MEREOLOGICAL COMPOSITION AND PLURAL QUANTIFIER SEMANTICS

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Abstract

Mereological universalists and nihilists disagree on the conditions for composition. In this paper, we show how this debate is a function of one's chosen semantics for plural quantifiers. Debating mereologists have failed to appreciate this point because of the complexity of the debate and extraneous theoretical commitments. We eliminate this by framing the debate between universalists and nihilists in a formal model where these two theses about composition are contradictory. The examination of the two theories in the model brings clarity to a debate in which opponents frequently talk past one another. With the two views stated precisely, our investigation reveals the dependence of the mereologists' ontological commitments on the semantics of plural quantifiers. Though we discuss the debate with respect to a simplified and idealized model, the insights provided will make more complex debates on composition more productive and deflationist criticisms of the debate less substantial.

Keywords: Composition, Mereology, Plural Logic, Ontology

1 Introduction

The Special Composition Question (SCQ) asks "When is it true that $\exists y$ the xs compose y?" (van Inwagen 1990, 30), or in other words

SCQ: Under what conditions do some objects compose an object?

Two positions dominate the literature on the SCQ: According to mereological nihilism, the answer to the SCQ is *Never*, i.e. no objects compose. According to mereological universalism, the correct answer is *Always*, i.e. any objects compose. This disagreement about the right way to answer the SCQ leads to radically different ontologies, in the sense of Quine (1951), nihilism and universalism[1] disagree about what objects exist: Nihilists claim that there are no objects with parts and the only things that exist are atoms, partless subatomic objects, see (Dorr 2005), (Hossack 2000), (Sider 2013). Universalists assume that besides atoms – if there are any[2] – there are not only ordinary objects, like animals, artifacts, and planets, but also such seemingly extra-ordinary objects as a "trout-turkey",[3] or "an object whose parts are [Michael Rea's] left tennis shoe, W. V. Quine and the Taj Mahal" (Rea 1998, 348), or "the object composed of the moon and … six pennies" (van Cleve 2008, 321).

The aim of this paper is *not* to settle the dispute between nihilists and universalists, but to present an analysis of the two positions which gives a better understanding of the role plural quantifier semantics has in the debate. In order to do this, we will present the two theories within a framework that allows us to spell out their differences in precise logical terms. This framework is a model which does not allow for any alternative view besides nihilism and universalism. It is important to note that this model is only used as a tool: It allows us to keep out certain discussions, for instance, about the existence of trout-turkeys, and helps us to focus on what is at stake in the discussion. In other words, we are isolating variables so as to bring clarity to the debate. We take many of the ongoing disputes in mereology to be the result of the debating parties talking past one another, so to speak. The model will help us eliminate the "noise" which surrounds the discussion on composition.

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[1] We drop 'mereological' from here on, for the sake of better readability.

[2] Universalism is, contrary to nihilism, not committed to the existence of atoms since it is consistent with the assumption that there are gunky objects, i.e. objects whose parts all have themselves proper parts.

[3] A trout-turkey "… is the front half of a trout plus the back half of a turkey, which is neither fish nor fowl [but] part fish and part fowl" (Lewis 1991, 7–8)

2 Framing the Analysis

Arguments against nihilism are often taken to be arguments in favor of universalism, and *vice versa*. However, they are not contradictories but contraries. By recalling Aristotle's square of opposition, this becomes clear: The central claim of nihilism has the form of a universal negative sentence, *No S are P*. Universalism's central claim is a universal affirmative sentence, *Every S is a P*. A universal negative sentence and the according universal affirmative sentence are contraries of each other, i.e. they cannot both be true, but both may be false. The negation of nihilism is the claim *Some objects compose* and the negation of universalism is *Some objects do not compose*. The conjunction of these two particular statements, i.e. the claim that some objects compose and some objects do not compose, is the feature shared by the so-called "[m]oderate answers" (van Inwagen 1990, 61) to the SCQ.

Although some have suggested a moderate answer to the SCQ, e.g. (Carmichael 2015), (van Inwagen 1990), and (Merricks 2001), the predominant view in the literature on the SCQ is that it is an "all or nothing"-question. We will follow this view here since it allows us to see the key differences between the two theories in a much clearer way. We will analyze nihilism and universalism in the context of a model that does not allow any alternative with respect to the SCQ besides those two. This model will be created on the basis of a classical first-order logic with identity and an *Atomistic Extensionality Mereology*. Before we spell out the specifications of the model, we present the line of thought that underlies it informally.

2.1 From Contraries to Contradictories

The purpose of the model we present is to exclude the possibility of an alternative answer to the SCQ besides nihilism and universalism, i.e. the model will not be consistent with the claim that some objects compose and some objects do not.[4] It follows from the model that nihilism holds iff universalism does not hold. Our strategy to get to this conclusion is to specify the model in such a way that the model validates the following claims:

(1) There are at least n many objects and at most m many objects

(2) Nihilism holds in the model iff there are exactly n many objects

(3) Universalism holds in the model iff there are exactly m many objects

(4) There are at most n many objects iff it is not the case that there are at least m many objects

From these claims it follows that nihilism holds in the model iff universalism does not hold in the model, or in other words: Nihilism and universalism are contradictory positions within the model. How can we get to a model that validates these claims?

