**Exploring Mathematics and Noumenal Realm through Kant and Hegel**

**Abstract**

This paper discusses the philosophical basis of mathematics by examining the perspectives of Kant and Hegel. It explores how Kant’s concept of the synthetic a priori, grounded in the intuitions of space and time, serves as a foundation for understanding mathematics. The paper then integrates Hegelian dialectics to propose a broader conception of mathematics, suggesting that the relationship between space and time is dialectically embedded in reality. By introducing the idea of a hypothetical transcendental subject, the paper attempts to overcome a potential limitation of Kant’s framework, particularly regarding the application of mathematical truths to pre-human reality. This synthesis of Kantian and Hegelian thoughts offers a lens through which the connection between mathematics and reality can be understood, while also acknowledging limitations in both philosophical systems.

Keywords: dialectics; philosophy of mathematics; noumena; absolute spirit; absolute idea

**1. Introduction**

There are radical arguments that mathematics explains everything today. For instance, Tegmark (2008) asserts that “our universe *is* mathematics” (p. 1). His argument is debatable, but there is no denying that mathematics matters enormously. However, what is mathematics? Also, how to explain its sheer efficiency in science?

When discussing the foundations of mathematics, we typically encounter formalism, logicism, and intuitionism. *Formalism* was propounded by David Hilbert, who sought for “a formalization of all of mathematics in axiomatic form” (Zach, 2023, Section 0). Under his formalist program, “mathematical propositions and proofs … turn into formulas and derivations from axioms according to strictly circumscribed rules of derivation” (Section 1.3). His program sought to remove Kant’s intuitionistic elements from mathematics. *Logicism* was pursued by Bertrand Russell and Alfred Whitehead. Their school dictates that mathematics can be reduced to logic and that all mathematical truths are ultimately logical truths. “Russell ... emphasized that a chief aim of the logicist project is to show that arithmetic and real analysis are not grounded in Kantian intuition” (Goldfarb, 1982, p. 692). *Intuitionists*, on the other hand, hold that mathematics is constructed by the human mind. They “hoped to found mathematics ... on our a priori intuition of time” (Maddy, 2012, p. 486). They believed that “mathematics is true of the world, and we can know this a priori, because the world ... is partly shaped by our temporal form of intuition.”

Given the influence of Kantian philosophy on the three schools, it is essential to consider Kant’s ideas when researching the philosophical basis of mathematics. Accordingly, this paper will investigate them and present two brief philosophical theses for mathematics under the Kantian context. Section 2 provides the first thesis that defines mathematics. Section 3 presents the second one that explains the reason for its efficiency. Section 4 discusses one worry about Kantian philosophy – that is, how to understand the world before the era of the human species. To address the worry, Section 5 examines Hegelian philosophy. In Section 6, we will discuss Hegel’s conceptions of space and time. Building on these, Section 7 offers a renewed thesis for mathematics. Section 8 indicates issues with Hegelian philosophy. Then, the paper concludes that ultimate reality is not fully knowable.

**2. Kantian Philosophy of Mathematics**

In the *Critique of Pure Reason* (1998), Kant asserts that “there are two pure forms of sensible intuition as principles of *a priori* cognition, namely space and time” (p. 157). “By means of outer sense ... we represent to ourselves objects as outside us, and all as in space. In space their form, magnitude, and relation to one another is determined.” Further, “[s]pace is a necessary representation, *a priori*, which is the ground of all outer intuitions” (p. 158). It is “in this *a priori* necessity” that the “apodictic certainty of all geometrical principles and the possibility of their *a priori* construction are grounded.”

Meanwhile, regarding “inner sense,” Kant notes that “everything that belongs to the inner determinations is represented in relations of time” (p. 157). If the origin of geometry can be attributed to our intuition of space, what branch of mathematics could be said to be based upon the intuition of time? Kant states that “arithmetic forms its concepts of numbers through successive addition of units in time, but above all pure mechanics can form its concepts of motion only by means of the representation of time” (Kant, 2004, p. 35). Regarding this, Shabel (2021) argues that “a formal intuition of time is inadequate to explain the general and abstract science of number” (Section 2.3). Accordingly, Shabel believes that “Kant declares mechanics to be the mathematical science that is to time what geometry is to space.”

