$ is a correct formalization of a proposition *A* if and only if a free verbalization of $\phi(A)$, $V(\phi(A))$, and *A* are equivalent (have the same meaning).
- *varying-*$\mathfrak{I}$ *criterion* (V$\mathfrak{I}$CR): $\phi(A)$ is a correct formalization of a proposition *A* if and only if the schematization of *A*, *sch(A)*, and $\phi(A)$ have the same truth value relative to all interpretations / realizations.
- *fixed-*$\mathfrak{I}$ *criterion* (F$\mathfrak{I}$CR): $\phi(A)$ of a logical language L with a fixed interpretation $\mathfrak{I}$, $<\mathrm{L}, \mathfrak{I}>$, is a correct formalization of a proposition *A* if and only if $\Phi(A)$ is an effective translation of *A* or of a rephrasing of *A* in respect to $<\mathrm{L}, \mathfrak{I}>$.
- *TC*: $\phi(A)$ is a correct formalization of proposition *A* if and only if $\phi(A)$ and *A* have the same truth values with respect to all interpretations in terms of conditions of truth or falsehood of *A* that are suitable according to the realization.
- *TC'*: $\phi(A)$ is a correct formalization of a proposition *A* if and only if $\phi(A)$ and *A* have the same truth values with respect to suitable realizations. Realizations are suitable if and only if they do not restrict the space of possible interpretations in terms of conditions of truth and falsehood of *A*.

I will argue that the formal representation of informal reasoning highly depends on the implicitly assumed notion of a correct formalization. This, I will demonstrate by referring to the so called "De Morgan argument" ("All horses are animals. Therefore: All heads of horses are heads of animals.").

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1. Introduction

Logical formalization is a standard tool for reconstructing informal arguments as well as for the logical analysis of ordinary language. The question of identifying criteria for correct formalization arises as soon as one considers alternative formalizations. The recent debate on criteria of adequate formalization demonstrates that this question is more difficult to answer than one might suppose from the familiar practice of formalization, cf. Brun(2003) as well as Blau(1977), Epstein(1990), Epstein(1994), Sainsbury(1993), Löffler(2006), Kleinknecht(2008), Baumgartner/Lampert(2008) and Lampert/ Baumgartner(2010). In particular, two questions arise: (i) how should established formalizations be reconstructed? and (ii) are the reasons underlying these formalizations persuasive? In this paper, these questions are discussed with respect to formalizations of the following well-known and allegedly trivial argument:

Premise (P): Horses are animals.
Conclusion (C): Heads of horses are heads of animals.

This argument is used in standard textbooks of logic to prove the insufficiency of monadic predicate logic. It is called “De Morgan's Argument” although De Morgan never used it in this form, cf. Merrill(1977) and Brun(2003), p. 189, footnote 1. In this paper, I refer to the discussion of its established formalization, which can be found in standard textbooks of logic such as Copi(1979), p. 131f., Lemmon(1998), p. 131f., Quine(1982) p. 168, 173, 251 and Suppes(1999), p. 93f. It is as follows:

$\phi(P)$: $\forall x\,(Hx \rightarrow Jx)$
$\phi(C)$: $\forall x(\exists y\,(Hy \wedge Ixy) \rightarrow \exists y(Jy \wedge Ixy))$

Realization 1:

Hx: *x* is a horse.
Jx: *x* is an animal.
Ixy: *x* is a head of *y*.

A formalization assigns a logical formula plus a realization to ordinary propositions. A realization interprets the categorematic parts of the formula, namely, its names, propositional variables and predicates.

According to the standard formalization, De Morgan's argument is valid. However, this formalization is hardly ever argued for. The detailed reasons for it are not obvious. In particular, the formalization of the conclusion is questioned. Presuming *realization 1*, Wengert(1974) suggests the following formalization:

$\phi(C)'$: $\forall x \forall y((Hy \wedge Ixy) \rightarrow (Jy \wedge Ixy))$

As will be seen, this alternative formalization raises the question of how extensively the formalization should take into account the relation of the concepts *horse* and *animal*. This is considered by Brun(2003). He discussed both of the mentioned alternatives and suggested a model of formalization according to which they are both correct. As soon as one acknowledges that questions of logical formalization imply conceptual analysis, it can also be questioned whether the formalization of the premise in terms of a formally invalid general implication is correct. At the end of this paper, an alternative formalization will be provided that formalizes the conclusion as well as the premise of De Morgan's argument in terms of a tautology.

All of the different formalizations are based upon different formalization models. These models imply different aims, differences within semantics and different formalization criteria. These differences will be spelled out in the following sections. I will only consider first-order logic formalizations without identity. Furthermore, I will confine the discussion to criteria for the correct formalization of single propositions. Criteria of correct formalization may be defined either depending on relations of implication or depending on interpretations of first-order formulae. With respect to the former, one distinguishes between the correctness and completeness of a formalization, cf. Baumgartner/Lampert(2008), p. 103-111. I will abstain from addressing completeness as well as from defining correctness with respect to relations of implication. Instead, I will only consider criteria of correctness with respect to interpretations of first-order formulae. The criteria of correctness of the different models of formalization discussed here differ with respect to the understanding of what is meant by an interpretation of a logical formula. In addition to the criteria of correctness, the literature discusses criteria that refer to a certain structural similarity of the surface of the formalized proposition and its formalization. I will abstain from addressing these kinds of criteria as well. I will begin with Wengert's position, as it makes clear that the traditional formalization of De Morgan's argument requires justification.

2. Interpretations as free verbalizations

Who says “Heads of horses are heads of animals” does not merely mean $V(\phi(C))$ but $V(\phi(C)')$:

$V(\phi(C))$: “Every head of some horse is a head of some animal.”
$V(\phi(C)')$: “Every horse that has a head is an animal that has that head.”

In contrast to $V(\phi(C)')$, $V(\phi(C))$ is still true even if some (or all) horses were not animals but the heads of those beings were still the heads of animals. This difference also exists between the formalizations $\phi(C)$ and $\phi(C)'$. In contrast to $\phi(C)'$, there are models of $\phi(C)$ in which not all objects that satisfy H and are related to some object by I also satisfy J. $V(\phi(C))$ is a free verbalization of $\phi(C)$, and $V(\phi(C)')$ is a free verbalization of $\phi(C)'$. By use of the realizations, free verbalizations translate the logical formula into comprehensive ordinary propositions. From the fact that *(C)* is not equivalent to $V(\phi(C))$ but to $V(\phi(C)')$, Wengert drew the conclusion that *(C)* is correctly formalized by ϕ(C)' and not by ϕ(C). He implicitly put forth the following criterion of correct formalization:

criterion of verbalization (CRVERB): $\phi(A)$ is a correct formalization of a proposition A if and only if a free verbalization of $\phi(A)$, $V(\phi(A))$, and A are equivalent (have the same meaning).

