Research Article

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Individuals, Existence, and Existential Commitment in Visual Reasoning

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Abstract: This article examines the evolution of the concept of existence in modern visual representation and reasoning, highlighting important milestones. In the late eighteenth century, during the so-called golden age of visual reasoning, nominalism reigned supreme and there was limited scope for existential import or individuals in logic diagrams. By the late nineteenth century, a form of realism had taken hold, whose existential commitments continue to dominate many areas in logic and visual reasoning to this day. Physical, metaphysical, epistemological, and linguistic positions underlie both nominalist and realist views. Since the paradigmatic works on visual reasoning in the 1990s, formal diagram systems have been developed that revive either the nominalist or realist perspectives. Unlike in the nineteenth century, these are not motivated by philosophical views. Nevertheless, they may still have an impact on many areas of philosophy and science outside logic.

Keywords: existential import, realism, nominalism, individuals, philosophy of mathematics, logic diagrams, history of logic

1 Introduction

In modern logic, it is common to read expressions like $\exists x\ Ax$ as asserting that "there exists (at least) one x such that Ax" or "for at least one individual, call it x, it is true that Ax." With these interpretations, logic and ontology have become closely related, a relationship that was often foreign to antiquity, the Middle Ages, and the early modern period. Existence, a fundamental theme of traditional ontology or metaphysics, has become a significant mode of expression in formal logic, shaping our understanding of logical expressions such as $\exists x\ Ax$. Individuals are no longer regarded, as in traditional ontology, as the members of a lowest species in the tree of a category. Rather, in modern logic, individuals are now considered the actors in a bound variable to which predicates are assigned. This is the perspective that has become established at the latest with Quine.[1]

However, $\exists x\ Ax$ was not frequently associated with existence even in the first half of the twentieth century. For a long time, there were logicians who considered the aforementioned interpretation of $\exists x\ Ax$ problematic and did not permit any existential import in such expressions.[2] Disputes about the precise interpretation persist to this day. While some advocate for the existential interpretation,[3] others see deficiencies that have yet to be addressed.[4] These disagreements are unsurprising given the significant implications of this interpretation. For instance, this reading necessitates a changed perspective on traditional logic. In logic,

1 Priest, "The Closing of the Mind: How the Particular Quantifier became Existentially Loaded Behind our Backs."
2 Reicher, *Referenz, Quantifikation und ontologische Festlegung*, 137ff.
3 Inwagen, "Meta-Ontology."
4 Orenstein, "Is Existence What Existential Quantification Expresses?;" Bricker, "Ontological Commitment."

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existential assumptions demand the alteration of the square of opposition or the rejection of inferences that were long considered valid.[5] As a consequence of these changes in views in logic, since the early twentieth century, changed views in other areas have also been demanded, such as in ontology or semantics.[6]

The logical concept of existence, in the manner described earlier, was first introduced in textbooks from the nineteenth century and primarily reflected upon in philosophical writings. Therefore, it is not uncommon to encounter statements like the following in the literature:

> The problem of existential import developed along with the development of modern symbolic logic during the nineteenth century. The problem is peculiar to the standard predicate calculus.[7]

This is, at most, only half the truth. While it is true that symbolic logic rose to prominence in the nineteenth century, diagrammatic logic was dominant at that time. Probably due to the algebraisation of logic, the emergence of symbolic logic, and the crisis in intuition in mathematics and physics in the late nineteenth and first half of the twentieth century,[8] it is often forgotten that geometric representations dominated logic in the nineteenth century with the use of geometric shapes that, today, would be referred to as "logic diagrams."[9] In particular, the dominance of Euler diagrams was also confirmed by representatives of the algebra of logic and symbolic logic. As Ernst Schröder wrote in 1890,

> Since Leonhard Euler, in his *Letters to a German Princess*, made popular use of this visual mode of representation …, it has been used or at least referenced in all works on logic.[10]

Similarly, in 1894, John Venn, who used Euler's diagrams as a basis for his own diagrams, wrote,

> With one such scheme, namely that which is commonly known as the Eulerian, every logical reader will have made some acquaintance, since a decided majority of the modern familiar treatises make more or less frequent use of it.[11]

Even the late nineteenth-century movement to which Ernst Schröder belonged and which is now summarised under the term "Algebra of Logic" still made intensive use of visual reasoning. It was only in the first decades of the twentieth century that the knowledge and use of logic diagrams fell into obscurity. During this time, the existential interpretation of expressions like $\exists x\ Ax$, which had previously been applied to logic diagrams, also became established.

Even though numerous logicians and ontologists, who were socialised within the paradigm of algebraic logic, attempted to explain existential assumptions using the notations of the twentieth century, the introduction of the concept of existence in logic is a result of the era in which logic diagrams prevailed, the so-called golden age of visual reasoning.[12] Thus, this article can be understood as an extension to visual reasoning of the thesis of Graham Priest, who investigated the development of existential assumptions in symbolic logic.

It is not surprising that the use of individuals, objects, existential commitment, etc. has not yet been investigated more closely in logic diagrams, for several reasons. Even today, general knowledge of logic diagrams cannot be assumed in all areas of logic, as many logicians are socialised directly and exclusively with purely symbolic logic. This is the result of the early twentieth century, which also popularised many prejudices about diagrams. Although it is today widely accepted that diagrams can potentially have the same expressivity as symbolic or other forms of representation of logic,[13] the properties are different.[14] This also

5 Parsons, "The Traditional Square of Opposition;" Strößner, "Existential Import, Aristotelian Logic, and its Generalizations."
6 E.g. Carnap, *The Logical Structure of the World*, 7ff., 47ff.
7 Nedzynski, "Quantification, Domains of Discourse, and Existence," 138.
8 Johansen and Pallavicini, "Entering the Valley of Formalism."
9 Moktefi and Shin, "A History of Logic Diagrams."
10 Schröder, *Vorlesungen über die Algebra der Logik*, 155.
11 Venn, *Symbolic Logic*, 110.
12 Moktefi and Shin, "A History of Logic Diagrams."
13 Shin, *The Logical Status of Diagrams*; Jamnik, *Mathematical Reasoning with Diagrams*.
14 Shimojima, *Semantic Properties of Diagrams and Their Cognitive Potentials*.

complicates the application of expressions such as existential import to diagrammatic logic, since the discussion was coined in the symbolic logic of the twentieth century (but actually dates back to the age of diagrams).[15]

Nevertheless, this article breaks new ground, investigating the introduction of the concepts of existence and individuals into the paradigm of visual reasoning. To this end, Section 2 begins with an examination of the nominalists who significantly influenced the logic of the early nineteenth century. These nominalists oppose the introduction of existential commitments in logic. Then, in Section 3, we shed light on further developments over the course of the nineteenth century that led to the incorporation of existential assumptions in visual reasoning. In Section 4, we show that, by the end of the nineteenth century, the diagrams were already heavily laden with individual existential import, leading to a realism in logic that contradicted the original intention of the paradigmatic book of early modern visual reasoning. However, this made it possible to establish a reading of logic that could then be adapted by the strictly algebraic notations of the twentieth century. In Section 5, we then examine the logic system within the current paradigm of visual reasoning, and, finally, in Section 6, we provide an outlook on the unresolved questions.

2 Nominalism

As demonstrated by the quotes from Venn and Schröder, Euler's influence on the nineteenth century can hardly be overestimated. Euler, therefore, seems to have written the paradigmatic book for the nineteenth-century logic. However, it was Kantian philosophy that made Euler's logic socially acceptable. In this respect, one can also speak of an Euler-Kantian paradigm in the nineteenth century. In Subsection 2.1, we first look at Euler's logic and emphasise that Euler represents a strong nominalism that vehemently opposes the introduction of the concept of existence in language philosophy and logic. Then, in Subsection 2.2, we outline the history of Euler diagrams in early Kantianism and show that this nominalism was represented and intensified by Kant's first successors.

2.1 Euler's Logic

Euler's engagement with logic began during his time as a young student in Basel. Here, he was a respondent during the rededications of the Chair of Logic in 1722. This was probably his first encounter with the algebraic methods of logic of Jacob Bernoulli. Indeed, throughout his life, Euler focused on a distinct form of representation in logic, and manuscripts from the late 1730s and early 1740s documenting Euler's attention to the geometric method in logic have been preserved for posterity.[16] At this time, Euler was already a famous mathematician but he became particularly unpopular in many parts of Central Europe due to his polemical writings against the proponents of rationalism.

When Euler published his logic in his *Letters to a German Princess* in 1768, he revealed the strong influence of British empiricism and his critical view of the rationalism that was dominant at the time to the general public. The *Letters* cover a variety of topics from natural science and philosophy, and the philosophical section spans hundreds of pages demonstrating how Euler argues against rationalists like Descartes, Leibniz, Wolff, and their followers.[17] In particular, he criticises the doctrine of monads and is, therefore, counted among the "antimonadists" alongside Samuel Christian Hollmann, Jean-Pierre de Crousaz, Johann Franz Buddeus, Johann Heinrich Gottlob von Justi, Marquis d'Argens, and others.[18] However, Euler's criticism

15 Yi, *Venn and Existential Import*.
16 Kobzar, "Гносеология и логика Л. Эйлера."
17 Look, "Kant's Leibniz."
18 Feder, *Erklärung der Logik, Metaphysik und practischen Philosophie*, 127.

is not only directed against the assumption of monads but also against the resulting determinism, the idea of pre-established harmony, Leibnizian optimism, and the idea that the spirit or individual is presented as a point ("punctum physicum/metaphysicum" or monad[19]). The latter topic is of crucial importance for the interpretation of logic diagrams.

Euler perceives the process of knowing as an interplay of empirical, rational, and believed elements that knowing is based on experience. The following quotes show how Euler connects with the peripatetic principle *nihil est in intellectu quod non fuerit in sensu* and that he believes that the origin of knowing lies in experience (although he also admits that rational knowledge and testimonies can also be sources of insight).