Whether it is arithmetic or mechanics that naturally derives from the intuition of time, it seems clear that Kant prioritizes geometry over arithmetic. In the *Critique of Pure Reason*, Kant does not accord arithmetic the same level of explicit endorsement through the intuition of time that he provides for the “apodictic certainty” of geometry through the intuition of space.[[1]](#footnote-1) Moreover, Sutherland (2004) argues that “magnitudes are at the heart of Kant’s theory of mathematical cognition” and that Kant’s treatment of them is “strongly influenced by the Greek mathematical tradition” (p. 157). As well known, the “Greek mathematical tradition ... gave priority to geometry over arithmetic” (p. 158). For Kant, “universal arithmetic (algebra) concerned the ratios between magnitudes” (p. 164). In his late times, Frege expressed a somewhat similar view that “the concept of number requires pure temporal and *spatial* intuition in the Kantian sense” (emphasis added) (Hanna, 2001, p. 164). Meanwhile, Sir William Rowan Hamilton maintained that “since geometry is a science of space, and since time and space are ‘pure sensuous forms of intuition,’ algebra must be a science about time” (Burton, 2011, p. 634). Whatever the case, a Kantian thesis on mathematics can be briefly put as follows:

**Mathematics originates as a synthetic a priori[[2]](#footnote-2) discipline that deals with spatial constructs and numerical values, grounded in our intuition of space and time and developed in accordance with logic. (Thesis 1)**

However, there are several branches of mathematics that are seemingly independent of arithmetic and geometry. To name just a few, there are combinatorics, graph theory, set theory, etc. But none of them are entirely separable from the two rudimentary concepts of mathematics: shapes and counting.[[3]](#footnote-3)

First, combinatorics is “concerned with arrangements of the objects of a set into patterns satisfying specified rules” (Brualdi, 2010, p. 1). Although it has nothing to do with, say, Euclidean geometry, it still relies on the abstract spatial idea of “arrangements.” Moreover, the basic themes of combinatorics such as permutations and combinations are closely related to the concept of counting. Secondly, graph theory studies the properties and structures of graphs, which are collections of vertices (or nodes) connected by edges (or links).[[4]](#footnote-4) One would not be able to envision a concept of linking between vertices if she did not have intuition of space. Thirdly, set theory investigates “sets of *mathematical* objects, such as numbers, points of space, functions, or sets” (Hrbacek, K., & Jech, T., 1999, p. 1). Set theory defines natural numbers as follows:

1={∅}={0},

2={∅,{∅}}={0,1},

3={∅,{∅},{∅,{∅}}}={0,1,2},

and so on.[[5]](#footnote-5)

We face one question: Does the above set-theoretic construction of natural numbers, which causes no apparent conflict within Zermelo-Fraenkel set theory, rightfully precede the intuitive concept of counting? According to Hanna (2001), the late Frege’s view that “number requires pure temporal and spatial intuition” could have “the same logical force as a well-founded or non-paradoxical set theory -- only without the controversial axioms required by, say, the Zemelo-Fraenkel theory” (p. 164). Therefore, despite the formalist merits of the set-theoretic construction of numbers, the Kantian approach provides a more fundamental basis.

Given this foundational understanding of mathematics, we will explore in the subsequent section how these abstract, a priori principles could translate into the effective applications of mathematics in the real world.

**3. Kantian Explanation on Unreasonable Effectiveness of Mathematics**

Through a Kantian framework, this section attempts to explain Wigner (1960)’s “unreasonable effectiveness of mathematics in the natural sciences.” Wigner remarks that “the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious” (p. 2). Einstein’s general relativity provides a good example. Wald (1984) comments that “general relativity has made a number of strikingly successful predictions concerning the spacetime structure of our universe” (p. 118). One of them is black holes. Given Einstein’s gravitational field equations, Schwarzschild considered the case of spacetime surrounding a spherically symmetric, non-rotating mass (e.g., a star) in the absence of any other matter or energy. In this scenario, the “stress-energy tensor” in the Einstein field equations is set to zero, which describes a “vacuum” solution. Schwarzschild derived the following solution to the Einstein field equations:[[6]](#footnote-6)

