

# Composition and Identities

MANUEL LECHTHALER

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For Shadi



## *Statement*

This thesis is solely the work of its author. No part of it has previously been submitted for any degree, or is currently being submitted for any other degree. To the best of my knowledge, any help received in preparing this thesis, and all sources used, have been duly acknowledged.

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MANUEL LECHTHALER

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# *Abstract*

Composition as Identity is the view that an object is identical to its parts taken collectively. I elaborate and defend a theory based on this idea: composition is a kind of identity. Since this claim is best presented within a plural logic, I develop a formal system of plural logic. The principles of this system differ from the standard views on plural logic because one of my central claims is that identity is a relation which comes in a variety of forms and only one of them obeys substitution unrestrictedly. I justify this departure from orthodoxy by showing some problems which result from attempts to avoid inconsistencies within plural logic by means of postulating other non-singular terms besides plural terms. Thereby, some of the main criticisms raised against Composition as Identity can be addressed. Further, I argue that the way objects are arranged is relevant with respect to the question which object they compose, i.e. to which object they are identical to. This helps to meet a second group of arguments against Composition as Identity. These arguments aim to show that identifying composite objects on the basis of the identity of their parts entails, contrary to our common sense view, that rearranging the parts of a composite object does not leave us with a different object. Moreover, it allows us to carve out the intensional aspects of Composition as Identity and to defend mereological universalism, the claim that any objects compose some object. Much of the pressure put on the latter view can be avoided by distinguishing the question whether some objects compose an object from the question what object they compose. Eventually, I conclude that Composition as Identity is a coherent and plausible position, as long as we take identity to be a more complex relation than commonly assumed.



## *Preface*

Our world contains a large number of material objects with parts. I am typing these lines by hitting some keys, which are part of a keyboard. The keyboard lies on a tabletop, which is part of a table standing in the philosophy department. On the shelf beside the table, there are several books, which have pages as parts. We call such objects ‘composite objects’ and say that they are composed, or made up, of their parts.

The aim of this thesis is to develop and defend an account of composition as a kind of identity. Thereby, it connects to the position called “Composition as Identity”. The two central claims I am arguing for are that an object is identical to its parts taken collectively and that this identity relation is sensitive to the way the parts are arranged. I will present a formal system for these claims, which avoids some major problems of Composition as Identity discussed in the literature and brings out the intensional character of Composition as Identity. My aim is to give a detailed presentation of this view, with its costs and benefits singled out, and indicate open problems, which are still to be faced.

Talk about identity and parthood is ubiquitous, which may suggest that we have a good understanding of these two notions. However, philosophers are still baffled by ancient puzzles such as the “the Statue and the Piece of Clay” or “the Ship of Theseus”. The fact that we are still troubled with these paradoxes shows that they are more than just means to puzzle first year philosophy students or unusual – and as I can tell from my own experience, rarely successful – icebreakers at parties. They show that this oversimplified picture of identity and parthood is mistaken and in need of clarification.

Composition is what brings the notions of identity and parthood together, and hence, the key to shed light on these puzzles is to find out what composition is. The recent development of plural logic promises to give us the adequate logical tools to analyze composition with the help of formal methods. It allows us to finally use *logical* methods in order to tackle the ancient *metaphysical* problems. Since it is only fair to say that the development of plural logic is still in its initial phase, there are many open questions and problems to be explored.

Part I, "Composition as Identity", is a discussion of the motivations for and criticisms of the claim that a composite object is identical to its parts, as well as an analysis of different ways this assumption is spelled out in the literature. Eventually, I suggest elaborating on the point that composition is a kind of identity. This hypothesis hinges on understanding identity as a relation, which comes in different kinds.

The idea of the varieties of identity is argued for in Part II, "From Plurals to Identities". There, I will develop a formal system, which makes use of well-established principles and concepts of plural logic, i.e. a logic that allows the use of singular and plural terms. I show then that the traditional strategy to avoid certain inconsistencies, which follow from the basic principles of plural logic and innocuous empirical assumptions, has serious difficulties. An examination of these derivations and some commonly upheld principles on identity will lead me then to a denial of the thought that there is only one kind of identity. This makes it at least possible to hold on to a restricted version of "substitution" – the inference rule that allows substituting co-referring terms – such that it is still applicable within some contexts. By reflecting upon the lessons, which have been drawn from substitution failures in singular contexts, I conclude that some plural terms are non-rigid designators. Further, I suggest that predicates which are collective in an argument place are non-extensional in that argument place. Thus, substituting non-rigid designators in these argument places should not be considered as a reliable inference: In the case of predicates which are intensional in an argument place, only rigid designators can be substituted. On the basis of the theory of "Articulated

Reference”, I propose that only terms, whose reference is articulated in the same way, can be substituted when we encounter predicates that are hyperintensional in an argument place. This allows us to reply to one of the criticisms raised against Composition as Identity, namely that objects necessarily have the parts they actually have.

With the aim to overcome the shortcomings of the conventional systems of plural logic, some of its doctrines are dismissed, while additional principles as well as further concepts will be introduced instead. These are then used to promote a formal system, which allows to define a general identity relation and its nine kinds. The different kinds of identity relations are distinguished syntactically and semantically, based on the kinds of terms they take as arguments and on the number of objects these refer to. After presenting some theorems of this formalism, two further objections to Composition as Identity can be met. The assumption that there are different kinds of identity relations might appear to be a high price to pay for a defense of Composition as Identity, yet the benefits outweigh the costs.

Eventually, we see in Part III, “Arrangement Matters”, that the previously developed view cannot deal with examples where the parts of a composite object are rearranged in such a way that they compose a different object. In order to meet this criticism put forward against Composition as Identity, I propose to modify the system from Part II. By taking the way objects are arranged as a condition for which object they compose these difficulties can be overcome. Thereby, the idea that Composition as Identity is built upon an intensional view gets further support and criticisms of it that are implicitly asking it to be extensional dissolve. All that remains is the questions of arrangement. Moreover, this account of composition provides us with reasons to embrace the view that any objects compose some object.<sup>1</sup> The final chapter concludes with some remarks on why the account of composition I develop does not turn out to be

1. In certain places, I will simply say that *objects compose*. This should be understood as short for *some objects compose some object*.

a disguised form of mereological nihilism, which claims that no objects compose.

Much of what I say about some objects composing *an* object applies *mutatis mutandis* to some objects composing *some* objects. However, I will not provide a complete account of composition, which can handle these cases due to its complexity and the space available. The analysis of composition given here is incomplete in a further respect by considering *material* objects only. Whether there are composite non-material objects, and how or under what conditions they compose are interesting questions but will be ignored in what follows. Some of the examples I provide may suggest the contrary, but they are intended to be read from a materialist standpoint. I shamelessly assume that composition is an irreflexive relation, i.e. no object composes itself. In my opinion, we cannot make sense of “self-composition”, which is why I am happy to exclude it from my analysis. Further, although plural logic will be introduced in Part II, we will rely on some of its resources already in Part I, where we will occasionally use plural variables ‘*uu*’, ‘*vv*’, ‘*ww*’, ranging over plural terms, such as ‘Anna and Frege’ or ‘the authors of *Principia Mathematica*’,<sup>2</sup> referring to more than one object at once.

I would like to give some final remarks concerning a methodological issue. Discussions of metaphysical theories are often confronted with the question how to evaluate the different positions. For instance, on the basis of what are we to decide whether it counts in favor of a metaphysical theory that it entails a certain other metaphysical view? Or, on the basis of what do we judge one metaphysical view to be better or more appropriate than another? As a quick look at the references I used already might suggest, my views are heavily influenced by the works of David Lewis and Willard Van Orman Quine. I take myself to be working broadly in the tradition of Lewis and Quine, where considerations of parsimony are lent considerable, though not exclusive, value. Yet, I should

2. For the ease of exposition, I will use the abbreviations ‘*PM*’ for ‘*Principia Mathematica*’, ‘*OD*’ for ‘*On Denoting*’, and ‘*Grundgesetze*’ for ‘*Grundgesetze der Arithmetik*’.

emphasize that there are ample points of disagreement, so that it would be unfair to these philosophers to say that they would agree with all the claims I am arguing for. Nevertheless, some of their views should be seen as a starting point of my investigation, in particular, the appreciation of the principle of parsimony, and so I hope to make a contribution to this bigger project with the theory we are about to develop.

Additionally, I should mention that, from a historical point of view, mereology *and* plural logic have been within a nominalistic framework. Since the principle of parsimony is a key point of nominalism, the following discussion is intended to contribute to the work that has been in this tradition. In order to avoid the impression that we are making arbitrary decisions at certain points in the following discussion, I think it is crucial to stress that considerations of parsimony will play an important role and that it will be one of the basic principles I shall use to evaluate different positions along the way we are about to go.

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## **Part I**

# **Composition as Identity**





## CHAPTER ONE

# *The Road to Composition as Identity*

A material object and its parts stand in a very special relation to each other. Take for instance, a broom and two of its parts, the stick and the brush. Whenever we move, burn, buy, or color the broom, we move, burn, buy, or color the stick and the brush, *and vice versa*. The broom is located wherever the stick or the brush is located, *and vice versa*. But how can we explain this “Intimacy of Parthood” (Sider 2007: 54)?

One way to understand the intimate relation that holds between a composite object and its parts is to take the object as being “nothing over and above” its parts. Here is where Composition as Identity has its starting point. At first sight, the idea of an object being nothing over and above its parts is as natural as it is puzzling. What does it mean for an object to be nothing over and above its parts? A plausible line of thought is, as Cotnoir (2014: 4) suggests, to take Composition as Identity to be an attempt to clarify this intuition: A composite object is *identical* to its parts taken collectively. This position stands in direct opposition to a long tradition, called “hylomorphism”. Most recently, hylomorphism is put forward by Fine (1999) and Koslicki (2008), and it appeals to the following view of Aristotle:

[T]he whole is not the same as the sum of its parts [...] [Y]ou may have the parts and yet not have the whole, so that parts and whole cannot be the same.

(Aristotle 1963b: 150a,16-22)

Based on this thought of Aristotle, the slogan “the whole is more than the sum of its parts” became popular.<sup>3</sup> Although I think that the reasons which led Aristotle and others to this claim are justified, I disagree with their conclusion. How can an object be anything more than its parts? After all, if I move the parts of a table from the corner to the center of the room, I move the table. Nothing is left in the corner. To say that an object is greater than its parts strikes me as subscribing to a sort of mysticism.

By the desire to avoid this enigmatic view on the relation between an object and its parts, Composition as Identity forces itself upon us. The denial of the above view is the claim that a composite object is nothing over and above its parts, which in turn is best explained by Composition as Identity. So let’s see in more detail what considerations speak for taking an object as being nothing over and above its parts and the interpretation Composition as Identity offers for this view.

## 1.1 From Parthood to Identity

Composition as Identity is the thesis that an object is identical to its parts, taken collectively.<sup>4</sup> Much of the motivation for this claim comes from intuition. This might look suspicious. People have different intuitions when it comes to metaphysical theories. Sometimes, and I think that is

3. As Harte (2002: 9, fn.2) notes, tracing back the history of this slogan is not an easy task. It is discussed explicitly, for instance, in (Nagel 1952), as well as in (Rescher and Oppenheim 1955: 94). For an overview on Aristotle’s hylomorphism, (see Cohen 2009: 202-7, Fine 1992, Koslicki 2006; 2008: §6, and Studtmann 2012: 71-4). Disagreements with Aristotle, and hence a tendency towards Composition as Identity, can be found in the writings of Plato (1892c: 204a; see also Harte 2002), Abaelardus (1970: 343-5), Hobbes (1839: 96-7), Leibniz (1902: 251), Locke (1952: ii. 284), Kant (1924: III.7), and Frege (1884: §46). For an overview on the discussion of Composition as Identity in medieval and early modern philosophy, (see Normore and Brown 2014).

4. To spell out what this claim actually means, is one the main tasks of our overall discussion here. Yet, to avoid confusion, let me point out that Composition as Identity, as it is usually understood, does *not* claim that an object is identical to *each one* of its parts *individually*. Further, we might think of this relation to be *similar* to the identity relation that holds between any object and itself. Although these remarks may seem a bit vague, I hope they give an initial hint about where we are about to go.

the case with Composition as Identity, the very same person might have opposing intuitions with respect to a metaphysical theory. However, one may ask *What else do we have to rely on when it comes to metaphysical theories?*

We do not have many principles which can help us to resolve a dispute in metaphysics, once we find out that a metaphysical dispute is the result of incompatible intuitions. In my view, ontological parsimony is, other things being equal, a suitable candidate. When we reach a stalemate in a metaphysical discussion and the only thing that seems to be left is to rely on diverging intuitions, I think, we should prefer the theory that comes with fewer ontological commitments. Why and under what conditions, ontological parsimony is a desirable feature of a theory is on its own a question which goes beyond the space I have available here. Yet, I should highlight now that it is one of my basic guidelines and it will influence some of the decisions which I will make in what is about to come. Although not everybody will agree that the principle of parsimony should enjoy such a status, it might at least help to see that I am not only relying on intuitions, but it will be possible to trace back on why I suggest to go one way, rather than another. It seems only appropriate that the principle of parsimony should play an important role, when dealing with Composition as Identity, since it allows us, as we will see soon, to keep down the ontological commitments of our theory of composite objects.

Although central for *my* belief in Composition as Identity, intuition cannot be the ultimate reason on which I wish to motivate my thesis. Hence, after identifying its intuitive motivations, I will present further reasons which aim to show why Composition as Identity deserves to be taken seriously.

### 1.1.1 Nothing Over and Above

One intuitive explanation for why material objects and their parts are so closely related to each other is to think that an object is nothing over and above its parts taken collectively. The broom is not an additional object besides the stick and the brush. It is not anything beyond them.

Recently, Baxter (1988a; 1988b; 2014), Bricker (2016), Bohn (2014), Cotnoir (2013a), Varzi (2000; 2014), and Wallace (2011a; 2011b; 2014) have, arguably, shown a sympathy for some kind of Composition as Identity.

Two examples, inspired from Baxter (1988a: 579 and 1988b: 197, 200), will help to see the line of thought underlying the idea of an object being nothing over and above its parts.

### **The piece of land and the six parcels**

A farmer owns a piece of land, which he divides into six parcels. He finds six buyers and offers to each a different parcel. After selling the six parcels, the farmer starts planting trees on the piece of land. Seeing this, the buyers confront him and ask what he is doing. The farmer tells them he is planting trees on his piece of land and goes on to argue that he only sold the six parcels, not the piece of land. Hence, he still has the right to plant trees on the piece of land. The buyers tell the farmer that he is wrong. He does not own the piece of land anymore, because he sold the six parcels to them. He does not own the piece of land anymore, since the six buyers are now the owners of the piece of land.

The buyers' argument strikes us as sound. The farmer is wrong in thinking that he still owns the piece of land. By selling the six parcels, he sold the piece of land. The piece of land is not a seventh plot of land besides the six parcels. It is nothing over and above the six parcels. Since each one of the buyers owns now a piece of land, they own it now together, just as if each one of them eats a sixth of a cake, then they have eaten the cake together.

### **The six-pack and the six cans**

Anne waits in the *six items or less* line in a grocery store. She has a six-pack of orange juice in her basket and reflects whether she can take a chocolate bar without having to change line. Anne is not sure. Is there only one object, the six-pack, in her basket? Or does the basket contain six objects, the six cans of orange juice? She decides not to buy the

chocolate bar and stays in the *six items or less* line. Anne is sure that she is buying less than six items. However, a shop assistant tells Anne that she has to go to a different check-out, since she is buying more than six items, the six-pack and the six cans of orange juice.

Although we may not be sure whether Anne is buying one object or six objects, we are inclined to disagree with the shop assistant. Anne is surely not buying more than six items. She is buying either one item, or six items.<sup>5</sup> We do not consider the six-pack to be something she is buying *in addition* to the six cans, as it would be the case, if Anne decided to buy the chocolate bar. The reason for this is that the six-pack does not count as something over and above the six cans. After all, Anne does not pay for the six cans *first*, and *then* for the six-pack later. By paying for the six cans, the six-pack is already paid, and *vice versa*.

### 1.1.2 The No Double-Counting Policy

The two examples help us to see an important point of the idea that an object is nothing over and above its parts. Why are the piece of land and the six-pack not a seventh object besides the six parcels and the six cans, respectively? In both cases, a mistake in counting the objects which are present happens: In the first example, we want to tell the farmer that he has to consider the plot of land *either* as one piece of land, *or* as six parcels. When he counts the piece of land as a seventh object, then he makes a mistake. In the second example, the shop assistant miscounts the objects when she thinks that the six-pack is a seventh object besides the six cans of orange juice. It would be different, if Anne were queuing in the *five items or less* line. Then, we can at least *understand* why the shop assistant tells Anne to go to a different cash-out, though we need not necessarily agree with the shop assistant about that: The shop assistant thinks that

5. One could argue, I think correctly, that Anne is buying three items, three pairs of cans. For the sake of the argument, we shall assume that there are no such intermediary cases and that the above question comes down to whether Anne buys one or six objects.

Anne has to go to a different cash-out. In her view, Anne is buying more than five objects because her basket contains six objects, the six cans. But, counting the six-pack as a seventh object is a mistake.

This is reflected in what is sometimes called the “*no double-counting policy*” (Cotnoir 2014: 5). The idea behind this policy is that the question *How many objects are there?* has several correct answers because there are different, equally legitimate ways to count objects.<sup>6</sup>

A similar line of thought is put forward by Ryle (1950: 22-3): If someone tells us she bought a left-hand glove, a right-hand glove and a pair of gloves, we *naturally* assume she got four gloves. If she were to reply that she bought only two gloves, then we would tell her that she “either [...] bought a left-hand and a right-hand glove or she bought a pair of gloves (but not both)” (Ryle 1950: 23). To count the two gloves *and* the pair as separate items appears as a category mistake. The two gloves and the pair of gloves are different kinds of things, similarly to the six-pack and the six cans. To include them in the same count is a mistake.

Yet, the no double-counting policy does *not deny* the existence of the piece of land or the six parcels. It only says that it is a mistake to count both, the piece of land *and* the six parcels, or more generally, an object and its parts as *distinct objects* – in the sense of being disjoint, or non-overlapping, (see Lewis 1999: 177-9). This counting policy has an important consequence when we examine the ontological presuppositions of a theory, which we will see next.

### 1.1.3 Ontological Free Lunches

If it is correct that there are different ways to count the objects there are, as has just been suggested, then there are different ways to “[...] draw up

6. We can ignore the limiting case of counting an atom. The above discussed idea does not work when we have only an atom in front of us. If that is the case, then there is only one correct answer. However, that there are different ways to count objects is thought to be valid on a global scale, i.e. there are different legitimate ways to count the objects in our actual world. We could reformulate our counting policy more carefully as *There are different ways to count non-atomic objects.*

an inventory of Reality [...]” (Lewis 1991: 81; see also Varzi 2000; 2014), i.e. to spell out the ontological commitments of a theory – assuming we are not dealing with theories talking about lonely objects.<sup>7</sup> Let’s get clear about what the ontological commitments of a theory are first, before we see how the no double-counting policy relates to it.

The commonly accepted view of what the ontological commitments of a theory are goes back to Quine:

A theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true. (Quine 1948: 33)

The ontological commitments of a theory consist in a list of those objects which must exist, if the theory is true. For instance, take the following example of a theory, *t*, which is the result of adding the following two sentences to classical first-order logic with identity:

- (1) Anne is smaller than Ben
- (2) If *x* is smaller than *y*, then *y* is not smaller than *x*

From (1) and (2) we can deduce with a few logical inferences that Anne is not identical to Ben. Hence, two objects – Anne and Ben – have to exist, in order for *t* to be true; *t* is committed to the existence of Anne and Ben. If they do not exist, *t* is false.

If we agree that we have several correct ways to count the objects a theory is dealing with, although that might still seem a bit counterintuitive, then there are different, correct ontologies for that theory corresponding to the different ways of counting. Take for instance our theory *t*. We have not specified what objects Anne and Ben are. Assume, they are two pieces of wood, depicted in figure 1.1.

7. Lewis (1983a) introduces the concept of a lonely object as an object that is the only object existing in a possible world. Discussions of lonely objects and lonely worlds, i.e. worlds where a lonely object exists, can be found in (Haufe and Slater 2009) and (French and McKenzie 2012).

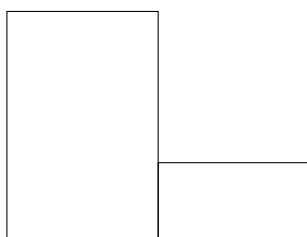


Figure 1.1: *Counting and Ontological Commitment*

The question *How many objects are depicted in figure 1.1?*, can then be answered with *Two, Anne and Ben*, or *One, the L-shaped piece of wood*. Both ways of counting match up with different ways to capture the ontological commitment of  $t$ . According to the first,  $t$  is committed to the existence of Anne and Ben. However, claiming that  $t$  is committed to the existence of the L-shaped piece of wood strikes us as equally correct. We intuitively tend to agree with Armstrong (1997: 13) that given the ontological commitment to the parts of an object, the object itself is an “[...] ontological free lunch”. Or to use another picture, once we paid our ontological taxes for the parts of an object, the ontological taxman cannot demand a payment for the composite object.<sup>8</sup> If a theory  $t$  is committed to the existence of, say, Anne and Ben, there is no additional commitment to the L-shaped object, since it is already accounted for in virtue of the commitment to Anne and the commitment to Ben. But we do not double count them.

This line of reasoning becomes even more plausible when we compare it with the following scenario: Assume, we were to extend  $t$  to  $t'$  by adding to the former the following two sentences:

(3) Claire is smaller than Ben

(4) Anne is identical to Claire

8. Zach Weber suggested this analogy to me in conversation. Tichý (1995: 175) uses a similar analogy. Weber and Tichý use this analogy to warn against blind faith in ontological parsimony.



An overzealous understanding of Quine's idea of ontological commitment will suggest that  $t$  and  $t'$  have different ontological commitments: While  $t$  is committed to the existence of Anne and Ben,  $t'$  is committed to Anne, Ben and Claire. But Anne is identical to Claire, so any ontological commitment to Anne *is* an ontological commitment to Claire, and *vice versa*. If Anne were not to exist, neither would Claire; if Claire were not to exist, Anne would not exist. The objects which have to exist for  $t'$  to be true are *the same* objects which have to exist for  $t$  to be true.

Taking the thesis that there are different ways to count the objects there are seriously, an analogous situation arises in connection with the commitment to objects and their parts. Being committed to the existence of the parts is being committed to the existence of the object they are parts of; and less controversially, *pace* Schaffer (2007; 2010), being committed to the existence of an object is being committed to the existence of its parts.<sup>9</sup>

Friends of Composition as Identity offer an explanation for why either an object or its parts are an ontological free lunch, which revolves around the belief that in the case of objects and their parts, things stand similar as with Anne and Claire. An object does not come with any additional ontological commitment simply because the object *is identical* to its parts:

Surely I am nothing over and above my particles: I am them, they are me. The 'are' of composition is just the plural of the 'is' of identity. [...] You might as well say: I know all about the life of Cicero, now what about Tully? (Lewis 1988: 71)

For the whole *is* all its parts taken together [...] stating an identity. (Baxter 1988b: 197)

An object *being identical* to its parts explains *why* there is no additional ontological commitment to the composite object, given a commitment to its parts. "It just *is* them. They just *are* it." (Lewis 1991: 81). If an object

9. The second claim above is important for theories which allow for gunky objects, i.e. objects whose parts all have at least one proper part. The term 'gunk' is introduced in Lewis (1991: 20). Gunk and gunky objects are discussed in more detail by Arntzenius (2012: §4), Forrest (1996; 2004), Hazen (2004), Nolan (2004), and Zimmerman (1996a; 1996b).

*is identical* to its parts, then the former does not come with an additional ontological commitment, given a commitment to the latter, since things stand similar as with the example of Anne and Claire and the theories  $t$  and  $t'$ : Being committed to the existence of the former simply is being committed to the latter, and *vice versa*.

We started this chapter with the intuition that an object is nothing over and above its parts. Composition as Identity offers us an interpretation of this idea. The no double-counting policy suggests that there are different ways to count objects and that it is a mistake to count an object *and* its parts. Composition as Identity tells us that given the ontological commitment to a composite object there is no further commitment to its parts, and *vice versa*. This entailment relation between Composition as Identity and a composite object being an ontological free lunch, given the ontological commitment to its parts, follows from the no-double counting policy and the fact that the ontological commitment of a theory is a list of those objects which have to exist in order for the theory to be true.

This may sound intuitive. Yet, we cannot but wonder how an object can be identical to its parts. After all, the parts are many and the object is one. What is this so-called “many-one identity”? It seems very unlike the cases of identity we encounter usually, where we have one object being identical to one object. What are we to make out of this many-one identity the friend of Composition as Identity is talking about? This and similar problems, which some authors take to arise from the concept of the identity at work within Composition as Identity, will be spelled out in detail in the next chapter. Meeting these criticisms of Composition as Identity will then be one of our major tasks in the next two parts.

## 1.2 The Overdetermination Argument

Composition as Identity is a way to take seriously the motivations of Merricks’ “Overdetermination Argument” (2001: §3). Also, Composition as Identity is a more commonsensical conclusion to draw from the Overde-

termination Argument. I argue that contrary to the intention on which the argument is built, it motivates Composition as Identity.

Merricks present the Overdetermination Argument in order to defend “[...] some sort of ‘biological anti-reductionism’ [...]” (Merricks 2001: 114).<sup>10</sup> According to this theory, there are no objects with parts, due to our decision to deal with material objects only. According to Merricks, there are no macroscopic inanimate objects:

[T]here are no books. Nor are there statues, rocks, tables, stars,  
or chairs. (Merricks 2001: 1)

The only existing things are humans, conscious animals – like dogs and dolphins – perhaps, other organisms, like trees and ants, and mereological atoms, i.e. objects with no proper parts, (see Merricks 2001: 114-5). The major step for Merricks to deny the existence of inanimate composite objects is a generalization of his Overdetermination Argument,<sup>11</sup> which concludes that if inanimate composites exist, then they do not cause any events. But there are no objects which do not cause any events. Hence, inanimate composite objects do not exist.<sup>12</sup>

This flies in the face of our ordinary conception of reality. The existence of books, statues and rocks is, from the standpoint of common sense, beyond dispute. It would be preferable to avoid Merricks’ conclusion. We shall see next that there is indeed a problem with the Overdetermination Argument. After presenting Merricks’ argument and its generalization, I will discuss this problem and suggest that the reasons Merricks presents to embrace his view lead us to Composition as Identity.

10. This position is for us equivalent to mereological nihilism. Mereological nihilism is a theory to which we have to pay particular attention, and we will come across it in several places. For a list of authors who show sympathy to this view, see section 1.4.1. In section 10.6, I will address the worry that the account of composition we are about to develop coincides with mereological nihilism.

11. The Overdetermination Argument follows the pattern of Kim’s “Supervenience Argument” against non-reductive physicalism, (see Kim 1993; 1999: 38-56; 2005: 13-22, 32-69).

12. Korman (2015: §10) and Thomasson (2007: §1) present a more detailed discussion of Merricks’ Overdetermination Argument than the one we are about to see.

### 1.2.1 Merricks' Overdetermination Argument

Merricks' Overdetermination Argument concludes that if baseballs exist, then they do not cause the shattering of a particular window. The argument goes as follows (Merricks 2001: 56):

1. The baseball—if it exists—is causally irrelevant to whether its constituent atoms, acting in concert, cause the shattering of the window.<sup>13</sup>
2. The shattering of the window is caused by those atoms, acting in concert.
3. The shattering of the window is not overdetermined. Therefore,
4. If the baseball exists, it does not cause the shattering of the window.<sup>14</sup>

13. The fact that Merricks' biological anti-reductionism denies the existence of windows is not a problem for the above argument. There is a way to formulate the argument such that an ontological commitment to the existence of windows is avoided. Merricks suggests that 'the shattering of the window' is an abbreviation for many scatterings, which he does not identify with the event of the shattering of the window, (Merricks 2001: 56, fn.1). I am not sure what Merricks' "scatterings" are. However, Merricks can avoid being committed to the existence of windows in the context of the above argument without having to rely on an explanation of scatterings by talking about the shattering of *window-wise arranged atoms* instead of the shattering of the window.

14. Contrary to what one might expect at first, we cannot use a similar line of argument in order to exclude the existence of atoms. Such an argument comes with highly problematic consequences. Since the Overdetermination Argument presupposes the truth of mereological atomism, as we will see in section 1.2.3, arguing against the existence of the baseball's atoms entails the claim that the baseball is an atom: If the baseball has no atoms as parts, then the baseball itself is an atom. *Pace* Contessa (2014), this claim is even more counterintuitive than Merricks' conclusion, which "[...] goes down like draught Guinness compared to the claim that baseballs are [atoms]" (Merricks 2001: 63). Moreover, eliminating alleged atoms in favor of "ordinary objects" ultimately forces an arbitrary decision upon us. For instance, we cannot hold on to the existence of a door *and* its handle. Either the door is an atom, then the handle does not exist, or the handle is an atom, but then the door does not exist. I cannot think of any way which helps us to make a plausible decision here. We can only make an arbitrary decision in this and similar cases, where the common sense view is that an inanimate macroscopic object is a proper part of an inanimate macroscopic object.

In order to evaluate the above argument, we have to clarify two central concepts, 'being causally irrelevant' and 'being overdetermined'.

Intuitively, an object  $x$  is causally irrelevant to whether some objects  $uu$  cause an event  $e$ , if the  $uu$  are causing  $e$  independently from  $x$ , i.e.  $x$  does not do any causal work for the  $uu$  in order to cause  $e$ . Here is an example to explain this thought:

Anne, Bill, and Chris form a Philosophy Club. In order to register their club, they have to name a president. They decide to choose the club's president in a democratic way and Anne gets elected unanimously. They cause Anne to be elected president with three to zero votes.<sup>15</sup> Each one of them, Anne, Bill, and Chris, is a partial cause for Anne being elected with three to zero votes. All three are *causally relevant* for the event of Anne being elected president with three to zero votes. On the other hand, Dan, who joins the Philosophy Club only *after* the election, is *causally irrelevant* for that event: He did not vote in the election; neither did he participate in any other way, for instance, by counting the votes, in the election; nor did he have any other causal connection to the election. It was not Dan who made them think about forming a Philosophy Club, or told them that they have to name a president in order to register the club.

In light of the above example, and analogous to the definition of Merricks (2001: 57-8), we define 'being causally irrelevant' as follows:

15. It is a central feature of the Overdetermination Argument that objects are taken to be causes of events. This diverges from the currently most popular views on causation which take the relata of the causation relation to be events, (see Davidson 1980: 693-5, Kim 1973: 222, Lewis 1986a: 216; 1986b: 558, Paul and Hall 2013: 7), or facts, (see Bennett 1988; 1995: 40-2, Mellor 1995: 156). Merricks tries to avoid this line of criticism by taking an object  $x$  being the cause of an event  $e$  simply to mean that  $x$  participates in an event  $e'$  which causes  $e$ . Yet, this only leads to further problems. In most cases, where an event  $e'$  causes an event  $e$  there are objects participating in  $e'$  which appear to be irrelevant for the causation of  $e$ . For instance, in the voting of the club's president  $e$ , the shoes of Anne, the spot on Bill's glasses, and the sun shining through the window participated in the election event. Yet, it is counterintuitive to take them as being causes for Anne being elected president. Not all objects which participate in an event causing another event qualify as causes for the latter. We shall flag this concern here and assume that the idea of objects being causes of events is feasible.

Def. An object  $x$  is causally irrelevant to whether some objects  $uu$  cause an event  $e$  iff

- (i)  $x$  is not among the  $uu$ , and
- (ii)  $x$  is not a partial cause of  $e$  alongside  $uu$ , and
- (iii) no one of the  $uu$  causes  $x$  to cause  $e$ , and
- (iv)  $x$  does not cause any of the  $uu$  to cause  $e$

Next comes the definition of ‘being overdetermined’. An event is said to be overdetermined iff it is caused by two events whereby either one of the two causes is irrelevant for the other to cause the event, (Paul and Hall 2013: 143). Applied to Merricks’ talk of *objects* as causes for events this amounts roughly to the claim that an event is overdetermined iff it is independently caused by *distinct* objects. Suppose, Anne throws a baseball towards the window. At the same time, Bill throws another baseball at the window. Both baseballs hit the window at the same time and the window shatters. Each one of the two baseballs causes the shattering of the window. Moreover, Anne and Bill acted independently, i.e. it is not the case that Anne saw Bill throwing the baseball and that led her to throw hers. Nor was it the other way round, that Anne’s behavior was in some way a reason for Bill to throw his baseball. For the sake of the example, we may imagine that they did not see or hear each other, or were connected in any other way. Then, each one of the baseballs is *causally irrelevant* for the other baseball to cause the shattering of the window. Hence, the shattering of the window is overdetermined by the two baseballs.

Since the Overdetermination Argument deals with the overdetermination of an event by *an* object, the baseball, and *some* objects, the baseball-wise arranged atoms, the following definition suffices for us:<sup>16</sup>

16. A complete definition of overdetermination would have to take into account that *an* object and *another object*, as well as *some* objects and *some other objects* may overdetermine an event. There is no need for us to develop such a sophisticated definition.

Def. An event  $e$  is overdetermined by some objects  $uu$  and an object  $x$  iff

- (i) the  $uu$  cause  $e$ , and
- (ii)  $x$  causes  $e$ , and
- (iii) the  $uu$  are causally irrelevant for  $x$  to cause  $e$ , and
- (iv)  $x$  is causally irrelevant for  $uu$  to cause  $e$ .

Having explained these two concepts, we can go on to examine the generalized version of the argument.

### 1.2.2 The Generalized Overdetermination Argument

There is nothing special about baseballs and window-shatterings. The Overdetermination Argument might as well deal with books, statues, or chairs, and book-wise, statue-wise, or chair-wise arranged atoms causing a revolution, a car to be dented, or a person to be injured. Hence, to think that the Overdetermination Argument generalizes is a consequential thought and gives us the following argument (Merricks 2001: 79-80):

1. An inanimate composite object  $x$ —if  $x$  exists—is causally irrelevant to whether its parts  $uu$ , cause an event  $e$ .
2. Any event  $e$  is caused by an atom  $y$ , or by some atoms  $uu$ .
3. No event  $e$  is overdetermined. Therefore,
4. If an inanimate composite object  $x$  exists, then  $x$  does not cause any event  $e$ .

With the last line of the above argument, Merricks goes on to argue that there are no inanimate composite objects. This claim can be reached with the help of the following assumption:

5. There is no object  $x$  which does not cause any event  $e$

This premise is inspired by what Kim calls “Alexander’s Dictum”:

*To be real is to have causal powers.*

(Kim 1993: 202, [italics in original])

It might appear to be a leap from Kim’s principle to premise 5., but they are based on similar ideas. Nevertheless, 5. can be motivated independently: Assume, I were to tell you some things about my sister. Her name is Anne. She is twenty years old. Her birthday is in May; and so on. But then, I tell you that she has never caused any event. She did not cause my father to celebrate on the day she was born, nor my mother to celebrate her third birthday. She never caused my father to make her breakfast, nor has she ever caused my mother to be worried because she did not come home before midnight. There is *absolutely no event* caused by Anne. You should take this to be sufficient evidence to assume that Anne does not exist. “Ockham’s Razor” demands not to assume the existence of more individuals than necessary. Therefore, it is only consistent to think that Anne does not exist. There is no need to postulate her existence in order to explain any event.<sup>17</sup>

The motivations behind Merricks’ argument, and more generally behind the arguments others present for mereological nihilism, deserve to be taken seriously. However, as I will argue next, these reasons do *not* lead us to deny the existence of composite objects, but rather that a composite object is nothing over and above its parts.

17. I flag three concerns about the above argument. First, Alexander’s Dictum excludes lonely worlds, see fn.7 in section 1.1.3. In such a world, there is *prima facie* no event caused by the lonely object. But, if we take conceivability as a condition for the existence of a possible world, the principle turns out to be false. Second, premise 2. *presupposes* the existence of atoms and excludes gunky objects, see fn.9 in section 1.1.3. Merricks’ attempt to allow for gunky objects fails. If there is a gunky object  $x$ , then its parts  $y_1$  and  $y_2$  must cause an event  $e$  which is not caused by  $x$ . This follows from Alexander’s Dictum. But,  $y_1$  is a gunky object, since every part of a gunky object is itself a gunky object. Hence,  $y_1$  has parts, which in turn must cause some event  $e'$  *not* caused by  $y_1$ ; and so on. Hence, if only one gunky object exists, Alexander’s Dictum works *against* the parsimony assumption which motivates it. Third, premise 3. is false. In section 1.2.1, we saw that there *are* overdetermined events. Hence, the premise must be that overdetermination is a non-abundant phenomenon; it happens only rarely.



### 1.2.3 An Alternative Conclusion

Others, for instance, Baker (2003: 598) and Lowe (2003: 706), have already challenged the Overdetermination Argument. The conclusion I draw from Merricks' argument begins with ideas presented by Sider (2003b: 772-3) and Thomasson (2006), but elaborates on their diagnosis of the argument. My claim is that the Overdetermination Argument is not sound because premise 1. is false: A composite object *is* causally relevant for whether its parts cause some event because the composite object just is its parts taken collectively. This is the conclusion I draw from comparing Merricks' example with examples of overdetermination discussed in the literature, (see, for example, McDermott 1995, Paul and Hall 2013: §3.5, and Schaffer 2003).

When we observe Anne and Bill throwing baseballs at the window, we want to say *Such a coincidence, both baseballs caused the shattering of the window!* We are surprised by the fact that two objects cause an event and each one of them is causally irrelevant for the other to cause it. That is a strange coincidence, and very unlikely to happen. However, assuming that there is part-whole overdetermination, there is nothing that surprises us or is a coincidence. On the contrary, we are inclined to say that an object can only cause events which are caused by its parts. How could the baseball break the window without the atoms arranged baseball-wise breaking the window, or *vice versa*? If the baseball causes the window to shatter, its atoms cause it; if the atoms cause the window to shatter, then the baseball causes the shattering as well. But how is this close connection between the baseball and the atoms arranged baseball-wise possible? And is the shattering *really* overdetermined, if it is caused by the baseball and its atoms? The asymmetry between the two examples suggests that it is not overdetermined.

It is important to stress that I do not reject premise 3. of the Overdetermination Argument. I agree with Merricks on this. The reason for why the shattering of the window is not overdetermined by the baseball and its atoms is not that the baseball does not exist, but that the baseball is

*causally relevant* for whether its atoms cause the shattering: It is identical to them. Hence, premise 1. is the weak link in the Overdetermination Argument. Likewise, we can show that this applies to the generalized Overdetermination Argument: Composite objects exist, because they are causally relevant for whether their parts cause an event. The composite object is identical to its parts. This suggests the conclusion that although there is no part-whole overdetermination, composite objects exist because they are identical to their parts.

Hence, we see that the Overdetermination Argument puts a spoke into the eliminativist's wheel. The lesson we learn from a closer inspection of the argument is *not* that composite objects do not exist, but that they are identical to their parts. Rejecting one of the claims on which the Overdetermination Argument is based, i.e. that Composition as Identity is false, (see Merricks 2001: 31), is a more reasonable conclusion to draw from Merricks' reflections on overdetermination. Thereby, the argument turns out to *support* Composition as Identity, rather than mereological nihilism or biological anti-reductionism.

### 1.3 The Uniqueness of Composition

A further reason to embrace Composition as Identity is that it guarantees that composition is unique. In other words, Composition as Identity entails that if some objects compose an object, then they compose *at most* that object. This is the principle of the Uniqueness of Composition (UNI):

UNI Any *uu* compose at most one object *x*

Taking an example from van Inwagen (1990: 39), let's suppose we build a house with some bricks. Then, the bricks compose the house. Intuitively, we think that they compose *only* the house. But, "[w]hat makes us think that we haven't built *several* things, each of them composed of all the bricks?" (van Inwagen 1990: 39). The assumption that the bricks compose a house does not entail that they compose the house only. This intuition is based on UNI.

The basic idea of UNI is captured in Classical Extensional Mereology by the extensionality principle: if  $x$  and  $y$  are composite objects, then they share all their parts iff  $x$  is identical to  $y$ , (Casati and Varzi 1999: 40). UNI and the extensionality principle are both under dispute, and we will eventually challenge them as well. For now, it suffices to note that a denial of UNI, together with the assumption that each one of the objects which compose an object is a part of that composite object, entails *prima facie* a denial of the extensionality principle.<sup>18</sup> Take again van Inwagen's example and suppose UNI fails in that case. Then, there are some objects, the bricks, which compose two distinct objects. Let's say they compose Anne's house and Bill's house. Hence, although the two houses are not identical, they have the same parts. The extensionality principle excludes this, and we see that denying UNI entails a denial of the extensionality principle. Moreover, a denial of the extensionality principle does not only allow for distinct objects to share all their parts, but also for a composite object to not share parts with itself. On the basis of this rather worrisome outcome – How can an object *not* have the parts it has? – embracing UNI, in some form or other, should be one of our ultimate goals.

Setting the worries about UNI discussed in the literature aside, it appears an intuitive assumption. That its denial forces us to deny the extensionality principle from Classical Extensional Mereology gives plausibility to UNI. So, we can conclude for now, with the caveat that the worries about UNI and the extensionality principle will be met eventually, that Composition as Identity entailing the principle of the uniqueness of composition motivates having a closer look at that theory.

## 1.4 Composition as Identity's Pioneering Role

I conclude this chapter with presenting further motivations for Composition as Identity. I will discuss two theories, mereological universalism

18. We will come back to criticisms of UNI in section 2.3. Cotnoir (2013b) challenges the extensionality principle.

and four-dimensionalism, and point out that they are best defended on the basis of Composition as Identity. However, I do not want *to argue for* mereological universalism or four-dimensionalism here. It suffices to highlight that a denial of Composition as Identity makes a defense of the two theories difficult, though not impossible. Since there are, arguably, plausible reasons to embrace either mereological universalism or four-dimensionalism, their dependence on Composition as Identity gives us good evidence to take the view seriously.

### 1.4.1 Mereological Universalism

According to “mereological universalism”, henceforth simply universalism, or the theory of unrestricted composition, any objects compose some object. Universalism is an answer to van Inwagen’s *Special Composition Question* (SCQ), which asks under what conditions some objects compose an object, or

SCQ Under what conditions is it true that  $\exists x$  the *uu* compose *x*?<sup>19</sup>

In search for a solution to the puzzles of material objects, van Inwagen suggests that we have to find an answer to the SCQ first, then “[...] the various components of the theory [of material objects] arise and fall into place quiet naturally [...]” (van Inwagen 1990: 20). Many agree on the central role of the SCQ, for instance the authors listed in the next paragraph, as well as Carmichael (2015), Hoffman and Rosenkrantz (1997: 179-87), Korman (2015), Markosian (2015), and Merricks (2001).

The two most popular answers to the SCQ are rather extreme. Mereological nihilism claims that under no condition some objects compose an object. Cameron (2010), Hestevolt (1981), Hossack (2000), Rosen and Dorr (2002), Schaffer (2007; 2010), and Sider (2013) show a tendency to embrace

19. Van Inwagen’s formulation of the SCQ is “[w]hen is it true that  $\exists y$  the *xs* compose *y*?” (van Inwagen 1990: 30). The difference between his formulation and the one above is merely a notational one, being a result of the differing uses of plural terms.

this view in various degrees. The other popular answer to the SCQ is universalism, which is arguably upheld by Armstrong (1989: 92), Bigelow and Pargetter (2006), Braddon-Mitchell and Miller (2006), Bricker (2016), Cartwright (1975), van Cleve (1986; 2008), Heller (1990), Hudson (2000; 2001), Lewis (1986c: 212; 1991: 7-10, 75-87, Quine (1981: 10), Rea (1998), Sider (2001), Thomson (1983: 216-7), and Varzi (2005; 2014). Its answer to the SCQ is that any objects whatsoever compose.

In the literature on composition, universalism and Composition as Identity are often found as a combined view. On the one side, people show sympathy for both views, for instance, Armstrong (1989: 92; 1997: 14-8), Bohn (2014), Bricker (2016), Lewis (1986c: §4; 1991: §3), or Varzi (2000; 2006; 2008; 2014). On the other side, some reject both views, for instance, Koslicki (2008), Merricks (2001), Simons (2006; 2016: 60), or van Inwagen (1990). This is not merely a coincidence. Bohn (2014), Harte (2002: 114), Merricks (2005: 629-30), and Sider (2007: 59-62) suggest that Composition as Identity entails universalism. Others have denied this link between the two positions, for instance, Cameron (2012) and McDaniel (2010). Whether there is indeed such a logical connection or not depends, I think, on the specific formulation of the two theories in question: Some versions of Composition as Identity entail a particular version of mereological universalism, but there are ways to spell out either one of the two theories such that universalism is not a consequence of Composition as Identity. Nevertheless, it cannot be denied that assuming Composition as Identity is a benefit for the universalist: If she denies Composition as Identity her theory creates an ontological explosion, forcing an enormous number of ontological commitments upon her.

We cannot yet give a fully satisfactory defense of universalism; that has to wait until chapter 10. But let me briefly outline an argument that aims to motivate universalism, the argument from vagueness. This argument is due to Lewis (1986c: 211-3)<sup>20</sup> and is elaborated by Sider

20. Heller (1990: 47-51) and van Cleve (2008: 328-31) follow the lines of thought of Lewis and Sider respectively.

(1997: 214-22; 2001: 120-32; 2003a). The argument aims to undermine the assumption that composition is a restricted relation, i.e. that some objects compose, and some objects do not.

The argument goes as follows: Suppose for *reductio*, composition is a restricted, non-empty relation,<sup>21</sup> i.e. there is a condition *C*, such that the following holds: the *uu* fulfill *C* iff the *uu* compose some *x*.<sup>22</sup> The only plausible suggestions for a restriction on composition are vague. If the restriction on composition is vague, then it is vague how many objects exist. It is not vague how many objects exist. Therefore, the initial assumption must be false and composition is unrestricted.

Let's have a look at each premise one by one. The first premise is the *reductio*-assumption, composition is restricted. The second premise claims that the only plausible suggestions for a restriction on composition are vague. Lewis discusses restrictions such as "[...] being in contrast with their surroundings more than they do with one another [and being] adjacent, stick together, and act jointly" (Lewis 1986c: 211). Surely, these conditions are all vague. Some objects are clearly in contrast with their surroundings more than they are with one another in some cases, and in some cases they are not. But there are situations where it is vague whether some objects are more in contrast with their surroundings than they are with one another. Suppose you have four navy blue colored pieces of confetti on a table. When the confetti are on the left end of the table, which is colored in sky blue, they are in clear contrast with the table. However, when they are on the right end of the table, which is navy blue, then they are not in clear contrast with the table. Further, think of the table as colored in such a way that it gradually turns from sky blue to

21. I added the clause that composition is a non-empty relation. This is an important piece for the argument to lead us to universalism. If we had not added it, the conclusion would leave us with a choice between universalism and nihilism. Lewis and Sider seem to take it for granted that composition occurs at least sometimes. However, this is disputed in the literature, as we have seen. We will come back to discuss nihilism in chapter 10.

22. In other words, the *uu* fulfilling *C* is a sufficient and necessary condition for them to compose some object.

navy blue. Then, there are certain parts of the table where it is not clear if the confetti are in clear contrast with its surface when placed there.

Other conditions for composition, which are discussed in the literature, lead to the same result. Take for instance the condition *being in contact*. Apart from contradicting common sense by denying the existence of scattered objects, for instance, suits, and galaxies, it is also a source for vagueness. There are cases where, say, two hydrogen atoms, are clearly in contact with each other; and there are cases where they clearly are not. Nevertheless, there are instances where it is vague whether they are in contact or not. Are the two hydrogen atoms in contact when they are one millimeter apart from each other? Or when they are one nanometer apart from each other? Van Inwagen's suggestion (1990: 82) that some objects compose iff their "activity [...] constitutes a life" is also a source of vagueness, as he concedes, (van Inwagen 1990: §17-19). Hence, his account confirms the above premise as well.

The third premise states that if the restriction on composition is vague, then it is vague how many objects exist. This is pretty straightforward. If it is vague whether some objects *uu* compose an object *x*, then it is vague whether the composite object *x* exists. Assume, it is vague whether two hydrogen atoms, and an oxygen atom compose a water molecule. Then, it is vague whether the water molecule exists. Hence, even if it were absolutely clear for all other objects whether they exist or not, it would still remain vague how many objects there are, since it is vague whether the water molecule exists.

Finally, we come to the fourth premise. It says that the number of objects is not vague. The number of objects cannot be vague, since the concept 'number of objects' can be formulated in a precise formal language, *pure* first-order logic with identity. This language does not allow for vagueness. The concept of number cannot be a source of vagueness. It does not make sense to say *The number five is a vague number*, or *The number of dogs in the park is vague*. In the latter case, it is not the *number* that might be vague, but the concept *dogs being in the park*. I think we can see that there is not much room for disagreement with the above premises.

However, if we grant the truth of the last three premises, then we have to reject the initial assumption that composition is restricted.

The argument from vagueness, though disputed, for instance by Korman (2010; 2015: §9), Koslicki (2003), Markosian (1998: 237-40) and Merricks (2007), is a good starting point to motivate universalism. Although I will ultimately present a more sophisticated version of universalism than the one we discussed here, I think the motivation for universalism carries over to Composition as Identity. Therefore, I will conclude our initial discussion of universalism with a brief examination of the relation between Composition as Identity and universalism.

The straightforward way to argue that any reason to embrace universalism is a reason to embrace Composition as Identity is by pointing out that the latter avoids a lot of the counter-intuitiveness which is put forward against the former. Universalism entails the existence of all kinds of objects, for instance, the object which consists of “[...] the front half of a trout plus the back half of a turkey [...]” (Lewis 1991: 7), “[...] an object whose parts are [Michael Rea’s] left tennis shoe, W. V. Quine and the Taj Mahal” (Rea 1998: 348), or “[...] the object composed of the moon and [...] six pennies” (van Cleve 2008: 321). When confronted with these objects, most people react not like van Cleve, whose “[...] reaction was not “How crazy!” but “Why not?” ” (van Cleve 2008: 5). Although we might not yet have an independent reason to think that these objects exist, we will now see why accepting their existence is not troublesome, if we assume Composition as Identity.

Since an object does not come with any additional ontological commitment over and above the commitment to its parts, our believe in the existence of, say, Lewis’ trout-turkey does not commit us to anything else than the existence of the front half of the trout and the back half of the turkey. We get, so to speak, the existence of the trout-turkey for free. On the other hand, if we were to *reject* Composition as Identity, we had to concede that the trout-turkey comes with an additional commitment and that looks like a bad outcome for the universalist. Composition as Identity takes away a lot of the pressure from universalism. In addi-



tion, because universalism is well motivated, given the argument from vagueness, holding on to Composition as Identity is a plausible option. For now we shall ignore that one might think of this close connection as speaking against Composition as Identity because universalism is troublesome. We will come back to address this worry in chapter 10.

### 1.4.2 Four-Dimensionalism

Four-dimensionalism is the view that material objects do not only have parts which exist in different places, but are temporally extended, i.e. have parts which exist at different times. These parts are called temporal parts, or time-slices. Hence, so the analogy goes, as there are different *spatial parts* of a composite object at different places, so there are *temporal parts* of an object at different times, (Sider 2001: §1). For instance, assume you are holding a broom out of your office window. The broom is at the same time inside and outside of your office. One of its parts, the brush, is outside of your office. Another part of the broom, the top third of the stick, is located inside of your office. This is no surprise. We find the parts of a material object in different places because it is spatially extended.

Similarly, we *appear* to encounter one and the same object at different times. Assume, that before noon, the broom was in one of the corners of your office. At noon, you held it out of the window. It was partially inside and partially outside the office. A few minutes after noon, you threw the broom on the street in front of your office window. Then, the broom is outside the office shortly after noon. However, so the four-dimensionalist's thought, it is not *the broom* which is first inside, then partially inside and partially outside, and finally wholly outside your office, but different *temporal parts* of the broom. The broom is a four-dimensional object which has these time-slices as parts. This line of thought helps us to explain how objects can exist at different times and have different properties at different times. Let me illustrate this line of thought with the help of the paradox of the statue and the piece of clay.<sup>23</sup>

23. Aristotle (1963a: 1028b,32-1029a,7) already discusses issues which are related to the

**The Paradox of the Statue and the Piece of Clay:**

You buy a piece of clay. The next day, you form a statue out of the piece of clay. Let's call the statue 'David' and the piece of clay 'Clay'. We are inclined to identify David and Clay. The atoms which compose David are the same atoms which compose Clay. David and Clay occupy the same regions of space. Yet, David did not come into existence until the second day. Clay already existed before David did. Suppose you decided to form a ball out of the statue one day after you formed David. David will not survive this process; It goes out of existence. Nevertheless, Clay still exists after this deformation. How can David and Clay be identical, if Clay existed before David and still exists even when David does not exist anymore?

The four-dimensionalist solution to this paradox is straightforward.<sup>24</sup> David and Clay are objects with temporal parts, sharing some of these temporal parts, namely the ones on the second day. Clay has a temporal part which exists on the day of your purchase while David does not. On the second day, the day when David comes into existence, David has a temporal part, and so does Clay. They have *the same* temporal part. This is the reason why we, mistakenly, identify David and Clay. It is not *them*, who are identical to each other, but *their temporal parts* which exist on the second day. Finally, on the third day when the remodeling is happening, David goes out of existence. Clay, who survives the deformation, does not share any temporal parts with David anymore since there is no

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following with an example of a statue and a piece of bronze. According to Stobaeus, (see Long and Sedley 1988: 170) the *paradoxical* character of the above line of reasoning was already known towards the end of the second century BCE. For a contemporary discussion of the paradox and possible solutions, consult some of the contributions in (Rea 1997b).

24. The most popular answer to the above paradox is to deny that David is identical to Clay and to allow for distinct object to be located at exactly the same place. Baker (1997), Doepke (1997), Fine (2003), Johnston (1992), Kripke (1971), Lowe (1995), Shoemaker (2003), and Thomson (1983) defend this as the solution to the paradox, (see also Wasserman 2015 for a more comprehensive list of defenders of the "standard account" (Burke 1992: 12), and a discussion of other possible replies, as well as Conee and Sider 2005: §7).

temporal part of David on the third day. However, some temporal parts of Clay exist on that day which is why Clay still exists.

Four-dimensionalism is not universally accepted, to put it mildly. The most widely held rival theory to four-dimensionalism, defended by Baker (2009), Fine (2006), Johnston (1992), and Merricks (1999b), is “three-dimensionalism”,<sup>25</sup> which assumes that material objects do not have temporal parts but are “wholly present” at each moment in time they exist. The details of the dispute between three- and four-dimensionalists are a subsidiary question for us. However, four-dimensionalism offers an – at least *prima facie* – attractive theory about change over time, and thereby presents us with solutions not only to the paradox of the Statue and the Piece of Clay, but also with a solution to the paradox of the Ship of Theseus, which makes it an attractive position. Moreover, there is a version of the argument from vagueness which can be modified in such a way as to argue for four-dimensionalism, (see Sider 2001: §4.9).

It is interesting that defenses of four-dimensionalism are sometimes paired with the assumption of Composition as Identity, (see Lewis 1986c, Sider 2001, Varzi 2006). As it is with universalism and Composition as Identity, this is not really a surprise. Composition as Identity takes away a lot of the ontological worries one might have with respect to four-dimensionalism. One of the difficulties in accepting four-dimensionalism is the claim that material objects are spread out in time. Yet, there appears little disagreement about the existence of what the four-dimensionalist calls “temporal parts”. The dispute looks rather – I think only *almost* – like a verbal one:<sup>26</sup> The three-dimensionalist just uses a different term, ‘material object’, for what the four-dimensionalist calls a ‘temporal part of a material object’.

But then, if the four-dimensionalist underpins her theory with Composition as Identity, not much work is left to convince the three-

25. Sider (2001: 3, fn.3) presents a more comprehensive list of supporters of three-dimensionalism.

26. McCall and Lowe (2003; 2006) and Miller (2005) take the two theories to be equivalent.

dimensionalist. If a four-dimensional object is identical to its times slices, then the three-dimensionalist has nothing to worry about accepting the claim that objects are spread out in time since she believes already in the existence of time slices, though she calls them by a different name. She already believes in the existence of four-dimensional objects. This line of thought should not be taken as a knock-down argument for three-dimensionalism, or suggest that the dispute is only a verbal one. All I want to say is that four-dimensionalism becomes more attractive if Composition as Identity is assumed. Since four-dimensionalism is an elegant way to explain how change over time is possible and solves some of the paradoxes of material objects, it gives us good reasons to have a closer look at Composition as Identity.

The aim of this chapter was to determine what motivates Composition as Identity. Composition as Identity is a way to help us understand the idea that a composite object is nothing over and above its parts. Moreover, the lesson to be learned from the Overdetermination Argument is that objects are identical to their parts. The uniqueness of composition is one of the consequences of Composition as Identity which makes it an attractive view. Finally, Composition as Identity takes a lot of the pressure from mereological universalism and four-dimensionalism. Since these are fruitful theories, further investigations into Composition as Identity are well motivated.

## CHAPTER TWO

# *Criticisms of Composition as Identity*

In this chapter, we will discuss four lines of argument directed against Composition as Identity. These are serious threats for our theory. However, we will see in the next chapter that different versions of Composition as Identity can meet some of these criticisms. Eventually, it will be our aim to develop a theory of Composition as Identity which can meet all of the reservations we are about to see.

We begin with a paradox, which results from the claim that a composite object is identical to its parts. Briefly put, the “Paradox for Composition as Identity” goes as follows: A composite object being identical to its parts contradicts the assumptions that any object is one, the parts of a composite object are many, and *being one* and *being many* are opposites of each other.

The second line of critique is due to Sider (2007: 56-9; 2014), who argues that Composition as Identity entails the so-called “principle of Collapse” (Sider 2014: 211). This is derivable from assumptions the friend of Composition is commonly thought to accept. Yet, it contradicts the comprehension axiom from plural logic and since Composition as Identity is best formulated within plural logic, this is worrisome.

A third criticism put forward against Composition as Identity is motivated by our intuition that the arrangement of objects is relevant for what object they compose. Since Composition as Identity reduces the identity of composite objects to the identity of their parts, it excludes the

possibility that the rearrangement of the parts of an object results in them composing a different object.

Eventually, I will conclude the chapter with an argument which aims to show that Composition as Identity entails mereological essentialism, (Merricks 1999a). According to mereological essentialism, objects could not be composed by different objects, nor can they be composed out of more or fewer objects than the objects they are actually composed of. It will be important for us to keep these four lines of attack against Composition as Identity in mind, when we are developing our account of the thesis in the following chapters.

## 2.1 The Paradox for Composition as Identity

We begin our examination with the question which, I think, comes immediately to one's mind when meeting Composition as Identity for the first time. It is related to the concept of many-one identity, i.e. an identity relation that holds between *many* objects and *one* object. This is one of the central notions of Composition as Identity, since it holds that the parts of a composite object are identical to the object they are parts of. We already encountered it at the end of section 1.1.3. However, this concept leads us immediately to the following question: *How can an object be identical to its parts, if it is one and they are many?* Although opponents as well as defenders rarely spell out this worry in detail, their reflections upon it show striking similarities:

One good reason to reject composition as identity is that it implies, obviously enough, that one thing (e.g. a whole) can be identical with many things (e.g. the whole's parts). But I think that one of the most obvious facts about *identity* is that while it holds both one-one (John is identical with Mr Smith) and perhaps even many-many (John and Mary are identical to Mr Smith and Ms Jones), it never holds one-many. [...] Identity cannot hold one-many. So composition as identity is false. (Merricks 2001: 21, [italics in original])

What's true of the many is not exactly what's true of the one. After all they are many while it is one. The number of the many is six, as it might be, whereas the number of the fusion is one. (Lewis 1991: 87)

The parts *are six in number* for example, while the whole *is one*. (Wallace 2011a: 808, [italics in original])

One is inclined to regard this as a knock-down argument against Composition as Identity. Let me explain with our example of the six-pack and the six cans from section 1.1.1 the rough line of thought behind a rejection of Composition as Identity based on this worry: According to Composition as Identity, the six-pack is identical to the six cans. Further, any object is one. Suppose an object were not one, but say, two. Then it would not be *an* object, but *two objects*, which is absurd because '1 = 2' is absurd. Therefore, the six-pack is one. Moreover, the six cans are six; hence they are many in the sense of more than one. This leads to two problematic conclusions: The six cans are one, and the six-pack is many. Each one of these conclusion is, together with the premises, in contradiction with the truism that 'being one' is the opposite of 'being many'.<sup>27</sup>

Let's spell out the general version of the paradox more carefully. The first premise is the claim of Composition as Identity: a composite object is identical to its parts. Next, we have the claim that a composite object is one. As we have just seen, any object is one. Hence, any *composite* object is one. The third premise says that the proper parts of an object are many. Our ordinary understanding of composite objects entails that a composite object has at least two *proper* parts.<sup>28</sup> This is due the "Sup-

27. A similar line of reasoning applies to examples where we have many objects composing *many* objects, e.g. twelve cans composing two six-packs, or four oxygen atoms composing two oxygen molecules: The two six-packs are identical to the twelve cans. The two six-packs are two. The twelve cans are twelve. Nothing is two and twelve. By substitution, the two six-packs are two and twelve, and the twelve cans are two and twelve.

28. Trivially, a composite object has many *improper* parts. By definition, a composite object is an improper part of itself, since  $x$  is an improper part of  $y$  iff  $x$  is a proper part of  $y$  or  $x$  is identical to  $y$ . Moreover, since it is a *composite* object, i.e. an object with at

plementation Principle”, which we generally assume<sup>29</sup> for the parthood relation, “*whenever an object has a proper part, it has more than one [...]*” (Casati and Varzi 1999: 38). It is a truism that ‘being one’ is the opposite of ‘being many’: Something is one iff it is not many. However, from the previously said it follows, by substitution, that a composite object is many, and the parts of the composite object are one. Yet, the composite object being one and being many stands in contradiction with the claim that nothing is one and many. Similarly, the composite object’s parts being many and being one contradicts the assumption that nothing is one and many. Hence, we get the following argument:

1. A composite object is identical to its parts
2. A composite object is one
3. A composite object’s parts are many
4. Something is one iff it is not many
- 5.a A composite object is many
- 5.b A composite object’s parts are one

The above argument is formally valid: 5.a follows from the first and the third premise; together with 2., it contradicts the fourth premise. 5.b can be derived from 1. and 2.; with 3., it contradicts again the fourth premise. Hence, we have a paradox: From apparently true premises, we reached apparently false conclusions, contradicting the premises, by the use of apparently valid logical inferences. What options do we have?

We can reject either one of the premises 1. to 4., accept the conclusions, or reject one of the inference rules used in the derivation. Since the only

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least one proper part, it has an improper part which is not identical to the composite object. Nothing is a proper part of itself. Hence, it has at least two improper parts.

29. Cotnoir (2013b) and Varzi (2016) discuss different ways of spelling out the thought that underlies the supplementation principle.



inference rule used besides “modus ponens”<sup>30</sup> is substitution, i.e. the rule that validates inferences of the form  $a = b, F(a) \vdash F(b)$  – or an extended version of this inference rule which one may adopt, if plural terms and variables are added to first-order logic with identity, see section 5.2. that amounts to rejecting substitution. Accepting either one of the conclusions is not possible, if we wish to hold on to all the premises and substitution, since that leads to inconsistencies. Hence, we can only reject one of the premises or substitution. But neither option is an easy route to go.

The opponent of Composition as Identity will not see a problem with the above argument. She will simply reject the first premise. However, this is not an option for us since it is the central claim of the theory we want to defend. Hence, the above argument is only a paradox under the assumption of Composition as Identity, which is why I call it “the Paradox for Composition as Identity”. If we *assume* Composition as Identity at the outset, then we cannot reject the first premise. Therefore, we are left with four options: Reject one of the premises 2., 3., or 4., or reject substitution. We shall have a look at each of the options in turn.

Rejecting premise 4. is the least plausible option we might choose. It means to deny that ‘being one’ is the opposite of ‘being many’. If we choose this strategy to solve the paradox, we have to find a definition of ‘being one’ which allows for an object, or some objects, to be one and many at the same time. However, that is an impossible task to set for ourselves. If someone tells us ‘being one’ is not the opposite of ‘being many’, we are likely to think the person does not understand the words she is using and we will correct her by pointing out that *by definition*, (see Yi 2014: 175), nothing can be one and many. Since I cannot see how we can give an account of ‘being one’ in a way that it is *not* the opposite of ‘being many’, holding on to premise 4. seems unavoidable.

Premise 2. states that a composite object is one. As we have seen with the example of the six-pack and the six cans above, this follows from the

30. Pace Beall (2009), Lycan (1993), and McGee (1985), I do not consider rejecting modus ponens as an option.

assumption that any object is one. Hence, if we want to reject premise 2., we have to accept that there is at least one object which is not one, but many. Supposing that we can make any sense of this claim – which I doubt – it is impossible to hold on to it, since it is contradictory. There cannot be at least *one* object that is not one. Let me add two remarks here. Such an object is not what Yi (2014: 169) calls a “plural object” since a “[...] *plural object* is a single object, some one thing, that is also many”. A plural object, if there is any, is one *and* many. However, the object we have to accept, if we deny that every object is one, is *not* one. Secondly, the most likely candidates one may take to be objects which are not one seem to be sets. Some sets, e.g. the set of logicians, have many members. Given the close relation that holds between a set and its members, we may mistakenly identify sets with their members, and think of some sets as not being one but many. However, this is a fundamental mistake. A set has to be distinguished from its members. It has them as its members, but it is *not* identical to them, (see Potter 2004: §2 or Suppes 1957: 179-80). Hence, we are well advised to hold on to the claim that every object is one, i.e. the second premise.

According to premise 3., the parts of a composite object are many. Given Composition as Identity, denying this premise amounts to a denial of the irreflexivity of the parthood relation, which says that no object is a proper part of itself. Here is how the friend of Composition as Identity ends up denying the irreflexivity of the parthood relation by denying premise 3.: Assume an object  $x$  is identical to its parts, and  $x$  does not have many parts, i.e.  $x$  has at most one part. It follows, by substitution, that  $x$  is a proper part of itself. Hence, the parthood relation cannot be irreflexive. The irreflexivity of the parthood relation is rarely disputed and attempts to undermine it, such as by Cotnoir and Bacon (2012), Kearns (2011), or Kleinschmidt (2011), rely on the use of scenarios which are themselves highly controversial. Therefore, I assume that denying premise 3. is not an easy way to solve the paradox.

Finally, the last option to consider is rejecting substitution which allows the inference from premise 1. and 3. to 5.a, and from 1. and 2. to

5.b. I will consider this in section 6.2, and a restriction on substitution will allow us to solve the paradox in chapter 8.<sup>31</sup> However, at this point, rejecting substitution would raise suspicion and we are very likely to be accused of making an *ad hoc* move. In chapter 6, we will see that substitution, or its second-order counterpart “Leibniz’s Law”, is ordinarily considered to be one of the defining features of the identity relation:

If  $x = y$ , then whatever is true of  $x$  is also true of  $y$ , and whatever is true of  $y$  is also true of  $x$ . (Suppes 1957: 102)

One is not prepared to abandon this basic logical principle easily. Hence, a rejection of or restriction on substitution cannot *only* be based on the desire of saving Composition as Identity. Since we have not yet met any additional reasons speaking against substitution, and no further options to avoid the derivation of the two contradictions from above, we have no plausible strategy to resolve the Paradox for Composition as Identity. In the next chapter, where different versions of Composition as Identity will be discussed, we will see that some of them, for instance the weak interpretation, (Lewis 1991: §3.6), or its stronger counterpart, (Wallace 2011a; 2011b), offer us ways to avoid the paradox. Yet for now, we have to acknowledge that the opponent of Composition as Identity will take this paradox as counting against our thesis.

## 2.2 The Principle of Collapse

Sider (2007: 56-9; 2014) argues that Composition as Identity leads to a collapse of the inclusion relation from plural logic and the parthood relation,<sup>32</sup> i.e. it entails “the principle of Collapse” (Sider 2014: 211):

31. This move is also a solution the following “modalized version” of the paradox:  $x$  is identical to the  $uu$ .  $x$  is necessarily one and the  $uu$  are possibly many. Hence,  $x$  is necessarily one and possibly many, and the  $uu$  are possibly many and necessarily one. But, being necessarily one is the opposite of being possibly many.

32. For the sake of exposition, we shall adopt here, and in section 8.6.2, where I will address this criticism against Composition as Identity, Sider’s framework, in particular the mereological notions of parthood, fusion and overlap.

An object is improperly among some objects iff it is an improper part of the fusion of these objects

This is worrisome for us because Collapse brings Composition as Identity in conflict with the principles of plural logic. Let me illustrate the problem with an example. The comprehension axiom from plural logic states, that if there is at least one object that has a property  $\Phi$ , then there are some objects such that each object which is  $\Phi$  is one of them, and nothing else is. Now, there is at least one object which has the property *being a logic book*. Therefore, there are some objects, *the logic books*, such that each one of them is a logic book, and nothing else is among them. It follows from Collapse, that anything that is among the logic books is a part of the fusion of the logic books. However, that is false since the fusion of the logic books has objects as parts which are *not* books, for instance, the first page of Prior's *Formal Logic*, or the fusion of Frege's *Begriffsschrift* and Leśniewski's *Lecture Notes in Logic*. Hence, Composition as Identity cannot hold on to all of the above assumptions. Thus, since plural logic is a necessary means to formalize the claim that an object is identical to its parts, it appears that Composition as Identity cannot hold on to the standard view on plural logic and Classical Extensional Mereology which is assumed to be the natural framework for Composition as Identity.

