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MEANS OR END?
ON THE VALUATION OF LOGIC DIAGRAMS

Resume: From the beginning of the 16th century to the end of the 18th century, there were not less than ten philosophers who focused extensively on Venn's ostensible analytical diagrams, as noted by modern historians of logic (Venn, Gardner, Baron, Coumet et al.). But what was the reason for early modern philosophers to use logic or analytical diagrams? Among modern historians of logic one can find two theses which are closely connected to each other: M. Gardner states that since the Middle Ages certain logic diagrams were used just in order to teach "dull-witted students". Therefore, logic diagrams were just a means to an end. According to P. Bernard, the appreciation of logic diagrams had not started prior to the 1960s, therefore the fact that logic diagrams become an end the point of research arose very late. The paper will focus on the question whether logic resp. analytical diagrams were just means in the history of (early) modern logic or not. In contrast to Gardner, I will argue that logic diagrams were not only used as a tool for "dull-witted students", but rather as a tool used by didactic reformers in early modern logic. In predating Bernard's thesis, I will argue that in the 1820s logic diagrams had already become a value in themselves in Arthur Schopenhauer's lectures on logic, especially in proof theory.
Keywords: symbolic logic, diagrammatic reasoning, visual reasoning, proof theory, Euler diagrams, Venn diagrams.

Йенс Лемански

СРЕДСТВО ИЛИ ЦЕЛЬ?
К ИСТОРИИ ОЦЕНОК ЛОГИЧЕСКИХ ДИАГРАММ

Резюме: С начала XVI в. до конца XVIII в. не менее десятка философов активно работали с наглядными аналитическими диаграммами Венна, что отмечается современными историками логики (Дж. Венном, М. Гарднером, М. Барон, Э. Кумэ и др.). Но что заставляло философов Нового времени использовать логические или аналитические диаграммы? Среди современных историков логики распространены два убеждения, тесно связанные между собой: М. Гарднер утверждает, что со времён средневековья некоторые логические диаграммы использовались просто для того, чтобы учить «твердолобых школяров». В таком случае логические диаграммы оказывались всего лишь средством для достижения цели. Согласно П. Бернарду, логические диаграммы не были оценены по достоинству до 1960-х гг., так что целью исследования логические диаграммы стали очень поздно. Настоящая статья посвящена вопросу о том, были ли логические (соответственно, аналитические) диаграммы в истории (ранне)новоевропейской логики всего лишь средством или нет. В противоположность

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Гарднеру, мы покажем, что логические диаграммы использовались не только как инструмент для «твердолобых школяров», но скорее как инструмент, которым пользовались реформаторы преподавания логики в ранненовоевропейский период. Отодвигая назад временнýю оценку Бернарда, мы покажем, что уже в 1820-х гг. логические диаграммы стали ценностью сами по себе в лекциях А. Шопенгауэра по логике, особенно в теории доказательств.

Ключевые слова: символическая логика, диаграмматическое рассуждение, визуальное рассуждение, теория доказательств, диаграммы Эйлера, диаграммы Венна.

1. Introduction

Nowadays logic diagrams — especially Venn's so-called "analytical diagrams" which share a family resemblance with Euler's diagrams and his own ones — are enjoying increasing popularity amongst philosophers, information scientists, mathematicians and many others. If one is to believe general introductions to logic, this appreciation of logic diagrams has not always been the case in history. The standard history of logic diagrams is as follows: Before the 18th century, there were just some tree diagrams, squares of opposition amongst others which had no significant role for logical reasoning and representation. However, in the 18th century Leonhard Euler invented analytical diagrams in the form of circles in order to make logical reasoning and representation more illustrative to non-logicians. In the following decades, analytical diagrams were continued by, inter alia, John Venn, Charles S. Peirce, Lewis Carroll, Warren McCulloch and now we use them in the form of Randolph-diagrams, Spider-diagrams, Karnaugh maps and so on in order to illustrate set containment, to calculate with many variables, and to verify proofs. According to the prevailing opinion, the benefits of analytical diagrams were only recognized in the last decades.

In his paradigmatic work *Symbolic Logic*, John Venn already alleged that there was a tradition before Euler using logic diagrams, specifically, the use of analytical ones. From the beginning of the 16th century to the end of the 18th century, there were no less than ten philosophers who focused extensively on analytical diagrams; each of these scholars having developed an original form of diagrammatic reasoning in logic. Yet, what was the reason for logicians in modern history to use logic diagrams at all?

Among modern historians of logic two theses, which are closely connected to each other, can be determined: some state that logic diagrams have been used since antiquity, or at least since the Middle Ages in order to teach "dull-witted students". Therefore, logic diagrams were just a means to an end in history, whereas others say that the appreciation of analytical diagrams in logic had not emerged before the 1960s. Thus it arose very late that logic diagrams become a value in itself.

In the present paper, I will at first illustrate the prevailing opinions of historians of logic concerning the valuation of logic diagrams in history (sect. 2). After that, I will focus on the question whether logic diagrams and particularly analytical ones, were just a means in the history of modern logic. Accordingly, I will combine the widely scattered information on the history of logic diagrams starting in the times of the printing press, until the time when logic diagrams first became the point to research for researchers. (1) In discussing the first thesis, I will argue that analytical diagrams were not only an auxiliary tool for "dull-witted students", but rather an instrument utilized by didactic reformers in early modern logic (sect. 3). (2) In addition to the second thesis, I will argue that already in Arthur Schopenhauer's lectures on logic in the 1820s, the value of logic diagrams increased, triggering a unique development in the history of how logic diagrams were received (sect. 4).

2. The prevailing opinion on the value of Euler diagrams

It is a well-known fact that logic diagrams were used since antiquity. Some historians are interpreting relevant passages of Plato (*Parm*. 131b) or Aristotle (*An. pr.* 43b) as an indication for the use of logical diagrams and in particular, Euler diagrams (cf. e.g. [Ueberweg 1871: 134 et seqq.; Stekeler-Weithofer 1986: 27–88; Flannery 1995: 1, 41]). Explicitly one can find a reference to logic diagrams in the works of Augustine of Hippo (*Conf*. IV 16), and since the Middle Ages, logic diagrams such as the square of opposition (quadrata formula), the bridge of asses (pons asinorum) and some kind of tree diagrams (arbor prophyriana/scientia), had become popular.

If one wants to know why these diagrams became popular during the Middle Ages, one can find a thesis in Martin Gardner's influential book *Logic Machines and Diagrams*. In this book, Gardner tells the story of the development from simple handwritten diagrams to complex digital computers, and within this account one can find an explanation for the invention of new logical diagrams in the middle ages:

> Jean Buridan, the fourteenth century French nominalist, was much concerned in his logical writings with finding middle terms, and his method became known as a "pons asinorum" ("bridge of asses") because *it helped dull-witted students* pass over from the major and minor terms to the middle ones. […] In later centuries, pons asinorum became a common phrase for Euclid's fifth proposition proving the base angles of an isosceles triangle to be equal, a bridge that *only stupid students* had difficulty in crossing. [Gardner 1958: 30; my italics]

Obviously this passage does not refer to all kind of diagrams, but rather only to the so-called "bridge of asses". Nevertheless, this passage was often quoted by historians of philosophy in order to explain that logical diagrams in general have been used since the Middle Ages for the sake of teaching dull-witted students. For example, in the history of logic diagrams written by Margaret E. Baron, Gardner's opinion about "pontes asinorum" has been adopted and linked with the claim that analytical diagrams were just used as a teaching aid (cf. [Baron 1969]). Let's allow this thesis (that considers Euler diagrams as a mere tool for dull-witted students) to be called the Gardner/Baron-thesis.

Another thesis which is closely connected to the first one was put forward by Peter Bernhard. Bernhard illustrates that visual reasoning and representation were rehabilitated by the so-called "Imagic turn" from the middle of the 20$^{\text{th}}$ century. Indications of this can be found in the work of Gardner, but also in the developments of the 1960s, such as the Visual Inference Laboratory (Bloomington/ Indiana), which lead to current imaging science. For Bernhard, especially since the enormous multiplication of the efficiency of data processing systems had enabled an increasingly better integration of graphic elements in user interfaces, this invigorated a new research interest in graphic modelling and processing of information.

Bernhard's thesis on the current positive valuation of logic diagrams depends mostly on the development in computer science since the 1960s. Furthermore, Catherine Legg has argued that there was a "picture shock" in the 19$^{\text{th}}$ century, peaking at the time of David Hilbert's and Alfred Jules Ayer's works; although over the last few decades logicians and mathematicians have recognized that logic diagrams are more than a heuristic aid (cf. [Legg 2013]) In addition to Bernhard and Legg, one can say that from the late 1960s to the early 1970s two other movements engaged with diagrammatic reasoning and representation have appeared: the "New Math Movement" (especially in set diagrams), and the "Proof without words movement" (with an interest in visual proofs with the help of diagrams). (Cf. e.g. the two representative works, [Murphy 1968: 8–25; Isaacs 1975].) Let us call this thesis the Bernhard/Legg-thesis.