ds² = - (1 - 2GM / c²r) c² dt² + (1 - 2GM / c²r)⁻¹ dr² + r²(dθ² + sin²θ dφ²)

where:

ds = spacetime interval

G = gravitational constant

M = mass of the non-rotating mass

c = speed of light

r = radius of the non-rotating mass

dt = infinitesimal time interval

dr = infinitesimal change in the radius

dθ = infinitesimal change in the polar angle (between the position vector of a point and the z-axis in spherical coordinates)

dφ = infinitesimal change in the azimuthal angle θ (around the z-axis).

When a particular mass satisfying the vacuum condition undergoes gravitational collapse, its radius can decrease to equal the “Schwarzschild radius” (2GM/c²). At this point, the time term “- (1 - 2GM / c²r) c² dt²” becomes zero (i.e., infinite time dilation), and the radial term “(1 - 2GM / c²r)⁻¹ dr²” becomes infinite (i.e., infinite spatial stretching) for any finite “dr.” This is known as the event horizon of a black hole. The existence of black holes has been empirically confirmed through observations.[[7]](#footnote-7) It is remarkable that mathematical manipulation of the equations led to the prediction and subsequent discovery of distant celestial bodies, whose existence is highly unintuitive. What explains this unreasonable effectiveness of mathematics?

From a Kantian perspective, mathematics corresponds to “reality” because it is fundamentally rooted in our processes of imagination,[[8]](#footnote-8) which actively structures and organizes the manifold[[9]](#footnote-9) through schemata. These schemata, as procedural rules, allow the imagination to construct mathematical concepts that are intrinsically linked to the spatio-temporal framework of our perception. Since space and time are the forms through which we experience the world, and mathematics operates within these forms, mathematical constructions are inherently aligned with the way reality is perceived and understood.

In a similar vein, Hall (2013) concludes that “imagination, via its operation of schematising … could be aptly understood as the mathematiser of reality” (p. 244). Moreover, the manifold could be thought of as “a blank sheet of paper covered with dots, in which case the (productive) imagination would be the function or power connecting the dots according to a rule-governed procedure or schema … allowing for the construction of, for example, geometric figures, arithmetically countable strokes and algebraic symbols.” Thus, we can formulate the following thesis:

**Mathematics *appears to* reflect reality because our imagination, at a preconscious level, provides a mathematical rendering of the manifold for our intellectual faculty, which operates within the intuitions of space and time. (Thesis 2)**

However, the thesis is not without its limitations. In the next section, we will briefly discuss one worry about it.

**4. One Worry about Kantian Philosophy**

According to Cummins (1968), “Kant asserted that neither the objects nor the forms of any intuitions have transcendental reality; they merely have *phenomenal (empirical) reality*. However, although neither outer nor inner intuition is apprehension of *independent (transcendental) reality*, both are related to bodies of synthetic apriori knowledge” (emphasis added) (p. 271). Indeed, Kant says:

“[I]f we remove our own subject or even only the subjective constitution of the senses in general, then all constitution, all relations of objects in space and time, indeed space and time themselves would disappear, and as appearances they cannot exist in themselves, but only in us” (Kant, 1998, p. 185).

In other words, space and time as well as physical objects intuited in their spatio-temporal form are not real independently of human cognition. This suggests that the mathematically derived truths of natural science might not have applied before the emergence of our species. However, it seems to go against our empirical “intuition” (or gut feelings). For instance, Žižek (2012) asks, “[I]s a dinosaur fossil proof that dinosaurs existed on Earth independently of any human observer, whether empirical or transcendental? If we can imagine transposing ourselves into the pre-historical past, would we encounter dinosaurs the way we reconstruct them today?” (p. 647). This question may be addressed in a simple way. In transcendental philosophy, it is nonsensical to ask what would have happened before human beings, because there were no cognitive agents who had intuitions of space and time. Similarly, Hanna (2001) notes: “If creatures minded like us had not existed, then purely logically speaking something might still have existed, but the assertion that it did exist would have been at the very least empirically meaningless and without a truth value” (p. 105).