According to this criterion, an “interpretation of a formula” is understood as the free verbalization of the formula by use of the realization. By “equivalence”, it is not only meant that A and $V(\phi(A))$ have the same truth value. This condition is also satisfied in the case of (C) and $V(\phi(C))$. Instead, “equivalence” is meant in terms of “identity of meaning (content)”. This kind of equivalence clearly depends on both the concepts contained in A as well as on the realization. Wengert alludes to this by comparing (C) with proposition (R), which is similar with respect to its grammatical structure:

(R): Children of mothers are children of fathers.

In this case, the following formalization is correct according to CRVERB:

$\phi(R)$: $\forall x(\exists y\,(Hy \wedge Ixy) \rightarrow \exists y(Jy \wedge Ixy))$

Realization 2:

Hx: x is a mother.
Jx: x is a father.
Ixy: x is a child of y.

The free verbalization is as follows:

$V(\phi(R))$: “Every child of some mother is a child of some father.”

$V(\phi(R))$ has the same meaning as (R). In contrast, the replacement of $\phi(R)$ with $\phi(C)'$ (= $\phi(R)'$) results in an incorrect formalization, because in this case, $V(\phi(R)')$ would result in “Every mother that has a child is a father that has that child”. Or, as Wengert(1974), p. 166 stated briefly, “Everybody's mother is his father”. Obviously this is not what is meant by (R).

According to CRVERB, different logical formalizations of grammatically similar propositions might be correct. According to this point of view, one of the benefits of formalization is that it can express differences in logical form that are not expressed by the syntax of ordinary language.

The following example is compatible with both formulae, $\phi(C)$ and $\phi(C)'$, in addition to the respective realization (cf. p. 20):

(S): Bets on winning numbers are bets on prime numbers.

(S) is ambiguous. CRVERB makes the two possible interpretations explicit: Either it means “Every bet on some winning number is also a bet on some prime number” or it means “Every bet on a winning number is a bet on a prime number”. The benefit of logical formalization according to CRVERB is found in expressing this difference in meaning through a syntactic difference in logical formulae.

In fact, CRVERB is frequently used as a tool for judging formalizations. However, CRVERB faces the following problems:

problem 1: No mechanical procedure exists to generate free verbalizations. One therefore runs the risk of generating a verbalization in light of the proposition to be formalized. In consequence, CRVERB cannot serve as an independent criterion. Even referring to an explicit paraphrase of the logical formula would not achieve a criterion. This is because it is generally not possible to judge the equivalence of a proposition to be formalized and an explicit paraphrase of a logical formula. In fact, the question “which of the alternative paraphrases is equivalent to *A*?” is as problematic as the question “which of the paraphrased alternative formalizations is correct?”

problem 2: Judgments of equivalence or identity of meaning are presumed to be primitive. However, as will be seen with respect to alternative formalizations of De Morgan's argument, such judgments require further explication or justification.

As Brun(2003), p. 200 notes, one runs the risk of shifting the problem of identifying a correct formalization to the problem of identifying a correct free verbalization. As long as no algorithm is available to precisely define how to generate verbalizations, one cannot speak of a *criterion* in a strict sense. Rather, one might speak of a device that does not yield a definite decision in any case.

However, CRVERB expresses a certain intuition: correct formalizations should not only correspond to the truth value of the proposition *A* to be formalized, but also to the *meaning* of *A*. According to this understanding, the logical formula represents the *form of the meaning* of *A*. The problem of CRVERB is how to explain the meaning of a proposition in this context. I will come back to this problem in sections 4 and 5.

3. Interpretations as realizations

The application of CRVERB to (C) of De Morgan's argument raises the question of how the traditional formalization of (C) by means of $\phi(C)$ can be justified. To argue that this formalization is based on the same criterion CRVERB but differs in understanding (C) in terms of $V(\phi(C))$ is not persuasive. Wengert drew the conclusion that the traditional formalization cannot be justified and must be given up in favor of $\phi(C)'$. Brun likewise did not provide an argument in favor of $\phi(C)$ and against $\phi(C)'$. In contrast to Wengert and Brun, this section shows how to reconstruct the traditional practice. I will distinguish between two different strategies of formalization that both justify the formalization of (C) by $\phi(C)$. The former is typical for philosophical traditions within logic, while the latter applies common strategies of formalization within mathematical logic. Although this strategy might not be as evident as others in the context of formalizing De Morgan's argument, I refer to it in order to illustrate different aims and criteria of logical formalization.

3.1 Varying interpretations

Within the philosophical tradition of logic textbooks, the aim of formalization is mostly to evaluate the formal validity of informal arguments. Examples of formalizations within this tradition go hand in hand with the distinction between form and content. Typically one refers to plain informally valid arguments, such as certain types of Aristotelian syllogisms, to motivate predicate logic as a necessary enhancement of propositional logic. Then, one notes that the validity of those arguments does not depend on the specific content of the concepts. This is done by replacing the predicates of the argument with other predicates with a different meaning. If one abstains from the specific predicates, one yields a schematization of the informal arguments. According to the standard view, the logical form of the proposition corresponds to this schematization. The aim of formalization in this tradition is to abstain from the specific meaning of the predicates. This is incompatible with CRVERB, as this criterion refers to the specific meaning of the predicates within the realization in order to judge the identity of meaning of *A* and $V(\phi(A))$. According to the traditional view, a given realization is just one of many other possible interpretations of $\phi(A)$ that must be considered. In order to judge the correctness of a formalization, one must refer to varying interpretations / realizations. The realizations / interpretations regulate how to substitute propositional variables, predicates and names within the schematization of the proposition to be formalized. One thereby compares the schematization of *A*, *sch(A)*, and $\phi(A)$ in accord with varying realizations / interpretations. In contrast to CRVERB, one refers to purely extensional judgments of equivalence: *sch(A)* and $\phi(A)$ must have the same truth value according to their respective interpretations. The interpretations of names, $\Im(t)$, of propositions, $\Im(A)$, and of predicates, $\Im(\varphi)$, are identified with their respective extensions: $\Im(t)$ is an object, $\Im(A)$ is a truth value, and $\Im(\varphi)$ is a class of objects.

Let us illustrate this approach by considering the formalization of (C). Like (R) and (S), (C) is an instance of *sch(C)*:

sch(C): All Xs of Ys are Xs of Zs.