Sider (2014: 212-4) presents his argument that Composition as Identity entails Collapse within a first-order logic, supplemented with plural variables,  $uu$ , an identity predicate,  $=$ , which reflects this extension of the language by being able to take singular and plural terms as arguments in each argument place, the improper inclusion predicate,  $\preceq$ , (read here as 'being one of/being among or being identical to')<sup>33</sup> a principle he calls "Plural Covering", and the improper parthood predicate,  $\leq$ , together with the principles of Classical Extensional Mereology.

Due to the use of the non-standard identity predicate, Sider suggests a generalized version of substitution,  $\alpha = \beta, \Psi(\alpha) \vdash \Psi(\beta)$ , whereby ' $\alpha$ ' and ' $\beta$ ' stand for singular as well as plural terms, (Sider 2014: 212). Thus,

33. We will formally introduce the improper inclusion relation in section 5.3.2.

the suggestion is that if  $\alpha$  is or are identical to  $\beta$  and,  $\alpha$  is or are  $\Phi$ , then we can conclude that  $\beta$  is or are  $\Phi$ .

Plural Covering is a principle which “[...] can be derived from the principles of mereology plus a plural comprehension principle [...]” (Sider 2014: 213) in the standard systems of mereology and plural logic. The idea of this principle is that if  $x$  is an improper part of  $y$ , then  $y$  is the fusion of some objects and  $x$  is one of these objects:

$$\forall x \forall y (x \leq y \rightarrow \exists uu (y = FU(uu) \wedge x \preceq uu))$$

The overlap relation is defined in a standard way, as sharing at least one improper part:

$$x \circ y =_{df} \exists z (z \leq x \wedge z \leq y)$$

Sider’s definition of a mereological fusion is non-standard, insofar as he uses plural variables and the inclusion relation in the definition, while standard accounts of mereology<sup>34</sup> are given in a singular language where these resources are not available, (see Casati and Varzi 1999: §3, Eberle 1970, Hovda 2009, Simons 2003: §1&2). However, this should not be a reason to worry for us. Since it is the common view that plural logic is the adequate framework to deal with plural terms from natural language, as we will see in chapter 4, and the central claim of Composition as Identity is that an object is identical to its parts *collectively*, Composition as Identity is in need of a plural language anyways. So, this deviation is not problematic.

$$x = FU(uu) =_{df} \forall y (y \preceq uu \rightarrow y \leq x) \wedge \forall y (y \leq x \rightarrow \exists z (z \preceq uu \wedge z \circ y))$$

The above definition captures the idea that the fusion of some objects is that object which contains each one of the objects as a part, and each part of the fusion overlaps at least one of the objects it is the fusion of.

34. An exception is Lewis (see his 1991; 1993), who presents Classical Extensional Mereology within a plural language.

Consider again our example of the piece of land and the six parcels from section 1.1.1: The piece of land is the fusion of the six parcels because the following two things hold: Anything that is among or identical to the six parcels is an improper part of the piece of land. Any object that is an improper part of the piece of land, e.g. one of the six parcels, or the northern half of the piece of land, overlaps at least one of the parcels.

With the definition of ‘fusion’ at hand, we can state the fusion principle from Classical Extensional Mereology, again in a slightly modified way because of the extended language we are dealing with, as follows:

$$\forall uu \exists x (x = FU(uu))$$

The fusion principle tells us that for any objects  $uu$ , there is some object  $x$  which is their fusion. This ultimately amounts to the principle of unrestricted composition, i.e. universalism.

The central thesis of Composition as Identity, is then:

$$\forall x \forall uu (x = FU(uu) \rightarrow x = uu)$$

This claim, if an object is the fusion of some objects, then it is identical to them, captures the idea of Composition as Identity thus: Any object is the fusion of its parts, so any object is identical to its parts. That is the core claim of Composition as Identity.

These definitions and principles entail Collapse:

$$\forall uu \forall x (x = FU(uu) \rightarrow \forall y (y \preceq uu \leftrightarrow y \leq x))$$

Here is a sketch of the derivation: Suppose,  $x$  is the fusion of the  $uu$ . It follows then from the definition of ‘fusion’ that any  $y$  which is among the  $uu$  is an improper part of  $x$ . Next, assume  $y$  is an improper part of  $x$ . From Plural Covering, we can deduce that there are some  $vv$  such that  $x$  is the fusion of the  $vv$  and  $y$  is one of the  $vv$ . With Composition as Identity, we get from this and the initial assumption that  $x$  is identical to the  $uu$  and  $x$  is identical to the  $vv$ . Substitution allows us then to conclude that the  $uu$  are identical to the  $vv$ , and since  $y$  is one of the  $vv$ ,  $y$  is one of the

$uu$ , by substitution, (Sider 2014: 213). Hence, we have Collapse: If  $x$  is the fusion of the  $uu$ , then for any  $y$ ,  $y$  is improperly among the  $uu$  iff  $y$  is an improper part of  $x$ .

As our example above indicated, Collapse causes trouble for Composition as Identity since it does not allow us to hold on to the definitions and axioms we have just spelled out together with the comprehension axiom from plural logic:

$$\exists xF(x) \rightarrow \exists uu\forall y(y \preceq uu \leftrightarrow F(y))$$

Confronted with the tension between the comprehension axiom and Composition as Identity, Sider (2014: 214-5) suggest the latter has to settle for a weaker form of the comprehension axiom and goes on to point out that this comes with further problems for Composition as Identity, (Sider 2014: 219-21). There is no need for us to go into the details about the consequences Collapse has, according to Sider, for Composition as Identity. It suffices to note that Composition as Identity, in conjunction with the unrestricted fusion principle, is not compatible with the standard picture of plural logic. Since we will develop an account of Composition as Identity which does not allow for the above derivation of the principle by restricting substitution, this will help us to avoid the Collapse principle. However, as I mentioned in the previous section, restricting substitution for the sake of saving Composition as Identity is a dodgy move. Hence, until we find independent reasons which motivate such a restriction, we have to acknowledge that because Composition as Identity violates certain principles of plural logic, this undermines the thesis that an object is identical to its parts.

## 2.3 Rearranging Parts

Let's turn to an objection against Composition as Identity which is as obvious as it is difficult to overcome. Indeed, I think it *cannot* be overcome by the standard accounts of Composition as Identity. The problem is related to the fact that Composition as Identity entails the uniqueness of

composition, see section 1.3. This principle leads to problems by considering that the arrangement of the composing objects is relevant with respect to the identity of the object they compose, and the fact that we can rearrange the parts of a composite object.<sup>35</sup> However, Composition as Identity reduces the identity of composite objects to the identity of their parts: A composite object  $x$  is identical to a composite object  $y$  iff  $x$  and  $y$  have the same parts. This means in turn, two objects  $x$  and  $y$  are *not* identical to each other iff there is at least one object  $z$  such that  $z$  is a part of  $x$  and  $z$  is not a part of  $y$ , or  $z$  is a part of  $y$  and  $z$  is not a part of  $x$ . In other words, distinct objects cannot be composed out of the same parts. This goes against our everyday experience. Intuitively, we take some objects to be distinct from each other even if they are composed out of the same parts. The following two passages give us a hint at why we may not always identify objects with the same parts:

It is completely obvious to those not in the grip of a philosophical theory that there is a vast and important difference between a heap of disassembled motorcycle parts, piled up, as they might be, at the Honda factory or in someone's garage, and the motorcycle in running condition that results from assembling these parts in a particular, fairly constrained, way.

(Koslicki 2008: 3)

If I am simply identical to my parts then I am them no matter how they are, or how they are arranged.

(Cameron 2014: 103)

Koslicki and Cameron reject Composition as Identity because it does not take into account that whether some objects  $uu$  compose an object  $x$  depends upon the way they are arranged. In other words, Composition as

35. The concept of rearrangement is notoriously vague. I do not intend to clarify this concept, but will rely on our intuitive grasp of it. Surely, what *re*-arrangement is will first and foremost depend upon what *arrangement* is. Although we will try to shed some light on the notion of arrangement in chapter 9, I will resist to take what might seem to be the next step, i.e. to spell out what rearrangement is. After all, it seems that is the task of the opponent of Composition as Identity, who aims to undermine the theory with the help of the counterexamples based on rearrangement, and not of the defenders of Composition as Identity.



Identity does not allow for some objects  $uu$  to compose distinct objects,  $x$  and  $y$ , if the  $uu$  are arranged in a different way. The standard accounts of Composition as Identity are, so to speak, “blind” with respect to the arrangement of the parts. I will explain this shortcoming of Composition as Identity with the help of a thought experiment:

Take a completed one thousand piece jigsaw puzzle of a picture of the Eiffel Tower and a heap of jigsaw puzzle pieces of the very same picture. Let’s call the former ‘Jig’ and the latter ‘Saw’. Jig is not identical to Saw. The two objects have different properties: Jig has a rectangular surface, while Saw has not. The parts of Jig are spatially connected, according to one way we may define ‘being disconnected’, while the parts of Saw are not spatially connected following the same definition. Jig depicts the Eiffel Tower, Saw does not. Now, if Jig is not identical to Saw, the uniqueness of composition tells us that they cannot have the same parts; if they had, they would be identical. Hence, the parts of Jig,  $uu$ , are not identical to the parts of Saw,  $vv$ . On the one hand, this is a desirable outcome. If Jig and Saw had the same parts but different properties, which means that Jig is not identical to Saw, then we cannot have both Jig and Saw. There is either the completed puzzle with one thousand pieces or the heap of one thousand pieces. In order for both to exist, we would need two thousand puzzle pieces – one thousand for the completed puzzle, and one thousand for the heap. But something is going wrong here.

For each puzzle piece  $x$  of Jig, there is a puzzle piece  $y$  of Saw which is *almost* indistinguishable from  $x$ , and *vice versa*. Take for instance the piece of Jig which is placed in the top left corner. There is a piece of Saw which has exactly the same shape, color, weight, and so forth. The only way to distinguish  $x$  from  $y$  appears to be their location. Now assume, for the sake of the argument, that we were not able to distinguish the puzzle pieces of Jig from their Saw-counterparts, i.e. those parts of Saw which are almost indistinguishable from them. Then, given the uniqueness of composition, Jig and Saw are identical, since they have the same parts. However, because Jig and Saw have differing properties, they cannot be identical.

The example helps to see the idea behind the above criticism. It would be easier to illustrate the problem, which the rearrangement of parts poses for the uniqueness of composition, with the help of the phenomenon of time – how can the puzzle pieces compose Jig now and Saw later? However, we decided to set questions that arise in connection with temporal considerations aside and they would lead us here too far astray.<sup>36</sup> However, we might as well use modal notions to clarify the above line of attack.

Consider this time Jig, the completed puzzle, only. Jig's parts *could* be piled up as a heap. There is a possible world  $w$ , where the puzzle pieces  $uu$ , which compose Jig in the actual world, are arranged in a different way than they are arranged in the actual world. Given the uniqueness of composition, the object the  $uu$  compose in  $w$ , say Saw, is identical to Jig. However, if Jig is a completed puzzle, then the  $uu$  cannot be arranged differently from how they are arranged in the actual world. Puzzle pieces compose a completed puzzle, only if they are arranged in the right way. If the puzzle pieces  $uu$  are just piled up as a heap in  $w$  and Saw is the object which the  $uu$  compose in  $w$ , then Jig cannot be identical to Saw in  $w$ , since they have different properties. Jig is a completed puzzle, while Saw is a heap of puzzle pieces.

36. A four-dimensionalist who endorses Composition as Identity can reply to the above question how the puzzle pieces can compose Jig now and Saw later, simply with *They do not*. According to four-dimensionalism, the puzzle pieces which are involved in the two cases of composition are not the same, but temporal parts of the same four-dimensional puzzle pieces. Thereby, one may think that four-dimensionalism is not in need to take the rearrangement of parts seriously. Yet, that is misleading. It only shows that four-dimensionalists can handle cases where we have an *apparent* rearrangement of parts of three-dimensional objects, (see Lewis 1988). The problem for four-dimensionalism returns in another form: If a four-dimensional object, for instance you, is identical to its temporal parts, and the arrangement of the parts does not matter to whether they compose you, then you are identical to the object which has all your temporal parts arranged in the opposite order, i.e. the object which “lives your life backwards”, and to the object which has all the temporal parts in the order you have, with the exception that the temporal part of your tenth birthday and the temporal part of your eleventh birthday switch places. Although our intuitions might get a bit lost here, that is a strange result. Intuitively, we are not inclined to identify you with these objects. You celebrate your tenth birthday before your eleventh, these objects do not.

With the help of modal notions, we can see that Composition as Identity entails, with the uniqueness of composition, that objects *necessarily* compose the objects they compose. Koslicki (2008: 113) calls this “Reverse Mereological Essentialism” and describes it as follows:

Reverse mereological essentialism [...] asserts that one and the same part cannot survive gaining or losing its whole, so to speak, i.e., the whole of which it is part. In other words, according to this thesis, no single object could survive, for example, *becoming* a part of a whole of which it is not already part or *ceasing* to be part of a whole of which it is part [.]

(Koslicki 2008: 114)

Reverse mereological essentialism goes against the way we ordinarily think about composition. It is obvious that the table in front of you can be cut into five pieces and each of its parts sent to a different country. Indeed, there is a possible world where it is the case that the three parts,  $x_1$ ,  $x_2$  are in London, Paris, and New York. UNI forces us to identify the object  $y$  which has  $x_1$ ,  $x_2$ , and  $x_3$  as parts in  $w$  with the table which is actually in front of you. However, the table in front of you and  $y$  have different properties: they are located in different places, the parts of the table are spatially connected, according to a certain way we may define spatial connection, the parts of  $y$  are not spatially connected, and so on.

To sum up, UNI entails a form of essentialism. This disagrees with our picture of reality. In order to overcome this shortcoming, we have to acknowledge that arrangement matters with respect to composition. Under the assumption that composition is identity, this amounts to the claim that arrangement matters with respect to *identity*. Taking identity to be sensitive to arrangement might come as a surprise, since it appears to add a condition to the identity relation where there is none needed. Many think of identity as a concept which “[...] is such a simple and fundamental idea that it is hard to explain otherwise than through mere synonyms” (Quine 1952: 208). Yet, this is an oversimplified picture of identity and I think we can defend the idea that the identity of composite objects also depends upon the arrangement of their parts.

## 2.4 Mereological Essentialism

The last objection we will discuss follows the critique against Composition as Identity of Merricks (1999a), who aims to show that it is committed to mereological essentialism. Mereological essentialism claims that objects have their parts essentially, i.e. could not have parts other than the ones they actually have. I shall first present the argument and illustrate its underlying thought with an example. In a second step, I will point out how mereological essentialism is in conflict with common sense. Eventually, I will conclude with a hint at how we shall counter Merricks' argument and thereby avoid being committed to mereological essentialism.

The main idea of the argument is the following. If a composite object is identical to its parts, and the parts are necessarily identical to themselves, i.e. they are identical to themselves in any possible world,<sup>37</sup> then the composite object necessarily has the parts it actually has, since they compose it in any possible world:

Now suppose that  $O$ , the object composed of  $O_1 \dots O_n$ , is identical with  $O_1 \dots O_n$ . From this, the fact that  $O_1 \dots O_n$  are identical with  $O_1 \dots O_n$  in every possible world, and the indiscernibility of identicals it follows that  $O$  is identical with  $O_1 \dots O_n$  in every possible world. (Merricks 1999a: 192-3)

An example might be helpful here. Take again the one thousand puzzle pieces, and Jig, the completed puzzle, from the previous section. The puzzle pieces compose Jig in the actual world. Each one of them is an actual part of Jig. The puzzle pieces are necessarily identical to themselves, i.e. the puzzle pieces are identical to the puzzle pieces in any possible world  $w$ . By substitution, we infer that the puzzle pieces are identical to Jig in any possible world  $w$ , since they are identical to it in the ac-

37. One way to avoid the conclusion that Composition as Identity entails mereological essentialism is to take the above identity to be a contingent identity. However, merely assuming this to be the case cannot be satisfactory, but is in need of an explanation: Why is it necessarily the case that  $x$  is identical to  $x$ , while  $x$  is only contingently identical to the  $uu$ ? I cannot see how the standard account of Composition as Identity can explain this asymmetry in a non-question-begging way.

tual world, and they are identical to themselves in any possible world  $w$ . Since objects which compose an object are parts of that object, each puzzle piece is a part of Jig in any possible world. The puzzle pieces are essentially parts of Jig. It could not be composed out of other objects, i.e. Jig could not have other parts than the ones it actually has.

However, mereological essentialism is not without supporters. While Abaelardus (1970: 423) and Leibniz (1981: 247) are among the earlier supporters, Chisholm (1973; 1875) and van Cleve (1986) belong to the more recent defenders of the theory. Nevertheless, “[mereological essentialism] is *prima facie* outrageous” (Wallace 2014: 111). It claims, for instance, that if one of the molecules which compose your copy of *Principia Mathematica* were different, you had a different book; if I removed a tire from my car, I had a different car; adding a grain to a heap of grains, gives us a new heap of grains. This is clearly at odds with the way we ordinarily identify objects. We regard the book that has all the molecules which compose your copy of *Principia* apart from one as the same as your copy; the vehicle that has all the parts my car has, except one of the tires as my car; the heap of grains that results by adding one grain to a heap of grains as the original heap. Surely, one may think that it is this ordinary way of thinking that leads us into trouble in the first place: If we were not to allow for objects to “survive” the change, addition or removal of parts, then we can avoid, for instance, the paradox of the Ship of Theseus:<sup>38</sup> The ship which results from substituting one old plank by a new plank is *not* the Ship of Theseus, but a different ship.

Yet, it would be desirable to have an account of composition which comes closer to our ordinary understanding of the identity conditions of composite objects. If we bite the bullet and accept mereological essentialism, then we have to explain how I can still be the owner of a car, if the

38. The Ship of Theseus already puzzled the ancient Greeks, (see Plutarch 1914: §23 and Plato 1892a: 58a-c). It was famously discussed and modified by Hobbes (1839: 136-7). Carmichael (2015: 486-9), Chisholm (1997), Hoffman and Rosenkrantz (1997: 163-5), Jansen (2011), Scaltas (1980), Sider (2001: 6-10), and Smart (1972) provide a more recent discussion of the paradox.

front left tire of my car gets removed. Since the two cars are not identical to each other, there appears to be no reason why I should still hold ownership of the car that has only three tires. After all, I own only one car before the tire is removed, so to claim that I own the other car after the tire is removed entails that I own two cars, one before and one after the removal.

Having said this, attempts to avoid mereological essentialism while holding on to Composition as Identity have already been made – most famously with Lewis' theory of counterparts (Lewis 1968; 1971; 1986c: §4). Contrary to this strategy, Wallace *defends* mereological essentialism with the assumption that objects have modal parts,<sup>39</sup> i.e. “[...] are stretched out across possible worlds” (Wallace 2014: 112). Although these theories are interesting ways to defend Composition as Identity, I will leave these strategies to others and aim for a version of Composition as Identity which avoids mereological essentialism while not being committed to the existence of counterparts. Which one of these modal theories would suit best for our account of composition is an interesting question, but we cannot deal with it here.

We have seen that the main claim of Composition as Identity has to meet serious criticisms. The Paradox for Composition as Identity and the derivation of Collapse put pressure on this view. Composition as Identity appears to commit us to reverse mereological essentialism and mereological essentialism. These criticisms make a defense of Composition as Identity not an easy business. Although we already encountered some ways of replying to them, we shall now move on to see how defenders of Composition as Identity elaborated on the main claim of the view in order to refine the theory under consideration.

39. The theory of modal parts is also called “five-dimensionalism”, (Evnine 2016: 40; Garrett 2006: 47; Rini and Cresswell 2012: 167). Graham (2015: 15, fn.5) observes correctly that ‘five-dimensionalism’ is a misnomer since it suggests that the belief in *modal* parts has to be held together with the belief in *temporal* parts. Yet, the two views are independent from each other.

## CHAPTER THREE

# *The Varieties of Composition as Identity*

'Composition as Identity' is a term used to refer to a group of theories which agree on a core claim, yet disagree on other claims. Lewis (1991: §3.6) introduces the term for the view that an object is identical to its parts and the positions defended by Armstrong (1978: 37-8) and Baxter (1988a; 1988b). Since then, different accounts of how to elaborate Composition as Identity's core claim have been developed. I follow the terminology of Cotnoir (2014: 9-11) and distinguish between weak, moderate and strong Composition as Identity.<sup>40</sup> The difference between these versions of the theory can be explained by taking them to be different interpretations of the slogan *Composition is identity*. Weak Composition as Identity interprets the 'is' in the slogan as expressing a *similarity* between the composition and the identity relation, while the other two versions assume that an *identity* holds between them. The disagreement between moderate and strong Composition as Identity rests on a different interpretation of 'identity'. According to strong Composition as Identity, there is only *one* identity relation and it is identical to the composition relation. Moderate Composition as Identity assumes that there are different *kinds* of identity relations and composition is one of them.

40. Wallace (2011a: 807) calls moderate Composition as Identity the "Stronger Composition Thesis".

We begin with a brief examination of weak and strong Composition as Identity and analyze the reasons which motivate the different elaborations of the core claim of Composition as Identity. Eventually, we will have a closer look at some of the moderate versions of the thesis. This latter group of theories will be especially important for us because the theory I defend belongs to this category of Composition as Identity theories and it will therefore be useful to see how it relates to its closest relatives within the family of accounts of Composition as Identity.

The aim of this chapter is *not* to present arguments which suggest that all of these theories are false. Rather, I would like to offer an overview on various, in particular moderate, interpretations of Composition as Identity<sup>41</sup> in order to facilitate a comparison between them and the account I develop here. Having said that, we will also see that due to the different difficulties the positions we are about to discuss have to face, it is worthwhile to consider an alternative interpretation of Composition as Identity, in particular one which can avoid these problems.

### 3.1 Weak Composition as Identity

The weak version of Composition as Identity is already disputed at length in the literature and there is a general agreement that it is too weak to be considered a serious alternative to the two stronger versions, (see Baxter 2014: 251-2, Cameron 2014: 93, Varzi 2014: 50-9, Wallace 2011a: 806, and Yi 1999a: 149-53). I agree with the received view on this point.

Weak Composition as Identity differs from the other interpretations by taking identity and composition to be very much alike or analogous but *not identical* relations. This view is commonly associated with Lewis (1991: §3.6),<sup>42</sup> and Sider (2007).

41. One of the interpretations of Composition as Identity, which would be interesting to discuss as well but for which I do regrettably not have the needed space available, is the one developed by Bohn (2009; 2014).

42. Authors argue about whether Lewis indeed defends weak Composition as Identity, (see Bricker 2016: 281-2). I think Lewis held different views at different times. In some



Taking composition and identity to be similar relations has some plausibility. One reason to resist identifying the two relations and to take, so to speak, “a step back” from the claim that composition *is* identity to composition is *only similar* to identity, comes from the problems arising in connection with substitution. Lewis expresses worries which are in line with our Paradox for Composition as Identity from section 2.1:

What’s true of the many is not exactly what’s true of the one.  
After all they are many while it is one. (Lewis 1991: 87)

Sider discusses similar substitution-related problems for Composition as Identity and claims that a rejection of substitution would “[...] arouse suspicion that their use of ‘is identical to’ does not really express identity” (Sider 2007: 57). Following Lewis and Sider, one may think weak Composition as Identity should be preferred over its alternatives, since it can avoid the paradox: A composite object is *not identical* to its parts, but stands in a relation which *is similar* to identity. Therefore, weak Composition as Identity can dismiss the first premise of the Paradox for Composition as Identity and avoid one of the criticisms put forward against it.

However, weak Composition as Identity does not come without any shortcomings. By taking composition to be only similar to identity, we do not gain much because similarity is a relative phenomenon. In a sense, *being a brother* is similar to *being a step-brother*, and *being taller than* is similar to *being identical to* – both relations are two-place relations. Yet, these similarity claims are not very useful or informative, since they do not have any explanatory power. This is why weak Composition as Identity comes only at a high price: The explanatory power of the core claim of Composition as Identity, a composite object is identical to its parts, is put on the line for the sake of avoiding problems which I take to arise in connection with substitution.

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places, (1988: 71-2), a moderate view is defended, while in (1991: §3.6), it seems a weak version of Composition as Identity evolves from a moderate account. We shall leave this question to the Lewis-scholars and focus on his weak interpretation.

We motivated Composition as Identity as an interpretation of the intuition that a composite object is nothing over and above its parts, the way it relates to the no double-counting policy and the thought that there is no additional ontological commitment to a composite object, given a commitment to its parts in chapter 1. However, its weak version cannot follow this line of thought. If composition is only *similar* to identity, how can a composite object come without any additional commitment besides the commitment to its parts? That a composite object comes without any further additional ontological commitment, given a commitment to its parts, and *vice versa*, can only be true, if the parts are *identical* to the whole. Why should there be no such additional commitment if an object and its parts stand only in a relation which is *similar* to identity? Hence, the weak version of Composition as Identity loses much of the theory's explanatory power. Since that is one of the main reasons to consider Composition as Identity as an interesting theory in the first place, the weak version turns out to undermine the core claim of the position.

### 3.2 Strong Composition as Identity

Strong Composition as Identity is the theory according to which composition is *identical* to the identity relation, whereby the latter behaves classically, i.e. "it is transitive, reflexive, symmetric, unambiguous, intuitive and obeys Leibniz's Law" (Wallace 2011a: 807). This version of the thesis is sometimes mistakenly associated with Baxter, (see, for instance, Lewis 1991: 83-4 and Sider 2007: 55, fn.12). As already noted by Baxter (2014), Cotnoir (2014: 10), and Yi (1999b: 149, fn.13), Baxter defends a *moderate* version of Composition as Identity. The only defender of strong Composition as Identity appears to be Wallace (2011a; 2011b; 2014).<sup>43</sup>

In comparison to its weak counterpart, strong Composition as Identity is able to preserve the explanatory power of the theory's core claim

43. Cotnoir (2014: 10) mentions Bohn (2009) as a further supporter of strong Composition as Identity.

by stating that a composite object is identical to its parts taken together *in the same way* that the composite object is identical to itself: The six-pack is nothing over and above the six cans because it is identical to them in the same sense as the six-pack is identical to itself. Hence, being committed to the six-pack amounts to being committed to the six cans, and *vice versa*. Being able to maintain this explanatory power makes it a more attractive theory than weak Composition as Identity.

*Prima facie*, strong Composition as Identity presents itself as a simpler theory than the moderate version. The latter argues that there are several kinds of identity relations and the composition relation is identical to one, or some, of them. The picture of the strong version is much simpler: There is one identity relation and *it* is the composition relation. By identifying composition with *the* identity relation, strong Composition as Identity gives us a much simpler account of composition, due to the fact that its account of identity is simple. However, I think this simplicity comes with serious difficulties.

There are two closely related points, which are difficult to overcome once composition is identified with what is commonly seen as *the* identity relation. Firstly, identity is a symmetric while composition is an asymmetric relation. Hence, composition cannot be identity understood classically. If composition is a symmetrical relation it does not only entail the reflexivity of composition, for which, I think, one *could* make a case, but it leads to the further claim that if some objects *uu* compose some object *x*, then *x* composes the *uu*. For instance, if some atoms compose a statue, then the statue composes the atoms. This is an absurd consequence. No object composes the objects which compose it.

The second problem also relates to the asymmetry of the composition relation and connects to the substitution problems discussed in the sections 2.1 and 2.2. Strong Composition as Identity cannot reply to these arguments that substitution is only valid if we have a certain kind of identity. Hence, substitution has to be restricted in some other way. Wallace

(2011a: 807-14) makes a strong case for such a restriction of substitution.<sup>44</sup> We do not have the space to investigate the details of the argument she offers, but we can summarize it briefly with the help of Wallace's description of what a counterexample to her account would have to look like:

Showing that there is a property,  $F$ , that the whole has that the parts (collectively) do not, or that there is a property,  $F$ , that the parts (collectively) have that the whole does not, would be in effect to fill in particular details of an [argument against strong Composition as Identity.] (Wallace 2011a: 812)

The restriction on substitution amounts then to the following, described in the second part of the above quote: Given  $x = uu$ , ' $x$ ' can be substituted for ' $uu$ ' in ' $F(uu)$ ' iff the  $uu$  have the property represented by ' $F$ ' collectively, i.e. the  $uu$  taken together are  $F$ .

Here is an example which matches the above description for a counterexample to strong Composition as Identity: Take a pile of cards which consists of two suits of cards. Each suit is composed of thirteen cards. Moreover, the twenty-six cards compose the pile, hence, they are identical to the pile. Yet, the pile does not compose the two suits, while the twenty-six cards do.

Let me clarify the example. According to strong Composition as Identity, given a pile of cards which consists of two suits of cards, the two suits of cards compose, hence are identical to, the pile of cards. Moreover, the twenty-six cards compose, hence are identical to, the two suits of cards. Since identity is a transitive relation, the twenty-six cards are identical to the pile of cards. But since the twenty-six cards compose the two suits, it follows with the above restricted version of substitution, that the pile of cards composes the two suits. Additionally, we can show that the two suits of cards compose the twenty-six cards, since the two suits compose the pile of cards, and the latter is identical to the twenty-six cards.

44. This goes clearly against the intention of Wallace and the initial idea of strong Composition as Identity, since the classical identity relation obeys substitution *unrestrictedly*. The restriction of substitution suggests that the composition relation cannot be identified with the classical identity relation.

However, that the pile of cards composes the two suits, and that the two suits of cards compose the twenty-six cards flies in the face of common sense, since these claims obviously violate our intuition of composition being an asymmetric relation. Although this line of critique against strong Composition as Identity might look like simply rehearsing the previous point, it has its origin in the idea that composite objects do not decompose uniquely. The pile of cards is composed of the two suits on the one hand, and it is composed of the twenty-six cards on the other hand. This leads to problems for strong Composition as Identity, since the twenty-six cards are *collectively* identical to the pile of cards, and they *collectively* compose the two suits of cards. Therefore, substitution is allowed and we can infer the false conclusions that the pile of cards composes the two suits of cards. Similarly, because the two suits of cards are *collectively* identical to the pile of cards, we can infer that they *collectively* compose the twenty-six cards. Hence, we can see that even a restriction of substitution which is formulated with such an utmost care as the one elaborated by Wallace is not able to help strong Composition as Identity to succeed. On the contrary, I think, it only shows that strong Composition as Identity is too strong and we should aim for a more moderate view.

### 3.3 Moderate Composition as Identity

Moderate Composition as Identity agrees with the strong interpretation of Composition as Identity on one point: composition is identical to identity. Yet, the moderate view assumes that there is only *one* identity relation, while the strong view claims that there are *several* identity relations. Adapting a more fine-grained view on the identity relation puts quite some pressure on moderate Composition as Identity. It stands squarely to how we think ordinarily about identity, because it contradicts the received view, (see, for example, McGinn 2000: 1), on identity, which says that there is only one identity relation:

Identity is a relation that is determined in such a specific way  
that it is impossible to see how different kinds of it may occur.  
(Frege 1903: 253 [my translation])

We will come back to this idea in section 6.1.4, where I argue against it. For now, I would like to outline some moderate accounts of Composition as Identity. This will help us to locate our account within the bigger discussion and to compare it to the views already defended. We begin our analysis with “[t]he progenitor of the modern version of the thesis” (Noonan and Curtis 2014: §8),<sup>45</sup> before turning to Bricker’s and Cotnoir’s views.

### 3.3.1 Baxter: Composition as Cross-Count Identity

Baxter’s theory of Composition as Identity rests on three central concepts: counts, aspects and location. With these, he argues for the claims that what exists is relative to a count and that there are three kinds of identity. We have already encountered the concept of counts in section 1.1. Recall, it is one of the central ideas of Composition as Identity that we can count objects in different ways. In our example of the piece of land and the six parcels, when we are asked *How many objects?*, we can either count the piece of land and reply *One*, or we can count the parcels and reply *Six*. The two different answers rely on different ways of counting, and the different ways of counting, in turn, rely on different ways to divide up the property of the farmer.

However, the farmer might have decided to divide up his property in a different way, as figure 3.1 depicts. Instead of dividing it into six parcels (top right), and selling each one of them to a different buyer, he could have sold the Northern and the Southern (bottom left), or the Western and the Eastern half to two buyers (bottom center). Moreover, he could have sold one of the six parcels to one buyer and the other five parcels

45. I will primarily focus on the views expressed in (Baxter 1988a; 1988b; 2014), as well as their excellent discussion in (Turner 2014).

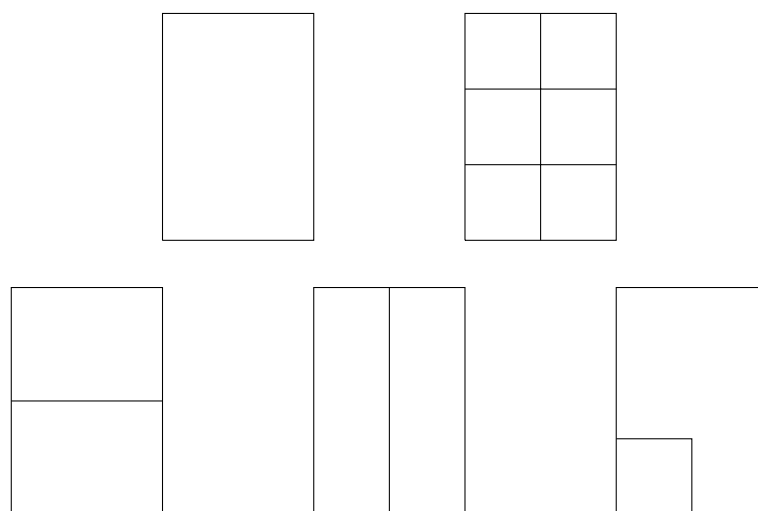


Figure 3.1: *The Piece of Land and the Six Parcels I*

to another buyer, dividing the piece of land in two parcels with different sizes, and so forth (bottom right).

The fact that there are different ways of counting objects leads Baxter to the claim that existence has to be relativized to counts:

[I]dentity (in the familiar sense), number, and existence are relative to what I call 'counts'. (Baxter 1988b: 193)

So what exists is relative to count. (Baxter 1988b: 201)

Although one might be skeptical about this claim, we can see what leads Baxter to this conclusion. Let me try to follow his line of thought by reconsidering our previous example: When the farmer divides up and sells his property, he has to settle for one way of counting. Although he can divide his property in different ways, he cannot hold on to two different ways of dividing it up at the same time. He has to sell either the piece of land as a whole, or the six parcels, or the North and the South half, and so on. He cannot sell the piece of land to one buyer *and* the six parcels to six buyers. Hence, what properties exist depends on the way the farmer divides up the piece of land. If he leaves it in one piece,

only the piece of land exists. If he divides it into six parcels, only the six parcels exist, and so on.

Baxter's "familiar identity" is – in a sense – the classical identity relation from first-order logic with identity. It is reflexive, symmetric, transitive and allows for substitution. However, it holds only *within* a certain count: "The familiar version of identity rules within counts" (Baxter 1988a: 193). In the count, where the farmer divides the piece of land in a North and a South half, the North half is identical to the North half, or the North half is identical to, say, the piece of land which is bought by the farmer's neighbor. Apart from the relativization to counts, familiar identity is classical identity.

To understand intra- and cross-count identity, we need the notion of aspects. They enter the stage when we encounter cases of self-differing. Suppose, Ben is the coach of his daughter's football team. Like any father, he wants to see his daughter in the starting line-up. However, he knows that his daughter is one of the weakest players and, like any other coach, he wants only the best players to start. Therefore, Ben differs from himself. He wants his daughter to be in the starting line-up and he does not want her to be in the starting line-up.

Baxter (1988b: 203-6; 1999) explains self-difference with the help of aspects: Ben-as-father wants his daughter to be in the starting line-up. Ben-as-coach does not want his daughter to be in the starting line-up. Ben-as-father and Ben-as-coach are identical to Ben insofar as they are aspects of him. Ben differs from himself because he has aspects, which differ from each other, by having properties which are not compatible with each other.<sup>46</sup> However, the identity relation holding between Ben and each one of his aspects cannot be the familiar identity relation on pain of contradiction. Baxter needs to assume that we have here another kind of identity relation at work, one not obeying substitution. It is "intra-count identity" (Baxter 1988a: 214, Turner 2014: 232-3). It becomes an

46. Ben does *not* have different properties *at different times*. He wants and does not want his daughter to be in the starting line-up *at the same time*, say, five minutes before the game starts.



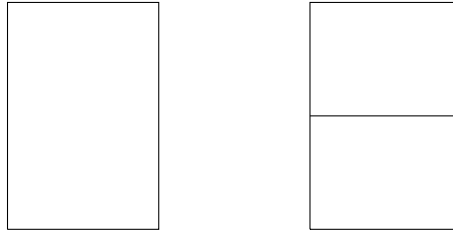


Figure 3.2: *The Piece of Land and the Six Parcels II*

important relation because Baxter takes a part to be an aspect of the object it is a part of. Hence, a composite object is intra-count identical to *each one* of its parts.

With the help of the concept of location, Baxter argues that an object's parts are aspects of it: An object's part is the *object-as-located-in-the-parts-place*. Add to our example of the farmer's piece of land that the equator divides the piece of land in a Northern and a Southern half, see figure 3.2. Then, the Northern half is identical to the piece-of-land-as-located-North-of-the-equator, and the Southern half is identical to the piece-of-land-as-located-South-of-the-equator.

This allows holding on to the claim that the piece of land is located North of the equator as well as South of the equator: The piece-of-land-as-located-North-of-the-equator is located North of the equator, and the piece-of-land-as-located-South-of-the-equator is located South of the equator. The contradiction – the piece of land is located North of the equator and it is located South of the equator, i.e. not located North of the equator – does not follow, since existence is relative to a count. We have to decide between two ways of counting: Either we count the two halves, then the piece of land does not exist, or we count the piece of land, then the two halves do not exist. In the first case, no contradiction can be derived: The Northern half is in the North, the Southern half is not, and the piece of land does not exist in this count. In the second count, the piece of land is in the North only insofar as it is intra-count identical to the Northern half, and it is not in the North only insofar as

it is intra-count identical to the Southern half. Intra-count identity does not allow for substitution, so no contradiction can be derived (see Baxter 1988a: 206-7).

Cross-count identity is an identity relation working across different counts. It comes in two forms. There is one-one cross-count identity holding between one object and one object; and there is many-one cross count identity holding between many objects and one object. The former is the kind of identity that holds between an object in one count and *the same object* in another count. Loosely speaking, one-one cross-count identity is the identification of an object across different counts and it behaves like the familiar identity relation.

Many-one cross-count identity is then the identity relation which holds between many objects from one count, and one object from another count, (Baxter 1988a: 193). It holds between an object and all its parts taken collectively, i.e. between an object from one count and all the objects which are intra-count identical to it in another count, (Baxter 1988a: 209). Many-one cross-count identity is what (Baxter 2014: 253) takes to be composition.

Because cross-count identity does not obey substitution, or only in a very restricted form, (see Baxter 1988a: 194, Turner 2014: 236), Baxter can meet the criticisms we discussed in the sections 2.1 and 2.2, the Paradox for Composition as Identity and the derivation of Collapse. However, there are some points in this account of Composition as Identity which are problematic.

Relativizing existence to counts strikes me as a mistake, or at least as misleading. When Composition as Identity claims that a composite object's being nothing over and above its parts amounts to there being no additional commitment to the object given the commitment to the parts, then there is no need for a relativization of existence. On the contrary, the fact that the parts exist gets identified with the fact that the composite object *exists*. What is relativized is the way we can spell out the ontological commitment: Either we are committed to the existence of the parts, in which case we get the composite object for free, or we are committed to

the existence of the composite object, in which case we get the parts for free.

Further, taking a part of a composite object as the object-as-located-at-the-parts-place “smuggles in” the parthood relation. The location of a part is a subregion of the location of its whole. The region a part occupies is a part of the region the whole occupies. Although the parthood relation holding between regions and the parthood relation holding between material objects must not necessarily be the same relation, (see McDaniel 2004; 2009), the important role played by regions within Baxter’s theory leads to the impression that the parthood relation holding between regions works as a surrogate for the parthood relation of material objects.

Finally, aspects are a mysterious concept. On the one hand, they can be understood as material objects, (Turner 2014: 226). However, if aspects are taken to be objects, their introduction does not bring us any step forward, since any explanatory work aspects do, can be done by objects in the ordinary sense. On the other hand, aspects might be understood as a result of our conceptualization of reality: Ben, insofar as I conceptualize him as a father, wants his daughter to be in the starting line-up; and insofar as I conceptualize him as a coach, Ben does not want his daughter to be in the starting line-up. But if aspects are an artifact of our conceptualization, then it is impossible for an object to be identical to one of its aspects or all its aspects taken together. Ben, the person, is surely not identical to my conceptualization of him. In lack of further ways to make sense of aspects, Baxter’s theory, especially when spelled out with aspects as a key notion, is difficult to maintain.

### 3.3.2 Bricker: Composition as a Kind of Identity

Bricker motivates mereological universalism, which he takes to be a trivial truth, on the basis of Composition as Identity:

The situation is untenable. Something that I take to be absolutely obvious is rejected by many, if not the majority, of my philosophical peers. [...] For any things whatsoever, there is

something that those things compose: a *fusion* of those things. Unrestricted Composition follows with perfect clarity from my understanding of the notion of composition.

(Bricker 2016: 264-5)

I will discuss three points of Bricker's account: his general identity relation, the key element for the derivation of Classical Extensional Mereology from the account of composition he holds, and the remarks on substitution.

Bricker's position is formulated within a plural logic containing a predicate for the general identity relation, ' $\doteq$ ', as a primitive. He takes it that we already have an intuitive understanding of what relation general identity is, (Bricker 2016: 269). This relation is taken to be an equivalence relation, i.e. it is reflexive, symmetric, and transitive. The standard identity predicate, '=', is understood to express those identity relations that can be expressed with the use of singular terms only, i.e. one-one identities. For instance, if we translate the predicate '=' with 'is one-one identical to', the sentences

- (1) Anne is generally identical to the daughter of Bill
- (2) Anne is one-one identical to the daughter of Bill

are logically equivalent. Besides one-one identities, there are three other kinds of identity relations, many-one, one-many, and many-many. Many-one identities are those identities where the general identity predicate takes a plural term on the left and a singular term on the right, ' $uu \doteq x$ '. One-many identities differ from many-one identities simply by the argument places singular and plural terms occupy, ' $x \doteq uu$ '. Finally, many-many identities are those identity relations, which are expressed with formulas where the identity predicate is flanked by two plural terms, ' $uu \doteq vv$ ', (Bricker 2016: 269).

Composition is then taken to be a kind of the general identity relation. With the general identity relation and composition thus characterized, fusions and improper parthood can be defined. The fusion of an object  $x$  or

some objects  $uu$  is the object  $y$  that is generally identical to  $x$  or the  $uu$ , respectively; and  $x$  is an improper part of  $y$  iff there are some  $uu$  such that  $y$  is many-one identical to the  $uu$ , and  $x$  is one of the  $uu$ , (Bricker 2016: 270). The general identity relation and the mereological notions of fusion and improper parthood make an introduction of further principles, which allow the deduction of mereological universalism, possible. We can spare a closer examination of these principles one by one. However, it is interesting to note that the key axiom in the derivation of universalism, called “E Pluribus Unum” (Bricker 2016: 271), ‘ $\forall uu \exists y (uu \doteq y)$ ’, gets introduced without much argument:

To derive **Unrestricted Composition**, we need the fundamental underlying idea that every many is also a one: every plurality of things coincides with some single thing.  
(Bricker 2016: 271)

Here lies one of the few shortcomings of Bricker’s account. Surely, Bricker’s opponent will put her foot down at this point and insist that Bricker begs the question in favor of universalism here. The complaint is not unwarranted and Bricker (2016: 274-5) anticipates this line of critique. It would be desirable to have some more reasons for why one should think that every many is also a one, i.e. why any objects whatsoever are many-one identical to some object besides taking it as “[...] natural ways of generalizing the plural framework” (Bricker 2016: 275). To put it in his own terminology, why is there for any plural term a singular term, such that both refer to the same portion of reality? This will be one of our tasks to accomplish in chapter 10 – to make sure that we have sufficient reason to hold on to *E Pluribus Unum*.

Since Bricker’s theory is a moderate version of Composition as Identity, it is not surprising that he takes substitution not to be generally valid, (Bricker 2016: 275-6). However, he is aware that restricting substitution is not a move that can be endorsed easily. Bricker shows that without a restriction of substitution “[...] composition as identity is dead” (Bricker 2016: 279). The attempt to do without a restriction he discusses tries to

relativize the properties expressed by predicates which are used in counterexamples to substitution, such as ‘are 52 in number’. Because Bricker notes further that such a relativization would be needed for the inclusion predicate from plural logic as well, since it has the defining feature on which such a relativization is based – according to his view – he takes it that we cannot escape the restriction of substitution.

In summary, Bricker takes the general identity relation to be a primitive concept coming in four different kinds. The derivation of Classical Extensional Mereology stands and falls with his axiom *E Pluribus Unum*. This will be one of the key elements we will rely on in chapter 10. Bricker’s theory is prone to the counterexamples of rearrangement we discussed in the previous chapter,<sup>47</sup> which marks an important difference to the account we will develop.

### 3.3.3 Cotnoir: Composition as General Identity

Cotnoir’s account of moderate Composition as Identity recognizes four kinds of identity: one-one, many-one, one-many, and many-many identity, (Cotnoir 2013a: 303). In parts, it shows close similarities to Baxter’s account by taking counts to play a central role within the theory and by taking the different kinds of identity to hold if two terms carve up, or refer to the same portion of reality in different ways:

In order to take many-one identity seriously, we need to suppose that we can refer to a portion of the world singularly or plurally [...] But no matter whether we carve a portion of it as one individual or many, it is still the same bit of reality.

(Cotnoir 2013a: 302)

The characteristic difference between Cotnoir’s position and other moderate views is that he aims to hold on to an *unrestricted* version of substitution. As one of the central parts of Cotnoir’s presentation is to give

47. In email-correspondence, Bricker told me that he is not troubled by the counterexamples based on rearrangement, since he is a four-dimensionalist. However, see my remarks in fn.36 on the problem of rearrangement for four-dimensionalism.

a formal semantics for Composition as Identity and we do not have the formal means to capture that here, we have to content ourselves with examining the *philosophical* claims of his theory.

Cotnoir presents his theory as a way for Composition as Identity to meet three challenges, the syntactical, the semantical, and the discernibility challenge.<sup>48</sup> They are the following:

How can many-one identities be grammatically expressed in English?

How can we make sense of many-one identities?

How can many-one identity be an identity relation, yet not obey substitution?

The first challenge goes back to van Inwagen (1994: 210-1), who exposes the core claim of Composition as Identity as being a grammatical mistake. From the standpoint of grammar, there is a mismatch in the claim ‘A composite object is identical to its parts’ between the verb ‘is identical to’, and the object ‘its parts’; the former is singular while the latter is plural. So, Composition as Identity cannot even be formulated correctly! How can it then be true?

I would like to highlight the final reply<sup>49</sup> Cotnoir offers for the syntactical challenge. He claims that the English language allows us “to singularize syntactically plural terms while maintaining plural reference” (Cotnoir 2013a: 299). In other words, for any plural term ‘uu’ there is a

48. All three challenges apply equally to one-many identities. However, there is no need to spell them out separately, since they stand or fall in each case together. Therefore, we can ignore one-many identity for the remainder of this section.

49. The easiest way to reply to the syntactical challenges is to move to a formal language where the singular-plural distinction can be avoided. This is what Sider (2007) does and the same strategy can be applied by Bricker (2016: 268). Another reply offered by Cotnoir is that we should take other languages into account as well, and not be biased by the constraints of the grammar of a particular language. The grammar of other languages, for instance of Nordic languages, allow for an acceptable formulation of many-one identities. Hence, we should not be misled by our prejudices based on English grammar or the grammar of any other language (Cotnoir 2013a: 296).

singular term 'x' such that the two terms are co-referring. So-called *what*-descriptions motivate this claim, (Cotnoir 2013a: 299):

I bought some things. The things I bought \*was/were expensive

I bought some things. What I bought \*was/were expensive

These two sentences show that the *what*-description, 'what I bought', allows us to refer to several objects, the things I bought, with the means of a grammatically singular term, 'what I bought'. The latter is, from the standpoint of grammar, a singular term asking for the singular, 'was expensive', but referring to many objects. Although the singularization of plural terms is a linguistic observation, I would like to flag it here, since it is the *syntactical* counterpart to Bricker's *E Pluribus Unum*. But we must keep in mind that if Cotnoir is right, and grammar allows us to *formulate* many-one identities, this does by not guarantee that they are true.

Cotnoir addresses the semantical challenge by providing a formal semantics. This is done with the use of set theory. Cotnoir (2013a: 31) calms the reader by pointing out that he is not endorsing the ontology of the semantics and that a semantics could be given with the means of superplural terms, i.e. terms that stand to plural terms as plural terms stand to singular terms. I do not intend to question whether it is possible for him to give a set theoretic semantics for his theory without taking on board its ontology. Rather, I should point out that giving a semantics with the help of superplural terms is, in my view, not less worrisome than giving it in set theoretic terms because the concept of superplural terms is highly problematic, as we will see in section 5.6. However, let's set this issue aside and see how the general identity relation is described.

Cotnoir then introduces the concept of 'being a partition':

Partitions are essentially ways of counting a domain. [...] A partition carves up reality into chunks or portions. It treats all the atoms in the same portion as a *single thing*.

(Cotnoir 2013a: 302)



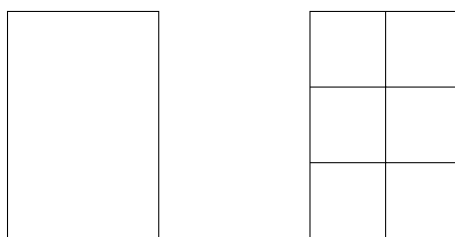


Figure 3.3: *The Piece of Land and the Six Parcels III*

With this concept, Cotnoir is eventually in a position to take counts as a means to refer to the same portion of reality in different ways.

General identity amounts then to referring to the same portion of reality, and a many-one identity holds iff a plural and a singular term refer to the same portion of reality in different counts, (Cotnoir 2013a: 303-5). Apart from the technical details provided, the *philosophical* point which is made here is much in line with Baxter's theory. However, when it comes to the discernibility challenge a quite different route is taken:

There are no failures of Leibniz's Law provided we do not switch our way of counting 'mid-sentence', as it were.

(Cotnoir 2013a: 322)

Hence, Cotnoir claims that his theory is able to work with an unrestricted version of substitution while holding on to Composition as Identity. He offers two different ways a semantics for such a theory can be provided. One of them relativizes truth to counts. For instance, relative to the "rectangle-count" on the left in figure 3.3, it is true that the piece of land is one object. Further, according to the same count, the six parcels are one object. They are identical to it because the term 'the six parcels' refers to the same portion of reality as 'the piece of land'. On the other hand, according to the "square-count" on the right in figure 3.3, it is true that the six parcels are six objects, and the piece of land is six objects. It is identical to them because 'the piece of land' refers to the same portion of reality as 'the six parcels', (Cotnoir 2013a: 309-10; see also, Hawley 2013: 325).

The problem then is that we have to accept the relativity of truth to counts. Bricker (2016: 277-9) has already raised concerns about this idea. It strikes me as a strange outcome that the piece of land and the six(!) parcels can be one object according to a certain count, and six objects according to another. It seems to me rather that the piece of land is *always* one object, and the six parcels are *always* six, *independently* from how we count them.

Cotnoir's account marks an important exception within the interpretations of Composition as Identity. He aims for an unrestricted version substitution, while holding on to the claim that there are several kinds of identity relations. His answer to the syntactical challenge deserves to get more attention than he paid to it. It strikes me that we may use our ability to singularize plural terms as a door-opener for Bricker's *E Pluribus Unum*. However, we cannot yet answer the question whether this is a legitimate move, and will have to wait until chapter 10 before we can come back to it.

### 3.4 Summary

We have now reached the end of Part I, so let's take stock and look ahead what comes next. We have seen that Composition as Identity gives us a reasonable interpretation for the intuition that an object is nothing over and above its parts. Further, it can be motivated with the help of the Overdetermination Argument and due to the fact that it entails the uniqueness of composition. Finally, it allows us to avoid difficulties for such fruitful theories as mereological universalism and four-dimensionalism. Yet, Composition as Identity is itself under pressure since it runs into to the Paradox for Composition as Identity and allows for the derivation of Collapse. Moreover, it is prone to the counterexamples from arrangement and comes dangerously close to mereological essentialism. Different versions of Composition as Identity aim to overcome these problems by elaborating on the theory's main claim. Weak versions of Composition as Identity lose the explanatory power of the

theory's core claim, while strong versions are in trouble due to the differing logical properties of the composition and the identity relation. Moderate accounts of Composition as Identity appear to be better suited to meet the criticisms since they are based on a more fine-grained notion of identity.

In the next chapters, we will spell out the ground for the theory of Composition as Identity which I claim can overcome the criticisms against Composition as Identity. Since plural logic is a necessary means to formulate such a theory, we will begin with an informal introduction of the basic concepts of plural logic in chapter 4. I will then present a formal theory which captures the standard conceptions of plural logic, only to show that they are mistaken and lead us to a revision of our conception of identity: Identity is not a unitary relation, but comes in a variety of ways. We will discuss some principles that are generally assumed to hold for the identity relation and show how we can hold on to some of them, while the idea that there is only one kind of identity relation is abandoned. Chapter 7 consists in spelling out some basic principles and making some decisions for a revision of the standard accounts of plural logic, before we will then finally present a plural logic that provides us with the means to capture the variety of identity in chapter 8.

One of the problems we have to resolve in these chapters is the tension between allowing for many-one identity and the claim that *being one* is the opposite of *being many*. These two views appear to be inconsistent with each other. Yet, we will be able to see that they can live in harmony with each other. The solution I will suggest hinges on the distinction between extensional and intensional contexts. I think that a many-one identity tells us two things. On the one hand, a composite object and its parts are *extensionally* the same. For instance, the six-pack of orange juice and the six cans of orange juice are the same piece of reality, or the same amount of matter. On the other hand, it makes a difference whether we refer to this piece of reality with 'the six-pack of orange juice' or with 'the six cans of orange juice' because they are *intensionally* not the same.



## **Part II**

# **From Plurals to Identities**



## CHAPTER FOUR

# *Plural Logic: Introduction*

In the previous chapters, for instance, when we discussed the Overdetermination Argument, reverse mereological essentialism, or the different forms of Composition as Identity, we occasionally relied on the resources of plural logic. We are now at a point where a more careful discussion of plural logic and its underlying principles is necessary. We could simply move on with the presentation of a formal system of plural logic and assume the truth of its underlying principles. However, a more careful examination of the basic ideas of plural logic not only does justice to the discussions in the literature, it also leads us to a revision of our ordinary views about identity. This revision of our initial view on identity leads us ultimately, or so I argue, to the view that composition is a kind of identity. Therefore, plural logic is not only a necessary means which we need to formulate the claim that a composite object is identical to its parts, but it gives us also evidence for the truth of this claim. Given this important role that plural logic plays for Composition as Identity, we will have to make sure that the basic principles and notions are spelled out with great care and its legitimacy is well-founded. Hence, I will give an informal introduction to plural logic, before a formal system of plural logic will be presented and discussed in the next chapter. We shall begin with some clarifications on what plural logic is and consider what motivations there are to use plural logic, even for someone not interested in Composition as Identity. As this is done, we will see why Composition as Identity is

in need of plural logic: *Any* theory about composition worth its salt is couched in plural logic, since composition is a phenomenon which can only be adequately captured with the resources of plural logic. Eventually, we conclude this chapter with the crucial distinction between singular and plural terms and some remarks on plural reference.

Plural logic is an extension of classical first-order logic ( $FOL^=$ ), as developed by Frege (1879), and presented, for instance, by Priest (2008: §12), Sider (2010: §4-5.3), and Smith (2003). Within  $FOL^=$ , the only terms we are allowed to use are terms which refer to *exactly* one individual. However, natural language contains so called “empty terms”, which do not refer to *at least* one object, e.g. ‘Pegasus’, as well as terms which do not refer to *at most* one object, e.g. ‘Whitehead and Russell’.

Several strategies for dealing with terms that do not refer to exactly one object have been developed. Some of these strategies are based on the idea that although natural language contains sentences with such terms, these sentences can be formalized within classical  $FOL^=$ , (for an excellent overview of such strategies, see Oliver and Smiley 2001; 2013: §3&4). However, there is the view that  $FOL^=$  cannot do justice to these terms and that an alternative logical system has to be developed. The systems which allow the use of empty terms are “free logics” (see Lambert 1997; 2003, or Morscher and Hieke 2001). While these allow the use of terms which fail to refer to an object, plural logics allow the use of terms which refer to *several* objects.<sup>50</sup>

Another way to avoid the step to plural logic is to consider predicates to be sensitive to number. This view denies, for instance, that there is one predicate ‘being British’ which takes singular terms, ‘Russell’, ‘Whitehead’, and plural terms, ‘Russell and Whitehead’, ‘Russell, Whitehead, and Moore’, as arguments. Instead there are many predicates ‘being British<sub>1</sub>’, ‘being British<sub>2</sub>’, ‘being British<sub>3</sub>’, ..., which differ

50. The distinction between free and plural logics is not exclusive. A logic can allow for both, the use of terms which fail to refer to any object *and* of terms which refer to more than one object. The systems developed by Oliver and Smiley (2013), and by Simons (2016) are such systems.



from each other depending upon the number of objects the terms which enter their argument place refer to. I agree with Oliver and Smiley that

[...] marking predicates for number is like marking them for person. We would never regard 'I am  $F$ ', 'you are  $F$ ', 'he/she/it is  $F$ ' as featuring three different predicates[.]  
(Oliver and Smiley 2013: 2)

Hence, I think that this way of undermining the legitimacy of plural logic should be dismissed, and we can move on to see how we to motivate the use of plural logic.

## 4.1 Two Kinds of Arguments for Plural Logic

Plural Logic is motivated by a dissatisfaction with formalizing natural language sentences containing plural terms within  $FOL^=$ .<sup>51</sup> The common means to paraphrase sentences containing plural terms within  $FOL^=$  is to use set theory, (see Black 1971, Hazen 1993, Levin 1992, and Quine 1973: 110-1), or mereology, (see Leonard and Goodman 1940, Link 1998, and Massey 1976). The idea of these two strategies is to paraphrase plural terms from natural language with the help of sets or mereological sums. Since this ultimately boils down to *paraphrasing away* plural terms by using singular terms, these strategies are often summarized under the header "singularism" (see Florio 2014, Hossack 2000, McKay 2006: §2, Oliver and Smiley 2013, and Rayo 2006).<sup>52</sup>

Within the literature, several arguments against the use of set theory and mereology as means to formalize sentences containing plural terms are presented. I group these arguments roughly in two categories: the non-substantial and the substantial arguments.

51. In the second half of the last century, the idea of a plural logic was taken up by Black (1971) and Simons (1982b) with the aim to spell out set theory in terms of plural logic. Boolos' discussions of plural logic (1984; 1985a; 1985b) as an interpretation of monadic second-order logic is often regarded as the *locus classicus* of twentieth century plural logic. For an overview on the history of plural logic, (see Oliver and Smiley 2013: §2).

52. Calling this view 'singularist' goes back to Lewis (1991: 65).

Non-substantial arguments claim that by using set theory and mereology we cannot adequately represent sentences containing plural terms. The general line of thought, (see also Cameron 1999: 133-4 and McKay 2006: 22-4), can be seen from the following two passages:

It is haywire to think that when you have some Cheerios, you are eating a *set*—what you’re doing is: eating THE CHEERIOS.  
(Boolos 1984: 448)

Sets are abstract and so cannot win things. And since, in general, a mereological sum has various decompositions, we would get the silly result that if Anthony and Bill won, so did their molecules (the sum of Anthony and Bill *is* the sum of their molecules).  
(Oliver and Smiley 2001: 299)

This sounds convincing: We do not eat sets, and molecules do not win races. A set is ordinarily considered to be an abstract object and we cannot eat abstract objects. Molecules are not the kinds of objects which participate in, let alone win, races. However, the arguments based on these lines of thought are merely polemics against the singularist position. They are not substantive criticisms because *there are* better ways to formalize sentences containing plural terms with the help of set theory and mereology.

The singularist’s reply to the criticism that set theory or mereology are not suitable for formalizing sentences containing plural terms leads us to what I take to be the real motivation for plural logic. But let’s have a closer look at the above criticism, before we come to the singularist’s defense of her strategy. Boolos<sup>53</sup> protests that the sentence

(1) I had some Cheerios for breakfast

cannot be adequately formalized by set theory or mereology. It seems Boolos is suggesting that the paraphrases we *have* to use when we want to formalize (1) by the means of set theory or mereology are

53. Oliver and Smiley’s criticism in their (2001) is similar to the one suggested Boolos and uses the example of Andy and Bill winning a race. It can be countered by the same reply I suggest singularists will give to the counterexample of Boolos.

(2) I had a set of Cheerios for breakfast

(3) I had a mereological sum of Cheerios for breakfast

which are inadequate, due to the reasons mentioned earlier: (2) implies that somebody was eating a set, and it follows from (3) that somebody had some molecules for breakfast. However, we can paraphrase (1) and (2) by

(4) I had some elements of a set of Cheerios for breakfast

(5) I had some parts of a mereological sum of Cheerios for breakfast

which do not, at least *prima facie*, seem inadequate. These paraphrases do not imply that someone had a set, or a mereological sum, or molecules for breakfast. But although singularism does not entail the implausible sentences (2) and (3), it is in trouble. The problems for the singularist result from the defense we have just seen.

Opponents of singularism, for instance Boolos (1984: 447) and McKay (2006: §2), argue that (2) and (3) come with ontological commitments which do not match with the ontological commitments of (1).<sup>54</sup> I consider these arguments to belong to the substantial arguments against singularism. They share the idea that set theory and mereology are not appropriate means to formalize sentences containing plural terms, because the resulting paraphrases carry ontological commitments to sets and mereological sums. But, so the arguments go, the truth of (1) neither depends on the question whether there is a *set* which has Anthony and Bill as its only elements, nor whether there is a *mereological sum* that has them

54. It might be argued that (2) and (3) are not adequate paraphrases *because* they come with additional ontological commitments when compared with (1). I can see that this point has a certain degree of plausibility. Nevertheless, I think the two points should be kept apart because it makes a difference whether we say that the paraphrases are not able to capture the meaning of the sentences *simpliciter*, or that they do not capture the meaning *because* they carry ontological commitments which do not correspond with the commitments of the original sentences. Anyway, both interpretations show that the singularist position is in trouble. Hence, the weaker position suffices to undermine singularism.

as parts. However, the truth of the set theoretic paraphrase, i.e. (2), depends on the existence of such a set, and the truth of the mereology-based paraphrase, i.e. (3), depends on the existence of such a mereological sum. Therefore, set theory and mereology are not appropriate means to formalize sentences containing plural terms. Because (1) carries ontological commitment only to cereals, which are material objects, we need a framework which allows us to come up with a formalization of (1) which does not bring any additional ontological commitments besides the commitments to material objects.<sup>55</sup> Plural logic is supposed to be such a framework. We will have a closer look at how plural logic manages to avoid these additional ontological commitments. Before we come to that, we shall elaborate the motivation for the use of plural logic a bit more.

## 4.2 Distributivity and Collectivity

In the first hours of elementary logic classes, we are trained to paraphrase away the plural terms of natural language by singular terms. We are told, for instance, that (6) is an adequate paraphrase of (7):

(6) Russell and Whitehead are British

(7) Russell is British and Whitehead is British

By being told to paraphrase away plural terms, we are taught to follow the singularist strategy. The shortcoming of this strategy remains mostly

55. We are here relying on an adequacy principle for logical paraphrases which follows the thought that “[t]o paraphrase a sentence [...] is, first and foremost, to make its ontic content explicit [...]” (Quine 1960: 242; see also Alston 1958 and Rayo 2007: 434-6). This is a compelling principle, given the definition of ontological commitment, see section 1.1.3: A difference with respect to ontological commitment results in a difference with respect to truth conditions. Since an adequate paraphrase of a sentence should have the same truth conditions as the sentence it is a paraphrase of, their ontological commitments should match. Given that paraphrases may not be able to *fully* capture the ontological commitments of a sentence, we could make our case for plural logic in a relativized form: A paraphrase of a sentence is more adequate than another paraphrase, if the former captures the ontological commitments of the paraphrased better than the latter.

unnoticed because the sentences used to convey the underlying idea of the strategy are well-chosen. The predicates used in these examples are all *distributive* in their argument places. The failure of this strategy, and ultimately of the singularist program, becomes obvious when we consider predicates which are *collective* in some of their argument places, as for instance ‘baking a cake together’, or ‘surrounding’.

Before we give the general definitions of ‘being distributive in an argument place’ and ‘being collective in an argument place’, let’s consider the simple cases of one-place predicates. A one-place predicate ‘ $\Phi$ ’ is distributive in its argument place iff it is analytically true that some objects  $uu$  are  $\Phi$  iff each one of the  $uu$  is  $\Phi$ . A one-place predicate ‘ $\Phi$ ’ is collective in its argument place iff it is not distributive in its argument place.

As we have seen, ‘being British’ is distributive in its only argument place, since (6) is true iff (7) is true. Similarly, ‘being a student’ and ‘being blue’ are distributive in their argument places. However, ‘baking a cake together’ is collective in its only argument place, because it meaningless, or at least not true, that some objects  $uu$  bake a cake together iff each one of the  $uu$  bakes a cake together. We shall come back to this example in a moment after spelling out our general definitions:<sup>56</sup>

$D_1$  An  $n$ -place predicate  $P$  is distributive in its  $i$ -th argument place iff, it is analytically true that a sentence  $S$  where  $uu$  occupies the  $i$ -th argument place of  $P$  is true iff the conjunction consisting of all the sentences where a name of each one of the  $uu$  occupies the  $i$ -th argument place of  $P$  is true.

$D_2$  An  $n$ -place predicate  $P$  is collective in its  $i$ -th argument place iff  $P$  is not distributive in its  $i$ -th argument place.

56. Since predicates with more than one argument place can be collective in one, but distributive in another, as for instance ‘surrounding’, we cannot define distributivity and collectivity for predicates *simpliciter*. Ben-Yami (2004: 21), Bohn (2012: 218, fn.18), McKay (2006: 5-6), Oliver and Smiley (2013: 3), and Rayo (2002: 439) use definitions which are similar to  $D_1$  and  $D_2$ .

When dealing with predicates which are collective in some of their argument places, singularism leads us to false sentences:

(8) Andy and Ben bake a cake together

(9) Andy bakes a cake together and Ben bakes a cake together

In analogy to the above instructions to paraphrase (6) by (7), we might paraphrase (8) by (9). However, (9) is not an adequate paraphrase of (8) because it does not make any sense.

Sometimes we encounter sentences where we have no clear way to decide whether a predicate is distributive or collective in a certain argument place.<sup>57</sup> Instead of (8) and (9), we could have

(10) Andy and Ben bake a cake

(11) Andy bakes a cake and Ben bakes a cake

Although (11) makes sense, it can be interpreted in a way that is incompatible with an interpretation of (10). We might understand (11) as telling us that Andy bakes one cake, and Ben bakes another cake. Hence, Andy and Ben bake two cakes. But that contradicts one of the possible interpretations of (10), according to which Andy and Ben bake *one* cake. In order to get hold of those incomplete ways of talking, illustrated by (10), we might talk about predicates as *being able to be interpreted distributively or collectively* in an argument place. Hence, 'baking a cake' can be interpreted as being distributive or as being collective in its only argument place. Yet, 'baking a cake together' can only be interpreted as being collective in its only argument place. Here are two further examples illustrating the inadequacy of the singularist strategy:

(12) Andy, Ben, and Chris surround Dan

(13) Andy surrounds Dan

57. Examples such as the ones above are also discussed by Díez (2010: 152, fn.5), McKay (2006: 11-2), and Yi (2005: 481).

(14) Eva and Fran are classmates

(15) Eva is/are classmates

Since paraphrases of (12) and (14), according to singularism, entail (13) and (15), we can see that this strategy ultimately fails.

There are good reasons to abandon singularism and to consider plural logic as a serious option. Moreover, the notion of predicates being collective in argument places helps us to understand why all answers to the composition questions use plural logic. This will be discussed in the next section.

### 4.3 No Plurals, No Theory of Composition

That Composition as Identity, or other theories about composition, rely on the use of plural logic is not a coincidence. The first argument place of ‘composing’ cannot be interpreted distributively: Some objects  $uu$  composing an object  $x$  does *not* entail that each one of the  $uu$  composes  $x$ .

Suppose ‘composing’ could be interpreted distributively. Then, we could deduce (17) and (18) from (16):

(16) The brush and the stick compose the broom

(17) The brush composes the broom

(18) The stick composes the broom

However, (17) and (18) are false. Assuming that the predicate ‘composing’ is distributive in its first argument place does not conform with our understanding of the predicate. It does, like the predicates used in (12) and (14), not allow for an interpretation where it is distributive in its first argument place. Hence, given that the composition relation is essentially collective in its first argument place, and plural logic is the right means

to deal with predicates which are collective in some of their argument places, *any* constructive<sup>58</sup> theory of composition is in need of plural logic.

