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SPECIAL ISSUE ARTICLE

Thin Mereological Sums, Abstraction, and Interpretational Modalities

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Abstract

Some tools introduced by Linnebo to show that mathematical entities are thin objects can also be applied to non-mathematical entities, which have been thought to be thin as well for a variety of reasons. In this paper, I discuss some difficulties and opportunities concerning the application of abstraction and interpretational modalities to mereological sums. In particular, I show that on one hand some prima facie attractive candidates for the role of an explanatory plural abstraction principle for mereological sums (in terms of pluralities of summed entities) are not really explanatory; on the other hand, singular abstraction principles (in terms of single summed entities) are materially inadequate. Nonetheless, explanatory criteria of identity and conditions of existence for mereological sums are provided by classical extensional mereology independent of abstraction principles. Thus, given classical extensional mereology, the reasons why, according to Linnebo, mathematical abstracted entities are thin also hold for mereological sums. Finally, I contend that interpretational modalities can be used to characterise the process by which a subject adds sums of previously admitted entities to the domain of quantification.

KEYWORDS

abstraction, criteria of identity, mereology, ontological commitment

1 | SUMS AND THINNESS

According to Linnebo, some objects are thin. The existence of absolutely thin objects does not make a substantial demand on the world, whereas the existence of relatively thin objects does not make any substantial *further* demand on the world with respect to the existence of some other objects. In

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metaphysics, the expectation that the ontological commitment to some entities is not a burden – or, even more frequently, that it is not an additional burden with respect to some other ontological commitments – is widespread and concerns a variety of entities ranging from properties and relations to holes and shadows, from epiphenomenal mental states to propositions. The ways in which the thinness of these entities is construed vary from case to case and among various philosophical methodologies. The entities at stake are characterised as constituting an *ontological free lunch*, or said to be *grounded* in some more fundamental entities, or even *identical* to already admitted entities. In some cases, the claims at stake explicitly support a generous ontology: if an ontological commitment is thin and does not cost much or anything at all, there is no reason to be stingy with it.

In *Thin Objects. An Abstractionist Account* (2018) Linnebo focuses on the claim that mathematical entities (such as numbers and sets) are thin, and he thereby develops a generous ontology of mathematical entities. Mathematical entities are thin because *abstraction principles* provide explanatory criteria of identity for them in terms of an equivalence or unity relation among other entities, the entities with respect to which the abstracted entities are relatively thin. The subsistence of the unity relation is also said to be sufficient for the identities at stake where this sufficiency is akin to metaphysical grounding (Linnebo, 2018, 18). Moreover, the abstractionist framework also provides conditions of existence for the mathematical entities at stake, again in terms of the entities with respect to which the abstracted entities are relatively thin.

To express the dynamic character of the abstraction of mathematical entities (and particularly of sets), Linnebo also resorts to a kind of modality originally discussed by Fine (2005, 2006): *interpretational modalities*. These modalities are connected with the expansions of our domain of quantification: "$\Diamond_{int}\phi$" is true if and only if "ϕ" is true for at least an expansion of the domain of quantification; " $\Box_{int}\phi$ " is true if and only if "ϕ" is true for every expansion of the domain of quantification. The two tools – abstraction principles and interpretational modalities – are applied by Linnebo conjointly: the expansions of the domain of quantification that are discussed in *Thin Objects* concern entities whose criteria of identity and conditions of existence are established by abstraction principles in terms of a unity relation among entities that were already present in the domain of quantification.

In his book, Linnebo makes some references to *mereological sums*. Mereological sums are often expected to be, in Linnebo's terminology, thin.[1] Also Linnebo refers to them in this vein (Linnebo, 2018, 4,9,11,17), in particular by expressing the expectation that abstraction principles are available for mereological sums and legitimise their relative thinness with respect to the summed entities. The main purpose of this paper is to investigate some candidate abstraction principles for mereological sums – some of them already discussed in the small literature about mereology and abstraction, some others new – and to show that they raise different kinds of problems. These problems affect in some cases their material adequacy and in some others their ability to motivate the alleged thinness of mereological sums. I will consider various candidates for the role of abstraction principle for mereological sums and, although some candidates will fare better than others, the upshot is that there are concerning issues with all of them.

I am nonetheless going to suggest that, if certain theories of parthood and composition – and in particular the so-called classical extensional mereology (CEM henceforth) – are assumed (a substantial assumption that I am not going to defend in this paper), then explanatory criteria of identity and conditions of existence in terms of previously admitted entities *are* available for mereological sums. According to Linnebo, the reason why abstraction principles legitimise reference to thin mathematical objects and, as a consequence, also their thin existence is that they provide explanatory criteria of identity and conditions of existence for them. If explanatory criteria of identity and conditions of existence are available for sums *independently of abstraction*, then sums can be deemed to be thin with as good reason as abstracted mathematical entities are. There

[1]Many doctrines, variously expressed and substantially different from one another, would accept the gist of this claim. Armstrong (1978, 36–38), Baxter (1988), Lewis (1991, 81) are among the most influential sources.

is therefore no spur to take an arduous detour through abstraction principles in sofar as a theory delivering the same reasons for thinness is available. Linnebo's interpretational modalities can then be used to characterise the process by which a subject or a community of subjects comes to introduce new, *thin* mereological sums into the domain of quantification for a language.

In this paper, only the relative thinness of sums and wholes with respect to summed entities and parts is discussed, almost without any mention of alternative, mereologically identified candidates to thinness: for example, the priority monism of Schaffer (2010) could perhaps be framed in broadly Linnebian terms by qualifying every proper part of the universe as relatively thin with respect to the universe. This omission is a declared limitation of the topic of this paper, which only concerns the application of Linnebo's tools and methodological principles to the thinness of wholes with respect to parts. The only implicit point of contact with other forms of mereological thinness is the following: in the discussion of how the thinness of sums can be motivated by mereological principles independently of abstraction principles, we will see that a well-known mereological theory (CEM) provides criteria of identity and conditions of existence for wholes in terms of summed entities or parts. In contrast, it is unclear what kind of mereological theory could provide criteria of identity and conditions of existence for parts in terms of wholes. Thus, the claims of *downward* mereological thinness (i.e., concerning parts with respect to wholes) are unlikely to be articulated through mereological theories providing criteria of identity and conditions of existence, whereas this can be done – I am going to argue – for claims of *upward* mereological thinness (i.e., concerning wholes with respect to parts). Obviously, this does not exclude claims of downward mereological thinness being otherwise articulated but suggests that the Linnebian tools discussed in this paper are not directly relevant for this project.

This paper is structured as follows. In section 2, I discuss some candidate *plural* abstraction principles for mereological sums (i.e., abstraction principles whose unity relation connects pluralities of entities, and in particular pluralities of summed entities) and contend that they risk lacking the explanatory or grounding value that Linnebo attributes to abstraction principles for mathematical thin objects. In section 3, I extract from Linnebo's discussion of physical bodies in Linnebo (2018) a candidate *singular* abstraction principle for mereological sums (i.e., an abstraction principle whose unity relation connects single entities) and contend that it is materially inadequate. The reasons for its inadequacy extend to singular abstraction principles for sums in general. In section 4, I maintain that the thinness of sums can be motivated by mereological principles providing explanatory criteria of identity and conditions of existence for them, independent of abstractionism. Finally, in section 5, I suggest that it is methodologically sound to apply interpretational modalities to this non-abstractionist route to thinness in order to illustrate how the expansions of the domain of quantification involving thin mereological sums work.

