Wittgenstein's programme of a New Logic

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1. Introduction

In his earliest writings, *Notes on Logic* (NL), *Moore Notes* (MN) and *Tractatus logico-philosophicus* (TLP), Wittgenstein calls his conception of logic self-confident "New Logic". He opposes his New Logic to the "Old Logic", which he identifies with the logic of Frege and Russell. From 1912 to 1914 he confronted Russell with his work on New Logic. Soon Russell accepted Wittgenstein as his "master" (Monk (1990), chapter 3). Russell and Whitehead wanted Wittgenstein to work over *Principia Mathematica* (PM) (cf. Pinsent (1990), p. 60). Finally, Russell expected that the elaboration of Wittgenstein's New Logic would displace PM as paradigm of modern logic. This expectation was not fulfilled. The explanation for this is at hand: Wittgenstein's conception of logic could only be realized in propositional logic but not in predicate logic. In fact, no suggestions according proofs of predicate logic can be found in TLP. This seems to confirm the common judgement that Wittgenstein's main contribution to logic consists in the development of truth-tables, while his conception of logic is not able to supply any substantial contribution beyond propositional logic (cf. Black (1964), p. 323, Anscombe (1996), p. 137, cf. also the footnote of the editors in *Cambridge Letters* (CL), p. 52).

Yet, it is not taken into account that Wittgenstein did not think of truth-tables as the proof method of his New Logic but of the so called "ab-notation", a logical notation he worked on intensively in 1913/14. It is this notation he identifies with the "new notation" in opposition to the "old notation" of Frege and Russell (NL, p. 93[1]). The method of truth-tables – "WF-schemata" in Wittgenstein's terminology – was already worked out by Wittgenstein in 1912 (cf. Shosky (1997), p. 20). Contrary to the method of truth-tables, Wittgenstein's intention by developing the ab-notation was to realize his conception of logic in the realm of predicate logic (cf. CL, letter 28, p. 4, against Biggs (1996), p. 27). The question in how far Wittgenstein's New Logic can be realized depends first and foremost on the question in how far his ab-notation is applicable to predicate logic.

Unfortunately, the notebooks from 1913/14 dealing with the ab-notation have not been received (cf. CL, letter 32, p. 58 and Biggs (1996), p. 11). Thus, one has to rely on the scanty remarks in NL, MN, CL from 1913/14. Furthermore, the understanding of the ab-notation even in the realm of propositional logic was hampered by the fact that all received diagrams of the ab-notation were reproduced mistakenly or not even printed in the first editions of NL, MN, CL. In addition to the misjudgement that the method of truth-tables displaced the ab-notation this accounts for the fact that Wittgenstein's ab-notation remained nearly disregarded up to now in the literature. Yet, Wittgenstein did not doubt the validity of the ab-notation for the whole realm of predicate logic. Merely the handling of identity within the ab-notation was an open question for him (cf. CL, letter 30, p. 53). Likewise, he does not confine his understanding of logical proofs to propositional logic in TLP and still speaks of the "Old Logic" in opposition to his "New Logic" (TLP 4.126, 6.125). It was not Wittgenstein's intention to work out in detail his conception of a New Logic in TLP, but he had no doubt on the feasibility of this project. As the editors of CL point out rightly this contradicts Church's theorem of the undecidability of predicate logic (cf. CL, p.52). However, one is unable to judge upon Wittgenstein's programme if one concludes from this that Wittgenstein's programme is doomed to failure. First of all, throughout his life Wittgenstein was critical about metamathematical proofs and their methods – these proofs are not independent of the conception of Old Logic. Furthermore, merits and anomalies of the Wittgensteinian paradigm can only be discussed in a logically and philosophically fruitful manner by elaborating it. This, in turn, presupposes an understanding of its main ideas. In what follows the objective of Wittgenstein's New Logic will be lined out in contrast to the Old Logic. The detailed elaboration of his programme of a New Logic is given in my book "Wittgenstein's New Logic", which works out Wittgenstein's ab-notation for first order logic.