This is not intended as an argument against singularism. The observation is that *if* plural logic is the correct framework to deal with predicates which have collective argument places, then any theory about composition has to rely on plural logic, since ‘composing’ is collective in its first argument place. I argued previously *that* plural logic is the correct framework to deal with predicates which have collective argument places.<sup>59</sup>

## 4.4 Plural Terms

We introduced plural logic as a logic which allows the use of plural terms. It is now time to spell out in more detail what *plural terms* are and how they are distinguished from singular terms. Intuitively, there are two ways to make this distinction: based on grammatical form, or on the actual number of referents. In agreement with Oliver and Smiley (2013: 74-5), I will briefly point out the shortcomings of these distinctions and define ‘is a singular term’ and ‘is a plural term’ based on “[...] the number

58. Interestingly, even the non-constructive account of composition, i.e. mereological nihilism, at least in its most competitive form, makes use of plural logic. “[S]tandard nihilism” (Tallant 2014: 1513, fn.1), uses the so-called “Paraphrase Strategy”. It is defended, e.g., by Merricks (2001: 2-20, 162-85), Rosen and Dorr (2002: 157), Schaffer (2007: 176), Sider (2013: 237-8), and van Inwagen (1990: 98-114). The Paraphrase Strategy is critically discussed, e.g., by O’Leary-Hawthorne and Michael (1996), Mackie (1993), Markosian (1998: 220-1), Sider (1993), Tallant (2014: 1512-3), Thomasson (2007: 160-1), Uzquiano (2004) and Wilkins (2016). The strategy claims that sentences which apparently commit to the existence of composite objects, for instance a chair, can be adequately paraphrased by sentences which do not come with such a commitment – by using the phrase ‘atoms arranged chair-wise’. Since these paraphrases are best formalized within plural logic because ‘being arranged chair-wise’ is collective in its only argument place, even nihilists need plural logic.

59. Although the predicate ‘composing’ can be used as an example to show that singularism fails, any other predicate that is not distributive in some argument place does the same work. Since there are such predicates, and using the composition predicate to argue for the adequacy of plural logic might raise the suspicion that we are arguing in circles, we are well advised to base our arguments for plural logic on predicates other than ‘composing’.



of things they are *capable* of [referring to]" (Oliver and Smiley 2013: 74).<sup>60</sup>

One might think that singular and plural terms differ due to their grammatical form. Singular terms have a singular grammatical form, while plural terms have a plural grammatical form. As natural as this distinction seems, it leads to problems because of "pluralia tantum nouns" and "singularia tantum nouns". The former appear only in the plural grammatical form, e.g. 'trousers', 'scissors', or 'clothes'. The latter have only a singular grammatical form, e.g. 'dust', 'fruit', or 'milk'.<sup>61</sup> If we distinguish singular from plural terms on the basis of grammar, this leads to two counterintuitive results. Pluralia tantum nouns refer, in certain contexts, to one object only, although they are grammatically plural, while singularia tantum nouns refer in some cases to many objects although they are singular:

(21) The scissors Andy buys are blue

(22) Ben eats all the fruit Chris gave to him

Considering (21) and that Andy might buy only one cutting instrument, shows that a classification of plural terms based on their grammatical form is inadequate. A similar inadequacy arises, if we examine (22) and assume that Chris gave an apple and an orange to Ben. A distinction between singular and plural terms on the basis of their grammatical form excludes these possibilities.

60. Oliver and Smiley talk about plural *denotation*. Yet, they might be happy to talk about plural reference instead, adopting the terminology which is used, for instance, by Ben-Yami (2004), Cameron (1999), Carrara and Martino (2015), Linnebo and Nicolas (2008), McKay (1994), Moltmann (2016), and Simons (1997), as this passage suggests: "Various authors say denotes, refers to, designates; we shall use 'denotes'" (Oliver and Smiley 2008: 22).

61. It seems that singularia tantum nouns are what are sometimes called "mass-expressions". According to the dominating view in the literature on mass-expressions, (see Gillon 1992, Krifka 1991, Nicolas 2016, and Weinreich 1966), terms can, exclusively and exhaustively, be distinguished into mass- and count-expressions, whereby the former have only one grammatical form, the singular. Although the apparent collapse of the two concepts is interesting, it is a subsidiary question for us.

Moreover, a grammatical distinction leads to another more pressing issue because we “[...] get misled by grammar” (Russell 1919b: 221). Whether we formalize an English sentence or its German translation should not make a difference to the logical form of the sentence. However, there are English pluralia tantum nouns whose German translations are not pluralia tantum nouns. The German translation of ‘scissors’, ‘Schere’ or ‘Scheren’, is not a plurale tantum noun<sup>62</sup> since it has a singular grammatical form as well. This means a distinction based on their grammatical form may lead to different formalizations of an English sentence and its adequate German translation. But if the German sentence is an adequate translation of the English, then they must have *the same* logical form. This suffices to dismiss a distinction between singular and plural terms on the basis of their grammatical form.

Another way to distinguish singular from plural terms is to take the number of objects they actually refer to. Such a distinction gives us three different categories of terms: empty terms, referring to no object; singular terms, referring to exactly one object; and plural terms, referring to more than one object. However, this distinction strikes me not as being satisfactory because we might question whether it is the logicians job to find out how many objects a term *actually* refers to: “Logicians deal with forms, not current affairs” (Oliver and Smiley 2013: 74). Hence, distinguishing terms on the basis of the number of objects they actually refer to does not seem the right way to do it either.

Finally, we come to what I take to be the most plausible way to draw the distinction between singular and plural terms. In agreement with Oliver and Smiley (2013: 74), we shall make a semantical and modal distinction as follows:

62. I could not find an example of a term which is a singulare tantum noun in one language but can be translated into another language by a term which is not a singulare tantum noun. This might be due to the above-mentioned similarity between singularia tantum nouns and mass-expressions. A discussion on the similarity between plural terms and mass-expressions can be found in (Cocchiarella 2009) and (Nicolas 2008; 2016: §9).

$D_3$  A term  $n$  is singular iff  $n$  cannot refer to more than one object  $x$  at once

$D_4$  A term  $n$  is plural iff  $n$  can refer to more than one object  $x$  at once<sup>63</sup>

Singular terms cannot refer to more than one object at once, and plural terms can refer to more than one object once.<sup>64</sup>

Hence, 'Zeus', 'Frege', and 'the men who are identical to Frege' are singular terms, while 'Zeus and Pan', 'the wives of Frege', and 'Frege and his wife' are plural.

The categorization of terms following the above definitions seems unusual at first. According to this distinction, some terms end up in a group where one might not suspect them to be. Usually, 'Zeus' and 'Zeus and Pan' are taken to be empty terms. However, following our definitions, the former is a singular and the latter a plural term. 'Zeus' is not capable of referring to more than one object at once, while 'Zeus and Pan' is. The fact that both terms *actually* fail to refer to any object does not matter. Figure 4.1 shows a singular term referring to an object (top left), and failing to refer to an object (middle left), as well as a plural term referring to more than one object (top right) and a plural term failing to refer to any object (middle right).

The term 'the men who are identical to Frege' is not capable of referring to more than one object. There can only be one man identical to Frege. It is

63. Although our overall aim is to give an account of composition as a kind of identity, and one may think that the question when an object is one and when an object is many cannot be answered before this account is given, the above definitions are not circular. We are, and will be throughout what follows, holding on to the claim that *every* object is one, and *no* object is many.

64. We used the phrase 'at once' in the definitions because terms might be used in different contexts and/or at different times to refer to different objects: If I use the term 'Socrates' when talking to my office mate, the term refers to the Greek philosopher; If I use it in a conversation with my father, it refers to the Brazilian football player. Further, we could relativize the definitions to a language, such that a term  $n$  cannot refer to more than one object in one language, but it can so in another language. This helps us to get a better understanding of the modalities that are in play and explains how it is possible that in the language used by philosophers 'Plato' is a singular term, while it is plural – referring to programs for electronic structure calculations – in the language of chemists.

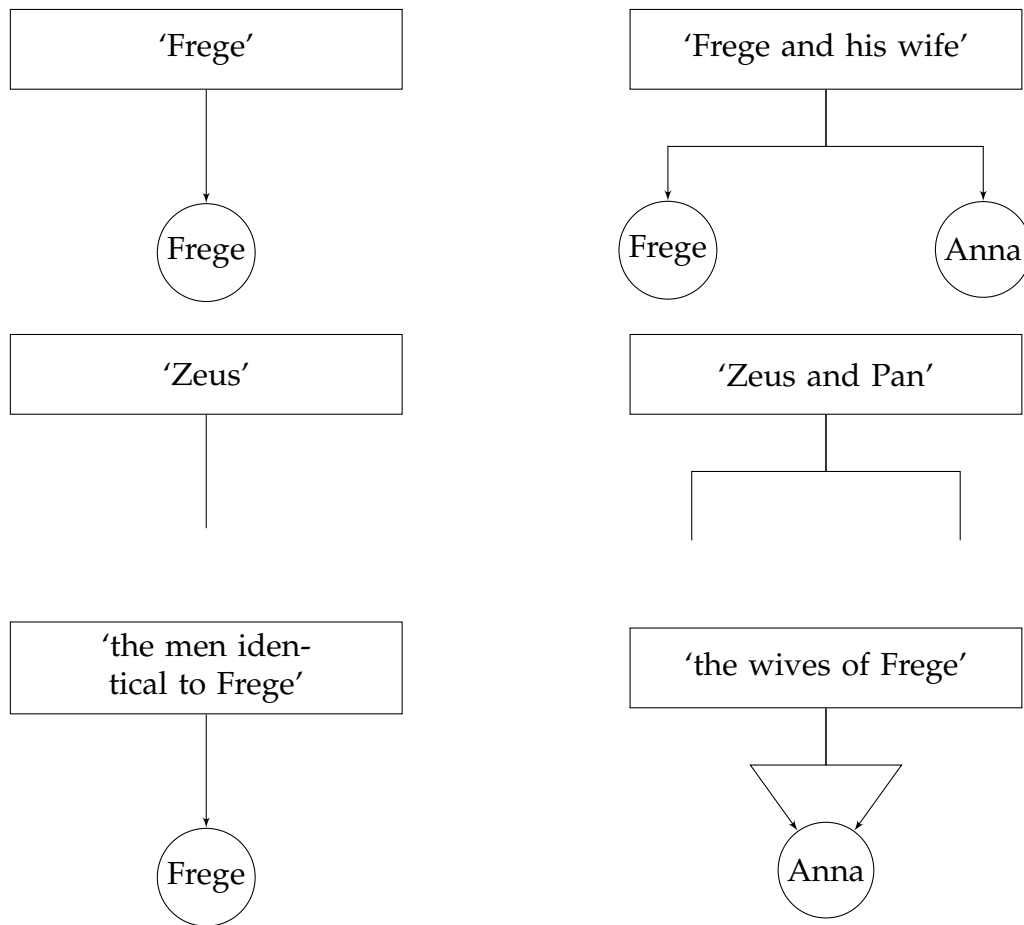


Figure 4.1: Singular and Plural Reference

logically impossible that Frege is identical to more than one man. Therefore, the term cannot refer to more than one object. Finally, 'the wives of Frege' is a plural term, although there is actually only one woman, Margarete Katharina Sophia Anna Lieseberg<sup>65</sup> who was ever married to Frege. But again, that the term *actually* refers to only one person does not affect our categorization of terms. It is *logically possible* that Frege had been married to more than one person, not only to Anna. Therefore, the term 'the wives of Frege' is capable of referring to more than one object. An illustration of the way these two terms refer is given in figure 4.1.

65. Please allow me to give her the nickname 'Anna'.

	Empty	Referring
Singular	the golden mountain Zeus the Queen of the US	the author of <i>OD</i> Frege the men identical to Frege
Plural	the golden mountains Zeus and Pan the Queens of the US	Frege and his wife the authors of <i>PM</i> Russell and the authors of <i>OD</i>

Table 4.1: Categorization of terms

Using this definition, it is possible for us to distinguish proper from improper singular terms, and proper from improper plural terms:

$D_5$  A term  $n$  is a *proper singular term* iff  $n$  is a singular term and  $n$  actually refers to exactly one object  $x$

$D_6$  A term  $n$  is an *improper singular term* iff  $n$  is a singular term and  $n$  actually refers to no object  $x$

$D_7$  A term  $n$  is a *proper plural term* iff  $n$  is a plural term and  $n$  actually refers to more than one object

$D_8$  A term  $n$  is an *improper plural term* iff  $n$  is a plural term and  $n$  does not actually refer to more than one object  $x$

Table 4.1 illustrates how we can categorize terms following the definitions just given.

We have seen that some intuitions about how to distinguish between singular and plural terms are either misleading or not successful. Since the singularist strategy, which claims that the term ‘Frege and his wife’ refers to exactly one object, cannot deal with collective argument places of predicates, distinguishing singular from plural terms according to the number of objects they are able to refer to is the best way we can choose. Keeping this modal-semantical distinction in mind, let’s see how plural reference works.

## 4.5 Plural Reference

Plural logic manages to formalize sentences containing plural terms without bringing in additional ontological commitments because of the way plural terms refer, i.e. plural reference. Plural reference is different from singular reference: Singular reference is, if successful, a relation between a term and *one* object. Naming with plural terms works differently. If a term is a *proper* plural term, i.e. it in fact refers to more than one object, then it is not a relation between a term and one object, but between a term and *many* objects.

A slightly modified analogy from Mill (1846: 23-4) might help to get a better understanding of this idea: Think of terms as labels for objects. Singular terms are labels with one adhesive area only. Each one of them can be used to label only one object at a time: 'Pegasus', 'Russell', and 'the men who are identical to Russell' cannot be stuck to more than one object. 'Pegasus' is a label where there is no suitable object to stick to in our actual world, though there is one in other possible worlds. Similarly, 'the men who are identical to Russell' cannot be stuck to more than one object because it has, due to logical reasons, only one adhesive area. 'Russell' can also be stuck to only one object at a time and our world is such that there is an object where it can be stuck to, although there are worlds where there is no such object.

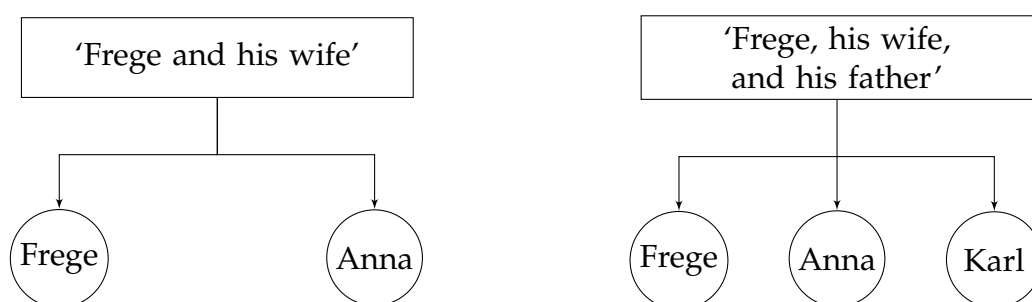


Figure 4.2: Plural Reference

Plural terms have more than one adhesive area and can be stuck to several objects at once: 'Zeus and Pan', 'the wives of Frege', and 'the

authors of *PM*' can be used to label more than one object. Although 'Zeus and Pan' and 'the wives of Frege' do not actually stick to more than one object, they are capable of doing so. The former does actually not stick to any object, due to the same reason that 'Zeus' fails to stick to anything. The latter sticks actually to exactly one object because there is only one object in our world it sticks to. Nevertheless, both terms *can* be used to label several objects at once, and there are possible worlds where they stick to more than one object. On the other hand, our world is such that 'the authors of *PM*' actually sticks to more than one object, although it fails to do so in other worlds.

This analogy suggests that singular and plural reference work differently, yet the difference is small enough to consider them as different kinds of reference. Here is Black's view on the similarity between singular and plural reference:

The notion of "plural" or simultaneous reference to several things at once is really not at all mysterious. Just as I can point to a single thing, I can point to two things at once—using two hands, if necessary; pointing to two things at once need be no more perplexing than touching two things at once.

(Black 1971: 629)

This brings us back to the motivation for plural logic. Because plural reference works in the way we have just hinted at, using plural logic to paraphrase sentences from natural language which contain plural terms does not result in paraphrases which come with any additional ontological commitments. A plural term simply refers to many objects at once, and not to anything over and above these objects, as figures 4.1 and 4.2 illustrate.

Since plural logic contains plural terms, there is no need to paraphrase a sentence from natural language which contains a plural term. It can directly be formalized in the language of the logical system. This makes plural logic a more desirable choice than set theory or mereology for a framework of paraphrasing those sentences. If we use one of the latter two as our framework, we need to paraphrase away the plural terms in

the sentences of natural language. The problem with these paraphrases is then, however, that they commit us to the existence of sets or mereological sums, which may not match up with the ontological commitments of the original sentence we started with. Therefore, plural logic is considered to be an adequate framework to deal with natural language sentences containing plural terms.

With our distinction of predicates being distributive and collective in argument places at hand, we might now wonder whether the predicate ‘refers to’ is distributive or collective in its second arguments place, i.e. is it analytically true that a term refers to some objects  $uu$  iff it refers to each one of them? Three positions emerge from the literature. On the one hand, we have the two exclusive views which claim that ‘refers to’ is distributive or collective, respectively, in its second argument place, and on the other hand, it is thought that it is indeterminate.

Simons defends the first view, i.e. that if a term ‘ $uu$ ’ refers to some objects  $uu$ , then it refers also to each one of the objects which are among the  $uu$ :

[W]hen an expression designates A and B and C ..., where these are individuals, this is to say no more than that it designates A and designates B and designates C[.]  
(Simons 1982a: 166)

Several authors oppose Simons’ view:<sup>66</sup>

[F]rom the premiss that ‘Bill and Ben’ refers to Bill and Ben, we may not deduce that ‘Bill and Ben’ refers to Bill.  
(Hossack 2000: 416)

The list ‘John, Mary, John, Alice’ has the same semantic value as the list ‘John, Mary, Alice’. Each list refers non-distributively to them, John, Mary and Alice.  
(McKay 2006: 68)

66. Further disagreement with Simons’ position can be found in (Higginbotham 2004: 271) and (Rumfitt 2005: 92-4).



According to this view, if a term ‘ $uu$ ’ refers to some objects  $uu$ , then it does not follow that it refers also to each one of the objects which are among the  $uu$ . Finally, we have a position which allows for both phenomena, distributive and collective plural reference. This view is defended by Oliver and Smiley:

So the extension of plain ‘denotes’ is indeterminate, and there is no fact of the matter whether ‘Anne, Charlotte, and Emily’ just denotes the three of them together, or also any things among them. (Oliver and Smiley 2013: 103)

The view of Simons strikes me as the correct one. If a plural term were to refer to some objects  $uu$ , yet not to each one of the objects which are among the  $uu$ , then I cannot see how the term can refer to them. For instance, if the term ‘Russell and Whitehead’, refers to Russell and Whitehead, how can it fail to refer to Russell, or fail to refer to Whitehead? It rather looks to be the opposite way. *Because* the term ‘Russell and Whitehead’ refers to Russell, and refers to Whitehead, it refers to Russell and Whitehead. In other words, plural terms are able to refer to many objects  $uu$  because they refer to each one of the objects among  $uu$ . Hence, I suggest to take the predicate ‘referring to’ as being distributive in its second argument place.

Before we come to the presentation of our system of plural logic, let me point out a pitfall which lies ahead of us. In what follows, we will sometimes use the term ‘plurality’ and say things like ‘The term ‘the blue objects’ refers to the plurality of blue objects’. This claim is misleading. The term ‘plurality’ suggests that a proper plural term refers to *exactly one* object, *a plurality*. As we have seen, proper plural terms do not refer to *one* object but *many* objects. The term ‘plurality’ is used as a technical term for the ease of exposition and it should not be understood to refer to *an object*, but to *many objects*. Although the use of the term ‘plurality’ is misleading, it allows us to say things shorter. Instead of ‘those objects which are all my logic books and no other object’, we can simply use ‘the plurality of my logic books’. However, any sentence containing the term ‘plurality’ can be rephrased by a sentence which does not contain

it. Highlighting this possible source for a misunderstanding marks the end of our informal introduction to plural logic. We are now prepared to go on with the construction of the logical system for our theory of composition, but let me first briefly summarize the main points of this chapter.

In this chapter, I introduced the basic ideas and motivations for plural logic. Plural logic is an extension of  $FOL^=$ , allowing the use of plural terms. Thereby, it is able to formalize sentences with predicates which are collective in some of their argument places. We defined plural terms as those terms, which are able to refer to more than one object. Eventually, I concluded with some remarks on plural reference.

## CHAPTER FIVE

# *The System FOP*

The aim of this chapter is to argue that the principles of plural logic are not consistent with the common views on plural logic, as for instance suggested by Linnebo (2014), McKay (2006), Oliver and Smiley (2013), and Yi (1999b). It will become apparent that the traditional view on identity cannot be defended within the context of a plural language and the principles of plural logic. The argument for this claim will be indirect: I will introduce a system of first-order plural logic, called the “logic of first-order plurals” (*FOP*), which follows the spirit of the traditional view on plural logic and systems built on these ideas, such as the ones that can be found in the works of the authors just mentioned. Then, I will go on to show that *FOP* leads to inconsistencies, if we assume certain, generally accepted, empirical truths. The lessons we can draw from the problems of *FOP* will then be explored in the next chapter.

The presentation of *FOP* will be done in the standard way: First, the vocabulary and the grammar of *FOP* will be given. Next, I will present the inference rules of the system and elaborate it by adding step-by-step definitions and axioms. Additionally, we will have a look at some theorems of *FOP*. The formal proofs for these can be found in the appendix to this chapter. Definitions and Axioms of *FOP* will be presented with the help of a metalanguage.

The object language of *FOP* is a fragment of its metalanguage. There is one key difference between the two languages: The metalanguage

of *FOP* contains a third kind of individual variable ‘ $\alpha$ ’, ‘ $\beta$ ’, ‘ $\gamma$ ’, ... , ranging over singular and plural terms of the object language.<sup>67</sup> The use of this variable in the metalanguage is merely a means to keep things short. For instance, the formula ‘ $\forall\alpha(\Phi(\alpha))$ ’ can be replaced by ‘ $\forall x(\Phi(x)) \wedge \forall uu(\Phi(uu))$ ’ in the metalanguage, and *vice versa*. Hence, the former formula is intended to represent the claim ‘Every object is  $\Phi$  and all objects are  $\Phi$ ’. Since we will consider the two formulas we have just seen as equivalent in the metalanguage, we have two additional inference rules within the metalanguage:

$$\alpha\text{-IN } \forall x(\Phi(x)) \wedge \forall uu(\Phi(uu)) \vdash \forall\alpha(\Phi(\alpha))$$

$$\alpha\text{-EX } \forall\alpha(\Phi(\alpha)) \vdash \forall x(\Phi(x)) \wedge \forall uu(\Phi(uu))$$

Moreover, the metalanguage contains variables, ‘ $\Phi$ ’, ‘ $\Psi$ ’, ‘ $\Sigma$ ’, ... which represent well-formed formulas of the object language.

## 5.1 The Language of FOP

The language of *FOP* consists in the primitive vocabulary of *FOP* and the grammar of *FOP*. The major difference in comparison to the languages of  $FOL^=$  is the use of plural constants, plural variables and the inclusion predicate in the primitive vocabulary. We discussed plural terms already at some length in the previous chapter and have already a basic understanding of plural constants and variables. The presentation of the primitive vocabulary will be followed by an informal introduction of the (proper)<sup>68</sup> inclusion predicate. A formal introduction of the predicate will be given in the next section. This section ends with the presentation of the grammar of *FOP*.

67. Bricker (2016) and Sider (2007; 2014) make also use of an “additional kind” of variable ranging over singular and plural variables.

68. We already encountered the *improper* inclusion predicate in our discussion of Sider’s Collapse principle. The proper and the improper inclusion predicate are, with the identity predicate, interdefinable, see section 5.3.2.

### 5.1.1 The Primitive Vocabulary of FOP

The basis for the language of *FOP* is the language of  $FOL^=$ , as presented by Priest (2008: §2), Sider (2010: §4-5.3), and Smith (2003). We extend the primitive vocabulary by adding plural individual constants, plural variables, and the proper inclusion predicate. Thus, the primitive vocabulary of *FOP* is the following:

- i. individual constants:
  - a. singular:  $a, b, c, \dots$
  - b. plural:  $dd, ee, ff, \dots$
- ii. individual variables:
  - a. singular:  $x, y, z, \dots$
  - b. plural:  $uu, vv, ww, \dots$
- iii. predicates:
  - a. identity (dyadic):  $=$
  - b. inclusion (dyadic):  $\prec$
  - c. further predicates ( $n$ -ary):  $F, G, H, \dots$
- iv. sentential connectives:
  - a. negation (monadic):  $\neg$
  - b. implication (dyadic):  $\rightarrow$
- v. universal quantifier:  $\forall$
- vi. punctuation marks:  $(, )$ <sup>69</sup>

69. As usual, outermost brackets are omitted for the ease of exposition, so that we may write, for instance, ' $\Phi \rightarrow \Psi$ ' instead of ' $(\Phi \rightarrow \Psi)$ '.

A few remarks on the inclusion relation seem appropriate at this point. Inclusion is the idiosyncratic relation of plural logic, as parthood and membership are for mereology and set theory, respectively. We may read ' $\prec$ ' as 'is or are among', or 'is one or are some of'. Here are a few examples to give us an initial intuition for how to translate ' $\prec$ ' into English:

- (1) Russell  $\prec$  Russell and Whitehead (the authors of philosophy books)
- (2) Russell is one of/is among Russell and Whitehead (the authors of philosophy books)
- (3) Russell and Whitehead  $\prec$  Russell, Whitehead, and Wittgenstein (the authors of logic books)
- (4) Russell and Whitehead are some of/are among Russell, Whitehead, and Wittgenstein (the authors of logic books)

It cannot be overemphasized that inclusion shall by no means be read as parthood or membership. This will become apparent when we come to the axioms of the system and consider some of its theorems in section 5.3.

It should be highlighted that *FOP* has a *typed* language, i.e. it contains two types of individual constants and variables, singular and plural. Not all plural logics are based on a typed language. Oliver and Smiley (2013: 209, 212-3) avoid the use of a typed language by introducing the predicates 'being at most one thing' and 'being many', which does in principle the same work for them as a typed language. Furthermore, the typing here differs from the typing in other typed languages where it is used to make an *ontological* distinction, as for instance in (Russell 1908), while we have here a *linguistic* distinction.

### 5.1.2 Grammar of FOP

We can now go on to define the terms of *FOP*. The definition that follows resembles the standard definition of terms in  $FOL^=$ . However, we have to keep in mind that the language of *FOP* contains beside singular individual constants and variables, plural individual constants and variables.

Therefore, we have here a definition of terms which is different from the standard definition in  $FOL^=$ :

- (a) Every individual constant of  $FOP$  is a term of  $FOP$ .
- (b) Every individual variable of  $FOP$  is a term of  $FOP$ .
- (c) Nothing else is a term of  $FOP$ .

Next comes the definition of  $FOP$ 's well-formed formulas (wff):

- (a) If  $F$  is an  $n$ -place  $FOP$ -predicate, and  $\alpha_1, \dots, \alpha_n$  are  $FOP$ -terms, then  $F(\alpha_1, \dots, \alpha_n)$  is a well-formed formula of  $FOP$ .
- (b) If  $\Phi$  is a well-formed formula of  $FOP$ ,  $\neg\Phi$  is a well-formed formula of  $FOP$ .
- (c) If  $\Phi$  and  $\Psi$  are well-formed formulas of  $FOP$ , then  $\Phi \rightarrow \Psi$  is a well-formed formula of  $FOP$ .
- (d) If  $\alpha$  is a  $FOP$ -variable and  $\Phi$  is a well-formed formula of  $FOP$ , then  $\forall\alpha\Phi$  is a well-formed formula of  $FOP$ .
- (e) Nothing else is a well-formed formula of  $FOP$ .

Based on this language, we can now go on to elaborate the logic  $FOP$ .

## 5.2 Inference Rules for FOP

As inference rules we assume

- (MP) If  $\vdash \Phi$ , and  $\vdash \Phi \rightarrow \Psi$ , then  $\vdash \Psi$
- (UG) If  $\vdash \Phi$ , then  $\vdash \forall\alpha(\Phi(\alpha))$
- (SI) If  $\vdash \alpha = \beta$  and  $\vdash \Phi(\alpha)$ , then  $\vdash \Phi(\beta)$

whereby ‘ $\vdash$ ’ should be read as *is derivable*. The inference rule (MP) is *modus ponens* which is simply adopted from  $FOL^=$ . The rule (UG) can only be applied in cases where the variable ‘ $\alpha$ ’ does not appear as a free variable in ‘ $\Phi$ ’. Please note that, although (UG) shows a close similarity to the inference rule called “universal generalization” from  $FOL^=$ , they come apart. Universal generalization tells us that if a sentence ‘ $\Phi$ ’ is derivable, then ‘ $\forall x(\Phi(x))$ ’ is derivable as well. (UG) tells us more: The derivability of ‘ $\Phi$ ’ does not only imply the derivability of ‘ $\forall x(\Phi(x))$ ’, but also of ‘ $\forall uu(\Phi(uu))$ ’. It seems a natural thought to expand universal generalization to (UG) due to the extension of the language. From here on, I will use the term ‘universal generalization’ to refer to (UG).

(SI) resembles substitution from  $FOL^=$ , though the two come apart for similar reasons (UG) and its counterpart from  $FOL^=$  have to be distinguished from each other.  $FOL^=$  does not contain any plural terms, so it only allows to substitute singular terms. The extension of the vocabulary of  $FOP$  makes an extension of substitution natural, in order to allow for the following inference:

1. Russell and Whitehead are identical to the authors of PM
2. Russell and Whitehead are British
3. The authors of PM are British

As we will see in sections 5.5 and 6.2, this extension of the inference rule is problematic. Yet, since nothing speaks against (SI) and it is a natural thought to adopt it as an inference rule once plural terms are added to the language, we hold on to it for now. From here on, I will use the term ‘substitution’ or ‘generalized substitution’ for (SI).

### 5.3 Concepts and Principles of FOP

We can now provide the first definitions for  $FOP$ . We begin with some logical concepts: the sentential connectives, the existential quantifier and



the unique description operator. This will be followed by a first group of axioms, which allow us to derive the theorems from  $FOL^=$  and their plural extensions.

### 5.3.1 Pluralized $FOL^=$

The definitions and axioms below are the result of taking the extension of  $FOP$ 's language into account. There is no need to spend too much time with these, since most of them are simply carried over from  $FOL^=$ . However, some differ from their  $FOL^=$  counterparts due to the extended language of  $FOP$ . Those cases will be highlighted.

We define the non-primitive sentential connectives conjunction, ' $\wedge$ ', disjunction, ' $\vee$ ', and equivalence, ' $\leftrightarrow$ ', as usual:<sup>70</sup>

$$(D1) \quad \Phi \wedge \Psi =_{df} \neg(\Phi \rightarrow \neg\Psi)$$

$$(D2) \quad \Phi \vee \Psi =_{df} \neg(\neg\Phi \wedge \neg\Psi)$$

$$(D3) \quad \Phi \leftrightarrow \Psi =_{df} (\Phi \rightarrow \Psi) \wedge (\Psi \rightarrow \Phi)$$

Furthermore, we define the existential quantifier in the traditional way:

$$(D4) \quad \exists\alpha\Phi =_{df} \neg\forall\alpha\neg\Phi$$

' $\exists$ ', like ' $\forall$ ', connects with singular and plural variables. Here we have a first difference to  $FOL^=$  with respect to introducing new notation. If ' $\exists$ ' is defined in  $FOL^=$ , then it is introduced as connecting with singular variables only. Since the language of  $FOP$  contains plural variables as well, we might want to formalize sentences, as for instance

(5) Some students surround the library

70. In order to keep the use of punctuation marks manageable, we introduce the following conventions with respect to the binding strength of our logical connectives: ' $\neg$ ' binds stronger than ' $\wedge$ '; ' $\wedge$ ' binds stronger than ' $\vee$ '; ' $\vee$ ' binds stronger than ' $\rightarrow$ '; ' $\rightarrow$ ' binds stronger than ' $\leftrightarrow$ '. Hence, ' $\Phi \rightarrow \Psi \wedge \Sigma$ ' can be written instead of ' $\Phi \rightarrow (\Psi \wedge \Sigma)$ ', but *not* instead of ' $(\Phi \rightarrow \Psi) \wedge \Sigma$ '.

which is why we need an existential quantifier that is able to connect with plural variables as well.

Moreover, (D4) illustrates that the use of three individual variables in the metalanguage is dispensable and *only* a way to keep things shorter. We could define our existential quantifier with the singular and plural variables only, by replacing (D4) with the following two formulas:

$$(D4a) \exists x\Phi =_{df} \neg\forall x\neg\Phi$$

$$(D4b) \exists uu\Phi =_{df} \neg\forall uu\neg\Phi$$

Due to the stylistic habit of writing the identity- and the inclusion-predicate between the terms which enter their argument places, we introduce the following two definitions:<sup>71</sup>

$$(D5) \alpha = \beta =_{df} (\alpha, \beta)$$

$$(D6) \alpha \prec \beta =_{df} \prec (\alpha, \beta)$$

With these definitions at hand, we add the following axioms to *FOP*:

$$(A1) \Phi \rightarrow (\Psi \rightarrow \Phi)$$

$$(A2) (\Phi \rightarrow (\Psi \rightarrow \Sigma)) \rightarrow ((\Phi \rightarrow \Psi) \rightarrow (\Phi \rightarrow \Sigma))$$

$$(A3) (\neg\Phi \rightarrow \neg\Psi) \rightarrow ((\neg\Phi \rightarrow \Psi) \rightarrow \Phi)$$

These three axioms allow us to derive the tautologies from the classical propositional calculus, (see Mendelson 1987: 29). In order to derive all tautologies from the predicate calculus as well, adding the following two axioms suffices, (see Mendelson 1987: 55-6):

$$(A4) \forall x\Phi \rightarrow \Phi(a/x)$$

$$(A5) \forall x(\Phi \rightarrow \Psi) \rightarrow (\Phi \rightarrow \forall x\Psi)$$

71. I will further simplify the notation from here on by dropping most of the initial universal quantifiers. (D5) and formulas thereafter should be understood as universally closed formulas.

where ' $\Phi(a/x)$ ' is the result of substituting any occurrence of ' $x$ ' in ' $\Phi$ ' by ' $a$ ', and in (A5), ' $\Phi$ ' does not contain any free occurrence of ' $x$ '. Since our variable ' $x$ ' stands for singular variables of the object language only, (A4) and (A5) do *not* allow the derivations of the following formulas

$$\forall uu\Phi(uu) \rightarrow \Phi(dd)$$

$$\forall uu(\Phi \rightarrow \Psi) \rightarrow (\Phi \rightarrow \forall uu\Psi(uu))$$

Since it seems only natural that the derivation of these formulas should be validated by the axioms of *FOP*, we add the plural counterparts of (A4) and (A5) to our set of axioms:

$$(A6) \quad \forall uu\Phi \rightarrow \Phi(dd/uu)$$

$$(A7) \quad \forall uu(\Phi \rightarrow \Psi) \rightarrow (\Phi \rightarrow \forall uu\Psi)$$

These have similar restrictions as we have on (A4) and (A5): ' $\Phi(dd/uu)$ ' is the result of substituting any occurrence of ' $uu$ ' in ' $\Phi$ ' by ' $dd$ ', and in (A7), ' $\Phi$ ' does not contain any free occurrence of ' $uu$ '.

In order to derive all theorems of  $FOL^=$ , we have to add one more axiom to the axioms of *FOP*. This is the law of identity:

$$(A8) \quad x = x$$

There is no need to assume a general or pluralized version of (A8), ' $\alpha = \alpha$ ' or ' $uu = uu$ ', to our axioms, since the plural counterpart of (A8) will be a theorem of *FOP*.

Finally, we define a unique description operator, ' $\iota$ '. This operator is used to formulate definite descriptions, such as 'the author of *OD*'. We shall follow orthodoxy, (see Copi 1967: 166, Sider 2010: 117-8, and Smith 2003: 345-6), which is based on (Russell 1905), and define ' $\iota$ ' contextually:

$$(D7) \quad \Psi(\iota\alpha\Phi(\alpha)) =_{df} \exists\alpha(\Phi(\alpha) \wedge \forall\beta(\Phi(\beta) \rightarrow \beta = \alpha) \wedge \Psi(\alpha))$$

Hence, the expression ' $\iota x\Phi(x)$ ' should be read as 'the object which is  $\Phi$ ', and we shall say that the object which is  $\Phi$  has a certain property  $\Psi$  iff there is at least one and at most one object, i.e. exactly one object, which has the property  $\Phi$  and it has the property  $\Psi$ . Similarly, ' $\iota uu\Phi(uu)$ ' should be read as 'the objects which are  $\Phi$ ', or 'the  $\Phi$ s'. Since pluralities will turn out to be unique, see (T13) below, we have the trivial claim that the objects which are  $\Phi$  are  $\Psi$  iff the objects which are  $\Phi$  are  $\Psi$ .

These definitions and axioms are the basis for *FOP*. They diverge from the definitions and axioms of  $FOL^=$  only insofar, as they take the extension of the language of *FOP* into account. What follows is a more obvious departure from  $FOL^=$ .

### 5.3.2 Inclusion and Improper Inclusion

We begin our departure from  $FOL^=$  by fixing the behavior of the proper inclusion predicate, ' $\prec$ '. It is commonly taken to be a strict partial order, i.e. it is asymmetric, transitive and irreflexive, (Oliver and Smiley 2013: 109). The first axiom we choose for *FOP*, the asymmetry of inclusion, is a lexical principle. Violations of the asymmetry of inclusion give rise to the suspicion of meaningless or a lack of correct understanding of the relation. According to the natural understanding of the predicates 'is one/are some of' and 'is/are among' the following sentences are analytically true:

- (6) If Russell is among the logicians (the British), then the logicians (the British) are not among Russell
- (7) If *Principia* and *Grundgesetze* are among my (logic) books, then my (logic) books are not among *Principia* and *Grundgesetze*

I take it that denying the truth of the above sentences is untenable, and hence suggest adopting the following axiom:

$$(A9) \alpha \prec \beta \rightarrow \neg(\beta \prec \alpha)$$

I ask the skeptic of (A9) to be patient. Presumably, the predicate she has in mind when putting (A9) into question is the *improper* inclusion predicate introduced below.

A further reason to accept (A9) is the fact that it entails the irreflexivity of the inclusion relation: No object is among itself and no objects are among themselves. Denying this principle contradicts our ordinary use of the predicate 'being among'. Take for instance the following sentences:

(8) Russell is among Russell

(9) My logic books are among my logic books

We do not want to accept the above sentences to be true, due to our ordinary use of the inclusion predicate. Naturally, the skeptic about the asymmetry of ' $\prec$ ' will disagree with me about this. However, I will here again ask her to be patient. I think this disagreement is based on the same misunderstanding as the disagreement about (A9).<sup>72</sup> Hence, we derive

(T1)  $\neg(\alpha \prec \alpha)$

which in turn speaks for accepting (A9). The derivation of (T1) from (A9) is straightforward: According to (A9), if  $a$  is among  $a$ , then  $a$  is not among  $a$ , for arbitrary  $a$ . Hence, no  $x$  is not among  $x$ . Similarly for arbitrary  $dd$ , if  $dd$  is among  $dd$ , then  $dd$  is not among  $dd$ . Therefore, no  $uu$  is among  $uu$ . These two claims entail (T1): No  $\alpha$  is among  $\alpha$ .

The last logical property of ' $\prec$ ' to discuss is transitivity. It is common practice to stipulate that the inclusion relation is transitive. I disagree. We will come back to the issues of the transitivity of inclusion in section 7.5, because we will then be in a position to see the problem with the following axiom. For now, we shall follow the standard view and adopt

72. With respect to the dispute about the reflexivity of inclusion, I think things stand similar as it is the case with the reflexivity of parthood. As Casati and Varzi (1999) and Lejewski (1957) show, the disagreement on the reflexivity of the parthood relation, (see Leonard and Goodman 1940, Rescher 1955: 9-10, Leśniewski 1992, Simons 2003: 25-41 and Tarski 1937) disappears, once we distinguish between the *proper* and the *improper* parthood relation. Presumably, things stand similar with the discussion about the reflexivity of inclusion.

$$(A10) \alpha \prec \beta \wedge \beta \prec \gamma \rightarrow \alpha \prec \gamma$$

as an axiom. It can be motivated by the truth of the following examples:

(10) If Russell is among the logicians and the logicians are among the philosophers, then Russell is among the philosophers

(11) If *Principia* and *Grundgesetze* are among my logic books and my logic books are among my philosophy books, then *Principia* and *Grundgesetze* are among my philosophy books

Finally, we introduce the technical predicate, ' $\preceq$ ', representing the *improper* inclusion relation, which is defined disjunctively:

$$(D8) \alpha \preceq \beta =_{df} \alpha \prec \beta \vee \alpha = \beta$$

Some authors use ' $\preceq$ ' as their primitive predicate, for instance Oliver and Smiley (2013: 211, 235), and Yi (1999b: 177-8), instead of ' $\prec$ '. The difference does not matter here, since they are, together with '=' interdefinable and one may choose either one of them.<sup>73</sup> (D8) shows us why some people might be skeptic about the asymmetry or irreflexivity of inclusion. I assume two parties can only disagree about these properties of inclusion, if one of them interprets it as ' $\prec$ ' and the other as ' $\preceq$ '.

Improper inclusion is a non-strict partial order (Oliver and Smiley 2013: 109), i.e. it is reflexive, antisymmetric and transitive. We can derive these properties from (T1), (A9) and (A10), once we have shown that identity is an equivalence relation. This will be done after the introduction of three further axioms.

### 5.3.3 Extensionality and Comprehension

Although proper inclusion<sup>74</sup> differs from proper parthood and membership, we have two axioms in *FOP* which resemble axioms from mereol-

73. Proper inclusion can be defined with ' $\alpha \prec \beta =_{df} \alpha \preceq \beta \wedge \neg(\alpha = \beta)$ '.

74. Henceforth, I will drop the qualification 'proper' for the sake of easier readability.

ogy and set theory. Those two axioms tell us a bit more about the behavior of plural terms and their relation to singular terms. First, we have the comprehension axiom, (see Linnebo 2014, McKay 2006: 129, Oliver and Smiley 2013: 242, and Yi 1999b: 180):

$$(A11) \exists x\Phi(x) \rightarrow \exists uu\forall x(x \prec uu \leftrightarrow \Phi(x))$$

According to this axiom, whenever there is some *object* (note the singular variable) having a certain property  $\Phi$ , then there are some *objects* (note the plural variable) such that each one of them has  $\Phi$ , and any object having  $\Phi$  is among them. At first sight, this axiom seems implausible: The existence of *one* object having a property  $\Phi$  appears not to guarantee that there are *some objects* which are  $\Phi$ . This initial skepticism against (A12) is understandable, but can be dismissed. If we keep in mind that our distinction between singular and plural terms is made according to the number of objects they *can* refer to, then it is clear that we want to hold on to the above axiom, since we may not know how many objects a plural term '*uu*' refers to. For instance, if we claim

(12) The guests at the party will get pizza and ice cream

then we want this sentence to be true even if only one person comes to the party.

Note that (A11) is not prone to a Russell-like paradox, (see Russell 1903: Appendix B, as well as Irvine and Deutsch 2016, Link 2004, and Russell 1987), i.e. it does not allow to derive the existence of a plurality among which there are all pluralities which do not contain themselves. The reason for this is that in formulating the above comprehension axiom we made use of the two different kinds of variables of our language. (A11) says, whenever there is *an object*  $x$  which has a property  $\Phi$ , then there are *some objects*  $uu$  such that for every *object*  $x$ :  $x$  is among the  $uu$  iff  $x$  is  $\Phi$ . The twofold use of the singular variable  $x$  is crucial. (A11) does *not* tell us anything about what follows from some objects  $vv$  having a property  $\Phi$ . In particular, it does not tell us that if some objects  $vv$

have a property  $\Phi$ , then there are some objects  $uu$  such that for all objects  $vv$ : the  $vv$  are among the  $uu$  iff the  $vv$  are  $\Phi$ .<sup>75</sup> Since we have spelled out the comprehension axiom by using singular and plural variables, the paradox can be avoided.<sup>76</sup>

Another axiom which shows a close similarity to axioms from mereology and set theory is the extensionality axiom.<sup>77</sup>

$$(A12) \quad \forall x(x \prec uu \leftrightarrow x \prec vv) \rightarrow uu = vv$$

Assuming this axiom is pretty plausible. Loosely speaking, it says that for any objects  $uu$  and  $vv$ , if it holds for every object  $x$  that if  $x$  is among the  $uu$ , then  $x$  is among the  $vv$ , and *vice versa*, then the  $uu$  and the  $vv$  are identical. Here is an example to illustrate the idea behind this axiom: If it is true for every object  $x$  that  $x$  is among the blue objects iff  $x$  is among the round objects, then the blue objects are identical to the round objects.

The comprehension and the extensionality axiom are powerful principles. Their introduction allows us to derive some important and interesting theorems of *FOP* as we will see next. Before we come to that, let's introduce the axiom of non-emptiness:

$$(A13) \quad \forall uu \exists x(x \prec uu)$$

This axiom reflects our stipulation to use only referring and no empty terms. It reassures that none of our plural terms turns out to be an empty

75. If we had this as an axiom, i.e. ' $\exists vv \Phi(vv) \rightarrow \exists uu \forall vv (vv \prec uu \leftrightarrow \Phi(vv))$ ', then a Russell-like paradox for plural logic could be derived in a few steps: From the irreflexivity of inclusion, it follows that there are some  $vv$  which are not among themselves. The supposed axiom entails then that there is a plurality of objects  $uu$  such that for all objects  $vv$ : the  $vv$  are among the  $uu$  iff the  $vv$  are not among themselves. Let this plurality be the  $dd$ . With (A6), we can then show that the  $dd$  are among the  $dd$ , i.e. the  $dd$  are among themselves iff the  $dd$  are not among themselves. This is then the Russell-like paradox for plural logic.

76. In so far as the distinction between singular and plural terms is a distinction between different *types* of terms, the solution to the paradox is, in spirit, similar to Russell's own solution (see Russell 1903: Appendix B; 1908, and Whitehead and Russell 1963: §2).

77. For Hossack (2014: 523), Linnebo (2014), McKay (2006: 125), Oliver and Smiley (2013: 110) and Yi (1999b: 180), (A12) is a theorem.



term. In other words, it helps us to make sure that there are no “empty pluralities”.

Before moving on to have a look at some of the theorems of *FOP*, we should have a look at a worry that might arise from the use of the proper inclusion predicate in the above three axioms. One might ask why we made the choice to use this predicate, and not the improper inclusion predicate. Or what might seem even more worrisome for us, someone might claim that the above axioms are *false*, if we formulate them with the proper inclusion predicate. This would be a serious problem, in particular with respect to the conclusions I will draw in section 5.6. Hence, let me anticipate some of the criticisms that might be raised at this point.

To begin with, it should be clear that the use of proper inclusion in the axioms above is, though initially motivated, not solely motivated on how we use the predicate ‘being among’ in natural language. Even if it turned out that this predicate is used in natural language either in the sense of the proper or the improper inclusion relation, this does not mean that we should use that predicate to axiomatize our *logic*. Hence, the choice for which inclusion predicate to use in the above axioms must be based ultimately on formal considerations.

The problem with rejecting the above axiomatization and replacing it with one that is based on the improper inclusion relation is that such an axiomatization entails, together with the negation of the axiomatization I suggest, “mixed identities”. These are sentences where the identity predicate is flanked by a singular term and a plural term. We will turn back to these kinds of identities in section 5.5.1. For now, it suffices to point out that the suggestion to use the improper inclusion predicate is very likely based on the intention to avoid mixed identities.

Alternatively, I think, someone might suggest that we should use the improper inclusion relation in the axioms, but that mixed identities are not well-formed expressions. This strikes me as an odd line of thought. Firstly, I cannot see what could be the motivation to build the axioms of our logic on the use of the improper inclusion predicate, and not the proper inclusion predicate, if the former is defined disjunctively with

the proper inclusion and the identity predicate, and then to claim that mixed identity statements are not well-formed. Secondly, we can see that the suggestion to use the improper inclusion predicate in the axioms becomes obscure, if it is claimed that mixed identities are not well-formed. Someone might suggest, in a similar vein, that a set theory with urelements should be axiomatized with the predicate 'being an improper element of', but add then next, that sentences that claim that an urelement is identical to a set are not well-formed expressions. This strikes me as an odd view and, in my opinion, things stand similar with the claim that the axiomatization of plural logic should be based on the use of the improper instead of the proper inclusion predicate.

Having said this, I should again highlight that there is, *prima facie*, nothing that prohibits the use of the improper inclusion predicate in the above three axioms. We have to make a choice here. Nevertheless, I think that we have seen that the use of the proper inclusion predicate is a rational option, and that an axiomatization based on the proper inclusion predicate may run into certain difficulties.

### 5.3.4 Some Basic Theorems of FOP

Our logical framework allows us to derive some interesting theorems. With the extensionality axiom, we can show that identity is an equivalence relation. This in turn makes it possible to show the above-mentioned properties of improper inclusion. Eventually, we can prove that there is a universal plurality, i.e. some objects such that every object is among them. The extensionality axiom guarantees that this plurality is unique, hence there is "*the* universal plurality".

To prove the logical properties of '=', we show first the plural counterpart of (A8), the law of identity. This gives us the generalized version of (A8) in our metalanguage. The pluralized version of (A8),  $uu = uu$ , follows from the extensionality axiom (A12), since it holds for any  $x$  that  $x$  is among the  $uu$  iff  $x$  is among the  $uu$ . Hence, '=' is reflexive:

$$(T2) \alpha = \alpha$$

This theorem gives us with (SI) the symmetry and transitivity of '=':

$$(T3) \alpha = \beta \rightarrow \beta = \alpha$$

$$(T4) \alpha = \beta \wedge \beta = \gamma \rightarrow \alpha = \gamma$$

The proofs in the appendix to this chapter exhibit that the derivations of (T3) and (T4) follow the basic strategy one uses when proving these properties for the identity predicate in  $FOL^=$ .<sup>78</sup>

These three theorems allow us then to derive the logical properties of ' $\preceq$ ' from the definition (D8) together with (T1), (A9) and (A10):

$$(T5) \alpha \preceq \alpha$$

$$(T6) \alpha \preceq \beta \wedge \beta \preceq \alpha \rightarrow \alpha = \beta$$

$$(T7) \alpha \preceq \beta \wedge \beta \preceq \gamma \rightarrow \alpha \preceq \gamma$$

Finally, we have a last group of theorems and a further definition to add to *FOP*, before we come to the semantics and the problems of *FOP*. The comprehension axiom allows us to derive from (A8), the existence of a plurality among which there are all objects:

$$(T8) \exists uu \forall x (x \prec uu)$$

(A8) entails that there is some object  $x$  which is identical to itself. From that we can infer with the comprehension axiom, that there is some plurality  $uu$  such that for every object  $x$ ,  $x$  is among  $uu$  iff  $x$  is identical to

78. It should be noted that the proofs for (T3) and (T4) are more complex than the proofs of their  $FOL^=$  counterparts due to the different kinds of variables we have. For instance, to proof (T3) we use  $\alpha$ -EX, the derivable inference rule conjunction-elimination, (T2) and (A4) to get ' $a = a$ '. With (SI) we get ' $b = a$ ' by assuming ' $a = b$ ', and conclude with the derivable inference rule conditional introduction and (UG) ' $\forall y (a = y \rightarrow y = a)$ '. We repeat this step with the assumption ' $a = dd$ ' accordingly and conclude ' $\forall vv (a = vv \rightarrow vv = a)$ '. These two conclusions allow us to derive ' $\forall x \forall \beta (x = \beta \rightarrow \beta = x)$ ' with  $\alpha$ -IN and (UG). The above steps are repeated in a similar manner, by deriving  $dd = dd$  from (T2), together with the assumptions ' $dd = b$ ' and ' $dd = ee$ ' in order to conclude ' $\forall uu \forall \beta (uu = \beta \rightarrow \beta = u)$ '. We conclude from this, with the previous and  $\alpha$ -IN, ' $\forall \alpha \forall \beta (\alpha = \beta \rightarrow \beta = \alpha)$ '.

itself. Since every object is identical to itself, we conclude that there is some plurality such that every object is among it.

But (T8) might seem odd: How can it be that it follows from *FOP* that there are *some objects*? Should the theorems of *FOP* not hold independently of whether there is any object at all? That is a reasonable worry to have here. Yet, the only move in the above sketch of the derivation which might seem contentious is the first one, from *Every object is identical to itself* to *There is an object which is identical to itself*. This inference, which actually divides up into two inferences, universal instantiation and existential generalization, does *not* involve any characteristic feature from *FOP*. It seems rather that *FOP* inherits the problem of existential import via the universal quantifier from  $FOL^=$ , which was already identified as an unwelcome result by Russell:

The primitive propositions of *Principia Mathematica* are such as to allow the inference that at least one individual exists. But I now view this as a defect in logical purity.

(Russell 1919a: 203)

The standard semantics of  $FOL^=$  assumes that there is at least one object, i.e. the domain is not empty, (see Kleene 1967: 84, Mendelson 1987: 46, Priest 2008: 264, and Sider 2010: 92). Hence, the axioms of  $FOL^=$ , to be precise the law of identity, ' $\forall x(x = x)$ ', imply the existence of at least *one object*. *FOP* is an extension of  $FOL^=$ , so it is not a surprise that the axioms of *FOP* entail the proposition that there are at least *some objects*. Rather, it would be surprising if *FOP* did *not* entail this proposition. The problem of existential import is not a homemade problem of *FOP*, but an oddity *FOP* inherits from  $FOL^=$ .

Now that we have concluded that there is at least one universal plurality, we can use the extensionality axiom to prove that the universal plurality is, like any plurality as we will see soon, unique.

Assume, there are some *uu* and some *vv* such that it holds for all objects *x*, *x* is among *uu* and *x* is among *vv*. From this, it follows that for any object *x*, *x* is among *uu* iff *x* is among *vv*. With the extensionality

axiom, we can then show that  $uu$  and  $vv$  are identical, or in other words, the universal plurality is unique:

$$(T9) \quad \forall x(x \prec uu) \rightarrow \forall vv(\forall x(x \prec vv) \rightarrow vv = uu)$$

Since (T8) tells us that there is at least one universal plurality and (T9) that there is at most one universal plurality, we can show that there is exactly one universal plurality. Together with the definition of ' $r$ ' this allows us to derive:

$$(T10) \quad \exists uu(uu = rv(\forall x(x \prec vv)))$$

The universal plurality is a noteworthy plurality, so that we might want to define a constant<sup>79</sup> for it:

$$(D9) \quad \omega\omega =_{df} uu\forall x(x \prec uu)$$

This definition gives us eventually the following corollary to (T10):

$$(T11) \quad \exists uu(uu = \omega\omega)$$

As hinted at above, not only the universal plurality but *any plurality* is unique. Following the above sketch of the proof for the uniqueness of the universal plurality, the proof for the uniqueness of all pluralities can be reconstructed easily and we can hold on to the following:

$$(T12) \quad \forall x(x \prec uu \leftrightarrow \Phi(x)) \rightarrow \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = uu)$$

(T12) and the comprehension axiom give us the interesting insight that if there is some object having a property  $\Phi$ , then there is exactly one plurality which contains all the objects which are  $\Phi$ :

$$(T13) \quad \exists x\Phi(x) \rightarrow \exists uu(uu = rv(\forall x(x \prec vv \leftrightarrow \Phi(x))))$$

79. I choose the two omegas, ' $\omega\omega$ ', because ' $\omega$ ' is the last letter of the Greek alphabet. In a sense, the universal plurality marks an end, since it contains all objects there are; it leaves out no object. The constant consists of *two* omegas to indicate that it is a *plural* constant.

We may use this theorem to introduce one more concept for *FOP*. Since we have shown that the existence of a plurality guarantees its uniqueness, it will be convenient to have some notational abbreviation for *specified* pluralities. That is the intention behind the following definition:

$$(D10) \exists x\Phi(x) \rightarrow \llbracket \Phi(x) \rrbracket =_{df} \text{nuu}(\forall x(x \prec \text{uu} \leftrightarrow \Phi(x)))$$

We may read ‘ $\llbracket \Phi(x) \rrbracket$ ’ as ‘the plurality of  $\Phi$ s’, ‘the objects which are  $\Phi$ ’, or just ‘the  $\Phi$ s’. Since the existence of the plurality of  $\Phi$ s depends upon the existence of at least one object that has the property  $\Phi$ , (D10) has to be a partial definition,<sup>80</sup> i.e. a conditional, in order to not be creative in the sense that the existence of an arbitrary plurality would follow from the definition alone.

Finally, (D10) allows us to derive the following plausible theorem:

$$(T14) \exists x\Phi(x) \rightarrow \forall y(\Phi(y) \leftrightarrow y \prec \llbracket \Phi(x) \rrbracket)$$

This theorem validates the intuition that if there is at least one object which is  $\Phi$ , then it holds for any object  $y$  that  $y$  is  $\Phi$  iff  $y$  is among the  $\Phi$ s. (T14) will turn out to be crucial for our two arguments against *FOP*, though the two arguments rely on different directions of the entailment.

## 5.4 Semantics for FOP

In this section, I will set out a semantics for the system *FOP*. Although we will not make use of the semantics of *FOP*, it might be useful to have a semantics in terms of getting a better understanding of the formalism. Standardly, semantics for a logic are provided with the means of set theory. This will not do for plural logic, since we do not want to take the ontological baggage of set theory on board and follow what “[...] many philosophers now regard as its canonical semantics [...]” (Florio and Linnebo 2015: 1). Hence, we shall use the resources of plural logic when we

80. More on partial definitions can be found, for instance, in (Carnap 1936) and (Soames 2010).

give a semantics for plural logic. I am aware that it is circular to use the logic for providing a semantics for that very logic. However, when logicians spell out the semantics for  $FOL^=$ , then this is done with the help of  $FOL^=$ . So, *tu quoque*. I think the circularity which is involved here is not vicious, but virtuous. It is only consistent to work all the way with plural logic, even when we give the semantics for plural logic itself.

Standard semantics for logical theories take as the domain-of-discourse a non-empty set. Since we do not want to build our logic upon set theory, we take the domain to consist of some objects  $uu$ , whereby we postulate that there is at least one object  $x$  in the domain  $uu$ . This postulate is the effect of not allowing for empty terms within our logic, which resulted in the axiom of non-emptiness, saying that for any plurality of objects there is at least one object which is among them.<sup>81</sup>

Besides the domain, I specify a valuation function  $v$  which assigns objects from the domain to terms, and relations on the objects from the domain to predicates. Each singular variable  $x$  and each singular constant  $a$  gets assigned one object from the domain. The assignment of plural variables and constants are *objects* from the domain, whereby  $v$  may assign more than one as well as only one object to a plural term. The value of an  $n$ -place predicate is an  $n$ -place relation, whereby the value of  $=$  is the identity relation and the value of  $\prec$  is the proper inclusion relation. Satisfaction is defined standardly – a negation of a formula is satisfied by a valuation iff the valuation does not satisfy the unnegated formula, and a valuation satisfies an implication of a formula  $\Psi$  by a formula  $\Phi$  iff it does not satisfy  $\Phi$  or it satisfies  $\Psi$  – whereby it is worth noting that the valuation  $v$  satisfies  $F(\alpha_1, \dots, \alpha_n)$  iff the relation which  $v$  assigns to  $F$  holds of the *objects*  $v$  assigns to  $\alpha_1, \dots, \alpha_n$ .

The logical truth of a formula  $\Phi$  is defined as  $\Phi$  being satisfied by any valuation, and a formula  $\Phi$  following from some formulas  $\Psi_1, \dots, \Psi_n$  is defined as  $\Phi$  is satisfied, if  $\Psi_1, \dots, \Psi_n$ , for any valuation. Below follows a

81. Plural logics which allow for the use of empty terms are not in need of such a postulate and can allow for an empty domain-of-discourse, (see Oliver and Smiley 2013: 194-5 and Simons 2016: 65).

more formal summary of the semantics, given with the means of plural logic itself, of our logical system *FOP*:

- (i) Domain: The domain are some objects
- (ii) Valuation  $v$ 
  - (a) For each singular variable  $x$ ,  $v(x)$  is an object from the domain.
  - (b) For each plural variable  $uu$ ,  $v(uu)$  is an object, or some objects from the domain.
  - (c) For each singular constant  $a$ ,  $v(a)$  is an object from the domain.
  - (d) For each plural constant  $dd$ ,  $v(dd)$  is an object, or some objects from the domain.
  - (e) For each  $n$ -place predicate  $F$ ,  $v(F)$  is an  $n$ -place relation on the objects from the domain; in particular,
    - (I)  $v(=)$  is the identity relation.
    - (II)  $v(\prec)$  is the inclusion relation.
- (iii) Satisfaction:
  - (a)  $v$  satisfies  $F(\alpha_1, \dots, \alpha_n)$  iff  $v(F)$  holds of  $\alpha_1, \dots, \alpha_n$ .
  - (b)  $v$  satisfies  $\neg\Phi$  iff  $v$  does not satisfy  $\Phi$ .
  - (c)  $v$  satisfies  $\Phi \rightarrow \Psi$  iff  $v$  does not satisfies  $\Phi$ , or  $v$  satisfies  $\Psi$ .
  - (d)  $v$  satisfies  $\forall xF(x)$  iff  $v$  satisfies  $F[a/x]$  for every  $a$ , whereby  $F[a/x]$  is the result of substituting  $a$  for any occurrence of  $x$  in  $F(x)$ .
  - (e)  $v$  satisfies  $\forall uuF(uu)$  iff  $v$  satisfies  $F[dd/uu]$  for every  $dd$ , whereby  $F[dd/uu]$  is the result of substituting  $dd$  for any occurrence of  $uu$  in  $F(uu)$ .
- (iv) Logical truth and consequence:
  - (a)  $\Vdash \Phi$  iff all valuations  $v$  satisfy  $\Phi$
  - (b)  $\Psi_1, \dots, \Psi_n \Vdash \Phi$  iff for all valuations  $v$ , if  $v$  satisfies  $\Psi_1, \dots, \Psi_n$ , then  $v$  satisfies  $\Phi$ .



## 5.5 Two Identity-Problems in FOP

The system *FOP*, based on the views of Linnebo (2014), McKay (2006), Oliver and Smiley (2013), and Yi (1999b), allows us to show that the traditional views on plural logic run into trouble. In particular, the principles of *FOP* are not compatible with the traditional views on identity. In order to show that, I will present two arguments which give us convincing evidence that substitution is causing problems. The plural logician has to make a decision: Either substitution is abandoned altogether, or the idea that identity is unitary is dismissed, whereby a restricted version of substitution can be saved. I will take the second route and argue that this is the most plausible option we can choose in the next chapters.

Here is a sketch of the argument that follows: First, I will deduce two contradictions within *FOP* which follow from plausible empirical assumptions. Then I will discuss a possible attempt to avoid the derivation of these contradiction within *FOP*. This solution to the challenge I pose for *FOP* is based on two tightly connected ideas which are problematic: an infinite hierarchy of terms and the concept of superplural terms. Roughly speaking, the idea of this “conservative strategy” is to mimic the strategy presented by Tarski (1935; 1944), i.e. to avoid the derivation of contradictions by stipulating a hierarchy of terms. However, as we will see, this strategy cannot be applied within plural logic, since the concept of “superplural terms” cannot be defended in a convincing way and the move to a hierarchy of terms does not solve all the issues which arise for *FOP*.

### 5.5.1 The Problem of Mixed Pluralities

The first problem for *FOP* are what I call “mixed identities”. A mixed identity is a sentence which is adequately formalized by a formula where the identity predicate is flanked by a singular term on one side, and by a plural term on the other. Thus, the following sentences are mixed identities:

- (13) Russell is identical to the authors of *OD*
- (14) Anna is identical to the wives of Frege
- (15) The Queen of Spades is identical to the cards I memorized

These sentences surely sound odd, but that is no argument against their truth.<sup>82</sup> Nonetheless, one might think that we can reject these with the help of considerations about possible worlds. Take for instance (13). An attempt to argue against the truth of this sentence might be to claim that although there *could* be more than one author of *OD*, this only shows that at possible worlds where Russell co-authored this essay, Russell *is among* the authors of *OD*, while he is *identical* to the *author* at the *actual* world. Yet, this way of arguing against (13) misses the point. From the fact that Russell wrote, i.e. is an author of, *OD* at the actual world, the existence of the authors of *OD* at the actual world is entailed by the relevant instance of the comprehension axiom, which states that if there is an author of *OD*, then there are some authors of *OD*. Moreover, this instance of the comprehension axiom does not tell us whether there are *other* possible worlds where the authors of *OD* exist, though there surely are, since it is concerned with the the *actual* world only. Finally, although the plural term ‘the authors of *OD*’ might occasionally be used to *suggest* – for instance in everyday conversations – that there is more than one author of *OD*, it does not *mean* that there is more than one author of *OD*, since our account of plural terms allows for them to refer to exactly one object, see again section 4.4.

In contrast to the mixed identities above, let me point out that

- (16) Russell is identical to the men identical to Russell

is *not* a mixed identity, but an “ordinary” identity claim, because ‘=’ is flanked by two singular terms, see section 4.4.

82. See also, the remarks at the end of section 5.3.3, where we have already addressed the objection that mixed identities are artifacts, which result from using the proper inclusion predicate in the axioms of plural logic.

Mixed identities lead to contradictions within *FOP* by taking (13) and (17) Russell is an author of *OD*

as empirical truths. (17) follows trivially from Russell being *the* author of *OD*. Hence, we can construct the following argument:

1. Russell is identical to the authors of *OD*
2. Russell is an author of *OD*
3. Russell is among the authors of *OD*
- 4.a Russell is among Russell
- 4.b The authors of *OD* are among the authors of *OD*
- 5.a Russell is not among Russell
- 5.b The authors of *OD* are not among the authors of *OD*

1. and 2. are given assumptions. Since there is an author of *OD*, 3. follows from 2. and (T14). We derive the problematic lines 4.a and 4.b from 1. and 3. with substitution. Each one of them contradicts an instance of the irreflexivity of ' $\prec$ ' which we have in the lines 5.a and 5.b.

What possibilities do we have to avoid the derivation of the above contradictions and to save *FOP*? There are not a lot of options here: The assumptions in the lines 1. and 2. are empirical truths. The theorems involved in the derivation, (T1) and (T14), follow from the principles of *FOP*. *Modus ponens*, which is also used in the above derivation, has to remain untouched. This leaves us with only two more options: Either we reject substitution and avoid thereby the derivation of the lines 4.a and 4.b, or we reject line 1., i.e. deny (13). My suggestion is to opt for the first option. However, what might speak for the second?

Supporters of *FOP* will try to hold on to (SI) by rejecting the first line of the above proof. This defense might be justified by the thought that mixed identities are not well-formed expressions, i.e. that the identity

predicate cannot be flanked by a singular and a plural term. However, this goes against one of the fundamental ideas of plural logic which we encountered at the very beginning of this chapter: Any predicate, which can take singular terms as arguments in a certain argument place, can take plural terms as arguments in that argument place as well.

However, there is another, more general line of response which may be entertained in order to avoid the derivation of the above contradictions while leaving (SI) untouched in *FOP*. This strategy, I will call it “the conservative strategy”, aims to avoid contradictions which do not only follow from mixed identities, but from plural identities in *FOP* as well. We shall turn back to it after considering the problem of plural identities for *FOP*.

### 5.5.2 The Problem of Plural Identities

A plural identity is a sentence which is adequately formalized by a formula where the identity sign connects two plural terms. Contrary to mixed identities, not all plural identities are problematic for *FOP*. For instance

(18) Russell and Whitehead are identical to Russell and Whitehead

(19) The books on my shelf are identical to my logic books

do not raise any challenges. The difficulties for *FOP* arise from claims like the following:

(20) The 26 cards are identical to the two suits of cards

(21) The five companies are identical to the three battalions

(22) The 12 cans of orange juice are identical to the two six-packs of orange juice

These three sentences are, given the right context, true.<sup>83</sup> However, with the principles of *FOP*, they lead to a violation of the irreflexivity of ' $\prec$ '. Consider (20) uttered in the following situation: (a) There are exactly 26 cards on the table in front of me. (b) There are exactly two suits of cards on the table in front of me. (c) The term 'the 26 cards' refers to the 26 cards which are on the table. (d) The term 'the two suits of cards' refers to the two suits of cards which are on the table in front of me.

In this context we can derive a contradiction from (2) within *FOP*, if we make the (plausible) assumption that no card is a suit:<sup>84</sup>

- (23) If something is a card, then it is not a suit of cards
1. The 26 cards are identical to the two suits of cards
  2. If something is a card, then it is not a suit of cards
  3. Something is among the 26 cards iff it is a card
  4. Something is among the two suits of cards iff it is a suit of cards
  5. Something is among the two suits of cards iff it is a card
  6. Something is a card iff it is a suit of cards
  7. There is a card *a*
  8. *a* is a card and *a* is not a card

83. One might be tempted to reject, say, (20) by claiming that cards are different kinds of objects than suits of cards. However, it is hard for me to make sense of such a line of thought. Surely, cards and suits are different kinds of objects. But, it strikes me that there is no relevant *ontological* difference, in order to claim that they belong to different *ontological categories*, as it is sometimes claimed in set theory about urelements and sets. Thus, there is no reason to think of suits of cards as in some sense "second-order objects" in contrast to cards. Moreover, claiming that cards and suits of cards, or companies and battalions, belong to different kinds of objects, goes against considerations of parsimony.

84. It appears even to be an analytical truth that no a card is a suit of cards. Imagine you teach someone a game of cards and you tell her *It's your turn. Throw a card!* But then the rookie throws a suit of cards. What do you think? Either she wants to pull my leg, or she does not understand the meaning of 'being a card'. Hence, assuming the above claim seems uncontentious to me.