By interpreting these short passages based on general historical claims regarding the history of logic diagrams, I am not certain whether I do justice to Gardner, Baron, Bernhard or Legg. But for the heuristic purpose of an examination of the history of Euler diagrams in (early) modern logic, I will use both theses as I have interpreted them above. Both theses are closely connected to each other and complementary, since the result of the Gardner/Baron-thesis is the point of departure from the Bernhard/Legg-thesis: the scepticism regarding logic diagrams until the middle of the 20^{th} century. Thus, we can conclude that the prevailing opinion on the value of Euler diagrams is the following: until the middle of the 20^{th} century, logic diagrams were just auxiliary means or heuristic tools, whereas since the middle of the 20th century logic, and especially analytical diagrams become more and more an own end and purpose of logical research.

3. Valuation of logic diagrams until the 1820s

In the current section, I would like to verify whether the Gardner/Baron-thesis can be substantiated. However, I will not start in the Middle Ages, but rather in the times of the printing press. There are several reasons for choosing this method: although there are some promising candidates (cf. i.a. [Nolan 2002: 282; Edwards 2006]), it is not fully clarified by current research whether we find any analytical diagrams in textbooks of the Middle Ages. However, I have some doubts as to whether there were Euler-Venn diagrams in the Middle Ages. Secondly, as far as I know there are not any thoughts on the use and valuation of analytical diagrams in the work of those promising candidates (and it is already difficult to find considerations on this topic in modern and early modern logic). Thirdly, the history of analytical diagrams starting from the times of the printing press is better explored at present.

For that reason, I have collected the information regarding the history of analytical diagrams from relevant literature [Venn 1894: 504–527 (= chap. XX. II); Ziehen 1920: 227–236 (= § 54); Gardner 1958; Baron 1969; Coumet 1977; Bernhard 2001: 69–80 (cf. also index); Moktefi, Shin 2012] and created a repository of historical texts which can be found via the following link: http://blog.fernuni-hagen.de/euler-venn-diagrams/.

According to this repository, a list can be found of at least ten logicians before Euler and a lot of successors who anticipated or continued the tradition of visual and diagrammatic reasoning and representation in logic. Having read these texts in chronological order these texts regarding to the question of what was the reason for them to use analytical diagrams at all. I should say at once that all logicians until the 1820s state — if they are even reflecting on what they do — that diagrams are just the auxiliary means to the end of verbal logical reasoning, and only the Gardner/Baron-thesis appears to be justified. Yet, as we will see in the following, I have not found any hint in early modern logic texts that logical diagrams are anywhere used in order to teach dull-witted students. In the following sections, in each case some quotes will be provided on what logicians such as Erhard Weigel (sect. 3.1), Gottfried Ploucquet, Johann Heinrich Lambert (3.2), Leonhard Euler (3.3) and Johann Gebhard Ehrenreich Maaß (3.4) have said on their use of analytical diagrams. As a result (sect. 3.5), I will give a hypothesis on the use of analytical diagrams which presents an argument against the Gardner/Baron-thesis.

3.1. Erhard Weigel (1625–1699)

One of the first logicians in the history of logic who reflected extensively on the use of analytical diagrams was Erhard Weigel. Since 1653, Weigel was professor for mathematics in Jena and among his students were Gottfried Wilhelm Leibniz and Johann Christoph Sturm, who are well known for their use of logic diagrams (cf. [Lemanski forthcoming]).

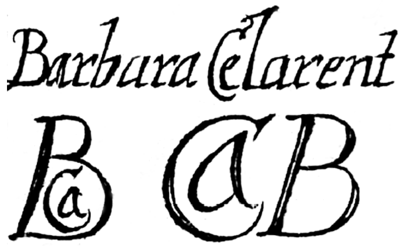
Analysing chronologically, we find analytical diagrams in Weigel's books not before 1693, in his *Philosophia Mathematica*. In this *opus magnum*, Weigel uses especially initial letters such as A, B and C that are connected or separated due to their relation in order to depict syllogisms. Weigel calls his invention "Logometrum" [Weigelus 1693: I, 122; II, 105]. For instance, we see in fig. 1 Weigel's illustration of the modi

Barbara ($\forall x Ax \rightarrow Cx, \forall x Cx \rightarrow Bx \vDash \forall x Ax \rightarrow Bx$)

and

Celarent ($\neg\exists x Bx \wedge Cx, \forall x Ax \rightarrow Cx \vDash \neg\exists x Bx \wedge Ax$)

(cf. [Weigelius 1693: II, 165]).



Although these diagrams were published in 1693, Weigel has reflected on the use of the Logometrum already in his book *Idea Matheseos universae*, published in 1669. In this book there is a passage in which Weigel describes his invention of the Logometrum and reflects on its value and purpose:

§ 17. Factum hinc est, ut veteres Mathematici quantorum abstractam rationem non abstracte, nec, ut directa methodus exigit, catholicis propositionibus, & quae sunt kath auto; sed quasi concrete tantum, indirecta methodo, per lineas & figuras, *tanquam per clariorem speciem doctrinae gratia tradiderint*, quod ex Euclidis libro tum secundo, tum quinto, nemo non agnoscit.

§ 17. From here it is a fact that the old mathematicians have taught the abstract ratio of quantities not in an abstract manner and also not, as it is required by the direct method, in universal and through itself certain propositions. But rather in a quasi-concrete manner, due to an indirect method, viz. by using lines and figures, *for the sake of a clearer form of doctrine*; this method is commonly accepted, in accordance with Euclids second and fifth book [sc. *Elements*].

§ 18. Data mihi hinc est occasio cogitandi, *annon ad alia quaedam generaliora facilius tradenda* similiter adhiberi possint lineae vel figurae: Et illico vim earum in ipsis logicis Syllagisationibus alioquin abstractissmis expertus sum. [Weigelius 1669: 46; my italics]

§ 18. Thus the opportunity was bestowed upon me to reflect whether or not — *for the purpose of an easier explanation of certain other more general relations* — it is possible to use lines and figures in a similar manner; and immediately I had proved the strength of those [sc. lines and figures] at precisely theses Syllogisations, which are otherwise very abstract.

In § 17 of the quote, Weigel uses two combined *argumenta auctoritatis* as a justification for his own logical invention which is presented in § 18 of the quote. The first *argumentum* is as follows: the "old mathematicians" have taught the abstract ratio of quantities (rationes quantorum), in a concrete manner in order to make their doctrine clearer. The second *agumentum auctoritatis* is a justification of the old mathematician's method, a concrete method which is commonly accepted since it is in accordance with Euclid. It stands to reason that Weigel indicates with "rationes quantorum" to Eudoxos's doctrine of proportions (abstract), which is especially given in the fifth book of Euclid's *Elements* (concrete).

When Weigel introduces his own invention of analytical diagrams in § 18, this invention is directly justified by the method of the "old mathematicians" and indirectly justified by Euclid. Since if Weigel's usage of the Logometrum in logic is as tangible as the method of the old mathematicians in the doctrine of propositions, and if the old mathematicians are as exact in their presentation as Eudoxos with his doctrine in the *Elements*, then Weigel indicates that both premises of his logic are as tangible as Euclid's geometry. A further analogy between Weigel and Euclid can be seen in the factual justification and evaluation of analytical diagrams: the old mathematicians have used visual geometry "for the sake of a clearer form of doctrine". In a similar way, Weigel explains that he has used the Logometrum "for the purpose of an easier explanation of certain other more general relations [sc. in logic]". Thus, analytical diagrams have no ends in themselves, but rather they are a means for simplification.

Finally, the question then arises whether Weigel's Logometrum is restricted to special diagrammatic forms. § 18 indicates that he uses concrete geometrical forms (in accordance with Euclid) in order to make Syllogism's abstracts appear more tangible. Weigel speaks about "lines and figures", perhaps indicating one-dimensional line diagrams, or some other figures which could be two-dimensional. Since Leibniz has used line diagrams together with circle diagrams in order to represent syllogisms (cf. [Wolters 1980: 132]) and Sturm has used circle diagrams, explicitly on behalf of Weigel, it is likely that Weigel's idea of the Logometrum is not restricted to initial diagrams. The following sentences shed light on Weigel's idea:

agnovi tandem, non gratis Aristotelem in Syllogismis tradendis usum esse vocibus Geomtrarum, (πέρας, σύνδεσμος, σχῆμα) sed omnes Syllogismorum modos per schemata figurasque geometricas multo facilius discerni, quam per *Barbara*, *Celarent*,

Finally, I have discovered that Aristotle had not only good reasons for using terms of the geometers (boundary, connection, scheme) in order to describe the traditional syllogisms, but also that all modi of the syllogisms can be learned more easily by means of geometrical schemes and figures than by *Barbara, Celarent* multoque succinctius demonstrari (vulgo reduci) posse, quam per τὸ *Phoebifer axis obit terras athramque quotannis*: adeò quidem ut, vera sit an falsa syllogisandi forma per nudam coincidentiam vel discoincidentiam sive distantiam figuralem ipsarum saltem literarum initialium cujusque termini (non enim opus est ut sint circuli vel Triangula) [...] [Weigelius 1669: 46]

and can be demonstrated (or reduced) in much shorter form than by *Phoebifer axis obit terras aethramque quotannis*; so much so that — equally whether it may be a correct or incorrect form of syllogisation —, hereby it will be possible that they [sc. the modi of syllogisms] can be found out immediately and in an instant by means of the mere coincidence or discoincidence, i.e. by means of the figurative distance of at least the initial letters itself or its boundary (viz. it is not important that they are circles or even triangles) [...]