2 | PLURAL ABSTRACTION OF MEREOLOGICAL SUMS

Linnebo (2018, 17) compares the thinness of mereological sums with that of impure sets, and this comparison can be expected to provide a hint about how to formulate abstraction principles for sums, namely by analogy with abstraction principles for impure sets. Impure sets would be thin relative to the urelements of their transitive closure, whereas mereological sums would be thin relative to the summed entities. To exemplify, Linnebo claims that, with respect to the existence of the books in his office, the existence of both the impure set of these books and of their mereological sum can be thought *to require little or nothing from the world*, and this is what their relative thinness would consist in. This corresponds to the fact that "it is possible to formulate abstraction principles for sets and mereological sums" (2018, 19).

A large part of Linnebo's book concerns abstraction principles for sets. These abstraction principles are inspired by Frege's notorious *Basic Law V* but move away from the latter's features that lead to paradoxes. In particular, Linnebo distinguishes between two components concerning the criteria of identity and the conditions of existence, respectively, and contends that set abstraction occurs in steps and is in this sense dynamic. Concerning the conditions of existence, given any plurality of entities in a domain of quantification (and so at any step in the process of expansion of the domain of quantification), it is possible to make a further step such that there exists a set whose members are the members of the plurality[2]:

$$\Box_{\text{int}} \forall xx \Diamond_{\text{int}} \exists y \boldsymbol{Set}(xx, y) \quad \text{(Set Abstraction : Conditions of Existence)}$$

Regarding the criteria of identity, the results of the operation of set formation applied to pluralities are identical if and only if the pluralities have the same members in an interpretationally necessary way, that is, no matter what further abstraction steps are carried out (Linnebo, 2018, 62):

$$\boldsymbol{Set}(\mathrm{uu}, \mathrm{x}) \wedge \boldsymbol{Set}(\mathrm{vv}, \mathrm{y}) \rightarrow (\mathrm{x} = \mathrm{y} \leftrightarrow \Box_{\text{int}} \forall \mathrm{z}(\mathrm{z} \prec \mathrm{uu} \leftrightarrow \mathrm{z} \prec \mathrm{vv}))$$

$$\text{(Set Abstraction : } \times \text{Criteria of Identity)}$$

What about mereological sums? How could an abstraction principle be formulated? Let us set aside, for the sake of simplicity, the need to avoid paradoxes. This simplification is likely to be innocuous in the theoretical landscape we are assuming. Uzquiano (2006) showed that a Russell-like paradox emerges from the interaction of CEM with set theory if: (a) absolutely unrestricted quantification is available in both set theory and CEM; (b) CEM is assumed as the true, exhaustive theory of parthood; and (c) the ZFCU theory of sets (i.e., the Zermelo-Fraenkel theory plus the axiom of choice with urelements) is assumed as the true, exhaustive theory of membership. In Linnebo's approach, assumption (a) is emphatically rejected for set theory (e.g., Linnebo 2018, section 3.6): quantification over sets is unrestricted only relatively to a specific stage of the expansion of the domain. Dynamic abstraction for what concerns sets seems thus to remove a pivotal assumption of Uzquiano's paradox.[3] No Russell-like paradox, as far as the extant literature shows, concerns CEM in and of itself, independently of its interaction with a certain conception of sets.[4]

Once the antiparadoxical, dynamic aspects are laid aside, the gist of the problem is to pinpoint the suitable equivalence, unity relation, and its *relata*. What could instantiate the partial equivalence relation in a suitable abstraction principle? The first hypothesis is that the partial equivalence relation is instantiated by the pluralities of summed entities (as the unity relation of coextensionality in (Set Abstraction: Criteria of Identity) is instantiated by the pluralities of members *uu* and *vv*). Coextensionality is surely not a good candidate for the abstraction of sums, inasmuch as pluralities with different members can nonetheless have the same mereological sum. The plurality that includes the legs, seat, and back of a chair and the plurality of chemical molecules in the chair are not coextensional with each other (e.g., the seat is a member of the former plurality but not of the latter); nonetheless, they have the same sum (the chair). Thus, a plural abstraction principle for sums in terms of coextensionality among pluralities of summed entities would provide wrong criteria of identity for sums.

A better, albeit prima facie underspecified, candidate comes from the debate about composition as identity (CAI), in which it is rather common to claim that a sum and the summed

[2]The variables "*xx*," "*yy*" … are plural variables. The modal operators "$\Diamond_{int}$" and "$\Box_{int}$" in (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) express interpretational modalities. I leave interpretational modalities in the background until section 5. "***Set***" expresses an operation of set formation going from the plurality of members to a set.

[3]Also in the extant applications of interpretational modalities to mereology, such as Botti (2021) and Lando (2020), quantification is not absolutely unrestricted. I will come back to these applications in section 5.

[4]See Cotnoir and Varzi (2021, section 5.4.1) for a discussion of the reasons why a Russell-like paradox does not affect CEM.

entities *are the same portion of reality* (Lewis, 1991, 81).[5] We can set aside CAI in this context: indeed, if we subscribed to CAI and thereby contended that a sum is identical to the summed entities, we would have a clear sense of sums being thin objects with respect to the summed entities (i.e., they *are* the summed entities); therefore, there would be no need to resort to abstraction principles. Independently of CAI, we can try to employ the relation of *being the same portion of reality* and the idea that it subsists between a sum and the summed entities, and therefore also (by transitivity) among different pluralities of entities having the same sum. The chair would be the same portion of reality of its legs, seat, and back – and would also be the same portion of reality of its molecules. The relation of being the same portion of reality is transitive. Thus, the legs, the seat, and the back would also be the same portion of reality of the molecules.

An abstraction principle could then claim that sums are identical if and only if the summed entities are the same portion of reality[6]:

$$\Sigma(uu,x) \wedge \Sigma(vv,y) \rightarrow \left(x=y \leftrightarrow uu \backsim_{por} vv\right) \qquad \text{(Sum Abstraction : Same Portion of Reality)}$$

This abstraction principle provides materially adequate criteria of identity for sums. It works even if the value of at least one of the plural variables "*uu*" and "*vv*" is a single entity. In this case (Sum Abstraction: Same Portion of Reality) correctly makes the single-argument sum of the chair identical to the multi-argument sum of its legs, seat, and back if and only if they are the same portion of reality.

However, this approach, in the absence of a further specification, does not provide the kind of asymmetric metaphysical explanation that characterises abstraction principles for thin mathematical entities according to Linnebo (see in particular Linnebo [2018, ch.4]). With respect to Frege's famous example in *Die Grundlagen der Arithmetik* (section 64), it is at least prima facie plausible that the parallelism of lines metaphysically explains or grounds the identity of their directions. In order to play an analogous role in (Sum Abstraction: Same Portion of Reality), the relation of being the same portion of reality should be specified in independent terms: clearly, if being the same portion of reality consisted in having the same sum, then no explanatory value with respect to the identity of sums could be gained.[7] It is also doubtful that it is possible for a non-idealised agent to know that, if two pluralities are the same portion of reality, then they have the same sum, as Linnebo's*epistemic constraint* requires for abstraction principles that really make the abstracted entities (in this case, the mereological sums) thin (2018, 16). The right-to-left direction of (Sum Abstraction: Same Portion of Reality) (i.e., the conditional that, if the pluralities are the same portion of reality, then their sums are identical) is indeed a theory-laden claim that builds on a relation (*being the same portion of reality*) discussed in a very specific literature; in the absence of specification and clarification of it in other terms, it is difficult even to assess whether a non-idealised subject could know that, if two pluralities of entities are the same portion of reality, they have the same sum. In general, in order to assess whether the relation of being the same portion of reality is fit for the purpose of formulating a suitable abstraction principle for sums, we need to characterise it in more explicit terms.