Strictly speaking, schematizations may presume changes in structure and wording of the respective propositions, cf. p. 11 below. Furthermore, we must presume a correlation of the schemas X,Y,Z and the terms of the realization. Thus, in our example, the interpretation of X and Ixy, of Y and Hx and of Z and Jx are correlated. Considering the correctness of $\phi(C)'$ for (C), it does not suffice to evaluate whether *sch(C)* and $\phi(C)'$ have the same truth value according to *realization 1*. In fact, one may admit that (C) is to be understood in terms of $V(\phi(C)')$. However, judging the correctness of $\phi(C)'$ also requires considering whether *sch(C)* and $\phi(C)'$ have the same truth value according to *realization 2*. This is not the case; (R) is true while *realization 2* is not a model of $\phi(C)'$. For this reason, $\phi(C)'$ is not a correct formalization of (C). In contrast, the truth value of *sch(C)* corresponds to the truth value of $\phi(C)$ relative to both realizations. Whereas $\phi(C)$ represents the form of (C), $\phi(C)'$ does not represent the form of (C); $\phi(C)'$ does not correspond to *sch(C)*. One erroneously considers semantic aspects if one chooses $\phi(C)'$. However, this model of formalization requires that logical formalization must be independent of the specific meaning of the categorematic parts of propositions.

The traditional formalization of De Morgan's argument is based on the following criterion:

varying-$\mathfrak{I}$ criterion (V$\mathfrak{I}$CR): *$\phi(A)$* is a correct formalization of a proposition *A* if and only if the schematization of *A*, *sch(A)*, and *$\phi(A)$* have the same truth value relative to all interpretations / realizations.

In contrast to CRVERB, the intention of this criterion is to evaluate the correctness of formalizations merely with respect to formal equivalence. Any implication based on conceptual relations must not be considered. De Morgan's argument can still be proven as (formally) valid according to this approach. However, the class of arguments that are proven as valid according to this strategy is significantly smaller than it is according to formalization strategies not based on schematization. The following example illustrates this:

argument B: Heads of horses are heads of animals. The horse Fury has a head. Therefore: Fury is an animal.

According to a pre-theoretic, informal understanding of validity, *argument B* is valid. This means that the truth of the premises is incompatible with the falsehood of the conclusion. How to explicate this validity is another question. The traditional strategy of formalization in compliance with V$\mathfrak{I}$CR does not make it possible to explicate *argument B* as a valid argument in terms of first-order logic. The reason for this is that its schematization allows for invalid instances such as the following:

argument C: Children of mothers are children of fathers. The mother Jane has a child. Therefore: Jane is a father.

In contrast, CRVERB allows one to prove the validity of *argument B* by means of first-order logic without thus implying that *argument C* is valid. This shows that V$\mathfrak{I}$CR goes hand in hand with a loss of the power to explicate the validity of arguments by means of first-order logic.

Other examples that are praised as achievements of the logical analysis of ordinary language show that V$\mathfrak{I}$CR trivializes the problem of formalizing ordinary propositions. Consider, for example, Davidson's analysis of action sentences:

argument D: Ann strolls slowly. Therefore: Ann strolls.

Davidson's strategy makes it possible to formalize this argument as valid according to first-order logic:

$\phi(D)$: $\exists x\ (Fx \wedge Gx \wedge Ixa)$ |- $\exists x\ (Fx \wedge Ixa)$

Realization 3:

Fx: *x* is a stroll.
Gx: *x* is slowly.
Ixy: *x* is conducted by *y*.
a: Ann.

However, this formalization is not correct according to VℑCR. This can be seen by replacing "slowly" with "allegedly".

There are two strategies for avoiding this objectionable consequence of trivializing formalization: (i) the limitation of admissible interpretations and (ii) standardization. Strategy (i), for example, is applied if one rules out the substitution of "slowly" by "allegedly" in *argument D*. The same strategy is applied in the following case:

argument E: John loves a human. Therefore: Some human exists.
argument F: John seeks a unicorn. Therefore: Some unicorn exists.

Argument E is valid, whereas *argument F* is not. The validity of *argument E* can be proven by the following formalization:

$\phi(E)$: $\exists x\ (Fx \wedge Gax)$ |- $\exists x\ Fx$

Realization 4:

Fx: *x* is a human.
Gxy: *x* loves *y*.
a: John.

According to CRVERB, this is a correct formalization of *argument E*. However, $\phi(E)$ with a respective realization is not a correct formalization of *argument F*, because "John seeks a unicorn" and "Some unicorn exists that John seeks" are not identical in meaning. In contrast to the latter, the former does not imply the existence of some unicorn.

According to VℑCR, however, one cannot make this argument. The replacement of "love" with "seek" and "human" with "unicorn" results in an invalid argument. Thus, $\phi(E)$ cannot be a correct formalization according to VℑCR unless one rules out this alternative interpretation. Indeed, this interpretation is commonly excluded by arguing that “to seek” is not a predicate obeying the principle of extensionality. This is demonstrated by the fact that “*x* seeks *y*” might be true even in case that no *y* exists to be sought. Yet, this raises the problem of how to define a criterion for distinguishing between admissible and inadmissible interpretations. This question also becomes relevant with respect to the possibility of formalizing paradoxes. Is “*x* is a member of *x*”, for example, an admissible predicate as one assumes by formalizing Russell's Paradox within predicate logic? The question is whether the two commonly presumed semantic principles, namely, the principle of extensionality and the principle of bivalence, suffice to rule out inadmissible interpretations. I will return to this question in section 6.

In addition to strategy (i), strategy (ii), namely standardization, is indispensable for applying V$\Im$CR. Standardization indicates the rephrasing of ordinary propositions to the effect that their surface grammar becomes more similar to logical formulae. For example, one may rephrase *argument D* as follows: “Some event *x* exists such that *x* is a stroll and *x* is slowly and *x* is conducted by Ann. Therefore: Some event *x* exists such that *x* is a stroll and *x* is conducted by Ann.” However, the question of a correct formalization is clearly shifted to the question of correct standardization. It is also apparent that standardization is necessary if one refers to schematization. Consider, for example, (C), (R) and (S): rephrases are necessary to schematize all of these by “All Xs of Ys are Xs of Zs”.

If one dispensed with strategy (ii), only arguments that are explicit paraphrases of formally valid inferences could be proven as logically valid. As a consequence, formalization would become an unprofitable endeavor. Nearly all pre-theoretically valid arguments had to be classified as “semantically valid”. This is not in fact what is done. In sum, if one applies V$\Im$CR, strategies (i) and / or (ii) are almost always used. However, if this is conceded, one can hardly sustain the distinction of formally and semantically valid arguments. Instead, one should regard reducing the validity of arguments, if possible, to formally valid formalizations as a first task of logical formalization.

The problems of V$\Im$CR result from the reference to schematizations, which are indispensable if one intends to pass judgment upon the correctness of formalizations due to varying interpretations. Like CRVERB, all of the following models of formalization do not depend on schematizations and do not vary realizations.

3.2 Fixed interpretations

As in the philosophical tradition of logic, mathematical logic refers to realizations in terms of interpretations and presumes a purely extensional conception of interpretations. However, in contrast to the philosophical tradition, it is referred to a logical language that assigns a fixed interpretation not only to so-called “logical constants” but to *all* constituents of the formal language. The question of formalization thus becomes a question of translation (or encoding).