This quote demonstrates that Weigel is aware of various forms of logic diagrams, speaking explicitly of circles, triangles or initial letters, while reflecting on the function of analytical diagrams defined by the terms "coincidence" and "discoincidence". Furthermore, one can find some line diagrams in Weigel's work which are described by the logical vocabulary of the Logometrum (cf. [Weigelius 1693: II, esp. 104–105, 112]). It is also interesting to note that Weigel repeats his remark concerning the purpose and value of analytical diagrams, in contrast to the scholastic method, whereby syllogisms can "be learned more easily by means of geometrical schemes and figures than by *Barbara, Celarent*", and they can be "demonstrated (or reduced) in much shorter form than by *Phoebifer axis obit terras aethramque quotannis*".[2] Even if analytical diagrams are much easier and shorter than the mnemonic terms and verses of the scholastic method, Weigel does not say that analytical diagrams have a purpose.

Ultimately, Weigel appears as a didactic reformer who justifies his new logical method by referring to the antique authority of Euclid. The analytical diagrams he had in mind are not an end to research, but rather an auxiliary tool for reasoning. They are easier and shorter than the scholastic methods. Nevertheless, Weigel denied that those diagrammatic forms of reasoning and representation are a kind of "pons asinorum". On the contrary, he claimed that the Logometrum and some other diagrams used in the *Principia Mathematica* are a "pons sapientum" [Weigelius 1693: II, 10, 62, 172 et seq.].

3.2. Gottfried Ploucquet (1716–1790) and Johann Heinrich Lambert (1728–1777)

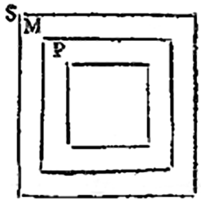
Although line diagrams can be found already in the works of Bartholomäus Keckermann, Johann Heinrich Alsted, Weigel and Leibniz, in the historiography of geometrical logician, Johann Heinrich Lambert, he is normally said to be the originator of line diagrams in the manner of analytical diagrams (cf. [Keckermannus 1601: 91 (= III, I 3); Keckermannus 1611: 426 (= III, I 6); Alstedius 1614: 395 (= VII, IV 1)]). Lambert's line diagrams were published 1764 in his *Neues Organon, oder Gedanken über die Erforschung und Bezeichnung des Wahren*. In the same year, Lambert became the government building officer of the Prussian Academy of Sciences in Berlin and was in close contact with Leonhard Euler.

During the 1750s and 1760s Gottfried Ploucquet had also worked on analytical diagrams. Since 1750, Ploucquet was Professor of Logic and Metaphysics at the University

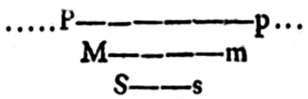
[2] For an explanation of the mnemonic verse "Febiferaxis obit terras..." cf. [John of St. Thomas 1955: 119].

of Tübingen. In Ploucquet's book from 1759, *Fvndamanta Philosophiæ Speculativæ*, we can find three nested squares in order to illustrate the *nota notae* principle:

Ex intuitione patet, P esse prædicatum omnis M, & M esse prædicatum omnis S. Sed prædicatum prædicati est prædicatum Subjecti. P itaque est prædicatum omnis S, id quod ita exprimitur: Omne S est P. [Ploucquet 1759: 25 (= § 71)]	By intuition it openly emerges that P is the predicate of all M, & M is the predicate of all S. But the predicate of the predicate is the predicate of the subject. P is therefore the predicate of all S, so that is vividly demonstrated: All S is P.



Despite this quote, some years later a dispute concerning the original invention of analytical diagrams arose between Ploucquet and Lambert. In 1762, Lambert had found in the municipal library of Zurich, called *Wasserkirche* (Water Church), an "old scholastic logic, or […] a Commentarius about the logic of Aristotle" with logical "figures in woodcuts", which are used in order to illustrate "many terms and conditions".[3] The fact that he even sent a letter to Zurich six years later in order to inspect the book again, can be taken as an indication that the logical lines in his *Neues Organon* were motivated by this "scholastic logic". An example of these lines as a relation of terms can be seen in the following drawing which is similar to Ploucquet's ones:



The dispute between Lambert and Ploucquet on the origins of Eulerian diagrams began in 1765, with the short article on Georg Jonathan von Holland's *Treatise on Mathematics* 1765, in which a comparison between Lambert's method and the logical calculation invented by Ploucquet ("von Hrn. Prof. Ploucquet erfundene logikalische Rechnung") can be found (cf. [Holland 1764]). In this treatise of Holland, the logical calculus of Ploucquet was favored. However, Lambert welcomed the fact that Holland had attempted to fix the "epochs of such arithmetic operations, so that, if once they will achieve the true perfection and usefulness, there will be a not such a bitter dispute concerning its invention, as it had happened due to the differential calculo" between Newton and Leibniz (cf. [Lambert 1765: 152]). Furthermore, Lambert asserted that he had invented the geometrical method in logic at least one year before the drafting of the *Neues Organon* — apparently he figured that Ploucquets method has emerged not before 1763/64. Although it was not until around the year 1770 that Lambert noticed that books were using analytical diagrams in about 1700 (cf. [Lambert 1771: 128, furthermore XIII, XXI]).

[3] Johann Heinrich Lambert an Johann Jakob Steinbrüchel, 14.4.1768, in: [Bernoulli 1782: 403–408].

Despite issues with dates, a consideration of the purpose and advantage of analytical diagrams is given by Lambert in his review of Holland's treatise:

Des Herrn [Georg Jonathan von] Hollands Abhandlung [Über die Mathematik, 1764] habe ich hierauf, und mit vielem Vergnügen durchgelesen. […] Besonders vergnügte mich die auf der 28sten Seite befindliche kurze Anmerkung, welche ungefähr sagen will: man könne nur in der Geometrie zu einer völligen Gewisheit gelangen, aber es sey dieselbe der allgemeinen Sage nach den meisten Menschen zu schwer, und unter den Wissenschaften die schwerste; und hieraus könne man den Schluß machen, wie sehr man sich den anderen Wissenschaften mit dem Schein der Wahrheit und leeren Worte begnüge. In der That sieht man dieses besonders an solchen Metaphysikern, welche die Geometrie nach den Begriffen ihrer Metaphysik einrichten wollen. Man hat da noch *Mittel, die Ungereimtheiten zu entdecken, weil die Geometrie die Fehlschlüsse bald verräth*. [Lambert 1765: 150; my italics]

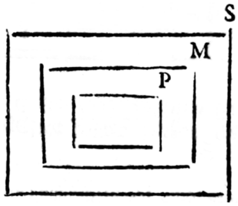
I have read the treatise [On Mathematics, 1764] of Mr. [Georg Jonathan von] Holland with a lot of pleasure. […] Particularly, I have had great satisfaction from the short comment on page 28, that roughly says: one can only come to complete certainty only in geometry, but it is, as is generally known, too difficult for most people and also the most challenging science of all. On this basis one can draw the conclusion of how much all other sciences content themselves with the shadow of truth and with idle words. Indeed, one can especially see this as these metaphysicians try to build up geometry with the concepts of metaphysics. There, one can adapt *the means in order to discover the inconsistencies, since geometry soon reveals fallacies*.