Let us then consider two ways in which the relation of being the same portion of reality can be independently characterised: in terms of *atomic parts* and in terms of *spatiotemporal location*. Both specifications are hinted at in the literature about CAI in which the relation is commonly discussed, but we should keep in mind that in order to serve its current purpose the relation of

[5]Wallace (2011), Cotnoir (2014), and Carrara and Lando (2021) are introductions to the debate about CAI.
[6]"Σ" is a many–one relational constant expressing the relation of mereological sum between summed entities and their sum. This relation is actually an operation in CEM. "$\backsim_{por}$" expresses the relation of being the same portion of reality.
[7]However, the informativity of a circular characterisation of the relation of being the same portion of reality in terms of the relation of having the same sum is defended in Bøhn (2009, p. ix). See also Hawley (2013) for a specific discussion of the difficulty of analysing the relation of being the same portion of reality, with specific reference to the theory of general identity of Cotnoir (2013).

being the same portion of reality should be detached from the doctrine of CAI, which warrants the thinness of sums in a more direct way.

For what concerns *atomic parts*, the theory of general identity of Cotnoir (2013), in the context of a defence of CAI, contends that a sum and the summed entities, as well as different pluralities of summed entities having the same sum, are generally identical. General identity consists in its turn (2013, 303) in the fact that the sets of atomic parts of the summed entities (a set for the atomic parts of each summed entity) have the same set-theoretical union. From this – bypassing general identity, building upon the ensuing analysis of being the same portion of reality in terms of having the same atomic parts, and getting rid of sets in favour of pluralities – we could extract the following abstraction principle, according to which the sums of the pluralities of entities *uu* and *vv* are identical if and only if the same atoms are parts of at least one of *uu* and are parts of at least one of *vv* ("*A*" expresses the property of being mereologically atomic, that is of having no proper part):

$$\Sigma(uu,x)\wedge\Sigma(vv,y)\rightarrow(x=y\leftrightarrow\forall z(Az\rightarrow(\exists w(w\prec uu\wedge z\ P\ w)\leftrightarrow\exists w(w\prec vv\wedge z\ P\ w))))$$
$$\text{(Sum Abstraction : Same Atomic Parts)}$$

The unity relation between *uu* and *vv* would then be the relation of having the same atomic proper parts. For entities entirely decomposable in atomic parts, the material adequacy of this principle depends on the idea that no two composed entities have the same atomic constituents, and this holds in CEM, which – in the terminology of Goodman (1956) – is hyperextensional. However, the adequacy of the ensuing conditions of identity for sums depends on the assumption that *everything* is entirely decomposable in atomic parts: entirely gunky entities (i.e, entities all parts of which have proper parts) will have no atomic parts at all, and the disastrous upshot of (Sum Abstraction: Same Atomic Parts) would be that all gunky entities are identical to one another.[8] Also the conditions of identity for partially gunky entities (i.e., entities some parts of which are such that all their parts have proper parts) will be severely screwed inasmuch as they would turn out to be identical at the sufficient condition of having the same atomic parts independently of their gunky parts.

Atomism – the doctrine that rules out gunk and according to which everything is decomposable in atomic parts – is notoriously independent of CEM and is, in general, a doctrine in need of an independent motivation. Also the hypothesis of restricting (Sum Abstraction: Same Atomic Parts) to entities that are decomposable in atomic parts would draw a specious and arbitrary line of distinction inasmuch as mereological summation works in the same way for gunky and non-gunky entities. There is thus no evidence of a significant corresponding distinction among sums, and a principle of abstraction only for sums of atomically decomposable entities would therefore fail to cut the issue at any significant joint. It is therefore impossible to use (Sum Abstraction: Same Atomic Parts) as a foundation for the thinness of sums.

The second attempt to specify the relation of being the same portion of reality in more explicit and explanatory terms involves *spatiotemporal colocation*.[9] The resulting abstraction principle contends that two pluralities of entities are identical if and only if they are exactly located at the same spatiotemporal region. A theory of exact spatiotemporal location for pluralities of entities is needed and is easily attained once a theory of exact location for single entities is made available.[10] A plurality of entities is exactly located at the union of the locations at which the various members of the plurality are exactly located. Given this plural variety of

[8]The condition $\forall z(Az\rightarrow(\exists w(w\prec uu\wedge z\ P\ w)\leftrightarrow\exists w(w\prec vv\wedge z\ P\ w)))$ is indeed always satisfied for x and y gunky: if the entire reality is gunky, then there are no atoms and the universally quantified conditional is trivially true; if reality is not entirely gunky, then in any case no atom is part of one of *uu* or part of one of *vv* and so trivially the same atoms are parts of one of *uu* and parts of one of *vv*.

[9]See, for example, Bøhn (2009, p. 10).

[10]Parsons (2007) distinguishes and defines various relations of location, including exact location.

exact location, we will be able to define the relation $\backsim_{colo}$ (exact colocation for pluralities), and this relation will be a partial equivalence relation. We therefore obtain the following candidate abstraction principle for sums (the principle differs from (Sum Abstraction: Same Portion of Reality) only insofar as ' $\backsim_{por}$' is replaced with ' $\backsim_{colo}$'):

$$\Sigma(uu,x) \wedge \Sigma(vv,y) \rightarrow (x=y \leftrightarrow uu \backsim_{colo} vv) \qquad \text{(Sum Abstraction : Exact Colocation)}$$

However, the following two problems affect (Sum Abstraction: Exact Colocation).

a. The approach cannot be applied if one or more of the summed entities lacks/lack a spatiotemporal location. However, mereology has often been applied to sets (Lewis (1991)) or properties (e.g., in Paul (2002)). If these entities have no spatiotemporal location and are nonetheless arguments of the operation of mereological summation, then the unity relation cannot be applied to them, which means that the abstraction principle would not provide general criteria of identity for mereological sums and would therefore fail to explain what a sum, in general, is. This first problem is obviously avoided if mereology is expected to concern only concrete entities, located in spacetime, but this limitation needs an independent motivation.
b. Even once we restrict our attention to spatiotemporally located entities and thereby avoid problem a., (Sum Abstraction: Exact Colocation) risks endowing mereological sums with controversial, theory-laden criteria of identity. In particular, the principle excludes colocated distinct entities, such as those widely discussed in the debate about material constitution: a statue and the colocated portion of clay are colocated and will come out identical, thereby excluding dualism about material constitution already at this level. However, a principle that is expected to be explanatory with respect to the criteria of identity for sums and to motivate their status of thin objects should avoid a commitment to a metaphysical thesis (monism about constitution) that is independent of strictly mereological principles. As underlined in particular by Varzi (2008), dualists about constitution can adopt an extensional mereology (such as CEM), provided that they claim that the statue and the clay, besides being distinct, also have distinct proper parts. Thus, CEM is compatible with both dualism and monism about constitution, and this neutrality should be retained in an abstraction principle for mereological sums.[11]

Problem b. is also connected with the expected explanatory value of (Sum Abstraction: Exact Colocation). An abstraction principle in terms of spatiotemporal colocation *externalises the burden of explaining* the criteria of identity for sums to another subfield of metaphysics, namely the theory of spatiotemporal location. This is arguably the source of its lack of neutrality; to connect the identity of sums with the plural colocation of the summed entities is to make the concept of sum dependent on the interface between mereology and something else. Problem a. could also be traced back to this externalisation of the explanation. Indeed, once the burden of explanation is externalised to the theory of spatiotemporal location, it is unsurprising that sums lacking a spatiotemporal location end up neglected.

