Liar paradox mirroring our reasoning as Hegel's quasi-speculative sentence

**ABSTRACT**

This paper explores the liar paradox and its implications for logic and philosophical reasoning. It analyzes the paradox using classical logic principles and paraphrases it as "affirmation of the falsity of the very affirmation." The study draws connections between the liar paradox and Hegel's speculative sentence and suggests it functions as a "quasi-speculative sentence." Additionally, it examines parallels with the logocentric predicament and the determinist's assertion, highlighting their paradoxical nature. Through these analyses, the paper aims to illuminate the fundamental paradoxes in our reasoning and logic.

Keywords: liar paradox, speculative sentence, logocentric predicament, determinism, dialectics

**Introduction**

Philosophers have long grappled with the intricacies of paradoxes. Among these, the liar paradox stands out for its persistent defiance of straightforward resolution. The proposition "This sentence is false" encapsulates a self-referential loop that oscillates between truth and falsehood. Several philosophers treated it as a meaningless statement. For instance, Sobel (2008) argues that “it does not say *anything at all*” (emphasis in original, p. 136). This argument does no injustice to the sentence. After all, it does not convey any substantive meaning.

Nevertheless, no one would deny its significance in the realm of philosophy and even mathematical logic. For instance, Gödel notes “a close relationship” between his proof strategy for the incompleteness theorems and the “‘liar’ antinomy” (Gödel, 1992, p. 40). Furthermore, the liar paradox gave rise to fuzzy logic and paraconsistent logic.

However, instead of delving into its logical intricacies, this paper will focus on the similarities between the alternating appearances of truth and falsity within the liar sentence and some paradoxical features of our philosophical reasoning. Specifically, this paper will present an adamant claim that the liar paradox mirrors the structure of our reasoning.

To justify its claim, we will first analyze the liar sentence in Section 1 by using two rules of classical logic: the principle of bivalence and the law of excluded middle. Based on this analysis, this paper will offer the following paraphrase of the liar sentence: “*Affirmation* of the falsity of *the very affirmation*.” It will serve as a useful phrase for illustrating paradoxical aspects of our reasoning.

In Section 2, we will investigate Hegel’s speculative sentence. As Houlgate (1986) notes, however, “Hegel does not write much about the speculative sentence” (p. 145). Nevertheless, “it is clearly an important idea because it is the kernel of his theory of what constitutes non-metaphysical philosophical language, the mode of language appropriate to dialectical method.” This paper proposes that the liar sentence can partially achieve Hegel’s aim as a “quasi-speculative sentence.”

Section 3 discusses the logocentric predicament, which was first mentioned by Sheffer (1926) in his review of Russell’s *Principia Mathematica*. Drawing on an extensive discussion in Chapter 3 of Hanna’s (2006) *Rationality and Logic*, the paper derives an analogue to the liar paradox: “logical affirmation of the groundlessness of logic.”

In Section 4, we will discuss Lee’s (forthcoming, 2024) formulation of a determinist’s assertion and how this can bear resemblance to the liar paradox. In parallel with the liar sentence, the paper derives: “*Affirmation* of the determinacy of the world events including *the very affirmation*.”

By using the liar sentence as an analogy with regard to the logocentric predicament and determinism, the paper will shed light on the underlying structures of our reasoning.

**1. Liar sentence**

Before we analyze the liar sentence, let us assume that the two following rules of classical logic hold.

(1) The principle of bivalence

: There are only two *truth values* available for any proposition: T (true) and F (false). That is, there is no such thing as “only half true.”

(2) The law of excluded middle

: A *proposition* is either true or false. There is no middle ground.

We will also assume:

(3) The copula “is”[[1]](#footnote-1) in the propositional form “S is P” is used to indicate affirmation (assertion) of a particular property represented by P.

(4) S can exist as a lower-level sentence (e.g., “A is B”) within “S is P,” and a truth value of “S” can be alleged by plugging the truth value into P. For instance, we can state: “‘A is B’ is true.”[[2]](#footnote-2)

Now, as an assumption, let us assert that the truth value of the liar sentence is F. Then, this assertion can be propositionally expressed as:

“This sentence is false” is false.

What does “this sentence” refer to? Naturally, it refers to “this sentence is false.” To simplify, let us simply call it K. Then:

K = K is false.

For simplicity, consider the following version of the statement on the right-hand side of the equation:

“K2 is false2” is1 false1.