It is striking that this quotation speaks about analytical diagrams as a means, and not as an end. For Lambert, geometrical diagrams are an auxiliary tool in order to detect logical fallacies. However, Lambert's claim that he had invented analytical diagrams is anachronistic for various reasons. Firstly, we can find analytical diagrams in early modern philosophy long before Lambert and Ploucquet: in printed books, the first analytical diagrams can be found in a chapter on syllogism in the second book of *De censura veri et falsi* in 1531, by Juan Luis Vives (cf. [Vives 1531: fol. 57^{v}]). And in 1589, Nicolaus Reimarus Ursus published circle diagrams in his *Metamorphosis Logicae* (cf. [Raymarus Vrsus 1589: 32 et seqq.]). The second reason relates to the fact that Ploucquet had published his square diagrams already by 1759, so he was some years ahead of Lambert. Thus, in a response to Lambert, Ploucquet did not hesitate to "fix the epochs" more precisely:

Im Jahr 1758 kam ich auf den Gedanken die Schlüsse zu zeichnen, und in Figuren vorzustellen, um dieselbe auf eine anschauende Erkenntniß dergestalt zu bringen, daß der *genze Schluß mit einem Blik*, ohne an Folgen zu gedenken, *übersehen*, mithin *aller Zweifel wider die Untrüglichkeit der Schlüsse gänzlich aufgehoben werde*. Wenn z. Ex. alles M ein P. und alles S ein M ist: so ist, wenn man das Prädikat

In the year 1758, I had the idea to depict conclusions and to present them in figures, in order to transform them into a visible knowledge, so that *the whole syllogism can be overseen in one view*, without thinking of the consequences; and so that *the doubt against the infallibility of syllogisms is repealed*. If, for example, all of M is P. and all of S is M, and if one interprets the predicate in einem bejahenden Saz als einen Theil von dem Begrif des Subjekts betrachtet, das P in M und das M in S enthalten. Folglich kann die Construction diese seyn [Ploucquet 1766: 157; my italics]:

of an affirmative sentence as a part of the concept of the subject, P is contained in M and M is contained in S. Thus, the construction can only be the following:



This quote is not only interesting in regard to the given date of the said innovation, but also with regard to the reflection on the purpose and advantage of analytical diagrams. Even for Ploucquet, analytical diagrams are a means in order to make syllogistic reasoning easier and more certain. Similar to Weigel and Lambert, it seems that for Ploucquet logic diagrams cannot replace verbal syllogisms and they cannot be used without an accompanying explanatory. But they can help in avoiding fallacies since the observer is able to grasp a syllogism in just one view.

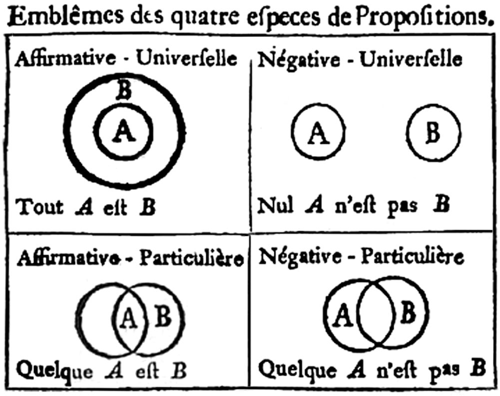
3.3. Leonhard Euler (1707–1783)

Although Lambert and Ploucquet dispute on who was the inventor of geometrical diagrams in logic, nowadays spatial logic diagrams are commonly named after Leonhard Euler. Euler's first manuscripts on analytical diagrams were written long before 1757.[4] Later, he presented circle diagrams in his *Lettres à une princesse d'Allemagne sur divers sujets de physique et de philosophie*, which was written in 1760–62 and published in 1768 in Saint Petersburg. The analytical diagrams which are known today as Euler diagrams are presented in a longer passage on language, intermingling seamlessly with some letters on logical reasoning.

For Euler, language — and especially notions — are "formed by abstraction" from sensible impressions and "are the source of all our judgments and of all our reasoning" [Euler 1833: 338 (= L. CII)]. Consequently, from a current point of view, Euler's philosophy of language may be interpreted as a traditional form of compositionality which starts with conceptual atomism and connects, in a bottom-up manner, atomistic concepts to molecular judgements and judgments to holistic inferences. Within this triadic model — consisting of concepts, judgments and inferences — the molecular relational role of concepts in judgments play a significant role. For Euler, the formula of molecular relational concepts consists of "four species" ("quatre especes"): (1) Every A is B, (2) No A is B, (3) Some A is B and (4) Some A is not B. The two notions, A and B, which can be found in all species, and "contain an infinite number of individual objects, we may consider it as a space in which they are all contained" [Euler 1833: 339 (= L. CII)]. If each concept, represented by variables such as A and B, has a specific space that comprehends a specific number of individual objects, the relation between those concepts can also be

[4] Санкт-Петербургский филиал Архива РАН Ф. 136. Оп. 1. № 134, 42 (cf. [Кобзарь 2005]).

illustrated by spaces or circles. Therefore, Euler illustrated the four species of relations of concepts in judgments in the following way:



After introducing these four types of concepts in judgments, Euler reflects on the benefits and value of his diagrams:

> Ces figures rondes, ou plûtôt ces espaces (car il n'importe quelle figure nous leur donnions), *sont très propres à nous faciliter nos réflexions sur cette matière*, & à nous découvrir tous les mysteres dont on se vante dans la Logique, & qu'on y démontre avec bien de la peine, pendant que par le moïen de ces signes *tout saute d'abord aux yeux*. [Euler 1768: 96–101 (= L. CIIf.); my italics]

> These circles, or rather these spaces, for it is of no importance of what figure they are of, are *extremely commodious for facilitating our reflections on this subject*, and for unfolding all the boasted mysteries of logic, which that art finds it so difficult to explain; whereas, by means of these signs, *the whole is rendered sensible to the eye*. [Euler 1833: 340 (= L. CIII); my italics]

Although this is the only existing small quote on the value of logic diagrams in the work of Euler, it is of much interest regarding the Gardner/Baron-thesis. On the one hand, the quote reveals that Euler interprets his diagrams as a means for an easier and shorter form of logical reasoning. Furthermore, it is remarkable that Euler does not stress a direct analogy between logical reasoning and his diagrams, for he only speaks about the *reflection* on logical reasoning with the help of diagrams. This can be interpreted as a sign of metalogical reasoning using Eulerian diagrams, viz. they cannot replace or compensate the conceptual reasoning itself. But on the other hand, the quote does not indicate that Euler sees these diagrams as an auxiliary tool to teach dull-witted students. The benefit of circle diagrams as a metalogical tool can be found in the last subsentence of Euler's quote: with diagrams the whole syllogism, logical reasoning is given in only one look.

3.4. Johann Gebhard Ehrenreich Maass (1766–1823)

Since 1791, Johann Gebhard Ehrenreich Maaß was an extraordinary professor of philosophy at the University in Halle. He not only taught philosophy, but also mathematics and rhetorics. Similar to the contemporary logician Johann August Heinrich Ulrichs who also used some line diagrams,[5] he was sceptical of the transcendental philosophy of Kant and makes use of analytical diagrams. The textbook in which we find analytical diagrams in triangular shapes is entitled *Grundriß der Logik* and has been printed in four editions between 1793 and 1823.

The *Grundriß der Logik* is divided in three parts, (1) pure logic, (2) applied logic and (3) practical logic. In the introduction of the book, Maaß claims that pure and practical logic are organized in a systematic manner and are complementary. In contrast, the applied logic rests on "empirical principles" borrowed from psychology. In this context, which accounts for a large part of the introduction, while Maaß compares his logic diagrams with his predecessors, but also criticizes the method of Euler and Lambert:

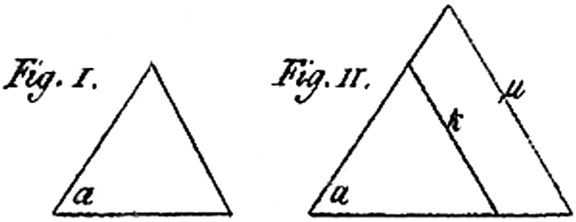
> Vorzüglich erwarte ich hier [sc. in der angewandten Logik] das Urtheil der Kenner über die neue Art, die Verhältnisse der Begriffe, die Urtheile und Schlüsse durch Zeichnungen anschaulich darzustellen (§. 365–381.). Bekanntlich haben Euler und Lambert das nämliche versucht. Die Eulerische Erfindung ist nicht brauchbar; die Lambertsche ist zwar viel vollkommener, allein noch immer fehlt den Zeichen, deren sich Lambert bediente, die vollständige Analogie mit dem Bezeichneten. [Maaß 1793: IX et seq.]

> Here [sc. in the applied logic], I especially expect the criticism of the experts on the new kind of illustrative presentation of the ratios of concepts, judgments and conclusions by drawings (§. 365–381.). As is well known, Euler and Lambert have tried the same thing. The Eulerian invention is not useful; The Lambertian is indeed much more perfect, but the signs, which are used by Lambert, are still missing the perfect analogy with the signified.

The point, which Maaß criticizes, is still seen as problematic, especially in modern ontology (cf. [van Invagen, Sullivan 2016: sect. 2.2]). Subsequent to the quote, Maaß claims that Lambert tries to illustrate two metaphors which are mutually exclusive: (1) A concept is "falling under" another one, and (2) concepts have an "extension." Only if both metaphors can be illustrated in one diagram, a perfect analogy between the sign and the signified emerges. Lambertian line diagrams illustrate the metaphor of extension by the specific length of each line, on the one hand, and the metaphor of falling under by the lines written among each other, on the other hand. However, Lambertian's line diagrams cannot perfectly illustrate both metaphors in equal measures. The argument against Lambert's lines (which can, for Maaß, actually apply to Euler in a greater extent) is that the extensional dimension of diagrams says that i.e. A and B are identical, whereas the lines written among each other illustrate that both concepts are repectively different to each other.