The objective of this model of formalization is to prove the truth of propositions by their derivation from axioms (or the falsity of propositions by deriving their negation from axioms). This objective would be satisfied if any true proposition is formalized by $P \vee \neg P$ (and any false one by $P \wedge \neg P$). However, the problem is that this would presume knowledge of the truth values. This is exactly what one wants to find by deriving the proposition or its negation within an axiomatic system. To do so, one must translate the proposition into a proposition of the presumed logical language. A logical language L is understood as a pair consisting of the recursively defined formulae, *L*, and the fixed interpretation, $\Im$, of all expressions of the alphabet of *L*. Formalization consists in translating ordinary propositions into propositions of a presumed logical language with a limited vocabulary. Thus, the objective is to prove the truth or falsehood of as many propositions that can be expressed by L as possible. These proofs are carried out within an axiomatic theory T that comprises only a limited number of non-logical axioms. Gödel has shown that this ideal cannot be realized to its full extent for basic arithmetic if basic arithmetic is formalized according to this model of logical formalization. In order to address this conception of formalization, predicates of ordinary language must first be expressed by predicates of L. This is done according to the following criterion:

express-criterion (ECR): A predicate *Px* of ordinary language is expressed by $\varphi(x)$ of L if and only if for all *x*

- if an object satisfies *Px*, then $\varphi(x)$ is true,
- if an object does not satisfy *Px*, then $\neg\varphi(x)$ is true.

The application of ECR does not presume that the truth value of *Px* is known for every object. Rather, it is presumed that one can judge whether *Px* and $\varphi(x)$ have the same extension according to $\Im$.

In the case of formalizing De Morgan's argument along these lines, one must presume a logical language with the predicates *Hx*, *Jx* and *Ixy* and their fixed interpretations in terms of *realization 1*. Thus, $\Im(H)$ is the extension of “_ is a horse”, $\Im(J)$ is the extension of “_ is an animal”, and $\Im(I)$ is the extension of “_

is a head of _”. According to ECR and L, the predicates “*x* is a head of a horse” and “*x* is a head of an animal”, which occur in (C), are to be translated as follows:
x is a head of a horse $=_{\text{Def}}$ $\exists y\ (Hy \wedge Ixy)$
x is a head of an animal $=_{\text{Def}}$ $\exists y\ (Jy \wedge Ixy)$

In contrast, $\forall y(Hy \wedge Ixy)$ or $\forall y(Jy \wedge Ixy)$ would not satisfy ECR.

To translate (C) into an expression in L, (C) must be rephrased as a proposition that can be effectively translated to L. (C) is of the form “Ys are Zs”. Propositions of this form are translated to general implications of the form “For all *x*, if *x* is *Y*, then *x* is *Z*”. This results in the following rephrase of (C), (C*) “For all *x*, if *x* is a head of a horse, then *x* is a head of an animal”. To translate (C*) to L, the definitions of “*x* is a head of a horse” and “*x* is a head of an animal” must be applied. This results in $\phi(C)$: $\forall x\ (\exists y\ (Hy \wedge Ixy) \rightarrow \exists y\ (Jy \wedge Ixy))$. This is an effective translation of (C**) “For all *x*, if some *y* exists such that *y* is a horse and *x* is a head of *y*, then some *y* exists such that *y* is an animal and *x* is a head of *y*”. (C**) is obtained from (C) by expressing (C) by means of the vocabulary of L. The formalization of (C) by $\phi(C)$ is based on the following criterion:

fixed-ℑ criterion (FℑCR): $\phi(A)$ of a logical language L with a fixed interpretation $\mathfrak{I}$, $\langle L, \mathfrak{I}\rangle$, is a correct formalization of a proposition *A* if and only if $\Phi(A)$ is an effective translation of *A* or of a rephrasing of *A* in respect to $\langle L, \mathfrak{I}\rangle$.

$\phi(C)$ (or $\neg\phi(C)$) is to be proven by derivation from axioms. For this sake, it suffices to introduce $\forall x(Hx \rightarrow Jx)$ as a non-logical axiom. This axiom expresses the relation of $\mathfrak{I}(H)$ and $\mathfrak{I}(J)$. The class of horses is a subclass of the class of animals. As a consequence, (C) is true as $\phi(C)$ is derivable from the axiom $\forall x\ (Hx \rightarrow Jx)$. Thus, the following criterion is satisfied:

capture-criterion (CCR): A proposition *A* is captured by an axiomatic theory T if and only if
- if *A* is true, then $T \vdash \phi(A)$,
- if *A* is false, then $T \vdash \neg\phi(A)$.

Thus, the aim of this model is satisfied in the case of De Morgan's argument as $\phi(C)$ follows from T, namely, $\forall x\ (Hx \rightarrow Jx)$.

This model of formalization does not intend to represent the meaning of a proposition or its truth conditions. This can be seen by the fact that ECR is still satisfied if arbitrary formulae that are true according to the fixed interpretation are added by conjunction. “*x* is the head of an animal”, for example, could also be expressed by $\exists y(Jy \wedge Ixy) \wedge \exists y \exists z(Hz \wedge Iyz)$. The same holds in the case of CCR and theorems of T that are added by conjunction. ECR and CCR are purely based on extensional considerations, cf. Smith(2007), p. 33-36.

In contrast to philosophically motivated models of formalization, the use of formalization within mathematical logic is not motivated by logical analysis of ordinary language or by proofs of the formal validity of informal arguments. Proofs of validity are only considered within the framework of an axiomatic theory T. If some informally valid argument cannot be proven as formally valid, this is not a deficiency of the formalization but rather of T. For example, the conclusion of *argument B*, cf. p. 9, follows trivially from the second premise and the axiom that all horses are animals. This can be shown by the fact that the formalization of the second premise implies Ha (= “Fury is a horse”). From this and the formalization of the axiom, $\forall x(Hx \rightarrow Jx)$, the formula Ja (= “Fury is an animal”) follows. Within the framework of T, the problem does not arise that the conclusion is not derivable. As it is not referred to schematization, the problem of distinguishing between formally and semantically valid arguments does not arise either.

As a matter of fact, philosophical requirements of a theory of formalization are not fulfilled by this mathematical model of formalization. One reason for this is that the problem of formalization is trivialized due to the presumption of an effective translation of the propositions in questions or of their rephrasing. Controversial formalizations of propositions that cannot be effectively translated into a logical language cannot be resolved by referring to F$\mathfrak{I}$CR. For example, one cannot argue with respect to F$\mathfrak{I}$CR why “Smith died because he ate tomato sorbet” is not correctly formalized by $P \wedge Q$ with “$\mathfrak{I}(P)$ = Smith died” and “$\mathfrak{I}(Q)$ = Smith ate tomato sorbet”. However, this model of formalization does not claim to fulfill such aims of other models of formalization. Only propositions that can be expressed within the vocabulary of L and that can be translated effectively to L are considered. By making use of logical formalization within mathematics, the only propositions that are considered are more or less standardized and are capable of being effectively translated to a logical language with a suitable vocabulary. A problem of formalization in which one has to identify the correct logical form of the propositions in the first place is not recognized within the framework of this model.