Obviously, such a model has to be relatively small, similar to the one discussed in (Black 1952). As a first step, we stipulate for the model that there are exactly two atoms, i.e. two objects which do not have any parts.[5] Furthermore, we have to exclude gunky objects, i.e. objects whose parts all have themselves parts, from the model. Finally, we stipulate that if an object with at least one part[6] shares all its parts with an object, then they are identical. Thereby, we exclude that the model contains distinct objects with parts, which share all their proper parts, i.e. we stipulate that our model does not resemble the model of a non-extensional universe depicted in figure 1 on the next page, if we interpret lines

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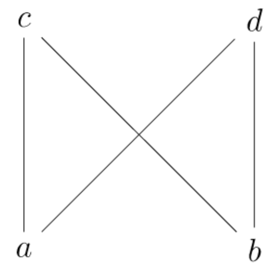
[4] Although Lewis (1986, 211-3) and Sider (2001, 120-32) use the "Argument from Vagueness", which concludes that composition is unrestricted, it has – if it is sound – the effect as our model has: ruling out any alternatives besides nihilism and universalism.

[5] We use the term 'part' here in the sense of *proper* part in contrast to *improper* part. See the next section for a formal definition of 'improper part'.

[6] This condition is needed in order to avoid that all atoms turn out identical, due to the fact that any two atoms have the same parts, none.

going upwards as depicting the parthood relation.



[fig.1] A model of a non-extensional universe.

This is already enough to give us the claims we need as we will see in the next section. Before we move on to the formal presentation of the model and the proof for our claim that nihilism and universalism are contradictories in our model, we wish to highlight the importance of the claim (1): It guarantees that both nihilism and universalism can each hold on their own in the model and avoids begging the question against one of the two theories.

2.2 The Model

Our formal framework consists of classical first-order logic with identity and Atomistic Extensional Mereology (AEM). We take *proper parthood* as our primitive relation, and define *improper parthood*, *being an atom* and *overlapping* as follows:

(D1) $x \leq y =_{df} x < y \lor x = y$

(D2) $Ax =_{df} \neg\exists y(y < x)$

(D3) $x \circ y =_{df} \exists z(z \leq x \land z \leq y)$

AEM[7] is based on three axioms: the transitivity and asymmetry of *proper parthood*, as

[7] Our presentation of AEM is similar to that in (Casati and Varzi 1999). Like Casati and Varzi, we will drop the initial universal quantifier. All formulas are to be understood as universally closed.

well as an atomistic supplementation principle:

(A1) $x < y \wedge y < z \rightarrow x < z$

(A2) $x < y \rightarrow \neg(y < x)$

(A3) $\neg(x \leq y) \rightarrow \exists z(Az \wedge z \leq x \wedge \neg(z \leq y))$

The axiom (A3) allows us to derive the principles called "Atomicity", stating that any object has an atom as improper part, and "Strong Supplementation", which in turn gives us "Extensionality".[8] Hence, it guarantees that our model does not contain any gunky objects and that objects with at least one part are identical iff they have all the same parts. Finally, we restrict the model's domain of discourse to two atoms, i.e. the following formula holds in our model:

(A4) $\exists x \exists y(x \neq y \wedge \forall z(Az \leftrightarrow (x = z \vee y = z)))$

Now we can derive the four claims that, following the strategy outlined above, allow us to show that nihilism and universalism are contradictories within our model. The first conjunct of the first claim we need, states that there are at least two objects, and it follows immediately from (A4):

(T1) $\exists x \exists y(x \neq y)$

According to the second conjunct of the first claim

(T2) $\forall x \forall y \forall z \forall w(x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w)$

there are at most three objects. It can be shown to follow from the axioms with the help of two lemmas

(L1) $Ax \wedge Ay \wedge Az \rightarrow x = y \vee x = z \vee y = z$

(L2) $\neg Ax \rightarrow \forall y(\neg Ay \rightarrow x = y)$

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[8] See (Casati and Varzi 1999, 38-42 & 47-9) and (Simons 1987, 29-30 & 41-4) for more on these three principles and how they are related to (A3).

whereby (L1) tells us that there are at most two atoms, and (L2) says that there is at most one object which is not an atom. The former follows immediately from the definitions and (A4), while a derivation of the latter relies on all of the above axioms.