The first predicate (is false1) negates the lower-level proposition (i.e., “K2 is false2”). Therefore, based on the law of excluded middle, the only possibility is that “K2 is false2” does not hold. This is satisfied in one of the two ways:

(i) “K2 is [not false2].”

(ii) “K2 [is not] false2.”

In the first instance, the property of falsity has been negated. In the second instance, the copula has been negated.[[3]](#footnote-3) For our purposes, we will consider only the first instance. Now, what is the outcome of the negation of falsity? It should be undoubtedly “true,” based on the bivalence principle. Thus, K2 is true. This can be alternatively understood as “affirmation of the truth of K.” However, upon affirming its truth, we find:

“K3 is false3” is true2.

The predicate “is true2” now affirms the lower-level proposition “K3 is false3.” Hence, K3 is false. Following this reasoning, we observe:

K1 is false1. (Level 1)

“K2 is false2” is false1.

K2 is true2. (Level 2)

“K3 is false3” is true2.

K3 is false3. (Level 3)

“K4 is false4” is false3.

K4 is true4. (Level 4)

...

We see that truth values alternate in the following order:

F1 -> T2 ->F3 -> T4 -> ...[[4]](#footnote-4)

F and T alternate as we move through the lower levels.[[5]](#footnote-5) It is crucial to note that a property (true/false) at a particular level is decided by affirmation of its antithesis at a higher level. For example, the property of falsity3 at Level 3 is decided by the predicate “is true2” at Level 2. Similarly, the property of truth2 is decided by the predicate “is false1.” Thus, F and T are inextricably linked. What is remarkable is that all these alternating Fs and Ts are contained within the single liar sentence, which can be indefinitely developed as follows:

[[[[...] is false4] is false3] is false2] is false1.

We can paraphrase it as:

“Affirmation of the falsity of affirmation of the falsity of …”

How does the above phrase follow? In “This sentence is false,” the copula “is” serves to indicate affirmation, as previously noted in assumption (3). Thus, the sentence can be changed to “affirmation of the falsity of this sentence1.” Regarding “this sentence1,” we see:

“this sentence1”

= This sentence2 is false

= affirmation of the falsity of this sentence2.

Therefore, “affirmation of the falsity of this sentence1” can be extended to:

“Affirmation of the falsity of affirmation of the falsity of this sentence2”

Or more briefly:

“**Affirmation of the falsity of the very affirmation**.”

Note that “this sentence” is omitted from the phrase. This omission is justified because the subject is only a placeholder. Specifically, it merely serves to lengthen the phrase endlessly. Its additional function is for the copula “is” to be placed as a verb within the liar sentence.

The above key phrase will illuminate an important aspect of our reasoning. Before exploring this further, we must first discuss its significance in the context of Hegel’s speculative sentence. We will then see how the image of the alternating Ts and Fs in the liar paradox can place it within the context of Hegelian dialectics.

**2. Speculative sentence**

In *Phenomenology of Spirit*, Hegel introduces speculative sentences such as “God is being” and “The actual is the universal” (2018, p. 39). In these sentences, the “difference between subject and predicate”[[6]](#footnote-6) … “is destroyed by the speculative judgment.” Further, “[t]hinking loses its fixed objective basis which it had in the subject, when, in the predicate, it was thrown back to the subject, and when, in the predicate, it returns not into itself but into the subject of the content.” Hegel aims to demonstrate that a speculative sentence “has within itself the dialectical motion necessary to present consciousness as alive and self-developing through its determinate shapes to the organic whole of spirit as ‘absolute knowing’” (Verene, 2007, p. 10).

However, Bowman (2013), in *Hegel and the Metaphysics of Absolute Negativity*, quotes Rainer Schäfer’s argument that a single proposition is “not able to express the dialectical, internally self-reverting movement of the grammatical subject to the predicate and from the predicate … back to the grammatical subject” (p. 252). Hence, “there cannot actually be a speculative sentence as a sentence.” In a similar vein, Houlgate (1986) notes that “the *Logic* cannot be expressed by one speculative sentence alone, even if that sentence is the most concrete definition of reason as dialectical self-determination” (p.150).

This paper argues that a single liar sentence can partially exhibit this dialectical feature. In Section 1, we observed that F and T continue to alternate through endless different levels as the liar sentence begins to unfold. These movements result from the recursive re-evaluation of the truth value of the subject. Although neither F nor T functions as a subject or predicate, the liar sentence resembles a speculative sentence in a significant way. To discuss this point in detail, we will consider a few more references below.