> All this knowledge is acquired only in so far as the objects make an impression on some one of our senses.[20]

> We undoubtedly derive them [i.e., the ideas], in the first instance, from real objects, which strike our senses;[21]

For Euler, sensory perception is the foundation of all knowledge. Sensory experience is the realm of existence, as the things we perceive can have a real effect on us. Only rationalists, idealists, and egoists dare to doubt existence in the realm of perception. However, these are just philosophical games with doubt that have no relevance in everyday life. After all, the watchdog does not doubt the existence of the burglar nor does the soldier doubt the existence of his superior.[22]

> The knowledge supplied by our senses is undoubtedly the earliest which we acquire; and upon this the soul founds the thoughts and reflections which reveal to it a great variety of intellectual truths. In order to better comprehend how the senses contribute to the advancement of knowledge, I begin by remarking, that the senses act only on individual things, which actually exist under circumstances determined or limited on all sides.[23]

Language is merely an abstraction of sensory perception and logic, in turn, is an abstraction of language. Due to this abstraction, which only humans are capable of,[24] Euler makes a clear distinction between perception and language. Language is always abstract and general, while perception is always concrete and individual. Based on this distinction, in his philosophy of language, Euler adheres to a strict nominalism that not only doubts the existence of general concepts but also questions the reference between proper names and individuals.

The concept of a tree is general and refers to all types of trees, such as cherry trees, apple trees, etc. "Now, *the tree* which corresponds to my idea of tree exists nowhere," Euler writes.[25] With this, Euler clearly speaks out against any realism or Platonism. If we have a general concept or an idea, it can be applied to a variety of existing objects such as trees, but the concept itself is not the tree. One might think that the species, such as cherry-trees, apple-trees, etc., that Euler lists refer to individuals but even this is excluded by Euler's strict nominalism: "In like manner, when I speak of a cherry-tree, it too is a general notion, which comprehends all the cherry-trees that exist."[26] For Euler, truly existing things are called a "single thing" or "individual," and these do not exist in language, only in perception.

One might think that Euler interprets proper names in such a way that they have a 1:1 correspondence with a single thing or individual. However, this is not the case either, and he states "I remark, first, that we have scarcely a word in any language whose signification is attached to one individual object."[27] Euler does not elaborate further on this sentence but, from the context, it becomes clear that the 1:1 correspondence and, thus, the reference of proper names to individuals can, at best, be a coincidence. Therefore, it is possible for a

19 Busche, *Leibniz' Weg ins perspektivische Universum.*
20 Euler, *Letters*, I, 271.
21 Ibid., I, 366.
22 Ibid., I, 322.
23 Ibid., II, 23.
24 Ibid., II, 80.
25 Ibid., I, 333.
26 Ibid.
27 Ibid., I, 334.

proper name to apply to a single person by chance although proper names are inherently general concepts that can only apply to a multitude of people or objects. Euler illustrates this with a remarkable example:

> The name, Alexander the Great, is applicable to one particular person but then it is a compound name. There may have been many thousands of Alexanders, and the epithet great extends to an infinite number of things.[28]

It is possible that a proper noun such as "Alexander the Great" can only be applied to one person, but potentially no proper noun is limited to just one individual. Instead, it always refers to all possible objects that can be labelled with this word with good reason. A good reason is determined by custom and use of the name and the correspondence theory of truth.[29] For example, it is incorrect to refer to Alexander as "Aristotle" if the name "Alexander" and not "Aristotle" is customarily used to refer to the individual in question.

This means that the proper name refers to all individuals who are called, were called, will be called "Alexander the Great" or who could be given this name with good reason. This does not exclude the possibility that other proper names may also be attributed to an individual. For example, "Alexander the Great" could be called "the former king of Macedonia," but there is no identity between these two names, as the "the former king of Macedonia" could also be the individual that is normally called "Demetrius II." Euler does not use so many examples, but his abstract descriptions and his Alexander example make it clear that the so-called proper name is never reserved for a single individual, but can refer to several.

Euler only differentiates concepts by whether they typically apply to a few objects (proper names), slightly more objects (species), or many objects (genera). In this respect, the extension of a concept is relative, never absolute. Only an omniscient being would know how many "Alexander the Greats" there have been, are, and will be. However, since we do not possess the information of an omniscient being, we must assume that all types of concepts can always be applied to an imprecisely determined number of objects.

Euler later transferred his nominalist philosophy of language to logic, which is not just about words but often only about symbols: "These signs or words represent, then, general notions, each of which are applicable to an infinite number of objects."[30] For Euler, formal logic is still an abstraction from language, as it reduces language to rule-guided quantified propositions (Aristotelian logic) or rule-guided propositions including connectives (Stoic logic). Moreover, Euler does not even allow the correspondence theory of truth in logic to apply here, since logic has no relation to the sensory world.[31] So, it is sensuality (faculty of experience), not logic (faculty of intellect), that decides whether something exists.

In the field of Aristotelian logic, Euler introduced his renowned diagrams.[32] They are intended to simplify the handling of logic and also enable an algorithmic decision procedure for syllogistics.[33] Euler's diagrams consist of circles. Each circle represents a concept, and the position of the circles in Euclidean space depicts the quantified relationship between the concepts. Thus, two circles form a judgement and three circles depict a syllogism. In total, there are four different diagrams representing the four judgements of syllogistics. Figure 1a represents the universal affirmative judgement "All A are B," Figure 1b shows the universal negative judgement "No A is B," Figure 1c shows the particular affirmative judgement "Some A are B," and Figure 1d shows the particular negative judgement "Some A are not B."

Euler's diagrams have often been interpreted from the perspective of (1) existential import and (2) set theory. However, both interpretations are not compelling and do not follow from the context. (1) One can get the impression that the position of the letter A in Figure 1c and d plays the role of at least one individual and that the diagrams therefore suggest objectual or existential import. However, Euler's original diagram system does not distinguish between Figure 1c and d.[34] The displacement of the letter appears to be a later addendum

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28 Ibid., I, 335.
29 Ibid., I, e.g. 334, 385.
30 Ibid., I, 337, similar to 333, 339.
31 Ibid., I, 385.
32 Moktefi and Shin, "A History of Logic Diagrams."
33 Bernhard, *Euler-Diagramme*.
34 Kobzar, "Гносеология и логика."

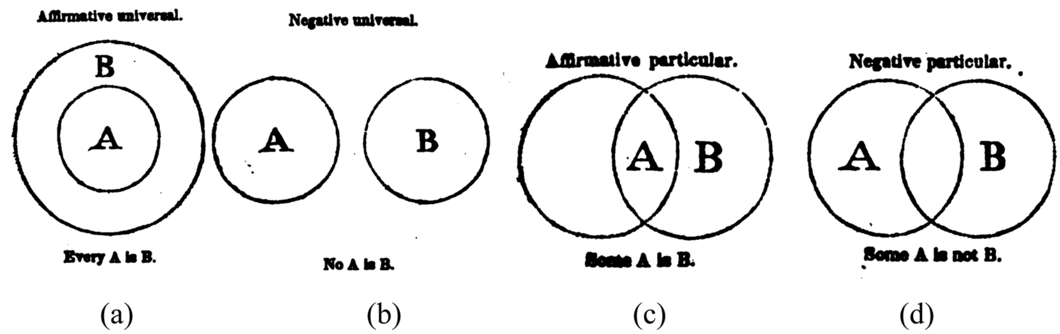

Figure 1: Euler's circle diagrams.

introduced only to facilitate reading and learning. Some modern interpreters point out that Euler does not address the position of the letter anywhere in the text and that the position is even misleading, as it results in errors that can be solved without taking the letter into account.[35]

(2) Euler's diagrams were often interpreted as set diagrams and there are, indeed, some statements in Euler's text that suggests that the circles can represent a set of objects, individuals, or meanings.[36] Considering the context of logic and, especially, the previously presented philosophy of language, this interpretation is unlikely. It would be more reasonable to say that the circles, like the concepts, could be applied to an unspecified and potentially infinite set of elements. For Euler, it is not important whether the idea exists that is represented by the circle at all.[37] Even empty, fictitious, or impossible concepts can be represented by circles. Therefore, the size and extent of the concepts or circles do not matter, only the relative position of at least two circles to each other.

It makes sense to exclude objectual or existential import from Euler's logic altogether (or that it is nonsensical to speak of existential import when referring to Euler). A single circle by itself has no meaningful function in logic other than representing a concept or an idea. What is more important is the position of the circle relative to the other circles in a diagram. This can be understood as a rule or convention that governs the translation of language into geometry. If we have a diagram, as shown in Figure 1a, we can translate the diagram into the logical proposition "All *A* are *B*." Conversely, the logical statement "All *A* are *B*" can be translated into the diagram shown in Figure 1a. The symbols of geometry are there solely to allow simple calculations with logical expressions. In this respect, Euler's diagrammatic logic is just a consistent continuation of his nominalist philosophy of language.

Using the four basic diagrams in Figure 1, syllogisms can now be algorithmically checked for their validity and which conclusions necessarily follow from two premises can be determined. The first method was called the validity test and the second the inference test.[38] Although Euler does not deviate significantly from traditional syllogistics, he develops a precise method where only the translation rules and the algorithm need to be learned. Euler's diagrams are, therefore, a significant advancement over scholastic methods in which syllogisms must be checked using rules of reduction. Since this procedure was very cumbersome, often only the valid syllogisms were taught with the help of mnemonic techniques, and then the syllogisms to be tested were compared with those learned by heart to see if they matched.

Euler's procedure is simple and its method is usually interpreted as a validity test. In this case, there are three judgements, two of which are premises and one of which is the conclusion. First, all possible diagrams for the premises are drawn. Second, a check determines whether the conclusion is shown in all the diagrams.

35 Waszek, "Rigor and the Context-Dependence of Diagrams."

36 Moktefi, "Is Euler's Circle a Symbol or an Icon?"

37 Euler, *Letters*, I, 331.

38 Bernhard, *Euler-Diagramme*.

If every diagram shows the conclusion, then the syllogism is correct. If a diagram correctly displays the premises but not the conclusion, then it is not a valid syllogism.[39]

Euler diagrams have both disadvantages and advantages. One of the disadvantages is that, when using particular judgements, more than one diagram must be constructed to verify the validity of the syllogism. Euler referred to this disadvantage as “uncertainty,”[40] and the literature also uses terms such as “imperfect knowledge” or “indeterminacy.”[41] This can be easily demonstrated with the following syllogism: “Some A are B, and all A are C, therefore some C are B.” In this syllogism, it is uncertain whether B is entirely within C or not. For this reason, one must draw two diagrams, Figure 2(a) and (b). Only if both diagrams also display the same conclusion, is the inference from the premises valid and the syllogism correct.

