Intertranslatability and Ground-equivalence *

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Abstract

When are logical theories equivalent? I discuss the notion of ground-equivalence between logical theories, which can be useful for various theoretical reasons, e.g., we expect ground-equivalent theories to have the same ontological bearing. I consider whether intertranslatability is an adequate criterion for ground-equivalence. Jason Turner recently offered an argument that first-order logic and predicate functor logic are ground-equivalent in virtue of their intertranslatability. I examine his argument and show that this can be generalized to other intertranslatable logical theories, which supports the following: intertranslatability implies ground-equivalence. I also argue, however, that this ground-equivalence argument can be challenged as it faces a dilemma. The dilemma arises because the argument allows two distinct readings, the 'internal' and the 'external' reading. I argue that the argument turns out to be unsuccessful in both readings. The upshot of this dilemma in both philosophy of logic and metaphysics is considered.

1 Introduction

Some intuitions suggest that a pair of logical theories can be equivalent. Russellian and Polish presentations of classical logic are deemed equivalent provided that they are mere notational variants (French, 2019). Classical and intuitionisic logic are arguably not (Wigglesworth, 2017).

The recent debate on the criteria of theoretical equivalence in logic largely revolves around such pairs of logical theories which are deemed equivalent or inequivalent on an intuitive basis. For instance, the intuition that classical and intuitionisic logic are inequivalent has been treated as a data point that an adequate explication of the theoretical equivalence needs to accommodate.[1] I suggest that a question remains as to the nature of this equivalence though. What does it mean for two theories to be equivalent? Does it merely try to capture the intuitive notion of equivalence as we conceive it?

This question can be sidestepped by relativizing the notion of equivalence instead of analyzing the notion of equivalence simpliciter.[2] We can ask whether two theories are equivalent under a

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[1] One of the recent debates is between the syntactic and the semantic approaches to theoretical equivalence in logic. They disagree about whether the equivalence criteria that appeal to syntactical terms can adequately account for such intuitively equivalent pairs of logical theories. See Wigglesworth (2017) for a defense of the semantic approach, and Dewar (2018) and Woods (2018) for the syntactic approach.

[2] Analogously, for scientific theories, it is plausible to ask whether two theories are only 'empirically equivalent' or 'equivalent in a stronger sense' following Quine (1975).

certain background theory. I focus on the notion of *ground-equivalence*, i.e., the equivalence of logical theories under metaphysical grounding.[3] There are multiple reasons for focusing on ground-equivalence: First, given that grounding is widely associated with metaphysical explanation, it is expected to shed light on the *explanatory* nature of logical theories. It accords with the spirit of logical anti-exceptionalism that has driven the recent debate on theoretical equivalence in logic (Hjortland, 2017; Wigglesworth, 2017); if a scientific theory can explain another theory, then so can logical theories. Second, ground-equivalence is *ontological* in that grounding is closely associated with ontological dependence (see Tahko and Lowe, 2020, sec. 5). We expect ground-equivalent theories to have the same ontological bearing, if they have ontological bearing at all. Hence, ground-equivalence seems to be a useful notion in approaching the problem of theoretical equivalence in logic.

The aim of this paper is to examine a specific approach that takes *intertranslatability* between logical theories to be a sufficient criterion for ground-equivalence.[4] I examine Jason Turner's (2017) argument for the ground-equivalence between classical first-order logic without individual constants (FOL) and predicate functor logic (PFL), which is based on their intertranslatability (§2). It is argued that Turner's argument for ground-equivalence is a special case of a more abstract argument that can be extended to other intertranslatable logical theories. I show how Turner's argument can be generalized to arbitrary pairs of intertranslatable logical theories under certain conditions (§3). If successful, this will show that intertranslatability indicates ground-equivalence; intertranslatable logical theories will have, e.g., the same ontological commitment. That is, we can reach a substantial ontological conclusion by using syntactic notions such as intertranslatability.

I also argue, however, that this ground-equivalence argument does not succeed. It faces a dilemma, which shows that ground-equivalence between intertranslatable theories fails to be demonstrated. Hence, we are not entitled to conclude that a pair of logical theories are ground-equivalent based on their intertranslatability (§4). I finish this paper by discussing the upshots of this conclusion in various subfields of philosophy(§5).

[3]See, e.g., Rosen (2010) and Fine (2012) for introduction to metaphysical grounding. Metaphysical grounding remains a contested issue (e.g., whether it is a relation or an operator). The present paper only appeals to some widely agreed-upon features of metaphysical grounding, e.g., hyperintensionality (cf. footnote 24), more or less treating it as a placeholder notion. Also, the present paper is not exclusively about *logical grounding*, which may be a subset of metaphysical grounding (Correia, 2014). One of the primary motivations behind logical grounding is that logical grounding tracks some rules of inference, but the use of grounding in this paper does not necessarily share the same motivation (also see footnote 28).

[4]This paper follows the conventional 'syntactic' characterization of intertranslatability (Barrett and Halvorson, 2016; Dewar, 2018; Woods, 2018), which, roughly speaking, takes 'translation' to be a mapping from consequences to consequences such that the composition of translation functions preserves the equivalence. The rest of the paper will use terms such as 'truth', 'truth value', etc. for the sake of presentation, but they do not mean anything beyond what can be accommodated under the present characterization of intertranslatability (cf. Barrett and Halvorson's (2016, 477-478) definitions of 'translation' and 'intertranslatable').

2 Turner on FOL and PFL

In this section, I offer my reconstruction of Turner's (2017) argument for the ground-equivalence of FOL and PFL, clarifying some key assumptions that were left implicit in the original presentation.

Turner's original argument is driven by his concern against Dasgupta's (2009; 2017) *generalism*, a metaphysical position stating that "fundamentally speaking at least, there are no such things as material individuals" (Dasgupta, 2009, p. 35).[5] One of the objections to generalism Dasgupta anticipates is the following: FOL, a 'standard' logic in metaphysics, seems *ontologically guilty*; the truth of a quantified sentence in FOL (e.g., '$\exists x(Fx \& Gx)$') is guilty of being committed to the existence of individual objects. That is, ground-theoretically speaking, it is grounded in the truth of atomic sentences (e.g., '$Fa \& Ga$'[6]) which involves individual objects. Hence, Dasgupta worries that FOL is inadequate for articulating generalism; it commits you to fundamental truths involving individual objects, which generalism attempts to dispense with.

His solution is to adopt an *ontologically innocent* system, i.e., the truths of which are *not* grounded in truths involving individual objects. One strong candidate is PFL.[7] It lacks the elements that are taken to make FOL ontologically guilty, such as first-order variables, quantifiers, etc. At the same time, PFL is still as expressively powerful as FOL is in virtue of its predicate functors, e.g., the cropping functor 'c'. For example, the FOL sentence '$\exists x(Fx \& Gx)$' can be translated to the PFL sentence '$c(F \wedge G)$', which has no 'guilty' term.[8] Hence, FOL and PFL are not ground-equivalent and yet are intertranslatable. Accordingly, it is argued that PFL can be offered as an ontologically innocent substitute of FOL, which we may use to articulate generalism.[9]

Turner (2017) argues otherwise. FOL and PFL need to be ground-equivalent given their intertranslatability. Specifically, if the truth of the FOL sentence '$\exists x(Fx \& Gx)$' is grounded in the truth of '$Fa \& Ga$', then so is the truth of the PFL sentence '$c(F \wedge G)$'; given that '$\exists x(Fx \& Gx)$' can be translated to '$c(F \wedge G)$' and vice versa, they should have the same ontological bearing.[10] Therefore, since FOL and PFL are intertranslatable, it cannot be the case that FOL is ontologically guilty but PFL is ontologically innocent. Switching from FOL to PFL does not help articulate generalism since they are ground-equivalent.