The theorems (T1) and (T2) show us that our model validates the claim that there are at least two objects and at most three objects:[9]

(i.) $\exists x \exists y(x \neq y) \wedge \forall x \forall y \forall z \forall w(x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w)$

Next come the characterizations of nihilism and universalism within our model. The thesis of nihilism boils down to the claim that there are exactly two objects: According to nihilism, no objects compose. Given that there are exactly two atoms, this assumption is equivalent to the claim that there are exactly two objects. Similarly, the thesis of universalism, stating that any objects compose, becomes equivalent to the claim that there are exactly three objects. Thus, taking *N* and *U* to represent *Nihilism holds* and *Universalism holds*, respectively, we hold on to the following two propositions:

(ii.) $\exists x \exists y(x \neq y) \wedge \forall x \forall y \forall z \forall w(x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w) \rightarrow$
$\rightarrow (N \leftrightarrow (\exists x \exists y(x \neq y) \wedge \forall x \forall y \forall z(x = y \vee x = z \vee y = z)))$

[9] Please note that (i.) is *not* an instance of the following theorem from classical extensional mereology: If there are n atoms, then there are $2^n - 1$ objects, see (Simons 1987, 17). Classical extensional mereology *presupposes* universalism and if this theorem were to hold in our model, then it would ultimately amount to the claim that there are *exactly* three objects. Theorem (i.) on the other hand says that there are *at least* two and *at most* three objects. In other words, it tells us that there are either *exactly* two objects, or *exactly* three objects. This is not a trivial claim. The theorem is only derivable because of the assumption of AEM and (A4), the specification of the model. If we were not to assume AEM, the theorem cannot be derived because (A4) cannot exclude that there are four objects, since it allows for the existence of a gunky object or two distinct composite objects which share all their parts. If we were not to assume (A4), the theorem may fail since there might be only one object, an atom, or there might be more than three objects, for instance, four atoms.

(iii.) $\exists x \exists y(x \neq y) \wedge \forall x \forall y \forall z \forall w(x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w) \rightarrow$
$\rightarrow (U \leftrightarrow (\exists x \exists y \exists z(x \neq y \wedge x \neq y \wedge y \neq z) \wedge \forall x \forall y \forall z \forall w(x = y \vee x = z \vee x = w \vee$
$\vee\ y = z \vee y = w \vee z = w)))$

As a final step, we prove that there are at most two objects iff it is not true that there are at least three objects.

(iv.) $\forall x \forall y \forall z(x = y \vee x = z \vee y = z) \leftrightarrow \neg \exists x \exists y \exists z(x \neq y \wedge x \neq y \wedge y \neq z)$

With these four claims at hand, we can show that, within our model, nihilism and universalism are contradictories:

(T3) $N \leftrightarrow \neg U$

There are at least two and at most three objects (i.). If there are at least two objects and at most three objects, then, by definition, nihilism holds iff there are exactly two objects (ii.), i.e. there are at least two and at most two objects. Hence, it follows from (i.) that nihilism holds iff there are at most two objects. If there are at least two and at most three objects, then, by definition, universalism holds iff there are exactly three objects (iii.), i.e. there are at least three and at most three objects. Hence, it follows from (i.) that universalism holds, iff there are at least three objects. It is a logical truth that there are at most two objects iff it is not the case that there are at least three objects (iv.). Hence, with (ii.) and (iii.), nihilism holds iff universalism does not hold.

As one might have expected, it can be shown that nihilism and universalism are contradictory positions in our model. This shows that our model is a legitimate framework within which nihilism and universalism can be treated as contradictories. Therefore, we can ignore intermediary positions, which will help us to see what are the key elements of the disagreement between nihilism and universalism. Moreover, given excluded middle, evidence for one position will count as evidence against the other.

The above two formulas (ii.) and (iii.) contain what we understand as lying at the heart of the disagreement. This can be seen if we have a closer look at the first conjuncts which follow the equivalence-signs of the formulas: According to nihilism, the formula

(N∀) $\exists x \exists y(x \neq y) \wedge \forall x \forall y \forall z(x = y \vee x = z \vee y = z)$

holds, while according to universalism

(U∀) $\exists x \exists y(x \neq y) \land \neg \forall x \forall y \forall z(x = y \lor x = z \lor y = z)$

or, what comes eventually down to the same, nihilism embraces

(N∃) $\exists x \exists y(x \neq y) \land \neg \exists x \exists y \exists z(x \neq y \land x \neq y \land y \neq z)$

while universalism holds on to

(U∃) $\exists x \exists y(x \neq y) \land \exists x \exists y \exists z(x \neq y \land x \neq y \land y \neq z)$

Accordingly, we take the disagreement between nihilism and universalism to be related to those parts of the formulas which involve the variable *z*, or to put it in other words, about the alleged object for which the variable *z* stands. Before we turn to this analysis of the dispute, we shall discuss some reservations about the use of the just presented model as a means for examining the dispute between the two answers to the SCQ.

2.3 Reservations About the Model

The just presented model plays a central role for our overall argument, since it allows us to treat nihilism and universalism as contradictory positions. Therefore, it is appropriate to question whether the use of this model is legitimate. Considering other discussions on composition and related topics, three objections might be raised against our use of the model. We shall present and answer them in turn.