However, as I will explain in section 6, this also marks the problem of this model of formalization, even in applying it to mathematical propositions. The grammatical form of declarative sentences is seen as sufficient reason for their capability of being true or false and thus of their capability of being logically formalized. Whatever can be effectively translated to a logical language seems to be logically correct. Logical formalization is not a means of identifying fallacies that stem from taking the apparent form of ordinary propositions as their real, logical form. As we will see, it cannot be excluded on this basis that paradoxes relying on meaningless, inadmissible interpretations are expressed by apparently meaningful logical formulae. This problem is shared by F$\Im$CR and V$\Im$CR. Neither one provides a sufficient criterion for distinguishing admissible and inadmissible interpretations. As a consequence, they cannot distinguish between proofs by reduction, which prove the incompatibility or falsehood of axioms under the presumption of a correct formalization, and paradoxes, which rely on incorrect logical formalizations of meaningless propositions. We will come back to this in section 6.

4. Interpretations as restricted truth conditions

The following two models of formalization to be discussed in this and the following section adhere to the philosophical tradition of formalizing ordinary propositions independent of a logical language with a fixed interpretation and independent of axioms of a theory T. As opposed to the philosophical tradition of logic textbooks, the aim of these two models is not restricted to the proof of the validity of informal arguments. Instead, they are concerned with the logical analysis of ordinary language. They share the intuition underlying CRVERB: logical formalization serves to logically analyze the meaning of ordinary propositions. However, this is made precise within a framework of semantics that understands interpretations of a logical formula as an expression of *truth conditions* of meaningful propositions. In this respect, they differ from the semantics underlying the models described in the previous section. Both of the following models presume that formal and semantic validity cannot be reasonably distinguished. As a consequence, they account for the internal relation between the concepts *horse* and *animal* by formalizing De Morgan's argument.

In contrast to Wengert and the tradition of logical textbooks, Blau and Brun articulated criteria of formalization. Their criterion of correctness is as follows, cf. Brun(2004), p. 208:

TC: $\phi(A)$ is a correct formalization of proposition *A* if and only if $\phi(A)$ and *A* have the same truth values with respect to all interpretations in terms of conditions of truth or falsehood of *A* that are suitable according to the realization.

TC differs from V$\mathfrak{I}$CR in essentially two respects: (i) it refers to interpretations in terms of conditions of truth and falsehood, and (ii) it understands the truth conditions / interpretations as a function of the realization, cf. Brun(2003), p. 209-211. While V$\mathfrak{I}$CR considers the truth value of *different* propositions, *TC* refers to truth values of the *same* proposition with respect to *different* conditions. This determines a conception of interpretations that deviates significantly from that presumed in V$\mathfrak{I}$CR or F$\mathfrak{I}$CR. Interpretations articulate truth conditions of formulae and propositions. They represent *possible* extensions of categorematic parts, namely, possible extensions of predicates, possible truth values of atomic propositions and possible references of names. In contrast, V$\mathfrak{I}$CR and F$\mathfrak{I}$CR presume that any interpretation refers to the actual extension of a predicate, atomic proposition or name. For example, if one interprets *P* by “Paris is the capital of France”, then $\mathfrak{I}(P) = T$ according to traditional semantics, as Paris is indeed the capital of France. According to *TC*, however, $\mathfrak{I}(P)=T$ and $\mathfrak{I}(P)=F$ are two possible truth values of the atomic proposition “Paris is the capital of France”. Thus, according to this semantics, the interpretations vary in terms of possible extensions of predicates, atomic propositions and names, while the predicates, atomic propositions and names are assigned to fixed expressions by the realization. According to this understanding, the question in each case is how the truth value of the proposition to be formalized and the truth value of the formula depend on the respective possible extensions of the categorematic constituents.

According to Blau's and Brun's model of formalization, it may happen that certain extensions of the categorematic parts are unsuitable interpretations, as they constitute unsuitable conditions of truth or falsehood due to their meaning. For example, the realizations of the formalizations $\phi(C)$ and $\phi(C)'$ both refer to the concepts *horse* and *animal*. Due to the meaning of these two concepts, interpretations in which the class of horses is not a subclass of the class of animals are unsuitable. This takes into account the conceptual, internal relation of horses and animals according to which it is impossible that some horse is not an animal. This is no possible condition that allows one to judge the truth or falsehood of (C). Interpretations in terms of varying possible extensions of fixed concepts (propositions, names) only determine consistent conditions of truth or falsehood if “unsuitable interpretations” are not taken into account. That is why *TC* refers to suitable interpretations as a function of the realizations. From this, it follows that, in the case of formalizing (C), the logical difference between $\phi(C)$ and $\phi(C)'$ is of no consequence: the interpretations that are models of $\phi(C)$ but not models of $\phi(C)'$ are unsuitable. For this reason, Brun qualifies both $\phi(C)'$ and $\phi(C)$ as correct formalizations of (C). Both $\phi(C)$ and $\phi(C)'$ have the same truth value that (C) has with respect to all suitable interpretations.

Compared to CRVERB, this conception has the advantage that it explicates the equivalence or identity of meaning of the formalization and of the proposition in question in terms of identity of truth conditions. Regarding V$\Im$CR, *TC* does not refer to the problematic schematization of ordinary propositions. However, the formalization of (C) reveals a fundamental problem of this conception. Non-equivalent formulae, such as $\phi(C)$ and $\phi(C)'$, are equivalent with respect to the restricted space of possible interpretations. This shows that this conception is incompatible with the traditional semantics of first-order formulae. Thus, formulae become tautologies that are no tautologies according to common standards of first-order logic. Finally, not only the interpretations but also the truth conditions and logical relations of first-order formulae become a function of the realizations and, thus, of the proposition to be formalized. In fact, one does not explicate the truth conditions and logical relations of ordinary propositions by means of formalizations *within first-order logic*. This problem is labeled "the problem of suitable interpretation"', cf. Baumgartner/Lampert(2008).

Another problematic consequence of the impossibility of identifying only one correct formula out of multiple non-equivalent formulae is the impossibility of deciding upon the validity of arguments in certain cases. For example, this problem arises in the case of formalizing *argument B*, cf. p. 9. The formalization of (C) by $\phi(C)$ results in an invalid formalization (according to standard first-order logic), while the formalization of (C) with $\phi(C)'$ results in a valid formalization. A "problem of validity" arises from this and further assumptions of this model of formalization, namely, the problem of identifying the validity of certain valid arguments, cf. Lampert/Baumgartner(2010). This problem complements the so-called "problem of invalidity" identified by Massey(1975). This problem results from the fact that formally invalid formalizations may be correct for valid arguments, while no criterion is available to constrain the class of potentially correct formalizations to be finite. Thus, it is impossible to conclude the invalidity of the formalized argument from a correct and invalid formalization. In the following section, I propose a modification of *TC* that solves all of the aforementioned problems.