It will not be discussed here whether this argument is convincing or not — however it seems to be one of the first important criticisms of Eulerian diagrams, indicating that for Maaß diagrams are just an illustration of verbal reasoning and representation. Maaß himself tries to solve this problem by combining the extensional function of Eulerian diagrams with the function of falling under in Lambertian line diagrams by using triangles:

[5] Cf. [Vlrich 1792: 171]. In 1790s, analytical diagrams were also used by Johann Gottfried Kiesewetter [Kiesewetter 1793]; W. L. G. Freyherr von Eberstein [Eberstein 1794].



In his applied logic, Maaß comes back to his reflection on logical diagrams after having made some general points upon applied logic, conceptual formation and semiology. As an introduction to his new method of logic, Maaß defines in § 364 that a full analogy between the sign and the signified is given if all what applies to the sign is also true of the signified. After that definition he deliberates the benefits and value of these signs, so providing in the following paragraphs examples of the full analogy between the sign and the signified by describing his analytical diagrams:

> Ein solches Zeichen stellt uns das Bezeichnete gleichsam vor Augen, und *befördert* also ungemein die *Deutlichkeit und Evidenz* der Erkenntnis von dem letzeren: […] Inzwischen hat doch das Zeichen, außer dem angeführten Nutzen, auch noch den, daß es *die Erfindung neuer Wahrheiten erleichtert*, indem es uns das Bezeichnete in allen seinen Verhältnissen gleichsam *mit einem Blicke übersehen* läßt. [Maaß 1793: 245 et seq.]

> Such a sign places the designated before our eyes, and therefore, exceptionally *supports* the *clarity and evidence* of knowledge of the latter: […] By now, the sign has, except the above mentioned advantage, also the benefit that it makes the *invention of new truths more easier*, so that the signified can be *overseen in one view* with all it relations.

Additionally, from Maaß's quote it can be deduced that analytical diagrams are a means in order to support clarity and evidence. Particularly, the fact that via diagrams the signified designated can be overseen in one view, recalls Euler's reflections on the purpose of his own diagrams. A new insight can be deduced from the idea that diagrams make the "invention of new truths easier". It is necessary to pay attention to the exact wording: Maaß does not say that Eulerian diagrams are an auxiliary tool in order to invent new truths; rather, he states that *this invention* becomes easier by using diagrams. Thus, the role diagrams or signs in general play in the case of invention it has not been explicitly conceived.

All in all, Maaß's reflection on Eulerian diagrams is far from an instruction on dull-witted students. His criticism of Euler and Lambert indicates that the use of and the consideration of logic diagrams become a more and more specialized issue with more and more criteria that have to be fulfilled by authors using diagrams in logic.

3.5. Conclusion and hypothesis

The so-called Gardner/Baron thesis states that logic diagrams were used by logicians in early modern philosophy just for the purpose of teaching dull-witted students or as a mere teaching aid. Therefore, logic diagrams are merely a means and lack a *raison d'etre*. At a close reading, the Gardner/Baron thesis is restricted chronologically to the

Middle Ages and systematically to the bridge of asses (pons asinorum). However, there are reasons to apply their thesis to analytical diagrams in (early) modern philosophy. On the one hand, the Gardner/Baron thesis is closely connected to the Bernhard/Legg-thesis, claiming that logical diagrams became an end to research not long before the 1960s. That thesis can be interpreted as an *argumentum e contrario* of the Gardner/Baron-thesis. On the other hand, the writings of Gardner and Baron emphasize the connection between different kinds of logic diagrams, resulting from the transformations or complementation between logic diagrams which are only visibly different.

In section 3.1–3.4, I have tried to illustrate — namely on the basis of considerations by different authors between the 16th and 19th century — that Gardner and Baron are justified to interpret logic diagrams as a means, not as an end. The valuation of analytical diagrams can be found in the history of early modern philosophy and can be summarized briefly in the following manner.

Logic and analytical diagrams

- are for "an easier explanation" and "are shorter" than scholastic methods (Weigel),
- are a "means in order to discover the inconsistencies" (Lambert),
- have the benefit that "syllogism can be overseen in one view", and the "doubt is repealed" (Plocquet),
- have the benefit that "syllogism can be overseen in one view", they "support clarity and evidence", and make the "invention of new truths easier" (Maaß).

Although logical diagrams are predominantly seen in early modern philosophy as an auxiliary tool, we are not justified to claim that analytical diagrams are just used in order to teach students and especially dull-witted students, similar to the so-called "bridge of asses". In contrast to this, Weigel spoke of a "pons sapientium" in connection to his analytical diagrams.

But why were logical diagrams, and especially analytical diagrams, used in (early) modern logic at all, if they have no value in themselves? For further enquiry, I propose a hypothesis which could give an appropriate response to this question. By listing all logicians who used Eulerian diagrams in early modern philosophy, one finds a familiar resemblance between them. Most of the logicians using analytical diagrams in early modern philosophy are encyclopedists.

At first glance, this may seem ambiguous but to briefly elucidate this point in the following short definition: In my opinion, an encyclopedist (1) is a didactic reformer, (2) is interested in thematic systematization and (3) in universal knowledge. A further distinction is a direct or explicit encyclopedist, who uses the term "encyclopedia" for his own work, whereas an indirect or implicit encyclopedist is someone who is influenced by a direct or explicit encyclopedist, but without claiming the term "encyclopedia" for his own work.

By using these definitions and a list with all the early modern logicians who use logic diagrams, one can comprehend the familiar resemblance suggested here. Until the time of Maaß, direct encyclopedist are Vives, Keckermann, Alsted, Leibniz, Weigel and Lange (cf. e.g. [Leinsle 1988]). An indirect encyclopedist is Lambert, who refers to the direct encyclopedist Francis Bacon with his "Neues Organon".

While this is not the place to discuss the encyclopedist-thesis in detail, it is worth looking again into the quotes of Weigel (sect. 3.1). Although there are no indications of being a direct encyclopedist in these few quotes, he can be classified as a didactic reformer, since he uses Euler diagrams explicitly instead of scholastic memorial verses.

And that also applies to his scholars, Leibniz and Sturm, as well as to other philosophers at that time (cf. [Lemanski forthcoming]). A closer examination of Vives', Keckermann's, and Alsted's works, in which logic diagrams have been presented, will give us more evidence whether the encyclopedist-thesis is plausible or not.

4. Arthur Schopenhauer's valuation of analytical diagrams in the 1820s

Another indirect encyclopedist who uses logical diagrams in the 1810s and -20s was Arthur Schopenhauer. Similar to Lambert and Kant, who are also known for using analytical diagrams and referring to Francis Bacon (cf. [Kim 2008; Wolters 1980: 17 et seqq.]), Schopenhauer has a close connection to the English encyclopedist. In section 4.1, I will underline this connection between Schopenhauer's philosophy and the representational approach of Bacon's encyclopedia. Furthermore, I will try to show in sect. 4.2 that Schopenhauer was one of the first who was interested in the history of logic and analytical diagrams and that he has used analytical diagrams in his main work which were especially influenced by Euler. The main section will be 4.3, in which I present a longer quote of Schopenhauer lectures on logic who were held in the 1820s. This quote offers a valuation of analytical diagrams which go far beyond the opinions of his predecessors who were presented in sect. 3. In sect. 4.4, I will sketch some interpretation on Schopenhauer's argument. Based on Schopenhauer's considerations of the value of analytical diagrams, I am able to postdate the Bernhard/Legg-thesis, by which Eulerian diagrams become an end to logic until the 1960s, but rather already in the 1820s.

4.1. Schopenhauer encylopedistic representationalism

The connection between Bacon's encyclopedistic approach and Schopenhauer's philosophy can be seen in the latter's main work *The World as Will and Representation*, and especially in the paragraphs in which Schopenhauer reflects upon the task of his philosophy. The main paragraph where these topics are discussed is § 15: "The present philosophy," Schopenhauer reflects, "[…] attempts to say […] what the world is." Thus, "in order to present to rational knowledge the whole manifold of the world in general, according to its nature, condensed and summarized into a few abstract concepts" [Schopenhauer 1958: I, 35 (= § 15)].

Similar to Bacon's *Novum Organon* or to other modern philosophical works such as Ludwig Wittgenstein's *Tractatus logico-philosophicus*, Rudolf Carnap' *Der logische Aufbau der Welt* (*The Logical Structure of the World*), or maybe to contemporary metaphysics (e.g. Chalmer's, Sider's, Heil's books about the world), Schopenhauer's *World as Will and Representation* can be interpreted as a theory of representationalism with an explicit and logical conceptual framework. "Representationalism" here means the attempt to reproduce the non-conceptual and merely visible world in a conceptual text. Thus, representationalism is guided by the metaphor of the mirror: the philosopher's book becomes a mirror of the world.