A first worry might concern our use of the axioms (A1) to (A3), i.e. the transitivity and asymmetry of the parthood relation, and atomistic extensionality. These principles are under dispute in the literature: (Hossack 2000), (Moltmann 1998), and (Rescher 1955) present reasons to reject transitivity; (Cotnoir 2013) and (Thomson 1998) argue against asymmetry; and (Cotnoir 2013), (Nolan 2004) and (Zimmerman 1996) challenge the principle of atomistic extensionality. However, we do *not* presuppose that these axioms *are true*, but form a consistent position. All that is needed for developing the model, is that the conjunction of the above axioms form a consistent position.

Second, it might be raised that it is not appropriate to discuss nihilism within mereology.

After all, nihilism claims that the parthood relation is an empty relation. The nihilist does not need a theory of parthood. Similarly, it does not seem that someone who denies the existence of unicorns needs a theory about unicorns. This concern is more serious, but can be dismissed. The skeptic of unicorns might, *in principle*, not need a theory of unicorns. But as soon as she enters into a discussion with someone who believes in the existence of unicorns, it is unavoidable for her to spell out a minimal theory of unicorns in order to make sure that the discussion is not a merely verbal dispute: The two parties might simply disagree with each other because they are using the term 'unicorn' in a different way. The skeptic may talk about a horned horse, while the believer talks about fish. The nihilist is in a similar situation. In order to make sure that the dispute about parthood and composition is not merely a verbal dispute, the two parties have to make sure that they are talking about the same relation. Therefore, we think that it is legitimate to analyze nihilism on the basis of mereology, even if the nihilist claims that the parthood relation is empty.

Third, we have to face the criticism that our derivation of the contradiction between nihilism and universalism is based on the apparently illegitimate move of counting objects: The two views turn out to be contradictory with respect to the model because they disagree about the right answer to the question *How many objects are there?*, and because there is no alternative answer to this question, besides the two given by nihilism and universalism. However, questions of the form *How many objects are there?* are sometimes, see for instance (Frege 1884 62), (Lowe 1989), (Musgrave 2001, 41) and (Varzi 2000, 285), rejected as being not sensible. The reason for that is based on the thought that 'object' is not a count noun.

We acknowledge this point of criticism and note that it is obviously *not* an option for us to use the terms 'individual', 'entity', or 'thing' instead. They would fell prey to the same line of criticism. Yet, we do not share these reservations.

Two replies legitimate our approach of counting objects here. First, we can characterize the position of nihilism simply as the claim that everything is an atom. Within our model, there is only one alternative view to this claim, namely its negation: something is not an atom. Now the latter claim happens to be equivalent to the universalist position: If we have a model where there are exactly two atoms and where atomistic extensionality holds, then there is something that is not an atom iff any objects compose. Note that we do not need the claim that there is exactly one *non-atom*, which would require the ability of counting non-atoms. This is presumably equally illegitimate as counting objects following the above authors.

Second, consider our first theorem:

(T1) $\exists x \exists y(x \neq y)$

We paraphrase this formula with *There are at least two objects*. However, if the term 'object' is not a count noun, then what claim does the formula represent? Moreover, it seems impossible to make sense of (A3) and some of its immediate corollaries, for instance if there are two atoms, then there are at most four objects, when we cannot count objects.

As we can see, the reservations about our use of the model can be set aside. By using this model, we do not need to take a stand on the questions about the truth of the mereological axioms (A1)–(A3). Furthermore, it is appropriate to suppose that nihilism has to be formulated within a minimal mereological theory in order to make sure that the dispute does not turn out to be a merely verbal disagreement. Finally, we sketched a way how we can show that nihilism and universalism are contradictory in the model without using the term 'object' as a count-noun. Additionally, we noted that if we cannot use 'object' as a count-noun, it is difficult to paraphrase, and hence make sense of, the extensionality axiom of mereology and its corollaries. This shows that we can set aside these reservations against our use of the above model. We shall now move on to see how this model can be used to show that the ontological disagreement between nihilism and universalism is tightly connected to a disagreement about the underlying logic.

3 Extensional Semantics for Mereological Axioms

The relation between the logic and the world is classically understood by interpreting the variables extensionally and the existential quantifier as ontologically committing. In doing so, we adopt a semantics of the quantifiers that is objectual, as opposed to substitutional or free. Thus, in our framework we have the following logico-ontological theorem:

(Ont) $\forall x Fx \rightarrow \exists x Fx$

An immediate consequence is that the mereologists' theses are not only syntactical contradictions, but represent contradictory ontological views.

3.1 Quantification and Ontology

The ontological commitments may be avoided by adopting a substitutional or free interpretation of the quantifiers. However, in each case, doing so introduces other problems.