The first two lines are our assumptions (22) and (23). The lines 3. and 4. follow from (D10). The problematic line 5. is derivable from 1. and 4. with (SI), which is then used to get line 6. from 4. and the transitivity of the biconditional. 7. is an instance of the axiom of non-emptiness and is needed in order to conclude the contradiction in the final line.

We have again two ways to avoid the above conclusion: either we deny line 1. or we limit the use of substitution in order to avoid the derivation of the sixth line. I will suggest the latter, but what speaks for the first option? In the case of plural identities, the friend of substitution cannot claim that the identity in 1. is not a well-formed expression, since the predicate is flanked by two plural terms. Therefore, a similar line of response as in the previous subsection is not possible. Yet, there appears to be another line of response available. I call it the “conservative strategy”, and we will discuss it next.

## 5.6 The Conservative Strategy

The conservative strategy may look familiar. We can find a similar strategy applied in an attempt to solve the Liar Paradox, famously elaborated in (Tarski 1935; 1944). The idea behind the conservative strategy coincides with the motivation suggested there: We can outrun the problems. Leaving aside the doubts about the strategy of invoking a hierarchy of languages to solve the Liar Paradox, the shortcomings of the conservative strategy become apparent. Its major problem is the concept of superplural terms, which is a worrisome concept for me, and even postulating superplural terms is not sufficient in order to avoid a restriction of substitution or to explain why we should restrict this inference rule. Further, the conservative strategy leads us to embrace certain implausibilities. But let’s first have a look at what the idea of this strategy is.

The strategy relies on two steps: First, the above derivations are taken to be evidence for the existence of another kind of term, beside singular and plural terms: superplural terms. These are also called “superplurals” (Linnebo and Nicolas 2008, Rayo 2006), “higher-level plurals” (Ben-Yami

2013, Oliver and Smiley 2013), “plurally plural” (Hossack 2000, McKay 2006: 46), “pluplurals” (Rosen and Dorr 2002), or “perplurals” (Hazen 1997). The just listed authors motivate the introduction of this concept due to similar problems as the one we have encountered above. The basic idea is that terms come in a hierarchy: At the bottom of the hierarchy, we have singular terms. One level up, there are the plural terms. On the third level, we have the superplural terms. As I argue below, this leads to an infinite hierarchy, with superduperplural terms on the fourth level, and so on. Postulating the existence of this hierarchy is the first step.

The second step is then to stipulate that any sentence which is formalized by a formula where the identity sign connects a term from the  $n$ th level with a term from the level  $n + 1$  will always be false. Similarly, only sentences formalized by a formula where the inclusion predicate takes in its first argument place a term from the  $n$ th level and a term from the level  $n + 1$  in its second argument place *can* be true. All other sentences containing the inclusion predicate are false.

This strategy has the apparently nice side effect, that it allows to avoid deriving the contradiction from mixed *and* plural identities. It simply prohibits the first line of the proofs, i.e. the sentences

(13) Russell is identical to the authors of *OD*

(20) The 26 cards are identical to the two suits of cards

This makes it impossible to derive the problematic lines 5.a and 5.b, and 8., respectively, because the identity predicate connects two terms, ‘Russell’ and ‘the authors of *OD*’, and ‘the 26 cards’ and ‘the two suits’, from different levels on the hierarchy. So we can kill two birds with one stone: By invoking a hierarchy of terms, where the identity and the inclusion predicate operate along this hierarchy, the above two derivations can be avoided.

Alternatively, one could, as I have already briefly mentioned in section 5.5.2, assume a hierarchy of *kinds of objects*. However, beside the fact that it is difficult to make sense of such a claim, the ontological costs

which come with such a view speak against it. Hence, until there is no other way to clarify this view, or a way to argue that it comes without these ontologically worrisome consequences, I think we should reject this solution.

But things look worse for the conservative strategy than it appears. We shall take the opportunity to have a closer look at its basic concept of superplural terms by following the lines of thought of Linnebo and Nicolas (2008), and Oliver and Smiley (2005; 2013: 119-28, 273-9).

### 5.6.1 Motivating Superplural Terms

Before we investigate the reasons which are used to postulate superplural terms, here are some first attempts to explain the concept of superplural terms. They are thought to be ...

[...] related to plurals as plurals are related to singulars.  
(Hazen 1997: 247)

[...] terms that stand to ordinary plural terms the way ordinary plural terms stand to singular terms.  
(Linnebo and Nicolas 2008: 186)

[...] a product of iteration of the step from singular to plural.  
(Ben-Yami 2013: 84)

With these initial approximations to the concept at hand, let me flag some problems which come with superplural terms for *FOP* right from the start.

The axioms we presented above are not sufficient for *FOP*, if the system contains superplural terms. If superplural terms stand in the same relation to plural terms, as plural terms stand to singular terms, then we have to adopt a superplural comprehension, extensionality, and non-emptiness axiom accordingly. Second, a further primitive inclusion relation ' $\prec^2$ ', (see Oliver and Smiley 2013: 275-9 and Rayo 2006), which takes plural terms in its first and superplural terms in its second argument place has to be added to the vocabulary. Finally, we end up with an



infinite hierarchy of terms: If we assume that there are superplural terms, then there is no reason to deny that there is a next level on the hierarchy containing “*super-duper plural terms*” (Rayo 2006: 228), i.e. terms which stand to superplural terms as superplural terms stand to plural terms. But then again, why should there not be a fifth level, and a sixth? Why should there be a final level at all? This means we have not only to postulate infinitely many comprehension, extensionality, and non-emptiness axioms – for each level one from each – but also infinitely many primitive inclusion relations together with their appropriate asymmetry and transitivity axioms, due to the assumption of the infinite hierarchy of terms.

A problem for this strategy is its lack of parsimony: The use of infinitely many axioms and infinitely many primitive relations flies in the face of Occam’s Razor. However, other features of the resulting logic, e.g. theoretical fruitfulness or the gain of expressive power, might override considerations of parsimony. Yet, I will argue to the contrary that introducing superplural terms has several shortcomings and that abandoning the idea of a unitary identity relation is a better alternative.

The friend of superplural terms is not necessarily a friend of the conservative strategy. One can believe that there are superplural terms and reject the conservative strategy. Yet, from our point of view, i.e. looking for a way to resolve the above inconsistencies, there is no point in holding on to superplural terms, if the conservative strategy is rejected.<sup>85</sup>

### 5.6.1.1 Superplural Terms and Predicates

Linnebo and Nicolas ask

[...] whether the step from the singular to the plural can be iterated. Are there terms that stand to ordinary plural terms the way ordinary plural terms stand to singular terms?

(Linnebo and Nicolas 2008: 186)

85. Linnebo and Nicolas mentioned in email correspondence that their observation in (Linnebo and Nicolas 2008) is that substitution fails in some contexts where non-singular terms appear, for instance in the sentences (28a) and (28b) below.

Following Rayo (2006), they call these terms “superplural” and go on to introduce the concept of “superplural predicates”, namely as those predicates which “[...] can be predicated of superplural terms” (Linnebo and Nicolas 2008: 186). They present two principles to search for plural and superplural predicates:<sup>86</sup>

(P) Multigrade predicates are plural predicates  
(Linnebo and Nicolas 2008: 188)

(SP) [...] [S]pecial multigrade predicates that can take different numbers of *plural* arguments at the same argument place [...] are *superplural* predicates  
(Linnebo and Nicolas 2008: 193)

Both principles are based on the concept of multigrade predicates. Multigrade predicates are predicates which “[...] can take different numbers of arguments at the same argument place” (Linnebo and Nicolas 2008: 188). Multigrade predicates are for instance, ‘cooperating’ and ‘being French’:

(25a) Anne and Bob cooperate

(25b) Anne, Bob and Charlie cooperate

(26a) Chirac and Sarkozy are French

Taking the above two principles literally is misleading. Given the second principle, (P) is best understood as the idea that predicates which can take different numbers of *singular terms* as arguments at the same place are plural predicates. If we read (P) as *all* multigrade predicates are plural predicates, we cannot hold on to the distinction between plural and superplural predicates because the special multigrade predicates which are allegedly superplural predicates are after all multigrade, hence, plural predicates. Yet, the distinction between plural and superplural predicates

86. Linnebo and Nicolas introduce the first principle as principle (P), but provide us not with a label for the second principle. I labeled it (SP), short for ‘superplural’, since it will be useful to have a way to refer to it.

is crucial for the overall argument of Linnebo and Nicolas. Second, what Linnebo and Nicolas say about multigrade predicates suggests that being a multigrade predicate excludes being a plural or superplural predicate: Multigrade predicates can take different numbers of terms as arguments at the same place, while plural and superplural predicates cannot. For instance, if ‘cooperating’ is a plural predicate, then it does not take different *numbers* of singular terms as arguments at the same argument place in each of the above sentences, but different *terms* as arguments in its only argument place. Hence, I suggest interpreting (P) and (SP) as follows:<sup>87</sup>

(P') Predicates which appear to be able to take different numbers of singular terms as arguments at the same argument place are plural predicates.

(SP') Predicates which appear to be able to take different numbers of plural terms as arguments at the same argument place are superplural predicates.

(P') and (SP') give us strategies to look for plural and superplural *predicates*, but they do not suffice to detect plural and superplural *terms*. Although one might guess plural terms can simply be defined as those terms which are the arguments of plural predicates, and superplural terms as those terms which are the arguments of superplural predicates, these definitions are inappropriate: According to (P'), ‘being French’ is a plural predicate, since (26a) is analytically equivalent to

(26b) Chirac is French and Sarkozy is French

Since ‘Chirac’ is a prime example of a singular term – if *it* is not a singular term, I do not know what else it is – defining plural terms as those terms which enter the argument place of plural predicates, results in the claim that ‘Chirac’ is a plural term because it can be an argument of the predicate ‘being French’:

87. In email correspondence, Linnebo and Nicolas confirmed that these interpretations of their principles are legitimate.

(26c) Chirac is French

Linnebo and Nicolas (2008: 188) need a further distinction, the one between distributive and collective predicates.<sup>88</sup> Plural terms can then be defined as those terms which enter the argument place of *collective* plural predicates, and superplural terms are those terms which enter the argument place of *collective* superplural predicates.

In order to legitimize the introduction of superplural terms as a logically relevant category, Linnebo and Nicolas (2008: 193) present then two examples of collective superplural predicates:

(27a) These people, those people and these other people play against each other

(28a) The square things, the blue things and the wooden things overlap

where (27a) should be understood as involving “[...] a video game in which any finite number  $n$  of teams can play against each other in an  $n$ -way competition” (Linnebo and Nicolas 2008: 193).

What makes the above two predicates superplural? First, ‘playing against each other’ and ‘overlapping’ appear to be able to take different numbers of terms as arguments at the same argument place

(27b) These people, those people, these other people and those other people play against each other

(28b) The square things, the blue things, the wooden things and the heavy things overlap

and are collective predicates. Second, ‘these people’, ‘those people’, and ‘these other people’, as well as ‘the square things’, ‘the blue things’, and ‘the wooden things’ are plural terms, since they can enter the argument place of collective plural predicates:

88. In section 4.2, we saw that predicates are distributive or collective *in their argument places* and not *simpliciter*. Linnebo and Nicolas may, using our definition, define a predicate as ‘being collective’ iff it has at least one collective argument place.

(25e) These people (those people, those other people) cooperate

(29a) The square things (the blue things, the wooden things) form a circle

However, if the above line of reasoning is correct, we end up with a strange result: If ‘playing against each other’ and ‘overlapping’ are superplural predicates, and superplural terms are terms which enter the argument place of superplural predicates, then ‘these people and those people’ as well as ‘the square things and the blue things’ turn out to be superplural terms, because they can be arguments of these predicates as well. Linnebo and Nicolas seem to anticipate this worry, and try to make sure that it cannot be used to undermine their position:

Note that we do not claim that the English predicates ‘play against each other’ and ‘overlap’ are always superplural: clearly, they often function as ordinary plural predicates. Our claim is just that these predicates can also function superplurally. (Linnebo and Nicolas 2008: 193)

This restricted claim allows Linnebo and Nicolas to avoid the just mentioned critique against their argument, which appears to be *prima facie* the only serious worry they have to face. Before we move on to discuss Oliver and Smiley’s argument for superplural terms, let’s have a look at an point that follows from the above view.

It is interesting to note that one can easily develop a principle, analogous to (P’) and (SP’), to look for other higher-level plural predicates which take “*super-duper-plural* terms ” (Rayo 2006: 228) as their arguments:

(SDP) Predicates which appear to be able to take different numbers of *superplural* terms as arguments at the same argument place are super-duper-plural predicates.

As one might see, we can then go on to look for predicates which appear to be able of taking different numbers of *super-duper-plural* terms as arguments at the same argument place, and the terms which are the argument of these predicates are one level further up the hierarchy than

super-duper-plural terms are. And we can then go on to look for other predicates which appear to be able of taking different numbers of those terms as arguments at the same argument place, and so on. In a nutshell, once we introduce plural and superplural terms on the basis of (P') and (SP'), we are committed to the existence of an infinite hierarchy of terms. Yet, it should be noted that the principles Nicholas and Linnebo suggest are thought to be a means to look for certain kinds of predicates and terms. Whether the search for these is, or can be, successful is a different question. Yet, *prima facie*, it seems that we should always be able to create a certain kind of predicate or term, if we use the predicates and terms from a lower level of the hierarchy.

### 5.6.1.2 Plurally Exhaustive Descriptions

It is common practice to distinguish between proper names, 'Anne', and 'Russell', and definite descriptions, 'the neighbor of Bob', and 'the author of *OD*'. Once plural terms are introduced, it is only natural to assume that there are not only plural proper names, 'Russell and Whitehead' and 'Russell and Frege' but also plural definite descriptions, e.g. 'the authors of *PM*' and 'my children'.

Oliver and Smiley (2013: 119-28) distinguish between three kinds of plural descriptions: exhaustive, plurally unique, and plurally exhaustive: A description 'the *Fs*' is exhaustive iff it refers to "[...] the things, however many, that individually *F*." (Oliver and Smiley 2013: 121). 'The students who passed the exam' is an exhaustive description, since it refers to the people that individually passed the exam. In some contexts, an exhaustive reading of a plural description is not appropriate. For instance, 'the men who wrote *PM*' does not refer to the men who *individually*, but collectively wrote *PM*. Hence, it is a plurally unique description, whereby a plurally unique description 'the *Fs*' is understood as referring to those things which are the only things that collectively *F*. Finally, there are plurally exhaustive descriptions. Some collective predicates can be instantiated by more than one plural term. 'Being neighbors' is such a predicate.

It is satisfied not only by ‘Anne and Bob’, but by other plural terms as well. Anne and Bob are not the only neighbors there are. Hence, the description ‘the people who are neighbors’ refers not only to Anne and Bob but also to, say, Claire and Dan. Therefore, Oliver and Smiley suggest that a plurally exhaustive description, ‘the *F*s’, refers to all the objects which collectively are *F*.

Since plurally exhaustive descriptions are the reason which leads us into “[...] the murky waters of higher-level plural logic” (Oliver and Smiley 2005: 1062), let’s compare them with exhaustive and plurally unique descriptions: On the one hand, an exhaustive description refers to individuals which *individually* satisfy a predicate ‘*F*’, while plurally exhaustive descriptions refer to individuals which *collectively* satisfy ‘*F*’. On the other hand, a plurally unique descriptions refers to some objects which are the *only* objects that satisfy collectively a predicate ‘*F*’, while a plurally exhaustive description refers to *some* objects which collectively satisfy ‘*F*’.

Consider the following example to see why Oliver and Smiley think that plurally exhaustive descriptions are superplural terms: ‘The twin primes’ refers to some numbers which *collectively* satisfy the predicate ‘being a twin prime’. A twin prime is a prime number that is either two less or two more than a prime number. Hence ‘the twin primes’ refers to 3 and 5, 5 and 7, 11 and 13, 17 and 19, ..., but not to 7 and 9, or 9 and 11. If we were to analyze the term ‘the twin primes’ as a plural term, two difficulties would arise. The first problem is related to the transitivity of the relation ‘being properly among’:

1. 3 is among 3 and 5
2. 3 and 5 are among the twin primes
3. Therefore, 3 is among the twin primes.

The above inference is problematic because the conclusion is false: ‘Being a twin prime’ is a predicate that can only be satisfied by two numbers *collectively*, not an individual number alone. Hence, the transitivity of

‘being properly among’ has to be restricted in a way to invalidate the above inference.

Secondly, we can reach false conclusions when the order of expressions within non-singular terms is changed:<sup>89</sup>

1. 3 and 5, and 11 and 13 are among the twin primes
2. 3 and 5, and 11 and 13 are identical to 3 and 13, and 5 and 11
3. 3 and 13, and 5 and 11 are among the twin primes.

3 and 13 are not among the twin primes because they do not satisfy the predicate ‘being a twin prime’. The conclusion can be avoided if the term ‘the twin primes’ is taken to be a superplural term. Moreover, it gives Oliver and Smiley a way to restrict the transitivity of ‘being properly among’: It can not be applied when singular and superplural terms, or more generally terms, which belong to levels whose difference is bigger than one, are involved. This avoids the inference which leads to the conclusion that 3 is among the twin primes, because ‘3’ is a singular term, while ‘the twin primes’ is a superplural term.

Finally, does this account of superplural terms imply an infinite hierarchy of terms? Oliver and Smiley (2013: 276) give us only a hint: “Once one goes beyond the first level, there is no natural stopping place at any higher finite level”. Given their introduction of superplural terms, this is only consistent. If their introduction of superplural terms is successful, it is easy to get to superduperplural terms via superplurally exhaustive descriptions, whereby a superplurally exhaustive description is an exhaustive description that takes superplural terms as arguments. However, to find an example for a superplurally exhaustive description is a different task, and it seems that our intuitions are reaching their limits here.

With these accounts of superplural terms at hand, we will now see what speaks against them. The problems that arise with superplural

89. Florio and Nicolas (2015) discuss issues of the sensitivity to order for plural logic in more detail and present a different attempt to resolve these.



terms are worrisome for the friends of the conservative strategy, since their position stands and falls with them. Without superplural terms, there is no conservative solution to the problems with mixed and plural identities.

### 5.6.2 Against Superplural Terms

Many people have expressed worry over the idea of superplural terms. The main problems I will consider here are similar to observations made by Ben-Yami (2013) and McKay (2006: 46-53). One reason to reject superplural terms is that the idea of an iteration from singular to plural terms is incoherent. Start with our distinction of singular and plural terms from section 4.4:<sup>90</sup>

A singular term cannot denote more than one thing on any occasion, a plural term may denote several.

(Oliver and Smiley 2013: 74-5)

From this definition, it follows that there cannot be any superplural terms: If singular terms can refer to at most one object, and plural terms can refer to more than one object, then there is nothing left for superplural terms to refer to. This is due to the fact that the distinction is “[...] exclusive and exhaustive: plural is the opposite of singular” (Oliver and Smiley 2013: 75). The distinction does not allow for another category of non-singular terms besides plural terms. Superplural terms are just plural terms.

Distinguishing singular and plural terms with respect to the number of objects they *actually* refer to, does not help here. If we were to define singular terms as those terms that refer to one object, and plural terms as those terms that refer to more than one object, then we cannot allow for superplural terms either:

90. The distinction above, which is Oliver and Smiley’s allows for empty plural terms, like ‘the kings of Australia’. Together with their account of superplural terms, it seems that this implies that there are empty superplural terms. I wonder whether this is problematic and what an example for an empty superplural term might be.

[A] superplural expression should refer to *more than more than a single individual*. But what could that mean? [...] ‘more than more than a single individual’ is either meaningless or at best synonymous with *more than two individuals* [...]. Yet if the latter is the case, then our superplural expression is still a plural expression, referring to a plurality of at least three individuals. (Ben-Yami 2013: 82-3)

If we distinguish between singular and plural terms as suggested, this criticism is devastating, here. Neither one of the two ways of defining singular and plural terms escapes the above line of attack and allows introducing superplural terms. Yet, a third possibility seems available.

If my analysis of Linnebo and Nicolas’ view is correct, they can counter these criticisms by embracing neither one of the two definitions above, but the following: Plural terms are those terms which can enter the argument place of collective plural predicates. This understanding of their position is suggested by what Ben-Yami (2013: 82, fn.2) points out in connection with the above quoted passage: “In conversation, Linnebo said that he wouldn’t like to gloss ‘iteration’ this way [...]”. Hence, Linnebo and Nicolas should prefer the just suggested definition of plural terms instead.

But further problems arise with their claim that predicates can function plurally as well as superplurally. It is difficult to see how the ability of predicates to function plurally in some and superplurally in other contexts works. Even if it could be explained, it is problematic, since the introduction of plural terms relies on the assumption that we can decide for any predicate whether it *is* a plural predicate or not. Yet, if we find out that one and the same predicate can *function* plurally in some context while superplurally in others, the definition of plural terms becomes sensitive to context as well. But, a context sensitive definition of terms is highly implausible: A term is singular, plural or, superplural *independently* of the context within which appears. It either *is* singular, plural, or superplural. If we define plural and superplural terms as those terms which are arguments of plural and superplural predicates, respectively,

then we cannot claim that a predicate can play the role of a plural and a superplural predicate, depending on the context within which it appears: Why does this flexibility not apply to terms as well? Why can terms not function as plural terms in one context and as superplural terms in another?

In absence of a different way to demarcate singular from plural terms, we have to conclude that an adequate definition of superplural terms cannot be provided. Since the conservative strategy essentially relies on the concept of superplural terms, the strategy is inadequate. Turning back to the problems from *FOP*, this means that we have no good evidence to reject the first lines of the arguments from sections 5.5.1 and 5.5.2, and a restriction of substitution seems to be the only alternative left to avoid the contradictions. Hence, the standard views on plural logic, incorporated in *FOP*, lead us to a restriction of substitution, since relying on the conservative strategy ends up in difficulties. We shall now move and assume for the sake of the argument, that the concept of superplural terms can be explicated in an adequate way in order to see that the conservative strategy runs into further problems.

### 5.6.3 Limited Applicability

A second problem for the conservative strategy is that even *if* there were superplural terms, the strategy is not always applicable. There are still plural identities causing trouble. Here is an example: Six children Anne, Ben, Carla, Dino, Erika and Fritz are playing a game. They play in two teams. The boys are in one team and the girls in the other. Furthermore, Anne and Ben are English, Carla and Dino are Italian, and Erika and Fritz are German. Given this scenario, which is a modification of the example used by Ben-Yami (2013), the boys and the girls are identical to the English, the Italian, and the German children.<sup>91</sup>

91. The two terms, 'the boys and the girls' and 'the English, the Italian and the German children', should be understood as abbreviations for more complex terms referring to the six people above, not to *all* the boys and girls, or *all* the English, Italian and Ger-

Here again, the conservative strategy gets into trouble due to substitution, since the unrestricted rule validates the derivation of a false conclusion:

1. The boys and the girls are identical to the English, the Italian and the German children
2. The boys and the girls play against each other
3. The English, the Italian and the German children play against each other

The move to superplural terms and the use of a hierarchy of terms is of no help here. If it were correct, one of the terms 'the boys and the girls' or 'the English, the Italian and the German children' would have to be on a higher level than the other. But any decision we can make is arbitrary. There simply is no evidence that allows us to make an objective decision either way. The conservative strategy seems to work only in cases where we can suspect something like a structure of reality, as it might be assumed in the case of cards and suits of cards, or companies and battalions, so that it may be able to avoid problems arising from inferences involving sentences, such as

(20) The 26 cards are identical to the two suits of cards

(21) The five companies are identical to the three battalions

by arguing that the terms 'the 26 cards' and 'the two suits of cards', belong to different levels of the hierarchy. Similarly, the terms 'the five companies' and 'the three battalions', are on different levels of the hierarchy because companies make up battalions. Yet, we do not have any way to decide which one of the terms used in line 1. of the above argument is

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man children there are. We may suppose the six children are the only children in a scout camp and add read both terms as containing the further qualification 'in the scout camp'.

on a higher level: What is more “basic”, gender or nationality? Do English, German and Italian children make up boys and girls, or do boys and girls make up English, German and Italian children? These do not sound like sensible questions to ask, let alone questions that are capable of being answered in a sensible way. Therefore, the conservative strategy *cannot* give us any good explanation why we should *not* accept the above identity claim and it ultimately ends up with a contradiction, if we keep (SI). Although this incompleteness of the conservative strategy may not mean that it fails, it forces arbitrary decisions upon us. With the desire to avoid this arbitrariness, we may want to see what other options we have.

#### 5.6.4 Accepting Implausible Identities

Finally, the conservative strategy leads to a denial of certain intuitively true identity claims. We have already seen that according to the conservative strategy, sentences as for instance the above (20) and (21) are false. Moreover, we have previously seen that we cannot accept

(13) Russell is identical to the authors of *OD*

as true, if we adhere to the conservative strategy. The problem for the conservative strategy is not only that these intuitively true sentences have to be rejected, but also that

(30) Russell and the author of *OD* are identical to the authors of *OD*

turns out to be true. How does this sentence follow from the conservative strategy? A few steps suffice to show that it is a consequence of the above strategy: First, we know that Russell is the only author of *OD*. Hence, anyone who is among the authors of *OD* is identical to Russell. Second, anyone who is among the objects identical to Russell or the author of *OD* is identical to Russell, because Russell is identical to the author of *OD*. Therefore, it holds for any object that it is among the authors of *OD* iff it is among Russell and the author of *OD*. By use of the extensionality

axiom, we can conclude that Russell and the author of *OD* are identical to the authors of *OD*:

1. Russell is the author of *OD*
2. Russell is among the authors of *OD*
3. Russell is among Russell and the author of *OD*
4. Russell and the author of *OD* are identical to the authors of *OD*

The above derivation can be avoided by invalidating the substitution inference with the means of superplural terms. ‘Russell’ and ‘the author of *On Denoting*’ are singular terms. The term ‘Russell and the author of *On Denoting*’ is a plural term. It is not a superplural term because it is the result of the step from singular to plural; no iteration of the step from singular to plural is present. Finally, the term ‘the authors of *On Denoting*’ is a plural term, due to the same reasons. But, given that (13) is supposed not to hold, it is more than puzzling to me that (30) should be true. This seems to be a further indication that the conservative strategy is in trouble and the evidence against it is piling up. Hence, I suggest that we should consider alternatives to the conservative strategy, i.e. accept the first lines of the above arguments and look for a solution which tackles the problems right at the bottom of the alleged hierarchy.

### 5.6.5 Looking for Alternative Solutions

To sum up, we see that the conservative strategy avoids the contradictions only by denying intuitively true sentences such as (13), or even more worryingly, (20) and (21). This makes it hard to defend it. However, intuitions are not always a warrant for correctness and the conservative plural logician might argue that these intuitions are false. Nevertheless, there are more serious challenges for the conservative strategy: The concept of superplural terms is not able to do the work it was intended to do because the distinction between singular and plural terms does not allow for a third category of terms beside them. The conservative strategy

cannot always be applied. Certain pairs of terms cannot be located in the alleged hierarchy since there is no objective way to answer the question which one of the terms stands on a higher level in the hierarchy. Finally, the need to accept counterintuitive identity claims and to deny intuitively true identity claims, together with the loss of parsimony, which has to be accepted when we follow this strategy, make this strategy a rather unattractive alternative. These difficulties for the conservative strategy suggest that it might be reasonable to consider other solutions to these problems, and that rejecting substitution is a viable option after all. This will strike as a radical step to take, but we will make sure that it is well reflected in the next chapter.

## 5.7 Appendix: Proofs of Theorems

The following proofs of the theorems of *FOP* will be presented in the metalanguage of *FOP* as outlined at the beginning of the chapter. We will justify an instantiation of the axioms and definitions which contain the schematic predicate symbol ' $\Phi$ ' with *IN*. The assumption of a tautology from propositional logic, which can be deduced from the axioms (A1) to (A3), will be justified by *LT*. Besides *modus ponens* we will make use of the derivable inference rule

If  $\Phi \vdash \Psi$ , then  $\vdash \Phi \rightarrow \Psi$

which will be justified with *CI* – for “conditional introduction” – whereby we may use *CI1*, *CI2*, ..., if more than one conditional introduction is made in a proof. *SC*, *SC1*, *SC2*, ... indicate that a supposition for a conditional introduction is made. Further, we will use the derivable inference rules “universal elimination” (UE), “existential elimination” (EE), and “existential generalization” (EG) with the usual restrictions. The use of other derivable inference rules from  $FOL^-$ , such as *modus tollens* or *quantifier negation*, will be justified with *PL*, standing for “primitive logic”.  $\surd$  is used to indicate that the variable which has been universally generalized or existentially eliminated is arbitrary. The proofs will make use of the equivalence forms of the definitions and the universally closed versions of the formulas from the object language.

The metalanguage of *FOP* has two additional inference rules,  $\alpha$ -IN and  $\alpha$ -EX, specified at the beginning of this chapter. I will abbreviate some proofs by leaving out steps, which simply follow a repetitive pattern and indicate this by pointing out which inference steps are made. The left out inferences can be reconstructed easily by following the previous steps *mutatis mutandis* – reconsider the sketch of the derivation for the symmetry of = in section 5.3.4, fn.78.



(T1)  $\forall\alpha\neg(\alpha \prec \alpha)$ 

1.  $\forall\alpha\forall\beta(\alpha \prec \beta \rightarrow \neg(\beta \prec \alpha))$  [A9]
2.  $\forall\beta(a \prec \beta \rightarrow \neg(\beta \prec a)) \wedge \forall\beta(dd \prec \beta \rightarrow \neg(\beta \prec dd))$   
[1.;  $\alpha$ -EX, PL, UE]
3.  $\neg(a \prec a)$  [2.; PL,  $\alpha$ -EX, UE]
4.  $\forall x\neg(x \prec x)$  [3.; UG $\checkmark$ ]
5.  $\neg(dd \prec dd)$  [2.; PL,  $\alpha$ -EX, UE]
6.  $\forall uu\neg(uu \prec uu)$  [5.; UG $\checkmark$ ]
7.  $\forall\alpha\neg(\alpha \prec \alpha)$  [4., 6.; PL,  $\alpha$ -IN]

(T2)  $\forall\alpha(\alpha = \alpha)$ 

1.  $\forall x(x = x)$  [A8]
2.  $\forall uu\forall vv(\forall x(x \prec uu \leftrightarrow x \prec vv) \rightarrow uu = vv)$  [A12]
3.  $a \prec dd \leftrightarrow a \prec dd$  [LT]
4.  $\forall x(x \prec dd \leftrightarrow x \prec dd)$  [3.; UG $\checkmark$ ]
5.  $\forall x(x \prec dd \leftrightarrow x \prec dd) \rightarrow dd = dd$  [2.; UE]
6.  $\forall uu(uu = uu)$  [4., 5.; MP, UG $\checkmark$ ]
7.  $\forall\alpha(\alpha = \alpha)$  [1., 6.; PL,  $\alpha$ -IN]

(T3)  $\forall\alpha\forall\beta(\alpha = \beta \rightarrow \beta = \alpha)$ 

1.  $\forall\alpha(\alpha = \alpha)$  [T2]
2.  $b = a$  [SC1]
3.  $b = dd$  [SC2]
4.  $ee = a$  [SC3]
5.  $ee = dd$  [SC4]
6.  $a = a$  [1.;  $\alpha$ -EX, PL, UE]
7.  $a = b$  [2., 6.; SI]
8.  $\forall x(b = x \rightarrow x = b)$  [2.-7.; CI1, UG $\checkmark$ ]
9.  $dd = dd$  [1.;  $\alpha$ -EX, PL, UE]

10.  $dd = b$  [3., 9.; SI]  
 11.  $\forall uu(b = uu \rightarrow uu = b)$  [3.-10.; CI2, UG $\sqrt$ ]  
 12.  $\forall y\forall\beta(y = \beta \rightarrow \beta = y)$  [8., 11.; PL,  $\alpha$ -IN, UG $\sqrt$ ]  
 13.  $a = ee$  [4., 6.; SI]  
 14.  $\forall y(ee = y \rightarrow y = ee)$  [4.-13.; CI3, UG  $\sqrt$ ]  
 15.  $dd = ee$  [5., 9.; CI3, UG  $\sqrt$ ]  
 16.  $\forall uu(ee = uu \rightarrow uu = ee)$  [9.-15.; CI4, UG $\sqrt$ ]  
 17.  $\forall vv\forall\beta(\beta = vv \rightarrow vv = \beta)$  [14., 16.; PL,  $\alpha$ -IN, UG $\sqrt$ ]  
 18.  $\forall\alpha\forall\beta(\alpha = \beta \rightarrow \beta = \alpha)$  [12., 17.; PL,  $\alpha$ -IN]

(T4)  $\forall\alpha\forall\beta\forall\gamma(\alpha = \beta \wedge \beta = \gamma \rightarrow \alpha = \gamma)$

1.  $a = b \wedge b = c$  [SC1]  
 2.  $a = b \wedge b = ff$  [SC2]  
 3.  $a = ee \wedge ee = c$  [SC3]  
 4.  $a = ee \wedge ee = ff$  [SC4]  
 5.  $dd = b \wedge b = c$  [SC5]  
 6.  $dd = b \wedge b = ff$  [SC6]  
 7.  $dd = ee \wedge ee = c$  [SC7]  
 8.  $dd = ee \wedge ee = ff$  [SC8]  
 9.  $a = c$  [1.; SI]  
 10.  $\forall z(a = b \wedge b = z \rightarrow a = z)$  [1.-9.; CI1, UG $\sqrt$ ]  
 11.  $a = ff$  [2.; SI]  
 12.  $\forall ww(a = b \wedge b = ww \rightarrow a = ww)$  [2.-11.; CI2, UG $\sqrt$ ]  
 13.  $\forall y\forall\gamma(a = y \wedge y = \gamma \rightarrow a = \gamma)$  [10.,12.; PL,  $\alpha$ -IN, UG $\sqrt$ ]  
 14.  $\forall z(a = ee \wedge ee = z \rightarrow a = z)$  [3.; SI, CI3, UG $\sqrt$ ]  
 15.  $\forall ww(a = ee \wedge ee = ww \rightarrow a = ww)$  [4.; SI, CI4, UG $\sqrt$ ]  
 16.  $\forall vv\forall\gamma(a = vv \wedge vv = \gamma \rightarrow a = \gamma)$  [14.,15.; PL,  $\alpha$ -IN]  
 17.  $\forall x\forall\beta\forall\gamma(x = \beta \wedge \beta = \gamma \rightarrow x = \gamma)$  [13.,16.; PL,  $\alpha$ -IN, UG $\sqrt$ ]  
 18.-23.: Mimic the inferences in 9.-16. by beginning with  
     5., 6., 7. and 8. to infer:  
 24.  $\forall uu\forall\beta\forall\gamma(dd = \beta \wedge \beta = \gamma \rightarrow dd = \gamma)$  [20., 23.; PL,  $\alpha$ -IN, UG $\sqrt$ ]  
 25.  $\forall\alpha\forall\beta\forall\gamma(\alpha = \beta \wedge \beta = \gamma \rightarrow \alpha = \gamma)$  [17., 24.; PL,  $\alpha$ -IN]

(T5)  $\forall\alpha(\alpha \preceq \alpha)$ 

1.  $\forall\alpha(\alpha = \alpha)$  [T2]
2.  $\forall\alpha\forall\beta(\alpha \preceq \beta \leftrightarrow \alpha \prec \beta \vee \alpha = \beta)$  [D8]
3.  $a = a$  [1.;  $\alpha$ -EX, PL, UE]
4.  $a \preceq a \leftrightarrow a \prec a \vee a = a$  [2.;  $\alpha$ -EX, PL, UE]
5.  $\forall x(x \preceq x)$  [3., 4.; PL, UG $\checkmark$ ]
6.  $dd = dd$  [1.;  $\alpha$ -EX, PL, UE]
7.  $dd \preceq dd \leftrightarrow dd \prec dd \vee dd = dd$  [2.;  $\alpha$ -EX, PL, UE]
8.  $\forall uu(uu \preceq uu)$  [6., 7.; PL, UG $\checkmark$ ]
9.  $\forall\alpha(\alpha = \alpha)$  [5., 8.; PL,  $\alpha$ -IN]

(T6)  $\forall\alpha\forall\beta(\alpha \preceq \beta \wedge \beta \preceq \alpha \rightarrow \alpha = \beta)$ 

1.  $\forall\alpha\forall\beta(\alpha \prec \beta \rightarrow \neg(\beta \prec \alpha))$  [A9]
2.  $\forall\alpha\forall\beta(\alpha = \beta \rightarrow \beta = \alpha)$  [T3]
3.  $\forall\alpha\forall\beta(\alpha \preceq \beta \leftrightarrow \alpha \prec \beta \vee \alpha = \beta)$  [D8]
4.  $a \preceq b \leftrightarrow a \prec b \vee a = b$  [3.;  $\alpha$ -EX, PL, UE]
5.  $b \preceq a \leftrightarrow b \prec a \vee b = a$  [3.;  $\alpha$ -EX, PL, UE]
6.  $a \prec b \rightarrow \neg(b \prec a)$  [1.;  $\alpha$ -EX, PL, UE]
7.  $b = a \rightarrow a = b$  [2.;  $\alpha$ -EX, PL, UE]
8.  $a \prec b \wedge b \prec a \rightarrow a = b$  [4.-7.; PL]
9.  $\forall y(a \prec y \wedge y \prec a \rightarrow a = y)$  [8.; UG $\checkmark$ ]

10.-14.: Mimic the inferences in 4.-8. by introducing

'ee' for ' $\beta$ ' to infer:

15.  $\forall vv(a \prec vv \wedge vv \prec a \rightarrow a = vv)$  [14.; UG $\checkmark$ ]
16.  $\forall x\forall\beta(x \prec \beta \wedge \beta \prec x \rightarrow x = \beta)$  [9., 15.; PL,  $\alpha$ -IN, UG $\checkmark$ ]

17.-28.: Mimic the inferences in 4.-15. by introducing

'dd' for ' $\alpha$ ' to infer:

29.  $\forall uu\forall\beta(uu \prec \beta \wedge \beta \prec uu \rightarrow uu = \beta)$  [21., 27.; PL,  $\alpha$ -IN, UG $\checkmark$ ]
30.  $\forall\alpha\forall\beta(\alpha \prec \beta \wedge \beta \prec \alpha \rightarrow \alpha = \beta)$  [17., 29.; PL,  $\alpha$ -IN]

(T7)  $\forall\alpha\forall\beta\forall\gamma(\alpha \preceq \beta \wedge \beta \preceq \gamma \rightarrow \alpha \preceq \gamma)$

1.  $\forall\alpha\forall\beta\forall\gamma(\alpha \prec \beta \wedge \beta \prec \gamma \rightarrow \alpha \prec \gamma)$  [A8]
2.  $\forall\alpha\forall\beta\forall\gamma(\alpha = \beta \wedge \beta = \gamma \rightarrow \alpha = \gamma)$  [T4]
3.  $\forall\alpha\forall\beta(\alpha \preceq \beta \leftrightarrow \alpha \prec \beta \vee \alpha = \beta)$  [D8]
4.  $a \prec b \wedge b = c$  [SC1]
5.  $a = b \wedge b \prec c$  [SC2]
6.  $a \prec b \wedge b = ff$  [SC3]
7.  $a = b \wedge b \prec ff$  [SC4]
8.  $a \prec ee \wedge ee = c$  [SC5]
9.  $a = ee \wedge ee \prec c$  [SC6]
10.  $a \prec ee \wedge ee = ff$  [SC7]
11.  $a = ee \wedge ee \prec ff$  [SC8]
12.  $dd \prec b \wedge b = c$  [SC9]
13.  $dd = b \wedge b \prec c$  [SC10]
14.  $dd \prec b \wedge b = ff$  [SC11]
15.  $dd = b \wedge b \prec ff$  [SC12]
16.  $dd \prec ee \wedge ee = c$  [SC13]
17.  $dd = ee \wedge ee \prec c$  [SC14]
18.  $dd \prec ee \wedge ee = ff$  [SC15]
19.  $dd = ee \wedge ee \prec ff$  [SC16]
20.  $a \prec b \wedge b = c \rightarrow a \prec c$  [SI, CI1]
21.  $a = b \wedge b \prec c \rightarrow a \prec c$  [SI, CI2]
22.  $a \prec b \wedge b \prec c \rightarrow a \prec c$  [1.;  $\alpha$ -EX, PL, UE]
23.  $a = b \wedge b = c \rightarrow a = c$  [2.;  $\alpha$ -EX, PL, UE]
24.  $a \preceq b \wedge b \preceq c \rightarrow a \preceq c$  [3., 20., 21., 22., 23.;  $\alpha$ -EX, PL, UE]
25.  $\forall z(a \preceq b \wedge b \preceq z \rightarrow a \preceq z)$  [24.; UG $\sqrt{}$ ]
- 26.-30.: Mimic the inferences in 20.-24. by beginning with  
6. and 7. to infer:
31.  $\forall ww(a \preceq b \wedge b \preceq ww \rightarrow a \preceq ww)$  [30.; UG $\sqrt{}$ ]
32.  $\forall y\forall\gamma(a \preceq y \wedge y \preceq \gamma \rightarrow a \preceq \gamma)$  [25., 31.; PL,  $\alpha$ -IN, UG $\sqrt{}$ ]

33.-44.: Mimic the inferences in 20.-32. by beginning with  
8., 9., 10. and 11. to infer:

$$45. \forall vv \forall \gamma (a \preceq vv \wedge vv \preceq \gamma \rightarrow a \preceq \gamma) \quad [44.; \text{UG}\checkmark]$$

$$46. \forall x \forall \beta \forall \gamma (a \preceq \beta \wedge \beta \preceq \gamma \rightarrow a \preceq \gamma) \quad [32., 45.; \text{PL}, \alpha\text{-IN}, \text{UG}\checkmark]$$

47.-73.: Mimic the inferences in 20.-46. by beginning with  
12., 13., 14., 15., 16., 17., 18. and 19. to infer:

$$74. \forall uu \forall \beta \forall \gamma (uu \preceq \beta \wedge \beta \preceq \gamma \rightarrow uu \preceq \gamma) \quad [45.; \text{PL}, \alpha\text{-IN}, \text{UG}\checkmark]$$

$$75. \forall \alpha \forall \beta \forall \gamma (\alpha \preceq \beta \wedge \beta \preceq \gamma \rightarrow \alpha \preceq \gamma) \quad [46., 74.; \text{PL}, \alpha\text{-IN}]$$

(T8)  $\exists uu \forall x (x \prec uu)$

$$1. \forall x (x = x) \quad [A8]$$

$$2. \exists x \Phi(x) \rightarrow \exists uu \forall x (x \prec uu \leftrightarrow \Phi(x)) \quad [A11]$$

$$3. \exists x (x = x) \quad [1.; \text{UE}, \text{EG}]$$

$$4. \exists x (x = x) \rightarrow \exists uu \forall x (x \prec uu \leftrightarrow x = x) \quad [2.; \text{IN}]$$

$$5. a \prec dd \leftrightarrow a = a \quad [3., 4.; \text{MP}, \text{EE}\checkmark, \text{UE}]$$

$$6. a = a \quad [1.; \text{UE}]$$

$$7. \forall x (x \prec dd) \quad [5., 6.; \text{PL}, \text{UG}\checkmark]$$

$$8. \exists uu \forall x (x \prec uu) \quad [7.; \text{EG}]$$

(T9)  $\forall uu (\forall x (x \prec uu) \rightarrow \forall vv (\forall x (x \prec vv) \rightarrow vv = uu))$

$$1. \forall uu \forall vv (\forall x (x \prec uu \leftrightarrow x \prec vv) \rightarrow uu = vv) \quad [A12]$$

$$2. \forall x (x \prec ee) \quad [\text{SC1}]$$

$$3. \forall x (x \prec dd) \quad [\text{SC2}]$$

$$4. \forall x (x \prec ee \leftrightarrow x \prec dd) \quad [2., 3.; 2 \times \text{UE}, \text{PL}, \text{UG}\checkmark]$$

$$5. \forall x (x \prec ee \leftrightarrow x \prec dd) \rightarrow ee = dd \quad [1.; 2 \times (\text{UE})]$$

$$6. ee = dd \quad [4., 5.; \text{MP}]$$

$$7. \forall vv (\forall x (x \prec vv) \rightarrow vv = dd) \quad [2.-6.; \text{CI1}, \text{UG}\checkmark]$$

$$8. \forall uu (\forall \rho (x \prec uu) \rightarrow \forall vv (\forall x (x \prec vv) \rightarrow vv = uu)) \quad [3.-7.; \text{CI2}, \text{UG}\checkmark]$$

(T10)  $\exists vv(vv = nuu(\forall x(x \prec uu)))$

1.  $\forall \alpha(\alpha = \alpha)$  [T2]
2.  $\exists uu\forall x(x \prec uu)$  [T8]
3.  $\forall uu(\forall x(x \prec uu) \rightarrow \forall vv(\forall x(x \prec vv) \rightarrow vv = uu))$  [T9]
4.  $\Psi(\ulcorner \alpha \Phi(\alpha) \urcorner) \leftrightarrow \exists \alpha(\Phi(\alpha) \wedge \forall \beta(\Phi(\beta) \rightarrow \beta = \alpha) \wedge \Psi(\alpha))$  [D7]
5.  $\forall x(x \prec dd)$  [2.; EE $\sqrt{}$ ]
6.  $\forall x(x \prec dd) \rightarrow \forall vv(\forall x(x \prec vv) \rightarrow vv = dd)$  [3.; UE]
7.  $\forall vv(\forall x(x \prec vv) \rightarrow vv = dd)$  [5., 6.; MP]
8.  $dd = dd$  [1.;  $\alpha$ -EX, PL, UE]
9.  $\forall x(x \prec dd) \wedge \forall vv(\forall x(x \prec vv) \rightarrow vv = dd) \wedge$   
 $\wedge dd = dd$  [5., 7., 8.; PL]
10.  $\exists uu(\forall x(x \prec uu) \wedge \forall vv(\forall x(x \prec vv) \rightarrow vv = uu) \wedge$   
 $\wedge dd = uu)$  [9.; EG]
11.  $dd = nuu(\forall x(x \prec uu)) \leftrightarrow \exists uu(\forall x(x \prec uu) \wedge$   
 $\wedge \forall vv(\forall x(x \prec vv) \rightarrow vv = uu) \wedge dd = uu)$  [4.; IN,  $\alpha$ -EX, PL, UE]
12.  $dd = nuu(\forall x(x \prec uu))$  [10., 11.; MP]
13.  $\exists vv(vv = nuu(\forall x(x \prec uu)))$  [12.; EG]

(T11)  $\exists uu(uu = \omega\omega)$

1.  $\forall \alpha\forall \beta(\alpha = \beta \rightarrow \beta = \alpha)$  [T3]
2.  $\forall \alpha\forall \beta\forall \gamma(\alpha = \beta \wedge \beta = \gamma \rightarrow \alpha = \gamma)$  [T4]
3.  $\exists vv(vv = nuu(\forall x(x \prec uu)))$  [T10]
4.  $\omega\omega = nuu\forall x(x \prec uu)$  [D9]
5.  $dd = nuu(\forall x(x \prec uu))$  [3.; EE $\sqrt{}$ ]
6.  $dd = nuu(\forall x(x \prec uu)) \rightarrow nuu(\forall x(x \prec uu)) = dd$  [1.;  $\alpha$ -EX, PL, UE]
7.  $\omega\omega = nuu\forall x(x \prec uu) \wedge nuu(\forall x(x \prec uu)) = dd$  [4., 5., 6.; PL]
8.  $\omega\omega = nuu\forall x(x \prec uu) \wedge nuu(\forall x(x \prec uu)) = dd \rightarrow$   
 $\omega\omega = dd$  [2.;  $\alpha$ -EX, PL]
9.  $\omega\omega = dd$  [7., 8.; MP]
10.  $\omega\omega = dd \rightarrow dd = \omega\omega$  [1.;  $\alpha$ -EX, PL, UE]
11.  $\exists uu(uu = \omega\omega)$  [9., 10.; MP, EG]

- (T12)  $\forall uu(\forall x(x \prec uu \leftrightarrow \Phi(x)) \rightarrow$   
 $\rightarrow \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = uu))$
1.  $\forall uu\forall vv(\forall x(x \prec uu \leftrightarrow x \prec vv) \rightarrow uu = vv)$  [A12]
  2.  $\forall x(x \prec ee \leftrightarrow \Phi(x))$  [SC1]
  3.  $\forall x(x \prec dd \leftrightarrow \Phi(x))$  [SC2]
  4.  $\forall x(x \prec ee \leftrightarrow x \prec dd)$  [2., 3.; UE, PL, UG $\sqrt$ ]
  5.  $\forall x(x \prec ee \leftrightarrow x \prec dd) \rightarrow ee = dd$  [1.; UE]
  6.  $ee = dd$  [4., 5.; MP]
  7.  $\forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = dd)$  [3.-6.; CI1, UG $\sqrt$ ]
  8.  $\forall uu(\forall x(x \prec uu \leftrightarrow \Phi(x)) \rightarrow$   
 $\rightarrow \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = uu))$  [2.-7.; CI2, UG $\sqrt$ ]

- (T13)  $\exists x\Phi(x) \rightarrow \exists uu(uu = vv(\forall x(x \prec vv \leftrightarrow \Phi(x))))$

1.  $\exists x\Phi(x) \rightarrow \exists uu\forall x(x \prec uu \leftrightarrow \Phi(x))$  [A11]
2.  $\forall \alpha(\alpha = \alpha)$  [T2]
3.  $\forall uu(\forall x(x \prec uu \leftrightarrow \Phi(x)) \rightarrow$   
 $\rightarrow \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = uu))$  [T12]
4.  $\Psi(\iota\alpha\Phi(\alpha)) \leftrightarrow \exists\alpha(\Phi(\alpha) \wedge \forall\beta(\Phi(\beta) \rightarrow \beta = \alpha) \wedge \Psi(\alpha))$  [D7]
5.  $\exists x\Phi(x)$  [SC]
6.  $\forall x(x \prec dd \leftrightarrow \Phi(x))$  [1., 5.; MP, EE $\sqrt$ ]
7.  $\forall x(x \prec dd \leftrightarrow \Phi(x)) \rightarrow \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = dd)$   
[3.; UE]
8.  $\forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = dd)$  [6., 7.; MP]
9.  $dd = dd$  [2.;  $\alpha$ -EX, PL, UE]
10.  $\exists uu(\forall x(x \prec uu \leftrightarrow \Phi(uu)) \wedge \forall vv(\forall x(x \prec vv \leftrightarrow$   
 $\leftrightarrow \Phi(x)) \rightarrow vv = uu) \wedge dd = uu)$  [6., 8., 9.; PL, EG]
11.  $dd = uu(\forall x(x \prec uu \leftrightarrow \Phi(x))) \leftrightarrow \exists uu(\forall x(x \prec uu \leftrightarrow \Phi(x)) \wedge$   
 $\wedge \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow vv = uu) \wedge dd = uu)$   
[4.; IN,  $\alpha$ -EX, PL, UE]
12.  $\exists vv(vv = uu(\forall x(x \prec uu \leftrightarrow \Phi(x))))$  [10., 11.; MP, EG]
13.  $\exists x\Phi(x) \rightarrow \exists vv(vv = uu(\forall x(x \prec uu \leftrightarrow \Phi(x))))$  [5.-12.; CI]

(T14)  $\forall y(\exists x\Phi(x) \rightarrow (\Phi(y) \leftrightarrow y \prec \llbracket \Phi(x) \rrbracket))$

1.  $\forall\alpha\forall\beta(\alpha = \beta \rightarrow \beta = \alpha)$  [T3]
2.  $\Psi(\iota\alpha\Phi(\alpha)) \leftrightarrow \exists\alpha(\Phi(\alpha) \wedge \forall\beta(\Phi(\beta) \rightarrow \beta = \alpha) \wedge \Psi(\alpha))$  [D7]
3.  $\exists x\Phi(x) \rightarrow \llbracket \Phi(x) \rrbracket = nuu(\forall x(x \prec uu \leftrightarrow \Phi(x)))$  [D10]
4.  $\exists x\Phi(x)$  [SC]
5.  $\llbracket \Phi(x) \rrbracket = nuu(\forall x(x \prec uu \leftrightarrow \Phi(x)))$  [3., 4.; MP]
6.  $\exists uu(\forall x(x \prec uu \leftrightarrow \Phi(x)) \wedge \forall vv(\forall x(x \prec vv \leftrightarrow \Phi(x)) \rightarrow$   
 $\rightarrow vv = uu) \wedge \llbracket \Phi(x) \rrbracket = uu)$  [2., 5.; IN, PL]
7.  $\forall x(x \prec dd \leftrightarrow \Phi(x)) \wedge \llbracket \Phi(x) \rrbracket = dd$  [6.; EE $\sqrt{\phantom{x}}$ ], PL]
8.  $\llbracket \Phi(x) \rrbracket = dd \rightarrow dd = \llbracket \Phi(x) \rrbracket$  [1.;  $\alpha$ -EX, PL, UE]
9.  $(a \prec dd \leftrightarrow \Phi(a)) \wedge dd = \llbracket \Phi(x) \rrbracket$  [7., 8.; UE, PL]
10.  $\exists x\Phi(x) \rightarrow \forall y(\Phi(y) \leftrightarrow y \prec \llbracket \Phi(x) \rrbracket)$  [4.-10.; UG $\sqrt{\phantom{x}}$ , CI]



## CHAPTER SIX

# *The Varieties of Identity*

The previous chapter shows that within *FOP* a tension between the principles of plural logic and the principles regarding identity emerges: The assumption of certain empirical truths allows us to derive contradictions from the basic principles of plural logic. As we have seen, the standard hierarchical strategy cannot block these derivations and rejecting substitution is the only alternative we have left. In this chapter, we will reassess these arguments and have a closer look at those principles which are involved in the derivations of the above contradictions. Since the inconsistencies are obviously related to the identity relation, we shall begin with spelling out some basic, commonly assumed principles of the identity relation. It will turn out that substitution is only insofar responsible for the contradiction as another identity principle is – I will argue wrongly – assumed. This principle is the idea that identity is unitary, i.e. that there is only one kind of identity relation. We will examine the derivations of the contradictions from section 5.5 with the help of the six identity principles spelled out at the beginning of this chapter and see that the problematic steps in the derivations are the substitution inferences. Hence, we have a case of substitution failure in these derivations.

On the basis of this, I will argue against the idea of a unitary identity relation in order to guarantee that substitution does not universally fail. The extension of our language gives us good reason to assume that identity behaves in a different way than usually assumed, since it can appear

in contexts where we have plural terms. This leads to the idea that there are several kinds of identity relations. By considering other, e.g. modal or epistemic, contexts where substitution fails, I will draw the conclusion that the reason for the failure of substitution within plural contexts can be explained in a similar way as within modal and epistemic context: First, a predicate that is collective in one of its argument places is non-extensional in that argument place. Second, some plural terms are non-rigid designators. Hence, substituting plural non-rigid designators in argument places of predicates which are non-extensional in that argument place is not a reliable inference.

This helps us to show why Composition as Identity does not lead to mereological essentialism. Identity is collective, hence, non-extensional, in both of its argument places. The term ‘the parts of  $x$ ’ is a non-rigid designator which may refer to different objects in different possible worlds. Therefore, substituting a term for ‘the parts of  $x$ ’ in a sentence which says that  $x$  is identical to its parts is not legitimate, or in other words, we can show that although  $x$  is identical to its parts, this identity is not necessary.

In a second step, we shall turn to the phenomenon of hyperintensionality in plural logic. I will suggest that some predicates are hyperintensional in their argument places, i.e. substitution may fail, even if we are dealing with rigid designators. Although it would be an option to ban substitution *tout court* in these cases, I will present a more moderate solution on the basis of Ben-Yami’s theory of “Articulated Reference” (Ben-Yami 2013).

We will then turn back to the different kinds of identity and give a clear way to distinguish between them. Additionally, I shall present some examples of the various identity relations. Eventually, we shall address two objections that might be raised against my suggestion of defining the general identity relation with these different kinds of identity relations. First, one might think identity is a concept that is too basic to be defined. Hence, the aim of defining identity cannot be accomplished. Second, it might be objected that the relations which we discuss and present as kinds of identity relations are not genuine identity relations.

## 6.1 Six Identity Principles

Identity is often regarded as a well understood, easy to grasp concept:

Identity is utterly simple and unproblematic. Everything is identical to itself; nothing is ever identical to anything else except itself. (Lewis 1986c: 192)

However, we have good reasons to think that this suggestion is highly deceptive. Identity *may seem* to be a clear and fully understood relation, but only if we stay within the limited framework of singular languages. We have seen in sections 4.1 and 4.2 that this framework has to be extended. Thus, I think it is no surprise that not all judgements, which we have formed within this framework, will be confirmed outside of it. Similarly to fishermen, who decide to use more closely meshed nets, only to find out that there are fish who are smaller than any fish they caught before and are smaller than any fish they have ever imagined, we have to consider the possibility that some of our previously formed prejudices about identity may turn out to be false. However, the view I defend here is conservative with respect to identity when it is restricted to singular contexts. In that case, it is indeed simple and unproblematic as Lewis tells us.<sup>92</sup>

In this section, I will list and discuss six principles that we generally assume, based on what we have learned from the behavior of the identity relation in singular contexts. One might expect that these general conditions for being an identity relation should apply in plural contexts as well. As we will see, we have to abandon at least one of these principles. I will argue that the idea of a unitary identity relation is the one we should dismiss. Alternatively, we could reject two other principles in order to avoid the derivation of the two contradictions from the last chapter. However, those alternatives cannot be motivated sufficiently, which is why I decide to discard the thought of a unitary identity relation.

92. Though, as we will see in section 6.3.1, even in singular contexts, identity is not really as simple and unproblematic as it is often presented.

### 6.1.1 Reference and Identity

Let me begin with a principle which is usually not spelled out when people talk about identity, (see, for instance, Griffin 1977: 1-2). It seems to be so basic that one might think it is not worth spelling out. However, it is important for us to have it laid out, simply to understand which principles are responsible for the problems of *FOP*.

(=<sub>1</sub>) If two terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object, then ' $\alpha$  is/are identical to  $\beta$ ' is true

One reason why we cannot find the above principle in other discussions on identity is the fact that those discussions are usually conducted within a singularist language. However, even the singular version of =<sub>1</sub>

If two terms ' $x$ ' and ' $y$ ' refer to the same object(s), then ' $x$  is identical to  $y$ ' is true

is not as ubiquitous as its plausibility might suggest. While it is missing within most metaphysical discussions about identity,<sup>93</sup> we find it in some logic textbooks:

[...]  $\alpha = \beta$  is true iff the terms  $\alpha$  and  $\beta$  refer to the same object.<sup>94</sup> (Sider 2010: 108)

Two designators ' $a$ ' and ' $b$ ' are *co-referential* if they actually refer to the same thing: in that case, the identity claim ' $a = b$ ' will be true. (Smith 2003: 306-7)

If we were attaching meanings to our statements, the meaning of  $x = y$  would be that  $x$  and  $y$  are two names of the same identical object. (Rosser 1978: 163)

93. The above principle might be missing in the discussions on the principles of identity because of its triviality. I cannot see another explanation for that. The principle sounds circular. Whoever denies it ends up in a contradiction. This is why it is difficult to reject the principle, and may explain its absence in most metaphysical discussions.

94. Sider is here working in a language that contains singular terms only. Therefore, his claim translates into the language of *FOP* with ' $x = y$  is true iff the terms  $\rho$  and  $\sigma$  refer to the same object'. Similar translations apply to the other two quotes above.

Yet, it is absolutely uncontentious: If two terms refer to *the same object*, then the sentence where the identity sign connects those two terms is, trivially, true. For instance,

(1) Russell is identical to the author of *OD*

is true *because* 'Russell' and 'the author of *OD*', refer to *the same object*. When we extend our language with plural terms, we might want to embrace not only the above singularist principle, but the more general ( $=_1$ ), in order to conclude, for instance

(2) Russell and Whitehead are identical to the authors of *PM*

from the assumption that the two plural terms 'Russell and Whitehead' and 'the authors of *PM*' refer to the same objects. Singularists will deny the above principle because according to their view any plural term refers to *at most one object*. However, as we have seen in chapter 4, singularism is not a viable option.<sup>95</sup> Therefore, ( $=_1$ ) is more than just natural to assume. I think it is one of the most important criteria a relation has to meet in order to be an identity relation.

### 6.1.2 The Logical Properties of the Identity Relation

Identity is considered to be a reflexive, symmetric and transitive relation, (see McGinn 2000: 6, Smith 2003: 305, and Williamson 1990: 1). Thus, it is an equivalence relation. We have seen that the identity relation in *FOP* has these three properties:

( $=_2$ )  $\alpha$  is/are identical to  $\alpha$

( $=_3$ ) If  $\alpha$  is/are identical to  $\beta$ , then  $\beta$  is/are identical to  $\alpha$

( $=_4$ ) If  $\alpha$  is/are identical to  $\beta$  and  $\beta$  is/are identical to  $\gamma$ , then  $\alpha$  is/are identical to  $\gamma$

95. Thanks to Prof. Bricker for pointing out in an email-correspondence that a singularist might disagree with a pluralized version of the above principle.

The above three principles seem to be, more or less, on a par with our first identity principle with respect to their plausibility, or to put it differently:

It is intuitively obvious that the relation of identity is transitive, symmetrical, and totally reflexive. (Copi 1967: 159)

However, not all of the above three principles enjoy the same status in the literature, or so it seems. While I do not know of any arguments against  $(=2)^{96}$  and  $(=3)$ , there are some reservations with respect to  $(=4)$ . We will have a quick look at  $(=2)$ , since it is an axiom of *FPO* and the system we will develop in chapter 8, and  $(=4)$ , because it is the only one of the above three principles which has been challenged in the literature.

According to  $(=2)$ , every object is identical to itself. The roots of this principle extend back to Plato and Aristotle:

[...] either of them is different from the other, and the same with itself. (Plato 1892c: 185a)

Now 'why a thing is itself' is a meaningless inquiry (for < to give meaning to the question 'why'>) the fact or the existence of the thing must already be evident [...] but the fact that a thing is itself is the single reason and the single cause to be given in answer to all such questions [...]  
(Aristotle 1963a: 1041a,14-20)

Similar views about the law of identity are still widely upheld:

[T]here is the indisputable fact that everything is identical with itself [...]  
(Lowe 2002: 23)

96. Geldsetzer (2013: xxvii-xxviii, 73-76) seems to be the exception to the rule. Wittgenstein might be considered to reject the reflexivity of identity as well, when he says "[t]hat identity is not a relation between objects is obvious" (Wittgenstein 1922: 5.5301), and "to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing" (Wittgenstein 1922: 5.5303). However, this view of Wittgenstein seems to be a consequence of his, I think mistaken, thought that tautologies and contradictions are meaningless, (see Wittgenstein 1922: 4.462), which makes the law of identity meaningless. See also, (Wehmeier 2012) for an argument against identity being a relation.

[I]dentity is a very special relation. It is a relation that every object bears to itself and to nothing else. (Priest 2000: 64)

Everything is identical to itself [...] There is never any problem about what makes something identical to itself; nothing can ever fail to be. (Lewis 1986c: 192-3)

I suppose there is no need to discuss this principle any further. It just seems to be such a basic truism which we cannot explain any further and which cannot be justified in a non-question begging way.

Things stand different with ( $=_4$ ), the transitivity of identity. As the puzzles about composition and parthood show – in particular the Sorites paradox and the puzzle of the Ship of Theseus – doubts about the transitivity of identity have a long tradition.

Interestingly, dismissals of ( $=_4$ ) are often accompanied by a dismissal of the idea of a unitary identity relation: Geach (1972: §7) works with the concept of “relative identity”; Locke (1952: i. 439-70) uses “personal identity”; Priest allows for a transitive identity relation only under certain conditions, namely when “[...] the “middle” object is consistent” (Priest 2014: 20), which gives rise to the suspicion that there is *one* identity relation for ordinary objects, and *another one* for inconsistent objects.<sup>97</sup> We shall discuss these positions in more detail soon.

### 6.1.3 Substitution

We have already encountered substitution as an inference rule of *FOP*. Informally, we can state it as follows:

( $=_5$ ) If it is derivable that  $\alpha$  is/are identical to  $\beta$  and  $\alpha$  has/have the property  $\Phi$ , then it is derivable that  $\beta$  has/have  $\Phi$

( $=_5$ ), or its singularized version, is commonly assumed in logics with an identity predicate, (see Priest 2008: 272, Smith 2003: 326, and Zalabardo 2000: 134). Although substitution is closely related to “Leibniz’s Law”

97. See (Priest 1995: 365-6 and Priest 2014: §2, §5), for details about Priest’s views on identity.

(LL)  $\alpha$  is/are identical to  $\beta$  iff  $\alpha$  has/have any property  $\Phi$  which  $\beta$  has/have, and *vice versa*

the two have to be kept apart. (LL) is a biconditional, which consists of the conjunction of the “Indiscernibility of Identicals” (INID) and the “Identity of Indiscernibles” (IDIN):<sup>98</sup>

(INID) If  $\alpha$  is/are identical to  $\beta$ , then  $\alpha$  has/have any property  $\Phi$  which  $\beta$  has/have, and *vice versa*

(IDIN) If  $\alpha$  has/have any property  $\Phi$  which  $\beta$  has/have, and *vice versa*, then  $\alpha$  is/are identical to  $\beta$

(IDIN) is famously criticized by Black (1952), as well as by Moore (1948: 307) and Peirce (1933: 4.311). More recent doubts about this principle are expressed by Cortes (1976) and Zimmerman (1997). Our principle ( $=_5$ ) relates to (INID) and it is easy to mistake one for the other. However, there is an important difference. (INID) is a second-order principle: In the consequent of (INID), there is a second-order quantifier that cannot be eliminated. ( $=_5$ ), on the other hand, is an inference rule that contains an eliminable schematic variable. Therefore, we can use the latter and not the former within a first-order logic.

Nevertheless, arguments for or against substitution or (INID) apply in most cases, *mutatis mutandis*, to the other principle as well. It is well-known that both principles run into trouble in contexts of modality, intentionality and time, (see, for instance, Frege 1892, Kripke 1980, Priest 2014: 60-6, Quine 1955, van Inwagen 1981, and the references in section 6.3.1). These discussion about the failure of substitution, in particular discussions about the substitution failure in modal contexts, are important for our overall concerns. The lessons we can learn from these discussions are similar to the ones we can learn from the failure of substitution within

98. Sometimes either one of these principles is called ‘Leibniz’s Law’, (see for instance Akiba 2000: 3, Prior 1965: 186, or Smith 2003: 306). The above terminology is also used by Griffin (1977: 2) and Sider (2010: 124).



plural contexts. However, I would like to set this discussion aside for now. We will come back to it in section 6.3. For now, I would like to focus on elaborating the point that *substitution* causes the problems for plural logic, before we consider what to learn from the failure of substitution.

#### 6.1.4 Unitary Identity

A further common assumption on identity is that it is unitary, i.e. it is denied that there are different kinds of identity:

Identity is a relation that is determined in such a specific way that it is impossible to see how different kinds of it may occur.  
(Frege 1903: 254 [my translation])

My first thesis is that identity is *unitary* [...] There is no equivocation or vagueness in the notion of identity, and it operates as a determinate property not a determinable one.  
(McGinn 2000: 1)

Although this view is questioned, for instance by Alston and Bennet (1984), Blanchette (1999), Geach (1972: 238-49), and Lowe (1989), the following principle is still commonly accepted:

(=6) There is only one kind of identity relation

The views of the critics of this principle divide roughly into four groups, which suggest different ways to distinguish the kinds of identity relations from each other. For now, we will discuss two of these. We will return to the remaining in section 6.4.

**Numerical vs. Qualitative Identity:** Numerical identity is understood to be the dyadic relation that is instantiated by an object and itself, and nothing else. However, this relation has to be distinguished from qualitative identity. Qualitative identity is based on the idea of discrimination. Two objects are said to be qualitatively identical iff they cannot be distinguished from each other, i.e. share all their properties

with each other. Take for instance the suit that is now in my closet. It is numerically identical to the suit I bought a year ago; it is *the same* suit. However, the suit in my closet is not qualitatively identical to the suit I bought. They have different properties. The former has a slightly different shade of black than the latter. The former is used, the latter is not. On the contrary, we may take two new copies of the same book to be (loosely speaking) qualitatively identical, yet not numerically identical. Two new copies of *PM* are (almost, apart from a few differing properties, as for instance with respect to location, or numerical identity) indistinguishable. They share all their properties, such as color, number of pages, or letters on the blurb. Yet, they are not numerically identical. They are not the same copy of the book, but *two* copies of the same book.

This distinction is based on a denial of the ordinary interpretation of Leibniz's Law, which says that numerical identity is equivalent to qualitative identity, or substitution. Although I am sympathetic to the idea of restricting substitution, I think the distinction between numerical and qualitative identity is misleading. There is only one way I can make sense of the notion of qualitative identity, namely the identity of properties. Qualitative identity ultimately comes down to the identity of properties, (see McGinn 2000: 2-3). Two copies of the same book may share (almost) all their properties, but that only means that *the properties* they instantiate are identical: For each property which is instantiated by one copy, there is a property identical to that and it instantiated by the other copy, and *vice versa*. But that does not make *the copies* identical. They are merely similar to each other. Although the degree of their similarity is very high, this does not imply the identity of the two books in any way.

#### **Absolute vs. Relative Identity:**

Some authors, for instance Alston and Bennet (1984), Blanchette (1999), Geach (1972: 238-49), Griffin (1977), and Lowe (1989), defend the view that identity is not an absolute, but a relative relation. According to this position, it does not make sense to say that *x* and *y* are identical. This is regarded as an incomplete statement. What we should say instead is that

' $x$  is the same  $\Phi$  as  $y$ ', where  $\Phi$  is a count-expression, "[...] or else it is just a vague expression of a half-formed thought" (Geach 1972: 238).