2. Old vs. New Logic

In MN, p. 109[5] Wittgenstein describes the "procedure of the old Logic" as follows:

> This is the actual procedure of [the] old Logic: it gives so-called primitive propositions; so-called rules of deduction; and then says that what you get by applying the rules to the propositions is a logical proposition that you have proved.

This is just the common understanding of logical proofs in the sense of derivations within an axiomatic system. Frege's and Russell's systems satisfy this proof conception as well as modern sequence calculi do: A formula is proven by deducing it from the axioms applying derivation rules. Wittgenstein does not deny that logical true formula, tautologies, can be identified by this procedure. Yet, he emphasizes that their logical truth cannot be proven this way. He goes on to say:

> The truth is, it tells you something about the kind of proposition you have got, viz that it can be derived from the first symbols by these rules of combination […].

What is proven by the axiomatic proof procedure is simply the deducibility of the formulae from the axioms. This is not denied within the framework of classical logic either. It is an accepted truism that only by assuming the logical truth of the axioms and the correctness of the derivation rules the logical truth of theorems can be concluded from their deducibility. Not the content of Wittgenstein's remark that proofs within an axiomatic system are in need of a metalogical justification is illuminating but the fact that he opposes his conception of a New Logic to this common understanding of logical proofs. Through his life Wittgenstein opposed to the understanding of logical and mathematical proofs resting on axioms, because one has to rely on some metalogical, intuitive evidence if one does not only want to maintain the deducibility of theorems but their logical or mathematical truth. In PG, p. 297 (cf. TLP 6.1271) he says:

> Logic and mathematics are not *based on* axioms, […]. The idea that they are involves the error of treating the intuitiveness, the self-evidence, of the fundamental propositions as a criterion for correctness in logic.

Axiomatic proofs do not deliver a purely syntactical criterion for logical properties of arbitrary formulae of a formal system. The axioms are taken for granted without a formal proof. They hold an exceptional position within the system, but this position is not justified syntactically – the axioms are formulae within the system and do not differ essentially from other formulae. This can be seen by the fact that there are several correct and complete axiom systems for the same formal system and by the fact that not all axioms have some syntactical feature in common that identifies them as axioms. The common understanding of logical proofs in the sense of derivations from axioms depends on proofs of the logical truth of the axioms and of the correctness and completeness of a calculus relative to some prior given semantics. Such proofs cannot be carried out within formal logic. Thus, the question arises to the metalogical justification of an axiomatic calculus. Such a foundation necessarily exceeds the limitations of admissible evidence in logic. One objective of Wittgenstein's New Logic is to replace axiomatic proof procedures by a proof procedure that is not in need of such a metalogical foundation. In TLP 6.1265f. he says:

> It is always possible to construe logic in such a way that every proposition is its own proof.
>
> All the propositions of logic are of equal status: it is not the case that some of them are essentially primitive propositions and others essentially derived propositions.
>
> Every tautology itself shows that it is a tautology.

That logical propositions are "their own proof" or tautologies "show themselves" to be tautologies does not mean that there is no need for proofs in the sense of manipulations of formulae in order to identify tautologies as tautologies. It only means that this can be done by relying solely on the formulae themselves as starting points of the proof instead of relying on axioms. In this respect Wittgenstein was looking for something similar to tableaux procedures such as Beth's or Smullyan's procedure (cf. Beth (1962), Smullyan (1965)). Yet, contrary to these procedures New Logic does not only aim for a procedure in order to identify tautologies but for a procedure applicable to "every proposition", i.e. any predicate formula, in order to identify its truth conditions. In Wittgenstein's conception proofs in the sense of derivations of theorems from axioms are replaced by proofs in the sense of converting formulae to symbols of an ideal notation that allow to identify unambiguously tautologies and, generally, the truth conditions of any formula by the syntactical features of the ideal symbols. Again and again Wittgenstein stresses that one has to identify tautologies "from the symbol alone" (TLP 6.113) or that one can "[recognize] in a suitable notation […] the formal properties of propositions by mere inspection of the propositions themselves" (TLP 6.122). Axioms, i.e. formulae with an exceptional position within a logical system, are not needed in this conception, because presuming a sufficient notation, which identifies the truth conditions of all formulae likewise, every formula "is its own proof" (TLP 6.1265, cf. 6.127f.): The proof does not consist in a derivation of formulae from formulae of the same system but in a conversion of the formula in the symbols of an ideal notation according to a general procedure wholly depending on the syntax of the initial formula. Put concisely, the proof conceptions can be opposed as follows.