The substitutional interpretation of Ruth Barcan-Marcus, see (Marcus 1962), (Marcus 1972), and (Marcus 1978), removes the ontological commitment of the quantifiers by taking a quantified sentence to be true just in case some instance of the sentence is true. This removes the burden of ontology from the quantifier but a new set of problems immediately emerges. Such problems have resulted in the substitutional interpretation not gaining widespread adoption, but there are criticisms specifically relevant to our case.

Free logic achieves its goal of quantifying without ontological implications by, among other things, denying existential generalization. Much of the pressure placed on the nihilist comes from the use of this inference rule: $F(t/x) \vdash \exists xF$. So, by rejecting existential generalization via the adoption of free logic, the nihilist can avoid ontological issues from logic alone. But this only pushes the problems further down the line for the nihilist.

The nihilist position is a position about ontology. It is a claim about what does and does not exist. In our case, nihilism entails (N∃), i.e. the formula $\exists x \exists y (x \neq y) \wedge \neg \exists x \exists y \exists z (x \neq y \wedge x \neq y \wedge y \neq z)$. With respect to the two-atom-model, the left conjunct of (N∃) asserts that there exist two nonidentical atoms, but the free logician denies the existential import of the existential quantifier.

Using α and β as labels for the atoms, the nihilist may defend the view that two nonidentical atoms exist in the model by reasoning informally as follows. All atoms exist and α is an atom, thus, α exists. Using the free logicians existence predicate $E!x$, this argument may be represented thus: $A\alpha, \forall x(Ax \rightarrow E!x) \vdash E!\alpha$. This, however, is invalid. Validity requires an inference from $\forall x(Ax \rightarrow E!x)$ to $A\alpha \rightarrow E!\alpha$ which is denied by the free logician. See (Nolt 2014) for similar reasoning and detailed discussion.

We are not saying that these modifications to the axioms result in issues that are insurmountable for the mereologists. Rather, this shows that denying the axioms on which *our argument* is based by varying the interpretation of the quantifiers will only result in other difficulties. The mereologist must face the logical issues directly. This holds whether the mereologists concern themselves with higher-order logic or any other alternative logics.

3.2 The Disagreement Simplified

Considering the existential versions of their theses, (N∃) and (U∃), we take the first conjunct to be satisfied in virtue of the specification for the model. Given that it is a two-atom-model, both mereologists are committed to the truth of $\exists x\exists y(x \neq y)$. So the distinction becomes a disagreement concerning $\exists x\exists y\exists z(x \neq y \land x \neq y \land y \neq z)$. The objectual interpretation means this represents an assertion of the existence of three unique objects. The nihilist, will deny this. The universalist will accept this.

The universalist, given the constraints of the model, asserts that there are exactly three objects but the logic does not specify a distinction between the two atoms and the third object. The model only specifies two objects, so the interpretation of a third object remains an open question. The universalist's third object cannot be an atom. This is because it is a two-atom-model, so if a position asserts the existence of a third object, it cannot be an atom. This is just a theoretical constraint on the model. It seems natural, then, to ask the universalist for an account of this third object. If nothing else, ontological parsimony motivates this inquiry. So, in the absence of additional specifications for the model, the universalist position seems unwarranted.

The burden falls on the universalists to justify their position. In the absence of any justification, with the two-atom-model, it seems that the nihilist position should be the default. This follows if universalism is rejected but one of the two positions must be true in the model. Reflecting on such a simple model, we take it as obvious that there just are two atoms in the model. Not only is this by stipulation in the model, but there seems to be no advantage gained by accepting an ontological commitment to three objects if the task is to account for the number of objects in the world.

3.3 The Central Role of Plural Logic

In response, the universalist may modify their position, to assert the existence of only two objects. However, this would be a denial of universalism as defined here. We take the two-object-ontology to be representative of the nihilist view and such a view has problems as well.

For example, a nihilist ontology seems unable to accommodate a direct-reference semantic analysis of sentences containing linguistic collective predicates. With respect to the model, an example would be,

(T) The two atoms touch only one another.

A property like *touching one another* is not satisfiable by an atom since touching each other requires at least two objects. Thus, we take the predicate to be collective in the sense that it syntactically modifies a collection of objects – in this case, the two atoms.

We take this sentence to have the same structure as the Geach-Kaplan sentence

(GK) Some critics admire only one another.

cited by Quine and others in debating the logic of collective predicates, see (Quine 1982, 238) and (Quine 1974, 111). (GK) seems to require quantification over at least a pair of critics. This pair of critics or plurality of objects needs to be accounted for when symbolizing (GK). In (T), since the two atoms are, by definition, not an object, the predicate is taken to apply to something other than an object. Like (GK), to regiment this sentence in logic, then, the quantifiers must range over something other than just objects, or so the argument goes. Sentences like these are taken as not being expressible in first-order logic, (Boolos 1984, 432), (Hossack 2000, 430).