The advantage of Euler diagrams becomes particularly evident with universal judgements. For all diagrams that do not contain particular judgements, only the circles of the premises need to be drawn into one diagram. If this is done and the syllogism is valid, then the conclusion is already evident in this diagram. This means that one simply reads the conclusion from the diagram that displays the premises. The inference is, thus, information that one receives for free from drawing the premises. This advantage has also been referred to as a “free ride.”[42] We return to this idea again in Section 5.

Regardless of the advantages and disadvantages of Euler diagrams, it should have become clear that this geometric procedure is nothing more than a geometric method to check validity or draw consequences from information given. Euler is never concerned with existence in logic. Objects and existence are terms reserved for epistemology or metaphysics.[43] Furthermore, the geometric procedure does not take into account whether and how the circles refer or can be applied to individuals or existing entities. A circle can just as easily represent Pegasus, a round square cupola, Alexander the Great, or any other concept. The class or concept represented by a circle can affect *all* individuals, *some*, *exactly one*, or *none* but none of this is relevant in the calculation. Whether the concepts relate does not depend on the “quantifier” or the predicate but solely on perception and intuition. It is, therefore, a question of epistemology, not of logic. For Euler, calculation does not depend on the reference of the circles or on the existential commitment of the concepts involved. Only the spatial position of the two circles in relation to each other determines the quantity in the sense of syllogistic judgements and offers a method for checking the validity of syllogisms.

2.2 The Early Reception of Euler

Euler’s philosophical achievements are not widely known today. There are very few studies that deal in depth with Euler’s philosophical treatises and most focus on the critique of his doctrine of monads.[44] The reason that Euler is still rarely perceived as a philosopher today stems from the eighteenth century. Euler’s criticism of Leibniz, Wolff, and Descartes is profound. He wrote polemically against the prevailing philosophical schools of Central Europe and was subsequently ostracised. By the mid-eighteenth century, Euler was depicted in newspapers, poems, pamphlets, and treatises as a kind of calculating machine without spirit.[45] Euler was regarded as a highly gifted mathematician and a competent physicist, but, due to his criticism of dominant rationalism, he was also seen as an idiot who had no understanding of philosophy, culture, and religion. Euler’s philosophy of language and logic was even mocked as a parody or satire by rationalists.

39 Ibid.

40 Euler, *Letters*, I, 352.

41 Moktefi and Lemanski, “On the Origin of Venn Diagrams.”

42 Shimojima, *Semantic Properties of Diagrams and Their Cognitive Potentials*.

43 Euler, *Letters*, I, 306ff., 318ff.

44 For example, Knobloch, “Leonhard Euler als Theoretiker;” Neumann, “Den Monaden das Garaus machen;” Leduc, “Euler et le monadisme.”

45 Fleckenstein, “Vorwort.”

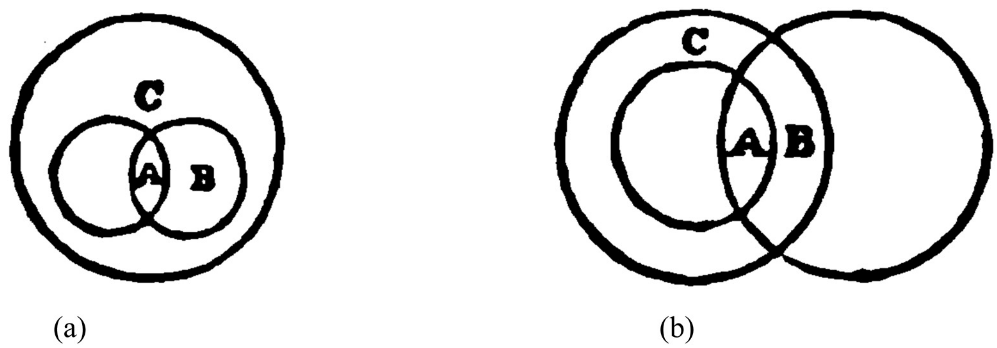

Figure 2: Proving inferences using Euler diagrams.

In both France and Germany, it was quickly decided to cleanse Euler's *Letters to a German Princess* of all philosophical content and only publish the letters on technical issues of optics, astronomy, and physics. Euler's work was, thus, subject to censorship, and it has yet to make a full recovery. The fact that Euler wrote several hundred pages on metaphysics, philosophy of mind, philosophy of language, epistemology, philosophy of religion, and much more is little known today. In the years between 1770 and 1800, when the censored editions of Euler's *Letters* were published, only one well-known philosopher popularised Euler's logic, Immanuel Kant.[46]

In his logic lectures from the 1770s onwards, Kant explicitly referred to Euler and adapted his method. In the 1780s and 1790s, numerous logic diagrams can be found in the manuscripts of Kant's lectures. However, Kant not only adopted Euler's method but he also tried to establish other forms of visual reasoning, for example, to make Eulerian diagrams usable in Stoic logic. Some of these diagrams from Kant's manuscripts were included in the so-called *Jäsche Logic* published in 1800.[47] This textbook became the paradigmatic standard work on logic in the nineteenth century until the works of Boole in England and Schröder and Frege in the German-speaking world. The extent to which Euler's nominalist philosophy of language is reflected in Kant has not been investigated in the current literature. So let us look out for nominalist theses in books after Kant that also feature Euler diagrams.

It was mainly Kant's students and the first generation of Kant-influenced authors who taught Euler's diagrams. From as early as 1790, Kiesewetter taught Kantian logic in Halle. In his logic textbook, we find the first Euler diagrams since the first German edition of the *Letters to a German Princess*. For Kiesewetter, it is certain that "all our concepts must be general by their nature."[48] Other students of Kant, such as Friedrich August Nitsch in England and Georg Samuel Albert Mellin in Germany, also published works containing Euler diagrams. It can be said that by the publication year of the *Jäsche Logic* in 1800, the Eulerian method had already been widely disseminated by Kantians.[49]

A milestone for the early Kantians is the logic of Karl Christian Friedrich Krause, which, in 1803, not only briefly repeated Euler's validity test but also introduced several new diagrammatic methods. Kant's critics, such as Johann Gebhard Ehrenreich Maaß, also tried to improve Euler diagrams, sometimes just to demonstrate that Kant was wrong in his forays into logic. Gottlob Ernst Schulze, Wilhelm Traugott Krug, and Jakob Friedrich Fries also used multiple Euler diagrams to familiarise the technique for educational purposes. Despite many good ideas, no real progress on Euler's visual reasoning can be found in any of the treatises. More importantly, none of the treatises address the topic of existence in any way that could be relevant to today's debate. In § 29 of his *Logic*, Krug intensely discusses why concepts are always general and also gives some passages from the *Jäsche Logic* that are similar to Euler's nominalism.

46 Lemanski, "Kant's Crucial Contribution to Euler Diagrams."
47 Kant, *Gesammelte Schriften* IX.
48 Kiesewetter, *Grundriss einer allgemeinen Logik*, 75, also 22.
49 Lemanski, "Kant's Crucial Contribution to Euler Diagrams."

Progress in visual reasoning and especially Eulerian diagrams started at the end of the 1810s. Schopenhauer and Krause were pivotal in the subsequent developments in logic and mathematics.[50] Both studied mathematics, came into contact with Euler diagrams early, and lived together in the same house in Dresden for several years. Krause's interest in the diagrams is combinatorial, while Schopenhauer's interest is geometric and driven by philosophical questions about the foundations of logic and mathematics.

Schopenhauer published some of the results of his work with Krause in 1819 in his main work *The World as Will and Representation* and then significantly expanded the digression on logic in his lectures around 1820. Krause began publishing his findings on logic from 1825. Both works are characterised by an intense engagement with Euler diagrams and Euler-similar diagrams. While Schopenhauer's ideas were received in mathematics by Friedrich Adolph Wilhelm Diesterweg in the early 1820s and then by Karl Friedrich Hauber and influenced the philosophical interpretations of Ignaz Denzinger, Carl Friedrich Bachmann, Moritz Wilhelm Drobisch, Bernhard Bolzano, and many others directly or indirectly, Krause's approaches only became known through his students in Germany and Spain in the 1830s and, especially, the 1840s.

The strength of Euler's nominalism during this period can be clearly seen in Schopenhauer's logic lectures. Schopenhauer, like Euler and the many previously mentioned Kantians, strictly differentiates intuition from concept. For Schopenhauer, intuitions or representations are determined by space, time, and causality. Here, we find individuals and existing things. However, language and logic are abstractions from intuition. Concepts are representations of representation and, therefore, always abstract and never individual or concrete. This applies to genera, species, and even proper names. As a strict nominalist, Schopenhauer thus denies not only that a concept is individual or concrete but also that it can refer to a singular referent. After all, only objects in the sensually given world are individual, but not language. Schopenhauer writes:

> I maintain … that the judgment is exclusively an operation of thought, not of intuition, and therefore remains exclusively in the domain of abstract concepts, not of individual things, and that, finally, a concept is always general, even if there is only a single thing that is thought by it, only one single intuition that gives it content or is a proof of it. My concept of this lectern is never this lectern itself: it remains an abstract, a universal. The concept never descends to the individual, to intuition; and in the judgment: 'Socrates is a philosopher,' one could very well think of more people, different in shape, size and other properties, who would nevertheless correspond to the concept of Socrates.[51]

Similar to Euler, Schopenhauer's nominalism asserts that language is detached from existential reality, which is received by us as intuition. For Schopenhauer, proper names also do not refer to a single object. In this sense, quantifying judgements, which are supposed to refer to *one*, *some* or *all* objects, are only relative to each other. They do not reflect the real conditions of reality. This is largely in line with Krug's views, which are also based on Kant. Euler's nominalism is, thus, reflected in Schopenhauer's philosophy of language, which directly or indirectly influenced the logic of the 1820s and 1830s.