Proof conception of Old Logic:

Axioms $\Rightarrow$ formula

The formula in question marks the end of the proof. It has to be a theorem in order to be provable. Proofs of the truth conditions of formulae not being theorems are not available in this conception.

Proof conception of New Logic:

Any Formula $\Rightarrow$ ideal symbol

The ideal symbol identifies the truth conditions of the initial formula. Wittgenstein exemplifies his proof conception in TLP 6.1203 for propositional formulae by introducing a notation using brackets that is similar to the ab-notation. One might also think of the truth-table method as a well known procedure that realizes this proof conception basically. In case of truth-tables the ideal symbol consists of the assignment of truth values, *T* and *F* respectively, below the main sentential connective to the truth values of the propositional variables in the left part of the truth-table. The objective of the ab-notation is to realize such a proof conception for predicate logic.

By the endeavour of Wittgenstein's New Logic it shall be demonstrated by purely logical means that an understanding of logic in the sense of an axiomatic theory, which is not based on purely syntactical grounds, is superfluous. It is not maintained that axiomatic proof systems are mistaken. However, in logic their form is misleading in so far it suggests that logic rests on some truth beyond symbols and their rule-governed manipulation and in so far it evokes problems as the foundation of axioms or the correctness and completeness of the axiomatic system, which, according to Wittgenstein's point of view, should be solved by changing the logical point of view rather than going beyond it. Thus, with the conception of New Logic a certain philosophical point of view concerning the understanding and foundation of logic is at stake. The ambitious objective is to justify stringently a Wittgensteinian understanding of logic by construing a logic system of an alternative form without delivering different logical results, i.e. without identifying truth conditions of formulae that they do not have according to classical logic.

Wittgenstein's proof conception brings forth that syntax and semantic do not fall apart as in classical logic. By the proof procedure the truth conditions of the formulae become obvious. In this respect it provides a semantic in the sense of a theory defining truth conditions of formulae. Thus, it is not in need to be justified by some prior, independent given semantics. This, of course, does not mean that it cannot be compared to classical semantics. Furthermore, it should be demonstrable that both concepts of semantics are compatible, because otherwise not both would concern the same logic. Yet, the truth conditions need not to be identified by some procedure or some considerations external to the syntactical manipulations of the proof procedure itself. Every step in the procedure is a step in clarifying the truth conditions and nothing more can and is to be done than defining the steps explicitly. In consequence, not the question of correspondence of syntax and semantic is in the focus of Wittgenstein's conception but the question how an ideal notation looks like that identifies truth conditions of the formulae unambiguously and how a procedure can be defined in order to convert formulae in the symbols of such an ideal notation.

Wittgenstein's conception differs significantly from the traditional point of view by regarding the syntax of predicate logic as deficient because of the fact that the truth conditions of predicate formulae cannot be identified by relying on its syntactical features. Repeatedly he identifies as the reason of his rejection of the syntax of predicate logic – the "old notation" – that syntactically different formulae might be equivalent. E.g. in NL, p. 102[3] he says (cf. NL, p. 93[1], TLP 5.43):

> If p = *not--not--p* etc.; this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that, in analyzing such propositions, we must not be guided by Russell's method of symbolizing.