Whatever this non-object is, it is something that the nihilist will reject an ontological commitment to. The nihilist, by definition, accepts the existence of objects only. In response, some nihilists, for example (van Inwagen 1990, 24–27), will take plural quantification to be ontologically innocent. Van Inwagen follows Quine (1982), (1986), by drawing an analogy between plural quantification and set-theoretic membership. In this case, such an interpretation would be that the two atoms are members of a set that are touching one another.

The universalist who wants to avoid commitment to a third object will do something similar by referring to the third object as the *sum* of the two atoms. For example, to define a sum as follows (Casati and Varzi 1999, 46):

(D4) $\exists x(Fx) \rightarrow (\sigma x[Fx] =_{df} \iota z(\forall y(y \circ z \leftrightarrow \exists x(Fx \;\; y \circ x))))$

Given this definition, the mereological sum of the two atoms in our two-atom-model, $\sigma x[Ax]$, is that object z which overlaps all and only those objects which overlap one of the two atoms. Although it follows from AEM that if such a sum exists, then there is *at most* one – due to extensionality – it does not guarantee that there is *at least* one such sum. This means for our two-atom-model that the existence of a third object is not guaranteed yet. Therefore, universalists have to postulate a sum-principle, see (Casati and Varzi 1999, 46), (Simons 1991, 286–87):

(Sum) $\exists x(Fx) \rightarrow \exists y(y = \sigma x[Fx])$

Generally speaking, (Sum) reassures that whenever there is at least one object that has a certain property F, then there exists the mereological sum of all those objects which have the property F. For instance, assume we find out that one of the atoms from our model is blue. Then we can conclude on the basis of (Sum), that there exists a mereological sum of all the blue objects.

Universalists will go on to define within our model the property of being identical to either one of the two atoms in the model.[10] Then they can conclude on the basis of (Sum) that there exists a third object beside the two atoms which is the sum of those two. Universalists will argue that this sum does not carry additional ontological commitment by appealing to the principle known as *composition as identity*:

(CAI) y is the sum of the Fs iff y is identical to the Fs

The identity in the above principle is a so-called "many-one identity", see (Baxter 1988), (Bricker 2016), (Lewis 1999, 195), (Turner 2013), i.e. an identity that holds between many things, the Fs, and one thing, the sum of the Fs. (CAI) contains, like (T) and (GK), a collective predicate, the many parts are *collectively* identical to the whole. This thought cannot be expressed by using singular terms only. Thereby, (CAI) commits the universalist to a framework which allows her to express this principle, or as Cotnoir puts it:

> [M]erely stating the thesis of CAI necessarily involves a plural formulation … [C]laims like 'They are it' and 'it is them' are irreducibly plural (Cotnoir 2014, 18)

Given our working definition of nihilism, the mereological positions that rely on (CAI) will count as nihilists in virtue of their common goal of maintaining an ontological commitment to at most two objects in the model. This deviation from the common use of the term 'nihilism' is a consequence of our decision to identify the nihilist position with the claim that there are exactly two objects in the model from section 2.

Thus, the nihilist in giving a holistic account of the ontology in the model, will require incorporating the theoretical machinery of plural logic. As such, the ontological innocence of plural quantification becomes a central concern for this family of mereological positions.

3.4 From Plural Quantifier Semantics to Mereology

Since the nihilist relies on plural quantification to express facts about the model, the

[10] Another property, being an atom, would do the same job here.

nihilist position *depends* on the ontological innocence of plural quantification. If the thesis that plural quantification is ontologically innocent is represented as

(OI) Plural quantification is ontologically innocent.

then we may express this dependency relation using the material conditional.

(A) $\neg PL \rightarrow \neg N$

That is, if plural quantification is not ontologically innocent, then the nihilist cannot maintain their commitment to at most two atoms in the model. If plural quantification is not ontologically innocent, then there are truths in the model, like (T), that will entail an ontological commitment to pluralities. Such a commitment results in a quantity of objects that exceeds the number set in the definition of the nihilist position.

The contrapositive of (A) expresses this in an illuminating way:

(A′) $N \rightarrow PL$

Thus, the nihilist requires plural quantification to be ontologically innocent. That is, if plural quantification is ontologically committing, then the nihilist cannot frame their position in a way that avoids commitment to the things they set out to deny.

There are interesting consequences for the universalist as well. Given the contradictory relation between (N) and (U) established in section 2 of the paper, we can infer from (A) with first-order logic:

(B) $\neg PL \rightarrow U$

This states that with respect to our framework, a mereologist who does not accept the ontological innocence of plural logic will need to embrace universalism in the sense defined. Finally, by transposition we have,

(B′) $\neg U \rightarrow PL$

This is an alternative way of expressing the idea in (A′). According to (B′), if a mereologist does not accept universalism, then she needs to defend the ontological innocence of plural logic.