5. Interpretations as unrestricted truth conditions

The "problem of suitable interpretations" arises from the fact that the space of possible interpretations is restricted by logical dependencies that are due to the realization's concepts. This problem does not arise if one claims that the concepts of a realization must be logically independent. This means that the space of possible interpretations is unrestricted. I call realizations satisfying this condition "suitable realizations". The semantics based on interpretations in terms of conditions of truth or falsehood must rely on their logical independence. Traditional semantics are based on the principles of bivalence and extensionality. The principle of bivalence states that any proposition is either true or false. The principle of extensionality states that the truth or falsehood of any proposition depends on nothing but the extension of the categorematic parts. Within semantics referring to truth conditions, these two principles must be complemented by the principle of logical independence. This principle claims that all interpretations that may be generated by combinatorial means are also possible. Thus, if *n* propositional variables occur in the realization, 2^n interpretations are admissible. If *u* predicates with arity k_1 ... k_u occur in the realization, $2^{i^{k1}} + \ldots + {}^{i^{ku}}$ interpretations must be possible with respect to a domain with *i* objects.[2] Any name can be interpreted by any of these *i* objects. According to this understanding, the principle of independence implies the principle of bipolarity, which states that any atomic proposition *may* be either true or false. The principle of independence is indispensable for any theory of formalization based upon both: (i) semantics of truth conditions and (ii) standard first-order logic (without identity). Any formula that is a tautology according to the standard extensional semantics is also a tautology according to this semantics and v.v. Within this conception, the suitable realizations assign meaningful expressions to the categorematic parts of the formulae. These assignments are fixed, while their interpretations in terms of their possible extensions vary without any restrictions. Only truth conditional semantics relying on the principle of independence make a logical analysis of ordinary language possible, which reduces and explicates informal logical dependencies within ordinary language to formal logical dependencies of first-order formulae. To satisfy this claim, no internal, logical dependencies must exist between the categorematic parts of the realizations.

The criterion of correctness established by this model results from modifying *TC*:

TC': $\phi(A)$ is a correct formalization of a proposition *A* if and only if $\phi(A)$ and *A* have the same truth values with respect to suitable realizations that do not restrict the space of possible interpretations in terms of conditions of truth and falsehood of *A*.

[2] In fact, we confine domains to enumerable numbers of objects, which also suffices according to the Löwenheim-Skolem theorem of traditional semantics. Referring to “non-denumerable” domains makes no sense according to a truth-conditional semantics. However, the important point is that no limitations due to the concepts of the realization are called for. Possible interpretations are solely defined by combinatorial means within the realm of enumerability.

Let us illustrate the application of this criterion by formalizing De Morgan's argument. Like Brun, it is assumed that the concept *horse* contains the concept *animal*. In contrast to all other models of formalization, *TC'* thus rules out realizations that contain both of these two concepts. Rather, *TC'* reduces this conceptual implication to a formal one. For the following, it is assumed that horses are defined as animals with a certain differentia specifica, say “tiptoeing equid”'. We also presume that it is possible to be a tiptoeing equid but not an animal. This may, in fact, be false, but it shall not be excluded by the meaning of the concepts. According to these assumptions, the following formalization of De Morgan's argument is correct according to *TC'*:

(P): Horses are animals.
$\phi(P)_{TC'}$: $\forall x((Gx \wedge Jx) \rightarrow Jx)$.

(C): Heads of horses are heads of animals.
$\phi(C)_{TC'}$: $\forall x((Gx \wedge Jx \wedge Ix) \rightarrow (Jx \wedge Ix))$

Suitable realization:

   Gx: *x* is a tiptoeing equid.
   Jx: *x* is an animal.
   Ix: *x* has a head.

It would also be possible to replace *I*x with the dyadic predicate *Iyx*, representing “*y* is a head of *x*”. Regardless of whether *y* would be bound by an existential quantifier or a universal quantifier, the resulting formula would be a tautology. However, in contrast to all of the other proposals for formalizing De Morgan's argument, it is not necessary to introduce a dyadic predicate to derive (C). According to *TC'*, both the premise and the conclusion of De Morgan's argument are tautologies if one presumes that horses are defined as animals. This seems unexpected, as De Morgan's argument suggests that (C) follows from (P) and not from any premise. However, the presumed conceptual relation between the concepts *horse* and *animal* is already contained in the conclusion. This is taken into account by $\phi(C)_{TC'}$ by virtue of representing the concept *horse* by $Gx \wedge Jx$ and the concept *animal* by *Jx*. According to this understanding, it is still adequate to say that heads of horses are heads of animals *because* horses are animals. However, this justification is not expressed by a formally valid inference from a premise that is assumed to be true. Instead, this internal relation is expressed within the formalization of (C). Thus, (P) is not a falsifiable proposition on which the truth of the conclusion depends. Instead, (P) articulates a conceptual relation that must be taken into account if the truth conditions of (C) are to be correctly represented. It is the correct formal representation of (C) takes into account the internal relation of the concepts *horse* and *animal* and this demonstrates (C)'s validity.

In contrast, the similar propositions (R), cf. p. 5, and (S), cf. p. 6, are not to be formalized by tautologies. A *TC'*-correct formalization of (R) is the following:

(R): Children of mothers are children of fathers.
$\phi(R)_{TC'}$: $\forall x(\exists y\, (Hy \wedge Ixy) \rightarrow \exists y(\neg Hy \wedge Ixy))$

Suitable realization:

Hx: *x* is a woman.
Ixy: *x* is a child of *y*.

This formalization presumes that no object exists that (i) is neither a woman nor a man and (ii) is a woman and a man. This is plausible on the basis of human beings as domains. Thus, men are definable by "*x* is not a woman".

(S): Bets on winning numbers are bets on prime numbers.
$\phi(S)_{TC'}$: $\forall x\, \forall y((Hy \wedge Ixy) \rightarrow (Jy \wedge Ixy))$

Suitable realization:

Hx: *x* is a winning number.
Jx: *x* is a prime number.
Ixy: *x* is a bet on *y*.

This formalization expresses that (S) means that any bet on a winning number is also a bet on a prime number. In this sense, the concepts *winning numbers* and *prime numbers* are logically independent, but as a matter of fact all winning numbers are prime numbers. On the other hand, if (S) means that whoever bets on a winning number also bets on a prime number, the following formalization with the respective realization would be *TC'*-correct:

$\phi(S)_{TC'}$: $\forall x(\exists y\, (Hy \wedge Ixy) \rightarrow \exists y(Jy \wedge Ixy))$

Thus, one should illustrate the limits of monadic first-order logic by referring to an argument based on (S) rather than referring to De Morgan's argument.