We already encountered the count-/mass-expression distinction *en passant* in section 4.4. The idea behind the distinction between count- and mass-expressions is that the former can be used to count objects, while the latter cannot. For instance, the terms 'chair', 'book', and 'window' are count-expressions, since we can count chairs, books, and windows. It makes sense to say that there are two windows, four chairs, and fifteen books in my office. On the other hand, mass-expressions cannot be used to count objects. The terms 'furniture', 'butter', and 'rain' are mass-expressions. It does not make sense to speak of "three furnitures", "five butters" or "ten rains". However, we can talk about three *pieces of* furniture, five *spoons of* butter, or ten *raindrops*. Mass-expressions can function as count-nouns only if they appear in connection with certain expressions as the previous examples suggest.

One of the motivations behind the idea of a relativized identity relation is to solve the paradox of the Statue and the Clay, which we encountered in section 1.4.2. Relativizing the identity relation makes it, in general, possible that  $x$  and  $y$  are the same  $\Phi$ , but not the same  $\Psi$ . This gives the following straightforward solution to the paradox of the Statue and the Clay: The statue  $x$  and the piece of clay  $y$  are the same *piece of clay*, but they are only the same *statue* until the squeezing happens. After the squeezing the relative identity relation *being the same statue* does not hold anymore between  $x$  and  $y$ .

Yet, this account of identity is shown to be flawed as well, (see Perry 1970; 1978, and Wiggins 1980: 15-44). Under the assumption of an adequate counterpart to (INID) with a relativized identity relation, we cannot hold on to the idea that identity is relative. If  $x$  and  $y$  are the same  $\Phi$ , but not the same  $\Psi$ , then  $x$  has a property which  $y$  lacks, namely being the same  $\Psi$  as  $x$ . But the relativized counterpart of (INID) tells us that if  $x$  and  $y$  are the same  $\Phi$ , then they share all their properties, which includes being the same  $\Psi$ . Hence,  $x$  and  $y$  must be the same  $\Psi$ . However, that is inconsistent with the basic assumption of a relativized identity relation

in the spirit of the above-mentioned authors. Second, it was suggested that the incompleteness in a statement as for instance

(3) What I bathed in yesterday and what I bathed in today are  
the same (Perry 1970: 182)

is not to be located with the identity relation, but with the singular terms 'what I bathed in yesterday' and 'what I bathed in today' used in the sentence. The problem then with the above sentence is that these terms do not completely determinate which object is identified here. If this indeterminacy of reference is removed, we can make sense of the sentence and there is no need to introduce a relativized identity relation.

Although we shall eventually dismiss the principle of a unitary identity relation, there is no need for us to get caught up in the above discussions. The problem arising in the context we are concerned with, i.e. avoiding the derivation of the contradictions from the previous chapter, is related to the phenomenon of plural terms and how they interact with the identity predicate. So let's turn back to these derivations and examine them in the light of the six identity principles just discussed.

## 6.2 Reassessing the Contradictions

With these principles spelled out, we can now better understand what views on identity are at work in the derivations of the contradictions from section 5.5. Three of our identity principles,  $(=2)$ ,  $(=3)$ , and  $(=4)$ , are not immediately related to the derivation of the contradictions. However,  $(=1)$ ,  $(=5)$ , and  $(=6)$  are relevant. Let's see what role they play in the derivations.

First,  $(=1)$  is the claim that the referents of co-referring terms are identical to each other. It is the principle which gives us the first lines of the arguments, claiming that Russell is identical to the authors of *OD*, and that the 26 cards are identical to the two suits of cards. It emerges from the discussion of  $(=1)$  in the previous that the principle is harmless. If two terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object, or objects, then  $\alpha$  is/are

identical to  $\beta$ . Note, that the derivation of the two contradictions does not rely on one of the two “mixed” versions of  $(=1)$  where we have one term referring to one object and one term referring to many objects. In the first argument, we have two terms referring to *the same object*, Russell, and in the second argument, two terms referring to *the same objects*, the 26 cards. Thereby, I think that in the light of  $(=1)$ , we get even further confirmation to accept these claims and to reject the conservative strategy.

Second, we have used substitution, relying on  $(=5)$ , in both arguments: In the first argument, to infer the lines 4.a and 4.b, and in the second argument, to infer line 5. Those are the places in the proof where we notice that something is going wrong. The previous lines do not seem to be worrisome. Therefore, one might think that  $(=5)$  is responsible for the derivation of the contradictions and should be abandoned. However, I think rejecting substitution altogether would mean to throw out the baby with the bathwater. If we take this route, then we lose the possibility to derive the symmetry and transitivity from the reflexivity of identity, i.e.  $(=3)$  and  $(=4)$  from  $(=2)$ . I reckon, we want to hold onto at least a restricted version of substitution, if possible. We have the possibility of revising  $(=5)$ , if we change our minds about  $(=6)$ , the claim that identity is unitary. So, let’s see how that principle is involved in the above derivation of the contradictions and explore the possibility of rejecting it.

Principle  $(=6)$ , is not used explicitly in *any one* of the lines of the above arguments individually. But we can see that it is relevant when we take a step back and look at the reasoning underlying the argument as a whole. For instance, consider line 1. in the first argument from section 5.5.1. It claims that Russell is identical to the authors of *OD*. We said this sentence follows trivially from the assumption that Russell is *the* author of *OD*, i.e. Russell is *identical* to the author of *OD*. As we see, there are two identities involved: one holding between Russell and the author of *OD*, and one holding between Russell and the authors of *OD*. We have then used substitution to infer the problematic lines 4.a and 4.b. This inference is only possible because we assumed that the two identities are the same kind of identity, i.e. we tacitly assumed  $(=6)$ . For consider, that identity is a rela-

tion which comes in different kinds, such that some of them obey substitution unrestrictedly, and some of them do not. Then, we have to make sure every time we make a substitution inference whether the necessary conditions for a valid substitution inference are given. With respect to the derivation in section 5.5.1, this means that we have to consider whether the identity relation in line 1. allows for substitution.

This is the solution I suggest in order to avoid the derivation of inconsistencies from the principles of plural logic. It has the advantage that we do not have to abandon  $(=5)$  altogether. We can still allow for substitution inferences in certain contexts, whereby we save  $FOL^=$  as a fragment of our logic. At the same time, we can avoid the derivation of the above contradictions by not allowing the use of substitution, as we will see. Moreover, that plural terms and their behavior in connection with the identity relation are responsible for the derivation of the contradiction is a reasonable explanation: If we were to eliminate all plural terms from  $FOP$ , then we could use substitution unrestrictedly – though see the remarks in section 6.3.1 – without being able to derive a contradiction.

However, rejecting  $(=6)$  has several consequences. First, we have to get clear about the *general identity relation*, i.e. that relation which the different kinds of identity relations are kinds of, and whether it follows the above-suggested identity principles. Principle  $(=5)$  will not hold for the general identity relation. If it were to allow for substitution unrestrictedly, then we would again be in a position to derive the above contradictions. But, what about the principles  $(=1)$  to  $(=4)$ ? We will see that those principles hold for the general identity relation in the system we will spell out in chapter 8. The logical properties of the general identity relation are reflexivity, symmetry, and transitivity, and that they can be shown to be theorems of the system presented there. Second, we have to get clear about the different kinds of identity relations: How many of them are there? How do they come apart? What makes them identity relations? We will answer the first two questions after discussing other contexts where substitution fails and considering that similar lessons can be drawn in the case of substitution failures in plural contexts.

## 6.3 The Non-Extensionality of Plural Logic

In section 6.1.3, I briefly hinted at the discussion about substitution failures in other contexts. Now that we have established that it is indeed a failure of substitution which led us to the contradictory conclusions in section 5.5, let's take a step back and reflect on this. We shall briefly have a look at the discussion on substitution failures in other contexts and the lessons that have been learned from them. I will then suggest that we can learn a similar lesson in the case of the failure of substitution within plural contexts. In a first step, this allows us to address the criticism from section 2.4 and we can show that Composition as Identity does *not* entail mereological essentialism. In a second step, I suggest to restrict the substitution of plural terms when dealing with predicates that are hyper-intensional in an argument place with the help of Ben-Yami's theory of "Articulated Reference" (Ben-Yami 2013).

### 6.3.1 Substitution Failures in Other Contexts

It is well-known that substitution fails in certain contexts,<sup>99</sup> (see Frege 1892, Kripke 1980, Griffin 1977: 2-9, Priest 2014: §5, Quine 1955, Rea 1997a: xv-xxiii, and van Inwagen 1981). For instance, when we encounter modal notions, substitution can lead us to false conclusions:<sup>100</sup>

1. Necessarily, eight is identical to eight
2. The number of planets is eight
3. Necessarily, the number of planets is eight

The first two lines of the above argument are true and the conclusion, which follows by substitution, is false: The number eight is necessarily identical to itself and the number of planets (in our solar system) is eight.

99. Quine calls these contexts "[...] *referentially opaque*" (Quine 1955: 142-3) [italics in original].

100. The argument below is inspired by an example of Quine (1955: 146-7).

Yet, the number of planets is not *necessarily* eight. There is a possible world where our solar system contains only seven planets. Hence, something must have gone wrong in the above argument. The problems which come with arguments of this kind are already discussed at length in the literature, for instance by Hintikka (1961: 127-8), Kripke (1980), Prior and Kenny (1963), and Quine (1947), and one of the suggestions to deal with them is to restrict substitution in modal contexts.

Moreover, substitution can a problem in epistemic contexts.<sup>101</sup>

1. Your brother is identical to the hooded man
2. You do not know who the hooded man is
3. You do not know who your brother is

The problem with the above argument is the following: Assume that the hooded man, who has covered his head, is your brother and he stands in front of you. You have no idea who the person standing in front of you is, i.e. you do not know who the hooded man is. Yet, you know who your brother is. Hence, the above two premises are true, although the conclusion is false, because you know your brother. But the conclusion follows from the first two premises by substitution. Issues with arguments of this kind are discussed by Eberle (1974), Priest (2002) and Quine (1956), and a restriction of the use of substitution has been suggested as one plausible way to avoid the derivation of the above conclusion. Since restrictions of substitution are already accepted in modal and doxastic contexts, a further restriction within plural contexts should be considered as an option.

One of the lessons that has been drawn from the substitution failure in modal contexts, famously advocated by Kripke (1980), is not to substitute non-rigid designators<sup>102</sup> in certain contexts. Hewitt (2012) suggests that

101. The above argument was already known to Eubulides, (see Priest 2002: 445 and the references there).

102. Rigid designators are terms referring to the same object in all possible worlds where that object exists. 'Russell', refers to Russell in all possible worlds where Russell exists. Non-rigid designators may refer to an object  $x$  in a world  $w_1$  and to a different object  $y$



the distinction between rigid and non-rigid designators should be made for plural terms as well. Given what we have seen, this is a sensible distinction to draw: Some plural terms are rigid, ‘Russell and Whitehead’, while others are non-rigid designators, ‘the authors of *PM*’. Yet, before we turn back to plural logic, let’s see in more detail what are the lessons that have been drawn from substitution failures in singular contexts.<sup>103</sup>

The substitution failures in the above presented contexts have led to the distinction between extensionality, intensionality and hyperintensionality. Some predicates<sup>104</sup> are such that the substitution of *any* co-referring terms in a particular argument place preserves truth.<sup>105</sup> These predicates are *extensional in that argument place*. Thus, ‘being British’ is extensional in its only argument place in

(4) Bill is British

since we can substitute *any* co-referring terms, for instance ‘your brother’, for ‘Bill’.

Contrary to that, we encounter predicates that are *non-extensional*, i.e. not extensional, in an argument place in a sentence. When dealing with these predicates, substitution is not always reliable inference. The predicates that are non-extensional in an argument place divide into two groups, predicates which are *intensional* and predicates which are *hyper-intensional* in an argument place. We will come back to the phenomenon

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in a world  $w_2$ , even if  $x$  exists in  $w_2$ . ‘The author of *OD*’ is such a term, since there is a possible world where Russell exists, yet somebody else wrote *OD*.

103. The following paragraphs are mainly slight modifications or elaborations of the remarks given by Nolan (2014: 151-2).

104. It will be convenient for our purposes to make the distinction we are about to see for *predicates* and their argument places in a particular sentence. Commonly, this distinction is drawn for *contexts* or *positions in sentences*, (see Nolan 2014: 151). Our distinction can be easily related back to these ways of drawing the distinction. First, it can be understood as a shorthand for *contexts which contain predicates* that have the relevant property in an argument place. Second, predicates and their argument places are positions in a sentence.

105. I use the phrase ‘preserves truth’ here as a shorthand for ‘the truth values of the two sentences which differ from each other only with respect to which one of the two co-referring terms they contain’ for the ease of better readability.

of predicates being hyperintensional in an argument place soon. First, what it means for a predicate to belong to the former group.

A predicate *is intensional in an argument place* iff it is not extensional, i.e. the substitution of co-referring terms may not preserve truth, and substituting *rigid designators* preserves truth. For instance, ‘being identical to’ is intensional in its first argument place<sup>106</sup> in

(5) Necessarily, eight is identical to eight

Substituting the non-rigid designator ‘the number of planets’ for one of the occurrences of the term *eight* above is, as we have seen above, not a legitimate inference. More generally, and we will come back to that point later, it has been noted that expressing modal phenomena can only be achieved with the means of sentences that – using the terminology I suggest here – contain predicates which are intensional in an argument place, (see Nolan 2014: 152).

Finally, a predicate is said to be *hyperintensional in an argument place in a sentence* iff it is neither extensional nor intensional in that argument place. Hence, if a predicate is hyperintensional, then even substituting *rigid* designators may not preserve truth. To illustrate this phenomenon, suppose Bill is your brother and unbeknownst to you, his friends nicknamed him ‘B’. This helps us to see that the predicate ‘being identical to’ is hyperintensional in its argument place in

(6) You believe Bill is your brother

because substituting rigid designators may not preserve truth, since you presumably do not believe that B is your brother.

If we now turn back to our definitions of plural terms and predicates being collective in an argument place from section 4.4, we will see that there is a reasonable explanation for the substitution failures in plural logic at hand.

106. To be precise, the predicate is intensional in *both* of its argument places in (5). But to illustrate the idea, it suffices to focus on one of the argument places.

### 6.3.2 Intensionality in Plural Logic

Recall our distinction between singular and plural terms. We defined singular terms as not being *able*, and plural terms as being *able* to refer to more than one object. With this modal distinction, we put a modal notion at the heart of our theory. Therefore, it is no surprise that since substitution fails in modal contexts, it may fail in some plural contexts as well. But, let me put the point a bit more carefully.

The claim I want to defend here is *not* that *all* plural terms are non-rigid designators. The plural term ‘Whitehead and Russell’ is a *rigid* designator. In every world  $w_1$  where Whitehead and Russell exist, the term refers to Whitehead and Russell. In a world  $w_2$  where Whitehead does not exist, but Russell exists, it refers only to Russell; in a world  $w_3$ , where Russell does not exist, but Whitehead exists, the term refers only to Whitehead; and in a world  $w_4$  where neither Whitehead nor Russell exist, it does not refer to any object. But more importantly, there is *no* possible world  $w_1$  where Whitehead and Russell exist, and the term ‘Whitehead and Russell’ refers to an object which is not either one of them, say Frege. In any possible world, where Whitehead exists, the term ‘Whitehead and Russell’ refers also to Whitehead; and in any possible world, where Russell exists, the term ‘Whitehead and Russell’ refers also to Russell.

Yet, this does not hold for all plural terms. Some plural terms are non-rigid designators. We already encountered ‘the authors of *PM*’ as a non-rigid designator, but there are many more. Take for instance, ‘the books on my table’, ‘the bins in front of Anne’s house’, or ‘the siblings of Ben’. There are two books on my shelf right now, a copy of Lewis’ *Parts of Classes* and a copy of Quine’s *Methods of Logic*. Yet, although ‘the books on my shelf’ actually refers to those two copies, there is a world, where these books exist but there are three, or two other books on my desk. Similarly, the four bins in front of Anne’s house might be somewhere else, and some other bins might stand in front of her house. There is a possible world where Ben has other siblings than the ones he actually has, and the people who are actually his siblings exist in that world.

These examples show that *some* plural terms are non-rigid designators. The reason for this is that we have defined plural terms modally as terms that are *able* to refer to more than one object. Some plural terms refer to the same objects in all possible worlds where these objects exist. These are not problematic. However, some refer to different objects in different possible worlds. Since we have allowed that there can be different referents of a term in different possible worlds, substitution has to be expected to fail when we encounter predicates that are intensional in an argument place: If the referents of the term can differ from each other, then substitution inferences are not legitimate. Hence, we can summarize our observation in a slightly more formal way: Because some plural terms can refer to different objects  $uu$  and  $vv$  in different possible worlds  $w_1$  and  $w_2$ , even if each one of the objects among  $uu$  and  $vv$  exists in  $w_1$  and  $w_2$ , some plural terms are non-rigid designators. Since we know non-rigid designators cannot be substituted, if a predicate is intensional in the argument place occupied by one of the terms, substitution has to be restricted in such cases to *rigid* designators only.

In singular contexts, substituting non-rigid designators may only be troublesome if we have a predicate that is *intensional* in the relevant argument place. Now, the question arises why we should consider the predicates involved in the plural substitution failures to be intensional in the relevant argument place. If they are not, it is irrelevant whether the substituted terms are rigid designators or not, since then substitution is legitimate anyways.

We can observe with respect to failures of plural substitution that they occur only in cases where we have predicates that are collective in an argument place. The substitution failures discussed in chapter 5 arise from substituting terms in the argument places of the improper inclusion predicate. Improper inclusion is collective in *both* of its argument places. Firstly, it is collective in its *first* argument place because there are some objects  $uu$  which are properly among some objects  $vv$ , yet not every object properly among the  $uu$  is properly among the  $vv$ . This is a consequence which, although it may seem a bit counterintuitive at first, follows from

plural logic's comprehension axiom: If some  $uu$  are properly among the  $vv$  and the  $vv$  are the plurality of objects which collectively  $F$ , then there might be some  $x$  properly among the  $uu$  which is not  $F$ , and hence is not properly among the  $vv$ .<sup>107</sup> We will come back to this issue in section 7.5, where we will discuss the partial transitivity of the proper inclusion predicate.

Further, proper inclusion is collective in its second argument place. This is pretty straightforward, as the following line of thought shows: Russell and Whitehead are properly among the logicians. Yet, they are *not* properly among each one of the logicians, since Russell and Whitehead are not properly among Frege.<sup>108</sup> Since Frege is properly among the logicians this shows that proper inclusion must be collective in its second argument place because it does not hold that for any  $uu$ , if the  $uu$  are among the  $vv$ , then the  $uu$  are among each one of the  $vv$ .

Secondly, I think we have good reasons to assume that predicates which are collective in an argument place are non-extensional in that argument place. The definition of 'being collective in an argument place' in section 4.2 relies in an important way on the notion of plural terms, as we can see from the fact that in  $FOL^=$ , any predicate is by default distributive: Without plural terms, there are no predicates that are collective in an argument place. Moreover, plural terms are defined *modally*: They *are able* to refer to more than one object. Hence, since modality is essentially an intensional phenomena, I think it is no surprise that the non-extensionality of modality carries over to predicates that are collec-

107. To illustrate the above thought, consider the following example. Suppose, two objects  $x$  and  $y$  weigh together more than a pound. Hence, they are properly among the objects that weigh more than a pound. Yet, if proper inclusion were distributive in its first argument place, then it would follow that each one of the two objects weighs more than a pound, which might not be the case.

108. The above sentence may sound ill-formed at first sight: How can *some* objects be properly among *an* object? Setting worries with respect to grammar aside, I think, this only highlights the point that it follows from the definition of predicates being collective in an argument place that the proper inclusion predicate cannot be distributive in its second argument place. Yet, we will turn back to the question whether singular terms can enter the second argument place of the inclusion predicate in section 7.2.

tive in their argument places. Thus, we might want to hold on to the conclusion that predicates which are collective in an argument place are non-extensional in that argument place.<sup>109</sup>

Finally, this shows why we encountered the failures of substitution in chapter 5: At least one of the substituted terms there is a non-rigid designator. The predicates involved in the substitution inferences are non-extensional in the relevant argument places. Hence, failures of substitution are to be expected. On the basis of these observations, we shall eventually only legitimize the substitution of *rigid* designator when we are dealing with predicates that are intensional in the relevant argument place. Yet, since not every predicate that is non-extensional in an argument place is also intensional in that argument place, we have now to turn to the phenomenon of hyperintensionality.

### 6.3.3 Hyperintensionality in Plural Logic

It does not suffice to restrict the use substitution to rigid designators only, since even the substitution of these terms may not preserve truth, as the following example shows:

1. Whitehead and Russell were born in that order
2. Whitehead and Russell are identical to Russell and Whitehead
3. Russell and Whitehead were born in that order

The predicate ‘being born in that order’ is collective, which is why I take it to be non-extensional, in its only argument place. However, the two terms ‘Whitehead and Russell’ and ‘Russell and Whitehead’ are both *rigid* designators, as we have seen in section 6.3.1. Hence, the above argument indicates that even substituting terms which are necessarily co-referring

109. I avoid the maybe more obvious seeming conclusion, that being collective in an argument place amounts to being *intensional* in that argument place. The reason for this is that, as we will see next, there are predicates that are collective in an argument place and hyperintensional, i.e. *not* intensional, in that argument place.

may not preserve truth. In other words, the predicate ‘being born in that order’ is *hyperintensional* in its only argument place, (see Nolan 2014: 151). This leads to two questions: Is there a principled way to identify predicates that are hyperintensional in an argument place? Can we use – and if we can, under what conditions – substitution when dealing with predicates that are hyperintensional in an argument place.

With respect to the first question, I think, we might have to rely on some of the hints that are provided by natural language. It seems that we can identify predicates that are hyperintensional in an argument place from natural language due to the occurrence of phrases such as ‘in that order’ or ‘in reverse order’. Yet, there are predicates, which are hyperintensional in an argument place and no expressions seem to immediately indicate this, as the following example shows:<sup>110</sup>

1. Anne and Bill, and Claire and Dan are married
2. Anne and Bill and Claire and Dan are identical to Anne and Dan, and Claire and Bill
3. Anne and Dan, and Claire and Bill are married

Therefore, we are not able to give a principled way to distinguish between predicates being intensional in an argument place and predicates being hyperintensional in an argument place. This is an unfortunate diagnosis, but in lack of further evidence we have to rely on our common sense intuitions here and be satisfied with the thumb-rule that predicates that exhibit a sensitivity to order in an argument place are hyperintensional in that argument place. I hope further investigations may help us to find a more reliable way to draw this distinction, but let’s move on to my suggestion for how substitution should be restricted in these cases.

110. One could argue that the substitution here fails, because we cannot change the order of the terms that are used to refer to the two couples. Yet, whether this is an appropriate answer under the assumption that plural terms are ontologically innocent is to be doubted.

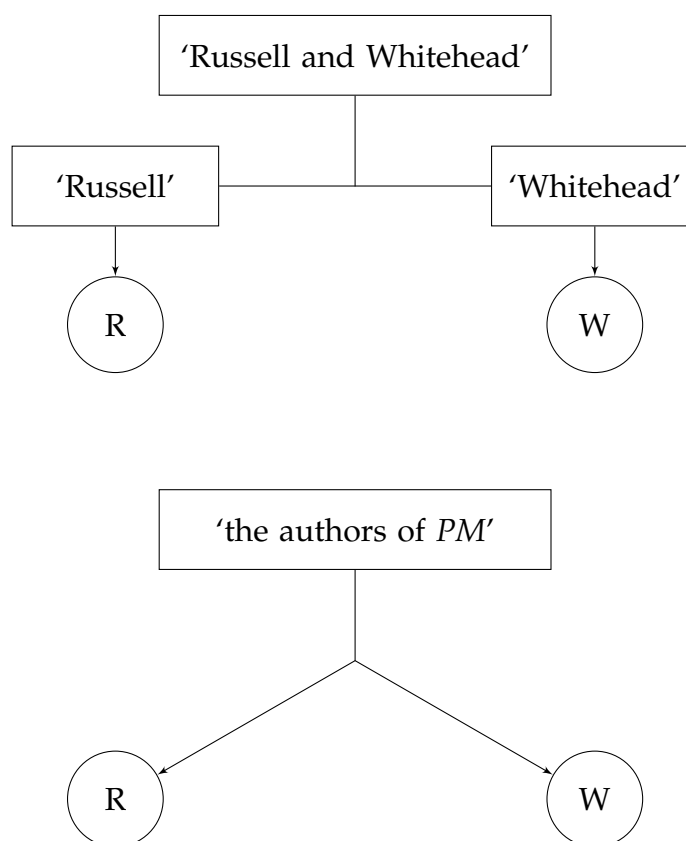


Figure 6.1: Articulated Reference

### 6.3.4 Substitution and Articulated Reference

My restriction of substituting terms, which occur in predicates that are hyperintensional in an argument place, is based on Ben-Yami's theory of "Articulated Reference" (Ben-Yami 2013). The basic idea of the articulation of reference is that some plural terms refer to their referents in virtue of containing expressions which refer only to some, but not all, of the term's referents. For instance, the term 'Russell and Whitehead' refers to Russell and Whitehead because it contains the terms 'Russell' and 'Whitehead', whereby both terms refer only to some and not all of the plural terms referents. In such cases, the reference of the term is said to be articulated. This does not hold for any plural term, as can be see



from figure 6.1. The reference of ‘the authors of *PM*’ or ‘the British’ is not articulated.

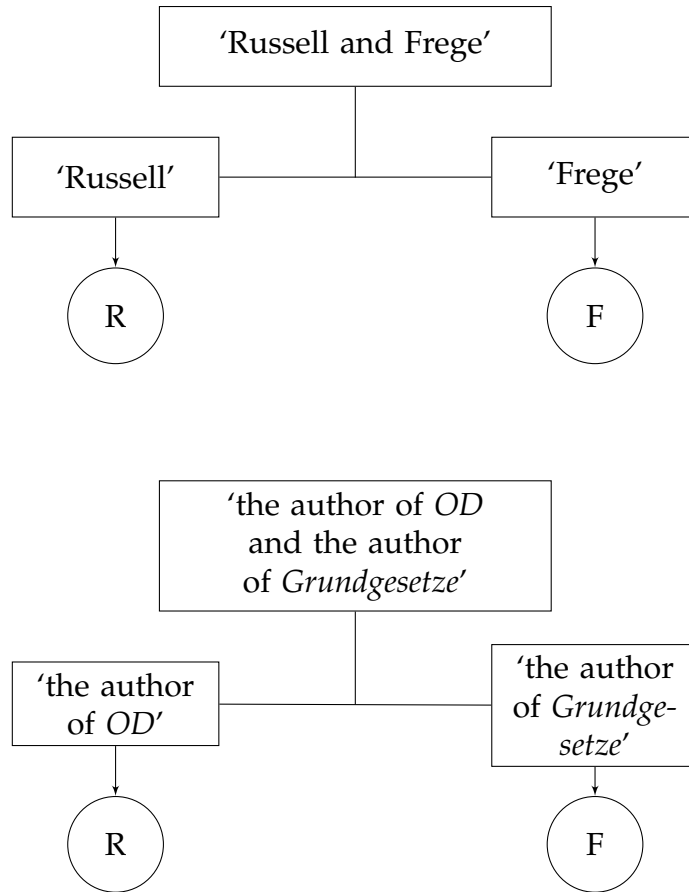


Figure 6.2: Articulating Reference in the same Way

Now, if we have two terms where their reference is articulated, one of two things can happen: Either their reference is articulated in the same way, or their reference is articulated in different ways. Here is where I again rely on the Black’s metaphor of reference as an act of pointing from section 4.5: The reference of two terms is articulated in the same way, if they “point” to their referents in the same way, see figure 6.2 for illustration. The reference of the two terms ‘Russell and Frege’ and ‘the author of *OD* and the author of *Grundgesetze*’ is articulated in the same way. Compare these two terms to the term ‘Frege and Russell’. Its reference

is articulated in a different way than the referents of the two other terms, since 'Frege and Russell' refers to Russell and Frege over-cross, as figure 6.3 indicates. If the reference of 'Frege and Russell' were to be articulated in the same way as the reference of 'Russell and Frege', or 'the author of *OD* and the author of *Grundgesetze*', then 'Frege' would refer to Russell, and 'Russell' to Frege.

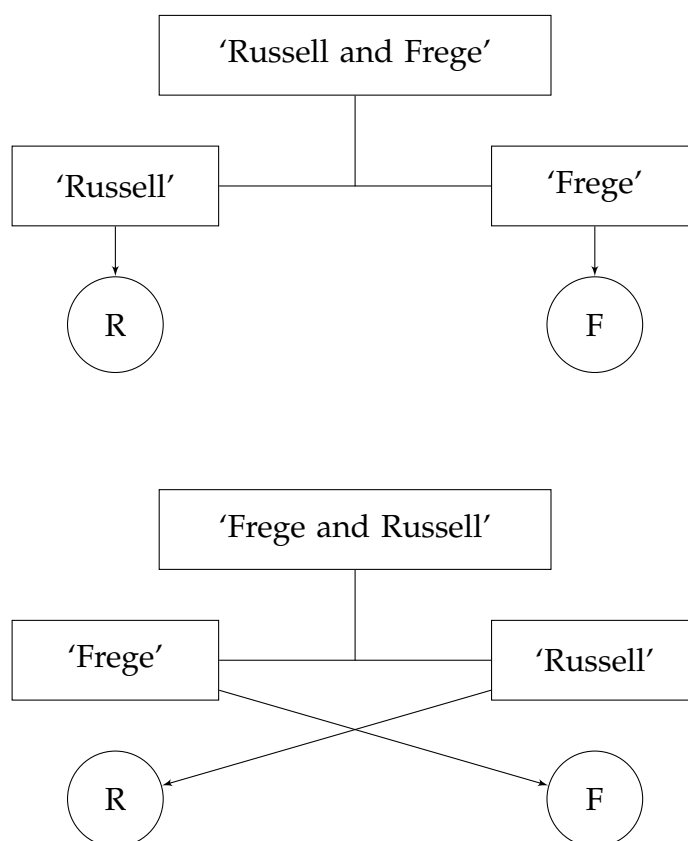


Figure 6.3: Articulating Reference in different Ways

The notion of articulated reference can now be used to do justice to the sensitivity to order we have encountered previously, since articulation of reference itself is sensitive to order. Hence, I suggest that when we encounter predicates that are hyperintensional in an argument place, then only terms whose reference is articulated in the same way can legitimately be substituted. I have to point out that this way of restricting

substitution is *not* the one suggested by Ben-Yami (2013). His suggestion is to restrict plural substitution *in general* to those terms whose reference is articulated in the same way. Without getting into too much detail, I would like to give a brief explanation for why that seems too strict to me.

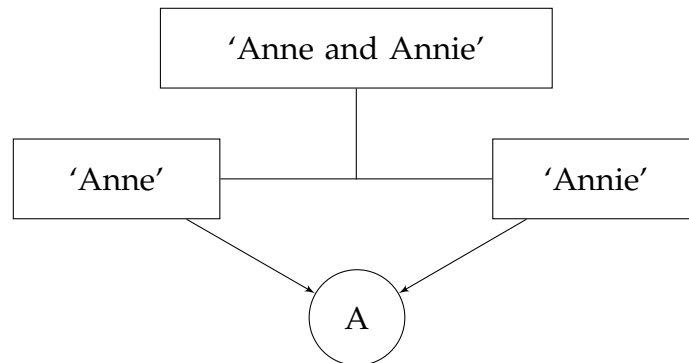


Figure 6.4: Not articulated Reference

Ben-Yami's restriction does not allow substitution in cases where the reference of the term is *not* articulated:

[T]he substituted terms need not only be co-referential, but they also need to *articulate* reference in the same way.

(Ben-Yami 2013: 92)

Hence, if we assume that 'Annie' is a nickname for Anne, then Ben-Yami cannot allow for the following inference

1. Anne is identical to Anne and Annie
2. Anne and Annie are identical to Annie and Anne
3. Anne is identical to Annie and Anne

because the reference of the substituted terms, 'Anne and Annie' and 'Annie and Anne' is not articulated: Both terms contain only expressions that refer to all of the referents of the terms, i.e to Anne, as illustrated in figure 6.4.

Contrary to that, my suggestion allows for this inference. ‘Being identical to’ is *intensional* in both of its argument places. Hence, we can substitute the two *rigid* designators, (see Nolan 2014: 151), from the second premise above. In a nutshell, we can see that the here suggested account is to be preferred over Ben-Yami’s because it allows for more intuitively valid inferences, while it manages to avoid the same troublesome inferences.

### 6.3.5 Composition as Identity without Mereological Essentialism

Let’s now turn back to the criticism from section 2.4 that Composition as Identity entails mereological essentialism. With the above analysis of some plural terms being non-rigid designators, and the claim that if a predicate is collective in an argument place then it is non-extensional in that argument place, we can address this point. Let’s illustrate the argument against Composition as Identity from section 2.4 in a slightly modified way: Suppose a composite object is identical to its parts. Then, the completed puzzle  $x$  is identical to the puzzle pieces  $uu$  it is composed of.  $x$  is necessarily identical to itself. By substitution,  $x$  is necessarily identical to the  $uu$ , i.e. necessarily  $x$  is not identical to the  $vv$ , if the  $vv$  are not identical to the  $uu$ . From that follows under the assumption of Composition as Identity, necessarily,  $x$  is not composed of the  $vv$ , or in other words,  $x$  could not have other parts than the ones it actually has.

We can see that a restriction of substitution avoids the above derivation which relies on the inference from necessarily,  $x$  is identical to itself to necessarily,  $x$  is identical to the  $uu$ . Previous reflections about substitution failures in other contexts and the lessons we can learn from the observations of these failure can be applied to the substitution failure in plural contexts and provide us with a sound explanation for why the above inference is invalid. We said that some plural terms are non-rigid designators and that these cannot be substituted in sentences, where a predicate is intensional in the relevant argument place. When we think,

for instance, about a puzzle and its parts, as well as the parts it could have, or the parts it could lack, then it emerges that ‘the parts of the puzzle’ is a non-rigid designator, referring to different objects  $uu$  and  $vv$  in different possible worlds  $w_1$  and  $w_2$ , although each one of the objects which are among the  $uu$  and the  $vv$  exists in both worlds.

Although our restriction on substitution avoids *this way* of deriving mereological essentialism from Composition as Identity, one might be worried that the appeal to non-rigidity is not able to exclude other ways of showing that mereological essentialism follows from Composition as Identity. *Prima facie*, the term ‘the *actual* parts of  $x$ ’ appears to be a rigid designator. Just like ‘the *actual* author of  $OD$ ’ may be taken to refer to Russell in any possible world where Russell exists,<sup>111</sup> ‘the *actual* parts of  $x$ ’ refers to the same objects in all possible worlds where these exist. Thus, it is a rigid designator. Yet, if  $x$  is a rigid designator, say, ‘Jig’, Jig being identical to its *actual* parts leads us again to mereological essentialism since Jig is necessarily identical to Jig, and we cannot avoid the substitution of the rigid terms. It follows that Jig is necessarily identical to its *actual* parts and we seem to be left with mereological essentialism again.

However, whether the above line of reasoning, which aims to reestablish the connection between Composition as Identity and mereological essentialism, is correct, can be doubted. There are good reasons to think that phrases such as ‘the *actual* author of  $OD$ ’ and ‘the *actual* parts of  $x$ ’ are non-rigid designators. I agree with Lewis, (see also Lewis 1970: 184-7), that the terms ‘actually’ and ‘actual’ are indexical terms:

[T]he meaning we give to ‘actual’ is such that it refers at any world  $i$  to that world  $i$  itself. ‘Actual’ is indexical, like ‘I’ or

111. This might be taken to follow from the stipulation of keeping the use of our language fixed. As Kripke (1980: 102-9, see in particular fn.51) notes, we can describe other possible worlds only with *our* language and we cannot adopt, for the sake of consistency, the language that might be used at that possible, non-actual, world we are talking about. Hence, when we claim that Anne could have been Beth, then this does not amount to the claim that at a possible world, Anne is identical to Beth.

'here', or 'now': it depends for its reference on the circumstances of utterance, to wit the world where the utterance is located. (Lewis 1973: 85-6)

Thus, expressions such as 'the actual author of *OD*', or 'the actual parts of *x*' must be indexical as well, and whether, for instance, it is true to say that Russell is identical to the actual author of *OD* will depend upon the circumstances under which this sentence is uttered. It is true, when we are talking about *our* actual world, but it may be false when we are talking about *another* possible world. But then, if these terms are indexical, then they are arguably non-rigid designators, since these terms do not have the same referents in all possible worlds, where their actual referents exist. For instance, when we use the term 'the actual author of *OD*', then it refers to Russell. But, if the very same term is used at a possible world where, say, Whitehead wrote *OD*, then the term refers to Whitehead, even if Russell exists at that world. Furthermore, we should note that the fact that Whitehead is the actual author of *OD* at this possible world is expressed in *our* language. Hence, Kripke's restriction that we can describe other possible worlds only with the help of our language, mentioned in the previous footnote, is met. This means that substituting this non-rigid designator may not be a legitimate inference. What's more, this applies also to plural terms which contain the notion of actuality. In particular, we should consider the term 'the actual parts of *x*' as a non-rigid designator and substituting it for '*x*' should not be considered as a valid inference.

Yet, this reply to the worry that our account of Composition as Identity leads to mereological essentialism can be challenged. Whether the notion of actuality is indeed an indexical notion is doubted, (van Inwagen 1980), as is the claim that indexical terms are non-rigid designators, (Kaplan 1989). Nevertheless, I would also like to point out that the above line of reasoning is a viable option to argue that Composition as Identity does not necessarily lead to mereological essentialism. There is an ongoing dispute in the literature on how to deal in a proper way with the notion of actuality and with indexical terms in modal discourse, (Davies

and Humberstone 1980, Humberstone 2004, Ninan 2013, Rabern 2013, Soames 2005, Yalcin 2015). Moreover, it remains to be seen whether a commitment to mereological essentialism is after all worrisome, if the debate on the notion of actuality turns out in such a way, that our account of Composition as Identity entails mereological essentialism. However, since it cannot be our aim here to give a final answer to these questions, we have to content ourselves with the more moderate claim that although the above way of separating mereological essentialism from Composition as Identity can be challenged, it is a live option.

All things considered, we can see that the intensional character of non-rigid plural designators and how it relates to the identity predicate being intensional in its argument places has important consequences for Composition as Identity. Its core claim, that a composite object is identical to its parts, with the non-rigid designator 'its parts' – which may refer to different objects in different possible worlds – opposes an extensional view of reality. Thereby, since Composition as Identity is built upon non-extensional considerations, criticisms of it asking it to be extensional dissolve. With this central thought about the status of Composition as Identity, let's move on to consider the different kinds of identity relations.

## 6.4 The Varieties of Identity

Let's turn back to the two questions from above: What are the different kinds of identity relations, and how can we distinguish them from each other? From what we have seen, there seem to be at least two good candidates for the kinds of identity relations: one that obeys substitution, and one that does not. However, it is good to have a more fine-grained distinction to begin with. I suggest two criteria, one *syntactical* and one *semantical*. This distinction brings together and elaborates two traditions to distinguish between different kinds of identity relations.

On the one hand, we can find in the literature on plural logic approaches that make a *syntactical* distinction between identity relations.

The idea there is to distinguish between singular and plural identities, depending on whether singular or plural terms are used to express the idea that an identity relation holds.<sup>112</sup> In addition to restricting plural identity to plural terms, McKay and Yi even introduce a special symbol to make the distinction between the two kinds of identity clear:

[T]he plural “identity” ( $\approx$ ) functions fully like singular  $=$ . Singular identity can be defined as a special case [...]  
(McKay 2006: 128-9)

[...] the plural (viz. neutral) cousin of the identity predicate “ $=$ ”, which is the refinement of the singular from of the predicate “to be” [...]  
(Yi 2005: 487)

On the other hand, we have in the literature on Composition as Identity a *semantical* distinction of identity relations. This distinction categorizes identity relations according to the number of objects which the terms entering the argument places of the identity predicate refer to. Consequently, we can find three different kinds of the identity relation: one-one, many-one and many-many:

So we need some other semantic treatment of many-one identity, preferably one that describes some more generally phenomenon that has many-many and one-one identity as special case.  
(Cotnoir 2013a: 301)

So striking is this analogy that it is appropriate to mark it by speaking of mereological relations – the many-one relation of composition, the one-one relations of part to whole and of overlap – as kinds of identity.  
(Lewis 1991: 84)

In chapter 1 as well as in section 3.3, we have seen that this view, or versions of it, are already discussed to some extent in the literature. For our purposes it will be helpful to make a more fine-grained distinction

112. Oliver and Smiley’s plural logic does not distinguish between a singular and a plural identity relation, though they define a second identity predicate, called “weak identity” for empty terms, (Oliver and Smiley 2013: 109-10, 191, 212).



of identity relations, instead of relying on either one of the above distinctions. Both attempts on their own seem not to be able to do fully justice to the variety of identity. Therefore, we will first spell out the two ways of distinguishing between the kinds of identity relation and then, in a second step, combine them with each other.

### 6.4.1 Two Criteria to Distinguish Identity Relations

We begin the elaboration of the aforementioned approaches with a refinement of the syntactical distinction. The general idea is that we can discriminate the identity relations on the basis of which kinds of terms are used to express that an identity holds. Since we have two kinds of terms, singular and plural, and the identity predicate has two argument places, we have four different possibilities: We can have two singular terms, or two plural terms in both argument places; Or, we can have a singular term in the first and a plural term in the second, or a plural term in the first and a singular term in the second argument place. It seems that a distinction between the last two possibilities is superfluous and we could ignore one of them. In fact, it will turn out that they reduce to each other. However, it would be a dodgy move to assume that right from the beginning, since we would then beg the question and *presuppose* that this relation is symmetric, which has to be shown.

Let's turn to the semantic distinction. The basic idea here is that we can distinguish the various identity relations on the basis of how many objects the terms, which are used to express an identity, refer to. We made a twofold distinction of terms according to the number of objects they refer to – one object, and more than one object, i.e. many objects – so we get again four possibilities: Both terms refer to one object, or both terms refer to many objects; Or, the term in the first argument place refers to one object and the term in the second argument place refers to many objects, or the term in the first argument place refers to many objects and the term in the second argument place refers to one object. As with the syntactic distinction, we will see that the symmetry of our general identity

	<b>One-One</b>	<b>Many-One</b>	<b>One-Many</b>	<b>Many-Many</b>
<b>Singular</b>	$x = y$	—	—	—
<b>Plural</b>	$uu \sim vv$	$uu \lesssim vv$	$uu \lesssim^* vv$	$uu \approx vv$
<b>Plural-Singular</b>	$uu \cong x$	$uu \simeq x$	—	—
<b>Singular-Plural</b>	$x \cong^* uu$	—	$x \simeq^* uu$	—

Table 6.1: *The Varieties of Identity*

relation makes considering both of the last two possibilities separately a superfluous exercise, though we will keep them apart for now.

Combining the above two ways to distinguish the different kinds of identity relations gives us 16 different combinations for our general identity relation  $\equiv$ , out of which nine are possible due to our definition of singular terms. Nine different identity relations might strike one as an odd and big number. Yet, as we have seen the number of the different identity relations simply results from the two different ways we distinguish the identity relations: the kinds of terms which enter the argument places of the identity predicate, and whether these terms are able to refer to more than one object or not. Table 6.1 gives an overview of the suggested kinds of identity relations and introduces a bit of notation, which will be introduced officially in chapter 8.

Let's introduce some terminology to keep things manageable from here on. I shall call sentences, which state that an identity relation is supposed to hold, and contain two singular terms "singular identities". If

two plural terms are used to express the thought that an identity holds, we shall call the sentence a “plural identity”. A “plural-singular identity” is a sentence where a plural term enters the first and a singular term the second argument place of the identity predicate, and a “singular-plural identity” where a singular term enters the first and a plural term the second argument place of the identity predicate. Moreover, we shall call sentences, where the terms used to express that an identity holds both refer to exactly one object “one-one identities”. If two terms each refer to more than one object in a sentence which claims that an identity relation holds, then we shall call it a “many-many identity”. Furthermore, a “many-one identity” is a sentence which expresses the thought that an identity holds and where the first argument place of the identity predicate is occupied by a term which refers to more than one object and the second argument place by a term which refers to one object. Finally, a “one-many identity” is a sentence which claims that an identity relation holds, and the first argument place of the identity predicate takes a term which refers to one object while it takes a term which refers to more than one object in its second argument place.

With the use of this terminology, we can now explain why out of the 16 different combinations only nine are possible. Why are all singular identities one-one identities? There can be no singular identity that is also a many-one identity, since that means there is a singular term, namely the one occupying the first argument place of the identity predicate, which refers to more than one object. However, we have excluded this possibility by our definition of ‘singular term’: A singular term is a term which *cannot* refer to more than one object. Due to the same reason, there cannot be a singular identity which is a one-many identity, since then the singular term in the *second* argument place were to refer to more than one object. Moreover, there are no singular identities which are many-many identities, since then we would have two singular terms each referring to more than one object. Similarly, no plural-singular identity is a one-many, or a many-many identity, since the singular term in the second argument place of the identity predicate cannot refer to more than one

object. Finally, there is no singular-plural identity that is also a many-one or a many-many identity because the singular term entering the first argument place of the identity predicate is not able to refer to more than one object.

It will be one of our main tasks in chapter 8 to spell out the differences between these nine relations. That these relations are *indeed* identity relations, will be discussed at the end of this chapter. Before we come to that, we have a look at some examples to get an initial idea about the differences of these relations.

### 6.4.2 Some Examples of the Variety of Identities

We have already seen a few identity claims in the previous chapters. Most of them belong to a kind of identity we are relatively familiar with, for instances, singular one-one and plural many-many identity, or are already extensively discussed in the literature, for instance singular one-one, plural many-many, or plural-singular many-one identity. We have less acquaintance with singular-plural one-one identities, which we already met in section 5.5.1:

(7) Anna is identical to the wives of Frege

(8) Russell is identical to the authors of *OD*

In order to get a better idea of the different kinds of identity relations which result from the just suggested distinction, some examples might be helpful for us. I will give two groups of examples, each group consisting of nine sentences, each sentence being a different kind of identity claim. Those examples will then be used in chapter 8 where the different kinds of identity relations will be defined.

(9) Russell is identical to the author of *OD*

(10) The authors of *OD* are identical to the authors of *Marriage and Morals*

- (11) The authors of *OD* are identical to Russell
- (12) Russell is identical to the authors of *OD*
- (13) Russell and Whitehead are identical to the pairs of men who wrote *PM*
- (14) Russell and Whitehead are identical to the pair of men who wrote *PM*
- (15) The pairs of men who wrote *PM* are identical to Russell and Whitehead
- (16) The pair of men who wrote *PM* is identical to Russell and Whitehead
- (17) Russell and Whitehead are identical to the authors of *PM*

Here we have again some sentences, notably (11), (12), (15) and (16), which will sound odd to many people. However, as I have remarked in section 5.5.1 with respect to the worries about the sentence (11), which equally apply to (12), and maybe their plural counterparts (15) and (16), one has good reasons to think that these sentences are not problematic, or meaningless constructions, although they may sound unnatural to many people.

According to our distinction of identity relations, each of the above sentences expresses a different kind of identity relation, as displayed in table 6.2. Why did we categorize these sentences as we did? Have a look at the terms which are used in the sentences (9) to (17) and ask the following two questions: What kinds of terms, i.e. how many objects *can* each term refer to, are used in a sentence? How many objects does each term *actually* refer to? The answers to these questions will automatically tell us what kind of identity we are dealing with. (9) is a singular one-one identity, because we have two singular terms, 'Russell' and 'the author of *OD*', each referring to exactly one object, Russell. (14), on the other

	<b>One-One</b>	<b>Many-One</b>	<b>One-Many</b>	<b>Many-Many</b>
<b>Singular</b>	(9)	–	–	–
<b>Plural</b>	(10)	(13)	(15)	(17)
<b>Plural-Singular</b>	(11)	(14)	–	–
<b>Singular-Plural</b>	(12)	–	(16)	–

Table 6.2: *The Varieties of Identity: Example I*

hand, is a plural-singular many-one identity, because ‘Russell and Whitehead’ is plural and ‘the pair of men who wrote *PM*’ is singular, and the former refers to more than one object, Russell and Whitehead, and the latter refers to exactly one object, the pair. In (17), we have the two plural terms, ‘Russell and Whitehead’ and ‘the authors of *PM*’, which each refer to more than one object, Russell and Whitehead. Hence, we have a plural many-many identity.

One might here of course raise the objection that I am begging the question. Whether, for instance, the term ‘the pair of men who wrote *PM*’ refers to one or many objects is debatable:

‘The pair’ and ‘the suit’ are not genuinely singular terms [...]  
 ‘Whitehead and Russell were a pair of logicians’ is a straight-  
 forward plural identity (Oliver and Smiley 2013: 274)

Or, one may worry whether we are here not resting too much weight on the accidents of the grammar and syntax of the English language.

This first dissent with the above categorization is warranted. How-

ever, since it is not my intention here to *justify* the above categorization, but merely to give some initial hint of where we will be going in the next chapters, we may flag this concern for now. A proper response to this worry will then be given in section 7.1. The second dissent is unwarranted. I do *not* claim that grammar tells us how many objects a term *actually* refers to. My observation is that the English grammar gets it right here and mirrors the logical structure of the sentences, unlike in the cases of singularia tantum nouns.

Hence, let's examine a second group of examples that illustrates the distinction of identity relations. For the sake of the example, imagine the following scenario: You are showing a magic trick to your niece. For that you need 26 cards. Thirteen of them are Spades, from 2 to Ace. Thirteen of them are Diamonds, from 2 to Ace. You have marked the Queen of Spades, which is all the magic behind the card trick. You put the marked card face down on the table and, *Abacadabra*, you know what card is on the table. Then, the following identities hold:

- (18) The card on the table is identical to the Queen of Spades
- (19) The marked cards are identical to the cards on the table
- (20) The cards on the table are identical to the Queen of Spades
- (21) The Queen of Spades is identical to the cards on the table
- (22) The 13 black cards are identical to the suits of Spades
- (23) The 13 black cards are identical to the suit of Spades
- (24) The suits of Spades are identical to the 13 black cards
- (25) The suit of Spades is identical to the black cards
- (26) The 26 cards are identical to the two suits

The explanation for why these sentences match with the different kinds of identities as the above table suggests, can be seen from the way we

	<b>One- One</b>	<b>Many- One</b>	<b>One- Many</b>	<b>Many- Many</b>
<b>Singular</b>	(18)	—	—	—
<b>Plural</b>	(19)	(22)	(24)	(26)
<b>Plural- Singular</b>	(20)	(23)	—	—
<b>Singular- Plural</b>	(21)	—	(25)	—

Table 6.3: *The Varieties of Identity: Example II*

justified the assignment of the examples from the first group. The categorization is illustrated in table 6.3.

In chapter 8, we will develop our system *LI*, which is able to capture the idea of the varieties of identity relations and serves as our basis for our theory of composition. We will now address two objections against the project lying ahead of us: the worry that identity is undefinable and the suspicion that the above relations are not identity relations.

## 6.5 On Defining Identity

The aim of chapter 8 is to present a formal system which takes *singular one-one* identity, ' $=$ ', and inclusion, ' $\prec$ ', as primitive relations. On the basis of these two relations, we will define the other eight kinds of identity relations. As I have mentioned earlier, some of these definitions will be revised in the light of the counterexamples to many-one identities based on rearrangement. However, we shall ignore these counterexamples in



the next two chapters and regard our definitions as a provisional starting point for an account that is able to meet these criticisms. Eventually, our definitions of the kinds of identity relations together with the primitive singular one-one identity relation will allow us to define the *general identity relation*, ' $\equiv$ ', disjunctively: It is that relation which holds between  $\alpha$  and  $\beta$  iff one of the nine kinds of identity relations holds between  $\alpha$  and  $\beta$ .

Yet, defining identity is often regarded with suspicion. The view the identity relation is "[...] too simple a notion to admit of logical definition [...]" (Reid 1850: 229), predates the development of modern formal logic. Frege (1884: §65) initially broke with this tradition, only to change his views about these matters later, (see his 1972; 1879; 1903). He argues that identity cannot be defined adequately because any definition of identity would turn out to be circular:

Since every definition is an equation, one cannot define equality itself. (Frege 1972: 327)

Frege takes a definition to be a statement that claims an *identity* between two concepts. Hence, any definition already uses the concept of identity. Therefore, a definition of identity can only be circular. More recently, similar arguments for the indefinability of the identity relation have been put forward by McGinn (2000: 7-9) and Bueno (2014: 328-9).

Are these worries justified and what does that mean for our enterprise of defining identity? I think these worries are to some degree well-grounded. However, this does not mean that we are heading in the wrong direction. Remember, Frege, McGinn, and Bueno are working within a singular language. Therefore, their worries apply only to the indefinability of *singular one-one identity*, i.e. the predicate 'being identical to' where it is flanked by two singular terms. Since we will *not* define *that* relation, but take it as one of our primitive relations, we are not ignoring the doubts about the indefinability of singular one-one identity. What we are defining is the *general* identity relation and the other eight non-primitive kinds of identity relations. Hence, the worries raised here

are relevantly similar to the objections raised against my suggestion to restrict substitution. Since all these objections are made from the standpoint of a singular language, they can only be directed against the claim that *singular identity* is indefinable and obeys substitution. We can agree with that, since we will take '=' as one of our primitive relations and hold on to unrestricted substitution in singular contexts. Therefore, we can reject this objection against the following attempt to define identity because it does not apply to the *general* identity relation.

## 6.6 Schmididentity Relations?

Some further critical reflections on our project of bringing together the above-presented kinds of identity relations to define general identity are necessary. One worry which might arise in connection with what was outlined in section 6.4 is that these relations are "not really" identity relations, but "schmididentity relations", i.e. relations that look like, but are in fact not identity relations, similar to the improper parthood relation or the improper subset relation. These may mistakenly taken to be kinds of identity relations. However, they are *not* identity relation, but include identity as a limiting case. So, the question we have to answer is: What makes each of our kinds of identity relations a genuine identity relation?

The answer to that question can be found in the identity principles we discussed at the beginning of this chapter. Our nine kinds of identity relations are proper identity relations because they collectively obey the first four principles:

- (=1) If two terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object(s), then ' $\alpha$  is/are identical to  $\beta$  is true'
- (=2)  $\alpha$  is/are identical to  $\alpha$
- (=3) If  $\alpha$  is/are identical to  $\beta$ , then  $\beta$  is/are identical to  $\alpha$
- (=4) If  $\alpha$  is/are identical to  $\beta$  and  $\beta$  is/are identical to  $\gamma$ , then  $\alpha$  is/are identical to  $\gamma$

The last three principles might be regarded as merely necessary conditions for a relation to be an identity relation. There are a lot of equivalence relations, i.e. relations that are reflexive, symmetric, and transitive relations: 'having the same hair color as' on the set of all human beings, 'being congruent to' on the set of all triangles, 'being from the same breed' on the set of dogs. Yet, these relations are not identity relations.

However, the central point is that all our nine kinds of identity relations obey the principle ( $=_1$ ). To show this, note first that the conditional in ( $=_1$ ) holds in the other direction, too:

If ' $\alpha$  is/are identical to  $\beta$  is true', then the two terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object(s)

This is obviously as trivially true as ( $=_1$ ): An identity claim ' $\alpha = \beta$ ' can only be true, if ' $\alpha$ ' and ' $\beta$ ' refer to the same object(s). Hence, ( $=_1$ ) really amounts to a biconditional:

( $=_1$ ) Two terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object(s) iff ' $\alpha$  is/are identical to  $\beta$  is true'

This biconditional allows us to see why the nine relations we distinguished from each other are in fact identity relations: First, if one of these relations holds between  $\alpha$  and  $\beta$ , then the terms ' $\alpha$ ' and ' $\beta$ ' refer to the same objects. Hence,  $\alpha$  and  $\beta$  is/are identical to each other, by ( $=_1$ ). Second, if  $\alpha$  and  $\beta$  is/are identical to each other, then the terms ' $\alpha$ ' and ' $\beta$ ' refer to the same object(s). Hence, one of the nine relations holds between  $\alpha$  and  $\beta$ . Therefore, we can see that our kinds of identity relations capture the variety of identity: Collectively, they give us necessary and sufficient conditions for an identity relation to hold. Hence, contrary to the improper parthood and improper subset relation, the above kinds of identity relation do not *have* identity as the limiting case, but they *are* the limiting case, identity, themselves. Thereby they will allow us to define the general identity relation.

As I outlined in section 6.2, I will eventually suggest that there are several kinds of identity relations and only one of them obeys substitution

unrestrictedly.<sup>113</sup> Surely, this will raise red flags, despite the evidence I have offered for our kinds of identity relations being genuine identity relations. As we have seen at the beginning of this chapter, the common view on identity is that it obeys substitution unrestrictedly. Hence, someone might object, if only singular one-one identity obeys substitution unrestrictedly, and the other eight relations do not, then the latter are not identity relations, because obeying substitution is a characteristic property of identity.

This view on identity is wide-spread and the wish to defend the claim that some kinds of identity relations do not obey substitution unrestrictedly may seem to many a fairly radical, if not even an impossible move. According to orthodoxy, any relation which does *not* obey substitution is *not* an identity relation. However, I do not share this conviction and think it is based on an overly confident view on what we know about identity.

Many take it that identity is already well-understood and that we are familiar with its properties. But, how can it then be that “[i]dentity is a popular source of philosophical perplexities” (Quine 1950: 621)? If identity is really so well-understood, why are we still struggling with many of the ancient puzzles, such as the Sorites paradox, the Ship of Theseus, or the Statute and the Clay? These paradoxes show that we do *not* yet have a complete understanding of identity, since they arise from questions about identity: How can the removal of one grain from a heap not make a difference whether it is *the same* heap, if we eventually end up with no heap at all? Is the ship of Theseus *identical* to the original or the reassembled ship? How can the statue be *identical* to the piece of clay, if the piece of clay already existed at a time when the statue did not exist? If we had a full grasp of the concept of identity, then these puzzles about identity should already be solved. Since this is not the case, I think we should not bump our fists on the table and insist that identity must obey substitution unrestrictedly. Rather, we should take these puzzles seri-

113. Strictly speaking, even singular one-one identity does not obey substitution unrestrictedly, as we have seen in section 6.3. However, for the sake of exposition, I shall assume that there are no issues with substitution for singular terms.

ously and consider the possibility that by working on solutions on them we might gain some new insights about identity. Therefore, we should be open to revise our cherished views on which properties are characteristic for identity. Moreover, and this relates back to some of my remarks in section 6.5, we should acknowledge that by adopting plural logic, we have just entered new territory and that some of the rules of  $FOL^=$  might have to be left behind. Hence, I think we should be a bit more modest about our knowledge of identity and consider the possibility that there are still things to learn about this puzzling relation.

I am aware that these arguments may not be accepted without protest. Nevertheless, I hope that the reasons for why I think that a relation can be an identity relation, even if it does not obey substitution unrestrictedly, have become clear. This will at least make it possible to have a fruitful discussion on whether obeying substitution unrestrictedly is a characteristic feature of identity or not, and should help to avoid to end up in a merely verbal dispute or a discussion where both parties are begging the question. So, let's take the next step, which will be to reconsider and clarify some concepts and principles for a logic that is able to capture the varieties of identity, before we move on to define the notion of general identity.



## CHAPTER SEVEN

# *Principles for a Logic of Identities*

Before we come to the presentation of the system for a logic of identities *LI*, we have to tighten some loose ends from the previous chapters and make some important decisions. We will clarify the status of those terms which are, as we have seen in section 5.6 mistakenly, considered to be superplural terms. I will argue that they are singular terms. Secondly, I will suggest that although, sentences where a singular term enters the second argument place of the inclusion predicate undoubtedly go against an initial understanding of our everyday use of the predicate 'being among', there are cases where it makes sense to say that an object, or some objects, are among another object. Thirdly, we will introduce and discuss a new concept, "bottom objects", i.e. objects which do not have any object among them. We will see that this concept is not as contentious as it looks at first sight and we can postulate the existence of bottom objects without being at risk to smuggle in ontological presuppositions about the ultimate structure of reality. Since we will allow for the inclusion sign to take singular terms on either side, we will investigate whether an object  $x$  being among some object  $y$  entails that there is a further object  $z$  among  $y$ . This will be a further assumption we shall adopt. Finally, we will re-examine *FOP*'s axiom (A10), the transitivity of inclusion. After discussing some of the problems this principle causes for *FOP* and some more general worries that arise from it, I will suggest that the axiom has to be revised.

Each of these claims I am about to argue for in the following sections might be considered troublesome. However, I think they are legitimate assumptions to make from where we are now. They will eventually allow us to elaborate a formal system which enables us not only to capture the varieties of identity, and so to avoid the derivations of the contradictions from section 5.5, but also to address the criticisms against Composition as Identity from the sections 2.1 and 2.2, i.e. the paradox for Composition as Identity and the derivation of Collapse.

Prior to tackling the above questions, I introduce two further technical concepts. It will be convenient for the following discussions to have the concepts of *improper* and *proper pluralities* at hand. An improper plurality is a plurality that contains only one object, as for instance, the plurality of the authors of *OD*. A proper plurality is simply the opposite of an improper plurality, i.e. one among which there are at least two objects.

$$(D11) \quad IP(uu) =_{df} \forall x \forall y (x \prec uu \wedge y \prec uu \rightarrow x = y)$$

$$(D12) \quad PP(uu) =_{df} \neg IP(uu)$$

It might seem that the distinction between proper and improper pluralities cannot be drawn, since pluralities are no more than a *façon de parler*, see the remarks at the end of section 4.5. We could have defined two other predicates in order to draw this distinction: ‘being many’ and ‘being one’, or Oliver and Smiley’s ‘being singular’ and ‘being a strict plurality’ (Oliver and Smiley 2013: 110-1). These pairs of predicates are very similar to each other. However, the two latter pairs take singular as well as plural terms as arguments, while we defined the above two for plural terms only. This is an important feature of this technical notion and will turn out to be useful for us.<sup>114</sup>

The distinction between proper and improper pluralities, which happens at the ontological level, mirrors the distinction between proper and

114. We can explicate the above two notions with the help of some modal terminology: Some *uu* are an improper plurality iff the *uu* are actually one, but possibly many. Some *uu* are a proper plurality iff the *uu* are actually many.



improper plural terms from section 4.4: A plural term '*uu*' is a proper plural term iff '*uu*' refers to a proper plurality. A plural term '*uu*' is an improper plural term iff '*uu*' refers to an improper plurality. We will rely on this connection between proper plural terms and proper pluralities on the one side, and between improper plural terms and improper pluralities in the next subsection.

## 7.1 Not Superplural but Singular Terms

Let's turn back to the terms which are, in my view, mistakenly understood to be superplural terms by the friend of the conservative strategy. In section 5.6, we have seen that terms, as for instance, 'the pair of men who wrote *PM*' or 'the suit of cards', due to the fact that the distinction between singular and plural terms is exhaustive, cannot be superplural terms. The immediate question arising from this is the following: If those terms are *not* superplural terms, are they singular or plural terms? My answer to this question is that they are *singular* terms. Let's have a look at the other option first and see where it leads us.

Oliver and Smiley (2013: 273-5) suggest that terms such as 'the pair',<sup>115</sup> or 'the suit' are plural terms because

[...] the pair really is the men, and the suit really is the cards,  
but 'the pair' and 'the suit' are not what they seem.  
(Oliver and Smiley 2013: 273)

I agree with the first part of the above sentence, but disagree with the second half. The pair is indeed the two men. However, I think that the terms 'the pair' and 'the suit' are really what they seem to be at a first sight, namely singular terms. Yet, Oliver and Smiley take these terms to be "pseudo-singular" (2013: 274). Pseudo-singular terms have a singular *grammatical* form but are, from a *logical* point of view, plural terms – ca-

115. Here and for the remainder of this section, the term 'the pair' is used, for the ease of exposition, occasionally instead of the term 'the pair of men who wrote *PM*'. When it is used in the ordinary sense, I will highlight this.

pable of denoting *many* objects. In the words of the terminology we have just introduced: The pair is a proper plurality. Hence, the above claim ultimately comes down to assuming that a pair of men is many, and not one. This is one way we could go. But I reckon, it is not the best way to choose.

The above justification for taking the term 'the pair' to be a singular term looks problematic. Recall, the paradox for Composition as Identity. There we ended up with two contradictions which followed from the assumptions that a composite object is one, its parts are many, and 'being one' is the opposite of 'being many'. We noted that abandoning one of these premises suffices to avoid the derivation of the two contradictions. If what Oliver and Smiley say in the above quote is correct, a solution to the paradox looks to be at hand and we can defend Composition as Identity: The first premise of the argument is false. A composite object is not one, but many. The composite object is not what it seems. After all, it is really its parts, and its parts are many.

By relating back to the paradox for Composition as Identity, we can see what is going wrong in the above line of reasoning. Taking the term 'the pair', and similar terms such as 'the suit of cards', or 'the six-pack', to be plural terms comes with strange consequences. Let's focus on 'the pair' in the general sense and suppose Oliver and Smiley are correct. Then, it strikes me that in principle *anything* we might take to be one, is in fact many. Take, for instance, a broom. We think of a broom as being one. The broom has a stick and a brush. Yet, isn't the broom the pair consisting of the stick and the brush, just as Russell and Whitehead are really the pair of men who wrote *PM*? But then, we were wrong in assuming that the broom is one and it turns out, from our assumption, that it is many. Or take another object, we think of as one, an apple. It has a left and a right half. If the pair of the two halves are many, then the apple turns out to be many as well, since it simply is the pair of the two halves.

Does this suggest that we should rather consider 'the pair' to be a singular term? I think a lot speaks for that. However, one may object that this is due to intuitions coming from a sympathy for Composition

as Identity. Yet, I think that the above examples illustrate that taking 'the pair' to be a plural term, referring to many objects, is not that straightforward as Oliver and Smiley suggest. In any case, it is not a viable option for us.

Is there another way to hold on to the claim that 'the pair' is a plural term? There seems to be only one option left, if we do not want to follow the above line of thought. If 'the pair' is a plural term, yet not a proper plural term, then it refers to an improper plurality, i.e. exactly one object. What are the possible candidates for that object? Presumably, we have only three answers that seem to be sensible: Russell, Whitehead, or the pair. However, neither of them is a suitable candidate after a closer analysis. It cannot be Russell. There is no reason why the term should only refer to Russell, and not to Whitehead, or only to Whitehead and not to Russell. So, if 'the pair' refers to Russell, then it refers to Whitehead; and if it refers to Whitehead, then it refers to Russell. However, if the term refers to an improper plurality, i.e. there is only one object among the things it refers to, then we end up contradicting the irreflexivity of inclusion. Here is why: Having excluded Russell and Whitehead as the objects which are among the pair, we are left with the pair as the only option. But then the pair is among the pair and that contradicts the irreflexivity of ' $\prec$ '. *The worse for (T1), you might think, then it has to go, together with the asymmetry of inclusion!*

This is of course another option here. Though it is not a very attractive one. Although it solves this problem, it leads only to further complications. Let's assume, for the sake of the argument, that 'the pair' is an improper plural term, referring to the improper plurality the pair. Then, what kind of term is 'the pairs (of men who wrote *PM*)'? Since we unmasked the notion of superplural terms and given the above line of argument, the term is presumably also an improper plural term. Yet, that is counterintuitive. The terms 'the pair' and 'the pairs' are not on a par. Rather, it seems that they stand in a similar relation to each other as 'Russell' and 'the authors of *OD*': Both terms refer to the same object, Russell. However, the authors of *OD* is an improper plurality and Russell

is among the authors of *OD*. This analogy is undeniable and it shows that abandoning the irreflexivity and asymmetry of ' $\prec$ ' is not a very attractive solution to resolve this problem either.

In conclusion, assuming that such terms as 'the pair' and 'the suit' are plural terms leads to several undesirable consequences. Therefore, I suggest we should consider the possibility of grammar getting it right in these cases: The pair is really one, and the suit is really one.

## 7.2 The Relata of $\prec$

The next decision to make concerns the inclusion predicate, to be precise its relata. We introduced ' $\prec$ ' in section 5.1 without specifying what its relata are, in other words, we did not answer the question *What kinds of terms can enter the argument places of the predicate?* We have to get clear about that now. The examples considering ' $\prec$ ' and the inclusion relation, which we have considered thus far, divide into three groups: First, we have sentences where the inclusion predicate takes a singular term in its first and a plural term in its second argument place, as for instance in

- (1) Russell is among Russell and Whitehead
- (2) Russell is among the logicians

Second, we had sentences where the inclusion predicate takes plural terms in both of its argument places:

- (3) Russell and Whitehead are among Russell, Whitehead, and Wittgenstein
- (4) *PM* and *Grundgesetze* are among my books

Some authors do not allow plural terms to enter the first argument place of the inclusion predicate. Linnebo (see his 2014: §1.1, point 2 of the definition of well-formed formulas) restricts ' $\prec$ ' such that " $\alpha \prec \beta$  is a formula

when  $\alpha$  is a singular term and  $\beta$  a plural term".<sup>116</sup> This way of limiting ' $\prec$ ' is counterproductive for the idea of plural logic. After all, the project of taking plural terms as logically relevant starts with the assumption that predicates are not sensitive to number, i.e. can take plural terms as arguments, if they can take singular terms as arguments. Hence, we need a justification for limiting ' $\prec$ ' in this way since natural language suggests, as the above examples show, that it takes plural terms in its first argument place. Without such a justification, the restriction is unwarranted.