Inasmuch as the externalisation of the explanation leads to troubles, an abstraction principle *within* mereology is a worth considering alternative. Also the explicitation of being the same portion of reality in terms of atomic parts and the ensuing (Sum Abstraction: Same Atomic Parts) is entirely formulated in the language of mereology and is in this sense *intramereological*. However, it crucially depends on atomism, a principle external to CEM and in general contentious and, as a consequence, ill-suited to ground criteria of identity for sums and ultimately explain what a sum is. We could then look for a more strictly intramereological principle, which

[11]Smid (2022) is an in-depth defence of the compatibility of mereological extensionality with colocation.

does not depend on atomism. We can begin by inspecting the following, standard definition of mereological summation[12]:

$$\Sigma(xx,y) \equiv_{def} (\forall z(z \prec xx \rightarrow z\ P\ y) \wedge \forall z(z\ P\ y \rightarrow \exists w(w \prec xx \wedge w^{\circ} z)))$$
$$(\text{Mereological Sum : Definition})$$

The gist of this definition is that a sum includes all the summed entities as parts and nothing extraneous to all of them. The absence of anything extraneous is rendered by saying that every part of the sum overlaps (i.e., shares a part with) one of the summed entities. A plural abstraction principle could introject this same gist and claim that, if two pluralities have the same sum, then every part of every member of the former plurality overlaps at least one member of the latter plurality[13]:

$$\Sigma(uu,x) \wedge \Sigma(vv,y) \rightarrow (x = y \leftrightarrow \forall z \forall w(z \prec uu \wedge w\ P\ z \rightarrow \exists j(j \prec vv \wedge j^{\circ} w)))$$
$$(\text{Sum Abstraction : Intramereological})$$

Given the kinship with (Mereological Sum: Definition), this abstraction principle might be expected to explain or ground what sums are and to provide explanatory criteria of identity for them. (Sum Abstraction: Intramereological) might thereby fulfil an expectation that is rather common in the neo-Fregean, abstractionist tradition to which Linnebo belongs – the expectation that the commitment to abstracted entities is as lightweight as definitions are in general lightweight.[14]

A doubt might concern the order of definitions. It is indeed possible to axiomatize CEM in a varieties of way, and some of these varieties instead choose summation as a primitive and define parthood in terms of it. For example, in Cotnoir and Varzi (2021, section 2.4.4) (where the axiomatization of Leonard (1930) is refined), a kind of summation is assumed as a primitive with certain features and then, by exploiting some of these features, parthood is defined in terms of it. The resulting system is then shown to be equivalent to CEM in its most standard axiomatizations, where parthood (or proper parthood, or overlap) is chosen as a primitive. For example, the following definition of parthood exploits the idempotence of mereological summation and the corresponding fact that x is part of y if and only if the sum of x and y is y itself:

$$x\ P\ y \equiv_{def} \forall zz(x \prec zz \wedge y \prec zz \wedge \forall w(w \prec zz \rightarrow (w = x \vee w = y)) \rightarrow \Sigma(zz,y))$$
$$(\text{Parthood : Definition in Terms of Summation})$$

However, as declared in section 1, we are trying to apply Linnebo's tools to an approach in which sums are thin with respect to parts – without trying to argue that this is the case or that other claims of mereological thinness, for example, those concerning summed entities with respect to their sums, are in contrast wrong. Under this assumption, it would be unwise to build upon an axiomatization of mereology in which summation is a primitive, thereby blocking the possibility of extracting from the *definition* of summation an explanatory abstraction principle. Once an axiomatization in which (Mereological Sum: Definition) is preferred, then summation

[12] "∘" expresses mereological overlap, that is the relation of sharing at least one part. The following is not the only definition of mereological summation we could build upon, but the outcome of the analysis would be similar for other common definitions. See Cotnoir and Varzi (2021) (in particular ch. 5) for an in-depth analysis of various definitions of summation.
[13] This abstraction principle is briefly discussed in Rosen (2020, section 6) and more extensively, in a marginally different form, in Russell (2017, section 2).
[14] Russell (2017, section 3) expresses this expectation for intramereological abstraction.

is indeed defined in terms of parthood and not vice versa, and the explanatory ambitions of (Sum Abstraction: Intramereological) are thus uncurbed by circularity. Linnebo's*epistemic constraint* is also plausibly respected: it is surely possible for a non-idealised subject to know that if the conditions on the right side of (Sum Abstraction: Intramereological) are respected, then the sums are identical.

There are thus some reasons to prefer an intramereological one, such as (Sum Abstraction: Intramereological), in the field of plural abstraction principles. However, (Sum Abstraction: Intramereological) is no less exposed than its rivals to a general problem concerning *plural* abstraction principles for sums in which unity relations connect different *pluralities* of entities. The problem is that the nature of sums and their explanatory criteria of identity do not systematically involve a *relation* among these various pluralities. If sums are in general thin with respect to the summed entities, it systematically happens that composed entities are sums of multiple pluralities of entities, and are then thin with respect to all of these pluralities. Thus, for example, the Benelux is the sum of the Netherlands, Belgium, and Luxembourg, as well as of some municipalities and of some molecules (under the assumption that the Benelux is a physical entity). However, this does not entail that the relative thinness of sums is warranted by a principle concerning *a relation among these pluralities*, such as the equivalence or unity relations at stake in (Sum Abstraction: Same Portion of Reality), in its specifications (Sum Abstraction: Exact Colocation) and (Sum Abstraction: Same Atomic Parts), and in (Sum Abstraction: Intramereological). Whereas a direction plausibly *is* what is common to mutually *parallel* lines, and the nature of a set is given by its members and so by the relation of coextensionality among pluralities of members, there is no reason to think that the nature of a sum be characterised by a relation among the various pluralities of entities of which it is a sum. The widespread expectation that sums are thin with respect to summed entities has nothing to do with a relation among pluralities of summed entities.

We should not get confused on this terrain by another line of thought. The reason why the abstraction principles for sums in terms of various pluralities of summed entities do not explain what sums are *is not* that sums have single, privileged decompositions in parts. This confusion might be enticing because, given the specific nature or kind of certain sums, there is a privileged plurality of summed entities – a privileged way of divvying up a sum with a specific nature or of a specific kind. Consider again the Benelux, which presumably belongs to the kind of multinational entities. Its nature and its belonging to this kind is clearly connected with its being the sum of the Netherlands, Belgium, and Luxembourg, that is, of the nations composing this multinational entity: one might argue that this subdivision grounds Benelux's being a multinational entity. By contrast, other subdivisions and corresponding pluralities of entities, such as the subdivision of Benelux in a plurality of municipalities or, if Benelux is considered a physical entity, in a plurality of molecules, are neither needed nor helpful to explain or ground Benelux being a multinational entity. This is the reason – the enticing and wrong line of thought might continue – why plural abstraction principles for sums are off the mark and unable to warrant the thinness of sums: because there is in many cases a single, privileged plurality of summed entities, and the other pluralities of summed entities should not be involved.

The temptation should be resisted because any explanation of the nature or identity of specific kinds of sums is besides the point here: the abstraction principle warranting the thinness of sums with respect to summed entities should not explain at all what are specific kinds of sums such as multinational entities. The purpose is instead to explain what mereological sums in general are in terms of those entities of which they are the sum; clearly most sums are the sums of multiple pluralities of entities. There is indeed a potential kind of overexplanation or overdetermination at stake: the most promising plural abstraction principle discussed above, (Sum Abstraction: Intramereological), is tightly connected with (Mereological Sum: Definition). That definition is indeed *enough* to explain what a mereological sum is; in this light, a composed entity is grounded or metaphysically explained once a single plurality of summed entities of which that composed entity is the sum (a single value of the plural variable "*xx*") is considered.