According to *Phenomenology of Spirit*, the philosophical (speculative) proposition “evokes the common opinion” that learns that “[the proposition] means something other than what it took itself to have meant, and this correction of its opinion compels knowing to come back to the proposition and now grasp it in some other way” (p. 40). Building on this, Kolman (2023) notes that a speculative sentence is to “express, in a perspicuous and positive way, the reflective ascent associated with the negative nature of our experience” (p. 392). In addition, “this negativity must be applied to itself, not only in acknowledgment of the existence of the other side of any difference, *but also of the other side of this differentiating itself*” (emphasis added).

Let us examine the liar sentence by using both Hegel’s and Kolman’s expressions. After reading the statement “this sentence is false,” the reader soon realizes that the subject within the sentence is equivalent to the whole sentence. Thus, she is repeatedly compelled “to come back to the proposition.” Then, she tries to “grasp it in some other way.” However, this sentence can never be fully grasped, because it refers to itself endlessly. The moment she believes that its truth value has been decided, she does not realize that it is negated by the predicate “is false” lurking in a lower-level sentence. Thus, any clear affirmation made “in a perspicuous and positive way” inevitably negates itself. By doing so, “this negativity” (“the falsity”) is made “in acknowledgment of the existence of” “the other side of this differentiating itself (“the very affirmation).” This recursive self-negation is succinctly illustrated in the paraphrased version of the liar sentence: “Affirmation of the falsity of the very affirmation.” This process closely resembles the incessant activity of the Absolute Idea.[[7]](#footnote-7)

Admittedly, the liar sentence is *not* a speculative sentence *per se*. First, Hegel himself never said that it is a speculative sentence. Instead, he asserted that only the conjunctive form of “µ and not µ” is true for the liar sentence.[[8]](#footnote-8) Second, the liar sentence does not illustrate any higher form of understanding where contradictions are resolved. This conflicts with our usual understanding of Hegel’s dialectics where thesis leads to antithesis and they both merge into synthesis. Given these observations, this paper proposes the term “quasi-speculative sentence” to describe the liar sentence. It will serve as a useful analogy for illustrating the paradoxical features of our reasoning that will be discussed in the subsequent sections. In the following section, we will first discuss the logocentric predicament.

**3. Logocentric predicament**

According to Sheffer (1926), “the attempt to formulate the foundations of logic is rendered arduous” by a “logocentric predicament” (p. 228). He explains, “In order to give an account of logic, we must presuppose and employ logic.” Hanna (2006) points out that Sheffer assumes that “epistemic noncircularity is a necessary condition of all legitimate explanations and justifications” (p. 55).[[9]](#footnote-9) However, Carroll (1895) questions whether logic can ever achieve non-circularity. Specifically, he believes that a *modus ponens* argument faces a regressive problem. The details of his discussion can be briefly reconstructed as follows.

P obtains Premise 1

P->Q obtains. Premise 2

Thus, Q. Conclusion

Carroll deems the above argumentation insufficient. He asserts that we need the following additional premise to reach the conclusion.

“If Premises 1 and 2 obtain, then Q.”

Essentially, he is suggesting that we must accept the entire exiting argument process (Premises 1 and 2 leading to the Conclusion) as a premise in order to reach the conclusion. Upon adding Premise 3, the argument becomes:

P obtains Premise 1

P->Q obtains. Premise 2

If Premises 1 and 2 obtain, then Q. Premise 3

Thus, Q. Conclusion

However, Carroll still believes that the above process is incomplete and requires yet another premise – namely, that if Premises 1, 2, and 3 obtain, then Q. This leads to infinite regress, where each step demands a further premise, and we never actually arrive at the conclusion.[[10]](#footnote-10)

Is there a problem with Carroll’s argument? According to Hanna (2006), the argument is based upon the following implicit assumptions:

“(1) Every valid deductive advance from the premises of an argument to its conclusion can be explained only by appeal to a principle of valid inference.

(2) That principle of valid inference must therefore itself be included as a true premise in the very same argument” (p.56).