Third, we encountered sentences where, in each case, one singular term entered both of the inclusion predicate's argument places:

(5) Russell is among Russell

(6) My logic books are among my logic books

However, we agreed in section 5.3.2 that these sentences are false. On what assumption was this claim based? The two sentences each contradict the irreflexivity of ' $\prec$ ', following from the asymmetry of ' $\prec$ ', which in turn we have identified as one of the central axioms of plural logic. Though, we have not yet considered the possibility that two *different* singular terms might flank the inclusion predicate or that a plural term enters the first and a singular term the second argument place. Can such sentences be true?

At a first glance, it seems counterintuitive to accept sentences of the form ' $x \prec y$ ' or ' $uu \prec y$ ' as true. It looks like our use of the predicate 'being among' does not allow for a singular term in its second argument place. However, let's have a closer look at these possibilities by examining sentences where an inclusion predicate is flanked by two singular terms first. Consider the two sentences

(7) Some object is among Russell

(8) Some object is among Whitehead

116. I substituted our metalinguistic variables ' $\alpha$ ' and ' $\beta$ ' for Linnebo's ' $t$ ' and ' $T$ ', respectively.

Those immediately follow, if we accept the truth of sentences, where singular terms – in that particular case, the terms ‘Russell’ and ‘Whitehead’ – enter the second argument place of inclusion. Yet, (7) and (8) may not immediately be acceptable. When we hear one of these sentences, then the feeling might arise that the person uttering the sentence has not completely finished expressing her sentence. The question *Some object is among Russell and what?* seems to come up immediately when we hear or read (7). It looks like those sentences make, at a first sight, no sense.

However, we can make sense of examples, if we take different singular terms. Namely those terms which are sometimes, as I just suggested, mistakenly taken to be superplural terms, for instance ‘the pair of men who wrote *PM*’ and ‘the suit of Spades’:

(9) Some object is among the pair of men who wrote *PM*

(10) Some object is among the suit of Spades

Those sentences do not immediately seem objectionable, although they might seem odd. However, if we accept them as true, what terms are possible candidates for entering the first argument place of ‘ $\prec$ ’ when we have ‘the pair of men who wrote *PM*’ in the second? If any term at all is suitable, then surely the terms ‘Russell’ and ‘Whitehead’ are. If anything is among the pair of men who wrote *PM*, then surely Russell is, and Whitehead is. Therefore, I think it is legitimate, if we utter the following sentences:

(11) Russell is among the pair of men who wrote *PM*

(12) Whitehead is among the pair of men who wrote *PM*

In addition to that, we get also an answer to our second question, whether we can accept sentences of the form ‘ $uu \prec x$ ’. If we assume that Russell is among the pair of men who wrote *PM*, and Whitehead is among the pair of men who wrote *PM*, then

(13) Russell and Whitehead are among the men who wrote *PM*

follows from that. Hence, I propose that we should not only accept sentences where we have singular terms in both argument places of the inclusion predicate, i.e. sentences which can be adequately formalized as ' $x \prec y$ ' in the language of *FOP*, but also such sentences as (13), whose formalization is ' $uu \prec x$ '. Permitting an inclusion predicate which takes singular terms in its second argument place, brings us to two further questions: Firstly, are there any objects that do not have any object among them? Secondly, can an object  $x$  have only one object  $y$  among it? Or must there be some other object  $z$  that is also among  $x$ ? We shall consider these two questions next.

### 7.3 Bottom Objects

The non-emptiness axiom of *FOP* makes sure that there are no empty pluralities. It stipulates that for any objects  $uu$  there is at least one object  $x$  which is among them. Should we postulate a similar axiom for singular terms, making sure that for any object  $x$ , there is an object  $y$  which is among  $x$ ? That is to say, should we take

$$\forall x \exists y (y \prec x)$$

as one of *LI*'s axioms? The question whether there are any objects having no objects among them arises only due to the fact that we just allowed for the possibility of singular terms entering the second argument place of ' $\prec$ '. Had we *not* allowed for that, the above formula would, trivially, turn out to be false. I suggest that we do not add the above formula to the axioms of our formal system and allow there to be objects not having any objects among them. I will call these objects "bottom objects", and they are defined as follows:

$$(D13) \quad B(x) =_{df} \forall y \neg (y \prec x)$$

The above question, whether it is reasonable to assume that there are any bottom objects, resembles the discussions about similar concepts from

set theory and mereology. The set theoretical counterpart to bottom objects are urelements<sup>117</sup> and their cousins in mereology are atoms.<sup>118</sup> It is important for us, that the assumptions we make about bottom objects will not force us to take a stance on whether there are atoms or gunky objects. Fortunately, we can achieve this by relativizing the notion of bottom objects to theories. We shall say that an object  $x$  is a bottom object of a theory  $t$  iff there is no object  $y$  such that according to  $t$ ,  $y$  is among  $x$ . Thereby, we can avoid taking sides in the dispute about atoms by assuming that according to any (non-empty) theory  $t$ , there are some bottom objects. Strong and weak atomists can embrace this principle without any doubts. They can take bottom objects simply to coincide with mereological atoms in atomistic theories – nothing is among an atom. Given this definition of bottom objects, the principle can – or to be precise must – even be accepted within theories that postulate a gunky universe, i.e. a world where *every* object is a gunky object. The reason is that even in such a theory there have to be some bottom objects.

The following line of thought explains how we get to this conclusion. Any non-empty theory  $t$  has to contain at least one singular term ' $x$ '. Here is why:  $t$  contains at least one term ' $\alpha$ ' because it is a non-empty theory. ' $\alpha$ ' is either singular or plural. If ' $\alpha$ ' is not singular, then it can be shown with the axiom of non-emptiness (A13) that  $t$  entails that there is an object  $x$  which is among  $\alpha$ . Hence,  $t$  will contain at least one singular

117. Urelements are objects that do not have any objects as members. Sometimes, for instance in (Zermelo 2010: 402), the empty set is taken to be an urelement. For Zermelo, urelements played a central role, (Zermelo 2010: 551). Fraenkel (1922), on the other hand, explicitly banned urelements from set theory.

118. In mereology, there are three possible answers with respect to the question of the existence of atoms, (Simons 2003: 41-2). On the one hand, "strong atomism" takes atoms to be the basic constituents of reality, claiming that any object is either an atom or has at least one atom as proper part. Van Inwagen (1990: 5) is one of the contemporary defenders of strong atomism. On the other hand, we can find the idea that *any* material object has at least a proper part, (see Zimmerman 1996a; 1996b), i.e. any object is a gunky object, see footnote 9 in chapter 1. The third alternative, "weak atomism", claims that there are atoms and gunky objects. However, this "[...] hybrid position [...] has rarely been seriously entertained" (Simons 2003: 42). Hudson (2007) gives an excellent overview of the present discussion on atoms and gunk, while Pyle (1995) investigates the history of the dispute.



term ' $x$ '. From the definition of bottom object, it follows that if  $x$  is not a bottom object in  $t$ , then there is an object  $y$  which is among  $x$ .  $y$  is either a bottom object or not. By assumption, it is not. Therefore, there is an object  $z$  which is among  $y$ .  $z$  is either a bottom object or not; and so on.

Now there are three possibilities: First, the chain of inclusion relations goes on *ad infinitum*. This might not seem to be a problem for the gunk theorist inasmuch as she does not worry about another infinite chain, the one of the parthood relation. Yet, it seems that the infinite regress of the inclusion relation is something she has to avoid. At some point, even the gunk theorist has to start her theory with some primitive terms. A particle physicist who believes that leptons and quarks are not mereological atoms but have proper parts, which in turn have again proper parts, and so forth, has to admit at some point, that she cannot name any further objects. Or think about it in a different way: Take a list of all the names – to keep things simple, let's focus on singular names only – the gunk theorist is using in her theory. Next, find out which names can be used to express truthfully that an inclusion relation holds between two objects. Surely this is not an impossible task, even if we assume that our world is gunky. The reason for this is that even when we assume the existence of gunky objects, it remains impossible for us to *use* infinitely many names. Whether we assume the existence of gunky objects or not does not affect that our abilities of reasoning and expressing ourselves are limited. This surely hints at a more complex problem for any theory that assumes the existence of gunky objects, or more generally, for any theory that assumes the existence of infinitely many objects: If the ontological commitments of a theory  $t$  is a list of those objects which have to exist for the theory to be true, and according to  $t$ , there exist infinitely many objects, then we will end up with an infinitely long list of ontological commitments. Although this is an issue we must not resolve here, let me point out that in the case of a gunky theory and its ontological commitments, this problem can be resolved, if Composition as Identity is assumed. It is not only the case that given the ontological commitment to the parts of a composite object, there is no additional commitment to the composite object. In

addition, it holds that given the ontological commitment to a composite object, there is no additional commitment to the existence of its parts. Hence, with Composition as Identity, the friend of gunky objects gets the existence of infinitely many objects without any additional commitment over and above the commitment to the composite object they are parts of.

But let's turn back to our question whether the notion of bottom objects is compatible with theories of gunk. My suggestion, that there cannot be an infinite chain of inclusion relations, even if the existence of gunky objects is assumed, is a pragmatic move: Although the gunk theorist believes in an infinite chain of the *parthood* relation, she cannot hold on to the claim of an infinite *inclusion* relation, because she has to use some primitive, singular terms. This is a linguistic limitation we have to face and which cannot be avoided. Even if we believe that there are infinitely many objects, we do not have the capacity to use all of their names. Since the notion of bottom objects is primarily intended to reflect a linguistic feature of a theory, it does not force us to beg the question against theories which postulate the existence of gunky objects.

Second, the chain of inclusion relations might be circular. We start with  $x$  not being a bottom object in  $t$ , which has  $y$  among it. Then,  $y$  is not a bottom object, and so forth, until we reach an object  $z$  which has  $x$  among it. This picture of reality is inconsistent with the principles we have agreed upon thus far, too. It contradicts our axioms (A9) and (A10), the asymmetry and transitivity<sup>119</sup> of ' $\prec$ ': If there are some objects  $x, y_1, y_2, \dots, y_n, z$ , such that there is chain of inclusion relations,  $y_1 \prec x \wedge y_2 \prec y_1 \wedge \dots \wedge z \prec y_n \wedge x \prec z$ , then it follows from (A10) that  $x$  is among  $z$  and  $z$  is among  $x$ . However, this is inconsistent with the asymmetry of the inclusion relation. Thus, there cannot be a circular inclusion-chain.

This leaves us with the third possibility; the chain of inclusion relations comes eventually to an end with some object  $z$ . In that case,  $z$  is

119. Although we will reject (A10) at the end of this chapter, we will hold on to a restricted form which states that ' $\prec$ ' is transitive when flanked by singular terms on either side. Hence, the restriction of ' $\prec$ ' will still allow for the inference which is sketched above.

a bottom object in  $t$ . Hence, our assumption that there are no bottom objects led us to its negation. Thereby, we have not only shown that a theory of gunk is consistent with the assumption that there are bottom objects, but even more, that it is impossible for a gunk theorist to avoid accepting bottom objects. The gunk theorist might regard them merely as fictitious objects, assuming their existence solely due to practical reasons. The particular details do not matter for us here. Important for us is only that a theory of a gunky universe is compatible with the view and that we are not presupposing an answer to the question of atomism.

Before we move on, let's pause for a moment and try to clarify the notion of a bottom object a bit further. At first sight, it might seem that the concept of a bottom object collapses with the mereological notion of an atom. Yet, I should stress two points. First, the introduction of the concept of bottom objects is mainly due to instrumental considerations and the fact that it gives our theory an algebraically neat structure, as we will see in chapter 8. In this respect, my considerations here are similar to the ones that are often taken into account when the notion of an empty individual, which is an object that is an improper part of any object, is postulated, (see, for instance, Carnap 1988: 35-9, and in particular Martin 1979: 82).

Second, the concept of mereological atoms is used to mark an *ontological* distinction, while the concept of bottom objects marks, first and foremost, a *linguistical*, and only in a derivative way an ontological, distinction. Unlike mereology, plural logic is primarily a theory that is based on a logico-linguistical distinction – the distinction between singular and plural *terms*. Naturally, this carries over to the basic relation, the inclusion relation, of plural logic and also to the concept of a bottom object, which is defined with the help of the inclusion relation. Hence, with the notion of a bottom object, I intend to capture a linguistic phenomenon of theories: a theory  $t$  uses a term  $x$ , which names an object  $y$ , and  $t$  does not use any term  $x'$ , which names an object  $y'$ , and  $y$  properly includes  $y'$ . With this in mind, let's move on to introduce a further concept which we will need in the next chapter.

The concepts of urelements and atoms can be used to introduce a further relation. In set theory,  $x$  is called a *minimal element of a set*  $y$  iff  $x$  is an element of  $y$  which does not have itself any elements. For set theories which are based on urelements, this means, trivially, that some object  $x$  is an urelement of a set  $y$  iff  $x$  is a minimal element of  $y$ . Similarly, we have in mereology the notion of an *atomic part of an object*, which is defined as follows:  $x$  is an atomic part of  $y$  iff  $x$  is a part of  $y$  and  $x$  is an atom. Interestingly, we can use our notion of *being a bottom object* to define a relation that is analogous to ‘being a minimal element of a set’ and ‘being an atomic part of an object’.

This relation will be important for the next chapter, where we will use it to define several kinds of identity relations. Its central idea is the following: A bottom object  $x$  of an object or some objects  $\alpha$  is a bottom object which we encounter, if we start with  $\alpha$  and keep asking *What’s among that?* until we get the answer *Nothing*. You can imagine this as a dialogue between two people, A and B. A asks *What is among these two decks of cards?* B answers – *Well, the red deck and the blue deck*. Then, A asks the rather odd sounding question *And what is among the blue deck?* Now, B seems to be in a position where several answers appear to be equally legitimate: *The four suits of cards, the two red suits and the two black suits, The cards from 2 to 9 and the cards from 10 to Ace, The 2 of Spades, the 3 of Spades, . . .*, and so on. But let’s suppose. B replies *The the two red suits and the two black suits*. A keeps on asking *And what is among the black suit?* B answers *The Queen of Spades*. *And what is among the Queen of Spades?* B finally says *Nothing*. According to (D13), the Queen of Spades is (for B) a bottom object. Moreover, the notion of *being a bottom object of* tells us that the Queen of Spades is a bottom object of the two decks of cards, but also of the blue deck, the four suits of the blue deck, and the two black suits of the blue deck for B.

In our formal language, we can define the relation ‘being a bottom object of’ as follows:

$$(D14) \quad BO(x, \alpha) =_{df} B(x) \wedge \exists y (x \preceq y \wedge y \preceq \alpha)$$

Thus,  $x$  is a bottom object of some object or some objects  $\alpha$  means that  $x$  is a bottom object and there is some  $y$  such that  $x$  is improperly among  $y$  and  $y$  is improperly among  $\alpha$ . The above definition might seem unnecessarily complicated, especially the use of a second singular variable ' $y$ ' in the definiens. The reason for this lies in the partial transitivity of inclusion, which we will discuss in the next section. Eventually, we might want to say that a bottom object is a bottom object of itself. Thus, since we use the improper inclusion predicate in the definiens, the worst that can happen is the following:  $x$  and  $y$  turn out to be identical, hence, the definiens simply collapses to ' $x$  is a bottom object and  $x$  is improperly among  $y$ ', in that case. This is not a disturbing outcome.

Before we move on to discuss the question whether the inclusion relation is transitive, note the following theorems that can be derived from (D14). First, 'being a bottom object of' is an antisymmetric, transitive, and not reflexive, i.e. neither reflexive nor irreflexive, relation. Second, every bottom object, and only they, are bottom objects of themselves.

Since postulating the existence of bottom objects does not exclude the existence of atoms or gunky objects, we can be reassured that we are not presupposing any answer to the question of atomism. Moreover, it emerged from our considerations on why the existence of bottom objects is consistent with theories of gunky objects that it is advisable to add a principle that states the existence of bottom objects. Hence, I think we should avoid adding a non-emptiness axiom for singular terms to our principles but should instead consider the following:

$$\forall\alpha\exists y(BO(y, \alpha))$$

Given what we have observed thus far about bottom objects, it strikes me that the claim, represented by the above formula, that any object either is a bottom object or has a bottom object among it, is acceptable. Hence, it will be one of the axioms of *LI*. This concludes our discussion about bottom objects and we turn back to the second question raised before we introduced this: Can an object  $x$  have only one object  $y$  among it?

## 7.4 Supplementation

By allowing for singular terms to enter the second argument place of the inclusion relation and motivating the thought that in some cases it can be true to say that there is an object  $x$  properly among an object  $y$ , the question arises whether it is possible that some object has only one object properly among it. A similar question arises in mereology, where one reflects whether it is possible for an object  $x$  to have one object  $y$  as proper part only, (see Casati and Varzi 1999: 38-42, Simons 2003: §1.5 and Varzi 2016: §3.1-3.3).

Given the idea of bottom objects and the thought that any object is either a bottom object or has a bottom object among itself, it appears odd to think that there can be only one object  $y$  properly among an object  $x$ . In that case, it seems difficult to distinguish between  $x$  and  $y$  in the first place. However,  $x$  cannot be identical to  $y$ , if the latter is among the former. That is excluded by the asymmetry of the inclusion predicate. Hence, we should adopt an axiom that avoids this odd outcome. We shall adopt the term ‘supplementation axiom’ for this principle, which is also used in mereology.

Let’s consider the two candidates for supplementation axioms discussed by Casati and Varzi (1999: 38-42),<sup>120</sup> and label them accordingly as “weak supplementation” and “strong supplementation”. An analogous version to the weak supplementation principle from mereology amounts in the terms of plural logic to the following claim: If  $x$  is properly among  $y$ , then there is a  $z$  such that  $z$  is improperly among  $y$  and there is no  $w$  such that  $w$  is improperly among  $z$  and  $z$  is among  $x$ . On the other hand, the version based on the strong supplementation from mereology amounts to, if  $x$  is not improperly among  $y$ , then there is a  $z_1$  such that  $z_1$  is improperly among  $x$  and there is not  $z_2$  such that  $z_2$  is among  $z_1$  and  $z_2$  is among  $y$ . In formal terms:

120. Further possible ways to spell out the above idea might be based on considering other principles discussed in (Casati and Varzi 1999: 38-42), (Simons 2003: §1.5) and (Varzi 2016: §3.1-3.3).

$$(WeS) \quad x \prec y \rightarrow \exists z_1(z_1 \preceq y \wedge \neg \exists z_2(z_2 \preceq z_1 \wedge z_2 \preceq x))$$

$$(StS) \quad \neg(x \preceq y) \rightarrow \exists z_1(z_1 \preceq x \wedge \neg \exists z_2(z_2 \preceq z_1 \wedge z_2 \preceq y))$$

I shall consider taking the strong supplementation as one of the principles on board. Analogous to its mereological counterpart, it has the advantage of capturing the idea that if some object  $x$  is *not* among an object  $y$ , then there must be remainder, some object  $z$  which makes the difference between  $x$  and  $y$ , (see Varzi 2016: §3.2). On the other hand, the weak supplementation principle tells us only that if  $x$  is properly among  $y$ , then there is some object  $z$  which makes the difference between the former two. It does not tell us anything about the objects  $x$  and  $y$ , which are absolutely distinct, i.e. if there is no object  $z$  which is among both, or if some, but not all objects which are among  $x$  are among  $y$ . It seems natural to assume in both cases that there is some remainder which makes the difference between  $x$  and  $y$ . Since (StS) is able to cover these cases, while (WeS) is not, the former is more natural to choose.

## 7.5 The Partial Transitivity of Inclusion

Finally, the last question we have to answer before we are prepared to develop our formal system concerns *FOP*'s axiom (A10), the transitivity of the inclusion predicate. Within the standard systems, this axiom does not cause any harmful troubles. Yet, the standard conception of plural logic makes the transitivity of ' $\prec$ ' an odd axiom and this suggests that we might have a closer look at it. I will suggest that the revision of our view on plural logic includes rephrasing the transitivity of ' $\prec$ '. Hence, we should consider that the inclusion predicate is transitive only if it operates between singular terms, but that it fails to be transitive when a plural term enters one of its argument places.

Let's first have a look at the oddities that arise from (A10) within the standard conception of plural logic. These arise from the conservative strategy from section 5.6: If an infinite hierarchy of terms, with infinitely many inclusion relations ' $\prec^1$ ', ' $\prec^2$ ', ' $\prec^3$ ', ... is postulated, one wonders

what is the point of having an axiom that postulates the transitivity of inclusion.<sup>121</sup> The idea of the transitivity of inclusion can be cashed out in two different ways in the standard framework. Both of them come with major shortcomings.

First, a transitivity axiom gets postulated for each inclusion relation:<sup>122</sup>

$$\alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^1 \gamma_3 \rightarrow \alpha_1 \prec^1 \gamma_3$$

$$\beta_2 \prec^2 \gamma_3 \wedge \gamma_3 \prec^2 \alpha_4 \rightarrow \beta_2 \prec^2 \alpha_4$$

⋮

In that case the axioms become pointless. They are vacuously true because their antecedents are necessarily false. Recall, the hierarchical solution is based on the claim that only sentences where an inclusion predicate takes a term from level  $n$  in its first and a term from level  $n + 1$  in its second argument place can be true. This implies that any sentence, which is adequately represented by an instance of an antecedent of the above formulas, is false. Whenever ' $\alpha_n \prec^n \beta_{n+1}$ ' represents a true sentence, ' $\beta_{n+1} \prec^n \gamma_{n+2}$ ' cannot represent a true sentence. If ' $\beta_{n+1}$ ' is a term from the level that can enter the second argument place, i.e. the first conjunct can be true, then ' $\beta_{n+1}$ ' cannot also be from the right level to go into the first argument place of the predicate, i.e. the second conjunct cannot be true.

Since the traditionalist might not want to blow up her theory with unnecessary axioms, she might try a different approach in the light of the above problem. She may revise the axioms of transitivity in such a way that the antecedent of the implication contains inclusion predicates from

121. See for that also the worries expressed by McKay (2006: 135-9).

122. The subscripts of the variables and constants in the formulas above have a twofold purpose: First, they indicate the level of the terms over which the variables range. Second, their use allows us to construct several distinct terms by the use of the same Greek letter.



different, neighboring levels. Hence, the antecedent of the first transitivity axiom might be ' $\alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^2 \gamma_3$ '. Thereby, the above problem of triviality can be avoided.

Nevertheless, this modification of the axioms comes with other problems. Although we can save the antecedents of the transitivity axioms from being trivially true, we cannot come up with the right consequents for them. If we have ' $\alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^2 \gamma_3$ ' as the antecedent of the first transitivity axiom, what inclusion predicate shall go into the consequent? It cannot be ' $\prec^1$ ', since ' $\gamma_3$ ' is not one of the terms that can be on its right side. Neither, can it be ' $\prec^2$ ', because ' $\alpha_1$ ' cannot be found on the level where the terms which can enter the first argument place of ' $\prec^2$ ' are located. In this context, one might be tempted to increase the number of inclusion relations. Given ' $\alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^2 \gamma_3$ ' and that among the infinitely many inclusion predicates, ' $\prec^1$ ', ' $\prec^2$ ', ' $\prec^3$ ', ..., which operate along the hierarchy of terms, there is not a single one that can connect ' $\alpha_1$ ' and ' $\gamma_3$ ', the traditionalist might stipulate that there is a further infinity of inclusion predicates, ' $\prec^1$ ', ' $\prec^2$ ', ' $\prec^3$ ', ..., such that they can take a term from level  $n$  in their first and a term from the level  $n + 2$  in their second argument place.<sup>123</sup> Thus, the transitivity axioms can be rephrased in the following way:

$$\begin{aligned} \alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^2 \gamma_3 &\rightarrow \alpha_1 \prec^1 \gamma_3 \\ \beta_2 \prec^2 \gamma_3 \wedge \gamma_3 \prec^3 \alpha_4 &\rightarrow \beta_2 \prec^2 \alpha_4 \\ &\vdots \end{aligned}$$

Be that as it may, the traditionalist has still not enough inclusion predicates at her disposal, if she wants to keep the unrestricted version of (A10). Her predicates cannot represent the inclusion relation that holds between ' $\alpha_1$ ' and ' $\alpha_4$ ', which given the idea that the transitivity of ' $\prec$ ' is

123. Note, that adding different inclusion predicates to the vocabulary has the consequence that not only different versions of the transitivity axiom have to be postulated, but in addition to that different versions of the asymmetry of inclusion, as well as of the comprehension, extensionality and the non-emptiness axiom.

unrestricted, should follow from ' $\alpha_1 \prec^1 \beta_2 \wedge \beta_2 \prec^2 \alpha_4$ '. To put things short, we can see – already from the traditional picture of plural logic – that the transitivity of ' $\prec$ ' is problematic. The traditionalist will unavoidably run into problems with (A11) due to the postulation of an infinite hierarchy of terms and an infinite number of inclusion predicates. Although we can avoid *these* problems, since we deny the existence of an infinite hierarchy of terms, we have to face another problem, which has its origin in the transitivity of ' $\prec$ '.

As we have seen in section 5.3.2, certain contexts suggest that the inclusion relation is transitive:

- (14) If Russell is among the logicians and the logicians are among the philosophers, then Russell is among the philosophers
- (15) If *PM* is one of the logic books and the logic books are some of the philosophy books, then *PM* is one of the philosophy books

However, we should resist concluding that ' $\prec$ ' is transitive:

- (16) If Rogers is among Rogers, Hammerstein, and Hart, and Rogers, Hammerstein, and Hart are among the people who wrote a musical, then Rogers is among the people who wrote a musical
- (17) The Queen of Spades is among the 52 cards, the 52 cards are among the objects which weigh more than 50 grams, the Queen of Spades is among the objects which weigh more than 50 grams

The problem with assuming the transitivity of inclusion is then the following. Although the antecedents of (16) and (17) are true, their consequents are false. If their consequents were true, then the following sentences could be derived from them together with the comprehension axiom:<sup>124</sup>

124. (18) follows from the consequent of (16) and the comprehension axiom, since if someone wrote a musical, then  $x$  is among the people who wrote a musical iff  $x$  wrote a musical. An analogous derivation for (19) from (17) is at hand.

- (18) Gilbert wrote a great comic opera
- (19) The Queen of Spades weighs more than 50 grams

These sentences are false: Gilbert never wrote a great comic opera alone, but only in collaboration with Sullivan. Rogers only wrote musicals together with Sullivan. A card does (usually) not weigh more than ten grams. Hence, we can see that postulating the transitivity of inclusion is problematic. In particular, if we have cases that fit one of the following two descriptions: First, an object,  $z$ , is among some objects,  $uu$ , and those are among some further objects,  $vv$ . The problem that can arise in those cases is that the latter mentioned plurality,  $vv$ , is a plurality of objects having a certain property  $\Phi$ . The former mentioned plurality,  $uu$ , might be among the  $vv$ , because they are *collectively*  $\Phi$ . Yet, that does not guarantee that each one of the objects among  $uu$  is  $\Phi$ . Second, some objects  $uu$ , are among some objects,  $vv$ , and furthermore, the latter are among some other objects,  $ww$ . Now again,  $ww$  is a plurality of objects having a certain property  $\Phi$ . Although  $uu$  is among the objects which are  $\Phi$ , because they are *collectively*  $\Phi$ , it is not the case that each one of the objects among  $uu$  is  $\Phi$ . The following examples illustrate the problem:

- (20) If the Italians are among the Europeans, and the Europeans are among the ethnic groups which have a population that exceeds 500 million people, then the Italians are among the ethnic groups that exceed 500 million people
- (21) If 2 and 4 are among the even numbers, and the even numbers are among the numbers whose sum exceeds 10, then 2 and 4 are among the numbers whose sum exceeds 10

As the examples and the previous analysis suggest, the obvious problem with the transitivity of the inclusion relation arises in connection with predicates that are collective in some of their argument places. So, it seems that we have two options: Either we reject the transitivity of ' $\prec$ ' altogether, or we restrict it in such a way that the problematic cases can

be avoided. The first option may be the safer option, yet I suspect that it is too radical. We might try to keep at least a restricted version. But, how can we restrict (A10) in order to avoid the above-mentioned counterexamples?

Since the counterexamples to (A10) only arise in connection with collective argument places, we have the option to restrict the axiom to singular terms only. By doing so, we can make sure that we will never run into a situation where some objects  $uu$  have collectively a property  $\Phi$ , i.e. are among  $vv$  which is the plurality of  $\Phi$ s, while each one of  $uu$  does not have said property, i.e. is not among the  $\Phi$ s. Thus, we will have a restricted version of the transitivity of the inclusion relation  $LI$ , which adheres to the thought that ' $\prec$ ' is transitive only in purely singular contexts, and may be *non-transitive*, i.e. neither transitive nor intransitive, if a plural term enters one of its argument places. Hence, we will reject (A10) from *FOP* and replace it with the following axiom, containing singular terms only:

$$x \prec y \wedge y \prec z \rightarrow x \prec z$$

We have now cleared the ground for the system  $LI$ , a logical framework that will be able to capture the idea that identity is a relation which comes in a variety of forms. At the beginning of this chapter, I argued for the claim that terms such as 'the pair of men who wrote *PM*', or 'the suit of cards', are singular terms. The former is not able to refer to more than one object, but refers at most to one object, the pair. With respect to the relation of the inclusion predicate, I suggested that we should be tolerant and allow for any term to enter each one of its argument places. Although this sounds counterintuitive at first, our decision to consider such terms as 'the pair of men who wrote *PM*', or 'the suit of cards' as singular terms makes this step easier to take. 'Russell is among the pair of men who wrote *PM*' or 'Russell and Whitehead are among the pair of men who wrote *PM*' are sensible sentences to utter. After introducing the predicates 'being a bottom object' and 'being a bottom object of', I provided reasons to assume that for any object  $x$ ,  $x$  is either a bottom object or has

a bottom object among it. Although this might look like a disguised form of mereological atomism, taking this principle as an axiom for our logic (LI) has been justified. The concept of being a bottom object is first and foremost a pragmatic notion, relative to a theory. What is a bottom object in one theory, must not be a bottom object in another theory. Finally, we formulated a supplementation axiom, which guarantees that objects, which are not bottom objects, have at least two different objects among them, and motivated a principle stating that the inclusion predicate is transitive only when it connects two singular terms. These principles will now be added to a fragment of *FOP* such that we can develop the system *LI*.



## CHAPTER EIGHT

# *A Logic for Identities: LI*

The goal of this chapter is to construct a formal system, *LI*, which allows us to capture the idea that identity comes in different varieties, whereby this assumption results from our analysis of the derivations of the contradictions from section 5.5. This system is an elaboration of *FOP*. The language of *FOP* and *LI* are the same, but given our previous considerations about identity, a different interpretation of '=' is intended. While in the former system, '=' is taken to represent *the* identity relation, which is what we have identified as the source of the contradictions, it is interpreted as representing a *kind* of identity relation, namely the singular one-one identity relation, in *LI*. Using the same symbol in the two different systems, yet interpreting them differently, allows us to carry over the inference rules and definitions, as well as some of the axioms and theorems of *LI*. However, since the symbol '=' is intended to be interpreted differently in the two systems some of the derivations of the theorems of *FOP* will not be possible in *LI*.

After spelling out the basis of *LI*, i.e. its language and inference rules, as well as the definitions and axioms, some of which carry over from *FOP*, while others are based on the considerations from the last chapter, we will define the predicates representing the different kinds of identity relations. These will then be used to define the predicate for the general identity relation ' $\equiv$ ' simply as the disjunction of the nine kinds of identity. Moreover, we will show that the reflexivity, asymmetry, and transitivity

of  $\equiv$  are theorems of *LI*, and sketch the proof for the derivations of these theorems and other lemmata. Eventually, we will show that *LI* can be used as a formal framework to provide us with a solution to the paradox from Composition as Identity and to avoid the derivation of Collapse. However, we are not yet able to reply to the second group of criticisms Composition as Identity has to face. These will be discussed in the next chapter. The appendix to this chapter contains some formal proofs for the theorems and lemmata of *LI*.

## 8.1 The Basis of LI

The object- and metalanguage of *LI* are the object- and metalanguage of *FOP*, see the introduction to chapter 5 and section 5.1. Please note again, that the primitive predicate '=' is intended to represent the *singular one-one identity relation*, as discussed in section 6.4. As a result of our discussion in chapter 6, we shall restrict substitution accordingly in a twofold way: If a predicate is intensional in an argument place, only rigid designators can be substituted in the relevant argument place. If a predicate is hyperintensional only terms whose reference is articulated in the same way can be substituted. Any terms can be substituted, if a predicate is extensional in the relevant argument place. Given the observations from section 6.3, I take it that these restrictions are justified, even if, as I have already remarked in section 2.1 and 6.1, this challenge to Leibniz's Law may be considered to be a big bullet to bite.

Since '=' represents singular identity, we can hold on to the inference rules of *FOP*, i.e. modus ponens, universal generalization, and a singularized version of substitution. I shall ignore, for the ease of exposition, that even in purely singular contexts, there may occur failures of substitution, as we have seen in section 6.3. We may assume, for simplicity's sake, that any singular term is a rigid designator. Thus, in *singular* contexts, substitution will be considered to be applicable *without* restrictions and we can hold on to the singularized version of substitution from section 5.2:



(SI) If  $\vdash x = y$  and  $\vdash \Phi(x)$ , then  $\vdash \Phi(y)$

We hold on to the definitions (D1) to (D10) of *FOP*, and add the definitions of ‘being an improper plurality’, ‘being a proper plurality’, ‘being a bottom object’, and ‘being a bottom object of’ from chapter 7 to *LI*:

(D11)  $IP(uu) =_{df} \forall x \forall y (x \prec uu \wedge y \prec uu \rightarrow x = y)$

(D12)  $PP(uu) =_{df} \neg IP(uu)$

(D13)  $B(x) =_{df} \forall y \neg (y \prec x)$

(D14)  $BO(x, \alpha) =_{df} B(x) \wedge \exists y (x \preceq y \wedge y \preceq \alpha)$

Apart from (A10), the unrestricted version of the transitivity of inclusion, which is replaced by the partial transitivity axiom (T $\prec$ ), and (A13), the extensionality axiom, which we abandon altogether, we can rely in *LI* on the axioms of *FOL*. (L=) is intended to represent the *singular* version of the law of identity, claiming that any *object* is *singularly* one-one identical to itself, and has to be kept apart from *FOL*’s (A4). Additionally, we have (BO), which guarantees that there is at least one bottom object for any object; (StS), the supplementation principle; and ( $\neq$ ), which says that no objects *uu* are singularly one-one identical to themselves, or to some object *x*, and allows for the syntactical distinction of our kinds of identity relations. This claim does not follow from the axioms or the definitions of the different kinds of identity relations.<sup>125</sup>

(L =)  $x = x$

(T $\prec$ )  $x \prec y \wedge y \prec z \rightarrow x \prec z$

(BO)  $\exists y (BO(y, \alpha))$

125. Since we define the different kinds of identity relations with the variables ‘ $\alpha$ ’ and ‘ $\beta$ ’, standing for singular and plural variables of the object language, we have to make sure to include this axiom to keep track of the *syntactical* distinction we want to make. This will be done by with the help of (L=) and ( $\neq$ ), since the former guarantees that any formula where ‘=’ takes the same singular variable in its argument places is true, while a formula where it is flanked by the same plural variable is false.

	<b>One-One</b>	<b>Many-One</b>	<b>One-Many</b>	<b>Many-Many</b>
<b>Singular</b>	$\alpha = \beta$	–	–	–
<b>Plural</b>	$\alpha \sim \beta$	$\alpha \bar{\sim} \beta$	$\alpha \bar{\sim}^* \beta$	$\alpha \approx \beta$
<b>Plural-Singular</b>	$\alpha \cong \beta$	$\alpha \simeq \beta$	–	–
<b>Singular-Plural</b>	$\alpha \cong^* \beta$	–	$\alpha \simeq^* \beta$	–

Table 8.1: The Varieties of Identity: Generalized

$$(StS) \neg(x \preceq y) \rightarrow \exists z_1(z_1 \preceq x \wedge \neg \exists z_2(z_2 \preceq z_1 \wedge z_2 \preceq y))$$

$$(\neq) \neg(\alpha = uu)$$

The only theorem of *FOP* which can be adopted here, given the derivations from section 5.7, is the irreflexivity of inclusion:

$$(I\prec) \neg(\alpha \prec \alpha)$$

This is the basis from where we start our task to work out a formal system that captures the idea of a variety of identity relations. As a semantics of *LI*, we can use the semantics for *FOP* from section 5.4, with the only difference that the valuation of '=' for *LI* is the singular one-one identity relation and not, as in the semantics for *FOP*, the identity relation.

As I argued in chapter 6, there are – besides the general identity relation – nine different identity relations which are distinguished according to the kind of terms that are used to express that an identity holds, and the number of objects the used terms refer to. As a reminder of what lies

ahead of us consider table 8.1, which is a modified version of table 6.1, whereby the former uses the variables ' $\alpha$ ' and ' $\beta$ '.

From a dialectical point of view, it is nice to group our definitions along the lines of the semantical distinction of the kinds of identities, though this is simply a matter of convenience. Hence, we shall begin our series of definitions with the one-one identity relations.

## 8.2 One-One Identities

Let's start our series of identity definitions with defining some of the one-one identities. We are already familiar with our primitive relation, '=', and a few remarks about what differentiates it from the other kinds of identities should suffice here. First, a singular identity ' $\alpha = \beta$ ' is true iff ' $\alpha$ ' and ' $\beta$ ' are co-referring singular terms. Second, '=' is the only identity relation obeying substitution:

(=<sub>5</sub>) If it is derivable that  $\alpha = \beta$  and  $\alpha$  has the property  $\Phi$ , then it is derivable that  $\beta$  has  $\Phi$

Given that this relation is already well-studied in the literature, there is nothing much left for us to add here. So, we shall move on to define the other one-one identity relations.

The above examples for the three kinds of one-one identities give us already a first impression of their definitions. Recall the examples for plural one-one identities:

- (1) The authors of *OD* are identical to the authors of *Marriage and Morals*
- (2) The marked cards are identical to the cards on the table

What makes (1) a plural one-one identity? First, it is a plural identity because the two terms used to express the identity are plural. Second, it is a one-one identity because the two terms refer to the same object, Russell.

How can we capture this within plural logic? The answer is straightforward: A plural one-one identity is true iff it holds for the referents,  $uu$  and  $vv$ , of the two plural terms, ' $uu$ ' and ' $vv$ ', which flank the identity predicate that anything that is among  $uu$  is among  $vv$ , and *vice versa*. More generally, we can define a plural one-one identity in the language of *LI* as:

$$(D15) \quad \alpha \sim \beta =_{df} \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge IP(\beta) \wedge \\ \wedge \forall x(x \prec \alpha \leftrightarrow x \prec \beta)$$

Similarly, we can easily come up with a definition of plural-singular one-one identities by considering the above examples

(3) The authors of OD are identical to Russell

(4) The cards on the table are identical to the Queen of Spades

(3) is a plural identity because we have a plural term, 'the authors of OD' in the first and a singular term 'Russell' in the second argument place of the identity predicate. Furthermore, it is a one-one identity because both terms refer to the same object, Russell. Since both terms have the same referent, (3) is true. Hence the definition of a plural one-one identity states that a plural-singular one-one identity holds iff we have an improper plurality  $uu$  and  $x$  is among  $uu$ :

$$(D16) \quad \alpha \cong \beta =_{df} \neg(\alpha = \alpha) \wedge \beta = \beta \wedge IP(\alpha) \wedge \beta \prec \alpha$$

The definition of singular-plural one-one identities simply changes the arguments of the above definition: A singular-plural one-one identity holds iff  $x$  is among the improper plurality  $uu$ :

$$(D17) \quad \alpha \cong^* \beta =_{df} \alpha = \alpha \wedge \neg(\beta = \beta) \wedge IP(\beta) \wedge \alpha \prec \beta$$

Since our conjunction commutes, we will be able to show that a plural-singular one-one identity holds between  $uu$  and  $x$  iff a singular-plural one-one identity holds between  $x$  and  $uu$ . This is our first lemma:

$$(L1) \quad uu \cong x \leftrightarrow x \cong^* uu$$

These are all the one-one identity relations we need and we move on to the many-one and one-many identities.

### 8.3 Many-One and One-Many Identities

The definitions of the various kinds of one-one identities are pretty straightforward. Things are a little bit more difficult with the many-one identity and one-many identity relation, which we are about to define. However, our definition (D14) gives us the necessary means to overcome these difficulties.

We shall first have again a look at some of the examples from the previous chapters, before we give a formal definition of the different kinds of relations.

- (5) Russell and Whitehead are identical to the pair of men who wrote *PM*
- (6) The 13 black cards are identical to the suit of Spades

Here again, we have to ask what makes, for instance, (5) a true plural-singular many-one identity? First, it is a plural-singular identity because we have a plural term, 'Russell and Whitehead', in the first, and a singular term, 'the pair of men who wrote *PM*' in the second argument place of the identity predicate. Second, the identity which according to (5) holds is a many-one identity because the term in the first argument place refers to many objects, while the term in the second argument place refers to one object. Finally, why is (5) true? We can explain its truth with the relation *being a bottom object of*: The referents of the two terms 'Russell and Whitehead' and 'the pair of men who wrote *PM*' share their bottom objects. Whatever is a bottom object of Russell and Whitehead, is a bottom object of the pair of men who wrote *PM*, and *vice versa*.

Since the definition of plural-singular many-one identity is a central element of *LI* and the remaining definitions of this section will piggy-back on it, let's have a look at (6) as well. There, we have a plural term, 'the 13 black cards', which refers to many object, and a singular term, 'the suit of Spades', referring to one thing. The 13 black cards and the suit of Spades share all their bottom objects, because if some object is a bottom object of the cards, then it is a bottom object of the suit, and if it is a bottom object of the suit, then it is a bottom object of the cards.

Finally, before we come to the formal definition, let's have a look at a negative example. 'The seven cans' is a plural term that refers to many objects, and 'the six-pack' is a singular term referring to only one object. Nevertheless, the sentence

(7) The seven cans are identical to the six-pack

is *not* a true plural-singular many-one identity because there is at least one object which is a bottom element of the seven cans and not of the six-pack, for instance, the seventh can, or its pull tab, or one of the atoms of can number seven.

Our formal definition of plural-singular many-one identity goes as follows:

$$(D18) \alpha \simeq \beta =_{df} \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge \forall y (BO(y, \alpha) \leftrightarrow BO(y, \beta))$$

The examples we gave for plural many-one identities, for instance

(8) Russell and Whitehead are identical to the pairs of men who wrote *PM*

(9) The 13 black cards are identical to the suits of Spades

give us already a hint at the fact that the concept of plural-singular identity will be used in the definition of plural many-one identity: A plural many-one identity holds, if we have two plural terms '*uu*' and '*vv*' connected by the identity predicate, whereby '*uu*' refers to many objects and

' $vv$ ' refers to one object. Hence, we have a proper plurality referred to by the first, and an improper plurality referred to by the second term. A sentence meeting these criteria is true iff the object  $x$ , which is among the improper plurality  $vv$ , stands in a plural-singular many-one identity to the ' $uu$ '.

$$(D19) \quad \alpha \approx \beta =_{df} \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge IP(\beta) \wedge \\ \wedge \forall x(x \prec \beta \leftrightarrow \alpha \simeq x)$$

In the case of our example (8), the last criteria mentioned above is met because the only object that is among the pairs of men who wrote  $PM$  is the pair of men who wrote  $PM$  and this stands in a plural-singular many-one identity to Russell and Whitehead. Hence, we can conclude that Russell and Whitehead stand in a plural many-one identity to the pairs of men who wrote  $PM$ .

The next two definitions are then pretty straightforward. Analogous to the plural-singular many-one identity, we will say that a singular-plural one-many identity holds between  $x$  and  $uu$  iff  $uu$  is a proper plurality and  $uu$  and  $x$  share all their bottom elements with each other.

$$(D20) \quad \alpha \simeq^* \beta =_{df} \alpha = \alpha \wedge \neg(\beta = \beta) \wedge PP(\beta) \wedge \forall y(BO(y, \beta) \leftrightarrow BO(y, \alpha))$$

Finally, similarly to (D19), we will stipulate that a plural one-many identity holds between  $uu$  and  $vv$  iff  $uu$  is an improper plurality,  $vv$  is a proper plurality, and the only object that is among  $uu$  stands in a plural-singular many-one identity relation to  $vv$ .

$$(D21) \quad \alpha \approx^* \beta =_{df} \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge PP(\beta) \wedge \\ \wedge \forall x(x \prec \alpha \leftrightarrow \beta \simeq x)$$

We note from these four definitions that the concepts defined form two pairs of, in principle, interdefinable relations:  $uu$  and  $x$  stand in a plural-singular many-one identity to each other iff they stand in a singular-plural one-many identity relation; and,  $uu$  and  $vv$  stand in a plural many-

one identity to each other iff they stand in a plural one-many identity relation. As we defined them in a different way, these interrelations can be demonstrated and are reflected by the following two lemmata:

$$(L2) \quad uu \simeq x \leftrightarrow x \simeq^* uu$$

$$(L3) \quad uu \simeq vv \leftrightarrow vv \simeq^* uu$$

## 8.4 Many-Many Identities

Finally, we come to the definition of our last kind of identity relation, before we can define the general identity relation. Prior to coming to the formal definition, I would like to discuss some examples of plural many-many identities, which might raise the suspicion that it is not possible to come up with an adequate definition of plural many-many identity such that all of them turn out to be plural many-many identities. Consider the following three sentences:

(10) Russell and Whitehead are identical to the authors of *PM*

(11) The 26 cards are identical to the two suits

(12) The five suits and the 39 cards are identical to the two decks

Given the formal apparatus developed thus far, we may note that although the above sentences qualify as plural many-many identities, in each case we have two plural terms which both refer to more than one object, they all seem to be true in virtue of facts which suggest structural differences: (10) straightforwardly qualifies as true, due to the extensionality axiom; (11) will turn out to be true, due to the plural-singular many-one identities that hold between the first sixteen cards and the first suit and the second sixteen cards and the second suit; similarly, (12) will have to be explained, though not only, on the basis of a plural-singular many-one identity.



Although it is possible to distinguish between different kinds of plural many-many identity relations, it does not seem that this has an important advantage. Hence, since we have the possibility to define this relation in a way that the sentences (10) to (12) will turn out to fulfill the requirements to be a plural many-many identity, we might as well spare the troubles to come up with several definitions. The key concept of the definition is again the relation of being a bottom object of an object: A plural many-many identity holds between  $uu$  and  $vv$  iff both are proper pluralities and share their bottom objects:

$$(D22) \quad \alpha \approx \beta =_{df} \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge PP(\beta) \wedge \\ \wedge \forall x (BO(x, \alpha) \leftrightarrow BO(x, \beta))$$

It can be seen that our above examples qualify as plural many-many identities, since Russell and Whitehead, the 26 cards, and the five suits and the 39 cards share their bottom elements with the authors of *PM*, the two suits, and the two decks, respectively. This is a nice feature of (D22) and it shows in addition that there is no need to invoke a variety of many-many identities. Now that we have our nine kinds of identity relations defined, we can move on to the general identity relation.

## 8.5 General Identity

We have now a clearer picture of what the variety of identity looks like and we can define the general identity relation, which we represent with ' $\equiv$ '. Already at the beginning of this chapter, I indicated that this will be done disjunctively on the basis of the different kinds of identity relations:

$$(D23) \quad \alpha \equiv \beta =_{df} \alpha = \beta \vee \alpha \sim \beta \vee \alpha \simeq \beta \vee \alpha \simeq^* \beta \vee \alpha \simeq^* \beta \vee \alpha \simeq \beta \vee \\ \vee \alpha \simeq^* \beta \vee \alpha \simeq \beta \vee \alpha \approx \beta$$

Let's turn to the logical properties of the general identity relation. As it is to be expected, it is an equivalence relation, i.e. it is reflexive, symmetric,

and transitive. Since we have an unrestricted form of substitution only for '=', it cannot be applied unrestrictedly for '≡'.

The reflexivity of '≡' is shown in two steps. The first step simply consists in bringing together (L=), the law of identity, with (D23). Thereby, we can conclude that each  $x$  stands in the general identity relation to itself. Secondly, we show that this holds for all any objects  $uu$  as well. This is done by arguing by cases:  $uu$  is either a proper or an improper plurality. If  $uu$  is an improper plurality, then it stands in the plural one-one identity relation to itself. If  $uu$  is a proper plurality, then it stands in the plural many-many identity relation to itself. In both cases, it follows from (D23) that  $uu$  is generally self-identical.<sup>126</sup> Hence, we conclude that '≡' is reflexive:

$$(R\equiv) \alpha \equiv \alpha$$

To show the symmetry of '≡', we argue by cases. Thereby, we rely on the lemmata (L1) to (L3), as well as on the following lemmata

$$(L6) \forall x \forall y (x \equiv y \leftrightarrow x = y)$$

$$(L7) \forall uu \forall vv (uu \equiv vv \leftrightarrow uu \sim vv \vee uu \simeq vv \vee uu \simeq^* vv \vee uu \approx vv)$$

$$(L8) \forall uu \forall x (uu \equiv x \leftrightarrow uu \cong x \vee uu \simeq x)$$

$$(L9) \forall uu \forall x (x \equiv uu \leftrightarrow x \cong^* uu \vee x \simeq^* uu)$$

which can be derived from ( $\neq$ ) together with the definitions (D15) to (D23), and help us keeping track of the syntactical distinction for identity relations. In a first step, we use the latter four lemmata to show that there are only nine possible ways – the nine kinds of identity relations – for ' $\alpha \equiv \beta$ ' to be true. Our lemmata (L1) to (L3), together with (D23), cover already six of these cases: If  $\alpha$  is singular-plural one-one, plural-singular one-one, singular-plural one-many, plural-singular many-one,

126. Moreover, we notice that the remaining cases of plural identity, i.e. plural many-one and one-many identity, are impossible because  $uu$  cannot be both a proper and an improper plurality, which is what they presuppose.

plural one-many, or plural many-one identical to  $\beta$ , then  $\beta$  is generally identical to  $\alpha$ . We are left with three cases to prove.

The first case, singular one-one identity, is itself symmetric, which is shown in the usual way with the help of substitution and the singularized law of identity. Secondly, we show with (D15) that a plural one-one identity between  $uu$  and  $vv$  entails the plural one-one identity between  $vv$  and  $uu$  because our conjunction commutes. Finally, a similar line of argument works for plural many-many identity. Hence, the three remaining cases entail general identities, and we have thereby shown the symmetry of  $\equiv$ :

$$(S\equiv) \alpha \equiv \beta \rightarrow \beta \equiv \alpha$$

Ultimately, we come to the transitivity of ' $\equiv$ '. With two different kinds of terms and nine different kinds of the identity relation, there is a large number of cases for ' $\alpha \equiv \beta \wedge \beta \equiv \gamma$ ' we have to take into account. However, we can save us a lot of work, since each identity predicate can only be true if it is flanked by the right kind of terms, as the lemmata (L6) to (L9) show. This leaves us with 45 cases, see the tables 8.2, 8.3, 8.4 and 8.5 in the appendix. Eighteen of these are trivial, since they involve a contradictory antecedent.<sup>127</sup> Hence, we have twenty-five cases to prove. These are still too many to sketch them all here. But let me briefly outline the general procedure that underlies the proof of (T $\equiv$ ).

After using the lemmata (L6) and (L9) to exclude cases where we have identities like ' $uu = vv$ ', or ' $x \approx uu$ ', we proceed with excluding the cases where we have a contradictory antecedent. Then, we group the contingent cases according to the types of names that enter the argument places of the identity relations. We make a first rough distinction by grouping them into cases where we have only singular, only plural, one singular

127. Consider, for instance, case 1. from table 8.2, where we have ' $uu \approx vv \wedge vv \approx ww$ '. It is false, since  $vv$  cannot be both a proper and an improper plurality, and it has to be the former in order for the second conjunct to be true, and the latter for the first conjunct to be true. Similarly, ' $uu \sim vv \wedge vv \approx ww$ ' is excluded, since it would only hold, if  $vv$  were a proper as well as an improper plurality.

and two plural, and one plural and two singular terms. The first group consists of one case only, since if ' $\alpha$ ', ' $\beta$ ', and ' $\gamma$ ' are all singular terms, then we can only have singular one-one identities holding between  $\alpha$  and  $\beta$ , and  $\beta$  and  $\gamma$ , due to (L6). In the usual way, we show then that  $\alpha$  stands in the singular one-one identity relation to  $\gamma$  with the help of the substitution of singular terms, and derive then from (D23) that they also stand in the general identity relation to each other.

The second group of cases comprises eight, the third group twelve, and the fourth group six cases. In all cases, we can show that if  $\alpha$  stands in the general identity relation to  $\beta$  and  $\beta$  stands in the general identity relation to  $\gamma$ , then  $\alpha$  stands in the general identity relation to  $\gamma$ , which means that we can prove the transitivity of  $\equiv$ :

$$(T\equiv) \alpha \equiv \beta \wedge \beta \equiv \gamma \rightarrow \alpha \equiv \gamma$$

Some of the cases are quite interesting, although it looks at first that it is not easy to find an instance of them from natural language. Let's have a look at one of them:

$$\#11 \quad uu \approx vv \wedge vv \cong z$$

We can find out which kind of identity relation holding between  $uu$  and  $z$  is entailed by the above by simply looking at the first and the last term of the formula, as well as considering the kinds of identity relations which hold between the objects to which they refer and the objects referred to by ' $vv$ ': First, ' $uu$ ' is plural and ' $x$ ' is singular. So the identity relation that holds between them is one of the two plural-singular identities. Second,  $uu$  stands in a many-one identity relation to  $vv$ . So it is a proper plurality.  $z$  stands in a one-one identity relation to  $vv$ . Hence, the identity relation between  $uu$  and  $z$  must be a plural-singular many-one identity. Indeed, this follows from the above conjunction in *LI* and we can therefore conclude with (D23) that if the above holds, then the  $uu$  and  $z$  stand in a general identity relation to each other.

Can we find an example that applies to the real world matching up with the above case? Here is one: Russell and Whitehead are many-one

identical to the pairs of men who wrote *PM*. The pairs of men who wrote *PM* are one-one identical to the pair of men who wrote *PM*. Therefore, Russell and Whitehead are many-one identical to the pair of men who wrote *PM*.

Before we move on to see how we are now in a position to address the criticisms from the sections 2.1 and 2.2, I would like to draw your attention to three particular cases which are involved in the proof of  $(T\equiv)$ . Showing that these cases are derivable within *LI* is not that straightforward, and we have to make use of the full resources of the system. A brief discussion of the derivation of these cases allows us to note a particular feature of the system *LI*. It turns out that *LI* is “[...] hyper-extensional in Goodman’s sense: things built up from exactly the same [objects] are identical” (Casati and Varzi 1999: 49).

The above-mentioned cases are #15, #16, and #25, see tables 8.4 and 8.5:

$$\#15 \quad uu \cong y \wedge y \simeq^* ww \rightarrow uu \sim^* ww$$

$$\#16 \quad uu \simeq y \wedge y \cong^* ww \rightarrow uu \sim ww$$

$$\#25 \quad x \simeq^* vv \wedge vv \simeq z \rightarrow x = z$$

They represent the following English claims:

#15’ If some *uu* are one-one identical to *y* and *y* is one-many identical to the *ww*, then the *uu* are one-many identical to the *ww*

#16’ If some *uu* are many-one identical to *y* and *y* is one-one identical to the *ww*, then the *uu* are many-one identical to the *ww*

#25’ If *x* is one-many identical to the *vv* and the *vv* are many-one identical to *z*, then *x* is one-one identical to *z*

Consider #25 first. From the definitions of *LI* it follows that if *x* is one-many identical to the *vv*, and the *vv* are many-one identical to *z*, that *x* and *z* share all their bottom objects – that is, it holds for any *y* that *y* is

a bottom object of  $x$  iff it is a bottom object of  $z$ . This entails that  $x$  is one-one identical to  $z$ , which is the lemma (L12) below, derivable from the two lemmata (L10) and (L11):

$$(L10) \quad \forall x \forall y (BO(x, y) \rightarrow x \preceq y)$$

$$(L11) \quad \forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x \preceq y)$$

$$(L12) \quad \forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x = y)$$

According to (L10), any bottom object  $x$  of  $y$  is improperly among  $y$ . This follows from the restricted transitivity of ' $\prec$ ' and the definition of 'being a bottom object of'. (L11) tells us that if two objects  $x$  and  $y$  share their bottom objects, then  $x$  is improperly among  $y$ . This follows from (L10), the supplementation axiom and the axiom (BO), which guarantees that for any object there is at least one bottom object. Finally, we use two instances of (L11) and the antisymmetry of ' $\preceq$ ' to show that if  $x$  and  $y$  share their bottom objects, then they are one-one identical.

Case #25 can then be used to show that #15 holds: First we show that if  $uu$  is one-one identical to  $y$ , and  $y$  is one-many identical to  $ww$ , with substituting for the *singular* term ' $y$ ', that any object among the  $uu$  is many-one identical to  $ww$ .<sup>128</sup> Then we assume for an arbitrary  $x$  that it is many-one identical to  $ww$ . With #25 it follows then from the assumption that  $x$  is one-one identical to  $y$ , so that we can conclude with the previously shown that it holds for any  $x$  that  $x$  is among  $uu$  iff  $x$  is many-one identical to  $ww$ . It follows from the assumption that all the other conditions for a plural one-many identity between  $uu$  and  $ww$  hold. Finally, case #16 follows from #15 because the antecedents of both cases, as well as their consequents are logically equivalent according to the lemmata (L2) to (L4).

Before we move on to address two of the criticisms put forward against Composition as Identity, I would like to point out that it would

128. This follows immediately from the assumption together with the definition of plural-singular one-one identity and (L2).

be possible to define the notion of general identity and the different kinds of identity in (LI) with the notion of sharing bottom objects:  $\alpha$  and  $\beta$  are identical to each other iff  $\alpha$  and  $\beta$  have the same bottom objects. However, if we define the different kinds of identity in this way, it will be impossible for us to address the counterexamples from rearrangement put forward against Composition as Identity. The central issue with these counterexamples is that two distinct objects share all their bottom objects. Hence, to spare the trouble of redefining general identity and the kinds of identity, I decided to use the definitions just presented.

## 8.6 Meeting two Criticisms

*LI* finally allows us to reconsider two of the criticisms put forward against Composition as Identity: the paradox for Composition as Identity and the derivation of Collapse. As we have seen in the discussion of these two criticisms in the sections 2.1 and 2.2, both make use of an unrestricted version of substitution. Let's rehearse the paradox in some detail first, and observe how we are now in a position to avoid the contradictory conclusions. Then we shall go on to see that *LI* allows us to avoid the derivation of the Collapse principle by the use of substitution, but that the transitivity of the general identity relation makes things not that straightforward as they might seem.

### 8.6.1 Solving the Paradox for Composition as Identity

The paradox for Composition as Identity concludes that a composite object is many and a composite object's parts are one. These two claims contradict two premises of the paradox: A composite object is one and a composite object's parts are many. The derivation of the two conclusions is made by assuming, besides these two premises, that 'being one' is the opposite of 'being many', and Composition as Identity's core claim that a composite object is identical to its parts.

The system *LI* provides us with a framework that gives us the means to formalize and address the paradox for Composition as Identity. What remains to be done is to introduce predicates representing the composition relation, as well as the relations ‘being one’ and ‘being many’. In the spirit of Composition as Identity, we shall define ‘composing’ as a plural-singular many-one identity. Some objects  $uu$  compose an object  $x$  iff the  $uu$  are plural-singular many-one identical to  $x$ :

$$(D24) \ C(uu, x) =_{df} uu \simeq x$$

(D24) gives us only a partial definition of composition, since there are also cases where many objects compose *many* objects, for instance, twenty-six cards composing two suits of cards. As explained in the preface, I think that tackling the concept of plural composition goes beyond our scope here. However, the inability to explain plural composition is not a genuine feature of Composition as Identity and other accounts of composition will struggle with the same problem. So, let’s set the concept of plural composition aside and turn back to the solution for the paradox for Composition as Identity.

By taking ‘being one’ and ‘being many’ as opposites of each other, they turn out to be interdefinable. Hence, taking either one of them as a primitive predicate would suffice. However, the resources of *LI* allow us to define ‘being one’ already. We shall say that an object  $x$  or some objects  $uu$ , for short, a thing  $\alpha$ , is one iff  $\alpha$  is one-one identical to  $\alpha$ , or  $\alpha$  is an improper plurality. ‘Being many’ amounts then for some  $\alpha$  to not being one-one identical to  $\alpha$  or being a proper plurality.

$$(D25) \ O(\alpha) =_{df} \alpha = \alpha \vee IP(\alpha)$$

$$(D26) \ M(\alpha) =_{df} \neg O(\alpha)$$

Given these three definitions, we cannot only formalize the paradox of Composition as Identity, we can also see that its premises follow from extending *LI* with these three definitions:



1.  $\forall x \forall uu (C(uu, x) \leftrightarrow uu \simeq x)$
2.  $\forall x (\exists uu (C(uu, x)) \rightarrow O(x))$
3.  $\forall uu (\exists x (C(uu, x)) \rightarrow M(uu))$
4.  $\forall \alpha (O(\alpha) \leftrightarrow \neg M(\alpha))$

Premise 1. is our definition of composition written as a universally closed formula. From the definition of ‘being one’ and the Law of Identity, any object is identical to itself, it follows that *any* object is one. This entails trivially the second premise, any composite object is one. The third premise, any objects which compose some object are many, follows from the definitions of composition (D24), plural-singular many-one identity (D18), and the definition of ‘being many’: If some objects  $uu$  compose an object  $x$ , then they are many-one identical to it. The  $uu$  being many-one identical to  $x$  implies that the  $uu$  are a proper plurality. Hence, they are not an improper plurality. Moreover, since they cannot be one-one identical to themselves, it follows that they are not one, i.e. they are many. The final premise 4. follows from the definitions just given.

In order to get to the conclusions of the paradox, a composite object is many, and a composite object’s parts are one

- 5.a  $\forall x (\exists uu (C(uu, x)) \rightarrow M(x))$
- 5.b  $\forall uu (\exists x (C(uu, x)) \rightarrow O(uu))$

we have to use an unrestricted version of substitution, which allows us to substitute a singular term for a plural term and *vice versa*. In section 2.1, we inferred 5.a from 1. and 3. However, this inference is not valid in *LI* since it would involve substituting the singular term ‘ $x$ ’ for the non-rigid designator ‘ $uu$ , ‘ $x$ ’s parts’’. This is not legitimate, since the predicate ‘being many’ is obviously collective – it does not hold that some things  $uu$  are many iff any object among the  $uu$  is many – and thus, non-extensional in its only argument place.

Similarly, we cannot derive 5.b from 1. and 2. in *LI*, since that would have to be done by substituting the non-rigid designator '*uu*' for the singular term '*x*'. This is not allowed either, since the predicate 'being one' is, and that might strike as a surprise, collective and hence, non-extensional in its only argument place. Intuitively, one may think that 'being one' is a distributive predicate. However, that is not the case. If 'being one' were distributive, then it would follow for any *uu*, the *uu* are one iff it holds for any object *x*, if *x* is among the *uu*, then *x* is one. Now, since we agreed that every object is one, it is trivial that any object that is among the *uu* is one. This would then lead us, together with the above biconditional, to the conclusion that any plurality of objects *uu* is one. However, that would lead plural logic *ad absurdum*. Thus, the predicate 'being one' must be non-extensional in its only argument place, (see also Oliver and Smiley 2013: §7.4, for more on numerical predicates being collective in their argument places).

Hence, we can see that our system *LI* allows us to hold on to Composition as Identity and to show that the derivation of the paradox of Composition as Identity can be blocked: A composite object being identical to its parts does not entail that the composite object is many or its parts are one. Although the composite object is one and its parts are many, the paradoxical conclusions cannot be derived, since substitution can only be applied in singular contexts, i.e. in cases where we have an identity claim stating that *an* object is identical to *an* object. Therefore, we can hold on to the claim that a composite object is identical to its parts, and moreover, that any object is one.

### 8.6.2 Avoiding Collapse

We have seen in section 2.2 that within the traditional framework of Classical Extensional Mereology and the standard account of plural logic,<sup>129</sup>

129. Although we have not officially the mereological concepts, which are necessary in order to derive Collapse, at hand in *LI*, we shall use Sider's framework for the sake of exposition, as we did in section 2.2. We will see in section 10.6.2 how, given the account

Composition as Identity entails Collapse. The Collapse principle states that an object is improperly among some objects iff it is an improper part of the fusion of these objects. For instance, an object  $x$  is improperly among the plurality of logic books iff it is an improper part of the fusion of the plurality of logic books. This is problematic, since the fusion of the plurality of logic books has at least one improper part, for instance itself, which is not a logic book. Therefore, the derivation of Collapse puts pressure on Composition as Identity. It suggests that there are fewer pluralities than the comprehension axiom from plural logic tells us. Thus, we need a way to avoid the conclusion of this principle. By restricting the use of substitution as suggested in section 6.3 and chapter 8, we can avoid Sider's derivation of the principle, because it relies on a substitution inference, which is not legitimate in *LI*.

Let's briefly follow some of the steps of the derivation of Collapse we have encountered in section 2.2 to see where the principles of our new framework come into effect, and how it eventually help us to avoid Collapse: Suppose,  $x$  is the fusion of some  $uu$ . Then, it follows from the definition of 'fusion', that any  $y$  which is among the  $uu$  is an improper part of  $x$ . This is one direction of the biconditional we have to show. Next, assume that  $y$  is an improper part of  $x$ . With plural covering, we can deduce from this that there are some objects  $vv$ , such that  $x$  is the fusion of the  $vv$ , and  $y$  is one of the  $vv$ . First, this gives us, with Composition as Identity, that  $x$  is identical to the  $vv$ , since it is the fusion of the  $vv$ . Because we started with the assumption that  $x$  is the fusion of the  $uu$ , we get additionally, again with Composition as Identity, that  $x$  is identical to the  $uu$ .

Here is where our restriction comes into play for the first time. In Sider's original derivation from section 2.2, substitution is then used to conclude from the identity between  $x$  and the  $uu$  and the identity between  $x$  and the  $vv$  that the  $uu$  are identical to the  $vv$ . However, in *LI* this

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of Composition as Identity we develop there, an introduction of mereological notions might be done. Such a mereological system will be able to avoid Collapse, since it will be non-extensional because it is based on a non-extensional notion of identity.

inference is only valid, if the substituted terms are rigid designators, since identity is a predicate that is intensional in both of its argument places, so that substituting either the plural term  $uu$  for  $x$ , or the plural term  $vv$  for  $x$  may not be legitimate. For now, we shall not worry whether one of the two plural terms is a non-rigid designator – though we will come back to it soon – since a derivation of the above identity can be done in another way.

From the assumption that  $x$  is identical to the  $uu$  and to the  $vv$ , it follows that the  $uu$  are identical to the  $vv$ . This is case #15 in the proof for the transitivity of ' $\equiv$ ', and it shows us that within *LI* we can deduce that the  $uu$  are many-many identical to the  $vv$ , if the  $uu$  are many-one identical to  $x$  and  $x$  is many-one identical to the  $vv$ , without substituting one of the plural terms. Therefore, our restriction on substitution does not help us to avoid deriving *this* line of the proof and it looks like we end up with Collapse after all. But we are not yet there.

The derivation of Collapse makes use of substitution for a second time. From the above identity between  $uu$  and  $vv$ , so the proof goes, we can conclude that  $y$  is among the  $uu$ , since  $y$  is among the  $vv$ . However, this means that a plural term  $uu$  is substituted for a plural term  $vv$  in the second argument place of the inclusion predicate. Again, since the inclusion predicate is collective, and hence, intensional in its second argument place, this inference is only legitimate, if both of the plural terms are rigid designators. Because the derivation of Collapse is done in a purely formal manner, we have no *prima facie* evidence whether this is the case or not. However, since Collapse is a universally quantified formula, it suffices to show that there is only one counterexample in order to undermine the principle.

This can be done easily. We simply take a plural, non-rigid designator, say, 'the authors of *PM*', and another plural term, 'Russell and Whitehead'. Then, we simply follow the steps of the alternative derivation from above, until we reach the plural identity from above, the  $uu$  are identical to the  $vv$ , and the claim that states that  $y$  is among the  $vv$ . In the case of our example, the identity holds between the authors of *PM* and

Russell and Whitehead, and the second claim is that  $y$  is among Russell and Whitehead. However, we also see that we cannot substitute the two plural terms in that sentence because, as we have already pointed out previously, inclusion is collective in its second argument place, and since 'the authors of *PM* is a non-rigid designator, substitution is not a legitimate inference. Thus, we have shown that Collapse is not a consequence of Composition as Identity within the system *LI*. Consequently, a lot of pressure can be taken from Composition as Identity because we are able to avoid this worrisome principle.

Although the reason why Collapse does not follow from our account of Composition as Identity are *formal*, namely the restriction on substitution, there is a *philosophical* lesson we can draw that might be worth to highlight again, since it will still seem difficult to get use to it. As I have already emphasized at several occasions in chapter 6, one of the motivations to restrict substitution is that by including plural terms, and thereby the phenomenon of predicates being collective in argument places, into our logical framework, we have to consider that some of the lessons we learned in singular contexts may fail. In particular, I think, we have seen that there are good reasons to think that predicates which are collective in some of their argument places, such as identity in both, or inclusion in its second argument place, may behave differently than expected.

The aim of this chapter was to develop a formal system capable of incorporating the idea that identity is a relation that comes a variety of ways. We spelled out the basic framework at the beginning of this chapter, which was followed by the definitions of the different kinds of identity relation in the sections 2 to 4. After defining the general identity relation disjunctively, we proved that it is a reflexive, symmetric and transitive relation. Eventually, we saw that *LI* allows us to solve the paradox for Composition as Identity and to avoid the derivation of Collapse. Thereby, two of the criticisms we have discussed in chapter 2 can be met. However, we are still not in a position to address the counterexamples based on rearrangement. The system *LI* represents, as we will see in the next chapter, a view which is based on a picture of reality that is based

too much on extensional considerations. We shall make some first steps towards overcoming this view in order to be able to avoid being committed reverse mereological essentialism and thereby addressing another criticism against Composition as Identity.