3 Changes in the Nineteenth Century

By the time of the onset of Kantianism at the latest, Euler's logic, dressed in the guise of Kantian philosophy, dominated for almost 100 years. Nevertheless, changes already began to emerge from the 1820s onwards, bringing the concept of existence closer and closer to logic. In Subsection 3.1, we demonstrate that, while Euler provided the most influential interpretation of the diagrams in this period, there were different views on visual reasoning among rationalists. The merging of these interpretations with the Euler diagrams and the application of the Euler diagrams in semantics (Subsection 3.2) ultimately led to a re-evaluation of visual reasoning, individuals, and existence.

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50 Meixner, "Krause;" Moktefi, "Schopenhauer's Eulerian Diagrams."

51 Schopenhauer, *Vorlesung*, 249.

3.1 Uncertainty, Objectual, and Existential Commitments

An alternative design to Euler's logic diagrams can be found in Johann Heinrich Lambert. As Lambert emphasises in his preface, it was his idea to connect the results of British empiricism with the German rationalists. In particular, he popularised line diagrams in logic with his *Neues Organon* (*New Organon*) from 1764. These diagrams do not consist of circles that contain or do not contain each other but lines and points that can be arranged above or next to each other in order to visually represent propositions of Aristotelian and, to some extent, Stoic logic.

> Every general concept extends to all individuals in which it occurs. It therefore has a specific scope. If we imagine all these individuals in a row or line, the length of this line will figuratively represent the scope of the general concept.[52]

The individuals are represented by points since, for Lambert, both the points and the individuals have no extension.[53] Only the composition of several points results in a line. This line represents an accumulation of points just as the general concept represents the accumulation of individuals it designates. Similar to Euler, there are now four basic diagrams (in Figure 3) that, analogously to Figure 1, are intended to depict the four basic forms of judgement.

Two crucial differences from Euler become apparent here, one concerning logic and the other existence. From a logical perspective, Euler and Lambert's diagrams differ in that they express the indeterminacy and uncertainty in the diagrams differently. While Euler introduces indeterminacy only at the level of inference and then demands multiple diagrams for particular judgements in a syllogism, Lambert already indicates the indeterminacy at the propositional level using points. Lambert's notion of uncertainty is concerned first with the exact number of individuals that fall under the concept in question, whereas Euler is concerned with the possible cases of circle positions that can make the premises true. Lambert is of course also concerned with Euler's problem, but not the other way around, because Euler's diagrams are not supposed to represent individuals. Regarding Figure 3(c), Lambert writes the following.

> For since we retain only a letter without a line for the concept A, this indicates that we only know of some, and perhaps only of one individual A, that it is B. The points indicate the indeterminate.[54]

From this quote, we also see that Lambert presupposes an existential commitment. On the one hand, Lambert flirts with nominalism, since the individuals can be "numerically as infinite as the line."[55] A precise specification of the individuals is, therefore, omitted. Instead, the line serves as the principal form of representation. On the other hand, a realistic component is already being introduced. For example, the diagram depicted in Figure 3(c) assumes the existence of at least one individual designated as a member of the class A and that there is possibly even an infinite number of individuals designated as such. The exact specification of individuals is, in any case, indeterminate. Nevertheless, the existence or non-existence of specific individuals is assumed. Thus, the uncertainty in Lambert's logic obliges an existential assumption. However, this is also because Lambert regards the line as a compound of points.

We do not yet find the expression of "existence" in Lambert. Therefore, the exact assertion of existence, as outlined in Section 1, is not yet given here in the field of visual reasoning with the interpretation of $\exists x\ Ax$. Nevertheless, the phrase "perhaps only one individual" in this quote already points to such an interpretation. After all, Lambert is not saying that it is "perhaps only one part of line A" but that it is one "individual." This interprets the smallest part of the line as an individual and reinterprets the quantity "some" as a class term with "at least one." In fact, Lambert explicitly defends this reading against his rival Gottfried Ploucquet.[56]

52 Lambert, *Neues Organon*, 110.
53 Ibid.
54 Ibid., 114.
55 Ibid., 110.
56 Bök, *Sammlung der Schriften*, 207–14.

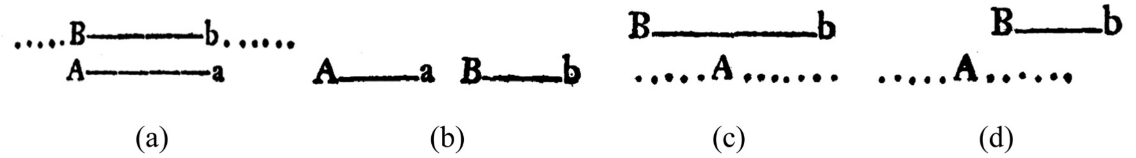


Figure 3: Lambert's line diagrams.

Euler would reject this perspective on geometric figures, considering it a metaphysical point of view of the rationalist doctrine of monads that deviates from geometry. Euler expounds on this point of view in various writings, such as his *Letters to a German Princess*.[57] For Euler, a line is not a collection of points nor does it contain any points as points lack extension. The constituents of a line consist of lines that extend infinitely. Euler, for example, uses the intercept theorem to prove this.[58] In addition to the aforementioned text passages of the *Letters*, Euler's beginning of *Anleitung zur Naturlehre* (*An Introduction to Natural Science*) and the third chapter of *Institutiones Calculi Differentialis* (*Foundations of Differential Calculus*) are particularly significant for this argument.

Euler also applies his doctrine of infinite division to the philosophy of language and logic, for the general concept is the extension of less general concepts, although the general concept is never the extension of individual things. Euler presents numerous arguments for this perspective and, in his *Letters*, he even accuses the rationalists of inventing logicism, which refers to the foundation of mathematics on logical laws of thought. He makes this accusation on the grounds they are aware that their application of mathematics to philosophy and logic reveals a deficiency in their mathematical understanding. One can hardly find a more severe critique of both Leibniz and Wolff.

Lambert's diagrams were positively received by empirically oriented philosophers, such as Aloyisius Havichhorst. Kantians like Mellin, in particular, contrasted Lambert's diagrams with those of Euler even though Euler's diagrams soon became prevalent. With the triumph of logic diagrams in Kantianism, Lambert's diagrams were championed by rationalists including Wilhelm Ludwig Georg von Eberstein and Maaß and contrasted to Euler's method.

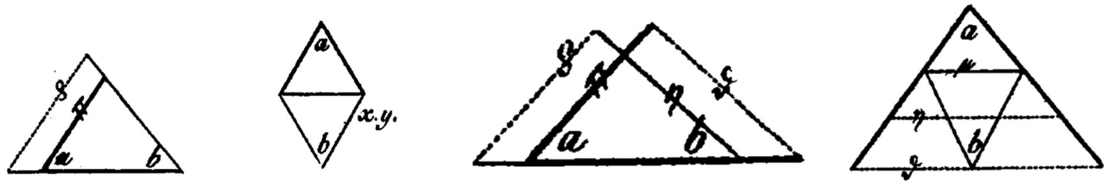
Maaß deserves special attention, as he essentially rejected the visual method in favour of logicism. Yet he tried to explain to the Kantians why one must at least apply Lambert's method to Euler's diagrams.[59] "Euler's is not usable," he writes, as it cannot represent the metaphor of the subordination of concepts.[60] Metaphorical expressions like "*A* is under *B*" can be represented by Figure 3(a) but not by Figure 1(a). Euler's method would only visualise the metaphor of containment. However, one can combine both metaphors by using triangles whose area corresponds to Figure 1 and whose line arrangement corresponds to Figure 3. The combination of containment and subordination yields the four diagrams for the fundamental evaluations presented in Figure 4.

According to Maaß, the area of the triangles is supposed to represent the extension of the concept, while the dotted sides indicate logical indeterminacy or uncertainty, which, again, existentially represent a set of individuals.[61] The Greek terms in Figure 4(a, c, and d) are therefore the possible extensions of *a* or *b*. Maaß explains it as follows:

> Furthermore, if the legs of the angle are extended and bounded by a series of points …, it means: only a certain set of objects is determined to belong to the sphere of the concept …, but it might be that some more objects belong to it, and it is also possible that it is not the case.[62]

Maaß introduced many interesting innovations to logic, especially with his diagrams. However, he does not use the diagrams to test the validity of syllogisms, rather to check invalid inferences according to the

57 Euler, *Letters* I, 304ff., II, 33ff.
58 Euler, *Letters* II, 33.
59 Bernhard, "The Remarkable Diagrams of Johann Maass."
60 Maaß, *Grundriß der Logik*, IX.
61 Ibid., 329ff.
62 Ibid., 321.



(a) (b) (c) (d)

Figure 4: Maaß's triangle diagrams.

Aristotelian-scholastic rules of syllogism. What is striking in the quote, however, is that he is dealing here with objectual commitment ("set of objects") and that these objects refer to individuals.

To the best of our current knowledge, Schopenhauer was the first to apply the Lambert–Maaß technique of dotted lines to Eulerian circle diagrams around 1820 to verify the validity of syllogisms.[63] Through this procedure, a harmonisation between Euler and Lambert emerged. As seen in Section 2.2, Schopenhauer supports Euler's nominalistic interpretation of the concepts and diagrams. However, he recognises that Lambert's and Maaß's point lines offer a visual advantage over Euler's diagrams. Before, one had to draw multiple Euler diagrams for each indeterminate judgment (Figure 2) and compare all of them to verify the inference. In contrast, the point lines now allow various possibilities to be drawn within a single Euler diagram.

In Figure 5, we see how Schopenhauer uses dotted lines to express the uncertainty and indeterminacy of particular judgments in Festino.[64] Thus, Schopenhauer leveraged the visual advantage of Lambert's diagrams but inadvertently also adopted the existential interpretation in which points represent individuals and, thus, express at least one existence. In other instances, Schopenhauer forgoes the dotted line, indicating its indeterminacy with semicircles that end on the circle of a larger circle (like the minor in the medius of Figure 5).