What this analysis shows is that the debates concerning plural logic should be resolved before the mereological dispute, since one's decision on the semantics of plural quantifiers entails a particular position on composition. Here 'priority' does not mean simply the antecedent of a conditional. In (A′) and (B′), the antecedent position is occupied by a mereological thesis. Rather, the entailments show that logical concerns are a priority in the sense that the mereologist cannot give a precise account of their position without having first made decisions concerning the logic in which to regiment their theoretical assertions. Choices concerning the semantics of higher-order logics govern the availability of options in the domain of mereology. For example, (A′) shows that the ontological innocence of plural logic is necessary for the nihilist. We infer from this that it is prudent for the nihilist to defend the ontological innocence of plural logic before defending nihilism as such.

3.5 Awaiting Insights from Logic

Though a central aim of the paper was to layout these entailment relations, it is worth briefly considering whether this analysis lends support to either position. We have seen that ontological parsimony favors the nihilist position, but these entailments highlight the nihilist's reliance on ontologically innocent plural quantification. Consequently, nihilists have employed two general approaches to translating sentences like (T) and (GK) into a formal system without ontological commitments beyond what is already required in classical first-order logic. The traditional view, associated with Quine, is to paraphrase the sentence into first-order logic and accommodate the plurality by interpreting the quantifiers as ranging over sets (Quine 1982). Hence, Quine's dictum that higher-order logic is "set theory in sheep's clothing" (Quine 1986, 66).

An alternative approach, offered by George Boolos, is to avoid introducing sets by interpreting the quantifiers as being plural (Boolos 1984) and (Boolos 1985). This is the point of departure for Boolos and others who reject Quine's introduction of sets. The concern is that Quine's account is not ontologically innocent and the introduction of sets is a violation of intuition. Opponents assert that although the Geach-Kaplan sentence requires an ontological commitment to critics, it does not (at least presumably) require an ontological commitment to *sets* of critics. Boolos proclaims, "It is haywire to think that when you have some Cheerios, you are eating a set - what you're doing is: eating THE CHEERIOS" (Boolos 1984, 448).

Both strategies have come under heavy criticism. See, for example, (Resnik 1988) and (Shapiro 2005). For a defense of higher-order logic, see (Shapiro 1991). Whatever one's view is on the matter, the thesis for this paper is that this debate on the appropriate logic for these types of sentences is a debate that must occur prior to the mereological debate. If plural quantification or set-theory is not ontologically innocent, then the nihilist and attenuated universalist positions are untenable.

In the absence of an alternative logic, or explaining away the apparent ontological commitments of set-theory or higher-order logic, it seems that neither position is favored by our analysis. This is expected. We claim that logic is a priority, so until the logical matters are settled, each mereological position is equally unsupported.

4 Closing Remarks

It strikes us that many mereological debates stem from the debating mereologists using the same terms but having different meanings in mind. This has been noted by others. Eli Hirsch, for example, has argued this point at length (Hirsch 2010). This paper represents our contribution to what we believe is a way forward from this. Part of the challenge to clarity in mereology is due to the level of analysis. The number of assumptions that vary independently for the mereologist is significant. Fixing variables is what we take to be fundamental to bringing clarity to the debates. Here we draw an analogy between the number of equations needing a solution and the number of independent variables. By fixing variables at the level of logic, there are less independent variables making the decision between mereological positions more tractable.

In addition to the methodological claims, we take simple denials of our assumptions to be ineffective for undermining our proposal. Avoiding the challenges with plural logic by means of adopting an alternative logic only changes the type of logic requiring analysis. It does not change the ordering of the debate that we propose. Regardless of alternative logic chosen, the logico-ontological decisions (fixing variables) must be done before adjudicating between nihilism and universalism.

References

Baxter, Donald L. M. 1988. "Many-One Identity." *Philosophical Papers* 17 (3): 193–216.

Black, Max. 1952. "The Identity of Indiscernibles." *Mind* 61 (242): 153–64.

Boolos, George. 1984. "To Be Is to Be a Value of a Variable (or to Be Some Values of Some Variables)." *The Journal of Philosophy* 81 (8): 430–49.

———. 1985. "Nominalist Platonism." *The Philosophical Review* 94 (3): 327–44.

Bricker, Phillip. 2016. "Composition as a *Kind* of Identity." *Inquiry* 59 (3): 264–94.

Carmichael, Chad. 2015. "Toward a Commonsense Answer to the Special Composition Questions." *Australasian Journal of Philosophy* 93 (3): 475–90.

Casati, Roberto, and Achille C. Varzi. 1999. *Parts and Places - The Structures of Spatial Representation*. Cambridge (MA): MIT Press.

Van Cleve, James. 2008. "The Moon and Sixpence." In *Contemporary Debates in Metaphysics*, edited by Theodore Sider, John Hawthorne, and Dean W. Zimmerman, 321–40. Oxford: Blackwell.

Cotnoir, Aaron J. 2013. "Strange Parts: The Metaphysics of Non-Classical Mereologies." *Philosophy Compass* 8 (9): 834–45.