In other words, it is meaningless to ask what happened “before” the human species, because time is meaningful only from a transcendental subject’s perspective. However, Meillassoux (2008) is not satisfied. He asks, “*what is* *it* exactly astrophysicists, geologists, or paleontologists are talking about when they discuss the age of the universe, the date of the accretion of the earth, the date of the appearance of pre-human species, or the date of the emergence of humanity itself? How are we to grasp the meaning of scientific statements bearing explicitly upon a manifestation of the world that is posited as anterior to the emergence of thought and even of life -- *posited, that is, as anterior to every form of human relation to the world*?” (pp. 9-10).

To answer the question, we can try assuming the legitimacy of establishing a viewpoint held by a *hypothetical* transcendental subject “before” humans came into existence. This assumption is necessary when considering that the “thought that something might have existed even if we had not already presupposes the cognitive capacities of creatures minded like us” (Hanna, 2001, p. 105). By introducing the hypothetical subject, we might be able to retain the idea that the world in the “pre-historical past” might have existed as it could be perceived by humans. So, to answer Žižek’s question, yes, we would be able to discover dinosaurs as speculated today if we time-traveled into the past. However, this scenario cannot escape from bringing in the idea that time might be transcendentally real, as we should assume that the world had been perceived through “virtual” intuitions of space and time. But this conflicts with orthodox transcendental philosophy. Yet, adhering to the existing doctrines of transcendentalism may not satisfy our growing sense that scientific realism deserves more consideration, if not full acceptance.

To reconcile between Kantianism and scientism, we must redefine the noumenal realm by incorporating our intuitions. This redefinition can involve introducing the hypothetical transcendental subject. If this existence can be justified, there is no reason to not suppose that the noumenal realm shares a certain spatio-temporal structure with our consciousness. This issue will be explored through a Hegelian framework in the next section.

**5. Hegel’s Absolute Idea as Alternative to Noumenon**

“[E]ternity is not before or after time, not before the creation of the world, nor when it perishes; rather is eternity the absolute present, the Now, without before and after. The world is created, is now being created, and has eternally been created; … Creating is the activity of the *absolute Idea*; … the true universal is the Idea, which … is eternal. The finite, however, is temporal” (emphasis added) (Hegel, 2004, 15).

Judging from Hegel’s text, the “absolute Idea” (or simply “Idea”) can be described as *super-temporal*. A finite human consciousness, which is marked by its temporality, can only detect contradictions when trying to comprehend the origin of the world. This is evident in Kant’s first antinomy.[[10]](#footnote-10) Meanwhile, no contradiction is seen from the standpoint located in the Idea, because it transcends our *temporal* realm. While we cannot experience the “before” and “after” at the same time, they can be seen simultaneously from “the absolute present.” The Idea can be viewed as Hegel’s counterpart to Kant’s “positive noumenon.[[11]](#footnote-11) Then, how could the super-temporal Idea create a phenomenal world (or “Nature”) for the hypothetical transcendental subject? The answer lies in the dialectic formula: thesis-antithesis-synthesis. According to Hegel, “Nature” is “the negative of the Idea” (Hegel, 2004, p. 19). More specifically, Nature is an externalized form of the Idea as its antithesis. Then what is the synthesis of the Idea and Nature? It is the Absolute Spirit (or simply “Spirit”), which is the full realization and self-manifestation of the Idea. Hegel says:

“Absolute Spirit implies eternal self-identical existence that is transformed to another and knows this to be itself; the unchangeable, which is unchangeable in as far as it always, from being something different, returns into itself. It signifies the sceptical movement of consciousness, but in such a form that the transient objective element at the same time remains permanent, or in its permanence has the signification of self-consciousness” (Hegel, 1894, p. 377).