But then this also holds for *each* plurality of entities of which that is the sum. What *does not* matter is a relation among these pluralities, and the real problem affecting (Sum Abstraction: Intramereological) and the other plural abstraction principles for sums stems from this. There is no reason why a relation of mereological intimacy – as well as, for example, the relation of spatiotemporal colocation – should explain what a mereological sum is. A definition of mereological summation such as (Mereological Sum: Definition) already explains what a sum is, and for each instance of mereological summation this involves a single plurality of entities. Then, most sums are sums of various pluralities of entities, and so the nature of sums is arguably grounded or explained in multiple ways. Nonetheless, there is no need to *also* involve a relation holding among the various pluralities.

3 | SINGULAR ABSTRACTION OF MEREOLOGICAL SUMS

An abstractionist might concede that it is wrong to involve a relation among various *pluralities* of summed entities in an explanatory abstraction principle for sums. The unity relation in the abstraction principle could instead connect *the single summed entities*, as much as parallelism (the unity relation for directions) is a relation among single lines and not among plurality of lines. The resulting principle would thus be radically different from Linnebo's plural abstraction principle for sets. We have seen in section 2 that for any instance of summation a sum should be expected to be relatively thin with respect to the summed entities, without any specific role for an equivalence relation among various pluralities of summed entities, and that this is the ultimate reason to be sceptical about plural abstraction principles for mereological sums. Singular abstraction principles would avoid this kind of scepticism inasmuch as their unity relation might connect the summed entities one with another within *single* instances of summation.

To obtain a singular abstraction principle for sums, we should first of all pinpoint a relational condition at which entities are parts of the same sum. Some hints can be found in Linnebo's*Thin Objects*, although he never explicitly presents them as attempts to provide an abstraction principle for sums; the hints are instead discussed in the context of a toy model for *reference to physical bodies* and therefore in the attempt to pinpoint an abstraction principle for physical bodies. The purpose of this toy model is to show how criteria of identity can play a role in reference, as occurs in Linnebo's account of reference to mathematical thin objects (Linnebo, 2018, 26). In the toy model, a robot is taught how to refer to physical bodies in its environment. "The parcels of matter with which the robot interacts through its rudimentary forms of perception are spatiotemporally connected in some appropriate way" (2018, 27), and the robot needs to be instructed about this appropriate way of spatiotemporal connectedness. This appropriate way concerns three-dimensional solidity, natural and relatively well-distinguished spatial and temporal boundaries, and the disposition to move as a cohesive unit. Linnebo's connectedness is not a merely topological relation: in the eyes of Linnebo, it involves a rather substantial kind of unity (this is captured by "the appropriate way" in which the parcels of matter need to be connected in order to constitute a body).

According to Linnebo, the thus characterised relation of spatiotemporal connectedness is "clearly symmetric. For if two parcels of matter *u* and *v* specify the same body, then so do *v* and *u*" (2018, 28). Linnebo adds that "a similar defense can be given of transitivity" and thus concludes that a partial equivalence relation is indeed at stake. The following abstraction principle thus provides criteria of identity for bodies in terms of appropriate connectedness of the parcels of matter constituting them:

$$b(u) = b(v) \leftrightarrow u \sim_{conn} v \qquad \text{(Body Abstraction : Appropriate Connectedness)}$$

The $\sim_{conn}$ relation is a one–one relation of appropriate spatiotemporal connectedness among individual parcels of matter (and not a many–many relation like, e.g., $\backsim_{colo}$).

There are at least two underdeveloped aspects in Linnebo's toy model (as is to be expected from a toy model). First, it is not clear what parcels of matter are; perhaps they are spatiotemporally non-extended physical entities. Some of the reasons supporting prudence about atomism discussed in section 2 could then be used to doubt that parcels of matter exist.[15] Second, the equivalence relation (i.e., connectedness *in an appropriate way*) could be variously specified, and it is arguably not trivial to specify it so that, for example, all the parcels of matter constituting the body of Barack Obama are connected in the appropriate way and are not analogously connected with the parcels of matter in the air surrounding Obama; this would be important in order to obtain good criteria of identity for bodies. Moreover, the way that Linnebo illustrates the symmetry of the relation of spatiotemporal connectedness in the above quote – and in particular claims that its transitivity can be analogously illustrated – is indeed circular: "for if two parcels of matter u and v specify the same body …" Consider the extension of this illustration to transitivity: if three parcels of matter, u, v, and w, are such that u and v specify the same body, and v and w also specify the same body, then u and w also specify the same body. This indeed presupposes that u, v, and w specify one and the same body; otherwise, the hypothesis that u and v specify a body b_1 and v and w specify a distinct body b_2 could not be ruled out. In other words, Linnebo's treatment presupposes a (possibly incomplete) partition of the parcels of matter in physical bodies, and thus a (possibly partial) equivalence relation among them.

Even if these lacunae and difficulties concerning physical bodies were met, it would be wrong to interpret b as a function from single entities to *their* sums, that is, as a function associating each single entity u to a sum of entities, one of which is u. First, there is no shared expectation that the parcels of matter in a sum are interconnected in Linnebo's appropriate way. In CEM, (Unrestricted Composition) establishes that any plurality of entities has a sum:

$$\forall xx \exists y \Sigma(xx, y) \qquad \text{(Unrestricted Composition)}$$

Thus, there exists the sum of Donald Trump and of the Taj Mahal; but the parcels of matter in this sum will not be connected in an appropriate way, concerning three-dimensional solidity, natural and relatively well-distinguished spatial and temporal boundaries, and the disposition to move as a cohesive unit.

This problem is moreover not solved either by rejecting (Unrestricted Composition) or by relaxing the condition of approriateness on interconnectedness. In CEM, (Unrestricted Composition) delivers plenty of mereological sums, but even much more restrictive stances about composition will make parcels of matter part of multiple sums: it is for example enough to concede that both a tire in a car and the car itself exist in order to obtain that any parcel of matter that is part of the tire is also part of the wheel. Thus, b cannot be interpreted as a function from parcels of matter to sums and cannot be replaced in this role by any other function because there is no functional correlation between parcels of matter and sums.

Moreover, mereological sums are normally allowed to overlap one another: given Unrestricted Composition, the mereological sum s of France and Switzerland and the mereological sum t of Switzerland and Liechtenstein will exist and overlap each other; and even given more moderate stances on composition, the body of conjoined twins will be overlapping sums. The interpretation of (Body Abstraction: Appropriate Connectedness) as an abstraction principle for sums would implicitly commit us to the scarcely plausible thesis of the pairwise disjointness (i.e., lack of overlap) of mereological sums. If every parcel of matter specifies a single sum, then no parcel can be part of two sums; therefore, no two sums can overlap each other.

It is instructive to see why the problem cannot be solved even if the kind of connectedness at stake is radically weakened by dropping Linnebo's substantial conditions of unity, such as the

[15]In the literature about mereological gunk, Zimmermann (1996) and Arntzenius and Hawthorne (2005) are especially relevant, inasmuch as they discuss the possibility of gunk on the basis of arguments concerning physical bodies.

disposition to cohesive motion. Consider the bare kind of spatiotemporal connectedness that consists in simply being either numerically identical or at some spatiotemporal distance, no matter how great. This relation is plausibly symmetric and transitive. In this sense, it is possible to envisage individuals closed under spatiotemporal connectedness. Lewis' possible worlds are indeed such entities: "whenever two possible individuals are spatiotemporally related, they are worldmates" (1986, 70). Lewis' worlds are maximal entities that include as parts whatever is at some spatiotemporal distance from them. In the context of Lewis' pluriverse, bare spatiotemporal connectedness is an equivalence relation in terms of which criteria of identity *for possible worlds* can be given. This also corresponds to Lewis' notorious claim that worlds are pairwise mereologically disjoint (1986, section 4.2). However, this kind of spatiotemporal connectedness does not deliver any sensible criterion of identity for mereological sums in general, inasmuch as mereological sums (in contrast to Lewis' possible worlds) *are not spatiotemporally separated* from one another.