The key point is that the respective formalization is understood as an explication of the truth conditions of the proposition to be formalized. It is by no means presumed that the formalization of an ordinary proposition must be unambiguous. Rather, logical formalization presents a means for unambiguously expressing the respective meaning. It is also not assumed that the mentioned

TC'-correct formalization of De Morgan's argument is the only possible correct explication of (P) and (C). Rather, the formalization depends on the assumed relation of the concepts *horse* and *animal*. If one does not assume that these two concepts are logically dependent, *TC'* claims a different formalization. If it is assumed that it is not meaningless that horses exist that are not animals, the common formalizations might well be *TC'*-correct. With this assumption, Wengert's formalization, $\phi(C)'$, is *TC'*-correct if it means that any horse with a head is an animal with that respective head. The traditional formalization, $\phi(C)$, is correct if it simply means that whenever something is the head of a horse it is also the head of an animal. Of course, the stronger claim follows from the assumption that all horses are animals. However, this does not mean that this stronger claim is inferred.

The purpose of logical formalization according to this model is to explicate the truth conditions of ordinary propositions. This model explains precisely what is meant by a formal representation of the meaning of propositions; namely, the representation of their truth conditions by a logical formula according to suitable realizations. It is important to note that the space of possible interpretations is defined independent of and prior to the categorematic parts of suitable interpretations. The form of propositions representable by first-order logic is thus not defined by ordinary language; one needs to refer neither to the grammatical surface nor to some assumed deep structure of ordinary propositions. Instead, the form of propositions is provided by first-order logic, namely, by its ability to represent truth conditions. Logical formalizations determine whether some ordinary proposition has the form of a proposition with precisely defined truth conditions according to logic.

The vagueness of ordinary language is no objection against the conception of logical formalization in terms of the logical analysis of ordinary language. Rather, this vagueness motivates the task of logical formalization. It is also not presumed that ordinary propositions must have definite truth conditions with respect to context and speaker. The purpose of logical formalization is to express possible meanings of ordinary propositions. This is compatible with the fact that no purported formalization expresses what is meant by some proposition. In this case, what is not meant by the proposition is at least clarified. This leaves the question open as to whether it is at all possible to express the meaning of a proposition within first-order logic. In any case, logical formalization fulfils the purpose of explicating the meaning of propositions by providing the means to precisely explain their truth conditions.

TC' overcomes the problems of the other models of formalization. In contrast to *TC*, the problems of validity and invalidity are solved in addition to the problem of suitable interpretations. This is because by claiming identical truth conditions with respect to suitable realizations, valid arguments are formalized correctly if and only if their formalization is formally valid. The problems of VℑCR result from the reference to schematization. The semantics according to which interpretations represent conditions of the truth and falsehood of propositions make this reference superfluous. In contrast to VℑCR, *TC'* reduces informal logical dependencies of ordinary concepts and propositions to formal relations within logic. Furthermore, in contrast to VℑCR and FℑCR, TC' is not in need of a criterion for admissible interpretations. Any interpretation that is possible according to combinatorial means is also admissible, and it represents a condition that allows one to judge the truth value of a proposition. If this claim is not satisfied, then the claim of suitable realizations is also not satisfied. As a consequence, the formalization is not *TC'*-correct. If some proposition does not have truth conditions that can be represented within logic according to *TC'*, it is meaningless according to logic. The crucial advantage of the truth conditional semantics, as compared to the semantics that VℑCR and FℑCR rely on, is that they do not refer to *actual* extensions, but more basically to *possible* extensions. Thus, it is possible to use logical formalization as a means of determining the meaning of propositions rather than simply assuming it. Furthermore, standardizations are not assumed but instead result from paraphrases of *TC'*-correct formalizations. In contrast to CRVERB, judgments of equivalence are not assumed to be primitive. Rather, they are based upon explications of truth conditions, which, in turn, refer to the construction of possible interpretations. It is even possible to refer to an effective, mechanical procedure to generate verbalizations in terms of explications of truth conditions. At least this is possible in so far as it is possible to not only enumerate single models (conditions of truth) and counter-models (conditions of falsehood), but also to identify the class of models and counter-models by certain distributive normal forms, cf. Lampert(2006).

The presumed semantics of *TC'* as well as the objective of explaining the truth conditions of ordinary propositions are rooted in the philosophical tradition of logical analysis of ordinary language. The most decisive articulation of this conception can be found in Wittgenstein's *Tractatus*. The main reason that this model of formalization is rarely articulated and is not applied to all consequences lies in the strong claim of suitable realizations. The principle of logical independence seems to be unsatisfiable with respect to logical dependencies of ordinary language expressions. Thus, like VℑCR, *TC'* runs the risk of unduly restricting the realm of informal arguments that can be formalized within first-order language. For example, should one decline to formalize “All men are beings. All beings are mortal. Therefore, all men are mortal” by means of a syllogismus barbara merely because the respective realization is not suitable? Or should one question the formalization of an inference by means of a modus ponens simply because, by adding an arbitrary premise, the principle of logical independence is no longer satisfied? Finally, does *TC'* not rely on the unrealistic presumption that propositions of ordinary language can be analyzed as a truth function of logically independent, bipolar atomic propositions?

Wittgenstein dealt with questions like these in his *Notebooks from 1914-16*. How can logic be applied to ordinary language, he asked, if one cannot carry out the complete analysis of ordinary propositions, cf., for example, Wittgenstein(1995), diary entries from 3.9.-7.9.1914, 11.10.1914, 20.6.-22.6.1915? On the one hand, Wittgenstein assumed in the *Tractatus* that the real, logical form of ordinary propositions can only be revealed by analyzing them in a truth function of logically independent, bipolar atomic propositions, cf. Wittgenstein(1995), remark 5. On the other hand, he assumed that logic can be applied to unanalyzed propositions in so far as their categorematic parts can be treated as if they were primitive and logically independent, cf., for example, Wittgenstein(1995), entries form 11.10.1914[2], 21.6.1915[10]. The mentioned disagreeable consequences can be avoided if the demand of reducing informal logical dependencies to formal logical relations is met in a pragmatic and context-dependent way. From the possibility of identifying further informal logical dependencies by formal ones, it does not follow that one must do so. One may well treat expressions of the realization as if they were primitive and logically independent, even if they might be capable of some further analysis. The application of *TC'* achieves neither more nor less than making explicit the internal relations that follow from this assumption. Any further detailed analysis can only reveal more internal relations and thus invoke a more thorough understanding of the meaning of the proposition to be formalized. However, no further detailed analysis can revise identified internal relations that are already identified by a superficial analysis. For example, one can accept the traditional formalizations of De Morgan's argument, $\phi(C)$ or $\phi(C)'$, as an expression of the internal relation between the premise (P) and the conclusion (C) that results even if one does not consider the relations between the concepts *horse* and *animal* as internal. A more thorough analysis considers this relation within the formalization of the conclusion (C). This results in $\phi(C)_{TC'}$, which does not derive the conclusion of the argument from some external relation of the class of horses and the class of animals, but rather from the internal relation of the concepts *horse* and *animal*. This does not reject the justification of (C) by (P) but makes explicit that this conceptual relation is already implied by (C) itself.