This somewhat abstruse explanation can be interpreted as follows. While achieving absolute self-identity throughout eternity, the Spirit can also transition into something else and yet identify itself with this something. It is also unchangeable in that it perennially (“unchangeably”) returns to itself after the transition. Moreover, it represents the speculative movement[[12]](#footnote-12) of consciousness (which can be partially shared by finite consciousness). Specifically, from the Spirit’s perspective, objective reality, which appears transient to finite consciousness, appears permanent. Compare this with our finite consciousness. From our standpoint, the sense of “now” persists only for a short time. Throughout this temporal expanse, we are quite certain we are the same persons that we are. Although the exact neural state of our consciousness at the beginning of this time must differ from that at the end of this time, we maintain a sense of continuity throughout. However, the Spirit differs from our finite consciousness in that it retains this sense of “now” throughout eternity. Therefore, its activity can be said to be *pan-temporal*.

Adopting this principle, we can connect between the temporal dimension of our realm and the higher source of all things (i.e., the Idea). Specifically, we can argue that Nature (i.e., the universe) and the Spirit (which we are a part of) share a particular relationship with time. Under this scheme, space and time can be treated as more “real” than they were under the Kantian scheme. This suggests that mathematics can connect with reality because it is partially embedded within reality. As such, Hegel’s speculative philosophy bridges between the Kantian construction of reality and contemporary scientific endeavors. Based on this renewed understanding, we can establish a new thesis for mathematics. However, before we do that, we will discuss space and time from a Hegelian perspective in the next section.

**6. Hegelian Approach to Space and Time**

(1) Hegel on Space

“The first or immediate determination of Nature is Space, the abstract *universality of Nature's self-externality*, self-externality’s mediationless indifference. It is a wholly ideal *side-by-sideness* because it is self-externality; and it is absolutely *continuous*, because this *asunderness* is still quite abstract, and contains no specific difference within itself” (Hegel, 2004, p. 29-30).

Hegel’s text suggests space is the most basic and immediate way that Nature takes form. However, it is unclear how space can be regarded as the “abstract universality” of Nature’s self-externality. Hegel likely would have meant that space is an externalized form of Nature. But how could it be “abstract universality”? This could suggest that space is a universal concept that applies everywhere in its pure form. At the same time, it has “mediationless indifference,” meaning that space lacks internal distinctions. Moreover, space is simply a continuous “side-by-sidedness” where parts exist next to each other without any deeper mediation. Now, let us examine what Hegel says about time.

(2) Hegel on Time

“Time, as the negative unity of self-externality, is similarly an out-and-out abstract, ideal being. It is that being which, inasmuch as it is, is not, and inasmuch as it is not, is: it is Becoming directly intuited; this means that differences, which admittedly are purely momentary, i.e. directly self-sublating, are determined as external, i.e. as external to themselves” (Hegel, 2004, p. 34).

The “negative unity of self-externality” likely indicates that time is the antithesis of space. However, even as time exists, simultaneously it does not exist. Or even as time does not exist, it exists. Here, Hegel seems to anticipate dialetheism. Further, “[time] is Becoming directly intuited.” In other words, a temporal transition is something that is immediately perceived or understood by us without reasoning or mediation. In the previous section of this paper, we contrasted the short time span of the “now” that we experience with the eternal time span of the “Now” occupied by the Spirit. The time discussed in the above texts pertain to the former and not the latter.

(3) Space and Time

Regarding the conceptual sequence of space and time, Hegel says:

“The truth of space is time, and thus space becomes time; the transition to time is not made subjectively by us, but made by space itself. In pictorial thought, space and time are taken to be quite separate: we have space and *also* time” (Hegel, 2004, p. 34).

In the previous section, we saw how the phenomenal world (“Nature”) came into being through the thread of dialectics within a temporal hierarchy: Super-temporal – pan-temporal – temporal. Does this contradict what Hegel said above? No. Again, the “time” in Hegel’s remarks relates to our phenomenal realm. Thus, there is no inconsistency.

(4) Motion in Space and Time

“In motion, space posits itself temporally and time posits itself spatially ... Motion falls into the Zenonian antinomy, insoluble if the places are isolated as points of space, and the time-moments as points of time; and the solution of the antinomy, i.e. motion, is reached only when space and time are grasped as in themselves continuous,[[13]](#footnote-13) and the moving body as being at once in and not in the same place, i.e. as being at once in another place; just as the same point of time at once is and is not, i.e. is at once another point of time” (Hegel, 2004, pp. 134-135).