Commonly, the language of predicate logic is regarded as an ideal language in contrast to natural language, because it is set up recursively and it is unambiguous in so far every formula expresses a certain truth function of atomic propositions. However, according to Wittgenstein's point of view this is not sufficient, because identical truth functions can still be expressed differently. In this sense, the syntax of predicate logic shares a deficiency with natural language. The problem is not primarily that signs of different types are equivalent, but that no general syntactical criterion exists to identify equivalent symbols as equivalent (cf. NL, p. 94[3], p. 99[2], p. 101[7]). This gets manifest by considering equivalent formulae differing in several respects, such as the following formulae:

(1) $\exists x_1 \forall x_2((Q \wedge \forall x((\exists y \exists z Ixyz \wedge \neg Q) \vee (\forall x_3 \exists x_4 Hx_3x_4 \wedge \neg Q))) \vee ((\neg Fx_2 \wedge Gx_1) \vee Hx_2x_1))$

(2) $\neg \forall y \exists x \neg((\neg Fx \wedge Gy \wedge P) \vee (\neg Fx \wedge Gy \wedge \neg P) \vee Hxy)$

(3) $\exists y \forall x Hxy \vee \exists y(\forall x(\neg Fx \vee Hxy) \wedge Gy)$

According to classical logic it is possible to prove their equivalence by deducing one from the other. However, it is not possible to identify a syntactical feature that (1) to (3) have in common in virtue of that they are equivalent. The fact that the truth conditions cannot be identified by means of the syntax of predicate formulae also becomes evident if one considers non-equivalent formulae: The differences of their truth conditions cannot be identified by syntactic criteria. Moreover, mostly it cannot even been proven syntactically that the formulae are not equivalent.

In the framework of Wittgenstein's New Logic not laying down axiomatic calculi with certain metalogical properties is the first task of logic but solving the equivalence problem.

Equivalence problem: The equivalence problem is the problem to define a mechanical procedure such that the same symbol is assigned to every predicate formula of a class of equivalent formulae and different symbols are assigned to non-equivalent predicate formula in a finite number of steps.

To solve this problem, syntactical differences of equivalent formulae must me minimized systematically.

The symbols assigned to the formulae – in case of the ab-notation the "ab-symbols" – shall identify the truth conditions of predicate formulae. This means that the ab-symbols can be paraphrased by a mechanical procedure such that they denote common features of the models and counter-models of the initial formula. This, in turn, implies the possibility of construing the totality of models and counter-models from the ab-symbol of a formula without reckoning single interpretations. The understanding of logical proofs in the framework of New Logic can be characterized as follows:

Logical Proof: A proof according to the conception of New Logic consists in the application of a mechanical procedure assigning an ab-symbol to a predicate formula identifying its conditions of truth and falsehood unambiguously.

In how far this proof-conception will be realized is, in turn, to be measured against the extent of the solution of the equivalence problem and against the possibility of construing the totality of models and counter-models of a formula given merely its ab-symbol. The complete realization of this proof-conception is the core problem of logic according to the Wittgensteinian view.

In fact, no satisfying answer to the question of the truth conditions of predicate formulae can be put forward in the framework of classical logical. Paraphrases of the formulae identify their truth conditions just as little as the formulae themselves. Derivations are only capable of identifying internal relations between formulae. And in the framework of classical semantics no general descriptions of the models and counter-models of a *predicate* formula can be delivered but only single models and counter-models (cf. Lampert (2006)). Even for subclasses of predicate logic that exceed propositional logic and monadic predicate logic no answer is given to the question of the truth conditions of a predicate formula in terms of a mechanical produced, finite expression explicating the truth conditions of the formula in a satisfying manner. This is not only deficient from the point of view of New Logic but from the perspective of everyone handling with predicate formulae and seeking to understand them. This deficiency should be resolved for as many subclasses of predicate logic as possible. Thus, the project to realize Wittgenstein's programme of a New Logic is motivated by philosophical as well as by logical grounds. And its feasibility should be measured by the question to what extent the elaboration of Wittgenstein's ab-notation for first order logic is able to solve the equivalence problem.

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