According to this model, the benefit of logical formalization consists in making explicit the truth conditions of ordinary propositions. How far one abstains from implied logical dependencies depends on the context and the aim of the respective formalization. Finally, by means of Russell's Paradox, we will illustrate the importance of relativizing inferences from formalizations to presumed criteria of formalization and to the grade of analysis.

6. Inadmissible interpretations

Within mathematical logic, Russell's Paradox is mostly understood as a refutation of the unrestricted comprehension axiom schema. Expressed within ordinary language, this axiom of “naïve” set theory is as follows:

UCAS: There exists a set *y* whose members are precisely those objects that satisfy the propositional function $\varphi(x)$.

According to F$\Im$CR and a logical language with the dyadic predicate $x \in y$ and its fixed interpretation as “*x* is a member of *y*”, UCAS is to be formalized as follows:

ϕ(UCAS): $\exists y \forall x(x \in y \leftrightarrow \varphi(x))$

The replacement of $\varphi(x)$ with “*x* is not a member of itself” and $\neg x \in x$, respectively, results in Russell's Paradox:

UCAS*: There exists a set *y* whose members are precisely those objects that satisfy the propositional function *x is not a member of itself.*
ϕ(UCAS*): $\exists y \forall x(x \in y \leftrightarrow \neg x \in x)$

From ϕ(UCAS*), a contradiction follows. In the framework of modern set theory, this is taken as a sufficient reason to conclude that UCAS is false. Russell's Paradox apparently demonstrates that there are concepts that do not define sets. According to F$\Im$CR, this reasoning is conclusive. According to modern mathematical logic, Russell’s Paradox refutes naïve set theory and calls for a different axiomatic system such as ZF that does not allow for Russell’s Paradox due to the axiom of separation. Likewise, according to V$\Im$CR, ϕ(UCAS*) is correct, and thus, UCAS* is contradictory.

Russell himself argued this way in *Principles of Mathematics*, Russell(1992), p. 102f. However, ϕ(UCAS*) is incompatible with the view that Russell and Whitehead advanced in *Principia Mathematica*, cf. Whitehead(1910), p. 75. Here, they analyzed $\in$ as an incomplete symbol. This analysis is incompatible with the understanding of $\in$ as a primitive symbol in terms of a dyadic predicate. According to the point of view put forth in the *Principia*, not all interpretations of $x \in y$ are admissible.

Within the framework of a theory of formalization, such a critique is opened up by *TC'*. $\in$ does not satisfy this criterion as its interpretation is not unrestricted. At least some interpretations do not lead to meaningful, bipolar propositions. Thus, correctly understood, Russell's Paradox is not a reduction to absurdity of the unrestricted comprehension axiom schema. Instead, the fault of the paradox lies in the formalization of "*x* is a member of *y*" in terms of a dyadic, logical predicate that articulates a primitive relation between objects. $x \in y$ is not a proper propositional function identifying sets. According to this point of view, the solution of the paradox consists in an alternative formalization of propositions about sets, for example propositions such as UCAS. However, it is not necessary to abandon UCAS. The paradox, in terms of a reduction to absurdity of an assumption that seemed to be true, is due to a mistaken formalization.

Only by *TC'* can one distinguish between two kinds of "absurdity": (i) the absurdity relying on a proof by reduction and (ii) the absurdity relying on an inadequate logical formalization. Without *TC'*, there is no sufficient criterion for distinguishing between meaningful but inconsistent propositions and meaningless propositions that cannot be formalized within logic. In the first case, we have a contradiction within the axiomatic system, while in the second case, we have a contradiction to logic. Put more precisely, we have a contradiction to the principles of a logic that provides the means for representing propositions with well-defined conditions of truth and falsehood. In this case, the propositions to be formalized cannot be true because well-defined interpretations in terms of meaningful conditions of truth and falsehood have not yet been specified. In the former case, in contrast, the propositions cannot be true because all interpretations are counter-models.

What seems to be a proof by reduction according to a superficial analysis may show up as a paradox according to a more thorough logical analysis. In this case, it is not the falsity of some axiom that is proven, but rather the impossibility to represent it by a proposition with well-defined truth conditions according to first-order logic. In the case of Russell's Paradox, it is proven that the so-called "relation of membership" cannot be represented by an atomic propositional function expressing an external, primitive relation between objects. Such a relation presumes that the objects satisfying the relation are identifiable independent of the relation. This criterion is not satisfied in the case of membership if a set is considered to be (or not to be) its own member. Such an understanding contradicts the principle of logical independence. In consequence, one cannot represent "*x* is (not) a member of *y*" by means of a propositional function $\varphi(x)$. Thus, it is the substitution of $\varphi(x)$ with $\neg x \in x$ that must be rejected in the first place. Such a substitution mistakenly confuses the grammatical form with the logical form of propositions. According to this kind of critique, the meaning of UCAS cannot be represented by some logical formula and even less so by the non-tautologous formula ϕ(UCAS). This is due to the fact that it does not make sense to assume well-defined propositional functions in terms of first-order logic that do not identify sets. Within this conception, it is not even possible to interpret a proper propositional function of logic such that it does not identify a set. Whatever does not identify a set is not a proper propositional function. To be a member of a set means to satisfy some propositional function identifying that set. Thus, what one intends to say by (UCAS) is quite right. Yet, (UCAS) cannot be articulated as a meaningful proposition within the logical symbolism because doing so would presume mistakenly that the relation between sets and propositional functions is external. The problems of "naïve" set theory do not arise before one makes use of a logical formalization naively expressing membership as a primitive relation between objects. Only a superficial analysis, "bedeviled" by surface grammar of ordinary language, makes such a deficient logical formalization of set theory possible.

Within the framework of a theory of formalization, it is irrelevant to consider whether such a critique of the logical formalization of set theory is adequate. What is more important is that it is possible. It should not be excluded by presuming some model of formalization without discussing and arguing against its alternatives. In contrast to VℑCR and FℑCR, *TC′* articulates assumptions of logical formalization concerning the alleged meaning of ordinary propositions. Whether these assumptions are valid is unimportant for a theory of formalization. Yet, it is important to identify them as assumptions that can be questioned. Only *TC′* makes it possible to use logical formalization as a means of logically analyzing the meaning of propositions. In contrast, VℑCR and FℑCR presume that grammatically well-formed propositions have a truth value without considering their truth conditions. However, in order to assume that some proposition *is* true or false, it must be *capable* of being true or false. VℑCR and FℑCR do not consider this priority of the meaning of propositions over their truth value. Thus, they cannot rule out that nonsense is represented by logic.

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