[5] Diehl (2018) refers to this position as 'ontological nihilism' (cf. O'Leary-Hawthorne and Cortens, 1995).

[6] Note that '$Fa \& Ga$' itself is neither a sentence of the object language of FOL nor that of PFL; we defined 'FOL' as a classical first-order logic without individual constants in §1. Hence, the truth of '$Fa \& Ga$' cannot be expressed in either FOL or PFL, but it does not mean that we cannot meaningfully ask whether the truths of FOL and PFL are grounded in the truth of '$Fa \& Ga$'.

[7] See Quine (1960, 1976) for classic presentations of PFL.

[8] See Kuhn (1983) for more formal details about the intertranslatability between PFL and FOL.

[9] PFL's innocence can be contested on the basis of adopting a set-theoretic meta-theory, which will not be addressed here. See Dasgupta (2009, p. 66) for a possible response. Also note that Dasgupta's (2009) preferred system is slightly different from the classic formulation of PFL by Quine (1960, 1976); see Turner (2017, sec. 3) for the critique that is specific to Dasgupta's preferred system.

[10] In the following, for any sentence σ, ⌜the truth of σ⌝ and ⌜σ⌝ will be used interchangeably provided that it does not cause confusion in the given context.

Turner's argument consists of two steps: First, he constructs the analog of FOL, which is ground-equivalent to FOL but, at the same time, structurally resembling PFL. Second, based on some metasemantic principles, he argues that PFL and the analog of FOL are ground-equivalent. Transitively, FOL and PFL should be ground-equivalent.

The premise of his argument is that the existential quantifier '$\exists$' in FOL has two separable logical components: binding first-order variables and quantifying objects. We can construe λ-abstraction as a device that allows us to bind variables without quantification proper, e.g., '$\lambda x(Fx\&Gx)$' which we may read as 'is both F and G'.[11] On the other hand, Turner isolates the quantification proper by assigning a separate symbol '$\exists_p$', which we may read as 'There is something that'. Together, they will form the sentence '$\exists_p \lambda x(Fx\&Gx)$', which we may read as 'There is something that is both F and G'. Hence, the FOL sentence '$\exists x(Fx\&Gx)$' can be restated as '$\exists_p \lambda x(Fx\&Gx)$' by borrowing λ-abstraction language.

It indicates that we can isolate the logical component of '$\exists_p$' from the rest of FOL's logical components, such as those of λ-abstraction and Boolean operators. Turner suggests that we can rearrange these remaining logical components of FOL in a way that mimics PFL. That is, the set of FOL's logical operators can be converted in a way that structurally resembles that of PFL with the exception of '$\exists_p$'. He justifies this claim by arguing that predicate functors in PFL can play the role of the FOL operators just as well. For example, the unary complex predicate '$\lambda x(Fx\&Gx)$' can be converted to its PFL counterpart '$F \wedge G$' without losing or adding any logical components.

Turner thereby constructs the analog of FOL that structurally resembles PFL, which we may refer to as 'FOL_A'. That is, FOL_A is nearly identical to PFL except for having '$\exists_p$' instead of 'c'. For instance, the FOL_A sentence '$\exists_p(F \wedge G)$' differs from its PFL counterpart '$c(F \wedge G)$' only in that the former has '$\exists_p$' in place of 'c'. Still, FOL_A remains an analog of FOL in that FOL_A is ground-equivalent to FOL. It is because the construction of FOL_A from FOL does not lose or add any logical component. For example, FOL_A remains ontologically guilty in that it retains a 'guilty' term '$\exists_p$'.

Based on this construction, we can say that the FOL sentence '$\exists x(Fx\&Gx)$' is *synonymous* with its FOL_A counterpart '$\exists_p(F \wedge G)$'. By 'synonymous', in the present context, we can understand synonymous sentences as having the same 'logical meaning'; converting '$\exists x(Fx\&Gx)$' into '$\exists_p(F \wedge G)$' preserves the logical meaning in the sense that no logical components were lost or added.[12] The more significant point is that *synonymy entails having the same ground*; if two sentences have

[11] See Stalnaker (1977) for a defense of the distinction between variable-binding and quantification proper.

[12] While Turner does not explicitly invoke 'synonymy' or 'logical meaning' in his presentation of the argument, his inference relies on the presupposition that the truths of '$\exists x(Fx\&Gx)$' and '$\exists_p(F \wedge G)$' have the same ground because they share exactly the same logical components (i.e., quantification proper and variable binding) and non-logical vocabularies. (also see footnote 21). His presupposition can be aptly explained by claiming that '$\exists x(Fx\&Gx)$' and '$\exists_p(F \wedge G)$' have the same logical meaning. Thus, for the sake of presentation, we can take the logical meaning of a sentence to be determined by the logical components of the operators as well as the non-logical vocabularies in the sentence. 'Synonymy', in turn, can be defined as the sameness of the logical meaning.

the same logical meaning, then the truths of these sentences should not differ in their ground.[13] Therefore, based on their synonymy, it is argued that $\exists_p(F \wedge G)$ is grounded in $Fa\&Ga$ given that $\exists x(Fx\&Gx)$ is.

Hence, FOL_A, our analog of FOL, structurally resembles PFL but still is ground-equivalent to FOL. The remaining task is to demonstrate the ground-equivalence between FOL_A and PFL. Of course, the mere resemblance between FOL_A and PFL is insufficient for deriving their ground-equivalence, so more needs to be shown to achieve this task. Recall, however, that PFL sentences are intertranslatable with FOL sentences, e.g., '$c(F \wedge G)$' is intertranslatable with '$\exists x(Fx\&Gx)$'. It means that we can say more about the relationship between FOL_A and PFL sentences. Consider, for example, the PFL sentence '$c(F \wedge G)$' and the FOL_A sentence '$\exists_p(F \wedge G)$', which resemble each other as discussed above. In addition to this resemblance, they are also indirectly connected through the FOL sentence '$\exists x(Fx\&Gx)$' that plays an intermediary role in between them; '$\exists x(Fx\&Gx)$' is synonymous with '$\exists_p(F \wedge G)$' and, at the same time, is intertranslatable with '$c(F \wedge G)$'. Hence, for any pair of PFL and FOL_A sentences that resemble each other, there exists an FOL sentence that is intertranslatable with the PFL sentence and is, at the same time, synonymous with the FOL_A sentence. This indirect connection allows room for showing a further conclusion about FOL_A and PFL sentences.

Based on this indirect connection, Turner asserts the following about the relationship between FOL_A and PFL:

> Individualists will assent to sentences of [FOL_A] exactly when functorese generalists assent to sentences of [PFL] just like them but for the replacement of '$\exists_p$' with 'c'. (Turner, 2017, pp. 32-33)

That is, for example, '$\exists_p(F \wedge G)$' will be assented when and only when '$c(F \wedge G)$' is assented. His underlying reasoning seems to be as follows: Let us claim that a sentence θ_1 *corresponds* to a sentence θ_2 iff (i) θ_1 is intertranslatable with θ_2, (ii) θ_1 is 'synonymous' with θ_2 in the sense that the logical meaning is preserved, or (iii) transitively by (i) or (ii).[14] For instance, we can demonstrate that '$\exists_p(F \wedge G)$' corresponds to '$c(F \wedge G)$' in the following way: by (ii), '$\exists_p(F \wedge G)$' corresponds to'$\exists x(Fx\&Gx)$', and by (i), '$\exists x(Fx\&Gx)$' corresponds to '$c(F \wedge G)$', so it follows that, by (iii), '$\exists_p(F \wedge G)$' corresponds to '$c(F \wedge G)$'. Then Turner's above assertion can be justified by appealing to the following assumption, which links the notion of correspondence with the pattern of linguistic use:

Assumption 1 (Correspondence assumption). *If a sentence θ_1 of a language L_1 corresponds to θ_2 of L_2, then L_1 speakers assent to θ_1 iff L_2 speakers assent to θ_2.*

[13] Hence, it cannot be taken for granted that intertranslatability automatically implies 'synonymy' in the present sense. If it did, then Turner's argument would be question-begging.