## 8.7 Appendix: Proofs of Theorems

Consider the remarks preceding the proofs for *FOP* in section 5.7. The proof for the irreflexivity of  $\prec$

$$(I\prec) \neg(\alpha \prec \alpha)$$

can be carried out in *LI* in the same way as in *FOP*. Moreover, we can derive the singularized versions of the *FOP* theorems (T3), (T4), (T6) and (T7), the asymmetry and the transitivity of  $\preceq$ ,

$$(S=) \forall x \forall y (x = y \rightarrow y = x)$$

$$(T=) \forall x \forall y \forall z (x = y \wedge y = z \rightarrow x = z)$$

$$(A\preceq) \forall x \forall y (x \preceq y \wedge y \preceq x \rightarrow x = y)$$

$$(T\preceq) \forall x \forall y \forall z (x \preceq y \wedge y \preceq z \rightarrow x \preceq z)$$

by mimicking parts of the derivations from section 5.7 and using the corresponding singularized versions of the axioms of (LI).

$$(L1) \forall uu \forall vv (uu \sim vv \leftrightarrow vv \sim uu)$$

1.  $\forall \alpha \forall \beta (\alpha \sim \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge IP(\beta) \wedge \wedge \forall x (x \prec \alpha \leftrightarrow x \prec \beta))$  [D15]
2.  $dd \sim ee$  [SC1]
3.  $ee \sim dd$  [SC2]
4.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge IP(dd) \wedge IP(ee) \wedge \wedge \forall x (x \prec dd \leftrightarrow x \prec ee)$  [1., 2.;  $\alpha$ -EX, PL, UE]
5.  $\forall x (x \prec ee \leftrightarrow x \prec dd)$  [4.; PL, UE, UG $\sqrt{}$ ]
6.  $\neg(ee = ee) \wedge \neg(dd = dd) \wedge IP(ee) \wedge IP(dd) \wedge \wedge \forall x (x \prec ee \leftrightarrow x \prec dd)$  [4., 5.; PL]
7.  $ee \sim dd$  [1., 6.;  $\alpha$ -EX, PL, UE]

8.  $dd \sim ee \rightarrow ee \sim dd$  [2.-7.; CI]
9.  $\neg(ee = ee) \wedge \neg(dd = dd) \wedge IP(ee) \wedge IP(dd) \wedge$   
 $\wedge \forall x(x \prec ee \leftrightarrow x \prec dd)$  [1., 3.;  $\alpha$ -EX, PL, UE]
10.  $\forall x(x \prec dd \leftrightarrow x \prec ee)$  [9.; PL, UE, UG $\sqrt$ ]
11.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge IP(dd) \wedge IP(ee) \wedge$   
 $\wedge \forall x(x \prec dd \leftrightarrow x \prec ee)$  [9., 10.; PL]
12.  $dd \sim ee$  [1., 11.; UE, PL]
13.  $ee \sim dd \rightarrow dd \sim ee$  [3.-12.; CI]
14.  $\forall uu \forall vv (uu \sim vv \leftrightarrow vv \sim uu)$  [8.-13.; PL, UG $\sqrt$ ]

(L2)  $\forall uu \forall x (uu \cong x \leftrightarrow x \cong^* uu)$

1.  $\forall \alpha \forall \beta (\alpha \cong \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge IP(\alpha) \wedge \beta \prec \alpha)$  [D16]
2.  $\forall \alpha \forall \beta (\alpha \cong^* \beta \leftrightarrow \alpha = \alpha \wedge \neg(\beta = \beta) \wedge IP(\beta) \wedge \alpha \prec \beta)$  [D17]
3.  $\neg(dd = dd) \wedge a = a \wedge dd \cong a \leftrightarrow IP(dd) \wedge a \prec dd$  [1.;  $\alpha$ -EX, PL, UE]
4.  $a = a \wedge \neg(dd = dd) \wedge IP(dd) \wedge a \prec dd \leftrightarrow a \cong^* dd$  [2.;  $\alpha$ -EX, PL, UE]
5.  $\forall uu \forall x (uu \cong x \leftrightarrow x \cong^* uu)$  [3., 4.; PL, UG $\sqrt$ ]

(L3)  $\forall uu \forall x (uu \simeq x \leftrightarrow x \simeq^* uu)$

1.  $\forall \alpha \forall \beta (\alpha \simeq \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge$   
 $\wedge \forall y (BO(y, \alpha) \leftrightarrow BO(y, \beta)))$  [D18]
2.  $\forall \alpha \forall \beta (\beta \simeq^* \alpha \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge$   
 $\wedge \forall y (BO(y, \alpha) \leftrightarrow BO(y, \beta)))$  [D20]
3.  $\neg(dd = dd) \wedge a = a \wedge dd \simeq a \leftrightarrow PP(dd) \wedge$   
 $\wedge \forall y (BO(y, dd) \leftrightarrow BO(y, a))$  [1.;  $\alpha$ -EX, PL, UE]
4.  $a = a \wedge \neg(dd = dd) \wedge a \simeq^* dd \leftrightarrow PP(dd) \wedge$   
 $\wedge \forall y (BO(y, dd) \leftrightarrow BO(y, a))$  [2.;  $\alpha$ -EX, PL, UE]
5.  $\forall uu \forall x (uu \simeq x \leftrightarrow x \simeq^* uu)$  [3., 4.; PL, UG $\sqrt$ ]



(L4)  $\forall uu\forall vv(uu\approx vv \leftrightarrow vv\approx^*uu)$

1.  $\forall\alpha\forall\beta(\alpha\approx\beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge IP(\beta) \wedge \wedge\forall x(x \prec \beta \leftrightarrow \alpha \simeq x))$  [D19]
2.  $\forall\alpha\forall\beta(\alpha\approx^*\beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge PP(\beta) \wedge \wedge\forall x(x \prec \alpha \leftrightarrow \beta \simeq x))$  [D21]
3.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge dd\approx ee \leftrightarrow PP(dd) \wedge IP(ee) \wedge \wedge\forall x(x \prec ee \leftrightarrow dd \simeq x)$  [1.;  $\alpha$ -EX, PL, UE]
4.  $\neg(ee = ee) \wedge \neg(dd = dd) \wedge ee\approx^*dd \leftrightarrow IP(ee) \wedge PP(dd) \wedge \wedge\forall x(x \prec ee \leftrightarrow dd \simeq x)$  [2.;  $\alpha$ -EX, PL, UE]
5.  $\forall uu\forall vv(uu\approx vv \leftrightarrow vv\approx^*uu)$  [3., 4.; PL, UG $\sqrt$ ]

(L5)  $\forall uu\forall vv(uu \approx vv \leftrightarrow vv \approx uu)$

1.  $\forall\alpha\forall\beta(\alpha \approx \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge PP(\beta) \wedge \wedge\forall x(BO(x, \alpha) \leftrightarrow BO(x, \beta)))$  [D22]
2.  $dd \approx ee$  [SC1]
3.  $ee \approx dd$  [SC2]
4.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge PP(dd) \wedge PP(ee) \wedge \wedge\forall x(BO(x, dd) \leftrightarrow BO(x, ee))$  [1., 2.; UE, PL]
5.  $\forall x(BO(x, ee) \leftrightarrow BO(x, dd))$  [4.; PL, UE, UG $\sqrt$ ]
6.  $\neg(ee = ee) \wedge \neg(dd = dd) \wedge PP(ee) \wedge PP(dd) \wedge \wedge\forall x(BO(x, ee) \leftrightarrow BO(x, dd))$  [4., 5.; PL]
7.  $ee \approx dd$  [1., 6.;  $\alpha$ -EX, PL, UE]
8.  $dd \approx ee \rightarrow ee \approx dd$  [2.-7.; CI1]
9.  $\neg(ee = ee) \wedge \neg(dd = dd) \wedge PP(ee) \wedge PP(dd) \wedge \wedge\forall x(BO(x, ee) \leftrightarrow BO(x, dd))$  [1., 3.; UE, PL]
10.  $\forall x(BO(x, dd) \leftrightarrow BO(x, ee))$  [9.; PL, UE, UG $\sqrt$ ]
11.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge PP(dd) \wedge PP(ee) \wedge \wedge\forall x(BO(x, dd) \leftrightarrow BO(x, ee))$  [9., 10.; PL]
12.  $dd \approx ee$  [1., 11.;  $\alpha$ -EX, PL, UE]
13.  $ee \approx dd \rightarrow dd \approx ee$  [3.-12.; CI2]
14.  $\forall uu\forall vv(uu \approx vv \leftrightarrow vv \approx uu)$  [8., 13.; PL, UG $\sqrt$ ]

(R $\equiv$ )  $\forall\alpha(\alpha \equiv \alpha)$ 

1.  $\forall x(x = x)$  [L =]
2.  $\forall uu(PP(uu) \leftrightarrow \neg IP(uu))$  [D12]
3.  $\forall\alpha\forall\beta(\alpha \sim \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge IP(\beta) \wedge \wedge\forall x(x \prec \alpha \leftrightarrow x \prec \beta))$  [D15]
4.  $\forall\alpha\forall\beta(\alpha \approx \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge PP(\beta) \wedge \wedge\forall x(BO(x, \alpha) \leftrightarrow BO(x, \beta)))$  [D22]
5.  $\forall\alpha\forall\beta(\alpha \equiv \beta \leftrightarrow \alpha = \beta \vee \alpha \sim \beta \vee \alpha \cong \beta \vee \alpha \cong^* \beta \vee \alpha \simeq \beta \vee \alpha \simeq^* \beta \vee \alpha \approx \beta)$  [D23]
6.  $a \prec dd \leftrightarrow a \prec dd$  [LT]
7.  $BO(b, dd) \leftrightarrow BO(b, dd)$  [LT]
8.  $\forall x(BO(x, dd) \leftrightarrow BO(x, dd))$  [7.; UG $\sqrt$ ]
9.  $PP(dd) \rightarrow dd \approx dd$  [4., 8.;  $\alpha$ -EX, PL, UE]
10.  $\forall x(x \prec dd \leftrightarrow x \prec dd)$  [6.; UG $\sqrt$ ]
11.  $IP(dd) \rightarrow dd \sim dd$  [3., 10.;  $\alpha$ -EX, PL, UE]
12.  $dd \sim dd \vee dd \approx dd$  [2., 9., 11.; UE, PL]
13.  $dd \equiv dd$  [5., 12.;  $\alpha$ -EX, PL, UE]
14.  $\forall uu(uu \equiv uu)$  [13.; UG $\sqrt$ ]
15.  $\forall\alpha(\alpha \equiv \alpha)$  [1., 14.; PL,  $\alpha$ -IN]

(L6)  $\forall x\forall y(x \equiv y \leftrightarrow x = y)$ 

1.  $\forall x(x = x)$  [L =]
2.  $\forall\alpha\forall\beta(\alpha \sim \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge IP(\beta) \wedge \wedge\forall x(x \prec \alpha \leftrightarrow x \prec \beta))$  [D15]
3.  $\forall\alpha\forall\beta(\alpha \cong \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge IP(\alpha) \wedge \beta \prec \alpha)$  [D16]
4.  $\forall\alpha\forall\beta(\alpha \cong^* \beta \leftrightarrow \alpha = \alpha \wedge \neg(\beta = \beta) \wedge IP(\beta) \wedge \alpha \prec \beta)$  [D17]
5.  $\forall\alpha\forall\beta(\alpha \simeq \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge \wedge\forall y(BO(y, \alpha) \leftrightarrow BO(y, \beta)))$  [D18]
6.  $\forall\alpha\forall\beta(\alpha \simeq^* \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge IP(\beta) \wedge \wedge\forall x(x \prec \beta \leftrightarrow \alpha \simeq x))$  [D19]

7.  $\forall\alpha\forall\beta(\beta \simeq^* \alpha \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge \wedge\forall y(BO(y, \alpha) \leftrightarrow BO(y, \beta)))$  [D20]
8.  $\forall\alpha\forall\beta(\alpha \simeq^* \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge PP(\beta) \wedge \wedge\forall x(x \prec \alpha \leftrightarrow \beta \simeq x))$  [D21]
9.  $\forall\alpha\forall\beta(\alpha \approx \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge PP(\alpha) \wedge PP(\beta) \wedge \wedge\forall x(BO(x, \alpha) \leftrightarrow BO(x, \beta)))$  [D22]
10.  $\forall\alpha\forall\beta(\alpha \equiv \beta \leftrightarrow \alpha = \beta \vee \alpha \sim \beta \vee \alpha \cong \beta \vee \alpha \cong^* \beta \vee \alpha \simeq \beta \vee \alpha \simeq^* \beta \vee \alpha \simeq\beta \vee \alpha \simeq^*\beta \vee \alpha \approx \beta)$  [D23]
11.  $a = b \rightarrow a \equiv b$  [10.;  $\alpha$ -EX, PL, UE]
12.  $a = a \wedge b = b$  [1.; UE, PL]
13.  $a \equiv b \rightarrow a = b \vee a \sim b \vee a \cong b \vee a \cong^* b \vee a \simeq b \vee a \simeq^* b \vee a \simeq\beta \vee a \simeq^*\beta$  [10.;  $\alpha$ -EX, PL, UE]
14.  $\neg(a \sim b)$  [2., 12.;  $\alpha$ -EX, PL, UE]
15.  $\neg(a \cong b)$  [3., 12.;  $\alpha$ -EX, PL, UE]
16.  $\neg(a \cong^* b)$  [4., 12.;  $\alpha$ -EX, PL, UE]
17.  $\neg(a \simeq b)$  [5., 12.;  $\alpha$ -EX, PL, UE]
18.  $\neg(a \simeq^* b)$  [6., 12.;  $\alpha$ -EX, PL, UE]
19.  $\neg(a \simeq\beta)$  [7., 12.;  $\alpha$ -EX, PL, UE]
20.  $\neg(a \simeq^*\beta)$  [8., 12.;  $\alpha$ -EX, PL, UE]
21.  $\neg(a \approx b)$  [9., 12.;  $\alpha$ -EX, PL, UE]
22.  $a \equiv b \rightarrow a = b$  [13.-21.; PL]
23.  $\forall x\forall y(x \equiv y \leftrightarrow x = y)$  [11., 22.; PL, UG $\sqrt{}$ ]

(L7)  $\forall uu\forall vv(uu \equiv vv \leftrightarrow uu \sim vv \vee uu \simeq vv \vee uu \simeq^* vv \vee uu \approx vv)$

1.  $\forall uu\neg(uu = uu)$  [ $\neq$ ]
2.  $\forall\alpha\forall\beta(\alpha \cong \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge IP(\alpha) \wedge \beta \prec \alpha)$  [D16]
3.  $\forall\alpha\forall\beta(\alpha \cong^* \beta \leftrightarrow \alpha = \alpha \wedge \neg(\beta = \beta) \wedge IP(\beta) \wedge \alpha \prec \beta)$  [D17]
4.  $\forall\alpha\forall\beta(\alpha \simeq \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge PP(\alpha) \wedge \wedge\forall y(BO(y, \alpha) \leftrightarrow BO(y, \beta)))$  [D18]
5.  $\forall\alpha\forall\beta(\alpha \equiv \beta \leftrightarrow \alpha = \beta \vee \alpha \sim \beta \vee \alpha \cong \beta \vee \alpha \cong^* \beta \vee \alpha \simeq \beta \vee \alpha \simeq^* \beta \vee \alpha \simeq\beta \vee \alpha \simeq^*\beta \vee \alpha \approx \beta)$  [D23]

6.  $dd \sim ee \vee dd \simeq ee \vee dd \simeq^* ee \vee dd \approx ee \rightarrow a \equiv b$  [5.;  $\alpha$ -EX, PL, UE]
7.  $\neg(dd = dd) \wedge \neg(ee = ee)$  [1.; UE, PL]
8.  $dd \equiv ee \rightarrow dd = ee \vee dd \sim ee \vee dd \cong ee \vee dd \cong^* ee \vee dd \simeq ee \vee dd \simeq^* ee \vee dd \simeq ee \vee dd \approx ee$  [5.;  $\alpha$ -EX, PL, UE]
9.  $\neg(dd \cong ee)$  [2., 7.;  $\alpha$ -EX, PL, UE]
10.  $\neg(dd \cong^* ee)$  [3., 7.;  $\alpha$ -EX, PL, UE]
11.  $\neg(dd \simeq ee)$  [4., 7.;  $\alpha$ -EX, PL, UE]
12.  $\neg(dd \simeq^* ee)$  [5., 7.;  $\alpha$ -EX, PL, UE]
13.  $dd \equiv ee \rightarrow dd \sim ee \vee dd \simeq ee \vee dd \simeq^* ee \vee dd \approx ee$  [8.-12.; PL]
14.  $\forall uu \forall vv (uu \equiv vv \leftrightarrow uu \sim vv \vee uu \simeq vv \vee uu \simeq^* vv \vee uu \approx vv)$  [6., 13.; PL, UG $\sqrt$ ]

(S $\equiv$ )  $\forall \alpha \forall \beta (\alpha \equiv \beta \rightarrow \beta \equiv \alpha)$

1.  $\forall x \forall y (x = y \rightarrow y = x)$  [L0]
2.  $\forall uu \forall vv (uu \sim vv \leftrightarrow vv \sim uu)$  [L1]
3.  $\forall uu \forall x (uu \cong x \leftrightarrow x \cong^* uu)$  [L2]
4.  $\forall uu \forall x (uu \simeq x \leftrightarrow x \simeq^* uu)$  [L3]
5.  $\forall uu \forall vv (uu \simeq vv \leftrightarrow vv \simeq^* uu)$  [L4]
6.  $\forall uu \forall vv (uu \approx vv \leftrightarrow vv \approx uu)$  [L5]
7.  $\forall x \forall y (x \equiv y \leftrightarrow x = y)$  [L6]
8.  $\forall uu \forall vv (uu \equiv vv \leftrightarrow uu \sim vv \vee uu \simeq vv \vee uu \simeq^* vv \vee uu \approx vv)$  [L7]
9.  $\forall uu \forall x (uu \equiv x \leftrightarrow uu \cong x \vee uu \simeq x)$  [L8]
10.  $\forall uu \forall x (x \equiv uu \leftrightarrow x \cong^* uu \vee x \simeq^* uu)$  [L9]
11.  $a \equiv b \rightarrow a = b$  [7.; UE, PL]
12.  $a = b \rightarrow b = a$  [1.; UE, PL]
13.  $b = a \rightarrow b \equiv a$  [7.; UE, PL]
14.  $\forall y (a \equiv y \rightarrow b \equiv y)$  [11.-13.; PL, UG $\sqrt$ ]
15.  $a \equiv ee \rightarrow a \cong^* ee \vee a \simeq^* ee$  [10.; UE, PL]
16.  $a \cong^* ee \rightarrow ee \cong a$  [3.; UE, PL]
17.  $a \simeq^* ee \rightarrow ee \simeq a$  [4.; UE, PL]
18.  $ee \cong a \vee ee \simeq a \rightarrow ee \equiv a$  [9.; UE, PL]

- |  |                              |
|--|------------------------------|
| 19. $\forall vv(a \equiv vv \rightarrow vv \equiv a)$  | [15.-18.; PL, UG $\sqrt$ ]   |
| 20. $\forall x\forall\beta(x \equiv \beta \rightarrow \beta \equiv x)$                             | [14., 19.; PL, $\alpha$ -IN] |
| 21. $dd \equiv b \rightarrow dd \cong b \vee dd \simeq b$  | [9.; UE, PL]                 |
| 22. $dd \cong b \rightarrow b \cong^* dd$  | [3.; UE, PL]                 |
| 23. $dd \simeq b \rightarrow b \simeq^* dd$  | [4.; UE, PL]                 |
| 24. $b \cong^* dd \vee b \simeq^* dd \rightarrow b \equiv dd$                                      | [10.; PL, UG $\sqrt$ ]       |
| 25. $\forall y(dd \equiv y \rightarrow y \equiv dd)$   | [21,-24; PL, UG $\sqrt$ ]    |
| 26. $dd \equiv ee \rightarrow dd \sim ee \vee dd \simeq ee \vee dd \simeq^* ee \vee dd \approx ee$ | [8.; UE, PL]                 |
| 27. $dd \sim ee \rightarrow ee \sim dd$  | [2.; UE, PL]                 |
| 28. $dd \simeq ee \rightarrow ee \simeq^* dd$  | [5.; UE, PL]                 |
| 29. $dd \simeq^* ee \rightarrow ee \simeq dd$  | [5.; UE, PL]                 |
| 30. $dd \approx ee \rightarrow ee \approx dd$  | [6.; UE, PL]                 |
| 31. $ee \sim dd \vee ee \simeq dd \vee ee \simeq^* d \vee ee \approx dd \rightarrow ee \equiv dd$  | [8.; UE, PL]                 |
| 32. $\forall vv(dd \equiv vv \rightarrow vv \equiv dd)$  | [26.-31.; PL, UG $\sqrt$ ]   |
| 33. $\forall uu\forall\beta(uu \equiv \beta \rightarrow \beta \equiv uu)$                          | [25., 31.; PL, $\alpha$ -IN] |
| 34. $\forall\alpha\forall\beta(\alpha \equiv \beta \rightarrow \beta \equiv \alpha)$               | [20., 33.; PL, $\alpha$ -IN] |

(L10)  $\forall x\forall y(BO(x, y) \rightarrow x \preceq y)$

- |   |                            |
|---|----------------------------|
| 1. $\forall x\forall y\forall z(x \preceq y \wedge y \preceq z \rightarrow x \preceq z)$                              | [T $\preceq$ ]             |
| 2. $\forall x\forall\alpha(BO(x, \alpha) \leftrightarrow B(x) \wedge \exists y(x \preceq y \wedge y \preceq \alpha))$ | [D14]                      |
| 3. $BO(a, b)$   | [SC]                       |
| 4. $\exists y(a \preceq y \wedge y \preceq b)$  | [1.; $\alpha$ -EX, PL, UE] |
| 5. $a \preceq c \wedge c \preceq b$   | [4.; EE $\sqrt$ ]          |
| 6. $a \preceq c \wedge c \preceq b \rightarrow a \preceq b$   | [1.; UE]                   |
| 7. $a \preceq b$  | [5., 6.; MP]               |
| 8. $\forall x\forall y(BO(x, y) \rightarrow x \preceq y)$   | [3.-7.; CI, UG $\sqrt$ ]   |

(L11)  $\forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x \preceq y)$

- |   |                             |
|---|-----------------------------|
| 1. $\forall \alpha \exists y (BO(y, \alpha))$   | [BO]                        |
| 2. $\neg(x \preceq y) \rightarrow \exists z_1 (z_1 \preceq x \wedge \neg \exists z_2 (z_2 \preceq z_1 \wedge z_2 \preceq y))$       | [StS]                       |
| 3. $\forall x \forall y (BO(x, y) \rightarrow x \preceq y)$   | [L10]                       |
| 4. $\forall x \forall \alpha (BO(x, \alpha) \leftrightarrow B(x) \wedge \exists y (x \preceq y \wedge y \preceq \alpha))$           | [D14]                       |
| 5. $\forall z (BO(z, a) \leftrightarrow BO(z, b)) \wedge c \preceq b$   | [SC1]                       |
| 6. $\forall z (BO(z, a) \leftrightarrow BO(z, b))$  | [SC2]                       |
| 7. $\exists y (BO(y, c))$   | [1.; $\alpha$ -EX, PL, UE]  |
| 8. $BO(d, c)$   | [7.; EE]                    |
| 9. $d \preceq c$  | [3., 8.; UE, MP]            |
| 10. $\exists y (d \prec y \wedge y \preceq b)$  | [5., 9.; PL, EG]            |
| 11. $BO(d, c) \leftrightarrow B(d) \wedge \exists y (d \prec y \wedge y \preceq c)$   | [4.; $\alpha$ -EX, PL, UE]  |
| 12. $B(d)$  | [8., 11.; PL]               |
| 13. $B(d) \wedge \exists y (d \prec y \wedge y \preceq b)$  | [10., 12.; PL]              |
| 14. $B(d) \wedge \exists y (d \prec y \wedge y \preceq b) \rightarrow BO(d, b)$   | [4.; $\alpha$ -EX, PL, UE]  |
| 15. $BO(d, b)$  | [13., 14.; MP]              |
| 16. $BO(d, a)$  | [5., 15.; UE, PL]           |
| 17. $d \preceq a$   | [3., 16.; UE, MP]           |
| 18. $\exists z_2 (z_2 \preceq c \wedge z_2 \preceq a)$  | [9., 17.; PL, EG]           |
| 19. $\forall z (BO(z, a) \leftrightarrow BO(z, b)) \wedge c \preceq b \rightarrow \exists z_2 (z_2 \preceq c \wedge z_2 \preceq a)$ | [5.-19; CI]                 |
| 20. $\neg(c \preceq b \wedge \neg \exists z_2 (z_2 \preceq c \wedge z_2 \preceq a))$  | [6., 19.; PL]               |
| 21. $\neg \exists z_1 (z_1 \preceq x \wedge \neg \exists z_2 (z_2 \preceq z_1 \wedge z_2 \preceq y))$                               | [20.; UG $\checkmark$ , PL] |
| 22. $a \preceq b$   | [2., 21.; UE, PL]           |
| 23. $\forall z (BO(z, a) \leftrightarrow BO(z, b)) \rightarrow a \preceq b$   | [6., 22.; CI1]              |
| 24. $\forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x \preceq y)$                                   | [6., 22.; CI2]              |

(L12)  $\forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x = y)$

- |  |                        |
|--|------------------------|
| 1. $\forall x \forall y (x \preceq y \wedge y \preceq x \rightarrow x = y)$                      | [A $\preceq$ ]         |
| 2. $\forall x \forall y (\forall z (BO(z, x) \leftrightarrow BO(z, y)) \rightarrow x \preceq y)$ | [L11]                  |
| 3. $\forall z (BO(z, a) \leftrightarrow BO(z, b))$   | [SC]                   |
| 4. $a \preceq b$   | [2., 3.; UE, PL]       |
| 5. $BO(c, b) \leftrightarrow BO(c, a)$   | [3.; UE, PL]           |
| 6. $\forall z (BO(z, b) \leftrightarrow BO(z, a))$   | [5.; UG $\checkmark$ ] |
| 7. $b \preceq a$   | [2., 6; UE, PL]        |
| 8. $a \preceq b \wedge b \preceq a \rightarrow a = b$  | [1.; UE]               |
| 9. $\forall x \forall y (\forall z (BO(z, a) \leftrightarrow BO(z, b)) \rightarrow x = y)$       | [3.-8.; CI, UE]        |

Due to the large number of cases we have to prove to show the transitivity of the general identity relation

(T $\equiv$ )  $\forall \alpha \forall \beta \forall \gamma (\alpha \equiv \beta \wedge \beta \equiv \gamma \rightarrow \alpha \equiv \gamma)$

a complete proof accounting for all cases cannot be given. Most of the cases are trivial exploitations of the definitions. Since these exhibit a repeating pattern, it does not provide much insight to have them included here. The following tables give an overview of the cases which are to prove for (T $\equiv$ ). The first columns in the tables indicate what kind of terms can be found in the identity claims of the antecedent. The third displays the antecedent whereby the acronyms ‘OOI’, for one-one identity, ‘MOI’, for many-one identity, ‘OMI’, for one-many identity, and ‘MMI’, for many-many identity are used as a means to facilitate the paraphrase of the formulas. The fourth column in table 8.2 explains why the antecedent in a given line is contradictory, while the fourth columns in the other tables shows the kind of identity relation which is entailed by the antecedent in a given line. I conclude the appendix by providing the first steps for the complete proof, which relies on the individual cases and shows how the underlying strategy works.

		<b>Antecedent</b>	<b>Contradiction</b>
<b>All plural</b>	1.	$uu\bar{\sim}vv \wedge vv\bar{\sim}ww$ MOI & MOI	$IP(vv) \wedge PP(vv)$
	2.	$uu\bar{\sim}^*vv \wedge vv\bar{\sim}^*ww$ OMI & OMI	$PP(vv) \wedge IP(vv)$
	3.	$uu \sim vv \wedge vv\bar{\sim}ww$ OOI & MOI	$IP(vv) \wedge PP(vv)$
	4.	$uu\bar{\sim}vv \wedge vv \approx ww$ MOI & MMI	$IP(vv) \wedge PP(vv)$
	5.	$uu\bar{\sim}vv \wedge vv \approx ww$ MOI & MMI	$IP(vv) \wedge PP(vv)$
	6.	$uu\bar{\sim}^*vv \wedge vv \sim ww$ OMI & OOI	$PP(vv) \wedge IP(vv)$
	7.	$uu \approx vv \wedge vv \sim ww$ MMI & OOI	$PP(vv) \wedge IP(vv)$
	8.	$uu \approx vv \wedge vv\bar{\sim}^*ww$ MMI & OMI	$PP(vv) \wedge IP(vv)$
<b>One singular</b>	9.	$uu \sim vv \wedge vv \simeq z$ OOI & MOI	$IP(vv) \wedge PP(vv)$
	10.	$uu\bar{\sim}vv \wedge vv \simeq z$ MOI & MOI	$IP(vv) \wedge PP(vv)$
	11.	$uu\bar{\sim}^*vv \wedge vv \cong z$ OMI & OOI	$PP(vv) \wedge IP(vv)$
	12.	$uu \approx vv \wedge vv \cong z$ MMI & OOI	$PP(vv) \wedge IP(vv)$
	13.	$x \cong^* vv \wedge vv\bar{\sim}ww$ OOI & MOI	$IP(vv) \wedge PP(vv)$
	14.	$x \cong^* vv \wedge vv \approx ww$ OOI & MMI	$IP(vv) \wedge PP(vv)$
	15.	$x \simeq^* vv \wedge vv \sim ww$ OMI & OOI	$PP(vv) \wedge IP(vv)$
	16.	$x \simeq^* vv \wedge vv\bar{\sim}^*ww$ OMI & OMI	$PP(vv) \wedge IP(vv)$
<b>One Plural</b>	17.	$x \cong^* vv \wedge vv \simeq z$ OOI & MOI	$IP(vv) \wedge PP(vv)$
	18.	$x \simeq^* vv \wedge vv \cong z$ OMI & OOI	$PP(vv) \wedge IP(vv)$

Table 8.2: Cases for  $(T \equiv)$  with contradictory antecedent



		Antecedent	Consequent
All singular		$x \equiv y \wedge y \equiv z$	
	#1	$x = y \wedge y = z$ OOI & OOI	$x = z$ OOI
All plural		$uu \equiv vv \wedge vv \equiv ww$	
	#2	$uu \sim vv \wedge vv \sim ww$ OOI & OOI	$uu \sim ww$ OOI
	#3	$uu \approx vv \wedge vv \approx ww$ MMI & MMI	$uu \approx ww$ MMI
	#4	$uu \sim vv \wedge vv \sim^* ww$ OOI & OMI	$uu \sim^* ww$ OMI
	#5	$uu \sim vv \wedge vv \sim ww$ MOI & OOI	$uu \sim ww$ MOI
	#6	$uu \sim vv \wedge vv \sim^* ww$ MOI & OMI	$uu \approx ww$ MMI
	#7	$uu \sim^* vv \wedge vv \sim ww$ OMI & MOI	$uu \sim ww$ OOI
	#8	$uu \sim^* vv \wedge vv \approx ww$ OMI & MMI	$uu \sim^* ww$ OMI
	#9	$uu \approx vv \wedge vv \sim ww$ MMI & MOI	$uu \sim ww$ MOI
One singular	(a)	$uu \equiv vv \wedge vv \equiv z$	
	#10	$uu \sim vv \wedge vv \cong z$ OOI & OOI	$uu \cong z$ OOI
	#11	$uu \sim vv \wedge vv \cong z$ MOI & OOI	$uu \cong z$ MOI
	#12	$uu \sim vv \wedge vv \simeq z$ OOI & MOI	$uu \simeq z$ MOI
	#13	$uu \approx vv \wedge vv \simeq z$ MMI & MOI	$uu \simeq z$ MOI

Table 8.3: Cases for  $(T \equiv)$  with contingent antecedent I

Antecedent		Consequent	
<b>One singular</b>	(b)	$uu \equiv y \wedge y \equiv ww$	
	#14	$uu \cong y \wedge y \cong^* ww$ OOI & OOI	$uu \sim ww$ OOI
	#15	$uu \cong y \wedge y \simeq^* ww$ OOI & OMI	$uu \sim^* ww$ OMI
	#16	$uu \simeq y \wedge y \cong^* ww$ MOI & OOI	$uu \sim ww$ MOI
	#17	$uu \simeq y \wedge y \simeq^* ww$ MOI & OMI	$uu \approx ww$ MMI
	(c)	$x \equiv vv \wedge vv \equiv ww$	
	#18	$x \cong^* vv \wedge vv \sim ww$ OOI & OOI	$x \cong^* ww$ OOI
	#19	$x \cong^* vv \wedge vv \sim^* ww$ OOI & OMI	$x \simeq^* ww$ OMI
	#20	$x \simeq^* vv \wedge vv \sim ww$ OMI & MOI	$x \cong^* ww$ OOI
	#21	$x \simeq^* vv \wedge vv \approx ww$ OMI & MMI	$x \simeq^* ww$ OMI

Table 8.4: Cases for  $(T \equiv)$  with contingent antecedent II

(#1) is the above theorem  $(T =)$ .

(#2)  $\forall uu \forall vv \forall ww (uu \sim vv \wedge vv \sim ww \rightarrow uu \sim ww)$

1.  $\forall \alpha \forall \beta (\alpha \sim \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge IP(\alpha) \wedge IP(\beta) \wedge \wedge \forall x (x \prec \alpha \leftrightarrow x \prec vv))$  [D15]
2.  $dd \sim ee \wedge ee \sim ff$  [SC]
3.  $\neg(dd = dd) \wedge \neg(ee = ee) \wedge IP(dd) \wedge IP(ee) \wedge \forall x (x \prec dd \leftrightarrow x \prec ee)$  [1., 2.; UE, PL]

Antecedent		Consequent
<b>One plural</b>	(a) $x \equiv y \wedge y \equiv ww$	
	#22 $x = y \wedge y \cong^* ww$ OOI & OOI	$x \cong^* ww$ OOI
	#23 $x = y \wedge y \simeq^* ww$ OOI & OMI	$x \simeq^* ww$ OMI
	(b) $x \equiv vv \wedge vv \equiv z$	
	#24 $x \cong^* vv \wedge vv \cong z$ OOI & OOI	$x = z$ OOI
	#25 $x \simeq^* vv \wedge vv \simeq z$ OMI & MOI	$x = z$ OOI
	(c) $uu \equiv y \wedge y \equiv z$	
	#26 $uu \cong y \wedge y = z$ OOI & OOI	$uu \cong z$ OOI
	#27 $uu \simeq y \wedge y = z$ MOI & OOI	$uu \simeq z$ MOI

Table 8.5: Cases for  $(T \equiv)$  with contingent antecedent III

4.  $\neg(ee = ee) \wedge \neg(ff = ff) \wedge IP(ee) \wedge IP(ff) \wedge$   
 $\forall x(x \prec ee \leftrightarrow x \prec ff)$  [1., 2.; UE, PL]
5.  $a \prec dd \leftrightarrow a \prec ee$  [3.; PL, UE]
6.  $a \prec ee \leftrightarrow a \prec ff$  [4.; PL, UE]
7.  $\forall x(x \prec dd \leftrightarrow x \prec ff)$  [5., 6.; PL, UG $\sqrt{}$ ]
8.  $\neg(dd = dd) \wedge \neg(ff = ff) \wedge IP(dd) \wedge IP(ff) \wedge$   
 $\wedge \forall x(x \prec dd \leftrightarrow x \prec ff)$  [3., 4., 7.; PL]
9.  $dd \sim ff$  [1., 8.; UE, PL]
10.  $\forall uu \forall vv \forall ww (uu \sim vv \wedge vv \sim ww \rightarrow uu \sim ww)$  [2.-9.; CI, UG $\sqrt{}$ ]

(#25)  $\forall x \forall v v \forall z (x \simeq^* v v \wedge v v \simeq z \rightarrow x = z)$

1.  $\forall u u \forall x (u u \simeq x \leftrightarrow x \simeq^* u u)$  [L3]
2.  $\forall x \forall y (\forall z (B O(z, x) \leftrightarrow B O(z, y)) \rightarrow x = y)$  [L12]
3.  $\forall \alpha \forall \beta (\alpha \simeq \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge P P(\alpha) \wedge \wedge \forall y (B O(y, \alpha) \leftrightarrow B O(y, \beta)))$  [D18]
4.  $a \simeq^* e e \wedge e e \simeq c$  [SC]
5.  $e e \simeq a$  [1., 4.; UE, PL]
6.  $e e \simeq a \rightarrow \forall y (B O(y, e e) \leftrightarrow B O(y, a))$  [3.;  $\alpha$ -EX, PL, UE]
7.  $e e \simeq c \rightarrow \forall y (B O(y, e e) \leftrightarrow B O(y, c))$  [3.;  $\alpha$ -EX, PL, UE]
8.  $B O(b, e e) \leftrightarrow B O(b, a)$  [5., 6.; PL, UE]
9.  $B O(b, e e) \leftrightarrow B O(b, c)$  [4., 7.; PL, UE]
10.  $\forall y (B O(y, a) \leftrightarrow B O(y, c))$  [8.-9.; PL, UG $\sqrt{}$ ]
11.  $a = c$  [2., 10.; UE, PL]
12.  $\forall x \forall v v \forall z (x \simeq^* v v \wedge v v \simeq z \rightarrow x = z)$  [4.-11.; CI, UG $\sqrt{}$ ]

(#15)  $\forall u u \forall y \forall w w (u u \cong y \wedge y \simeq^* w w \rightarrow u u \simeq^* v v)$

1.  $\forall u u \forall x (u u \simeq x \leftrightarrow x \simeq^* u u)$  [L3]
2.  $\forall x \forall v v \forall z (x \simeq^* v v \wedge v v \simeq z \rightarrow x = z)$  [#25]
3.  $\forall u u \forall y \forall z (u u \simeq y \wedge y = z \rightarrow u u \simeq z)$  [#27]
4.  $\forall u u (I P(u u) \leftrightarrow \forall x \forall y (x \prec u u \wedge y \prec u u \rightarrow x = y))$  [D11]
5.  $\forall \alpha \forall \beta (\alpha \cong \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge I P(\alpha) \wedge \beta \prec \alpha)$  [D16]
6.  $\forall \alpha \forall \beta (\beta \simeq^* \alpha \leftrightarrow \neg(\alpha = \alpha) \wedge \beta = \beta \wedge P P(\alpha) \wedge \wedge \forall y (B O(y, \alpha) \leftrightarrow B O(y, \beta)))$  [D20]
7.  $\forall \alpha \forall \beta (\alpha \simeq^* \beta \leftrightarrow \neg(\alpha = \alpha) \wedge \neg(\beta = \beta) \wedge I P(\alpha) \wedge P P(\beta) \wedge \wedge \forall x (x \prec \alpha \leftrightarrow \beta \simeq x))$  [D21]
8.  $d d \cong b \wedge b \simeq^* f f \wedge f f \simeq a$  [SC1]
9.  $d d \cong b \wedge b \simeq^* f f$  [SC2]
10.  $d d \cong b \rightarrow b \prec d d$  [5., 9.;  $\alpha$ -EX, PL, UE]
11.  $b \prec d d$  [9., 10.; MP]
12.  $b \simeq^* f f \wedge f f \simeq a \rightarrow b = a$  [2.; UE]
13.  $b = a$  [8., 12.; MP]

14.  $a \prec dd$  [11., 13; SI]
15.  $dd \cong b \wedge b \simeq^* ff \wedge ff \simeq a \rightarrow a \prec dd$  [CI1]
16.  $ff \simeq a \rightarrow a \prec dd$  [10., 15.; MP]
17.  $\neg(dd = dd) \wedge IP(dd) \wedge b \prec dd$  [5., 9.;  $\alpha$ -EX, PL, UE]
18.  $a \prec dd \rightarrow b = a$  [4., 17.; UE, PL]
19.  $ff \simeq b \wedge b = a \rightarrow ff \simeq a$  [3.; UE]
20.  $ff \simeq b \leftrightarrow b \simeq^* ff$  [1.; UE]
21.  $\forall y(y \prec dd \leftrightarrow ff \simeq y)$  [9., 16., 18., 19., 20.; PL, UG $\sqrt{}$ ]
22.  $\neg(ff = ff) \wedge PP(ff)$  [6., 9.;  $\alpha$ -EX, PL, UE]
23.  $dd \simeq^* ff \leftrightarrow (\neg(dd = ff) \wedge \neg(ff = ff) \wedge IP(dd) \wedge PP(ff) \wedge$   
 $\wedge \forall y(y \prec dd \leftrightarrow ff \simeq y))$  [7.;  $\alpha$ -EX, PL, UE]
24.  $dd \simeq^* ff$  [17., 21., 22., 23.; PL]
26.  $\forall uu \forall y \forall ww (uu \cong y \wedge y \simeq^* ww \rightarrow uu \simeq^* vv)$  [9.-25.; CI2, UG $\sqrt{}$ ]

(T $\equiv$ )  $\forall \alpha \forall \beta \forall \gamma (\alpha \equiv \beta \wedge \beta \equiv \gamma \rightarrow \alpha \equiv \gamma)$

1.  $\forall x \forall y (x \equiv y \leftrightarrow x = y)$  [L6]
2.  $\forall uu \forall x (x \equiv uu \leftrightarrow x \cong^* uu \vee x \simeq^* uu)$  [L9]
3.  $\forall x \forall y \forall z (x = y \wedge y = z \rightarrow x = z)$  [#1]
4.  $\forall x \forall y \forall ww (x = y \wedge y \cong^* ww \rightarrow x \cong^* ww)$  [#22]
5.  $\forall x \forall y \forall ww (x = y \wedge y \simeq^* ww \rightarrow x \simeq^* ww)$  [#23]
6.  $a \equiv b \wedge b \equiv c \leftrightarrow a = b \wedge b = c$  [1.; UE, PL]
7.  $a \equiv b \wedge b \equiv c \rightarrow a = c$  [3., 6.; UE, PL]
8.  $\forall z (a \equiv b \wedge b \equiv z \rightarrow a \equiv z)$  [1., 7., 8.; UE, PL, UG $\sqrt{}$ ]
9.  $a \equiv b \rightarrow a = b$  [1.; UE, PL]
10.  $a = b \wedge b \cong^* ff \rightarrow a \cong^* ff$  [4.; UE]
11.  $a = b \wedge b \simeq^* ff \rightarrow a \simeq^* ff$  [5.; UE]
12.  $b \equiv ff \leftrightarrow b \cong^* ff \vee b \simeq^* ff$  [2.; UE, PL]
13.  $a \equiv b \wedge b \equiv ff \rightarrow a \cong^* ff \vee a \simeq^* ff$  [9.-12.; PL]
14.  $\forall ww (a \equiv b \wedge b \equiv ww \rightarrow a \equiv ww)$  [2., 13.; UE, PL, UG $\sqrt{}$ ]
15.  $\forall \gamma (a \equiv b \wedge b \equiv \gamma \rightarrow a \equiv \gamma)$  [8., 14.; PL,  $\alpha$ -IN]

⋮



## **Part III**

# **Arrangement Matters**





## CHAPTER NINE

# *Many-One Identity and the Arrangement-Condition*

Although the system presented in the previous chapter, and the principles it represents, allows us to avoid the derivations of the paradox for Composition as Identity and Collapse, it is not yet able to help us meet the criticisms discussed in section 2.3. The counterexamples against Composition as Identity discussed there are based on the rearrangement of the parts of a composite object and are still in need of being resolved.

In this chapter, I will outline a way to modify our system such that these counterexamples can be met. As explained earlier, I think the only way to address these issues is to take the arrangement of objects as an additional condition for composition. Since my aim is to defend Composition as Identity, this amounts to taking arrangement as a condition for identity.<sup>130</sup> This will sound like a quite extreme move and maybe cause some incredulous stares. However, as it is the case with the restriction of substitution, our fine-grained notion of identity allows us to relativize the previously said, that identity is a relation which is sensitive to arrangement, and it will hopefully help to see how an initial reluctance of understanding the identity relation as being sensitive to arrangement can

130. To my knowledge, Abaelardus is the only who agrees with me on *both* points, that an object is identical to its parts taken collectively, and that the arrangement of objects has to be taken into account when we ask what object they compose, (see Abaelardus 1970: 343-5, 550-1).

be overcome: One-one identities will remain unchanged. Thus, the fragment of the system which corresponds to  $FOL^=$  will not be affected by the revisions I will suggest in what follows. The only kinds of identity relations which I take to be sensitive to arrangement are many-one, one-many and many-many identities. Hence, identity being sensitive to arrangement may be seen as another consequence of extending our framework by allowing for the use of plural terms.

The idea of composition as depicted in this chapter may seem to suggest that the arrangement of some objects corresponds to the *structure* the parts of an object must have in order for the object to exist. However, this is a misleading impression. Arrangement is a collective *relation* and it is relevant with respect to the question *what* object some objects compose, or are identical to. Hence, it will eventually be the key to achieve one of the goals set out at the beginning of chapter 1, i.e. to avoid the need of taking an object as something over and above its parts, and the mysticism that usually comes with the notion of the structure of composite objects.

However, in a sense, it seems that this chapter leaves us with some loose ends, which I will pick up in the next chapter. The definition of many-one identity I will suggest, does not tell us under what conditions, some objects  $uu$  are identical to an object, or if some objects  $uu$  arranged motorbike-minus-the-frontwheel-wise are identical to a motorbike or not. We will deal with the first question, i.e. the Special Composition Question, in the first three sections of the next chapter, where I will suggest that once we embrace Composition as Identity, it is only natural to take the next step and to claim that any objects compose some object. In section 10.5, I will suggest that the question *which* object some objects compose is not necessarily a metaphysical question and thus, it need not be answered by our metaphysical theory.

## 9.1 Problems with the Transitivity of $\equiv$

We have seen in the previous chapter that the general identity relation turns out to be transitive: If  $\alpha$  is identical to  $\beta$ , and  $\beta$  is identical to  $\gamma$ ,

then  $\alpha$  is identical to  $\gamma$ . When we discussed some of the commonly held principles about identity in section 6.1, we agreed that being transitive is one of the defining characteristics of identity, although this is not beyond dispute, see the references at the end of sections 6.1.2 and in 6.1.4. However, the transitivity of ' $\equiv$ ' is problematic. In order to hold on to the transitivity of the general identity relation, we shall therefore modify our definition of many-one and one-many identity. This will be done by adding a third condition to the definiens which aims to capture the thought that the arrangement of the many things determines to which object they are identical to. Please note that this is not an *ad hoc* move to avoid the problems that arise from the counterexamples of rearrangement. On the contrary, I think that these examples show us that when we are dealing with many-one identities, ignoring the arrangement of the many leads to mistakes. This thought will be further underpinned at the end of this chapter when we will carry out our case study from chemistry.

The transitivity of ' $\equiv$ ', as defined in the previous chapter, is problematic. It shows us that Composition as Identity is "blind" to the rearrangement of parts and thus is prone to the counterexamples based on rearrangement: By reducing the identity of composite objects to the identity of their parts, Composition as Identity is not able to distinguish between a completed puzzle and a heap of puzzle pieces, if we ignore the distinctness of the puzzle pieces which compose the two. As I pointed out in section 2.3, this puts even more pressure on the friend of Composition as Identity, once modal notions are introduced, since it leads Composition as Identity to the implausible view that objects *necessarily* compose the object they compose.

To illustrate the problem of ' $\equiv$ ' in formal terms, consider case # 25 of the proof for the transitivity of ' $\equiv$ ':

$$\#25 \quad x \simeq^* uu \wedge uu \simeq y \rightarrow x = y$$

The above formula illustrates what underlies the argument from rearrangement: If the completed puzzle is one-many identical to the puzzle pieces, and the puzzle pieces are identical to the heap of puzzle pieces,

then the completed puzzle is identical to the heap of puzzle pieces. Therefore, we can see that *LI* does not help to address the arguments from rearrangement.

My suggestion is to modify the definition of plural-singular many-one, and singular-plural one-many identity – whereby the modification of the latter will simply rehearse the one of the former, so that we may ignore it from here on – in such a way that we can do justice to the idea that arrangement is relevant for composition by adding an “arrangement-condition” for many-one identity. Please note that our primitive singular one-one identity, ‘=’, and the definitions of mixed and plural one-one identities, (D15) to (D17), will remain as presented in the previous chapter. ‘Being arranged in a certain way’ is like ‘surrounding’ a predicate whose first argument place is collective, asking for a plural term to enter it. Hence, there is no need to think that one-one identities are sensitive to arrangement. After all, we can hardly make sense of a sentence claiming that *an object*, say my chair, is arranged in a certain way. Furthermore, our definitions of plural-plural many-one and plural-plural one-many identity, (D19) and (D21), will remain unchanged as well. We defined them with the help of plural-singular many-one identity, and changing this definition will suffice to give us the intended results for (D19) and (D21).

In the previous chapter, we defined many-one identity as the relation that holds between a proper plurality *uu* and an object *x* which share all their bottom objects. Besides these two conditions, a third condition reflecting the idea that the arrangement of the *uu* matters with respect to which object they are identical to is needed. In other words, our aim is to find an appropriate third condition to fill the blank in the definition below:

(D18')  $uu \simeq x =_{df}$

(i)  $PP(uu) \wedge$

(ii)  $\forall y (BO(y, uu) \leftrightarrow BO(y, x)) \wedge$

(iii) \_\_\_\_\_

## 9.2 The Arrangement-Condition for Many-One Identity

Let's approach the question what might be good candidates for this condition with the help of our example of Jig and Saw from section 2.3. Recall, the problem in this example is the following: Jig and Saw are composed out of the same puzzle pieces, *uu*. However, Jig and Saw are not identical because the former is a completed puzzle while the latter is a heap of puzzle pieces. How can we do justice to the idea that the way the puzzle pieces are arranged is what makes the difference whether they compose Jig or Saw?

A first attempt to answer this question is simply to say that what makes the difference to whether the puzzle pieces compose Jig or Saw is whether they are arranged jig-wise or saw-wise. If they are arranged jig-wise, then they compose Jig; if they are arranged saw-wise, then they compose Saw. In formal and more general terms, this amounts to the following third condition for many-one identity

(iii)  $A(uu, x)$

whereby the predicate '*A*' is paraphrased as 'are arranged ...-wise'. With the above as the third condition for many-one identity, the *uu* are many-one identical to *x* iff the *uu* are a proper plurality sharing bottom objects with *x*, and the *uu* are arranged *x*-wise.

Although this appears the straightforward and easiest answer for us, it is not very useful because I cannot see how we should make sense of it. What does it mean for some puzzle piece to be arranged jig-wise, or saw-wise? Or to give a further example, what does it mean for some pieces of wood to be arranged this-chair-wise? Please note, that the above condition (iii) does make use of a general term. These questions do *not* ask what it means for some puzzle pieces to be arranged *completed-puzzle-wise*, or *heap-wise*, or for pieces of wood to be arranged *chair-wise*. They are asking for the condition or conditions which must hold for some ob-

jects to be arranged *this-completed-puzzle-wise*, *this-heap-wise*, and *this-chair-wise*. Thereby, the question amounts to a question when some object are arranged like a *particular* object, *not* a *kind* of object. I do not know of any way to answer the former of these questions, and think that the second way of accounting for the importance of the arrangement of objects with respect to them being identical to some object is more promising.

So, let's turn to a second attempt to develop condition (iii) for our definition of many-one identity by making this condition less specific. Here is a further suggestion: If the puzzle pieces are a proper plurality and share their bottom objects with Jig, then we shall say that they are many-one identical to it, if the following holds: The puzzle pieces are arranged completed-puzzle-wise iff Jig is a completed puzzle. This allows us to meet the criticisms put forward against Composition as Identity which are based on the rearrangement of parts, since this condition makes it possible for us to distinguish between Jig and Saw, even though both of them share all their bottom objects with the puzzle pieces. When the puzzle pieces are arranged completed-puzzle-wise, then they compose Jig, which is a completed puzzle, and when they are arranged heap-wise, then they compose Saw, which is a heap of puzzle pieces. As we see, there is no danger that our theory identifies Jig and Saw, as other theories of Composition as Identity will, since our account of many-one identity reduces the identity of composite objects to the identity of their parts in a way that the arrangement of the parts is relevant with respect to what object they are identical to.

Although this looks like a promising way to modify our definition of many-one identity, a bit of reflection shows that the discussed example of Jig and Saw leads us to a difficult question: Why are 'being arranged completed-puzzle-wise' and 'being a completed puzzle', as well as 'being arranged heap-wise' and 'being a heap of puzzle pieces' the relations which make the difference in the above case of Jig and Saw? In other words, since these predicates will only suffice to resolve the issues in the discussed example, what is the *general* form of our condition (iii)? As

far as I can see, there are two possible ways we can choose here and they are, I think, equally suitable candidates for a generalization of the above example and qualify, therefore, as possible formulations for condition (iii). Firstly, we might want to say that it holds for any property  $\Phi$ , that  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. This is what I will take as the arrangement-condition for our theory. Given the assumption of Composition as Identity, this arrangement condition follows what one might think is a rather naive thought: Surely, the parts of, say, a book, must be arranged book-wise. How could some object be a book, if its parts are, say, arranged car-wise? Alternatively, one might as well limit this condition only to sortal predicates, such that for any *sortal* predicate  $\Phi$ ,  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. I will embrace the first generalization, but I cannot see that there are major difficulties with the second alternative.

Hence, the restriction I suggest is best imposed on many-one identity in asking for a correspondence between *any* property  $\Phi$  which is had by  $x$  and the arrangement of the  $uu$ :

For any  $\Phi$ ,  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise

This is, I think, the straightforward way to generalize the thought that underlies our above discussion of the example with Jig and Saw: Given that the puzzle pieces, as well as Jig and Saw share all their bottom objects, it should hold that if the puzzle-pieces are arranged completed-puzzle-wise, they compose a completed puzzle; and if the puzzle pieces are arranged heap-wise, they compose a heap of puzzle pieces. Furthermore, we can show that the counterexample to Composition as Identity from section 2.3, can be met: A definition of many-one identity which contains the above restriction on many-one identities gives us the means to distinguish between a heap of motorbike parts and a motorbike in running conditions, since the motorbike parts are identical to the former, only if they are arranged heap-wise, and they are identical to the latter, only if they are arranged motorbike-wise. Since the parts cannot be both, arranged heap-wise and arranged motorbike-wise, we can show that the

heap of motorbike parts is not identical to the motorbike in running conditions.

In formal terms, adopting the above restriction on many-one identities gives us the following revised definition of many-one identity

$$(D18') \quad uu \simeq x =_{df} PP(uu) \wedge \forall y (BO(y, uu) \leftrightarrow BO(y, x)) \wedge (\Phi(x) \leftrightarrow \Phi A(uu))$$

whereby ' $\Phi A$ ' represents the collective property *being arranged  $\Phi$ -wise*.<sup>131</sup>

(D18') would best be formulated with the help of a second-order logic, since the intention behind the second conjunct of the definition is to claim that if  $x$  and the  $uu$  share all their bottom objects, then it holds for *any* property  $\Phi$ ,  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. Since we do not have a second-order logic at our disposal but use the second-order variable ' $\Phi$ ' only as a schematic letter, we have to bear in mind that it is here used as a bound variable.

Yet, one might think that the property of being arranged  $\Phi$ -wise does only exist for particular  $\Phi$ s, and not for any  $\Phi$ . For instance, any object has the property of being self-identical. So, if  $x$  is identical to some  $uu$ , what does it mean for the  $uu$ , to be arranged self-identical-wise? As it is the case with the property of being self-identical, being arranged self-identical-wise is a property had by any *objects*  $uu$ : the property of being arranged self-identical-wise is a universal property. Any plurality of objects is arranged self-identical-wise. There is nothing particularly interesting about some objects having this collective property, as there is nothing special about *an object* being self-identical. Hence, I think it is plausible to assume that the *collective* property of being self-identical, i.e. the many-many identity relation ( $\approx$ ), is the collective property of being arranged self-identical-wise.

Note also that the above arrangement-condition (D18') is, within our logical framework, logically equivalent to the following claim:

131. Also, given the strength of the second conjunct in the universally quantified formula of the definition, the first conjunct of the definiens is very likely redundant.



For any  $vv$ ,  $x$  is among the  $\Phi$ s, and  $vv$  is the plurality of  $\Phi$ s iff the  $uu$  are arranged  $\Phi$ -wise

Hence, we can deduce

$$(QU) \quad uu \simeq x \leftrightarrow PP(uu) \wedge \forall y (BO(y, uu) \leftrightarrow BO(y, x)) \wedge \forall vv (x \prec vv \wedge vv \equiv \llbracket \Phi(y) \rrbracket \leftrightarrow \Phi A(uu))$$

which is an immediate corollary of (D18').

Yet, adding this arrangement condition raises the question how we can make sense of the “barbarism” (Unger 2014: 12), ‘being arranged  $\Phi$ -wise’. Ways to elucidate this notion are already discussed within the literature. I shall discuss some of them in the next section by distinguishing between two questions we can ask about arrangement, which I think have not yet been distinguished with sufficient clarity. Distinguishing between these two questions will help us to evaluate answers we might suggest to either one of the questions more appropriately and relate them to our account of composition.

### 9.3 Two Arrangement Questions

The modification of our definition of many-one identity with the help of the notion ‘being arranged  $\Phi$ -wise’ is in need of explication. I would like to shed some light on this notion by distinguishing between two questions from the literature, which have not been kept apart. On the one hand, we have the Special Arrangement Question (SAQ) raised by Tallant (2014: 1513), asking under what condition some objects are arranged  $\Phi$ -wise, for some  $\Phi$ . On the other hand, Brenner (2015) is dealing with a different question, though he aims to be dealing with Tallant’s SAQ, when he asks the following: For any  $\Phi$ , under what conditions are there objects arranged  $\Phi$ -wise? The two questions obviously come apart, since the quantifiers are shifted. The former asks *for any objects*  $uu$  when there is a  $\Phi$  such that they are arranged  $\Phi$ -wise. The latter asks *for any*  $\Phi$ , when

are there some objects  $uu$  such that they are arranged  $\Phi$ -wise. Let's have a look at the two questions in reverse order.

In order to be able to keep the two questions apart, we refine them by borrowing a bit of notation from second-order logic and labeling the question discussed by Brenner with 'BAQ' for "Brenner's Arrangement Question". Hence, we can pose it as follows:

BAQ For all  $\Phi$ , when is it true that  $\exists uu$  such that the  $uu$  are arranged  $\Phi$ -wise?

I agree with Brenner (2015: 1297) that there is no way to give a general answer to this question. There are many answers to this question, depending upon the predicate we choose for ' $\Phi$ '. For instance, the conditions under which there are some objects arranged hydrogen-molecule-wise, are different from the conditions under which there are some objects arranged heart-wise, or under which there are objects arranged planet-wise. In like manner, the conditions under which there are some objects arranged heart-wise are different from the conditions under which there are objects arranged planet-wise. Does that mean that BAQ is not a sensible question to ask?

In a sense, that is correct. We cannot expect to be able to answer such a question in a general way. However, that is not the end of the story. More specific versions of BAQ can be answered individually in different ways. If we take a particular predicate ' $\Phi$ ', say, 'being a hydrogen-molecule' and ask under what conditions some objects are arranged hydrogen-molecule-wise, then an answer to this question can be found. However, I think it becomes apparent that these questions do not fall within our area of competence, but have to be dealt with in other disciplines, namely those who are dealing with the study of the particular  $\Phi$ s the specific version of BAQ is concerned with. Hence, I reckon that it will be the chemist who answers the question under what conditions some objects are arranged hydrogen-molecule-wise, the anatomist who tells us when some objects are arranged heart-wise, and the astronomer who is looking for the conditions under which some objects are arranged planet-wise.

Hence, we may give different answers for different predicates, when we ask under what conditions some objects  $uu$  are arranged  $\Phi$ -wise. In other words, we will come up with a list of answers,  $BAA_1, BAA_2, BAA_3, \dots$ , to specific versions of BAQ which ask for particular predicates, ' $\Phi$ ', ' $\Psi$ ', ' $\Sigma$ ',  $\dots$ , under what conditions are there some objects  $uu$  arranged  $\Phi$ -wise,  $\Psi$ -wise,  $\Sigma$ -wise,  $\dots$ , respectively.<sup>132</sup>

$BAA_1$   $\exists uu$  such that the  $uu$  are arranged  $\Phi$ -wise iff  $p$

$BAA_2$   $\exists uu$  such that the  $uu$  are arranged  $\Psi$ -wise iff  $q$

$BAA_3$   $\exists uu$  such that the  $uu$  are arranged  $\Sigma$ -wise iff  $r$

However, we can then use this list to formulate an answer to BAQ itself by connecting these answers disjunctively:

BAQ For all  $\Phi$ , it is true that  $\exists uu$  such that the  $uu$  are arranged  $\Phi$ -wise iff  
( $p \vee q \vee r \vee \dots$ )

Setting this possible way of answering BAQ, let's compare it to Tallant's Special Arrangement Question:

SAQ [For all  $uu$ ,] when is it true that  $\exists \Phi$  the  $uu$  are arranged  $\Phi$ -wise?<sup>133</sup> (Tallant 2014: 1513)

Tallant asks under what condition some objects are arranged  $\Phi$ -wise, for *some*  $\Phi$ . He uses this question to show that standard nihilism fails, since it relies on an answer to this question, yet is not able to answer it in a satisfactory way. The SAQ arises for us as well, since we might want to know under what conditions some objects are arranged, say, table-wise,

132. I use here ' $p$ ', ' $q$ ', and ' $r$ ' as variables for sentences.

133. I made the above quote consistent with the notation of *LI* for the ease of exposition. The original formulation of Tallant (2014: 1513) "[W]hen is it true that  $\exists xx$  the  $xs$  are arranged  $F$ -wise?", is likely to contain a slip which can be spotted from the kind of the answers he discusses in §3 and his hint that the SAQ is modeled after van Inwagen's SCQ.

in order to be in a position to tell whether they are many-one identical to a certain table. Although Tallant asked the SAQ only quite recently, answers to it predate the question. Rosen and Dorr, as well as Merricks present a similar point of view when they defend the following claims:

If we put some things arranged house-wise on the corner, they would look and feel and act just like a house [...]

(Rosen and Dorr 2002: 158)

Atoms are *arranged statuewise* if and only if they both have the properties and also stand in the relations to microscopic upon which, if statues existed, those atoms' *composing a statue* would non-trivially supervene.

(Merricks 2001: 4, [italics in original])

I agree with Tallant's conclusion that Rosen and Dorr's nihilism, as well as Merricks' biological anti-reductionism need different answers to the SAQ than the one above.<sup>134</sup> Yet, we can sidestep this discussion. Important for us is whether *we* can adopt these answers within our theory of composition. As you might already suspect, we cannot. Adopting the above answers to the SAQ would make our theory circular: Composition is already defined with many-one identity and many-one identity, in turn, is defined with the help of the arrangement-condition. Hence, answering the SAQ, which asks under what conditions the arrangement-condition is fulfilled, cannot be done with the help of the notion of composition. So, we cannot just simply take the above answers, but have to look elsewhere.

Another answer to the SAQ can be reconstructed from van Inwagen's line of thought:

The *uu* are arranged chair- (table-) wise if they fill a chair- (table-) receptacle and satisfy certain other conditions [...]

(van Inwagen 1990: 109)

134. Alternatively, defending one of the above answers while holding on to nihilism or biological antireductionism requires the use of a counterpossible conditional. For a discussion of counterpossible conditionals, (see Bjerring 2014, Jago 2013, Mares 1997, Nolan 1997).

Chair-receptacles are, so van Inwagen (1990: 104-5), “[...] those regions of space that, according to those who believe in the existence of chairs, are occupied by chairs”. We can see that this account is not suitable for us either, since it would again make our theory circular.

Other answers to the SAQ can be modeled on answers to the Special Composition Question. Mimicking the brute answer to the latter, developed by Markosian (1998), one might think that it is simply a brute fact when there is a  $\Phi$  such that some objects are arranged  $\Phi$ -wise. Alternatively, we can use an organicist answer to the SAQ, modeled after van Inwagen (1990), and claim that if the activity of some objects constitutes a life, then there is a  $\Phi$  such that the  $uu$  are arranged  $\Phi$ -wise.

In my view, the best answer we can give to the SAQ is one that resembles the universalist answer to the SCQ, (see Tallant 2014: 1518), and it will eventually turn out to be one of the key elements to implement universalism within our account of composition:

For any objects  $uu$  there is a  $\Phi$  such that the  $uu$  are arranged  $\Phi$ -wise

This way of answering the SAQ has a lot going for it, since the SAQ is just another way of asking under what conditions some objects are arranged *in some way*. Surely, *any* objects are arranged in *some* way. Hence, I think a universalist answer to SAQ is the most natural way to reply to Tallant’s question. However, the above answer does not yet legitimize the claim that *any* objects compose some object. In order to arrive at this claim, we have to show that for any objects  $uu$  there is some object  $x$  such that they share all their bottom objects with  $x$ . Although it is not necessary for us to embrace this claim, given the thus far developed position, I think it is only natural to do so. I will come back to this point in the next chapter where I will motivate this claim and defend universalism against arguments from counterexamples. I will now go on to discuss a case study from chemistry where I will not only show how our account of composition can be applied, but also that the insights gained in scientific research suggest that arrangement matters for composition, before we shall conclude this chapter.

## 9.4 Isomers: A Case Study

It might seem that the following case study is merely an accumulation of more or less interesting observations, but does not really contribute to our overall project. After all, the name of the entities we are about to discuss, ‘isomers’, translates into English as ‘same parts’. Hence, the scientific terminology already reflects that we have here distinct objects with the same parts. However, the aim of this section is not to argue that there are objects which have the same parts. Rather, it serves to show that the account of Composition as Identity we have developed here can allow for such entities. I think this is a remarkable feature of the theory, since these objects usually pose problems for theories of Composition as Identity. A second, and probably more important aim of this section, is to illustrate the degree of precision we can achieve with respect to spelling out certain notions of being arranged  $\Phi$ -wise. Due to the fact, that arrangement already plays an important role in chemistry, and in particular in the study of isomers, as we will see soon, they are an ideal candidate to illustrate that the apparently vague notion of being arranged  $\Phi$ -wise can be explicated in a very precise way for certain  $\Phi$ s.

Isomers are chemical molecules composed out of the same kinds and number of chemical atoms,<sup>135</sup> but differing from each other only with respect to the way the atoms are arranged, (see Chauhan 2008: 43-7, Crowe and Bradshaw 2010: 257-89, and Johnson 2013: 154). They provide us with a scientific counterpart to the counterexamples of rearrangement. Suppose the two molecules  $x$  and  $y$  are isomers of each other. Further, disregard the distinctness of the atoms  $uu$ , and the atoms  $vv$  which compose  $x$  and  $y$ , respectively. Then this does not entail the identity of  $x$  and  $y$ , since if the  $uu$  and the  $vv$  are arranged appropriately, then  $x$  and  $y$  turn out to be different kinds of molecules with different properties:

135. From here on, I will simply talk about molecules and atoms instead of *chemical* molecules and *chemical* atoms, since the context avoids that misunderstanding might arise. Please note, that I do not want to suggest that chemical atoms are *mereological* atoms, i.e. objects with no proper parts.

If two or more substances have the same percent composition, they must have the same empirical formula, but this does not make them identical substances. To be identical they also must have identical structures and properties. Many substances called **isomers**, have the same formulas but differ in their geometrical structures and in their properties.

(Petrucci and Harwood 1993: 879)

Chemists call such compounds—with the same overall formula but different atomic arrangements—**isomers** of each other. [...] Two chemicals can differ, even when all the same atomic linkages are present, if the spatial arrangements are different.

(Breslow 1997: 8)

To give an example, take the chemical compounds which have the formula  $C_3H_8O$ , i.e. are composed by an oxygen atom,  $o$ , three carbon atoms,  $c_1, c_2, c_3$ , and eight hydrogen atoms,  $h_1, \dots, h_8$ . They can compose three different kinds of molecules: n-propyl alcohol (left), isopropyl alcohol (right), or methoxyethane (below), depending upon the way the atoms are arranged. Figure 9.1, based on the figures in (Johnson 2013: 153), illustrates this point.<sup>136</sup>

Isomers are of particular interest for us because molecules that are isomers of each other have different properties, which is a result of the different ways their atoms are arranged, (see Petrucci and Harwood 1993). In the case of the three molecules in our example, the different arrangement of the atoms results, among other things, in different boiling points: n-propyl alcohol and isopropyl alcohol have a boiling point around 89 °C, but methoxyethane boils at 7 °C, (Crowe and Bradshaw 2010: 282). Another pair of isomers are fumaric acid and maleic acid. They have different melting points. “[F]umaric acid has a melting point of 300 °C, while maleic acid has a melting point of 240 °C” (Crowe and Bradshaw 2010: 282). As a final example, consider the different melting points of n-

136. Figure 9.1 differs from the way molecules are depicted by scientists, since it is common to use capital letters to represent chemical atoms of the same kind. However, we need a way to discriminate the different chemical atoms from each other, which makes a deviation from scientific orthodoxy necessary.