From the 1840s onwards, these and some other methods became established, so that by the mid-nineteenth century, Ueberweg had provided a list of ways to represent uncertainty or indeterminacy.[65] Ueberweg's relatively popular textbook, which appeared in many editions in both German and English, partially adopted Euler's phrasing and partially adopted Lambert's interpretation. More in line with Euler, he reads Figure 1c, for example, as "At least a part of the circle *A* is *B*."[66] However, over the course of the book, he abbreviates this phrasing and writes more in Lambert's sense that "Some *A* are *B*," which means nothing other than "At least one *A* is *B*"[67]. To our knowledge, Ueberweg was the first to apply Lambert's realistic reading of line diagrams to Eulerian circle diagrams. As shown in Section 4, this reading of the Euler diagrams would have been influenced by Boole's definition of the class concept.

While Euler's reading neutrally describes the region of the circle, Lambert's realistic interpretation suggests that *A* represents an extension as a set of individuals who are *B*. "Some" means "at least one individual." This Lambertian interpretation continued to intensify until the end of the nineteenth century. However, as shown in Subsection 3.2, there is at least one other reason for suggesting a realistic interpretation of the diagrams with individuals and existential import.

3.2 Individuals with and in Euler Diagrams

The application of Euler diagrams to fields outside of logic in the nineteenth century has, to this day, been the subject of minimal research. However, one possible application for employing Euler diagrams in other

63 Moktefi, "Schopenhauer's Eulerian Diagrams."
64 Schopenhauer, *Vorlesung*, 287.
65 Moktefi and Lemanski, "On the Origin of Venn Diagrams;" Moktefi et al., "Representing Uncertainty with Expanded Ueberweg Diagrams."
66 Ueberweg, *System of Logic*, 377.
67 Ibid., 217.

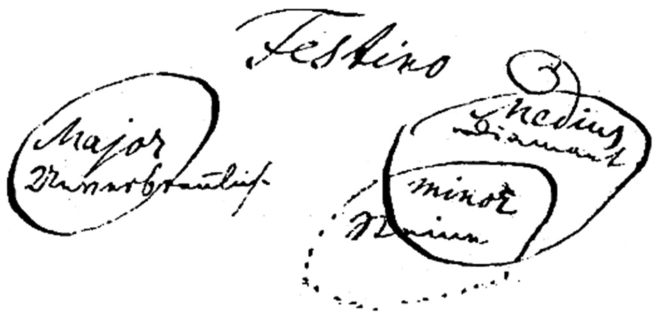


Figure 5: Schopenhauer's Festino diagram.

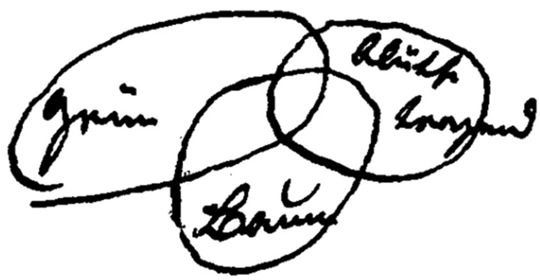


Figure 6: Schopenhauer's three circle diagram. https://sammlungen.ub.uni-frankfurt.de/schopenhauer/content/zoom/7198714.

academic domains is known from Schopenhauer's research and the work of the logicians and mathematicians he inspired, for example, Diesterweg, Denzinger, and Bachmann, who used Euler diagrams in areas such as semantics and ontology. For example, they used them to visualise equivalences in translations, analyse linguistic developments, and determine objects based on their properties.

In Schopenhauer's work, for instance, we find Euler diagrams that are used to determine objects by their characteristics. For example, we encounter multiple three-circle diagrams in his writings that describe linguistic developments, classifying objects that arise through the combination or elimination of properties.

Figure 6 depicts three concepts: tree, green, and flowering. Although Schopenhauer clearly holds a nominalistic stance, maintaining that concepts are always general while intuitions are specific and individual, he acknowledges that Figure 6 can be interpreted as showing that existing objects can be classified by the compartments created by the overlapping of the circles.[68] A cherry tree in spring, for example, might be represented by the compartment where the circles "tree" and "flowering" intersect, but not "green." An apple tree in early autumn, on the other hand, is probably green and a tree but is no longer flowering. A pear tree in winter is still a tree but probably no longer green and not yet flowering.

In some instances, Schopenhauer even blends the realistic interpretation with the nominalistic one. This mixing stems from his understanding of the concept as a metaphor, which he explains in analogy to comprehensible actions in perception. The concept grasps meanings just as the hand grasps objects. He states this due to the peculiar analogy between intuition or perception and the concept that even concepts can have meaning and language can bring about action.[69]

Euler diagrams now form a kind of connection between concept and perception. As shown in Figure 6, Euler diagrams can be used to classify objects from intuition and perception using properties. Schopenhauer never explicitly says it, but one could assume that he would have either viewed existence and non-existence as properties of concepts or used Figure 6 to suggest that he saw compartments as empty. In the first case, for

68 Schopenhauer, *Vorlesung*, 227.
69 Ibid., 226.

instance, in Figure 6, he could have replaced “flowering” with the property “non-existent” to denote a fictitious green tree. In the second case, he might also have stated that the compartment for “flowering” would be empty if he made that statement in winter. However, these interpretations introduce an entirely different dimension than the purely formal computability that Euler had in mind. With Schopenhauer’s metaphorical reading, he takes a step towards a realistic interpretation of Euler diagrams, as individuals with different properties can now be presented as contained in compartments, just as real objects are grasped by hand.

In 1828, Bachmann, who knew Schopenhauer from his main work and probably also personally, used also a metaphorical reading for logic and interpreted the areas of the diagrams such that there are objects encompassed by the respective diagram compartments. While Schopenhauer argues that a concept contains meanings in the way a container or hand encompasses objects, for Bachmann, it is the circles or “spheres” that directly encompass the objects. Schopenhauer’s understanding of Euler diagrams is, therefore, metaphorical or analogous, while Bachmann’s realistic interpretation has existential commitments in Figure 7. These objects are denoted by lowercase letters, while the uppercase letters represent general concepts.

> Some objects of sphere *A* do not lie in sphere *B*, which is different from *A*, namely *x*, *y*, etc., so it is clear that other parts of *A*, [i.e. *a*, *b*] lie in sphere *B*. Conversely, certain parts of *B*, = *b*, lie in the sphere *A*, and *a* = *b*, for the parts of *A* which lie in *B* are exactly the same parts of *B* which lie in *A*. Likewise there are other parts that do not lie in *A*, such as *z*, *u*, *v* and others.[70]

Bachmann’s quote represents one of the first realistic interpretations of Euler diagrams from the perspective of existential import. Unlike Ueberweg, however, Bachmann does not yet translate “some” as “at least one.” For Bachmann, circles are concepts like *A* and *B*, while objects are denoted by letters such as x, y, *b*, etc. Thus, concepts contain individuals in the same way that circles contain specific letters. From Bachmann’s perspective, every compartment of an Euler diagram represents objectual commitment. Each enclosed area contains a set of objects that can be expressed through letters. Consequently, Bachmann is the first to implicitly hold a view on at least one particular diagram, Figure 7, in which existence is presumed in every compartment. However, as Bachmann otherwise describes Euler diagrams without explicit reference to existential import, one should not attribute too much historical significance to Figure 7. Nevertheless, it is not as if Schopenhauer’s and Bachmann’s interpretations had no impact. Similar views, according to which Euler diagrams are populated with objects or individuals, as shown in Figure 7, can be found from the 1830s in the works of Bolzano, Matthias Amos Drbal, Wilhelm Kaulich, and others.[71]

When Bachmann’s Figure 7 is compared to Lambert and Euler, the difference becomes evident quickly. One might be inclined to say that Bachmann assumes existence everywhere, whereas Euler primarily uses the diagrams to denote non-existence. However, this is incorrect, the question of existence and non-existence in diagrams does not even arise for Euler. For Euler, the issue of existence or non-existence relates to sensory perception, not logic. Thus, Euler uses the geometric forms only in logic, not when considering questions of semantics or ontology as Schopenhauer does. One could argue that Euler’s diagrams are also sensory, and for this reason, non-existence dominates Euler’s work because no individuals are drawn into the compartments. Yet even on this, Euler would likely respond that while logic diagrams can be visually perceived, their rules only concern the relative position of the circles to one another and existence and non-existence play no role here. As shown in Section 5, recent research on Euler diagrams has proven this view.

In contrast to Lambert, it is evident that Bachmann has moved the individuals from the line to the plane or disk. X, y, *b*, etc. are not located on the circle line but within the circle’s disk, which is bounded by the circle line. This was implicitly addressed in Schopenhauer’s semantic diagrams but was made explicit by Bachmann. Bachmann’s perspective opens up an option for all who followed that was not available for Lambert. The area inside the circle becomes a space where specific individuals, such as x, y, a, can exist, and a space of possibilities where individuals or objects, such as p or q, could have existed. Bachmann does not directly point to this interpretative possibility but, indirectly, an interpreter of Figure 7 should be able to see, quite

70 Bachmann, *System der Logik*, 175.

71 Bolzano, *Wissenschaftslehre*, 450; Drbal, *Lehrbuch der propädeutischen Logik*, 66; Kaulich, *Handbuch der Logik*, 58.

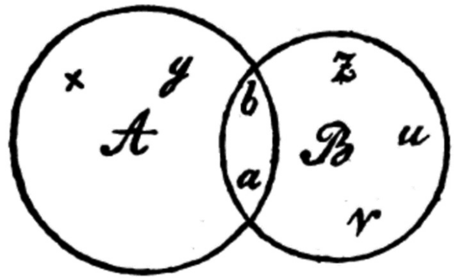


Figure 7: Bachmann's Euler diagrams including objects.

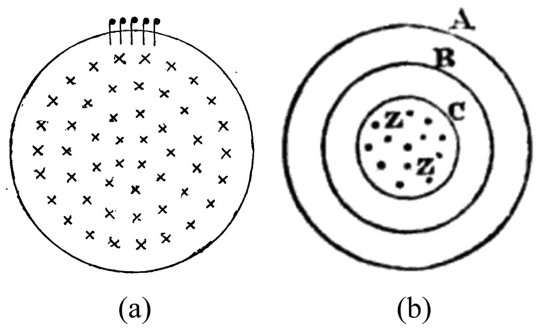


Figure 8: (a) Karslake's and (b) Shedden's Euler diagrams including individual points.

clearly, that we are aware of the existence of seven individuals and an indeterminate number of individuals that do not exist.