[14] $\theta_1, \theta_2, \ldots$ will be reserved for sentential meta-variables.

In other words, correspondence implies being truthfully assertable in the same set of possible situations. Hence, for any corresponding pair of FOL_A and PFL sentence (e.g., '$\exists_p(F \wedge G)$' and '$c(F \wedge G)$'), the FOL_A sentence is assented when and only when the PFL sentence is assented.

Given this interim conclusion, Turner invokes a sub-sentential metasemantic principle that he refers to as '**(*)**',[15] which can be presented as the following conditional:

Assumption 2 (**(*) principle**). *For any language L_1 with a term t_1 and a language L_2 with a term t_2, if* (Near-identity condition) *and* (Assent condition) *hold, then t_1 and t_2 have the same interpretation.*

The two conditions for (*) principle, Near-identity condition and Assent condition, can be stated as follows:

- **(Near-identity condition)**: L_1 and L_2 have all terms in common except that L_2 has t_2 in place of L_1's t_1 and all shared terms have the same interpretation in both languages.
- **(Assent condition)**: L_1 speakers will assent to a sentence with t_1 when and only when speakers of L_2 will assent to the corresponding sentence with t_2 substituted for t_1.

That is to say, Near-identity condition being a *ceteris paribus* clause,[16] Assent condition states that the use of t_1 in L_1 and the use of t_2 in L_2 follow exactly the same pattern. If these two conditions are met, then t_1 and t_2 will have the same interpretation.

FOL_A and PFL seem to satisfy both Near-identity and Assent condition: By construction, FOL_A is identical to PFL with the exception of having '$\exists_p$' instead of 'c', which makes them satisfy Near-identity condition. They also seem to satisfy Assent condition since their sentences are assented in the same set of possible situations in virtue of Correspondence Assumption. It follows that '$\exists_p$' in FOL_A and 'c' in PFL have the same interpretation.

Hence, '$c(F \wedge G)$' and '$\exists_p(F \wedge G)$' are not only nearly identical; 'c' and '$\exists_p$' turn out to have the same interpretation as well. Based on what we have shown, we can now appeal to the following assumption, which is a relatively weak claim that determines the sameness of ground:

Assumption 3 (**Grounding assumption**). *For any true sentence θ_1 and θ_2, if (1) θ_1 corresponds to θ_2, (2) θ_1 and θ_2 have the same syntactic structure, and (3) the interpretation of each sentential constituent of θ_1 coincides with its counterpart in θ_2, then the truth of θ_1 and the truth of θ_2 have the same ground.*[17]

[15] (*) principle has been presented in multiple forms, e.g., Turner (2011, p. 17) and Turner (2017, p. 33).

[16] Near-identity condition isolates the difference between L_1 and L_2 to the difference between t_1 and t_2; without Near-identity condition, the difference between L_1 and L_2 may abound. The condition that L_1 and L_2 are nearly identical makes (*) a "very weak principle of interpretation" (Turner, 2017, p. 33). If L_1 and L_2 were *not* nearly identical, Turner's argument would have to appeal to a much more radical metasemantic principle.

[17] Cf. Correia (2010, p. 266), whose account not only affirms what corresponds to Grounding assumption but also its converse, which is not assumed here.

Grounding assumption allows us to infer that $c(F \wedge G)$ and $\exists_p(F \wedge G)$ have the same ground: We know that, by construction, '$c(F \wedge G)$' and '$\exists_p(F \wedge G)$' correspond to each other, have the same syntactic structure, and have the same interpretations except for 'c' and '$\exists_p$'. Now, the above argument from (*) tells us that even 'c' and '$\exists_p$' have the same interpretation, which makes '$c(F \wedge G)$' and '$\exists_p(F \wedge G)$' satisfy all three conditions. Hence, it follows that $c(F \wedge G)$ is grounded in $Fa\&Ga$ insofar as $\exists_p(F \wedge G)$ is grounded in $Fa\&Ga$ too.

By generalization, Turner claims that PFL is ground-equivalent to FOL_A, and transitively, PFL is ground-equivalent to FOL as well. Hence, Turner argues, if FOL is ontologically guilty, then so is PFL. Switching from FOL to PFL does not help avoid its ontological commitment. It seems to dash the hopes of generalists advancing PFL as an 'innocent' alternative of FOL for articulating their metaphysical position; PFL seems to commit you to individual objects as much as FOL does.

As I mentioned at the beginning of this section, Turner's argument on FOL and PFL is specifically meant to undercut generalism. It is unclear if Turner intended to assert anything beyond the ground-equivalence of FOL and PFL. Nevertheless, in any case, I suggest that Turner's insight can be extended to other logical theories as well; we can draw from Turner's argument a point that pertains to the ground-equivalence of logical theories in general. In the next section, I present how Turner's argument can be generalized to other intertranslatable theories.

3 The Ground-equivalence Argument, Schematized

What is noteworthy about Turner's argument is that its reasoning is based on abstract principles with a more general scope. For instance, (*) principle, which plays a critical role in Turner's argument, is a metasemantic principle that applies to the determination of linguistic meanings in general.

For the sake of argument, I will not contest the assumptions employed in Turner's argument; I grant, for example, that (*) is indeed true about the interpretations of linguistic items. Instead of questioning them, I consider how these assumptions can be used to draw a more general conclusion about the ground-equivalence of logical theories. This generalized argument supports the following: *Non-hyperintensional logical theories are ground-equivalent if they are intertranslatable*. For instance, PFL and FOL are non-hyperintensional in that they do not contain any hyperintension-sensitive operator, which makes Turner's argument a special case of this generalized argument. If this ground-equivalence argument succeeds, we can ensure that many more intertranslatable pairs of logical theories, not just PFL and FOL, have the same ontological bearing in virtue of their ground-equivalence. Even more generally, the present argument may lend support to the general approach to theoretical equivalence that appeals to translatability (Barrett and Halvorson, 2016; Dewar, 2018; Woods, 2018).

I argue that each step of Turner's argument can be generalized to arbitrary pairs of intertranslatable logical theories given certain conditions, which can be presented by using a schematic formalism. As a setup, let $\mathcal{L}$ and $\mathcal{L}'$ stand for non-hyperintensional logical theories which are intertranslatable. The goal of the generalized argument is to show that $\mathcal{L}$ and $\mathcal{L}'$ are ground-equivalent.

For the sake of convenience, we can consider arbitrary sentences of $\mathcal{L}$ and $\mathcal{L}'$. Suppose that ϕ, a true sentence in $\mathcal{L}$, is intertranslatable with ϕ', a true sentence in $\mathcal{L}'$. Assume, however, that ϕ and ϕ' differ in their ground; there is something that grounds ϕ but not ϕ'. Let us call this truth g. Therefore, we presume the following:

$$\phi \text{ is grounded in } g.$$

$$\phi' \text{ is not grounded in } g.$$

Our first step is constructing an analog of $\mathcal{L}$, which we may refer to as $\mathcal{L}_A$. $\mathcal{L}_A$ has two desiderata: First, as an analog of $\mathcal{L}$, every $\mathcal{L}_A$ sentence should have a synonymous counterpart in $\mathcal{L}$ and vice versa. That is, given the $\mathcal{L}$ sentence ϕ, there should be a $\mathcal{L}_A$ setnence, which we may refer to as ϕ_A, with the following property:

$$\phi \text{ is synonymous with } \phi_A.$$

As defined in §2, the notion of synonymy roughly means having the same logical meaning, which in turn implies having the same ground. Hence, given the synonymy between ϕ and ϕ_A, the following biconditional should hold, which, by generalization, implies that $\mathcal{L}$ should be ground-equivalent to $\mathcal{L}_A$.

$$\phi \text{ is grounded in } g \text{ iff } \phi_A \text{ is grounded in } g.$$

The second desideratum of $\mathcal{L}_A$ is that $\mathcal{L}_A$ should be 'nearly identical' to either (i) $\mathcal{L}'$ or (ii) an analog of $\mathcal{L}'$.[18] For the sake of presentation, here I will focus on (i).