Philosophers took issue with the second assumption. Specifically, they held that “principles of valid deductive inference *for* a proof are not the same as true or logically true conditional premises *in* a proof” (emphasis in original, p. 57). That is, Carroll failed to see the distinction between the “object language” (logical premises) and the “metalanguage” (logical interference rules). However, despite acknowledging the plausibility of this critique, Hanna (2006) argues that “it is not at all clear that the philosophical problem [Carroll] was trying” to get us to notice was solved (p. 58). Specifically, Hanna asks, “by virtue of what logical resources are valid *metalogical* deductions to be explained or justified?” (emphasis in original). If we justify them through meta-metalogical proofs, then these would require meta-meta-metalogical proofs. This again traps us in infinite regress. Hence, we are led to make a rather radical conclusion that “logic is viciously circular, that is, *groundless*” (emphasis added, p. 65).[[11]](#footnote-11)

Upon closer reflection, we notice an interesting issue. By indicating the infinite regress problem in *modus ponens*, Carroll led us to conclude that logic is groundless by appealing to logic itself. In other words, logic is groundless on the ground of logic. Let us call this “logic’s groundlessness thesis.” For convenience, let us consider the following version of this statement:

“Logic1 is groundless on the ground of logic2.”

When a reader first considers Carroll’s argument, she sees logic as logic1. However, her reasoning process that led to the conclusion is based on logic2. Specifically, that logic1 is groundless is based on the following argumentation:

We cannot resolve infinite regress for *modus ponens*. (Premise 1)

If we cannot resolve infinite regress for *modus ponens*, *modus ponens* is groundless. (Premise 2)

Thus, *modus ponens* is groundless. (Conclusion)

This argumentative process exactly employs *modus ponens*. In other words, *modus ponens*2 was used to prove the groundlessness of *modus ponens*1. Thus, the thesis that logic is groundless on the ground of logic holds.[[12]](#footnote-12) This can be paraphrased as “logical affirmation of the groundlessness of logic.” This feature of our reasoning bears resemblance to the liar paradox. They appear similar in that they are both self-defeating.

In Section 1, when discussing the liar sentence, we saw a paradoxical situation where F1 led to T2, T2 led to F3, F3 led to T4, and so on. Similarly, when logic2 is used to prove the groundlessness of logic1, the groundlessness of logic2 will also be proved by logic3. This process will go on indefinitely.

However, the paradoxical nature of the liar sentence is not only reflected in the logocentric predicament but is also observed when a determinist asserts determinism. In the following section, we will examine how a liar-paradox-like situation may arise in the case determinism.

**4. Determinist’s paradox**

Regarding a determinist’s assertion of determinism, Lee (2024, forthcoming) provides the following formulation:

“[T]he determinist refers1 to:

The determinacy of all the events of the universe comprising the very event of referring2 to the determinacy of all the events” (p. 20).

Lee argues that “[w]hile the referring1 occurs dynamically, the referring2 exists within a static realm.” This is one of the two reasons that he argues that there is a discrepancy between the two acts of referring. The other reason is that the mind engaged in referring1 differs from the mind engaged in referring2, even though they should be identical. Specifically, “it is seen in hindsight” that the latter mind had been the former mind “all along without our knowing it.” His argument can be further explained as follows.

When a determinist asserts determinism, this suggests that her act of assertion has also been determined. From an external viewpoint, this seems totally reasonable. However, when she is involved in the act of assertion, there must be a brief time lapse where her mind separates itself from the objects of the world that are under her scope of determinism. When she realizes that the briefly separated mind was also part of the deterministic world, the state of mind where this realization takes place must be different from the briefly separated mind. When she understands that this realization was also a predetermined event, the state of mind where this understanding takes place must also differ from the state where the realization took place.[[13]](#footnote-13) Therefore, she can never achieve total identity between the mind engaged in referring1 and mind engaged in referring2. Thus, contrary to Lee’s statement, in our sensible realm, there can be actually no case where the two minds had been the same all along.

However, if perfect identity could be achieved between the two minds, this would result in the following paradoxical case:

“Affirmation1 of the determinacy of the world events including the very affirmation2.”

This phrase raises several questions. If “affirmation1” was a predetermined event, why would it stand out compared to the other predetermined world events? Additionally, if affirmation1 and affirmation2 are indeed identical, wouldn’t this create an infinite loop, similar to the liar paradox, from which there is no escape?

Nonetheless, the above discussion does not lead to a conclusion that determinism must be false. Lee (2024) does not question the truth of determinism. Rather, he only reveals an inherent discrepancy in a situation where a determinist purports to claim self-identity. This can support the notion that the “determinist’s paradox” resembles the liar paradox, where the liar perpetually self-negates her “affirmation.”