This metaphor can also be found in Schopenhauer's § 15 where he alludes to Bacon: The task of philosophy is "a complete recapitulation, so to speak, a reflection (german: Abspiegelung) of the world in abstract concepts, […]. Bacon already set philosophy this task". For Schopenhauer, this quote seems to be so important that he had repeated it in § 68 of his main work and it can also be found in other writings of him. For example, he denied normative approaches in philosophy and claims: "I [sc. Schopenhauer] generally […] reflect upon the world and show what everything is, and how it is connected; while everyone is left to his own discretion."[6]

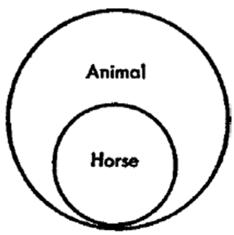
This attitude pertaining to representationalism can also be found in the works of Bacon and other encyclopedists. The encylopedistic approach of Bacon has often been interpreted with his list of 130 sciences in *Preparative toward a Natural and Experimental History* which was published together with the *Novum Organon* in 1620. Diderot has stated that this list was an important anticipation of the French encyclopedia (cf. [Diderot 1876: 133 et seq., 159 et seqq.]). In Bacon's *Novum Organon*, we also find the encylopedist idea (1) of reforming the Aristotelian logic and philosophy of science, (2) of systematic organization and (3) of universal knowledge (cf. [Blumenberg 1981]). According to his connection to Francis Bacon, Arthur Schopenhauer can be listed as an indirect encyclopedist, and he has used also logical diagrams in the 1810s and -20s. Similar to "encyclopedia", Schopenhauer used the term "system".

4.2. Eulerian Diagrams in Schopenhauer's work

Already in the first of the four books of his main work, *The World as Will and Representation*, we find some analytical diagrams, inspired by Euler. Whereas the whole work is a reflection of the world by using the term logic, all four books deal with either one of his two essential focus points, will and representation. Book I and III deals with representation, book II and IV with will. The first book (§§ 1–16) is in turn divided in two parts: cognizance (§§ 3–7) and Reason (§§ 8–16). Whereas the first part makes time, space (§ 3) and causality (§ 4) a subject of discussion, the second part is on language (§§ 9–13), science (§§ 14–15) and practical reason (§ 16). The section on language is divided in four sub-topics, namely logic (§ 9), eristic (ibid.), metalogic (§§ 10–12) and humor (§ 13), and of course the Eulerian diagrams are situated in § 9.

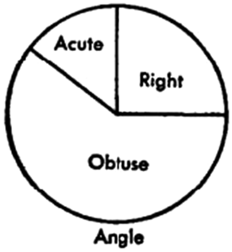
Overall, § 9 is much too short to give a complete recapitulation of logic in general. In just a few pages, Schopenhauer reduces all of logic to only conceptual relations which are presented and organized by five possible relations:

"(1) The spheres of two concepts are equal in all respects […],
(2) The sphere of one concept wholly includes that of another […],

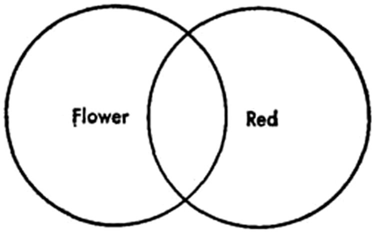


[6] Schopenhauer's Letter to Julius Frauenstädt: 31th May 1854, in: [Schopenhauer 1978: 343 (no. 332)]. Cf. also [Lemanski 2013].

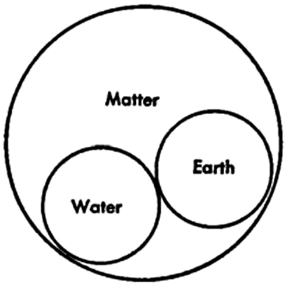
(3) A sphere includes two or several which exclude one another, and at the same time fill the sphere […],



(4) Two spheres include each a part of the other […],



(5) Two spheres lie within a third, yet do not fill it […].” [Schopenhauer 1958: I, 43 et seq. (= § 15)]



[Schopenhauer 1958: I, 41 et seqq.].

Only four relationships are illustrated by Eulerian circle diagrams since the first relation would be a circle which is hidden by another circle for both have the same extension. Thus, it does not make sense to illustrate the first relation, as the rest of § 9 does not provide more information on logic. Even though the second volume of the *World as Will and Representation*, which was published in 1844 for the first time, includes two chapters on judgments and syllogism, it is quite far from a complete recapitulation of logic — in all its many facets — as understood in those days.

Schopenhauer explains this lack of completeness by the fact that *The World as Will and Representation* is not the place to discuss logic in general. Since logic is an integral component of the world of reason, but this “schematism of concepts […] has been fairly well explained in several textbooks”, thus for the reader of his book “it is not necessary to load the memory with these rules" [Schopenhauer 1958: I, 44 et seq.]. Scientifically treated by itself alone and independently of everything else, logic should only "be taught at the universities" [Schopenhauer 1958: I, 46], proving that Schopenhauer's main work was not primarily written for a specific audience.

It is not widely known that Schopenhauer had written another version of his main work in 1820. This version was not addressed to a wide public, but rather to some of his students who had attended his lectures in Berlin in the 1820s. In this version, which was originally conceived to be taught at universities, Schopenhauer developed a form of logic which was 100 pages longer than the logical remarks in both volumes of his main work combined. Thus, Schopenhauer's Berlin lectures relate to § 9 of his main work as "great logic" (logica magna) to "minor logic" (logica minor). In the 130 pages of Schopenhauer's logica magna, we find a logic which is organized in general Aristotelian terms (concept, judgment, syllogisms) as well as Kantian (quantity, quality, relation, modality). Furthermore, all 130 pages are covered with Eulerian diagrams (cf. [Lemanski 2016]).

4.3. Schopenhauer's Valuation of Eulerian diagrams

In contrast to other logicians who had used analytical diagrams shortly after Euler, such as Maaß, Ulrich, Krause, Krug, Fries, Denzinger or Bachmann (cf. [Krause 1803; Krug 1806; Fries 1819; Denzinger 1824; Bachmann 1828]), Schopenhauer's lectures on logic are the only ones which can generally be regarded as a combination of Kant's *Jäsche-Logic* and Euler's *Letters to a princess* representing a unique form of logic in this period. But in contrast to Bachmann and all other mentioned logicians at that time, we find a reason for using Eulerian diagrams in Schopenhauer's lectures which continues also far beyond the quotes which were compiled in sect. 3.

Schopenhauer's reason to use analytical and Eulerian diagrams is given in a longer quote at the beginning of the section dealing with the composition of concepts into judgments. In this section, Schopenhauer claims that logic diagrams were at first given by Ploucquet, Lambert and Euler (cf. [Schopenhauer 1913: 270]). Then he makes a longer remark on the valuation of logic diagrams unique in the history of proof theory:

> Besonders werden diese anschaulichen Schemata uns die Erkenntniß der Regeln der Syllogistik sehr erleichtern, und *uns der Beweise der Regeln überheben*: nämlich Aristoteles gab für jede syllogistische Regel immer einen Beweis, was eigentlich überflüssig, sogar der Strenge nach unmöglich ist; denn der Beweis selbst ist ein Schluß und setzt folglich die Regeln voraus: man kann eigentlich diese Regeln nur deutlich machen und dann sieht die Vernunft ihre Nothwendigkeit sogleich ein, weil sie selbst der Ausdruck der Form der Vernunft, d. h. des Denkens sind.

> Principally, these illustratively schemes will be of great service to instruct in syllogistic rules and *prevent us from proving those rules*: since Aristotle has added to each syllogistic rule a proof, which is actually superfluous and in a rigid sense, quite impossible; since the proof is a syllogism for itself and therefore presupposes the rules: one can actually make these rules clear, so that reason itself realizes the necessity, since they [sc. the rules] are an expression of the form of reason, viz. thinking.

Was Aristoteles durch seine Beweise leistete, das werden uns die anschaulichen Schemata *viel besser*, und viel leichter leisten: denn, da sie eine ganz genaue Analogie zum Umfang der Begriffe haben; so lassen sie uns die Verhältnisse der Begriffe zu einander auf die leichteste Weise einsehn, nämlich anschaulich, und wir werden so die Nothwendigkeiten, welche aus diesen Verhältnissen entspringen, zur leichtesten Faßlichkeit bringen. Die Aristotelischen Beweise hat man schon längst aus der Logik weggelassen; aber man hat ihnen die Verdeutlichung durch anschauliche Schemata noch nicht so durchgängig substituirt, wie ich es thun werde. [Schopenhauer 1913: 272; my italics; similar 357]	What Aristotle has done by proofs, this will done by illustratively schemes *in a much better* and easier way: since they [sc. illustratively schemes] have an exact analogy to the comprehension of concepts; in that way, the relations between concepts can be seen in an easier light, and are especially revealing. In that way we will bring the necessities, which have their source in these relations, to the easiest comprehensibility. In Logic, the Aristotelian proofs have not been used for a long time; but no one has formulated them by using illustratively schemes in such a consistent manner as I will do throughout this chapter.