$\mathcal{L}_A$ is nearly identical to $\mathcal{L}'$ just in case $\mathcal{L}_A$ is identical to $\mathcal{L}'$ with just one exception; whenever an expression α' appears in an $\mathcal{L}'$ sentence, its counterpart sentence in $\mathcal{L}_A$ has an expression α in its place and vice versa. In other words, you should be able to yield ϕ_A by swapping every occurrence of α' in ϕ' with α and vice versa. Thus, the following condition needs to hold:

$$\phi' \text{ is identical to } \phi_A \text{ except for having } \alpha' \text{ in place of } \alpha.$$

What's at stake now is whether $\mathcal{L}_A$ that satisfies these two desiderata can be constructed in principle. I propose the following 'recipe' for constructing $\mathcal{L}_A$, which consists of three steps, provided that some theoretical conditions are met:[19]

[18] By an 'analog of $\mathcal{L}'$' in (ii), I refer to a logical theory that is an analog of $\mathcal{L}'$, which we may call $\mathcal{L}'_A$, in the same sense that $\mathcal{L}_A$ is an analog of $\mathcal{L}$. That is, $\mathcal{L}'_A$ should be ground-equivalent to $\mathcal{L}'$ by virtue of their sentential synonymy. This clause (ii) will be useful for handling difficulty in constructing $\mathcal{L}_A$ (see footnote 20).

[19] For example, this recipe requires that the meaning of an individual logical operator in $\mathcal{L}$ and $\mathcal{L}'$ can be determined independently of other operators ('molecularity') and the meaning of the operator can be divided into individually identifiable logical components ('modularity') (Golan, 2021). These conditions justify some of the key steps in the recipe, such as dividing the meanings of logical operators into 'smaller' logical components and rearranging them in a way that we can assign them as meaning to new operators.

First, divide the meanings of all the relevant logical operators from both $\mathcal{L}$ and $\mathcal{L}'$ into 'smaller' logical components. That is, we 'break down' the meanings of logical operators into smaller units. This will leave us with two lists of logical components, one from $\mathcal{L}$ and one from $\mathcal{L}'$ respectively.

Second, compare these two lists of logical components to see where they agree and disagree. That is, which logical components are unique to either $\mathcal{L}$ or $\mathcal{L}'$? While there is likely to be much agreement between these two lists, there is still bound to be some difference in logical components between $\mathcal{L}$ and $\mathcal{L}'$ insofar as ϕ and ϕ' differ in their ground; *some* logical component should be responsible for why ϕ is grounded in g but ϕ' is not. For the sake of presentation, let us call this logical component κ, which is a component of α' in $\mathcal{L}'$. That is, κ is responsible for the fact that ϕ', which has α' as its constituent, is not grounded in g. In contrast, κ is not a member of the logical components from $\mathcal{L}$; none of $\mathcal{L}$'s operators has κ as its logical component, so ϕ does not have κ as a constituent of its meaning.

Third, construct the logical operators of $\mathcal{L}_A$ by rearranging the logical components of $\mathcal{L}$ in a way that $\mathcal{L}_A$'s operators 'mimic' the logical operators of $\mathcal{L}'$. This requires two subtasks: First, $\mathcal{L}_A$ should have a new operator α that serves as a 'mirror image' of α' in $\mathcal{L}'$; α should be exactly like α' except that α does not have κ as its logical component. Second, given that $\mathcal{L}$ and $\mathcal{L}'$ only disagree about κ and have the rest of the logical components in common, $\mathcal{L}_A$ can borrow all the rest of the operators from $\mathcal{L}'$ except for α'. These subtasks may require a radical rearrangement of $\mathcal{L}$'s logical components; for example, it may require breaking down one $\mathcal{L}$ operator with multiple logical components into multiple $\mathcal{L}_A$ operators with each component. Nonetheless, this conversion from $\mathcal{L}$ to $\mathcal{L}_A$ preserves the logical components of $\mathcal{L}$; the logical components are merely rearranged, which makes $\mathcal{L}_A$ an analog of $\mathcal{L}$. At the same time, $\mathcal{L}_A$ 'mimics' $\mathcal{L}'$ since they share all the logical operators except for α and α'; the disagreement between $\mathcal{L}_A$ and $\mathcal{L}'$, which is responsible for the fact that ϕ_A is grounded in g but ϕ' isn't, is now 'isolated' to the difference in logical components between α and α' (i.e., whether it has κ or not).[20]

If successfully followed, this recipe will provide us with the newly constructed $\mathcal{L}_A$, which in-

[20]This recipe of $\mathcal{L}_A$ may appear to have a problem when the disagreement between $\mathcal{L}$ and $\mathcal{L}'$ stems from more than one logical component: Suppose that a theory T has two operators, O_1 and O_2, such that O_1 has one logical component κ_1 and O_2 has two components, κ_2 and κ_3, and a theory $T*$ has two operators, O_1* and O_2*, such that O_1* has one component κ_1* and O_2* has two components, κ_2* and κ_3. T and $T*$ have one logical component, κ_3, in common. The given recipe cannot help us construct an adequate analog of T; on the one hand, we should assign κ_1 and κ_2 to the new operator α, but on the other hand, the remaining component, κ_3, cannot 'mimic' any of the $T*$ operators. This seeming counterexample can be overcome by using clause (ii) of the second desideratum of $\mathcal{L}_A$, which invokes an analog of $\mathcal{L}'$ (see footnote 18). The analog of $\mathcal{L}'$, i.e., $\mathcal{L}'_A$, can serve as a 'buffer' between $\mathcal{L}_A$ and $\mathcal{L}'$; even when $\mathcal{L}_A$ that is nearly identical to $\mathcal{L}'$ cannot be directly constructed, $\mathcal{L}_A$ nearly identical to $\mathcal{L}'_A$ can still be constructed instead. In the present counterexample, for instance, consider an analog of $T*$, which we may call $T*_A$, with two operators, O_1*_A and O_2*_A, such that O_1*_A has two components κ_1* and κ_2* and O_2*_A has one component, κ_3; $T*_A$ and $T*$ share the same logical components. Whereas the present recipe cannot construct an analog of T nearly identical to $T*$, we can still construct an analog of T nearly identical to $T*_A$; that is, $T*_A$ serves as a 'buffer' between the analog of T and $T*$, which resolves the given counterexample. Introducing $\mathcal{L}'_A$ as a 'buffer' between $\mathcal{L}_A$ and $\mathcal{L}'$ in this manner helps the ground-equivalence argument since $\mathcal{L}'_A$ is ground-equivalent to $\mathcal{L}'$ by construction; the *reductio* conclusion that $\mathcal{L}$ and $\mathcal{L}'$ should be ground-equivalent can still be derived *mutatis mutandis*. I appreciate an anonymous reviewer for pointing out this concern.

cludes ϕ_A as its sentence.[21] Thus, given the intertranslatable pair $\mathcal{L}$ and $\mathcal{L}'$, we presume that there exists $\mathcal{L}_A$ that satisfies the above desiderata.