The realistic interpretation of the Euler diagrams in the sense of Bachmann gained traction in the German world first and, then, in the English-speaking world. Good examples can be found in William Henry Karslake's *Aids to the Study of Logic* from 1851 and in Thomas Shedden's *Elements of Logic* from 1864. Karslake illustrates the process of generalisation diagrammatically in Figure 8(a). He writes next to the diagram that "crosses representing the indefinite number of Individuals."[72] The circle, on the other hand, shows what Karslake defines as a "class idea." This is created by generalising the indefinite number of individuals into the class idea by means of abstraction. This abstraction is made from the individual attributes of these individuals.[73] In Figure 8(b), Shedden uses points, some of which are marked with a Z, to illustrate that individuals are supposed to be located everywhere within the circles.[74] Ernst Schroeder, in his lectures on the algebra of logic, even used the common term "sphere" synonymously with the concept of "point region." In his opinion, this was Euler's method.[75] Yet, Euler would probably have accused Shedden of not understanding geometry and of having fallen victim to the non-sensical doctrine of monads.

4 Realism

Paraphrases of assertoric judgements in syllogistic, in which existential commitments occur, can occasionally be found even before the second half of the nineteenth century. A prominent example of this is found in the

72 Karslake, *Aids to the Study of Logic*, 120.
73 Ibid., 33.
74 Shedden, *Elements of Logic*, 53.
75 Schröder, *Algebra der Logik*, 156.

work of Hermann Lotze, who formulated in 1843, “There is no *S* to which *P* would not belong.”[76] However, one does not find a conscious or even justified decision for using expressions, such as “there is…,” given by logicians of that time. In this respect, one should not overestimate such formulations. Only Leibniz had already made a conscious decision in favour of the concept of existence in syllogistics. Despite being published in the mid-eighteenth century, Leibniz’s text gained renewed attention through Venn.[77]

From the middle of the nineteenth century and especially with the introduction of Boolean logic, a way of speaking set in through which existence and many related metaphysical concepts found their way into logic. Boole’s interpretation of the concept of class in *An Investigation of the Laws of Thought* of 1854, for example, is famous. While a class used to be regarded as a collection of individuals, according to Boole a class is already designated if even a single individual exists in it. If it means all individuals, the concept of “universe” is applied and, if there is no individual, it represents the concept of “nothing.”[78] These ways of speaking soon became established in France and Germany through many studies such as the influential *Les logiciens anglais contemporains* by Louis Liard, which was translated into many European languages.

As early as the 1860s, in his well-known textbook of formal logic, which was published in several editions, Gustav Adolf Lindner reflected that there were at that time, as in the Middle Ages, two extreme views in logic. The actual view of the nominalists, for whom concepts, names and classes had no relation to reality and the realists, who vouched for the existence of concepts, names and classes using the existence of individuals.[79] It is not surprising, then, that Lindner is here revisiting the medieval debate on the problem of universals in the age of visual reasoning. The definition of a class with at least one individual, already indicated by Lambert and consistently implemented by Boole, no longer corresponded to traditional logic. If we use Porphyry as a reference point, then in traditional logic, there are always several individuals in a class (species infima, species specialissima) and the question of whether these individuals or the class exist or not is transferred from logic to metaphysics.[80] However, by the time of Boole’s realistic definition at the latest, existence had become a decision criterion in logic. It should be noted, of course, that some logicians do accept existential assumptions to test the validity of traditional logic, but later replace these assumptions with hypothetical judgements to avoid realism.[81]

We must now take a closer look at the position of the realists in the field of visual reasoning. The interpretation of the English logicians and existential imports have, at this time, been well researched, though certainly not fully. Similarly, some studies have shown how the Boolean interpretation of Euler’s diagrams favoured the development of the well-known Venn diagrams.[82] However, existence is a concept that was problematised, mainly, in early twentieth-century logic according to Alexius Meinong’s interpretation. The following, therefore, focuses primarily on Venn’s *Symbolic Logic* and on Meinong’s *Logic Lectures*, as we take a closer look at existential import.

4.1 Venn Diagrams

Today, John Venn is particularly well known today for his diagrams, which were later named after him. His symbolic or mathematical logic is primarily based on Boole and his successors.[83] Venn perceives diagrams as tools and, as discussed in the introduction, is aware that nearly all logicians of his time used this form of representation. He, therefore, dedicated a lot of attention to diagrams and also emphasised that, in symbolic logic, it does not matter if they are represented in letters or diagrams.[84] However, it is apparent from Venn’s book that he differentiated diagrammatic logic from symbolic logic in various sections.

76 Lotze, *Logik*, 143.
77 Yi, *Venn and Existential Import*.
78 Boole, *An Investigation of the Laws of Thought*, 28.
79 Lindner, *Lehrbuch der formalen Logik*, 21.
80 Barnes, *Porphyry’s Introduction*, 3, 37ff., 100ff.
81 E.g. Schröder, *Vorlesungen über die Algebra der Logik,* II, § 44.
82 Moktefi and Lemanski, “On the Origin of Venn Diagrams.”
83 Venn, *Symbolic Logic*, xvi ff.

Venn begins the introduction of his circle diagrams by searching for the appropriate combinatorics. Between n terms, the corresponding number of potential combinations of these terms is intended to be represented by intersecting circles with 2^n compartments. Thus, every diagram for n terms has the same number of compartments, which is why Venn refers to them as "primary diagrams." In this process, Venn realises that he starts with diagrams that resemble Euler's circles. Figure 1c and d depict the same circular positions in Euclidean space and have four compartments for two terms, namely, AB, $\overline{AB}$, $\bar{A}B$, and $A\bar{B}$, where the line above the letter indicates that this term is negated.

Venn now demonstrates that, even when illustrating eight compartments for three terms, a figure emerges that can also be found in Euler diagrams, specifically Figure 6. As this diagram is quite frequently found in literature, Venn also refers to it as a "three-circle diagram." In both cases, the primary diagrams for two or three terms now look like Euler diagrams. However, in contrast to Euler, the region outside of the circles is also considered a compartment. Furthermore, due to the combinatorial method, not every Euler diagram can be a Venn diagram. Figure 1a and b, for instance, show three compartments at most: the one containing the letter A, the one containing letter B, and then the area outside of A and B. Figure 1a and b can, therefore, never be interpreted as a Venn diagram.

Unlike many of his predecessors who also used diagrams for combinatorial purposes, Venn then sought to represent propositions with his primary diagrams. He did this by shading individual compartments. As with Bachmann's diagrams, all compartments within Venn's circles have existential import. The shading nullifies existence. Venn himself does not differentiate between existence and non-existence in the compartments but between occupation and non-occupation or emptiness.[85] If a compartment is shaded, it is not occupied with individuals but is empty. Otherwise, as with Boole, there is at least one individual in every non-shaded compartment or class.

With the help of the shading, propositions based on Aristotelian and Stoic logic could now be represented, that is, class logic (or term logic) and propositional logic. Let us take the compartment of Figure 1c, for example, containing A. If this compartment containing A was shaded, it would mean "No A is B" in Aristotelian logic. Now let us take the compartment of Figure 1d, for instance, containing A. If this compartment containing A was shaded, it would mean "All A are B."

Given this technique, in which existential import plays a significant role, it is also evident that Venn developed a realistic view of Euler diagrams that he hinted at in a footnote in his *Symbolic Logic*.[86] For Venn, Figure 1a merely illustrates that the compartment in Figure 1d containing A does not exist. In contrast, Figure 1b indicates that the compartment in Figure 1c containing A does not exist. As described above, Figure 1a and b only have three compartments, not four. The non-existence of a compartment in Euler's view is, thus, for Venn, synonymous with the non-existence of individuals in that compartment. In this manner, Venn transforms Euler's nominalism, which avoids any existential commitment, into a realism in which existential import is the primary criterion for understanding the technique of visual reasoning. This intensifies Bachmann's perspective by also explicating non-existence and ensuring the combinatorics are exhaustive. Venn is, thus, a good example of how the shift from strong nominalism to strong realism took place in visual reasoning in the course of the nineteenth century.

4.2 Höfler's and Meinong's Concept of Existence

A second approach in which Euler diagrams were interpreted realistically at the end of the nineteenth century can be found in the logic of Alois Höfler and Alexius Meinong. Many clues on the title page and in the introduction to *Logic* suggest that Höfler contributed more to this textbook than Meinong. However, it can be surmised that much content in this textbook can be traced back to Meinong, Franz Brentano, and Brentano's circle.

84 Ibid., 101.
85 Ibid., 23, 110.
86 Ibid., 118.

Meinong/Höfler begin their search for fundamental logic diagrams using a method reminiscent of what is now sometimes referred to as Question-Answer Semantics.[87] This method assumes two concepts. Meinong/Höfler use S and P but, for consistency within this analysis, we take A as the subject term and B as the object term again. Next, we pose four questions to clarify the relationship between A and B. All four questions demonstrate that Meinong had the distinction between existence and non-existence in mind, two attributes which he later reflects upon more precisely in *Logic*.

The four questions are as follows: (1) Are there A which are B? (2) Are there A which are not B? (3) Are there B which are A? (4) Are there B which are not A? [88] If we assume two possible answers to the questions, such as "yes" and "no" or 1 and 0, the following possibilities arise: If there is at least one individual that is both A and B, then question (1) is answered with a 'yes' or '1'. If there is an individual that is A but not B, then question (2) is also answered with a "yes" or "1." If such individuals do not exist, then both questions are answered with a "no" or "0." Based on this, sixteen relationship forms emerge that are meant to help determine the number of fundamental logic diagrams.