The dialectical relationship between space and time is evident in the statement that “space posits itself temporally and time posits itself spatially.” However, this relationship is explained with respect to a moving body. Under the context of motion, they are not isolated entities; they are interwoven. When the object moves, its position in space changes according to the passage of time. This temporal passage can also be delineated through the object’s motion.

Furthermore, whereas time is dynamic, space is static. In this regard, a spatial segment can be understood as a *static*, *simultaneous* representation of successive units in time. Likewise, time can be represented as *dynamic*, *successive* changes throughout space. In the next section, these notions will be taken into consideration when formulating a new thesis for the philosophy of mathematics.

**7. New Thesis for Philosophy of Mathematics**

**Mathematics is a synthetic a priori discipline that logically engages with spatial constructs and numerical values, grounded in our intuition of space and time, which are objectively embedded as a dialectical pair in the manifest world. (Thesis 3)**

Arguably, Kant’s greatest contributions to the philosophy of mathematics are (1) his clarification of mathematics as a synthetic a priori discipline and (2) demonstration of its connection to the phenomenal world through the intuitions of space and time. However, his philosophy was limited in scope due to its exclusive focus on explaining everything through a transcendental lens. By replacing Kant’s noumenon with Hegel’s Idea, we can now justify our mathematization of reality as that of a substantially real outside world. This carries an important implication that our mathematical construction is at least partially embedded in the universe.[[14]](#footnote-14) However, Hegelian philosophy, which the above thesis is based upon, is not without its limitations, either. We will investigate this issue in the following section.

**8. Limitations of Hegelian Philosophy**

According to Taylor (1975), “Absolute spirit is … higher than Spirit’s realization in objective reality which has not yet come to full self-consciousness” (p. 466). However, it is questionable whether the Absolute Spirit ever will come. One cannot help but feel that Hegel’s system of philosophy is overly ideal.

First, even if a state of absolute knowledge were achievable, it is difficult to imagine what state of mind we would actually experience in such a condition. Suppose nevertheless that our world did enter the stage of absolute knowledge. Even in this stage, we would be the finite components of the Absolute Spirit. Therefore, absolute knowledge would be still unattainable on an individual level. One could say that all the truths of the universe could be potentially knowable because finite consciousness partakes in the Absolute Spirit. This reminds us of a democracy where every citizen could potentially become president but only a select few ever do. Further, while citizens may actively care about how their state is run by their government, but most are completely in the dark about what it is doing on their behalf.

Second, Hegel’s philosophy might be too teleological. His system implies that history and reality are progressing toward a specific, rational end. While this provides a coherent picture of the world, it may not be realistic. For instance, evolutionary biology teaches us that life is not driven by any intrinsic purpose but is instead the product of random mutations and natural selection. Evolution is contingent and unpredictable, with no predetermined direction. Also, considering that we are potentially on the brink of nuclear annihilation, it is hard to believe that there is some teleological force driving our civilizations. Although this critical view might seem irrelevant to the topic of our discussion, we must note that teleology plays an important role in Hegel’s dialectics. In Hegel’s view, it is necessary that finite spirit emerge due to the “requirement that *Geist* be embodied” (Taylor, 1975, p. 89).

**9. Conclusion**

This paper has explored the philosophical underpinnings of mathematics through the contrasting perspectives of Kant and Hegel. It also discussed both the strengths and limitations of these approaches. Kant's framework, with its emphasis on synthetic a priori knowledge rooted in the intuitions of space and time, provides a strong foundation for understanding how mathematics applies to the phenomenal world. Building upon this, the paper sought to extend the foundation by legitimizing the realness of the outside world by using Hegel’s ideas.