Given this, we have three logical theories at hand, $\mathcal{L}$, $\mathcal{L}_A$, and $\mathcal{L}'$, which are related as follows: $\mathcal{L}$ and $\mathcal{L}_A$ are ground-equivalent because of their sentential synonymy; $\mathcal{L}_A$ and $\mathcal{L}'$ are nearly identical except for α and α'; and $\mathcal{L}'$ and $\mathcal{L}$ are intertranslatable by definition.

Now begins the second part of the ground-equivalence argument, which aims to show that $\mathcal{L}_A$ is ground-equivalent to $\mathcal{L}'$. At this point, we know that $\mathcal{L}_A$ sentences are nearly identical to $\mathcal{L}'$ sentences, but it is insufficient for guaranteeing ground-equivalence itself. We also know, however, that $\mathcal{L}$ plays an intermediary role between $\mathcal{L}_A$ and $\mathcal{L}'$; ϕ is synonymous with ϕ_A and is, at the same time, intertranslatable with ϕ'. Hence, based on the definition of 'correspondence' presented in §2, we can infer the following:

$$\phi_A \text{ corresponds to } \phi'.$$

Based on this inference, Correspondence assumption allows us to further infer that ϕ_A and ϕ' are assented in the same set of possible situations.

On the sub-sentential level, (*) principle can be applied to show that α' and α have the same interpretation; both Near-identity and Assent conditions are satisfied by $\mathcal{L}_A$ and $\mathcal{L}'$ by construction. Hence, in a similar vein, the following can be derived via Grounding assumption:

$$\phi_A \text{ is grounded in } g \text{ iff } \phi' \text{ is grounded in } g.$$

Thus, by generalization, $\mathcal{L}_A$ is ground-equivalent to $\mathcal{L}'$. When all the pieces are put together, however, we face a contradiction. Recall the premise that ϕ is grounded in g but ϕ' is not. By the biconditionals we have shown, it follows that ϕ is grounded in g iff ϕ' is grounded in g, which contradicts the premise. Therefore, by *reductio*, we are led to conclude that $\mathcal{L}$ and $\mathcal{L}'$ should be ground-equivalent; non-hyperintensional logical theories are ground-equivalent if they are intertranslatable.

This result generalizes the ground-equivalence claim demonstrated by Turner, which provides us a principled way to tell whether two logical theories are ground-equivalent. That is, intertranslatabilty can be a reliable indicator for comparing the ontological and explanatory aspects of logical theories in general. Moreover, as we will see in §5, this conclusion may have direct upshots in the neighboring subfields of philosophy. Hence, much is at stake for the ground-equivalence argument.

I also identified the key assumptions for the ground-equivalence argument, e.g., (*) principle, theoretical conditions for $\mathcal{L}_A$'s construction, etc. While some of the assumptions are disputed (see

[21] Turner's construction of FOL_A (§2) from FOL can be analyzed through the present recipe as follows: Given 'modularity' (see footnote 19), the disagreement between FOL and PFL can be narrowed down to quantification proper, which is a logical component of $\exists$. Thus, FOL_A should be equipped with '$\exists_p$', which is a 'mirror image' of 'c' in PFL except for having quantification proper as its component, while borrowing the rest of the operators from PFL. This way, the disagreement between FOL_A and PFL can be isolated to the difference in logical components between '$\exists_p$' and 'c'.

§5), I do not problematize them now. Instead, I suggest that this argument faces difficulty even when we grant these substantial assumptions. That is, the ground-equivalence argument does not demonstrate the ground-equivalence between intertranslatable logical theories, including that between PFL and FOL.

4 Dilemma

We saw, in the previous section, that the ground-equivalence argument makes a strong case for inferring ground-equivalence from intertranslatability. Nevertheless, I show that this argument can be challenged. My challenge is based on the observation that Assent condition of (*) principle allows two different readings. Recall that Assent condition concerns "a sentence with t_1" assented by L_1-speakers and "the corresponding sentence with t_2 replaced for t_1" assented by L_2-speakers. The ambiguity lies in the scope of "a sentence with t_1" and "the corresponding sentence with t_2 replaced for t_1" with respect to the given languages L_1 and L_2. We can ask the following: Is "a sentence with t_1" necessarily an L_1 sentence and "the corresponding sentence with t_2 replaced for t_1" an L_2 sentence? Two different answers to this question lead to two different readings of Assent condition, which give rise to their respective problems. Thus, given the indispensable role of (*) principle, the ground-equivalence argument confronts a dilemma. In the following subsections, I explain how we can read Assent condition and what problem we face in each reading.

4.1 The Internal Reading

Assent condition demands that "a sentence with t_1" is assented when and only when "the sentence with t_2 replaced for t_1" is assented. Since t_1 and t_2 are terms in L_1 and L_2 respectively, an intuitive way of understanding "a sentence with t_1" and "the sentence with t_2 replaced for t_1" is that they only refer to pure L_1 and L_2 sentences; a sentence with t_1 is necessarily an L_1 sentence and the sentence with t_2 replaced for t_1 is necessarily an L_2 sentence. Hence, Assent condition is satisfied if L_1 sentences with t_1 and their corresponding L_2 sentences with t_2 replaced for t_1 are assented in the same set of possible situations. Any sentence that does not properly fall under L_1 or L_2 is irrelevant. Call this the *internal reading*.

The ground-equivalence argument is apparently justified under the internal reading. As shown earlier, the $\mathcal{L}'$ sentence ϕ' corresponds to the $\mathcal{L}_A$ sentence ϕ_A, and by Correspondence assumption, they are assented in the same set of possible situations; Assent condition is satisfied and the ground-equivalence argument goes through.

The problem with the internal reading, however, is that it is too permissive. (*) principle, which plays an indispensable role in the ground-equivalence argument, becomes too coarse-grained to capture the grounding relation's hyperintension-sensitivity; under the internal reading, it cannot reflect the difference between the grounding and the grounded truths going beyond their modal status.

Metaphysical grounding is taken to be hyperintension-sensitive on the presumption that some truth can be explanatorily prior to another even when they hold in the same set of possible worlds.[22] Therefore, the ground-equivalence argument is expected to be sensitive to hyperintensional contents. However, Turner gives a caveat:

> ($*$) will only seem plausible if 'interpretation' in the consequent is understood in a coarse-grained way, so that intensionally equivalent interpretations have the same interpretation. (Turner, 2011, p. 19)

The consequent of (*) principle only tells that two terms t_1 and t_2 have the same extension in every possible world and no more.[23] This caveat seems indispensable since all Assent condition effectively states is that two corresponding sentences are assented in the same set of possible situations. When combined with Grounding assumption, it predicts that the pair of corresponding truths have the same ground, ignoring hyperintensional elements that should have been taken into account.[24]

A concrete example can be offered. Following Rosen's (2010, pp. 123-4) example, suppose that the truth that x is a square is grounded in the truth that x is an equilateral rectangle. Let the sentence 'Sx' express the former and 'Ex' the latter, each of which is a sentence of first-order extensional languages L_S and L_E respectively; L_S and L_E are identical except that L_S has 'S' in place of 'E' in L_E.

We can ask if the ground-equivalence argument applies to L_S and L_E: First, L_S and L_E satisfy Near-identity condition with regard to 'S' and 'E' by construction. Assent condition is also satisfied under the internal reading given that 'S' and 'E' are intersubstitutable *salva veritate* in every L_S and L_E sentence. By (*), it follows that 'S' and 'E' have the same interpretation. It implies that the truth of 'Sx' and the truth of 'Ex' have the same ground via Grounding assumption, which leads us to conclude that L_S and L_E are ground-equivalent.