The basic flaw in the attempt to transmute (Body Abstraction: Appropriate Connectedness) into a singular abstraction principle for sums is that in general sums *are not* a partition of reality in well-delimited and non-overlapping groups: the reason why no partial equivalence relation captures such a partition is that there is no such partition. The outcome of the analysis is therefore different from that in section 2. The pluralities of the entities that are the inputs of the sum operation are indeed surely partitioned in well-delimited and non-overlapping groups, at least in CEM and in any other mereology in which mereological summation is indeed an operation. All the pluralities to which the operation associates a certain sum belong to the same group; no two pluralities to which the operation associates distinct sums belong to the same group. Thus, it is not impossible to pinpoint a materially adequate plural abstraction principle for sums ((Sum Abstraction: Intramereological) is indeed materially adequate); the real troubles – as shown in section 2 – concern the expectation that these plural abstraction principles explain or ground the identity of sums. The troubles with singular abstraction begin earlier, inasmuch as the single summed entities are not partitioned in well-delimited and non-overlapping groups, to each of which no more than one sum corresponds.

4 | THIN SUMS WITHOUT ABSTRACTION

The philosophical purpose of Linnebo's (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) is to legitimise reference to sets and therefore the existence of sets themselves in the context of Linnebo's broadly Fregean thesis that to exist is to be a possible object of reference (Linnebo, 2018, ch. 2). The legitimization consists precisely in providing conditions of existence and criteria of identity for sets. It is useful to indulge in a counterpossible conditional. If no paradoxes affected naive set theory, a schematic principle of comprehension (according to which, for any formula, there is the set of the entities satisfying it) and the axiom of set-theoretic extensionality (according to which sets are identical if and only if they have the same members) would respectively provide conditions of existence and criteria of identity. They would be no less explanatory than (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) and, inasmuch as they are simpler than them, they would also abide by Linnebo's epistemic constraint mentioned above. In short, the schematic principle of comprehension and the axiom of extensionality would fit the bill and warrant the thinness of sets.

A mereological theory *can* deliver conditions of existence and criteria of identity for sums. CEM does it. CEM can be axiomatixed in various ways and on top of various logical cores. Let us focus on the axiomatizations that use plural logic (i.e., the logical tool also used in Linnebo's principles about sets). Either as an axiom or as a theorem, CEM will include the already discussed (Unrestricted Composition), according to which for every plurality of entity there is their sum:

$$\forall xx \exists y\, \Sigma(xx, y) \qquad \text{(Unrestricted Composition)}$$

It will also include the following (Extensionality of Proper Parts), according to which entities with proper parts are identical at the necessary and sufficient condition of having the same proper parts:

$$\forall x \forall y((\exists z(z\ PPx) \vee \exists z(z\ PP\ y)) \rightarrow (\forall w(w\ PP\ x \leftrightarrow w\ PP\ y) \leftrightarrow x = y))$$
$$\text{(Extensionality of Proper Parts)}$$

(Unrestricted Composition) endows sums with conditions of existence. (Extensionality of Proper Parts) endows them with criteria of identity. They can be expected to explain what a sum is: it is something that exists at the necessary and sufficient condition that the summed entities exist and whose identity is entirely determined by its proper parts. There is no reason why this should be less explanatory than the corresponding principles for sets (both Linnebo's (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) and the simpler forms counterpossibly discussed above), or why it should fail to abide by Linnebo's epistemic constraint.

It might be objected that (Extensionality of Proper Parts) does not establish criteria of identity for mereological atoms (i.e., entities with no proper part), inasmuch as (Extensionality of Proper Parts) is explicitly restricted to entities with proper parts and that atoms are sums nonetheless, namely sums of themselves. However, the need to warrant the relative thinness of sums with respect to the summed entities only concerns *proper* instances of mereological summation, that is, instances in which at least one of the summed entities is distinct from the sum. In improper instances – where there is a single summed entity, which is identical to the sum – the relative thinness is a trivial and uninteresting byproduct of identity: in Linnebo's preferred terminology about thinness, any sum of itself does not make any further substantial demand on the world with respect to itself, but this does not depend on and does not need to be warranted by (Extensionality of Proper Parts). It instead depends on the relatively trivial fact that identical entities are indiscernible and, as a consequence, make the same demands on the world. Thus, the fact that (Extensionality of Proper Parts) does not provide criteria of identity for atoms is not a limitation affecting its role of warranting the thinness of sums: atoms are trivial or improper sums, whose thinness with respect to themselves is warranted independent of (Extensionality of Proper Parts).

Thus, CEM already provides conditions of existence and criteria of identity for sums. Given the other pieces of Linnebo's methodology (which are not under discussion in this paper), this legitimises reference to them and their existence. Why bother looking for abstraction principles?

This simple line of thought is exposed to two perplexities: The first is that both (Unrestricted Composition) and (Extensionality of Proper Parts) are controversial principles. Gone are the days when several philosophers, from Quine and Goodman to Lewis, agreed that CEM was the true and exhaustive theory of parthood and composition, with a near-logical status. A quick look to the recent Cotnoir and Varzi (2021), in which the status quo of research in mereology is masterfully exposed, reveals that a large part of it consists in exploring mereologies in which either (Unrestricted Composition) or (Extensionality of Proper Parts) (or both) are dropped in a variety of ways, often motivated by metaphysical worries concerning for example gerrymandered or utterly monstrous sums to which (Unrestricted Composition) would commit us, or allegedly distinct entities (such as the statue and the portion of clay constituting it) that (Extensionality of Proper Parts) would force us to conflate.

As anticipated, this is not an objection I am going to address in this paper. The author of this paper is convinced that indeed CEM *is* the general, exhaustive theory of parthood, and this motivates his attempt to ground the alleged thinness of mereological sums directly on (Unrestricted Composition) and (Extensionality of Proper Parts), given the other aspects of

Linnebo's methodology – and in particular given the thesis that the availability of explanatory conditions of existence and criteria of identity is enough to legitimise reference to some entities and therefore their thin existence. Despite the contemporary lack of consensus about the status of CEM, there are still some defenders of what Fine (1994, p. 138) has polemically called "*mereological monism*," that is, the thesis that there is a single true and general theory of parthood and composition and that CEM is this theory (I have extensively tried to defend mereological monism, with some caveats, in Lando (2017)). In this paper, I do not argue in favour of mereological monism; I am only maintaining that CEM is sufficient to provide explanatory conditions of existence and criteria of identity for mereological sums without any need to resort to those troublesome abstraction principles for mereological sums at which Linnebo hints in his book. Any other mereology providing analogously explanatory conditions of existence and criteria of identity would equally fit the bill. A mereology in which either explanatory conditions of existence or explanatory conditions of existence *are not* provided does not fit the bill.

This limitation is alleviated by the fact that, as a matter of fact, many adversaries of mereological monism are motivated by the conviction that wholes *are not* thin with respect to their parts. In particular, they reject (Unrestricted Composition) because they maintain that something more than the mere existence of the summed entities is needed for a whole to exist,[16] or they reject (Extensionality of Proper Parts) because they maintain that the identity of a whole depends on something more than its proper parts.[17] Thus, the upshot is, so to say, a clean disagreement, at least between the two parties at stake. On the one side, mereological monists can adopt Linnebo'smetaontology and remark that CEM provides suitable conditions of existence and criteria of identity, and is thus enough to legitimise reference to sums and their thin existence. On the other side, their adversaries reject *both* those principles of CEM that in the eyes of mereological monists legitimise reference to sums and their thinness *and* the thinness of sums altogether.

