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Book Review

Karel Lambert, *Free Logics: Their Foundations, Character, and Some Applications Thereof*. (ProPhil – Projekte zur Philosophie Bd.1. Eine Schriftenreihe des Forschungsinstituts Philosophie/Technik/Wirtschaft an der Universität Salzburg. Herausgegeben von Edgar Morscher und Otto Neumaier). Sankt Augustin: Academia-Verlag, 1997. 156 pp. ATS 239. ISBN 3-89665-000-9.

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Abstract: Free logic is quite an interesting alternative to classical first order predicate logic with identity and has found many applications in philosophy. Karel Lambert is one of its founders and presents in this book a critical reassessment and defence of free logic as a fruitful alternative to classical predicate logic. After some historical preliminaries, I state his definition of a free logic and explain by means of the classical axiom scheme of universal instantiation how free logic makes logic free from existence assumptions. The variety of systems of free logic, published during the last 40 years, can be dealt with systematically by classifying

them into negative as well as positive and neutral free logics. This classification can be founded on the opposing solutions to the problem which truth-value elementary statements containing an empty singular term have. Furthermore, the opposing solutions to the problem, which referential status empty singular terms have, result in different semantics for free logic (e.g. in inner-domain-outer-domain-semantics or supervaluational semantics). I conclude that negative free logic is based on another concept of existence than the positive one. While in negative free logic by existence is meant self-identity, this cannot be meant in the positive one.

I

Karel Lambert published the very first system of free logic in 1963.[1] Lambert is one of the founders of free logic and presents in this book a critical reassessment and defence of free logic (p. 33). He discusses the existence assumptions made in the logical square of traditional term logic (pp. 48-58) and comprehends free logic – but not classical first order

predicate logic with identity – to be the natural continuation of a term logic made free of its existence assumptions (pp. 57-58).

What is a free logic? According to Lambert, a free logic is a logic free of existence assumptions with respect to its singular and general terms (respectively, predicates), but whose quantifiers have existential import as in classical first order predicate logic with identity. By the latter is meant that, e.g. the all quantifier ´$\forall x$` is read as ´For all existing things $x$ holds` (pp. 20, 35).

The programme of free logic aims at freeing logic from existence assumptions by making them explicit. For instance, in the classical axiom scheme of universal instantiation

UI $\quad \forall xA \rightarrow A(a/x)$,

there is an existence assumption hidden with respect to the singular term a (a singular term is an expression purporting to refer to exactly one thing (p. 33)). In classical predicate logic, every statement of the form UI is

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1 Lambert, K. 1963. ´Existential Import Revisited`, *The Notre Dame Journal of Formal Logic*, **4**, 288-292.

logically true. But in doing so, it is assumed that the singular term *a* refers to an existing thing. For, if the singular term *a* doesn't refer to an existing thing (that is, is empty), then no longer every statement of the form UI is logically true. That is, if *a* is empty, then there is a statement of the form UI that isn't logically true. This can be demonstrated as follows. Let *a* be empty, and consider the following statement of the form UI:

(1) If for all existing things *x* holds that *x* exists, then *a* exists.

Now it is trivially true that all existing things exist. Hence, the antecedence of (1) is true. Furthermore, the statement ´*a* exists` is false because *a* is empty and because, by definition, the statement ´*a* exists` is false if and only if (= iff) *a* is empty. Hence, the statement (1) is false and thus not logically true. Therefore, there is a statement of the form UI that isn't logically true if *a* is empty. In free logic, the existence assumption in question is made explicit as follows:

FUI $\forall xA \rightarrow (E!a \rightarrow A(a/x))$

Here the existence predicate is expressed by ´*E*!`; and, furthermore, it is assumed that the singular term *a* refers to an existing thing iff *E*!*a*. In this connection, Lambert makes mention of Frege's famous dialogue with Punjer (pp. 35-36, 41).

II

The historical development of free logic was motivated mainly by two problems, namely, the problem of empty singular terms, and that one of simple statements containing at least one such singular term (p. 59) (a statement of a language for predicate logic is simple iff it doesn't contain connectives and quantifiers). Lambert illustrates the problem of empty singular terms by means of an example drawn from the history of science. Formerly, the singular term ´Vulcan` was introduced by astronomers to refer to a planet which was supposed to cause the perturbations in the orbit of Mercury. Only later, it turned out that this planet doesn't exist at all. Hence, the singular term ´Vulcan` is empty. The problem is now whether an empty singular term doesn't refer to anything at all (that is, is irreferential), or whether it is nevertheless referring to something (that is, is referential). What is the referential status of such singular terms? The

adherents of free logic answer this question differently. They agree only upon the claim that an expression such as ´Vulcan` is an empty singular term. But some believe that it is irreferential, whereas others state that it is nevertheless referential because it is referring to a non-existing thing (pp. 59-60). That is why, in free logic, a singular term can either refer to an existing or non-existing thing, or to nothing at all (´or` has to be understood in the exclusive sense here). Furthermore, the non-emptiness of a singular term means either that it is referring to a non-existing thing or to nothing at all (but, of course, not both).

According to the previously mentioned definition, many logical systems are a free logic. These logical systems can be classified with regard to their respective answers to the problem of simple statements containing at least one empty singular term (pp. 61-63). In free logic, the problem which truth-value simple statements containing such empty singular terms get is answered differently. Are such statements true or false, or even truth-valueless (again ´or` is meant in the exclusive sense)?