Surprisingly, Schopenhauer combines an old view with a fresh outlook in terms of logical diagrams. It seems rather conventional for Schopenhauer that Eulerian diagrams are of "great help for the awareness of syllogistic rules" and that proofs can be done "more easily" by using those diagrams. Since the time of Weigel these views have been reported numerous times (cf. sect. 3), yet a completely new aspect refers to proof theory: Eulerian diagrams are not only a means for logical reasoning, but rather they shall replace the traditional way of proving syllogistic rules.

Schopenhauer's argument on proof theory is not wholly easy to plot. He explains how traditional proofs of syllogisms are problematic since they are ensnared in a *petitio principii*: Every traditional proof in the Aristotelian sense tries to verify a special form of syllogism. As a conclusion, the correctness and validity of a syllogism shall be given; yet in order to prove a syllogism a syllogism is required, and this required and presupposed syllogism plays a role in the major premise of proof. Thus, the conclusion is identical with the premise it presupposes, and therefore the whole traditional proof theory is problematic. Moreover, it could be argued, that even if proof verifies a conclusion with another syllogism as a major premise, the syllogism of the major premise has to become a conclusion if it is proved itself. However, even if this reduction can be accepted, it presupposes an initial proof in which conclusion and major premise are necessarily identical (formal or even material). Thus, avoiding a *petitio principii* leads to an infinite regress or to a dogmatic assumption.

Schopenhauer sees the solution for this problem in logic, especially in Euler diagrams: Syllogisms, rules or proofs can only "become clear". This making or becoming clear is not only much easier, but also much better than the traditional Aristotelian theory of proofs. Euler's diagrams are an improvement since they prove syllogisms without presupposing deductive or syllogistic reasoning itself, insofar as Eulerian diagrams seem to be the only solution to avoid the above mentioned tropes of Greek scepticism. Even if Schopenhauer repeats some of the advantages of logic diagrams known from history, he assigns analytical diagrams a role in proof theory, proving to be unique in the history of logic diagrams.

4.4. Some remarks on Schopenhauer's argument

It is difficult to resolve whether Schopenhauer is justified to appreciate Eulerian diagrams as an end to traditional proof theory. Most likely, it depends on which school of logic and philosophy one adheres to. However, since it cannot be elaborated on in detail, I will only provide two general observations concerning the (1) tradition and (2) continuation of Schopenhauer's argument.

(1) At first, Schopenhauer's sceptical argument is not unique. It has been advanced more or less by authors such as Sextus Empiricus, Francis Bacon, John Locke or John Stuart Mill who have criticized the fundamentals of Greek syllogistic reasoning.[7] Generally speaking, Aristotelians and Stoicists have argued that the proof of incomplete or imperfect syllogism can only be done with perfect syllogisms, e.g. Barbara and Celarent which depend itself on the *dictum de omni et nullo* or the five unproved Chrysippean syllogisms depending on the so-called *dictum de si et aut* (cf. [Barnes 2007: ch. 5]). For Sextus or Mill, and similar to Schopenhauer, the reduction or the dicta are problematic for several reasons, yet all lead to sceptical tropes (dogmatic, circular, infinite regress et al.). But whereas sceptical empiricists such as Bacon or Mill have replaced deduction with induction, Schopenhauer has used geometrical diagrams in order to avoid these tropes.

Thus, what is unique in Schopenhauer is his solution to scepticism. Nevertheless, the objection it might be raised that Schopenhauer is making a classical categorical mistake which is concerned with almost the exact categories illustrated by Aristotle: "One cannot, therefore, prove by crossing from another kind — e.g. something geometrical by arithmetic." (*An. post.* 75a38: "Οὐκ ἄρα ἔστιν ἐξ ἄλλου γένους μεταβάντα δεῖξαι, οἷον τὸ γεωμετρικὸν ἀριθμητικῇ.") However, Schopenhauer anticipates this objection with reference to the exact analogy between the conceptual and the visual, as Eulerian diagrams illustrative schemes "have an exact analogy to the comprehension of concepts".

However, is Schopenhauer justified in claiming such an analogy? A similar question is currently being discussed under topics such as "isomorphism", "homomorphism", and "Graphic-Linguistic Distinction" by modern logicians (cf. e.g. [Bernhard 2001: 62 et seqq.; Shimojima 1999]). Also in the history of logic diagrams we have found that Weigel and Maaß confirm that there is an analogy between conceptual logic and visual geometry. Furthermore, Weigel and others have claimed that there is an analogy between conceptual and visual reasoning, which can not only be found in the works of the old Mathematicians, but also in Aristoteles himself. (Cf. sect. 3.1. Furthermore [Weigelius 1669: 146; Weisius, Langius 1712: 248].) And indeed, in Aristotle we find some indications for geometrical, as well as conceptual descriptions concerning the fundaments of syllogisms (for example *An. Pr.* 24b25 et seqq.).

(2) Moreover, Schopenhauer's argument is not only restricted to logical proofs in Aristotelian syllogism. Jean-Yves Béziau has shown that Schopenhauer has applied a similar argument to that discussed in sect. 4.3 also to Euclidean geometry (cf. [Béziau 1993: 81–88]). Schopenhauer claims that geometry is also problematic if it is based on mere logical reasoning and although the argument is much more elaborated on in his Berlin lecture in the 1820s, it depends more so on the thesis given in his dissertation of 1813 and

[7] Cf. Sextus Empiricus, *PH*, II 156 et seqq.; Francis Bacon, *Distributio Operis; Novum Organum*, I 13 et seqq., I 54; John Locke, *Essay Concerning Human Understanding*, 4, XVII § 4; [Mill 1858: esp. 112–121].

in the *World as Will and Presentation* of 1818. In all three texts, Schopenhauer argues that Euclid had made the mistake to prove geometrical propositions with logical reasoning. For Beziau, Schopenhauer's argument is similar to Árpád Szabó's claim that Euclid has only preferred logical instead of visual reasoning, due to the influence of eleatic philosophy within his epoch. The better option would have been, if Euclid had focus on intuitive, visual or diagrammatic proofs instead of a systematic-deductive system of axioms which is unnecessary.

Applied to Geometry, Schopenhauer's argument may have anticipated Brouwer's intuitionism (cf. e.g. [Koetsier 2005]). Yet, Béziau claimed that there are important differences, especially concerning the role of the principle of sufficient reasons. Therefore, it seems more obvious when current researchers claim a stronger connection between Schopenhauer's argument and Wittgenstein's philosophy of mathematics. Similarly, over the last few years one can find more and more discussions as to whether Schopenhauer can be classified as a pioneer of the proof without words movement (cf. [Schreiber 2003]).

Nonetheless, points, (1) and (2), have to be assigned as general observations concerning the tradition and continuation of Schopenhauer's argument. For the topic of the present paper, it is more important that Schopenhauer considered logic diagrams as having value in themselves and being an end to logical research. The increase in the value of logic diagrams is based on Schopenhauer's opinion that spatial intuition has an a priori structure, on the one hand, but also that conceptual usage is based on the world a posteriori, as given in sect. 4.1.

5. Conclusion

I have examined two theses, (1) the Gardner/Baron-thesis, which argues that since the Middle-Ages some logic diagrams are just means in order to teach dull-witted students; (2) the Bernhard/Legg-thesis, which states that since the 1960s logic diagrams become the end to research. As a result of the previous sections, it can be said that both theses cannot be broadened to all logic diagrams or to the whole history of logic diagrams.

For (1), it is true that logic diagrams are used as a means in early modern philosophy. They were used for "an easier explanation" (Weigel), "in order to discover inconsistencies" (Lambert), since "syllogism can be overseen in one view" and the "doubt is repealed" (Plocquet) or because of the benefit that they "support clarity and evidence", enabling the "invention of new truths more easier" (Maaß). Since the presented quotes do not provide evidence for the fact that all — or even the majority — of logic diagrams were used in order to teach dull-witted students, this part of (1) is not generally true, especially not for analytical diagrams. A more likely scenario for the use of analytical diagrams is that they were used by didactic reformers with encyclopedic concerns, but admittedly, this hypothesis deserves to be studied in more detail.

For (2), it is true that logic diagrams have become the ends to research since the 1960s. But this does not mean that there are not any attempts in history to value the benefits of visual reasoning. In Schopenhauer's lectures of the 1820s, the first quote in history in which Eulerian diagrams were praised as the end of traditional proof theory, can be found. For Schopenhauer, there is a *petitio principii* in Aristotelian proof theory, since "the proof is a syllogism for itself and presupposes therefore the rules". For that reason, he considers the use of Euler diagrams to be better than the Aristotelian method of verifying syllogism.

Finally, attention should be drawn to the fact that all historical material provided in this article is based on inductive investigation: it may be possible that there are more quotes in the history of Euler diagrams, which can be addressed to either the Gardner/Baron-thesis or the Bernhard/Legg-thesis, as well as to other logicians who have used diagrams. However, the absolute certainty of having achieved "wholeness" can never be fully attained in historical investigations, as the material presented in the preceding pages is more than a contribution to the historiography of analytical diagrams, as new perspectives regarding old quotations on the use of diagrams can be found. It is desired that the quotes and illustrations from Weigel to Schopenhauer can confirm this assessment.