This conclusion, however, is problematic. We initially supposed that the truth of 'Sx' is grounded in the truth of 'Ex'. It follows transitively that the truth of 'Ex' grounds itself, which violates the irreflexivity of grounding.[25] That is, the internal reading of Assent condition leads to a conclusion that is inconsistent with a widely accepted feature of metaphysical grounding. Therefore, insofar as

[22] A classical example by Fine (1994) suggests, for instance, that the truth of '{Socrates} exists' holds in virtue of the truth of 'Socrates exists', which are yet necessarily equivalent.

[23] Cf. "Urmson's dictum" endorsed by (Hirsch, 2011, p. xi).

[24] Duncan et al. (2017) argue that grounding is not hyperintensional under "the standard predicate-fact view". Nevertheless, this is orthogonal to the present argument since the following example does not appeal to the authors' targets, i.e., intensionally equivalent names for the same truth or fact.

[25] See Jenkins (2011) and Schaffer (2012) for the views denying that grounding is a strict partial order. Still, I argue that the present example can be easily modified in a way that conforms to their alternative accounts (e.g., Jenkins' 'quasi-irreflexivity', Schaffer's 'differential transitivity').

we adhere to the widely shared view of grounding, L_S and L_E constitute a counterexample to the internal reading; it makes a wrong prediction about ground-equivalence.

Hence, the ground-equivalence argument turns out to be unreliable under the internal reading. Of course, it does not imply that the conclusion of the ground-equivalence argument is always wrong; the pair of L_S and L_E is just one counterexample, so there may be intertranslatable pairs of logical theories which are also ground-equivalent. For example, FOL_A and PFL may happen to be indeed ground-equivalent as Turner argued. Even so, this counterexample shows that the pair of FOL_A and PFL is in bad company. The ground-equivalence argument cannot be reliably used to demonstrate that logical theories are ground-equivalent, whether they are indeed ground-equivalent or not.

Notice that this recalcitrant conclusion could be prevented if the lexicon of L_S or L_E had any hyperintensional operator. Suppose that a sentential operator $\mathcal{H}$ is hyperintension-sensitive and is included in both L_S and L_E, so that sentences such as $\ulcorner \mathcal{H} S x \urcorner$ and $\ulcorner \mathcal{H} E x \urcorner$ should also be accounted for. It can no longer be guaranteed that Assent condition is satisfied. For 'E' and 'S' are only intensionally equivalent by definition, so they may not be intersubstitutable *salva veritate* within the scope of $\mathcal{H}$.

Such a maneuver is nevertheless blocked in the present context where $\mathcal{L}$ and $\mathcal{L}'$ are assumed to be non-hyperintensional, e.g., PFL and FOL. The internal reading confines the desiderata of Assent condition to sentences of $\mathcal{L}$ and $\mathcal{L}'$; any sentence that does not properly fall under $\mathcal{L}$ or $\mathcal{L}'$ is disregarded. Hence, no sentence with a hyperintension-sensitive term such as $\mathcal{H}$ can be taken into account. The internal reading's vulnerability exposed by the above counterexample, therefore, cannot be remedied in the ground-equivalence argument.

This exposition of the internal reading tells us that the internal reading is problematic because the desiderata of Assent condition are too limited. Only by incorporating sentences with hyperintension-sensitive terms can the ground-equivalence argument overcome this problem. It naturally leads to the other reading of (*) principle, the *external reading*.

4.2 The External Reading

The internal reading's pitfall suggests that an alternative reading of Assent condition is called for. Pure L_1 and L_2 sentences should not exhaust its desiderata, so the following should be taken into account as well: L_1 speakers assent to a sentence embedded with extra-L_1 terms iff L_2 speakers assent to the corresponding extra-L_2 sentence. That is, sentences that do not properly fall under L_1 or L_2 should be considered as well. The external reading of (*) principle demands that object languages be extended so as to accommodate such extra-L_1 or -L_2 elements, which makes Assent condition effectively more demanding.[26]

It was shown that the internal reading's problem can be avoided when a hyperintension-sensitive term is considered in addition to pure L_1 and L_2 elements. Hence, the external reading can overcome

[26] Cf. French's (2019) External Equivalence constraint for mere notational variance.

this problem by extending the object languages with a hyperintension-sensitive term. One natural approach is the following: Let $\mathcal{L}+$, $\mathcal{L}_A+$ and $\mathcal{L}'+$ be the extensions of $\mathcal{L}$, $\mathcal{L}_A$ and $\mathcal{L}'$ introducing other terms, including the binary sentential operator '*because*' that stands for the grounding relation. For example, if θ_1 and θ_2 are both $\mathcal{L}$ sentences, we can have the $\mathcal{L}+$ sentence $\ulcorner\theta_1$ *because* $\theta_2\urcorner$.

Since '*because*' is hyperintension-sensitive, the internal reading's problem can be resolved by considering $\mathcal{L}_A+$ and $\mathcal{L}'+$ sentences embedded with '*because*'. For instance, concerning the following pair of sentences, Assent condition requires under the external reading that $\mathcal{L}_A$ speakers assent to the first sentence when and only when $\mathcal{L}'$ speakers assent to the second sentence:

$$\ulcorner\phi_A \textit{ because } g\urcorner$$

$$\ulcorner\phi' \textit{ because } g\urcorner$$

It allows us to avoid the internal reading's problem; even though $\mathcal{L}_A$ and $\mathcal{L}'$ speakers may assent to ϕ_A and ϕ' in exactly the same set of possible situations, it does not imply that they will do the same for $\ulcorner\phi_A$ *because* $g\urcorner$ and $\ulcorner\phi'$ *because* $g\urcorner$. Hence, Assent condition under the external reading is no longer as permissive as it was under the internal reading.

Assent condition will be met only if $\ulcorner\phi_A$ *because* $g\urcorner$ and $\ulcorner\phi'$ *because* $g\urcorner$ are assented in the same set of possible situations. If Assent condition is met, then it will follow that α and α' have the same interpretation by (*) principle since Near-identity condition is already satisfied by the construction of $\mathcal{L}_A+$ and $\mathcal{L}'+$. If this is the case, then the ground-equivalence claim can be derived through Grounding assumption. Without falling into the internal reading's pitfall, the external reading seems to provide us the grounding-equivalence argument's intended conclusion.

There is little reason, however, to view that Assent condition is met by $\mathcal{L}_A+$ and $\mathcal{L}'+$. For we have not shown that the antecedent of Correspondence assumption is satisfied; a pivotal step for showing that $\mathcal{L}_A+$ and $\mathcal{L}'+$ meet Assent condition remains unfulfilled.

Correspondence assumption claims that two sentences will be assented in the same set of possible situations if they correspond to each other. It was shown earlier that ϕ_A corresponds to ϕ'; ϕ_A is synonymous with ϕ, which in turn is intertranslatable with ϕ'. By generalization, it was shown that $\mathcal{L}_A$ and $\mathcal{L}'$ satisfy the antecedent of Correspondence assumption under the internal reading.

Nonetheless, this cannot be extrapolated to the external reading. Unlike ϕ and ϕ', the intertranslatability between the $\mathcal{L}+$ sentence $\ulcorner\phi$ *because* $g\urcorner$ and the $\mathcal{L}'+$ sentence $\ulcorner\phi'$ *because* $g\urcorner$ fails to be demonstrated. Hence, the correspondence between the $\mathcal{L}_A+$ sentence $\ulcorner\phi_A$ *because* $g\urcorner$ and its $\mathcal{L}'+$ counterpart $\ulcorner\phi'$ *because* $g\urcorner$ cannot be established either. The antecedent of Correspondence assumption remains unsatisfied.