As noted in the previous section, the Hegelian system also carries limitations. It is perhaps more realistic to say that reality will mostly remain elusive from our grasp. The Absolute Spirit will probably never be achieved. The principles of reality in its entirety, if they could be viewed from the standpoint in the Absolute Idea, would not be expressible in linguistic form, because they can be understood only from a super-temporal viewpoint. In Kantian terms, they are beyond our judgment or sensibility. A corollary of this view is that even if mathematics could perfectly describe the physics of our reality, it would not be able to explain everything.[[15]](#footnote-15)

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1. This view is shared by Cummins (1968): “Geometry Kant straightforwardly associated with intuition. In contrast, …, indeed nowhere in the *Critique of Pure Reason*, did Kant shed much light on the basis of arithmetic or the nature of the truths associated with inner theory of intuition” (p. 271). [↑](#footnote-ref-1)
2. A synthetic a priori judgment produces a proposition where the content of the predicate is not deducible from that of the subject, yet it can be established independently of empirical investigation. For details, see Kant (1998, pp. 143-144). [↑](#footnote-ref-2)
3. This paper interprets Kantian philosophy of mathematics to embrace every branch of mathematics as long as it involves the concepts of shapes and counting. Therefore, even the theorems of non-Euclidean geometry or Cantor’s set theory can be considered to be synthetic a priori if they meet the following conditions in Hanna (2001, p. 279): (i) semantic experience-independence; (ii) intuition-dependence; (iii) consistent deniability; and (iv) restricted necessity in the special sense that a proposition is true in every humanly objectively experienceable world and lacks a classical truth value in any other possible world. Regarding (ii), non-Euclidean geometry relies on spatial visualization to some extent (though not in the strong sense that Kant suggested), and Cantor’s theory relies on the concept of counting, even though it is somewhat removed from our ordinary sense of time. [↑](#footnote-ref-3)
4. See Wilson (1996, p. 8). [↑](#footnote-ref-4)
5. See Hrbacek, K., & Jech, T. (1999, p. 39). [↑](#footnote-ref-5)
6. For details, see Wald (1984, pp. 118-124). [↑](#footnote-ref-6)
7. See Abbott et al. (2016); Event Horizon Telescope Collaboration (2019); and Genzel et al. (2010). [↑](#footnote-ref-7)
8. Imagination “functions automatically, without being self-consciously willed into action -- not in the sense that it is a mere mechanism” (Hanna, 2001, p. 39). Moreover, it is “essentially spontaneous, goal oriented, and vital.” Also, “as the engine of synthesis, it is also the very seat or ground of all consciousness and hence properly speaking *pre*conscious.” [↑](#footnote-ref-8)
9. Per Oxford Reference (2024), “the manifold is the unorganized flux presented to the senses, but not experienced, since experience results from the mind structuring the manifold by means of concepts. The nature of the unstructured manifold is unknowable (transcendental).” [↑](#footnote-ref-9)
10. See Kant (1998, pp. 470-475) [↑](#footnote-ref-10)
11. Kant considers the noumenon in both a positive and negative sense (Kant, 1998, pp. 360-361). Per Hanna (2001), “negative noumena fall outside our sensibility” (p. 106). But this “leaves open the possibility … that they are cognitively accessible to alien creatures with different forms of sensibility.” Meanwhile, “a noumenon in the positive sense” is “any fully thinkable yet non-sensible object, in so far as it could be cognized by a being possessing a faculty of intellectual intuition, or divine cognition, yet could not be cognized by a being possessing a finite sensory cognitive capacity like ours nor indeed by any sort of sensible cognizer, human or non-human.” [↑](#footnote-ref-11)
12. This speculative movement is also reflected in the Liar Paradox. [↑](#footnote-ref-12)
13. Hegel’s notion of continuity differs from that of contemporary mathematics. In Hegelian scholarship, continuity is synonymous with oneness or unity. For details, see Pinkard (1981, p. 459). [↑](#footnote-ref-13)
14. “Whatever is mathematizable can be posited hypothetically as an ontologically perishable fact existing independently of us. In other words, modern science uncovers *the speculative but hypothetical import* of every mathematical reformulation of our world” (Meillassoux, 2008, p. 117). [↑](#footnote-ref-14)
15. For example, Lee (2024) states: “Even if a [scientist] had all the information regarding her mind/body as well as [the universe] from a materialistic viewpoint, she might still fail to explain how her bodily composition gives rise to consciousness. Even a complete mathematical formulation of the neural correlates of consciousness might not fully elucidate its nature” (p. 25). [↑](#footnote-ref-15)