Thus, Meinong/Höfler adopted the Boolean class concept and its understanding of existence and non-existence. However, they made a mistake. While questions (1) and (3) may be phrased in different ways, they pertain to the same individuals. If there is at least one individual that is both A and B, then there is also at least one individual that is B and A. This would have become clear by considering the commutative law or illustrating it in a Venn diagram for two terms. Let us assume that Figure 1c and d display Venn diagrams. Questions (1) and (3) then refer to the compartment containing A in Figure 1c. Question (2) refers to the compartment containing A in Figure 1d. Question 4 refers to the compartment containing B in either Figure 1c or Figure 1d. So, instead of showing the four possible combinations for two terms, i.e., AB, $A\bar{B}$, $\overline{AB}$, $\bar{A}B$, and he shows only three because it repeats one of the possible combinations, i.e., AB, $A\bar{B}$, BA, $B\bar{A}$.

In doing so, Meinong/Höfler pose the same question twice. Yet they believe there are sixteen possible answers to his four questions. Unconsciously, they compensate for this mistake when they notice that it is contradictory to give different answers to questions (1) and (3). As such, they eliminated eight of the sixteen combination possibilities. Of the remaining eight combination possibilities, three more are eliminated when questions (1) and (2) or questions (3) and (4) are answered negatively. This leaves five possibilities that correspond to the four Euler diagrams and a diagram indicating the identity of two circles. These five diagrams are now referred to as "Gergonne relations" and were already known in the German tradition through Schopenhauer.[89] If we now take Meinong/Höfler's answers and use "1" for "yes" and "0" for "no," the four questions and the five correct answers become a bitstring semantic:[90] (I) 1011, (II) 1010, (III) 1110, (IV) 1111, and (V) 0101. Bitstring (I) corresponds to Figure 1a, (II) corresponds to the identity of two circles, (III) is Figure 1a with A and B swapped, (IV) corresponds to Figure 1c or Figure 1d, and (V) corresponds to Figure 1b.

Meinong/Höfler motivation for posing the four questions can be understood through their explanation of the four categorical judgements of syllogistics, which utilise the concept of existence.[91] Meinong/Höfler build on Brentano's approach, which simplifies the four forms of judgement in syllogistics into two modes using existence. Specifically, particular judgements are expressed as judgements like "There exists an A that is …," whereas universal judgements take the form of "There no A exists that is…." This corresponds roughly to the modern understanding of $\exists x\ Ax$ and $\neg\exists x\ Ax$. Furthermore, the predicate B is affirmed for i- and e-judgements ("… that is B," Bx) and negated for o- and a-judgements ("… that is not B," $\neg Bx$).

Meinong/Höfler take a different approach than Venn does. However, Venn first constructed combination possibilities based on the diagram and then applied existence or non-existence to the primary diagram, Meinong/Höfler started with existence and attempted to find the primary diagrams by asking about the

87 Schang, "Question-Answer Semantics."
88 Höfler and Meinong, *Logik*, 37ff.
89 Moktefi, "Schopenhauer's Eulerian Diagrams."
90 Demey and Smessaert, "Combinatorial Bitstring Semantics for Arbitrary Logical Fragments."
91 Höfler and Meinong, *Logik*, 108ff.

existence of specific individuals. As they formulated the question incorrectly, they arrived, yet again, at a realistic Euler interpretation in the sense of Bachmann. They failed to see that the region outside the circles can have meaning. This might have become clear if Meinong/Höfler's question (3) had asked whether there is something that is neither A nor B or $\overline{AB}$. This logical–combinatorial error was not corrected in later editions possibly because it is hard to imagine working with individuals or presupposing an existence that is not included or subsumed under one of the two classes in question, i.e., is neither A nor B.

However, Meinong/Höfler knew that, in Euler-similar diagrams, the region outside a circle represents the negation of at least the circle itself, as they explained in detail.[92] Here, however, one sees the limits of Meinong/Höfler's realism for they do not interpret the region outside a circle as a class occupied by at least one individual but as an infinite concept in the sense of Kant. Thus, the concept not-A, which lies outside the circle A, does not describe a class with at least one individual but an infinity of individuals. This shows how Meinong/Höfler contains a blending of nominalistic and realistic views in the area of visual reasoning at certain points.

5 Current Diagrammatic Systems

It is not surprising that Venn's approach has come to be considered a contrast to Meinong/Höfler's. After all, Venn avoided such mistakes as found in Meinong/Höfler and mix-ups and clearly distinguished between the primary diagram as a combinatorial method and the diagrammatic elements in the form of shading. However, Venn's diagrams are still trapped in the Bachmann perspective. If a compartment is not shaded, then there is automatically an objectual or existential import in this compartment. Existential commitments, thus, arise when they are not explicitly excluded. The result of Bachmann's perspective is that Venn did not represent particular judgements in syllogistics. Even though he already had a sense of how this might be done, overall, he was not satisfied with this method.[93]

In contrast to Bachmann and other precursors, Charles Sanders Peirce and Lewis Carroll established three states of the primary diagram. The primary diagram essentially shows nothing, and it only divides the universe and the classes within it. The primary diagram, therefore, has no existential commitment or is fundamentally free from existential import. In addition to using shading, a minus sign or ∘ to indicate non-existence, it is also possible to mark existence, for example, by dots, a plus signs, or × inscribed within the appropriate compartments. Both states must be explicitly drawn in the diagram.[94]

In principle, the method visualised in Peirce and Carroll is a combination of the syntactical rules of Euler and Venn. The primary diagram and non-existence follow Venn and the marking of existence is an interpretation of the process used in Euler's diagrams, Figure 1c and d, that is, by placing a letter, an asterisk, etc., into a specific region or compartment. While Euler's aim was solely to define particular diagrams and focus the viewer's attention by pointing to the intended region, such objects indicate existence in Peirce's and Carroll's compartments. Here, at the latest, Euler's nominalism is adapted to the age of existential import. These diagrammatic objects in Peirce or Carroll can also be connected to represent connections. Moreover, Peirce began to establish rules to indicate precisely how one diagram can be understood as a sequence of another diagram.[95]

As an example, a sequence of two diagrams can be shown here with Peirce, where × means that there is something in the universe of discourse, ∘ means that there is nothing, and the line represents the disjunction. The diagram on the left shows "All X are Y and some X are Y," from which the diagram on the right can be

92 Ibid., 102.
93 Moktefi and Pietarinen, "On the Diagrammatic Representation of Existential Statements with Venn Diagrams".
94 Ibid.
95 Bhattacharjee and Moktefi, "Revisiting Peirce's Rules of Transformation for Euler-Venn Diagrams."

inferred using the rule “Any sign of assertion can receive any accretion.”[96] The right diagram states: “All X are Y or some non-X are Y and some X are Y or all non-X are Y" (Figure 9).[97]

With a few exceptions, especially Peirce’s contribution to logic diagrams fell into obscurity for a long time in the twentieth century. Some of the most prominent exceptions, which at least indicate that they were familiar with Peirce’s basic ideas, came primarily from disciplines now considered to be part of the field of AI.[98] The 1980s and 1990s saw a significant renaissance of Peirce’s logic diagrams.[99] In 1994, Sun-Joo Shin successfully reapplied Peirce’s rules to Venn diagrams[100] and distinguished syntax and semantics. In the syntax, she defined diagrammatic elements and well-formed Venn diagrams and established the rules for transforming these diagrams. By using semantics, she then managed to prove the correctness and completeness of these formal systems. This marked a significant advancement in the field of visual reasoning, and as diagrams were no longer merely seen as a helpful visualisation, they were seen as a formal system in their own right. Soon, many more diagrammatic systems were developed, based on either Euler diagrams or entirely different types of diagrams.

Currently, many diagrammatic systems can be identified based on whether they allow existential import. It is now known that Euler diagrams, both with and without existential import, have comparable observational advantages.[101] While in symbolic logic, one relies on the assumption that every true formula must be derived from a given set of true formulas, diagrams have the advantage of also allowing true additional information to emerge within the diagram. Reading the diagram is sufficient to reveal this information without having to deduce it. This advantage has been referred to as a “free ride,” as in symbolic logic, we must “pay” for a logical operation to gain a benefit from it, whereas in diagrams, we receive the information freely.[102] Thus, in the current field of visual reasoning, not only are nominalist and realist claims recognised as being of equal value (in terms of expressivity and observational advantages), the formalist need is also united with the psychological aspects as strict rule-guided reasoning need not exclude the desire for intuitive cognition.

However, not every diagrammatic system possesses these observational advantages. This is so because some diagrammatic systems would become over-specified or undetermined if they allowed free rides. In the Euler diagrams presented in Section 1, we do have free rides but lose the precision of the diagrams, as shown in Figure 2. Both the diagrams in Figure 2 exhaust all possible cases of how circles can relate to each other according to the premises, and we then receive the conclusion freely. However, to get the free ride, we have to construct two diagrams that represent all the spatial positional relationships of the circles that the premises dictate. Furthermore, Euler even occasionally utilises the fact that a diagram like Figure 1a can represent more than just one judgement, e.g., “Some B are not A.” Newer diagrammatic systems based on Euler diagrams are aware of these problems and avoid this ambiguity.[103] However, all newer systems import existence in return, which Euler can avoid.

The definition of existential import in these new diagrammatic systems is similar to the historical developments described here but not identical. This is partly because most of today’s diagrammatic systems are no longer influenced by philosophical attitudes like nominalism and realism but by the technical requirements of logic. Previously, logicians held one of two positions, as the decision in favour of a particular logic had implications for various philosophical views including ontology, epistemology, and philosophy of religion. However, today’s logicians primarily prioritise the functionality of the corresponding logic rather than broader philosophical implications. This, of course, does not mean that modern logic is completely

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96 Peirce, *Collected Papers*, 4.362.

97 For details see Pietarinen, “Extensions of Euler Diagrams in Peirce’s Four Manuscripts.”

98 E.g. Trenchard, “On the Construction of Venn Diagrams;” McCulloch, “What is a Number.”

99 E.g. Sowa, *Conceptual Structures*; Lukose et al., *Conceptual Structures*.

100 Shin, *The Logical Status of Diagrams*.

101 Stapleton et al., “What Makes an Effective Representation of Information;” Stapleton et al., “The Observational Advantages of Euler Diagrams with Existential Import.”