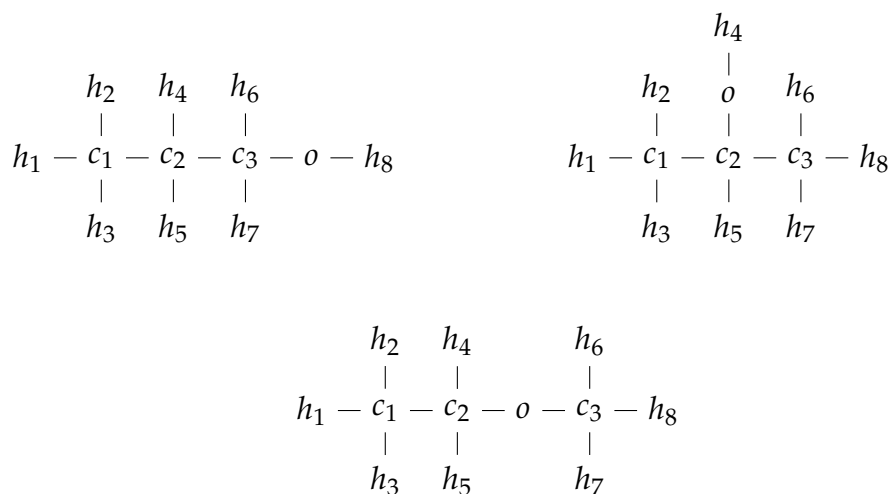


Figure 9.1: Isomers: Propanol

butane and isobutane. The former melts at  $-0,5\text{ }^\circ\text{C}$ , while the latter melts at  $-10.2\text{ }^\circ\text{C}$ , (Chauhan 2008: 43).

Our account of composition allows us to capture the phenomenon that the different arrangements of the atoms leads to the composition of different objects. Hence, we see that our version of Composition as Identity is able to meet the criticisms based on rearrangement. We can distinguish between the three molecules above *although* they are composed out of the same atoms because we reduce the identity of composite objects not only to the identity of their parts, but also to the way the parts are arranged. On the other hand, the naive version of Composition as Identity presented in chapter 8 identifies the three molecules. We shall first see how the latter theory ends up identifying the n-propyl alcohol molecule and the methoxyethane molecule, before we show that our just developed system can keep them apart.

For the sake of simplicity, let's take the atoms  $o, c_1, c_2, c_3, h_1, \dots$ , and  $h_8$  to be bottom objects of our theory. Recall, for an object  $x$  to be a bottom-object simply means that within our theory there is no object  $y$  which is among  $x$ . Hence, our supposition does not entail that the atoms under consideration are *mereological* atoms. If we wished, we could, in agree-



ment with the results of science, allow that there are some objects, for instance, protons, neutrons, and electrons, among each atom. However, we shall make this pragmatic decision to keep things manageable.

Then, the atoms  $o, c_1, c_2, c_3, h_1, \dots,$  and  $h_8$  are a proper plurality, since there is more than one object among them. Let's call this plurality 'the atoms', or simply ' $dd$ '. By definition, each one of the atoms  $o, c_1, c_2, c_3, h_1,$  and  $\dots h_8$  is among the atoms  $dd$ . This is illustrated in figure 9.2, where upward lines should be read as inclusion. Now, consider the two molecules isopropyl alcohol,  $a$ , and methoxyethane,  $b$ . Neither of them is a bottom object within our theory since there are objects, the atoms, which are among them. However, each one of them shares all its bottom objects with  $dd$ , since the atoms are the only candidates we have as being bottom objects for the two molecules. Hence, the atoms are many-one identical to the isopropyl alcohol molecule, and they are many-one identical to the methoxyethane molecule – represented by the arrow in the figure below – because they are a proper plurality and share all their bottom objects with each one of them. From these two many-one identities, the naive theory allows us to conclude that the two molecules are identical, due to the transitivity of identity, whereby this derivation relies on the use of case #25 for the proof of the transitivity of ' $\equiv$ '. Yet, the identification of the two molecules has to be avoided, since the two molecules have different properties and have to be distinguished from each other.

Let us now turn to our account and see how it makes it possible for us to keep the two molecules apart by relying on the concepts of 'being arranged n-propyl-alcohol-molecule-wise' and 'being arranged methoxyethane-molecule-wise' and how these two concepts can be spelled out. From what we have said in our example thus far, our definition of many-one identity does not tell us that the atoms  $dd$  are many-one identical to either one of the molecules, since we have not yet said anything about the arrangement-condition. However, what has been said up until now is that the first two conditions – the atoms are a proper plurality and they share all their bottom objects with either one of the two molecules – are given. Given that the molecule  $a$  is

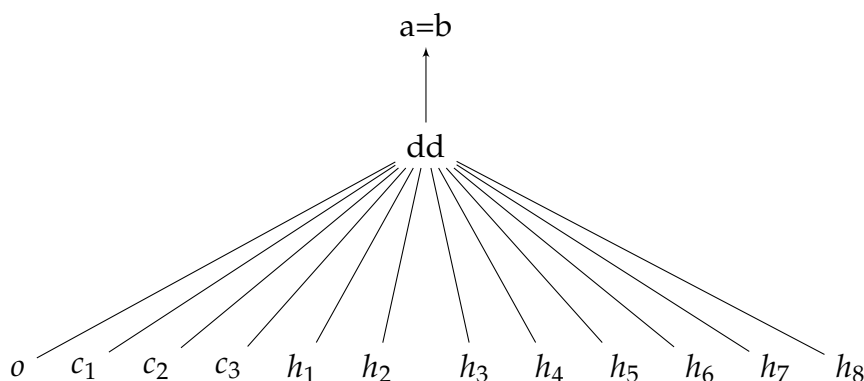


Figure 9.2: Identifying Isomers

an n-propyl alcohol molecule, the atoms are many-one identical to  $a$ , if they are arranged n-propyl-alcohol-molecule-wise. Since molecule  $b$  is a methoxyethane molecule, the atoms are many-one identical to  $b$ , if they are arranged methoxyethane-molecule-wise. As we will see next, the atoms cannot be both arranged n-propyl-alcohol-molecule-wise and arranged methoxyethane-molecule-wise. Hence, we can distinguish between the two molecules and will never end up in a situation as we have seen it previously, but rather with either one of the situations as depicted in figure 9.3 – if we ignore for a moment that our atoms can also be many-one identical to another object, if they are neither arranged n-propyl-alcohol-molecule-wise, nor methoxyethane-molecule-wise – whereby the broken arrows indicate that not both many-one identities can hold.

It remains for us to show how the relevant notions of ‘being arranged n-propyl-alcohol-molecule-wise’ and ‘being arranged methoxyethane-molecule-wise’ are spelled out and that no objects are both, arranged n-propyl-alcohol-molecule-wise *and* methoxyethane-molecule-wise. One of the central predicates we shall use to define these notions is ‘ $x$  is connected with  $y$ ’, which we in turn define with the topological notion ‘the distance between  $x$  and  $y$  is  $z$ ’. For the ease of exposition, we shall simply use a placeholder  $n$  for the distance between two atoms, since we might want to allow that the condition for when two objects are connected with

each other might differ with respect to the kind of objects considered. We could avoid this by not taking atoms as bottom objects, but protons, neutrons or electrons and define the connection for atoms as sharing an electron. However, in order to keep things simple, we shall avoid complicating matters further and settle for the universal placeholder  $n$ . Thus, we can define 'being arranged n-propyl-alcohol-molecule-wise' and 'being arranged methoxyethane-molecule-wise' as follows:

$o, c_1, c_2, c_3, h_1, h_2, h_3, h_4, h_5, h_6, h_7$  and  $h_8$  are arranged n-propyl-alcohol-molecule-wise iff

- (i)  $o$  is an oxygen atom,  $c_1, c_2,$  and  $c_3$  are carbon atoms, and  $h_1, h_2, h_3, h_4, h_5, h_6, h_7$  and  $h_8$  are hydrogen atoms; and
- (ii)  $c_1$  is connected to  $h_1, h_2, h_3$  and  $c_2$ ; and
- (iii)  $c_2$  is connected to  $h_4, h_5,$  and  $c_3$ ; and
- (iv)  $c_3$  is connected to  $h_6, h_7$  and  $o$ ; and
- (v)  $o$  is connected to  $h_8$ .

$o, c_1, c_2, c_3, h_1, h_2, h_3, h_4, h_5, h_6, h_7$  and  $h_8$  are arranged methoxyethane-molecule-wise iff

- (i)  $o$  is an oxygen atom,  $c_1, c_2,$  and  $c_3$  are carbon atoms, and  $h_1, h_2, h_3, h_4, h_5, h_6, h_7$  and  $h_8$  are hydrogen atoms; and
- (ii)  $c_1$  is connected to  $h_1, h_2, h_3$  and  $c_2$ ; and
- (iii)  $c_2$  is connected to  $h_4, h_5$  and  $o$ ; and
- (iv)  $o$  is connected to  $c_3$ ; and
- (iv)  $c_3$  is connected to  $h_6, h_7,$  and  $h_8$ .

Having spelled out the definitions in this way, we can see that no objects can be both arranged n-propyl-alcohol-molecule-wise and methoxyethane-molecule-wise. If some objects are arranged n-propyl-alcohol-molecule-wise, then the oxygen atom is connected to one carbon atom and one hydrogen atom. If some objects are arranged

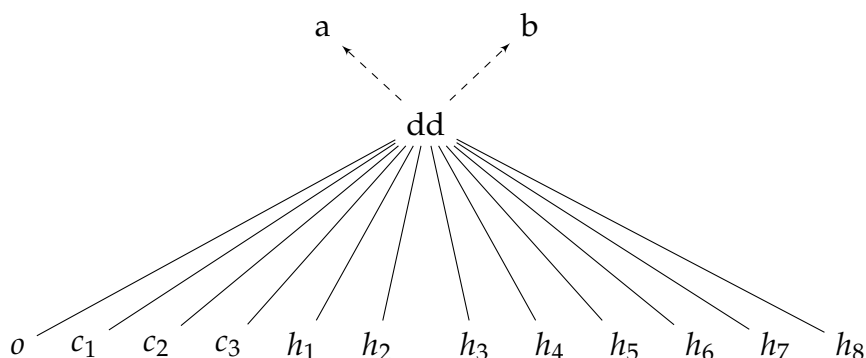


Figure 9.3: Distinguishing Isomers

methoxyethane-molecule-wise, then the oxygen atom is connected to two carbon atoms. Since the oxygen atom cannot be both, it is impossible for some objects to be arranged n-propyl-alcohol-molecule-wise and methoxyethane-molecule-wise. Hence, the two properties exclude each other and we can be reassured that our theory will help us to distinguish the n-propyl alcohol molecule from the methoxyethane molecule, although either one of them may be identical to the same plurality of atoms. Therefore, we have shown that our account of composition is able to meet the counterexamples based on rearrangement from section 2.3 and allows us to counter the criticisms which have been put forward against Composition as Identity.

## 9.5 Final Remarks on Arrangement and Composition

We conclude this chapter with some brief concluding remarks about arrangement and composition, which I locate somehow beyond the scope of our investigation here, yet think that they are noteworthy. On the one hand, I would like to reflect about cases where the rearrangement<sup>137</sup> of

137. As I already pointed out in section 2.3, the concept of rearrangement, and I think the concept of arrangement too, is vague. Yet, this seems to be an issue that is more

parts does *not* make a difference with respect to composition. Sometimes things stand differently from the way we have just seen and it is worth to think a bit more about these matters. On the other hand, it appears that by taking arrangement to be a condition for composition that we made it to be an external relation. Although we suppressed the thought that the way some objects  $uu$  are arranged might depend upon their relations to objects  $vv$  that are distinct from  $uu$ , this sounds like a plausible thought. I will argue that we can hold on to both views, relying on a distinction we will make in section 10.4, the one between the question *whether* some objects compose an object and *what* object they compose. The former is an internal question, i.e. whether some objects  $uu$  compose an object  $x$  depends solely upon the  $uu$ . Yet, the question what object  $x$  some objects  $uu$  compose can be influenced by objects distinct from the  $uu$ .

With some objects, it appears that *some* of their parts  $uu$  can be rearranged in ways such that the  $uu$  still compose the same object  $x$  as they did before they have been rearranged. In other words, sometimes rearrangement does *not matter*. Take a liter of water, and the  $H_2O$  molecules that compose it. We can rearrange the  $H_2O$  molecules in a way such that they still compose the liter of water: If we simply stir the water, then the molecules still compose the liter of water. I think this is an interesting observation. Yet, it always depends upon *which* parts are rearranged and in *which way* they are rearranged. For instance, if we were not to rearrange the *molecules* but the individual *atoms* of the liter of water by putting all oxygen atoms on one side and all hydrogen atoms on the other side, then we would *not* end up with a liter of water, but with an object consisting of two parts, whereby one of them consists entirely of oxygen atoms, and the other entirely of hydrogen atoms. This composite object is not a liter of water, since it does not have the same properties the liter of water has. The latter is potable while the former is not. Given that I put previously quite some weight on arrangement, I think it is at least noteworthy to observe that there are cases where arrangement does not matter.

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pressing for the opponents of Composition as Identity, than it is for its supporters.

A closely related point I would like to touch on briefly, is the worry that we made composition an external relation. By taking arrangement to be a condition for composition, it appears that we added a feature to composition which makes it look like an external relation, i.e. that whether some objects  $uu$  compose an object  $x$ , does not only depend upon the  $uu$  (and  $x$ ), but also on objects which are not among the  $uu$  (or different from  $x$ ). Van Inwagen (1990: 104-6) noted that the arrangement of some objects  $uu$  depends upon the way they are related to other objects, namely those objects with which they are in immediate contact. This led him to the conclusion that, for instance, some atoms – we can add arranged wood-wise – filling a chair-receptacle are not arranged chair-wise if they are surrounded by other atoms – arranged wood-wise. To make the point more succinctly, if there are some atoms  $vv$  arranged tree-wise, then there cannot be any objects  $uu$  among the  $vv$  which are arranged chair-wise. Trees do not have chairs as parts. Given our account of composition, one may think that this is a sensible restriction to adopt for us. After all, if the  $uu$  are arranged chair-wise, then they compose a chair. But one can sit or stand on a chair, or (usually) move it. So if the  $uu$  were to compose a chair, then they should be arranged being-able-to-be-sit-on, which they are not if they are among atoms arranged tree-wise.

I am confident that we can, contrary to van Inwagen, claim that the objects  $uu$  he is describing *are* arranged chair-wise without contradicting our account of composition. This can be done because we have to distinguish the question *whether* some objects  $uu$  compose an object  $x$ , and *what* object  $x$  they compose. We will do this in the next chapter, but let me show how this relates to the above worry that composition is an external relation. The picture of composition I defend here is not one that takes composition to be an external phenomenon in some sense, but it does in another. It is an external relation, insofar as the question *what* object  $x$  is composed by some objects *depends* upon the way they are arranged. Hence, it may depend upon the relation they bear to other objects, distinct from  $uu$ . Yet, *whether* some objects  $uu$  compose an object is an absolutely internal matter. Any objects  $uu$  compose some object, no

matter the relations they bear to other objects. Hence, we can see that we can allow for arrangement to be an external relation, while holding on to arrangement being relevant for composition and composition being an internal relation.

The aim of this chapter was to outline a strategy for how *LI* may be modified in order to be able to address the counterexamples based on rearrangement. We saw that *LI* is still prone to these counterexamples due to the transitivity of the general identity relation as defined in the previous chapter. Hence, we modified the definition for plural-singular many-one identity by adding an arrangement-condition. Next we distinguished Tallant's SAQ from Brenner's question and motivated a universalistic answer to the former question: Any objects are arranged  $\Phi$ -wise, for some  $\Phi$ . We engaged then in a case study from chemistry, where the concept of isomers was used to illustrate how an account of composition with an arrangement-condition can be applied and was further motivated. Eventually, I concluded with some remarks on arrangement, which are beyond our present scope.





## CHAPTER TEN

# *Composition as Identity and Answers to the SCQ*

I have repeatedly emphasized that mereological universalism does not follow from Composition as Identity. This has already been highlighted by other authors, for instance, Cameron (2012) and McDaniel (2010). However, we have also seen that Composition as Identity is often accompanied by universalism, see section 1.4.1. Given our system *LI*, it is only a small step to universalism. I shall now outline how these steps towards universalism can be done.

Although we will eventually talk briefly about the relation between our account of Composition as Identity and mereological nihilism, the step to universalism marks the actual endpoint of our discussion here. This is, I think, suitable. Once we accept the previously presented account of composition, it is hard to argue that there are some objects which do not compose an object. After all, the composition relation is characterized as an ontologically flyweight: A composite object does not come with any additional ontological given the commitment to its parts. Restricting the composition relation cannot be justified by relying on some mysterious account of composition, which makes the composite object something “greater” than the parts taken collectively. Yet, I would like to emphasize again that Composition as Identity makes universalism only plausible, but it does not necessarily lead to it. Figuratively speaking,

once we embrace Composition as Identity, we can feel a *pull* towards universalism, but we are not *pushed* over to it.

When we discussed Bricker's and Cotnoir's accounts of weak Composition as Identity, we flagged Bricker's *E Pluribus Unum*, the claim that any objects are identical to some object, and Cotnoir's observations about our ability to singularize plural terms. What we need to get to universalism from *LI* is Bricker's principle. In what follows, I will argue for *E Pluribus Unum* on the basis of our ability to singularize plural terms and by exploiting some of the considerations from the sections 7.1 and 7.2. Before we come to this argument, we shall first have a more detailed look at the relation between answers to van Inwagen's General and Special Composition Question, and in particular at the relation between Composition as Identity and universalism. Moreover, a defense of universalism against Korman's arguments from counterexamples will be presented, which relies on the distinction between the Special Composition Question and what I call the "Particular Composition Question". The former asks for the conditions under which some objects compose. The latter asks *what* object, or *what kind of object* some objects compose. These questions are not separated in the literature and it can be shown that by keeping these two questions apart arguments from counterexamples can be undermined. I reckon that the Particular Composition Question is, like Brenner's Arrangement Question, not a question, which can or should be answered by metaphysics.

## 10.1 From Composition as Identity to Universalism

Composition as Identity is an answer to van Inwagen's "General Composition Question" (GCQ):

GCQ What *is* composition? (van Inwagen 1990: 39)

This question asks for a *definition* of composition. Hence, an answer to GCQ will be of the form

For all  $x$  and for all  $uu$ , the  $uu$  compose  $x$  iff \_\_\_\_\_

Composition as Identity has the correct logical form in order to qualify as an answer to the GCQ:

CAI For all  $uu$  and for all  $x$ , the  $uu$  compose  $x$  iff the  $uu$  are many-one identical to  $x$

Interestingly, van Inwagen (1990: 46) points out that “any answer to the General Composition Question will “automatically” yield an answer to the Special Composition Question”. The Special Composition Question (SCQ) asks for the condition under which some objects compose an object, or in other words:

SCQ When is it true that  $\exists x$  the  $uu$  compose  $x$ ?<sup>138</sup>

We can see that the SCQ can indeed be answered in two logical steps, universal elimination and existential introduction, once an answer to the GCQ is at hand. Therefore, Composition as Identity entails the following answer to the SCQ:

C-I There is some object  $x$  which the  $uu$  compose iff there is an  $x$  such that the  $uu$  are many-one identical to  $x$

However, we can also see that this answer is not universalism. Universalism is the claim that for all objects  $uu$  there is an object  $y$  such that the  $uu$  compose  $y$ . In order to get from C-I to universalism, we have to assume Bricker’s “E Pluribus Unum”, from section 3.3.2, which says that any objects  $uu$  are identical to some object  $x$ , or in formal terms:

138. Van Inwagen (1990: 30) uses ‘ $y$ ’ as a singular and ‘ $xs$ ’ as a plural variable. I changed the above passage from van Inwagen in order to conform with the formalism we introduced previously.

EPU  $\forall uu\exists x(uu \simeq x)$

The difference between C-I and EPU cannot be neglected and it does not come as a surprise that MU does *not* follow from Composition as Identity: You can derive from your theory only what you have previously put in it and the universalist's claim has not yet been put into the Composition as Identity-package. Hence, we cannot derive it.

Universalism puts its focus on the *first* argument place of the composition relation. It is a theory about those objects which *compose* some object. Composition as Identity puts the focus on the *second* argument place of the composition relation. It is a theory about composite objects, i.e. those objects which *are composed*. If we give a list with the names of some objects, say the Eiffel Tower, the moon, and Frege's *Grundgesetze*, to a universalist, she will tell us that they compose an object. But she will not tell us anything about the relation these objects have to their parts. On the other hand, if we give the same list to a defender of Composition as Identity, she will only tell us that these objects are identical to their parts, and nothing about whether they compose some object. The only thing she can say is that *if* they compose some object, then it is identical to them.

However, I have pointed out that some authors, for instance, Harte (2002: 114) and Merricks (2005: 629-33), think that there is indeed such a close logical connection. And indeed, the two theories are often found together, see section 1.4.1. Therefore, I think if we want to take the road from Composition as Identity to universalism, then we have to be aware that it is not the only option there is for us. In other words, we have to supplement our account of composition with arguments to support the assumptions that are needed in order to adopt universalism. But let us first take a step back and reflect what might be the source for the false belief that Composition as Identity entails universalism.

I think the source for the mistaken view Composition as Identity to entail universalism has to be located in the fact that we are used to deal with singular one-one identity only. In  $FOL^=$ , we have the theorem that

every object  $x$  is identical to some  $y$ . This can be shown to follow from the law of identity,  $x = x$ , in a few steps. A pluralized version of this theorem can be shown in *LI* or more generally in systems of plural logic: Any objects  $uu$  are identical to some  $vv$ . However, it is easy to assume then that mixed versions of this theorem, any objects  $uu$  are identical to some  $x$ , and any object  $x$  is identical to some objects  $uu$ ,<sup>139</sup> should also be theorems of plural logic. However, they are not. Neither the standard systems of plural logic, nor our system *LI* has one of these two claims as a theorem.

Yet, accepting the claim that any objects are identical to one object, and hence universalism, “[...] is natural, if not inevitable for a defender of [Composition as Identity]” (Sider 2007: 62). We have already discussed some points which can be used to show that our version of Composition as Identity clears the ground for holding on to EPU in the chapters 1 and 7. Let me briefly rehearse the latter in the next section and elaborate on how they lead us to EPU.

## 10.2 E Pluribus Unum

In the sections 7.1 and 7.2, I argued that, given the problems of the conservative strategy, terms which are standardly taken to be superplural terms, should be understood as singular terms. Let me briefly reiterate: ‘the pair of men who wrote *PM*’ is not a superplural term, but a singular term. It is not a plural term, since ‘the pairs of men who wrote *PM*’ is already a plural term, and since the pair is *properly* among the pairs, the two cannot be identical (in the standard sense). Once we accept these terms as singular terms, we might want to allow such singular terms to enter the second argument place of the inclusion relation. Eventually, I concluded that Russell is among the pair of men who wrote *PM*, and so is Whitehead.

139. The latter is a consequence of Composition as Identity together with the assumption of a gunky universe, see fn.9 in chapter 1.

Given the account of composition we developed, we can now ask the following: Is there any proper plurality<sup>140</sup> of objects  $uu$ , which do not compose, i.e. are there any objects  $uu$  such that the  $uu$  are a proper plurality and there is no object  $x$  such that the  $uu$  and  $x$  share bottom objects and  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise? Or again in other words: Are there any objects  $uu$  such that there is no object  $x$  to which the  $uu$  are many-one identical?

Suppose, there were two objects, say, the Eiffel Tower and the moon, and there is no object  $x$  such that they compose it. Then, the claim

- (1) The Eiffel Tower and the moon do not compose any object

is true. For the sake of the argument, assume for a moment that any other objects compose some object. Hence, imagine that – maybe contrary to your present intuition – say, the Eiffel Tower and your car, as well as the moon and your car, compose some object. Under this assumption, the truth of (1) entails that the Eiffel Tower and the moon are the only two objects, the only *pair of objects*, which do not compose any object. Therefore, we might as well hold on to the following:

- (2) The Eiffel Tower and the moon are the pair of objects which do not compose any object

Within our theory, (2) amounts to the claim that the Eiffel Tower and the moon are not many-one identical to the pair of objects which do not compose any object. It follows from the definition of many-one identity that at least one of the following three claims is false:

- (3) The Eiffel Tower and the moon are a proper plurality  
 (4) The Eiffel Tower and the moon share their bottom objects with the pair of objects which do not compose any object

140. I take the claim of universalism to be that any *proper* plurality composes some object. If an *improper* plurality  $uu$  were to compose some object, then it would be a case of self-composition, since there is only one object  $x$  among  $uu$  and  $x$  appears to be the best candidate for which object is composed by the improper plurality  $uu$ .

- (5) The pair of objects which do not compose any object is  $\Phi$  iff the Eiffel Tower and the moon are arranged  $\Phi$ -wise

(3) is true by supposition, so one of the other two sentences must be false. Let's analyze the terms, which appear in the above sentences, with the help of our plural logic. The term 'the Eiffel Tower and the moon', which is used in both sentences, is a plural term because it refers to, hence is able to refer to, more than one object. In (2), we have further an apparently superplural term, 'the pair of objects which do not compose any object'. Since I argued that such apparently superplural terms are best understood as singular terms, I take 'the pair of objects which do not compose any object' to be a singular term. The justification for not taking it to be a plural term follows again the same line of thought as above: 'the pair ...' is singular, because 'the pairs ...' is a plural term, capable of referring to more than one object.

But then, if 'the pair of objects which do not compose any object' is a singular term, there must be an object to which the term refers to. The best candidate we have for being this object is *the pair of objects which do not compose any object*. Now, things stand similar as in our example from section 7.2 with Russell, Whitehead and the pair of men who wrote *PM*. Russell, as well as Whitehead, are the only objects which are among the pair of men who wrote *PM*. Analogously here, the Eiffel Tower and the moon are the only objects which are among the pair of objects which do not compose any object. The framework *LI* entails that an object inherits the bottom objects of the objects which are among it and has no bottom object which is not a bottom object of one of the objects which is among it.<sup>141</sup> Hence, the pair of objects which do not compose any object shares its bottom objects with the Eiffel Tower and the moon. Hence, (4) is true as well which means that (5) has to be false.

Although it is difficult to show the truth of (5), since it suffices to come across one property  $\Phi$  such that either the pair of objects is  $\Phi$  and the Eiffel Tower and the moon are not arranged  $\Phi$ -wise, or the Eiffel Tower

141. This follows from the definition of 'being a bottom object of', (StS) and (T $\leftarrow$ ).

and the moon are arranged  $\Phi$ -wise and the pair is not  $\Phi$ , we have good reasons to think that it is indeed true. In the previous chapter, we said that any objects  $uu$  are arranged in some way, i.e. for any objects  $uu$  there is a  $\Phi$  such that the  $uu$  are arranged  $\Phi$ -wise. In the case of the Eiffel Tower and the moon, what could such a property  $\Phi$  be?

When we think about spatial properties, 'being located in our solar system' or 'being partially located in Paris', might come to our mind. Hence, we can say that the Eiffel Tower and the moon are arranged located-in-our-solar-system-wise and partially-located-in-Paris-wise. If we accept these sentences as true, and I think one is inclined to do so even without a precise definition of these predicates,<sup>142</sup> we can see that the appropriate properties which the pair of objects which do not compose must have in order to verify (5) are instantiated by it: The pair is located in our solar system, it occupies a region of space which fully lies within the region of space occupied by our solar system, and it is partially located in Paris, since the region it occupies lies within the region of space occupied by Paris, but it also occupies a region which does not lie within that region.

What other properties  $\Phi$  are there such that we can agree on the Eiffel Tower and the moon being arranged  $\Phi$ -wise? I think there is quite a number of such properties: They are arranged heavier-than-ten-tons-wise, been-walked-by-humans-wise, orbiting-the-sun-wise, and so on. The appropriate properties  $\Phi$  to these arrangement properties are all instantiated by the pair of objects which do not compose any object: It is heavier than ten tons, has been walked by humans, and orbits the sun. Conversely, for any property  $\Phi$  which is had by the pair of objects which do not compose any object, there is a corresponding arrangement-property

142. We can hint at how to arrive at a precise definition of the predicates in question: Some objects  $uu$  are arranged located-in-our-solar-system-wise iff each one of the  $uu$  is located within our solar system. Some objects  $uu$  are arranged-partially-located-in-Paris-wise iff there is at least one object  $x$  among the  $uu$  which is located in Paris and there is at least one object  $y$  among the  $uu$  which is not located in Paris. We can leave the task to formulate definitions for the predicates 'is located in our solar system' 'is located in Paris' to the astronomers and geographers.



$\Phi A$ , such that the Eiffel Tower and the moon instantiate it: The pair is not spatially connected, the Eiffel Tower and the moon are not arranged-spatially-connected-wise; the pair has a volume bigger than one cubic kilometer, the Eiffel Tower and the moon are arranged having-a-volume-being-more-than-one-cubic-kilometer-wise; and so on.

We see from this that there are good reasons to expect that it holds for any property  $\Phi$ , the pair of objects which do not compose any object are  $\Phi$  iff the Eiffel Tower and the moon are arranged  $\Phi$ -wise. However, if that is true, then all of the above three sentences are true which means that (2) is false and, contrary to our supposition, the Eiffel Tower and the moon *are* many-one identical to some object.

We can apply the same line of reasoning to *any* objects whatsoever, no matter how different from each other they are, how big the distance between them is, how numerous they are, or how arbitrary they appear to be chosen: the Eiffel Tower, the moon, and your car; your car, the piano in Anne's living room, the first ten pages of my copy of Simons' *Parts*, and the mushrooms in Ben's fridge. In my view, we have good reasons to assume – and it is an assumption, it does not follow from our account of composition – that any objects are many-one identical to some object, and therefore, compose some object because as we have seen previously, any objects  $uu$  share their bottom objects with some object  $x$ . Moreover, as I argued in the previous paragraph, we have good reasons to think that it holds for  $uu$  and some  $x$  with which the  $uu$  share all their bottom objects, that for any property  $\Phi$ ,  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. At least, we have not come across any examples which point towards the contrary. Hence, we can rely on the evidence we get from these examples and assume that any objects are many-one identical to some object, until our opponents can present a counterexample.

As I said previously, the claim that any objects are many-one identical to some object does *not* follow from our theory. It rests on two key assumptions: Any objects  $uu$  share their bottom objects with some object  $x$ , and for any property  $\Phi$ ,  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. These two assumptions can be held or rejected from the point of view we have de-

veloped in the previous chapters. Yet, as I argued above these two claims fit nicely into the bigger picture of our theory and, if in doubt, it seems we should rather accept than reject them.

Finally, one might suspect that EPU brings back the problems of singularism discussed in chapter 4: Is EPU when it claims that for any  $uu$  there is an  $x$  such that the  $uu$  are many-one identical to  $x$  not a disguised form of singularism? This worry seems unnecessary. The problems for the singularist arise from her inability to paraphrase sentences containing plural terms with the means of singular terms in an adequate way. Although we use EPU, which functions in a way as a singularizing device, providing us with a singular term that is co-referring with a plural term, it does not “cancel-out” our plural terms. When needed or wanted, we can of course still rely on the use of plural terms and there is no need for us to paraphrase away, like the singularist does, the plural terms from natural language. Hence, we can use EPU without being afraid that we have to deal with the problems of singularism.

Setting this worry aside, we shall devote the rest of this chapter to trying to set your mind at ease about the connection between our account of Composition as Identity. Someone might still be worried about the convergence of our theory towards universalism and may regard it as a bad outcome for us that we end up that close to universalism. Thus, I will go on and argue that we can counter Korman’s arguments from counterexamples against universalism, which seems to be one of the main threats we have to be worried about at this point.

### 10.3 Arguments from Counterexamples

Mereological universalism claims that any objects compose some object. The immediate reply universalists have to face is that their position can be shown to be false due to counterexamples to their main claim. This view can be found in several places in the literature where authors aim to motivate a rejection of universalism due to the intuition that some objects do not compose. After stating that universalism entails that “[t]here is

an object composed of (i) London Bridge, (ii) a certain sub-atomic particle located far beneath the surface of the moon, and (iii) Cal Ripken, Jr." (Markosian 1998: 228), Markosian tells us the following:

My intuitions tell me that there is no such object, and I suspect that the intuition of the man on the street would agree with mine on this point. (Markosian 1998: 228)

A similar line of "critique" is presented by Elder, Berto and Plebani, and Koslicki:

Thus, on the assumption that the microparticles of physics are genuine objects, the doctrine holds that there is an object composed of seventeen microparticles in my left elbow, forty-three microparticles at the bottom of the Marianas Trench, one microparticle in the star Sirius, and the entirety of the Navy's latest Ohio-class submarine. Neither folk theories nor learned theories about how the world works find any need or use for such randomly assembled "objects" – to put it mildly. (Elder 2004: x)

[U]niversalism ontologically commits us to implausible things: scattered objects composed of disparate, unrelated things of different kinds, like the mereological sum of the right half of Lewis' left shoe plus the Moon plus the sum of all Her Majesty's ear-rings. (Berto and Plebani 2015: 187)

[I]t would follow that the material world is far more densely populated than we ordinarily assume it to be, with all manner of gerrymandered and intuitively bizarre mereological sums (such as the notorious "trout-turkey", whose parts are the, still undetached, upper half of a trout along with the, still undetached, lower half of a turkey). (Koslicki 2008: 40)

One might think these "arguments" show that universalism can be easily dismissed. Yet, that is a hasty conclusion to draw. After all, the just quoted passages merely pump intuitions against universalism and do not provide us any *argument* to show that the universalist is wrong. It was not until Korman spelled out his "Arguments from Counterexamples"

that the above intuition-pumps have been used to argue against unrestricted composition in a more systematic way, (see Korman 2015: §4). The argument from counterexamples against universalism goes as follows, (Korman 2015: 27):<sup>143</sup>

1. If universalism is true, then there are trogs<sup>144</sup>
2. There are no trogs
3. So, universalism is false

The argument is valid by *modus tollens*. But is it sound too?<sup>145</sup> Korman (2015: 28-90) puts the focus of his defense of the argument on premise 2. and on how universalists might try to argue against it. We shall have a brief look at how an attempt to undermine premise 2. in the spirit of Lewis' thoughts about quantifier restrictions might go and I will conclude that it is not an advisable strategy to defend universalism.

Following the idea of Lewis (1986c: 213; 1991: 79-80), which gained quite some popularity, (see Jubien 2001: 1-2, fn.2, Sider 2001: 218; 2004: 680-1, Sosa 1999: 142-3, Richard 2006: 173, and Rosen and Dorr 2002: 156-7), we might think the best way to reply to the argument from counterexamples is to claim that the argument equivocates and uses two different existential quantifiers. According to Lewis, we use – in everyday conversations – restricted quantifiers, which range only over a limited domain of all the objects there are:

143. Korman (2015: 28) presents a similar argument from counterexamples against nihilism. Roughly put, it goes as follows: Nihilism denies the existence of tables. There are tables. Hence, Nihilism is false.

144. Korman (2015: 27) introduces trogs as objects composed of a dog and a tree trunk of a tree.

145. There is an obvious problem with premise 1. because universalism *per se* does not tell us that trogs exist. From the context within which Korman presents the argument, it becomes apparent that *he* presupposes that at least one tree and at least one dog exists. But universalism does not assume that a dog or a tree trunk exists. Nevertheless, since this is a rather innocuous assumption, we shall grant it. Given this assumption, premise 1. simply appears to follow from the definition of universalism.

I am restricting my quantifiers [...] when I look in the fridge and say that there is no beer. I do not deny that there is beer outside the fridge, but I ignore it in my speech.

(Lewis 1986c: 136-7)

We could defend universalism in a similar vein: In premise 1., an existential quantifier is used unrestricted, or “wide open”, quantifying over everything there is. In premise 2., another existential quantifier is used, a restricted quantifier. This quantifier does not range over all the objects there are, but is limited to a domain which contains only some of the objects there are, leaving out others. Hence, when the universalist claims that trops exist, she is not contradicting the person on the street who claims that there are no trops. The apparent contradiction disappears, once we realize that the two speakers use different quantifiers.

This line of defense is not advisable. It not only begs the question in favor of universalism – it relies on the claim that the universalist’s quantifiers is unrestricted, and *her opponent* does not quantify over all the objects there are – but also raises the suspicion that “[...] disputes regarding the ontology of physical objects are verbal” (Hirsch 2011: 144). The nihilist has a similar strategy at hand: When she claims that there are no tables, she uses the “real” existential quantifier, and does not contradict common sense, because common sense uses an existential quantifier which quantifies over merely possible, or even impossible objects. Or, what amounts in my view to the same, the tension between the premises 1. and 2. might be denied by following Sider, who states that the universalist is talking a language which is different from the natural language we use in our daily conversations:

[...] perhaps my book, and other works of ontology, should not be interpreted as English, but rather as “Ontologese”, a language distinctive to fundamental ontology, in which the quantifiers are stipulated to mean something new.

(Sider 2004: 680)

However, the nihilist has the same option to rescue her position, she might want to speak “Tarskian” (van Inwagen 2014: 1), and point out that

premise 1. in the argument against her position is uttered in Tarskian. But then the interesting question becomes which language is the correct language to use when we want to talk about what object there *really* are. Yet, it remains unanswered and universalists and nihilist have ridiculed the whole discussion on composition. I think this is the wrong way to reply to the argument from counterexamples for both parties, universalists as well as nihilists. Hence, I shall develop my own defense of how we can best defend universalism against these arguments. So let's again have a closer look at the argument and see where we can tackle it.

## 10.4 The Particular Composition Question

The problem with the argument from counterexamples against universalism is that it is based on wrong expectations towards answers to the SCQ. Recall, the SCQ asks for conditions under which some objects  $uu$  compose an object  $x$ . Universalism tells us that any objects  $uu$  compose. However, universalism does not tell us *what* object, say, my dog and the trunk of the tree outside my office window compose. In other words, we have to distinguish the SCQ from what I call the "Particular Composition Question":

SCQ When is it true that  $\exists x$  the  $uu$  compose  $x$ ?

PCQ What object  $x$  do the  $uu$  compose?

To clarify the difference between the two questions, compare it to the following, structurally similar questions a set-theorist might ask, the Special, and Particular Intersection Question:

SIQ When is it true that  $\exists x$  the  $uu$  have  $x$  as their intersection?

PIQ What set  $x$  is the intersection of the sets  $uu$ ?

We know the answer to SIQ. For any sets  $uu$ , there is an  $x$  which they have as their intersection iff there is at least one object which is an element of

each one of the  $uu$ . For instance, the set of blue objects and the set of wooden objects have an intersection, because there is at least one object which is an element of the set of blue objects and an element of the set of the wooden objects. Even the correct answer – sharing at least one object as an element – to the General Intersection Question

GIQ What is it for some  $x$  to be an intersection of the  $uu$ ?

which mimics the GCQ does not suffice to answer PIQ for us. Surely, they imply a trivial answer to PIQ, the intersection of some  $uu$  is that set which contains all and only those objects which are elements of each one of the  $uu$ . Similarly, we can deduce a trivial answer to PCQ from answers to the SCQ and GCQ. Even if we know the conditions under which composition occurs, and what composition is, we are not in a position to know anything about composite objects, apart from the trivial facts that they are composite objects and have the objects which compose them as parts. Assume, our answer to the SCQ tells us that two atoms  $y$  and  $z$  compose some object  $x$ . All we know about  $x$  is that it is a composite object and has two atoms as parts. Moreover, even if we had besides the answer to the SCQ an answer to the GCQ, we could not give an answer to the PCQ.

To give a further example, consider the following three questions lawyers might ask:

SFQ When is it true that  $\exists x$  the  $uu$  have  $x$  as father?

GFQ What is it for the  $uu$  to have  $x$  as father?

PFQ Whom do the  $uu$  have as father?

Lawyers will tell us that they can spell out the answers to the first two questions: For any  $uu$ , there is an  $x$  such that the  $uu$  have  $x$  as father, if each one of the  $uu$  has the same male legal parent. Having a father is having a male legal parent. Yet, we cannot infer on the basis of those two answers, an answer to PFQ. Knowing the conditions under which some people have the same person as father, and knowing what having

a father means, does not tell us *who* is the father of some people. Similarly, knowing the conditions under which there is an object  $x$  which is composed by some  $uu$  and what composition is, does not suffice to know what object the  $uu$  compose.

By distinguishing the SCQ from the PCQ, we can see where the argument from counterexamples goes wrong. Universalism does not tell us *what* object  $x$  some objects  $uu$  compose, although it tells us *under what conditions* they compose *some* object. Hence, the counterexample misses the point. Universalism does not entail the existence of trogs but only that given the existence of a dog and a tree trunk, there is some object which they compose. Therefore, we can defend universalism against Korman's argument by dismissing its first premise.

The opponent of universalism will try to defend the argument from counterexamples against this critique. To use the famous example from Lewis (1991: 7-8), she might reply in the following way: Although universalism does not tell us *what particular* object the undetached upper half of a trout  $x$ , and the still undetached lower half of a turkey  $y$  compose, it entails the existence of a certain *kind* of object, a trout-turkey. So, universalism entails nonetheless that trout-turkeys exist. Yet, there are no trout-turkeys; so, universalism is false. If *this* line of thought underlies the argument from counterexamples, then we have to reconsider whether premise 2. is acceptable and on the basis of what the universalist's opponent can claim that there are no trout-turkeys. The answer to this question will depend upon what kind of object trout-turkeys are. So, let's try to find out what we can know about trout-turkeys and whether this shows that they do not exist.

Supposedly, one may want to defend the argument from counterexamples by claiming that there is another important composition question we have to consider besides the GCQ, SCQ, and PCQ:

What kind of object  $\Phi$  do the  $uu$  compose?

It is the answer to this question, so she might continue, which becomes a problem for universalism. According to universalism, the undetached



upper half of a trout  $x$  and the still undetached lower half of a turkey  $y$  compose an object  $z$ , and  $z$  is a trout-turkey. However, there are no trout-turkeys, which is why universalism is false.

We can see that this way of undermining universalism differs from the one discussed in the previous section. The point of critique raised here is not *that* some objects compose an object, or *what particular* object they compose, but *what kind* of object they compose. Hence, this reply hinges on what kind of object trout-turkeys are. But does the universalist tell us anything about that? What can we deduce from the universalists position about the kind of object trout-turkeys are? Indeed, there is not much we can know about the kind of objects trout-turkeys are, given what universalist have told us about them. Trivially, trout-turkeys are composite objects, but we cannot reject the existence of trout-turkeys because they are composite objects. Such a rejection would lead us straightforwardly to nihilism and that is not what is the intention of the argument from counterexamples. Therefore, I will offer two possible answers to the question what kind of object trout-turkeys are. It will turn out that neither of these justifies the premise that is needed for the argument from counterexamples.

Firstly, trout-turkeys are “scattered objects”, i.e. “[...] the region of space it occupies is disconnected” (Cartwright 1975: 175). By definition, a trout-turkey has two parts, the upper half of a trout and the lower half of a turkey. Even without a precise definition of ‘occupying a disconnected region of space’, we can see that a trout-turkey is a scattered object. One of its halves, the trout-half, occupies a region which is located close to, say, the region occupied by Motueka River, and the turkey-half occupies a region which is located close to the region occupied by Kiraka, the giraffe from Auckland’s zoo. However, we cannot agree with Korman that our everyday intuitions about which macroscopic objects exist are most of the time correct and reject the existence of trout-turkeys because they are scattered objects. These very same intuitions tell us that *there are* scattered objects, for instance, bikinis, suits, coin collections, copies of *PM*, solar systems, and so forth. Note further, Cartwright’s observation that if the

findings of science are correct, then all macroscopic material objects are scattered:

That there are scattered material objects seems to me beyond reasonable doubt. If natural scientists are to be taken at their word, all familiar objects of everyday life are scattered.  
(Cartwright 1975: 175)

It is evident that we cannot justify premise 2. by claiming that trout-turkeys do not exist because they are scattered objects. This would rule out the existence of other material objects which conservatives want to preserve and is therefore not an option.

A second attempt to hold on to the claim that there are no trout-turkeys may be based on the reason that the kind of objects trout-turkeys are is somewhat “gerrymandered” (Koslicki 2008: 40). The very details of this reservation against universalism can be spelled out in different ways. Yet, I think they all are based on the same line of thought, no matter whether the objection against such objects as trout-turkeys is that they belong to an “extraordinary” (Korman 2015: 1), “arbitrary” (Husserl 1901: 275-6), “implausible” (Berto and Plebani 2015: 187), “bizarre” (Markosian 1998: 229), or “randomly assembled” (Elder 2004: x), kind of objects. The driving force behind the argument from counterexamples relying on these reservations is the view that if there is no “ordinary”, non-arbitrary, plausible, . . . , *concept* of  $\Phi$ s, then there are no  $\Phi$ s. With respect to trout-turkeys, the idea becomes that trout-turkeys do not exist because the concept *trout-turkey* is not an ordinary, but arbitrary and implausible concept.

This line of putting pressure on universalism runs into problems. For instance, the notion of an ordinary object, and hence its opposite, being a gerrymandered object, is troublesome. On the one hand, what concepts count as ordinary changes over time. The concept of a smartphone became an ordinary concept only quite recently. A little bit less recently, the concepts of a nuclear reactor, a neutron or a strand of DNA became ordinary concepts. But how are such inventions and discoveries possible?

How can then people invent and discover objects that belong to a kind of objects for which we do not yet have an ordinary concept? That would be impossible, if objects which belong to a kind of objects for which we have no ordinary concept cannot exist. Thus, this way of justifying premise 2. should be dismissed.

We have equally good reasons to reject a defense of premise 2. which is aimed to be justified by the thought that trout-turkeys are objects which belong to an arbitrary kind of objects. What counts as arbitrary is a relative issue. If I were to name (and maybe I will do that indeed) my three future pets Ruth, Nelson, and Gottlob, it might seem arbitrary to you. But it is not. The three names are the authors' names of the first three books from the left on the shelf in the room next door. Maybe the ordering of these books on the shelf sounds arbitrary to you. But it is not. They are ordered according to their height. As you see, what is considered arbitrary is relative to our point of view. Hence, I think that arguing against the universalist and the existence of trout-turkeys because they belong to a kind of objects falling under an arbitrary concept does not work either. More generally, I think we have good reasons to suspect that all of the previously mentioned reasons to reject the existence of trout-turkeys will fail, due to similar reasons: What counts as a notion which represents implausible, bizarre, or randomly assembled objects is relative and changes over time. Hence, an argument which aims to show that trout-turkeys do not exist cannot rely on the above line of reasoning because the ancient idea of Plato that we should divide "[...] into species according to the natural formation, where the joint is, not breaking any part as a bad carver might" (Plato 1892b: 266a) – nowadays famous as "[to] carve reality at the joints" (Lewis 1983b: 346; see also Sider 2011: 8) – fails. As I just suggested, where we suspect these alleged joints to be is a relative matter and may change over over time. Hence, we do better without them and should not use them in our ontological arguments.

The failure of this line of defense for the second premise shows us that the opponent of universalism cannot reject the view by relying on the argument from counterexamples. From universalism, nothing follows

which suggests that there is something wrong with those objects which some consider to be extraordinary or arbitrary objects. Surely, they *seem* quite unlike chairs, tables and rocks, but chairs and tables are, in some sense, quite unlike rocks. Hence, unless we can find some further reasons to believe in the truth of premise 2. of the argument from counterexample, it should be rejected. Let's turn back to the PCQ and reflect briefly on whether it is a metaphysical question what object or what kind of object some objects compose. I will argue that it cannot be the metaphysician's task to answer the PCQ because it is a question which does not allow for a uniform answer and composition is a relation which happens "at different levels". In the final section of this chapter, we will then compare our account of Composition as Identity with mereological nihilism.

## 10.5 Who answers the PCQ?

Like the intersection-relation, composition is a very general relation, i.e. the extension of the predicate 'composing' is a very heterogeneous group of objects. For instance, oxygen atoms, as well as, pieces of metal, or rocks are good candidates to enter the composition relation. That is, from a common-sense point of view, it looks as if two oxygen atoms compose an oxygen molecule, pieces of metal the Eiffel Tower, and rocks Mount Everest. Oxygen atoms, pieces of metal, and rocks, on the one hand, and oxygen molecules, the Eiffel Tower and Mount Everest, on the other hand, form heterogeneous groups. In the sciences, these objects are studied by different disciplines: Chemists consider oxygen atoms and molecules, architects pieces of metal and the Eiffel Tower, and geologists rocks and Mount Everest as objects of the domain of their field of enquiry. This suggests that when we ask *What do these oxygen atoms compose?*, the person to answer this question is not the metaphysician, but the chemist. Similarly, it will be the architect and the geologist who answer the questions what some pieces of metal and what some rocks compose, respectively, and not the metaphysician. This suggestion can be reinforced, if we consider the duals to the SCQ, GCQ and PCQ by introducing the predicate 'de-

composing'. Let's define  $x$  decomposes into the  $uu$  with the  $uu$  compose  $y$ . Then, we can ask analogous to the SCQ, GCQ, and PCQ, the following three questions:

SDQ When are there some  $uu$  such that  $x$  decomposes into the  $uu$ ?

GDQ What is it for an object  $x$  to decompose into the  $uu$ ?

PDQ Into what objects  $uu$  does  $x$  decompose?

The answers to the SDQ and the GDQ follow trivially from our answers to the SCQ and the GCQ, whereby the argument places are simply reversed: The answer to the GDQ is the answer we give to the GCQ with reversed argument places. Since an object decomposes into some objects iff it is not an atom, and our answer to the SCQ will tell us under what conditions some object is not an atom, the answer to the SCQ entails an answer to the SDQ. But I would like to discuss the PDQ, the dual to PCQ, which asks what objects enter the second place of the decomposition relation, given that some object enters its first.

It is important to note that there are at first glance different, equally legitimate answers to the PDQ, if we take some composite object  $x$  in the first argument place of the decomposition relation. Take, for instance, the Eiffel Tower. It decomposes into a top and a lower half, but also into a left and a right half, pieces of metal, mereological atoms, and so forth. Or take a liter of water. It does not only decompose into two half liters of water, but also into (roughly)  $3.345 \times 10^{25}$  water molecules, or (roughly)  $3.345 \times 10^{25}$  oxygen atoms and  $6.69 \times 10^{25}$  hydrogen atoms.<sup>146</sup> But these answers are not answers which will be provided by a metaphysician. On the contrary, nobody will turn to a metaphysician in order to find out whether a liter of water decomposes into oxygen and hydrogen atoms. Conversely, we might doubt that it is reasonable to ask the metaphysi-

146. Water has a density of  $1g/l$  and the atomic mass of water is  $18g/mol$ . Hence, one liter of water contains  $55.56mol$  of water. Since there are  $6.022 \times 10^{23}$  molecules in a mole, there are roughly  $3.345 \times 10^{25}$  water molecules in a liter of water.

cian for an answer to the PCQ. To illustrate this, let's consider another example.

Although we might ask the metaphysician *whether* some rocks compose, it is not sensible to ask the metaphysician *what* object, or *what kind* of object they compose. For instance, assuming that we believe in the existence of some atoms located in the Kuiper belt, and that they compose an object, let's call it 'Pluto'. Then, it does not seem to be a reasonable thing to ask the metaphysician whether these objects compose a planet or a dwarf planet. The person we should ask is the astronomer, not the philosopher. Hence, I agree to a large degree with the following reflection of Koslicki:

The question of which kinds there are I take to be one that is not answered by the mereologist proper, but by the ontologist at large, in conjunction with other domains, such as science and common sense, which turn out to contribute to the question, "What is there?", or, more specifically, to the question, "What *kinds* of objects are there?" (Koslicki 2008: 171)

However, one might wonder about the consequences which follow from delegating the PCQ to other departments: If we agree that the PCQ does not necessarily fall into the metaphysician's area of competence, are we thereby not taking away the SCQ from her too? After all, how can it be possible that the metaphysician can not tell us *what* some objects compose, yet she is supposed to tell us *whether* they compose an object. If composition is indeed such a general relation, might we not rather ask the specialist to tell us about the conditions under which the objects from her domain of research compose? In other words, should we ask the chemist, architect, and geologist about the conditions under which oxygen atoms, pieces of metals, and rocks compose?

It may seem that delegating the PCQ to the individual sciences ultimately means that the SCQ will be answered by them as well. Let's suppose that were true. This would only make sense, if the SCQ would be answered in different ways by the different sciences: The chemist tells us the conditions for *chemical composition*, the architect for *architectural*

*composition*, and the geologist for *geological composition*. Yet then, we can ask for the conditions under which *any* objects, no matter of what kind they are – oxygen-atoms, pieces of metal, rocks – compose some object. This brings us again back to the original SCQ we encountered previously and it falls into the philosopher's area of competence. Therefore, we can see that even if the PCQ has not to be answered by metaphysics, but will rely on the special insights gained by people working in other areas of research, the SCQ will still remain a *metaphysical* question.

We have seen that our account of Composition as Identity provides a suitable starting point for universalism. The assumption that any objects are identical to some object, i.e. the principle "E Pluribus Unum", can be motivated within our theory on the basis of the considerations about the singularization of plural terms we discussed at the beginning of chapter 7. The examples we discussed in this context support the truth of this principle, since they suggest that we will find for any objects  $uu$  some object  $x$  with which they share all their bottom objects, and  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise. After presenting Korman's argument from counterexamples against universalism and rejecting a reply which aims to explain away the contradiction between universalism and conservatism, I argued that the argument is based on mistaken expectations towards universalism and metaphysics more generally. Universalism tells us only *whether*, not *what* object some objects compose. By spelling out the difference between these two questions and introducing the PCQ, I tried to make this point more tangible. In addition, we observed that rejecting the existence of those objects, which according to universalism exist, by claiming that they belong to a gerrymandered kind of objects cannot be maintained, because what is conceived as a gerrymandered kind of object is relative and changes over time. Eventually, I concluded by proposing the idea that the PCQ will be answered with the help of the insights from research which is done outside of philosophy departments. Since we have discussed the relation between Composition as Identity and mereological universalism at length, let's now turn to see how our account of composition relates to another answer to the SCQ.

## 10.6 Composition as Identity and Nihilism

Composition as Identity and nihilism are usually located on opposite ends of the spectrum of theories on composition we can choose from. Yet, Calosi (2016) argues that strong Composition as Identity is equivalent to nihilism. It is not my intention to investigate the relation between *strong* Composition as Identity and nihilism. Rather, I would like to ask whether the account of Composition as Identity I developed in the previous chapters ends up being in a similar close relation to nihilism as Calosi claims strong Composition as Identity is. I think one may indeed get the impression that my account of Composition as Identity is in fact boiling down to nihilism. My view on composition shares certain features with the nihilist's view; but they come apart. I will next discuss two points which might mistakenly lead to the conclusion that my understanding of composition is nihilism in a disguised form, before I present four reasons which show that I am not selling nihilism under a different label here.

### 10.6.1 What Composition as Identity shares with Nihilism

There are two features that, on a first glance, may lead one to think that my account of composition is mereological nihilism in universalist's clothes. Both theories are based on the use of plural logic and stress the importance of arrangement. Although we can show why both theories rely on the use of plural logic and why this does not mean that the theories come down to the same position, the central importance of arrangement within both theories cannot be neglected. Let's have a look at the two points in turn, before discuss some reasons, which give us sufficient evidence that the theory of composition we have seen in the previous chapters is not a version of nihilism.

The first commonality between my theory and nihilism is obvious. My account of composition cannot be expressed without plural logic. I identify the composition relation with a kind of the identity relation,



whereby I claimed that some objects compose an object iff they are identical to it. This many-one identity was defined within the framework of a plural logic. Standard nihilists employ the notions of plural logic within their paraphrase strategy. It tells us that ordinary sentences, which appear to be committed to the existence of composite objects, are adequately paraphrased by sentences, which are committed to the existence of atoms only. For instance, (1) gets paraphrased by (2), and (3) by (4):

- (1) The Eiffel Tower exists
- (2) Some atoms arranged Eiffel-Tower-wise exist
- (3) The Eiffel Tower is heavier than the Piscatory Ring
- (4) The atoms arranged Eiffel-Tower-wise are heavier than the atoms arranged Piscatory-Ring-wise

Without the use of plural logic, nihilism becomes untenable since nihilism without the paraphrase strategy amounts to strict atomism: The only things there are, are mereological atoms. The strict atomist does not have any further claims to offer which can tell us anything about the world. She is limited to claims about the existence of atoms and their *distributive* properties. Without plural logic, she cannot tell us anything about reality by the means of claims, which contain predicates with collective argument places. This makes the project of strict atomism an impossible enterprise. Although it trumps most of its rival theories when it comes to ontological parsimony, it cannot compete with their explanatory power. Even standard nihilism has more explanatory power than strict atomism. However, the principle of parsimony asks us to choose the more parsimonious of two theories only under the condition that they are *otherwise equal*. A difference in explanatory power shows us that standard and strict nihilism are *not* otherwise equal, which suffices for abstaining from choosing the more parsimonious theory. Hence, we see that the nihilist better invokes the paraphrase strategy, and thereby, builds her theory on the use of plural logic.

Yet, that Composition as Identity and nihilism both rely on the use of plural logic should not lead us to confuse the two theories. As seen in section 4.3, any competitive theory on composition will make use of plural logic, since composition is collective in its first argument place and plural logic is the right framework to deal with such predicates. Composition as Identity and nihilism share the need for plural logic with *any other* theory that can be taken as a serious contestant to shed light on the question what composition is or under what conditions composition occurs. Therefore, the use of plural logic should not suggest that theory of composition I have presented here and nihilism cannot be kept apart.

Secondly, both contain the idea of arrangement as a central feature. As we have just seen, nihilists paraphrase talk about composite objects with talk about the arrangement of atoms. My account of composition identifies composition with identity, whereby I argued that the kind of identity that is the composition relation is sensitive to arrangement. Consider again (D18’):

Some objects  $uu$  are many-one identical to  $x$  iff the  $uu$  are a proper plurality, the  $uu$  share their bottom objects with  $x$ , and  $x$  is  $\Phi$  iff the  $uu$  are arranged  $\Phi$ -wise.

Due to the criticisms against Composition as Identity based on the rearrangement of parts, we introduced the third condition in the definition of many-one identity as a means to get hold of the idea that arrangement matters with respect to the question to which object some objects are many-one identical. Together with the claim that some objects compose an object iff they are many-one identical to it, we made arrangement a central feature for composition and also a central part of our theory on composition. I think there is no way to explain away that arrangement plays a central role in both theories. But identifying the two theories on the ground of this shared common feature is too hasty. I think there are sufficient reasons that speak for keeping them apart. So let’s see what these are.

### 10.6.2 Why Composition as Identity is not Nihilism

Next, I will lay out why it is a mistake to think that the version of Composition as Identity discussed previously amounts to nihilism. The main reasons for distinguishing between the two positions are the following: Nihilism claims that parthood and composition are empty relations, is committed to mereological atomism, and is pushed by considerations of parsimony. Composition as Identity can define the parthood and composition relation, does not rely on mereological atomism, and shows us why we do not have to be nihilists to be ontologically parsimonious.

One major difference between my version of Composition as Identity and nihilism relies on the fact that the former is able to *define* the parthood and the composition predicate, while the latter claims that they are empty. As we have seen, the predicate ‘the *uu* compose *x*’ is defined in our theory. A definition of parthood is straightforward: *x* is a proper part of *y* iff there are some *uu* such that the *uu* compose *y* and *x* is among the *uu*.

In comparison, the nihilist does *not* offer us a definition of parthood or composition, but *eliminates* them from the stock of non-empty relations, when she claims that ‘*x* is a part of *y*’ and ‘the *uu* compose *x*’ are always false. Surely, there is some *prima facie* reason to think that the nihilist is here making a similar move as we do: Neither we, nor the nihilist uses the composition or the parthood relation as a primitive relation. However, I am not entirely sure whether nihilists can really do without one of these predicates as a primitive. I cannot see how they can define either one of the predicates without the other. Yet, when the nihilist utters her central claim, no object has a part, or there are no composite objects, she needs these predicates. Please note that claiming the only things which exist are atoms does not work either, since ‘being an atom’ is defined as not having any proper parts and trading the two predicates for the predicate ‘being an atom’ is no good trade. But let’s grant that nihilists have a way to formulate their central claim without either taking parthood or composition to be a primitive relation.

The difference between the two positions can nevertheless not be neglected: On the one hand, composition is defined. On the other hand, composition is claimed to be an empty relation. The difference between defining a predicate and taking it to represent an empty relation is clear. We can define 'is left-handed' in a theory where we use 'is right-handed' as a primitive predicate, or *vice versa*. However, this does not mean that we thereby claim that there are no left-handed people, i.e. that 'is left-handed' is an empty relation. If that were the case, then not only would Anselm's ontological proof have to conclude that God does not exist because he defines the concept of God, but the value of any definition might be doubted, if defining a relation amounts to the claim that there is no object which satisfies the property of the definiendum. Hence, since we should not confuse a definition of a predicate ' $\Phi$ ' with the claim that the relation represented by ' $\Phi$ ' is an empty relation, we should not relate our account of Composition as Identity to the nihilist's view on composition.

Furthermore, nihilism depends upon the truth of mereological atomism, i.e. that any object is either an atom, or has an atom as proper part – whereby the second disjunct will always be considered to be false by the nihilist. To put it the other way round, nihilism cannot be true, if there is at least one gunky object. If there were a gunky object, then it would have a proper part, which is what the nihilist denies. On the other hand, my version of Composition as Identity can work with a pragmatic version of atomism. By introducing the concept of bottom objects and claiming that any object is a bottom object or has a bottom object among it, we made a claim which appears to come dangerously close to atomism. Yet, as I stressed in section 7.3, bottom objects should not be confused with mereological atoms. What counts as a bottom object is relative to the theory within which we are working and depends upon the linguistic choices we make there. What counts as an atom, or not, is not relative, but an absolute, ontological fact. In no way, does this depend upon our linguistic choices. Hence, when I claim that any object is a bottom object or has a bottom object among it, this does not amount to the atomistic claim that any object is an atom or has an atom as proper part.

In addition, nihilism is pushed by the principle of parsimony. Due to the desire of not postulating the existence of more objects than necessary, nihilists think the existence of composite objects has to be denied. As we have seen with Composition as Identity at hand, Occam's Razor does not force us to shave off composite objects from our ontology. Composite objects are just their parts taken collectively. Given the ontological commitment to the existence of the parts, there is no additional ontological commitment to the existence of the composite object, and *vice versa*. Hence, postulating the existence of the composite object amounts to postulating the existence of the parts, and denying the existence of the composite object is denying the existence of its parts. As you see, when the nihilist uses the principle of parsimony, then Occam's Razor cuts in two ways: Not only does it shave off the composite object, but also its parts.

Moreover, if atomism is true, then the nihilistic view amounts to one particular way of dividing up reality besides the other possibilities we have according to Composition as Identity. The nihilists claim that only atoms exist is simply the strictest way of drawing up an inventory of the world. However, that does not mean that there are not other ways such an inventory can be drawn, and *a fortiori*, it does not amount to the claim that there are no composite objects. Finally, the nihilist seems to think of composition as an ontologically heavily loaded relation, which is difficult for me to make sense of. It might be that composition is thought to be some relation which 'adds something to reality'. However, I think the picture of composition we developed understands composition as an "ontological flyweight", which ultimately turns out to be what van Inwagen denied, namely a "[rearrangement of] the furniture of the earth without adding to it" (van Inwagen 1990: 124).



## *Conclusion*

Although Composition as Identity provides us with a plausible interpretation of our intuitions about the close relation between a material object and its parts, the criticisms which have been put forward against this view are challenging. Problems which arise for Composition as Identity such as the Paradox for Composition as Identity and the derivation of Collapse, as well as the theory's tight connection to mereological essentialism and reverse mereological essentialism are difficult to overcome for the versions of Composition as Identity which can be found in the literature. I argued that these challenges can be met, if we take composition to be a relation which comes in a variety of forms and add an arrangement-condition to the definition of many-one identity.

The problems of conservative strategies to resolve inconsistencies within the standard systems of plural logic suggest that identity is best understood as a relation which comes in different kinds, whereby only one of them obeys substitution. The view that identity is a unitary relation which always allows for substitution is a consequence of mistakenly paraphrasing away plural terms by singular terms, and hence avoiding plural logic altogether, or of the assumption that there are other non-singular terms besides plural terms. I argued against both views and showed that they are flawed. Once we abandon the narrow-minded view of singularism and part ways with the conservative views on plural logic, we are in a position to see that the singular one-one relation which allows for substitution, whereby I assumed for the sake of the argument that any singular term is a non-rigid designator, is the exception to the norm and that other kinds of identity relations do not obey this rule of inference

unrestrictedly.

I suggested that the lesson we can learn from the substitution failures in plural logic are analogous to the lessons that have been learned from its failures in modal and epistemic contexts: When dealing with predicates that are non-extensional in an argument place, substituting non-rigid designators in that argument places is not a legitimate inference. I argued further that predicates which are collective in an argument place are non-extensional in that argument place. After distinguishing between predicates that are intensional and predicates that are hyperintensional in an argument place, I proposed two revisions for when substituting plural terms is legitimate in cases where we encounter such predicates. In the case of predicates that are intensional in an argument place, plural terms can be substituted if they are rigid designators. On the other hand, with predicates that are hyperintensional in an argument place, we can substitute plural terms only, if their reference is articulated in the same way.

Further, I showed that criticisms against Composition which are based on the idea of rearrangement can be met, if an arrangement condition is added to the definition of many-one identity. Hence, it turns out that we have to acknowledge that identity is a relation which is more complex than commonly thought. Yet, the complexity of this notion of identity allows us to provide a fruitful theory of composition which makes an assumption of universalism natural. Worries about the implications which come with the view that composition is unrestricted have been dispelled by distinguishing between the questions whether some objects compose an object from the question what object or what kind of object some objects compose. Eventually, I drew a clear distinction between the account of Composition as Identity defended here and nihilism. Although they appear to have some common ground, several reasons show that identifying the two theories is a mistake.

I reckon that radical skeptics of Composition as Identity might remain relatively unmoved by the arguments I provided. Nevertheless, I am confident that they will agree with me on the point that Composition as



Identity can be defended against the challenging criticisms we have discussed and that further arguments are needed in order to undermine the it. Surely, one might be tempted to ask at what cost has Composition as Identity been defended. As we have seen, it is necessary to reject the view that identity is a unitary relation. This will strike some as a price which is too high to pay. However, I think the benefit outweigh the cost. Moreover, by extending our formal language in order to allow for the use of plural terms, the idea of a variety of identity relations becomes necessary. That these relations are in fact *identity* relations, and not merely like identity relations, is grounded on the first of the identity principles we discussed: If two terms ' $\alpha$ ' and ' $\beta$ ' are co-referring, then ' $\alpha$  is/are identical to  $\beta$ ' is true.

The ones who might have been undecided on questions about composition, even if not fully convinced by my arguments, will hopefully have seen that Composition as Identity is a position which deserves to be taken seriously. Finally, I hope that the friends of Composition as Identity will have experienced pressure as well as relief. On the one hand, I suppose they agree with me on the seriousness of the criticisms which are raised against Composition as Identity. On the other hand, I think their mind could be set at ease by developing a framework which gives us a way to reply to these criticisms. Thus, I hope that I succeeded in presenting one way for interpreting and defending the claim that an object is identical to its parts.

We have to concede that the account of Composition as Identity we developed here makes the view a far more complicated position than it looks at first glance. The simplicity of the claim that a composite object is just identical to its parts taken collectively sounds attractive. Yet, by taking identity to be a relation which comes in a variety of ways and making it sensitive to arrangement, we have shown that Composition as Identity is a far more complex theory than one may initially think. However, to think that identity is a simple relation is naive. The puzzles about parthood and composition concern identity already in the first place: Is the ship which results from substituting new planks for all the original

planks *identical* to the original ship? Is a heap of grains *the same* heap after one of the grains is removed? How can a piece of clay *be identical* to a statue if the former exists prior to the latter? People have been puzzled by these lines of reasoning for ages and to think that they can be solved with a simple theory is naive. Yet, as we have seen, Composition as Identity can provide us with a fruitful and coherent basis to resolve these puzzle, if we do justice to the complexity of identity.

What are the next steps to take from here? Underpinning this account of composition with a modal and/or temporal framework promises to lead to interesting insights and to strengthen the theory. Our account is build upon a modal distinction between singular and plural terms. Whether and how this can brought in line with a theory of transworld identity seems a worthwhile inquiry. Similarly, turning back to our discussion on four-dimensionalism I wonder whether the step to four-dimensionalism from the just presented framework is as straightforward as the step to universalism. Also, since we used at some points the tools of second-order logic, it might be worthwhile to consider rephrasing our theory with the use of a second-order logic.

With respect to plural logic, there are several questions which arise. First and foremost, it would be interesting to see whether it is possible to provide some sort of model theoretic semantics for our account of Composition as Identity. This would not only facilitate communicating the theory but also provide us with the means to provide counterexamples to arguments, such as the Paradox from Composition as Identity, in order to show that they are invalid and not only that a certain derivation of the paradox can be blocked.

From a methodological standpoint, we could investigate several questions. As I have already pointed out in the preface, the question what counts as success for a metaphysical theory is a rather difficult one to ask. Also, although there a lot of work has already been done on the principle of parsimony and the simplicity of theories as a theoretical virtue, it seems that further investigations, in particular with respect to Composition as Identity, would be promising.

Finally, from a historical point of view and turning back from where we started our discussion, it would be interesting to see whether our account can be seen as a link to bring together the views, often regarded as opposites, on composition of Plato and Aristotle. Yet, I think an even more interesting project consists in a reexamination of Abaelardus' views on composition. His ideas on the importance of arrangement are unusual. I think they are even more surprising given his inclination to embrace mereological essentialism.

The different nature of these questions shows that a one-dimensional analysis of composition, identity, and parthood has to fall short. In order to clarify these concepts, we have to engage in questions and problems from various philosophical disciplines and make use of the different tools they offer us. I hope our discussion has been a step towards this.



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