This leads us to the second perplexity, one of a broadly sociological nature. The thinness of sums is an often discussed thesis in metaphysics, which – as mentioned in section 1 – is usually supported with the help of arduous and complicate strategies such as those employed in the debate about CAI. Can these theoretical efforts be dispensed with by simply adopting CEM, a relatively old theory that was at disposal well before seminal works such as Lewis (1991) and Baxter (1988) triggered the contemporary debate about CAI?

I contend that this is indeed the case but under an already declared presupposition: Linnebo's broadly Fregean metaontological thesis that the availability of suitable conditions of existence and criteria of identity is enough to legitimise reference to some entities and warrant their thin existence. If you agree with this metaontological thesis, there is no reason why it should be reserved to abstraction principles: abstraction principles are not the only way to set forth suitable conditions of existence and criteria of identity. Inasmuch as the source of ontological legitimization and the ground of thinness *is* the availability of suitable conditions of existence and criteria of identity, abstraction principles are surely an important route for making them available. However, in the case of mereological sums, this route is complex and perilous, whereas CEM is well known and deemed by mereological monists to be the general and exhaustive theory of parthood and composition. If you accept mereological monism, you can adopt Linnebo's metaontology without the arduous detour through abstraction principles.

5 | INTERPRET`ATIONAL MODALITIES WITHOUT ABSTRACTION

Let us take stock, in order to set the stage for appreciating how another tool in Linnebo's toolbox, namely interpretational modalities, can be profitably applied to the thinness of

[16]This happens for example in Koslicki (2008, section VII.2.2).
[17]See for example Thomson (1983, 1998) and Cotnoir (2010).

mereological sums, independently of abstractionism. In section 2, I raised several concerns about plural abstraction principles for sums and in particular about their explanatory role, in virtue of which they could warrant the thinness of sums. In section 3, I maintained that singular abstraction principles for sums cannot work because sums do not correspond to any partition of the summed entities, and thus to any unity relation. In section 4, it turned out that, in the context of Linnebo's metaontology and in particular of his idea that explanatory conditions of existence and criteria of identity warrant thin existence, CEM already warrants the thinness of sums. In section 2, we also saw that Linnebo's abstraction principles for sets (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) resort to *interpretational modalities*, in order to express the dynamic character of set-theoretical abstraction and avoid paradoxes.

In this last section, I would like to suggest that interpretational modalities can also be useful for expressing some aspects of the thinness of sums, also if the more direct way to thinness articulated above in section 4 – in which standard principles of CEM provide explanatory conditions of existence and criteria of identity for sums, thereby warranting their thinness, given Linnebo's metaontology – is chosen.

This application of interpretational modalities does not concern the need to avoid paradoxes and is methodologically similar to two extant attempts to apply interpretational modalities to mereology. These two attempts are nonetheless different from the present one because in one case a different route to the thinness of sums is chosen, whereas in the other there is no commitment to the thinness or ontological innocence of sums. Namely, in the first attempt, Botti (2021) sets forth an original variation of CAI in terms of interpretational modalities, specifically by construing parts as interpretationally possible existents with respect to wholes. These interpretationally possible existents would provide a metaphysically informative analysis of wholes. In this paper, we are by contrast laying CAI aside inasmuch as CAI warrants the thinness of sums in a more direct (and more controversial) way; moreover, the paper is deliberately focused on the thinness of sums with respect to summed entities (and not on the alleged thinness of parts with respect to wholes). After Botti, in Lando (2020) – the second attempt – I also proposed an application of interpretational modalities to semantic compatibilism and (Unrestricted Composition), but I explicitly shunned any commitment to the thinness of sums (2020, 198). Abstractionism is involved in neither of the two attempts, and an analogous detachment of interpretational modalities from abstractionism characterises the present application of interpretational modalities to the direct way to the thinness of sums proposed above in section 4.

In Linnebo's reconstruction of set theory, given a certain plurality of entities, it is interpretationally possible that there is a set whose members are those entities. (Set Abstraction: Conditions of Existence) and (Set Abstraction: Criteria of Identity) provide explanatory conditions of existence and criteria of identity for the resulting sets. This legitimises reference to them and also quantification over them. It is therefore possible to expand the unrestricted domain of quantificaton so as to include them. The sets are relatively thin objects with respect to their members. It is worth noting that the reason of sets' relative thinness with respect to their members *is not* that their existence is interpretationally possible but the fact that they have suitable (i.e., explanatory and respecting the epistemic constraint) conditions of existence and criteria of identity in terms of their members. What the interpretational modal operators express is that it is then possible to include them in the domain of quantification. In other words, the thinness and the interpretationally possible existence of sets are both warranted by the availability of explanatory conditions of existence and criteria of identity.

At a certain point, Linnebo glosses interpretational modal operators in terms of abstraction: "we may think of 'necessarily' as meaning 'no matter what abstraction steps we carry out, it will remain the case that'" (2018, 61). However, there is no reason to keep interpretational modalities indissolubly connected with abstraction steps. Interpretational modalities concern what is possible and necessary once the unrestricted domain of quantification becomes more inclusive. Abstraction is not the only process through which the domain of quantification can become more inclusive.

In another application of interpretational modalities, *thin mereological sums* are added in the unrestricted domain of quantification. Mereological sums are thin because of (Extensionality of Proper Parts) and (Unrestricted Composition). These principles also legitimise reference to sums and the expansion of the domain of quantification so as to include them.

In this picture, the purpose of interpretational modal operators is to characterise a way in which, from an epistemic viewpoint, a subject or some subjects *can* come to interpret their language in such a way that the unrestricted domain of quantification begins including some sums that were previously not included. Some entities were unavailable in the domain of quantification *and then* become available. The expansion of the domain is legitimised by the availability of explanatory criteria of existence and conditions of identity for the new entities. The fact that these conditions of existence and criteria of identity are provided by CEM and not by some abstraction principle does not make the application of interpretational modality less appropriate.

The temporal ingredient ("*and then*") requires some comments. The expansions can be imagined to follow the epistemic condition of a quantifying subject (i.e., the subject who utters a quantified sentence) or of a community of users of a language to which the quantified sentences belong (in the following discussion, for the sake of simplicity, I mostly focus on a single quantifying subject and set aside the community alternative). In his account of expansions involving mathematical entities, Linnebo never countenances a temporal process of expansion. However, nothing in the concept of interpretational modality excludes the hypothesis that expansions occur in time and are a real sequence of temporal consecutive steps, involving a real subject. When this subject either attains an epistemic contact with new entities (perhaps through a form of Russellian acquaintance) or has some practical reasons to begin paying attention to something that the subject previously disregarded, these entities come to be at disposal in the subject's unrestricted domain of quantification.

In the case of sets and other mathematical thin objects, abstraction principles provide explanatory conditions of existence and criteria of identity. For mereological sums, these are provided by CEM (or by another mereology providing explanatory criteria of existence and conditions of identity for sums, but I will lay this alternative aside in what follows). Thus, given some entities to be summed that are already in the domain of quantfification, the subject is entitled to interpret their language so as to include *their relatively thin sum* in the domain of quantification.