**Conclusion**

This paper’s main ideas can be briefly illustrated as follows:

Liar sentence

* Affirmation of the falsity of the very affirmation

Logic’s groundlessness thesis

* Logical affirmation of the groundlessness of logic

Determinist’s paradox

* Affirmation of the determinacy of the world events including the very affirmation

As noted by Verene (2007), Hegel aimed to delineate through a speculative sentence “the dialectic motion necessary to present consciousness as alive” (p. 10). However, even if the speculative sentence could provide “the most concrete definition of reason as dialectical self-determination” (Houlgate, 1986, p. 150), it would still be mere textual representation unless read and examined through “consciousness.” The lone liar sentence, if read and understood by a philosophical reader, generates an infinite display of Fs and Ts traversing endless levels. Although this in itself is not an accurate representation of Hegelian dialectics, it closely mirrors the paradoxical features of our reasoning, particularly in relation to the logocentric predicament and the determinist's paradox. Therefore, the author believes that the liar sentence merits the label “Hegel’s quasi-speculative sentence.”

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1. The copula can also be interpreted to indicate “existence” rather than “affirmation.” For instance, “Heidegger cites the logical copula ‘is’” as “a clue to this connection” “between logic and what he maintains is the central question of metaphysics, the question of being” (McNulty, 2023, p. 21). In that case, using a bit of a stretch, the liar sentence could be interpreted to mean “This sentence exists in the category of false sentences.” [↑](#footnote-ref-1)
2. In formal logic, we use the symbol “T(S)” to express this proposition. However, for the purposes of discussion, we will stick to the form “‘A is B’ is true.” [↑](#footnote-ref-2)
3. The second instance can be paraphrased as “negation of the truth” rather than “affirmation of the falsity.” [↑](#footnote-ref-3)
4. This ordering evokes an image of Hegel’s “negation of negation.” [↑](#footnote-ref-4)
5. Level 1 is the highest level, and there is no limit to how low the levels can go. [↑](#footnote-ref-5)
6. Regarding “S is P,” we say “is P” is a predicate in classical logic. Meanwhile, in Hegel’s speculative philosophy, only P is a predicate. For details, see Houlgate (1986, p. 146). [↑](#footnote-ref-6)
7. In *Less Than Nothing*, Žižek (2012) describes “the mad self-referential play of the Absolute Idea.” He further describes the absolute immanence of a criterion for the “Hegelian truth” where “a statement is compared with itself, with its own process of enunciation.” [↑](#footnote-ref-7)
8. For more details, see d'Agostini, F., & Ficara, E. (2021). [↑](#footnote-ref-8)
9. The subsequent discussion on Carroll’s (1895) argument is based on Hanna’s (2006) *Rationality and Logic*. [↑](#footnote-ref-9)
10. As a Hegelian metaphysician, McNulty (2023) argues that “Subjective Logic” (traditional logic) depends on “Objective Logic” (ontology) and that this provides “Hegel’s resolution of the logocentric predicament” (p. xi). This idea can align with Hegel’s dialectics. [↑](#footnote-ref-10)
11. However, Hanna (2006) does not accept the conclusion that logic is “groundless.” He overcomes this issue through “the logical faculty thesis.” For details, see Chapter 6 in *Rationality and Logic*. [↑](#footnote-ref-11)
12. This external standpoint on logic is also based on logical reasoning. As such, our external standpoint on logic exists within the periphery of logic. This shows that we can take only a “perennially transient external standpoint” on logic. Specifically, in the short moment when we decide that logic is groundless, we are not aware that this judgment is based on logical reasoning. When we transition into a new conscious state and conclude that the judgment was based on logical reasoning as well, the conclusion will again be revealed to be based on logical reasoning. [↑](#footnote-ref-12)
13. This process is similar to the truth value alternation that we saw regarding the liar paradox: F1 -> T2 ->F3 -> T4 -> ... For instance, Lee (2024) states, “Before the determinist decides on the determinacy of the events of the universe, these events must first be placed within her scope of thoughts” (p. 26). Then, as she declares determinism, she realizes that the entire process” “is also part of the deterministic scenario.” “Subsequently, she concludes from a transient God’s-eye perspective that every time she declares determinism, this would have also been predetermined.” This notion parallels the “perennially transient external standpoint” described in Footnote 13. [↑](#footnote-ref-13)