———. 2014. "Composition as Identity: Framing the Debate." In *Composition as Identity*, edited by Aaron J. Cotnoir and Donald L. M. Baxter, 3–23. Oxford: OUP.

Dorr, Cian. 2005. "What We Disagree About When We Disagree About Ontology." In *Fictionalism in Metaphysics*, edited by M. E. Kalderon, 234–86. Oxford: OUP.

Frege, Gottlob. 1884. *Die Grundlagen der Arithmetik - Eine logisch mathematische*

Untersuchung über den Begriff der Zahl. Breslau: Verlag von Wilhelm Koebner. (Reprinted and translated by J. L. Austin. 1968. *The Foundations of Arithmetic - A logico-mathematical enquiry into the concept of number*. Oxford: Basil Blackwell, second revised edition).

Hirsch, Eli. 2010. *Quantifier Variance and Realism: Essays in Metaontology*. Oxford: OUP.

Hossack, Keith. 2000. "Plurals and Complexes." *British Journal for the Philosophy of Science* 51: 411–43.

Van Inwagen, Peter. 1990. *Material Beings*. Ithaca: Cornell University.

Lewis, David K. 1986. *On the Plurality of Worlds*. Oxford: Basil Blackwell.

———. 1991. *Parts of Classes*. Oxford: Basil Blackwell.

———. 1999. "Rearrangement of Particles: Reply to Lowe." In *Papers in Metaphysics and Epistemology*, edited by David K. Lewis, 187–95. Cambridge: Cambridge University Press.

Lowe, E. J. 1989. *Kinds of Being: A Study of Individuation, Identity and the Logic of Sortal Terms*. Oxford: Blackwell.

Marcus, Ruth Barcan. 1962. "Interpreting Quantification." *Inquiry* 5 (1-4): 252–59.

———. 1972. "Quantification and Ontology." *Noûs 6 (3)*: 240–50.

———. 1978. "Nominalism and the Substitutional Quantifier." *The Monist* 61 (3): 351–62.

Merricks, Trenton. 2001. *Objects and Persons*. Oxford: OUP.

Moltmann, Friederike. 1998. "Part Structures, Integrity, and the Mass-Count Distinction." *Synthese* 116 (1): 75–111.

Musgrave, Alan. 2001. "Metaphysical Realism Versus Word-Magic." In *Realismus, Disziplin, Interdisziplinarität*, edited by Dariusz Aleksandrowicz and Hans Ruß, 29–54.

Amsterdam, Atlanta: Editions Rodopi.

Nolan, Daniel. 2004. “Classes, Worlds and Hypergunk.” *The Monist* 87 (3): 303–21.

Nolt, John. 2014. “Free Logic.” In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Winter 2014. http://plato.stanford.edu/archives/win2014/entries/logic-free/.

Quine, W. V. O. 1974. *The Roots of Reference*. Lasalle (IL): Open Court.

———. 1982. *Methods of Logic*. Cambridge (MA): Harvard University Press.

———. 1951. “Ontology and Ideology.” *Philosophical Studies* 2 (1): 11–15.

———. 1986. *Philosophy of Logic*. Cambridge (MA): Harvard University Press.

Rea, Michael. 1998. “In Defense of Mereological Universalism.” *Philosophy and Phenomenological Research* 58 (2): 347–60.

Rescher, Nicholas. 1955. “Axioms for the Part-Whole Relation.” *Philosophical Studies* 6 (1): 8–11.

Resnik, Michael D. 1988. “Second-Order Logic Still Wild.” *The Journal of Philosophy* 85 (2). JSTOR: 75–87.

Shapiro, Stewart. 1991. *Foundations Without Foundationalism: A Case for Second-Order Logic*. Cambridge: Cambridge University Press.

———. 2005. “Higher-Order Logic.” In *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by Stewart Shapiro. Oxford: OUP.

Sider, Theodore. 2001. *Four-Dimensionalism: An Ontology of Persistence and Time.* Oxford: OUP.

———. 2013. “Against Parthood.” In *Oxford Studies in Metaphysics*, edited by Karen Bennett and Dean W. Zimmerman, 8:237–93. Oxford: OUP.

Simons, Peter. 1987. *Parts - A Study in Ontology*. Oxford: OUP.

———. 1991. “Free Part-Whole Theory.” In *Philosophical Applications of Free Logic*, edited by Karel Lambert, 285–306. Oxford: OUP.

Thomson, Judith J. 1998. “The Statue and the Clay.” *Noûs* 32 (2): 149–73.

Turner, Jason. 2013. “Existence and Many-One Identity.” *Philosophical Quarterly* 63 (250): 313–29.

Varzi, Achille C. 2000. “Mereological Commitments.” *Dialectica* 54 (4): 283–305.

Zimmerman, Dean W. 1996. “Could Extended Objects Be Made Out of Simple Parts? An Argument for ‘Atomless Gunk’.” *Philosophy and Phenomenological Research* 56 (1): 1–29.