Consider the following three simple statements (according to Lambert, they also don’t contain connectives and quantifiers in a depth-analysis):

(2) Vulcan exists.

(3) Vulcan is identical with Vulcan.

(4) Vulcan rotates.

Adherents of free logic take it mostly for granted that (2) is false because ´Vulcan` is empty. However, there is a huge disagreement with regard to the statements (3) and (4). Some believe that (3) and (4) are false because a non-existing thing cannot have any properties at all; hence, in particular, it cannot be self-identical or rotate. In a negative free logic, therefore, all simple statements containing at least one empty singular term are false (p. 62). Others state that at least the statement (3) is true because of the Leibnizian concept of identity, whereupon two things are identical iff they have the same properties. It follows from this definition that every statement of the form $a = a$ is true no matter whether the singular term *a* is referential or not. That is why, the statement (3) is true, according to this account (p. 93). In a positive free logic, therefore, at least one simple statement containing at least one empty singular term is true (p. 62). Since in a negative free logic the statement (3) is false, identity is, for this reason, not understood in the classical sense – namely, it is reflexive only

for existing things (p. 85). Lambert argues this to be a heavy philosophical disadvantage of a negative free logic (pp. 92-93). Finally, others believe that the statements (3) und (4) are truth-valueless because of the Fregean principle of compositionality, whereupon the truth-value of a statement is only a function of the referents of its parts (and its form). They assume, furthermore, that empty singular terms are irreferential. Now, if a part of a simple statement is irreferential, then the whole statement is truth-valueless. In a neutral free logic, therefore, all statements containing at least one empty singular term are truth-valueless (except, perhaps, for statements like (2)) (pp. 62-63).

III

Lambert indicates an axiomatic system for a positive as well as a negative free logic (pp. 39, 83) – calling them ´PFL` and ´NFL`, respectively – and underpins both systems by an adequate semantics (pp. 63-66, 69-78, 86-87). The system PFL is even based on two adequate semantics differing from each other in their respective answers to the problem of empty singular terms. The first semantics, the so-called ´inner-domain-outer-domain-semantics` (pp. 63-66), introduces, besides an inner do-

main of existing things, an outer domain of non-existing things. Such an account admits of understanding empty singular terms like ´Vulcan` as expressions referring to a non-existing thing, namely, an element of the outer domain. In such a semantics, every simple statement containing an empty singular term has a truth-value. The second one, the so-called ´supervaluational semantics` (pp. 69-78), rejects such a bisection of the domain and proceeds, instead of that, from a single domain of existing things. According to this account, singular terms like ´Vulcan` are ir-referential expressions referring to neither an existing nor a non-existing thing. However, in such a semantics, some statements containing an empty singular term are truth-valueless (e.g. (4)).

Lambert stresses several differences between the two systems PFL and NFL (pp. 87-95). I would like to add to these differences another one that I consider being remarkable. Two statements of the forms

(5) $a = a$

and

(6) $E!a$

are only in NFL (p. 85), but not in PFL, logically equivalent (that is, mutually deducible). Indeed, in PFL, from a statement of the form (6) a statement of the form (5) is deducible, in fact, trivially, because (5) is an axiom scheme of PFL (p. 39); however, from a statement of the form (5) one of the form (6) is not. For this reason, one cannot deduce in PFL from (3) that Vulcan exists. Since only in NFL, but not in PFL, two statements of the forms (5) und (6) are logically equivalent, one also can define only for NFL, but not for PFL, existence by self-identity as follows:

D_{NFL} $\quad E!a \leftrightarrow_{\mathrm{Df}} a = a$

Since (3) is false in every negative free logic, clearly, one also cannot deduce in NFL from this definition that Vulcan exists. Because of this difference, the system NFL is based on another concept of existence than the system PFL. While in NFL by existence is meant self-identity, this cannot be meant in PFL. In other words: In PFL self-identity is only a necessary condition for existence, but not a sufficient one.

IV

Two chapters each on free description theories as well as on applications of free logic – e.g. to the so-called ´attributive terms`, to the theory of partial functions, and to naive set theory – finish this excellent introduction to free logic (pp. 109-153). Lambert indicates Hintikka's and Brittan's investigations of Descartes' cogito-ergo-sum-argument as another example of a fruitful application of free logic (pp. 46-47). He points for further such applications to a book edited by him in 1991.[2] Free logic is supposed to be the best foundation of modal predicate logic by many philosophers (e.g. Kit Fine, James Garson, etc.) (pp. 45-46). It has proven to be of use to investigations in philosophy of language, epistemology, and metaphysics. Lambert's exposition is well balanced between motivating remarks and formal elaboration of the ideas. His book presupposes only a minimal familiarity with predicate logic. Several misprints don't lessen the merit of this volume to introduce a broader readership to free logic; in fact, these misprints are corrected in a list added to the book.

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[2] Lambert, K. (ed.). 1991. *Philosophical Applications of Free Logic*, Oxford: Oxford University Press.

Finally, I would like to suggest some further corrections. On p. 38, one reads that $\exists xA$ and $\forall xA$ were parts of *A*. In spite of this, it is meant that $\exists xB$ and $\forall xB$ are parts of *A*. On p. 54, the general term is not ´Things the same as *a* exist`, but ´Things the same as *a*`. On p. 89 (line 9), read ´NFL` instead of ´PFL`.

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