102 Shimojima, *Semantic Properties of Diagrams and Their Cognitive Potentials*.

103 Hammer and Shin, “Euler’s Visual Logic.”

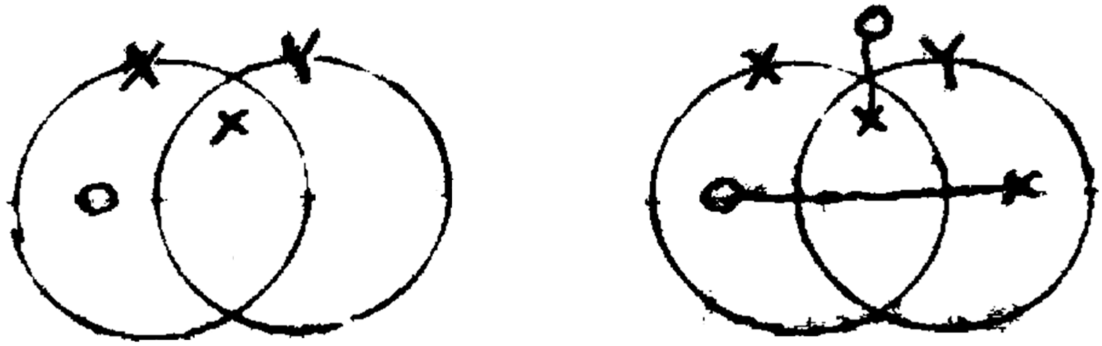

Figure 9: Peirce diagrams taken from Harvard Houghton Library Ms 479.

free of philosophical views. It is still possible to occasionally encounter nominalistic or realistic assumptions within these systems and some approaches also interpret Euler's original diagrams in a more realistic manner.[104]

The perspective of the nominalists and the contemporary diagrammatic systems, which deliberately eschew existential import, can be most effectively linked to the domain of "free logic." This is a possibility if free logic is understood as an abbreviation for the expression "free of existence assumptions with respect to its terms, general and singular."[105] However, there are differences in detail, which are probably due to the fact that diagrams have different representative properties than the symbolic notations that have popularised existential commitments and the strategies to avoid it such as free logic. A review of historical sources reveals that logicians such as Meinong, who have made significant contributions to the development of both diagrammatic reasoning and free logic, cannot be easily categorised.[106] These are some of the reasons why most current logicians who use diagrams avoid references to expressions such as "free logic."[107]

Many recent studies have overlooked the philosophical–metaphysical attitudes that motivated the logicians discussed in the preceding sections. It is not uncommon, for example, for line diagrams, such as those depicted by Lambert in Figure 3, to be interpreted in a nominalistic manner and, thus, equated with Euler diagrams without existential import.[108] Recently, there have also been several realistic approaches to Euler diagrams that have employed them in a Gentzen-style.[109] Yet, as good as these systems are, none of them came close to Euler's original diagrams, as some adopted Venn's realism by including shadings and others, similar to Bachmann, read existential import everywhere in the Euler diagrams and also employed geometric shapes that cannot be identified as Euler diagrams.

What we have outlined thus far is only a fraction of the extant research on visual reasoning in which Euler or Venn diagrams are used. In addition to existence and non-existence, we currently find, for example, different forms of non-existence in Euler and Venn diagrams. This can be achieved through non-classical diagram systems, which allow for the representation of different forms of negation, primarily concerning the regions of a diagram. Alternatively, certain forms of negation can be used to refer to specific components within a compartment.

An example of the first case is the combination of Euler and Venn diagrams for intuitionistic logic, in which the omission of compartments is given a different function than the shading of a compartment.[110] An example of the second case is the introduction of the absence of individuals in compartments. This

104 Hammer, *Logic and Visual Information*, 71; Moktefi, "Is Euler's Circle a Symbol or an Icon?"
105 Lambert, *Free Logic*, 124.
106 Max, "Zur Entwicklung der freien Logik (free logic)."
107 Bhattacharjee et al., "Logic of Diagrams."
108 Stapleton et al., "What Makes an Effective Representation of Information."
109 Mineshima et al., "A Diagrammatic Inference System with Euler Circles;" Linker, "Sequent Calculus for Euler Diagrams."
110 Linker, "Intuitionistic Euler-Venn Diagrams;" Linker, "Natural Deduction for Intuitionistic Euler-Venn Diagrams."

complements the classical negation of compartments with the negation of individuals in these compartments. While in Figure 7, for example, we could only explain the absence or non-existence of p and q by the fact that p and q were not represented in the diagram, formal systems can be constructed for the second case in which $\bar{p}$ and $\bar{q}$ can be made explicit in the diagram. This can then be used to construct formal systems that integrate the idea of absence from traditional Indian logic.[111]

All these systems are based on the diagrammatic approaches of Euler and Venn, even if they are extended by other elements and ideas, such as those introduced by Peirce and Carroll. What we have omitted here, however, are all the diagrammatic approaches that were introduced by authors before Euler and after Venn, for example, *CL* diagrams and existential graphs.

6 Conclusion and Outlook

In the previous sections, we have seen how, in the field of visual reasoning, the increasingly influential realist conception was able to assert itself against the waning prevalence of the nominalist conception. Nominalists object to the introduction of terms such as "objects," "existence," and "individuals" on the grounds that these are the province of epistemology or metaphysics. They regard language and logic as a distinct domain. One possible explanation for this stance is that many nominalists, such as Euler or Schopenhauer, do not limit their research to the domain of logic (or mathematics). Instead, they espouse a holistic philosophical position that precludes the transfer of concepts from one domain to another.

Priest's thesis, which was mentioned in the introduction, thus also appears to be correct with regard to diagram logic: for a long time, existential import was avoided in the history of diagrams, and it was not until the nineteenth century that a perspective emerged that would have been regarded as realistic from an earlier vantage point. Precursors who occasionally spoke about objects or individuals in logic, and thus introduced metaphysical or epistemic concepts into logic, had already emerged in isolated incidents. But there were of course also critics who were aware of this mixture. Euler's struggle against Leibniz and his successors is exemplary of this.

The results of the historical development from nominalism to realism between the years 1760 and 1900 can be explained by examining the diagrams. In the nominalist sense, Euler only used bearing relations of circles in Euclidean space in Figure 1 and explained his logic of the relation of containing and not containing. In Figure 8, we see how in the middle of the eighteenth century the realist interpretation prevailed in which the circular surfaces were occupied by individuals, while the points symbolised existential import.

This development from nominalism to realism is well explained by Figures 1, 3, 5 and 8. Figures 1 and 3 represent the two different views in logic at this time. In Figure 1, Euler insists that there are no points to be seen at all, as lines do not consist of points and terms do not have to be occupied by individuals. Lambert, in Figure 3, interprets the line as a collection of points representing individuals, which gives the lines existential import. However, both views, nominalism, which denies existential import, and realism, which advocates existential import, are not motivated by the efficiency of logic. In particular, logical attitudes in the eighteenth century are motivated by whether scholars outside of logic accept monads. Thus, logic is a building block in a coherent edifice of thought involving metaphysics, epistemology, philosophy of language, philosophy of nature, and philosophy of religion.

In Figure 5, we see how the points of Lambert's line diagrams with existential import were integrated into Euler's circle diagrams. This was not actually intended to transfer the idea of existential import but the technique of indeterminacy from line diagrams to circle diagrams. Existential import was, thus, only a by-product due to the Lambertian interpretation. However, it soon became successful. In Figure 8, we see how the points were transferred from the circle line to the surface for the first time. With this, Euler diagrams could be

111 Bhattacharjee et al., "Venn Diagram with Names of Individuals and Their Absence;" Bhattacharjee et al. "The Representation of Negative Terms with Euler Diagrams."

interpreted completely realistically. By the end of the nineteenth century, Euler diagrams were then transformed into the combinatorically exhaustive Venn diagrams and existence and non-existence were recovered as the maxim of interpretation. Existence was expressed by dots, dashes, circles, plus signs in the circular area, non-existence by shading, minus signs, etc.

Currently, all the traditional forms of interpretation of visual reasoning mentioned here are being explored by researchers who are not limiting themselves to only one direction or fighting another. One reason for this is probably that the connections between logic and other areas of philosophy and science are no longer considered by many logicians. In Euler's case, nominalism grew out of epistemology and the philosophy of language and influenced logic. Euler's aversion to interpreting circles as a composition of points in logic, i.e., existential import, was also motivated by the metaphysical aversion to monads, which was, in turn, underpinned by views in geometry and influenced physics.

However, a network of relationships such as one finds in Euler's work and probably also in Leibniz's work is rarely referred to or constructed today. Of course, this does not mean that logic is no longer connected to other fields of study but only that this connection is not always reflected. The fact that logic, ontology, epistemology, physics, etc. are closely related can be well demonstrated by the diagrammatic logics discussed in the previous sections, as well as by the discussions on symbolic logic that have intensified since the twentieth century.

However, the account presented here is by no means exhaustive, and, moreover, it raises many further unresolved questions that can be divided into technical, philosophical, scientific–historical, and diagrammatic questions. Technically, for example, it remains unresolved whether a natural system of deduction of Euler diagrams can be found that can satisfy Euler's nominalistic claims. The extent to which this goes hand in hand with modern strategies that have developed in symbolic logic to avoid existential import, such as free logic, is also worth discussing. Philosophically, many of the logicians presented here have not yet been sufficiently interpreted. In particular, there is still a lack of insight into Euler's metaphysics, philosophy of language, philosophy of logic, and many other areas. Historically, the main questions explore the extent to which the diagrams outlined here have also had an effect on symbolic logic. It is, after all, particularly striking that the talk of "existential import" was established precisely at that point in the nineteenth century when Leibnizian logic was gaining ground and the nominalism of the Kantian-Eulerian paradigm was fading. However, it remains unclear whether there was already a debate about individuals, existential import, etc. in visual reasoning before Euler. Diagrammatically, it is also unclear to what extent certain attitudes towards intuition and geometry impact logic diagrams. In turn, this also raises the big question of the extent to which logic diagrams or logic in general influence other areas of philosophy and science and vice versa. This article has attempted to make a small contribution on these issues.

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