The failure is due to the incorporation of extra-$\mathcal{L}$ and -$\mathcal{L}'$ elements in the construction of $\mathcal{L}+$ and $\mathcal{L}'+$, e.g., '*because*'. The intertranslatability has been proved only for the translation procedure between pure $\mathcal{L}$ and $\mathcal{L}'$ sentences; it does not accommodate extra-$\mathcal{L}$ or -$\mathcal{L}'$ expressions such as '*because*'. Moreover, since the given translation procedure is intended for the non-hyperintensional languages $\mathcal{L}$ and $\mathcal{L}'$, it essentially falls short of reflecting the hyperintensional difference within the opaque context created by '*because*'. We only know that ϕ and ϕ' are intertranslatable, not $\ulcorner\phi$ *because* $g\urcorner$ and $\ulcorner\phi'$ *because* $g\urcorner$, and we do not know if $\ulcorner\phi$ *because* $g\urcorner$ corresponds to $\ulcorner\phi'$ *because* $g\urcorner$. It implies that the correspondence between $\ulcorner\phi'$ *because* $g\urcorner$ and $\ulcorner\phi_A$ *because* $g\urcorner$, which is what we need for the external reading, cannot be demonstrated either. Since its antecedent remains unsatisfied, Correspondence assumption cannot be employed to derive its intended conclusion. Therefore, the ground-equivalence argument stalls at this step under the external reading.

One may argue that my counterargument is too hasty. It may be argued that the translation procedure can be revised so as to accommodate sentences embedded with '*because*' as well. For example, it can be stipulated that $\ulcorner\phi$ *because* $g\urcorner$ is intertranslatable with $\ulcorner\phi'$ *because* $g\urcorner$ under the revised procedure; the stipulation will ensure that $\ulcorner\phi$ *because* $g\urcorner$ corresponds to $\ulcorner\phi'$ *because* $g\urcorner$ and $\ulcorner\phi_A$ *because* $g\urcorner$, which makes Correspondence assumption applicable. However, this move begs the question insofar as the translation procedure is truth-preserving (see footnote 4): If we assume that $\ulcorner\phi$ *because* $g\urcorner$ and $\ulcorner\phi'$ *because* $g\urcorner$ are intertranslatable and therefore have the same truth value, then, by definition, it amounts to the claim that $\mathcal{L}$ and $\mathcal{L}'$ are ground-equivalent.[27] Notice that this is the very conclusion that the ground-equivalence argument aims to demonstrate though; the intended conclusion of the argument is presupposed even before the argument proceeds. Hence, this possible objection is question-begging in the sense that the ground-equivalence argument plays no role in establishing the ground-equivalence of $\mathcal{L}$ and $\mathcal{L}'$.

Such a problem is not exclusive to $\mathcal{L}+$, $\mathcal{L}_A+$ and $\mathcal{L}'+$ which have '*because*' in their lexicon. For example, consider extending $\mathcal{L}$, $\mathcal{L}_A$ and $\mathcal{L}'$ with an arbitrary hyperintensional operator $\mathcal{H}$. If the external reading is to succeed, the revised translation procedure should ensure that $\ulcorner\mathcal{H}\phi\urcorner$ and $\ulcorner\mathcal{H}\phi'\urcorner$ are intertranslatable, which entails that they have the same truth value. Then the original translation procedure between $\mathcal{L}$ and $\mathcal{L}'$, which allows translating ϕ to ϕ' and vice versa, should not only be truth-preserving but also hyperintension-preserving. Otherwise, we cannot guarantee that $\ulcorner\mathcal{H}\phi\urcorner$ and $\ulcorner\mathcal{H}\phi'\urcorner$ have the same truth value. Nevertheless, this requirement is too demanding given the present characterization of translation in terms of truth-preservation, which is a standard approach in the literature. The existence of a translation procedure between $\mathcal{L}$ and $\mathcal{L}'$ only ensures, at best, that ϕ and ϕ' are intensionally equivalent since they will then have the same truth value in any model. The intertranslatability between ϕ and ϕ' does not guarantee that $\ulcorner\mathcal{H}\phi\urcorner$ and $\ulcorner\mathcal{H}\phi'\urcorner$ have the same truth value; we cannot expect the translation procedure between $\mathcal{L}$ and $\mathcal{L}'$ to be hyperintension-preserving. In other words, the revised procedure, which extends the original translation procedure with $\mathcal{H}$, fails to guarantee that $\ulcorner\mathcal{H}\phi\urcorner$ and $\ulcorner\mathcal{H}\phi'\urcorner$ are intertranslatable. Hence,

[27] Note that the meaning of 'truth' (and 'falsity') need not be univocal across $\mathcal{L}$ and $\mathcal{L}'$. Insofar as truth (and falsity) in a language is taken to imply assentability (and rejectability) in the given language, intertranslatability implies that $\ulcorner\phi$ *because* $g\urcorner$ can be assented in $\mathcal{L}+$ iff $\ulcorner\phi'$ *because* $g\urcorner$ can be assented in $\mathcal{L}'+$. Since $\mathcal{L}+$ and $\mathcal{L}'+$ mean the same things by the non-logical expressions '*because*' and '*g*', the biconditional can still lead to the intended ground-equivalence claim ($\ulcorner\phi$ *because* g iff ϕ' *because* $g\urcorner$). I appreciate an anonymous reviewer for pointing out this concern.

given what we assumed about $\mathcal{L}$ and $\mathcal{L}'$, revising the translation procedure does not help the external reading.

In sum, I argue that the external reading of Assent condition has its own problem. It solves the internal reading's pitfall by introducing a hyperintension-sensitive operator, but it is argued that the introduction of such a hyperintensional element backfires.

5 Conclusion and Discussion

In this paper, I showed that Turner's ground-equivalence argument on FOL and PFL's ontological innocence can be generalized to other intertranslatable logical theories. At the same time, I argued that the ground-equivalence argument faces a critical dilemma; the internal reading elicits a problematic consequence inconsistent with the widely shared view of grounding, and the external reading's introduction of hyperintension-sensitive terms is in tension with intertranslatability. Hence, the ground-equivalence between intertranslatable logical theories cannot be taken for granted.

Where does this conclusion leave us? First, we need to properly situate the ground-equivalence argument in the philosophy of logic literature. Roughly speaking, the ground-equivalence argument is a syntactic attempt at the problem of theoretic equivalence in logic (see footnote 1), even though we leave open whether ground-equivalence captures the intuitive concept of equivalence simpliciter. Moreover, there have been recent attempts to explicitly connect the syntactic approach with the notion of 'having the same ground'.[28] The ground-equivalence argument presented here could be understood as a part of this concerted approach, aiming to defend the adequacy of intertranslatability as a criterion for ground-equivalence.

The dilemma (§4), however, undercuts this defense of the syntactic approach offered by the ground-equivalence argument. Of course, the ground-equivalence argument does not represent the syntactic approach as a whole. Moreover, the present conclusion does not outright reject the ground-equivalence of intertranslatable theories. For example, one may maintain that intertranslatability implies ground-equivalence as a brute foundational fact. Nonetheless, the present dilemma shows that the independent support for this ground-equivalence claim is ill-founded, so we may not have enough reason to believe in this claim.

The presented conclusion also has a wide-ranging upshot in metaphysics. First, given that the ground-equivalence argument first started as Turner's argument in the metaphysics of individual objects, we can infer the following *contra* Turner: We are not entitled to conclude that PFL is as ontologically guilty as FOL is based on their intertranslatability; unlike FOL, PFL can well-articulate generalism. Many recent works against Turner's argument attempted to challenge its specific assumptions, e.g., (*) principle (Diehl, 2018; Lee, 2022). Instead, the present paper shows that

[28] E.g., Poggiolesi (2020) presents a system that decides whether two formulas are *hyper-isomorphic* (i.e., have the same logical grounds) under classical logic (also see footnote 3 on logical grounding).