In order to capture how these possible expansions of the domain of quantification work, interpretational modal operators can be added in (Unrestricted Composition), the principle providing criteria of existence for sums, thereby obtaining the following principle, according to which, at each step of the expansion, for every plurality of entities in the unrestricted domain of quantification, it is interpretationally possible that their sum exists:

$$\Box_{int}\forall xx\Diamond_{int}\exists y\Sigma(xx,y) \qquad \text{(Interpretationally Modalized Unrestricted Composition)}$$

An adequate epistemic account of the interpretational possibilities at stake should also illustrate the circumstances in which a subject actually expands the domain of quantification by including the sum of some previously admitted entities.[18] This expansion, according to (Interpretationally Modalized Unrestricted Composition), is always possible provided that the summed entities already belong to the domain of quantification. But when is the possibility actualized by a subject?

[18]Both Botti (2021) and Lando (2020) provide several details about these epistemic aspects, in the context of their applications of interpretational modalities to mereology.

There are fathomable practical reasons why a subject could begin paying attention to sums of previously admitted entities and thereby exploit the possibility disclosed by (Interpretationally Modalized Unrestricted Composition). Fine, who first introduced in (2005; 2006) interpretational modalities as a tool to expand the unrestricted domain of quantification, has suggested in (2007) a reason why a principle in the vicinity of (Interpretationally Modalized Unrestricted Composition) might be accepted: no matter how disparate some entities are, there are (sometimes admittedly remote) reasons why the need could arise for the subject to consider their mereological sum and thereby include this sum in the domain of quantification. Fine's example of these remote reasons concerns a car and a bouquet of flowers and a change in a community's religious views from one generation to another, but it can be easily adapted to arbitrary summable entities and/or to a single individual changing their religious views:

> We may imagine that some future religious sect holds the view that cars are endowed with souls who migrate to a neighbouring bouquet of flowers after a gestation period of nine months (stranger religious views have been held). […] We may explain why it is correct for us to deny the existence of car-bouquets and yet also correct for the future generations to affirm their existence by appeal to a difference in what each of us has introduced into the ontology. (Fine, 2007, 165)

Fine has expressed his opposition to mereological monism and to various principles of CEM in several works, including Fine (1994), Fine (1999), and Fine (2010). Nonetheless, his example can be read from the perspective of a mereological monist who argues for the thinness of sums in the way I suggested in section 4. The reason why the interpretationally possibly existent sums, such as Fine's car-bouquets, are thus thin is that (Unrestricted Composition) provides conditions of existence for them, whereas (Extensionality of Proper Parts) provides criteria of identity for them. No abstraction principle is involved. The principles at play are nonetheless formulated only in terms of already admitted entities. Thus, there are good reasons to claim that the corresponding expansions of the domain do not make substantial, further demands on the world. Sums are therefore relatively thin objects, independent of abstraction.

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REFERENCES

Armstrong, D. (1978) *Nominalism & realism. Universals & scientific realism: volume I*. Cambridge: Cambridge University Press.

Arntzenius, F. & Hawthorne, J. (2005) Gunk and continuous variation. *The Monist*, 88(4), 441–465.

Baxter, D. (1988) Many-one identity. *Philosophical Papers*, 17, 193–216.

Bøhn, E. (2009) Composition as identity. A study in ontology and philosophical logic. PhD thesis. Amherst (MA): University of Massachusetts.

Botti, M. (2021) Composition as analysis: the meta-ontological origins (and future) of composition as identity. *Synthese*, 198(Supplement 18), 4545–4570.

Carrara, M. & Lando, G. (2021) Mereology and identity. *Synthese*, 198(Supplement 18), 4205–4227.

Cotnoir, A. (2010) Antisymmetry and non-extensional mereology. *The Philosophical Quarterly*, 60(239), 396–405.

Cotnoir, A. (2013) Composition as general identity. In: Bennett, K. & Zimmermann, D. (Eds.) *Oxford studies in metaphysics: volume 8*. Oxford: Oxford University Press, pp. 295–322.
Cotnoir, A. (2014) Composition as identity: framing the debate. In: Cotnoir, A. & Baxter, D. (Eds.) *Composition as identity*. Oxford: Oxford University Press, pp. 3–23.
Cotnoir, A. & Varzi, A. (2021) *Mereology*. Oxford: Oxford University Press.
Fine, K. (1994) Compounds and aggregates. *Noûs*, 28(2), 137–158.
Fine, K. (1999) Things and their parts. *Midwest Studies in Philosophy*, 23(1), 61–74.
Fine, K. (2005) Our knowledge of mathematical objects. In: Gendler, T. & Hawthorne, J. (Eds.) *Oxford studies in epistemology: volume 1*. Oxford: Oxford University Press, pp. 89–110.
Fine, K. (2006) Relatively unrestricted quantification. In: Rayo, A. & Uzquiano, G. (Eds.) *Absolute generality*. Oxford: Oxford University Press, pp. 20–44.
Fine, K. (2007) Response to Kathrin Koslicki. *Dialectica*, 61(1), 161–166.
Fine, K. (2010) Towards a theory of part. *The Journal of Philosophy*, 107(11), 559–589.
Goodman, N. (1956) A world of individuals. In: Bochensky, J., Church, A. & Goodman, N. (Eds.) *The problem of universals. A symposium*. Notre Dame (IN): University of Notre Dame Press, pp. 13–31.
Hawley, K. (2013) Cut the pie any way you like? In: Bennett, K. & Zimmermann, D. (Eds.) *Oxford studies in metaphysics: volume 8*. Oxford: Oxford University Press, pp. 323–341.
Koslicki, K. (2008) *The structure of objects*. Oxford: Oxford University Press.
Lando, G. (2017) *Mereology. A Philosophical Introduction*. London: Bloomsbury.
Lando, G. (2020) Expansionism and Mereological universalism. *Theoria*, 86(2), 187–219.
Leonard, H. (1930), Singular terms. PhD thesis. Cambridge (MA): Harvard University.
Lewis, D. (1986) *On the plurality of worlds*. Oxford: Blackwell.
Lewis, D. (1991) *Parts of classes*. Oxford: Blackwell.
Linnebo, Ø. (2018) *Thin objects. An abstractionist account*. Oxford: Oxford University Press.
Parsons, J. (2007) Theories of location. In: Zimmermann, D. (Ed.) *Oxford studies in metaphysics: volume 3*. Oxford: Oxford University Press, pp. 201–232.
Paul, L. (2002) Logical parts. *Noûs*, 36(4), 578–596.
Rosen, G. (2020) Abstract objects. In: Zalta, E. (Ed.) *The Stanford encyclopedia of philosophy. Spring 2020 edition*. Stanford (CA): Metaphysics Research Lab, Stanford University. Available from: https://plato.stanford.edu/archives/spr2020/entries/abstract-objects/ [Accessed 17th December 2020].
Russell, J. (2017) Composition as abstraction. *The Journal of Philosophy*, 114(9), 453–470.
Schaffer, J. (2010) Monism: the priority of the whole. *The Philosophical Review*, 119(1), 31–76.
Smid, J. (2022) Mind the gap. *Journal of Philosophical Logic*, 51, 57–73.
Thomson, J. (1983) Parthood and identity across time. *The Journal of Philosophy*, 80(4), 201–220.
Thomson, J. (1998) The statue and its clay. *Noûs*, 32(2), 149–173.
Uzquiano, G. (2006) The price of universality. *Philosophical Studies*, 129(1), 137–169.
Varzi, A. (2008) The extensionality of parthood and composition. *The Philosophical Quarterly*, 58(230), 108–133.
Wallace, M. (2011) Composition as identity: part 1. *Philosophy Compass*, 6(11), 804–816.
Zimmermann, D. (1996) Could extended objects be made out of simple parts? An argument for atomless "gunk". *Philosophy and Phenomenological Research*, 56(1), 1–29.

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