Turner's argument against generalism faces a problem even when we grant its substantial assumptions. We can thereby have a novel defense of generalism in the metaphysics of individual objects.

It is also noteworthy that Turner's argument effectively functions as a 'collapse argument' that 'collapses' the meaning distinction between '$\exists_p$' and 'c'. Collapse arguments have received much attention in metaphysics in the context of *quantifier variantism*: Roughly put, quantifier variantists maintain that different existential quantifiers in different languages can have the same inferential pattern without sharing the same meaning. This is taken to support the deflationist conclusion that many ontological disputes are merely verbal disputes (see, e.g., Hirsch, 2011; Sud and Manley, 2021). In response, many critics of quantifier variantism attempted to 'collapse' the alleged meaning difference between these quantifiers, thereby undercutting the deflationist conclusion that ontological disputes are merely verbal.[29] Since the generalized ground-equivalence argument is more or less applicable to all types of expressions, the critics of quantifier variantism may be tempted to utilize the ground-equivalence argument to 'collapse' the meaning difference between the quantifiers.

Again, the presented dilemma shows that this possible 'collapsing' attempt is ill-founded; you are not entitled to conclude that two different quantifiers have the same ontological bearing just based on their common inferential pattern. This is analogous to the aforementioned conclusion about the same ontological bearings of 'c' in PFL and '$\exists_p$' in FOL_A; while they have the same inferential pattern, we saw that the former can be 'innocent' unlike the latter. Hence, the present dilemma can provide an unexpected defense of quantifier variantism, at least insofar as the 'collapse arguments' are concerned.

The present conclusion may have ramifications in other domains as well. Among many cases, Barrett and Halvorson (2016) have considered intertranslatability as a criterion for theoretical equivalence in sciences, and Putnam (1967) attempted to deflate various disagreements in the foundation of mathematics based on intertranslatability. I conjecture that the present discussion may have a negative upshot for such intertranslatability-based approaches in general, but more elaboration is left for future work.

References

Barrett, T. W. and Halvorson, H. (2016). Glymour and Quine on Theoretical Equivalence. *Journal of Philosophical Logic*, 45(5):467–483.

Correia, F. (2010). Grounding and truth-functions. *Logique et Analyse*, 53(211):251–279. Publisher: Centre National de Recherches de Logique.

Correia, F. (2014). Logical Grounds. *The Review of Symbolic Logic*, 7(1):31–59. Publisher: Cambridge University Press.

[29] See Harris (1982) for the first presentation of the 'collapse arguments' and Dorr (2014) for the modern renewal with emphasis on quantifier variantism. For some recent discussion, see, e.g., Warren (2021) and Sider (2023).

Dasgupta, S. (2009). Individuals: An Essay in Revisionary Metaphysics. *Philosophical Studies*, 145(1):35–67.

Dasgupta, S. (2017). Can We Do without Fundamental Individuals? Yes. In Barnes, E., editor, *Current Controversies in Metaphysics*, pages 7–23. Routledge.

Dewar, N. (2018). On translating between logics. *Analysis*, 78(4):622–630.

Diehl, C. (2018). A Language for Ontological Nihilism. *Ergo, an Open Access Journal of Philosophy*, 5(37).

Dorr, C. (2014). Quantifier Variance and the Collapse Theorems. *The Monist*, 97(4):503–570. Publisher: Oxford University Press (Oup).

Duncan, M., Miller, K., and Norton, J. (2017). Is Grounding a Hyperintensional Phenomenon? *Analytic Philosophy*, 58(4):297–329.

Fine, K. (1994). Essence and Modality. *Philosophical Perspectives*, 8(1994):1–16.

Fine, K. (2012). Guide to Ground. In Correia, F. and Schnieder, B., editors, *Metaphysical Grounding*, pages 37–80. Cambridge University Press.

French, R. (2019). Notational Variance and Its Variants. *Topoi*, 38(2):321–331.

Golan, R. (2021). Is there a neutral metalanguage? *Synthese*, 198(Suppl 20):4831–4858.

Harris, J. H. (1982). What's so logical about the "logical" axioms? *Studia Logica*, 41(2):159–171.

Hirsch, E. (2011). *Quantifier Variance and Realism: Essays in Metaontology*. Oxford University Press, Oxford, New York, new to this edition: edition.

Hjortland, O. T. (2017). Anti-exceptionalism about logic. *Philosophical Studies*, 174(3):631–658.

Jenkins, C. S. (2011). Is Metaphysical Dependence Irreflexive? *The Monist*, 94(2):267–276.

Kuhn, S. T. (1983). An axiomatization of predicate functor logic. *Notre Dame Journal of Formal Logic*, 24(2):233–241.

Lee, C. (2022). The structuralist approach to underdetermination. *Synthese*, 200(2).

O'Leary-Hawthorne, J. and Cortens, A. (1995). Towards Ontological Nihilism. *Philosophical Studies*, 79(2):143–165.

Poggiolesi, F. (2020). Grounding Rules and (Hyper-)Isomorphic Formulas. *Australasian Journal of Logic*, 17(1):70. Publisher: Victoria University of Wellington.

Putnam, H. (1967). Mathematics without foundations. *Journal of Philosophy*, 64(1):5–22. Publisher: Journal of Philosophy Inc.

Quine, W. V. O. (1960). Variables Explained Away. *Proceedings of the American Philosophical Society*, 104(3):343–347.

Quine, W. V. O. (1975). On Empirically Equivalent Systems of the World. *Erkenntnis*, 9(3):313–328.

Quine, W. V. O. (1976). Algebraic logic and predicate functors. In *The ways of paradox and other essays*, pages 283–307. Harvard University Press.

Rosen, G. (2010). Metaphysical Dependence: Grounding and Reduction. In Hale, B. and Hoffmann, A., editors, *Modality: Metaphysics, Logic, and Epistemology*, pages 109–36. Oxford University Press.

Schaffer, J. (2012). Grounding, transitivity, and contrastivity. In Correia, F. and Schnieder, B., editors, *Metaphysical Grounding: Understanding the Structure of Reality*, pages 122–138. Cambridge University Press.

Sider, T. (2023). The New Collapse Argument Against Quantifier Variance. *The Monist*, 106(3):342–361.

Stalnaker, R. (1977). Complex Predicates. *The Monist*, 60(3):327–339.

Sud, R. and Manley, D. (2021). Quantifier Variance. In Bliss, R. L. and Miller, J. T. M., editors, *The Routledge Handbook of Metametaphysics*, pages 100–17. New York: Routledge.

Tahko, T. E. and Lowe, E. J. (2020). Ontological dependence. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2020 edition.

Turner, J. (2011). Ontological Nihilism. In Bennett, K. and Zimmerman, D. W., editors, *Oxford Studies in Metaphysics*, pages 3–54. Oxford University Press.

Turner, J. (2017). Can We Do without Fundamental Individuals? No. In Barnes, E., editor, *Current Controversies in Metaphysics*, pages 24–34. Routledge.

Warren, J. (2021). Quantifier Variance, Semantic Collapse, and "Genuine" Quantifiers. *Philosophical Studies*, 179(3):745–757. Publisher: Springer Verlag.

Wigglesworth, J. (2017). Logical anti-exceptionalism and theoretical equivalence. *Analysis*, 77(4):759–767.

Woods, J. (2018). Intertranslatability, Theoretical Equivalence, and Perversion. *Thought: A Journal of Philosophy*, 7(1):58–68.