References

Кобзарь 2005 — *Кобзарь В. И.* Элементарная логика Л. Эйлера // Логико-философские штудии. Вып. 3. СПб.: Издательство СПбГУ. С. 130–152.

Alstedius 1614 — *Alstedius J.-H.* Logicæ Systema Harmonicum […]. Herbornæ Nassoviorum, 1614.

Bachmann 1828 — *Bachmann C. F.* System der Logik. Leipzig, 1828.

Barnes 2007 — *Barnes J.* Truth, etc. Six Lectures on Ancient Logic. Oxford, 2007.

Baron, 1969 — *Baron M. E.* A Note on the Historical Development of Logic Diagrams. Leibniz, Euler and Venn // The Mathematical Gazette 53 (384), May 1969. P. 113–125.

Bernhard 2001 — *Bernhard P.* Euler-Diagramme. Zur Morphologie einer Repräsentationsform in der Logik. Paderborn, 2001.

Bernoulli (ed.) 1782 — Joh[ann] Heinrich Lamberts deutscher gelehrter Briefwechsel, Vol. 1. / Ed. *Bernoulli J.* Berlin s.a., 1782.

Béziau 1993 — *Béziau J.-Y.* La Critique Schopenhaurienne de l'Usage de la Logique en Mathématiques // O Que Nos Faz Pensar 7, 1993.

Blumenberg 1981 — *Blumenberg H.* Die Lesbarkeit der Welt. Frankfurt a.M., 1981.

Coumet 1977 — *Coumet E.* Sur l'histoire des diagrammes logiques, 'figures géométriques' // Mathematiques et Sciences Humaines 60, 1977. P. 31–62.

Denzinger 1824 — *Denzinger I.* Institutiones Logicæ, vol. II. Leodii, 1824.

Diderot 1876 — *Diderot D.* Œuvres complètes / ed. by J. Assézat, M. Tourneux. Paris 1876. Vol. XIII.

Edwards 2006 — *Edwards A. W. F.* An Eleventh-Century Venn Diagram // BSHM Bulletin: Journal of the British Society for the History of Mathematics 21, 2006. P. 119–121.

Euler 1768 — *Euler L.* Lettres à une princesse d'Allemagne sur divers sujets de physique & de philosophie, 2 vol. Saint Petersbourg, 1768. Vol. 2.

Euler 1833 — *Euler L.* Letters of Euler on Different Subjects in Natural Philosophy. Adressed to a German Princess / transl. and ed. by David Brewster, 2 vol. New York, 1833.

Flannery 1995 — *Flannery K. L.* Ways into the Logic of Alexander of Aphrodisias. Leiden, New York et al., 1995.

Fries 1819 — *Fries J. F.* System der Logik. Heidelberg, 1819.

Gardner 1958 — *Gardner M.* Logic Machines and Diagrams. New York, Toronto, 1958.

Holland 1764 — *Holland G. J.* Abhandlung über die Mathematik, die allgemeine Zeichenkunst und die Verschiedenheit der Rechnungsarten: Nebst einem Anhang, worinnen die von Hrn. Prof. Ploucquet erfundene logikalische Rechnung gegen die Leipziger neue Zeitungen erläutert und mit Hrn. Prof. Lamberts Methode verglichen wird. Tübingen, 1764.

Isaacs 1975 — *Isaacs R.* Two Mathematical Papers without Words // Mathematics Magazine 48 (4), Sep., 1975. P. 198.

John of St. Thomas 1955 — *John of St. Thomas*. Outlines of Formal Logic / transl. by F. C. Wade. Marquette, 1955.

Keckermannus 1601 — *Keckermannus B*. Systema Logicæ. Sompendiosa methodo […]. Hanoviae, 1601.

Keckermannus 1611 — *Keckermannus B*. Systema Logicæ. Tribus Libris Adornatvm, […]. Hanoviae 1611.

Kiesewetter 1793 — *Kiesewetter J. G*. Grundriß einer reinen allgemeinen Logik nach kantischen Grundsätzen […]. Frankfurt/Leipzig, 1793.

Kim 2008 — *Kim S.-H*. Bacon und Kant. Ein erkenntnistheoretischer Vergleich zwischen dem *Novum Organum* und der *Kritik der reinen Vernunft*. Berlin, New York, 2008.

Koetsier 2005 — *Koetsier T*. Arthur Schopenhauer and L. E. J. Brouwer: A Comparison // T. Koetsier, L. Bergmans (eds.). Mathematics and the Divine. A Historical Study. Amsterdam, 2005. P. 569–593.

Krause 1803 — *Krause K. C. F.* Grundriss der historischen Logik für Vorlesungen. Jena, Leipzig, 1803.

Krug 1806 — *Krug W. T.* Denklehre oder Logik (System der theoretischen Philosophie. Teil 1). Königsberg, 1806.

Lambert 1765 — *Lambert J. H*. Neue Zeitung von gelehrten Sachen 1765:1 (3. Januar) // A. F. Boͤk (ed.). Sammlung der Schriften, welche den logischen Calcul Herrn Prof. Ploucquets betreffen, mit neuen Zusaͤzen. Frankfurt, Leipzig, 1766. P. 149–156.

Lambert 1771 — *Lambert J. H*. Anlage zur Architectonic, oder Theorie des Einfachen und des Ersten in der philosophischen und mathematischen Erkenntniß. Vol. 1. Riga, 1771.

Legg 2013 — *Legg C*. What is a logical diagram? // S.-J. Shin, A. Moktefi (eds.). Visual Reasoning with Diagrams. Basel, 2013. P. 1–18.

Leinsle 1988 — *Leinsle U. G*. Reformversuche protestantischer Metaphysik im Zeitalter des Rationalismus. Augsburg, 1988.

Lemanski 2013 — *Lemanski J*. The Denial of the Will-to-Live in Schopenhauer's World and His Association of Buddhist and Christian Saints // A. Barua, M. Koßler, M. Gerhard (eds.). Understanding Schopenhauer through the Prism of Indian Culture. Philosophy, Religion and Sanskrit Literature. Berlin, 2013. P. 149–187.

Lemanski 2016 — *Lemanski J*. Schopenhauers Gebrauchstheorie der Bedeutung und das Kontextprinzip. Eine Parallele zu Wittgensteins Philosophischen Untersuchungen // Schopenhauer-Jahrbuch 97, 2016. P. 171–211.

Lemanski forthcoming — *Lemanski J*. Logic Diagrams in the Weigel and Weise Circles. Forthcoming.

Maaß 1793 — *Maaß J. G. E*. Grundriß der Logik. Zum Gebrauche bei Vorlesungen. Halle, 1793.

Mill 1858 — *Mil, J. S.* A System of Logic, Ratiocinative and Inductive. New York, 1858.

Moktefi, Shin 2012 — *Moktefi A., Shin S.-J.* A History of Logic Diagrams // D. M. Gabbay, J. Woods *et al.* (eds.). Logic. A History of its Central Concepts. Oxford, Amsterdam *et al.*, 2012. P. 611–682.

Murphy 1968 — *Murphy P. J*. The New Mathematics Made Simple. London, 1968.

Nolan 2002 — *Nolan C*. Music Theory and Mathematics // The Cambridge History of Western Music Theory / ed. by T. Christensen. Cambridge, 2002. P. 272–304.

Ploucquet 1759 — *Ploucquet G*. Fvndamenta Philosophiæ Speculativæ. Tübingae, 1759.

Ploucquet 1766 — *Ploucquet G*. Untersuchung und Abaͤnderung der logikalischen Constructionen des Hrn. Prof. Lambert // A. F. Boͤk (ed.). Sammlung der Schriften, welche den logischen Calcul Herrn Prof. Ploucquets betreffen, mit neuen Zusaͤzen. Frankfurt, Leipzig, 1766. P. 157–202.

Raymarus Vrsus 1589 — *Raymarus Vrsus N.* Metamorphosis Logicae […]. Argentorati, 1589.

Schopenhauer 1913 — *Schopenhauer A*. Philosophische Vorlesungen. Vol. I. // Sämmtliche Werke / Ed. by P. Deussen and F. Mockrauer. München, 1913.

Schopenhauer 1958 — *Schopenhauer A*. The World as Will and Representation. Vols. 1 and 2 / transl. by E. F. J. Payne. Mineola, 1958.

Schopenhauer 1978 — *Schopenhauer A*. Gesammelte Briefe / ed. by A. Hübscher. Bonn, 1978.

Schreiber 2003 — *Schreiber A*. Vorsicht, Mausefalle // Mitteilungen der DMV 11 (1), 2003. P. 58–59.

Shimojima 1999 — *Shimojima A*. The Graphic-Linguistic Distinction. Exploring Alternatives // Artificial Intelligence Review 13, 1999. P. 313–335.

Stekeler-Weithofer 1986 — *Stekeler-Weithofer P*. Grundprobleme der Logik. Elemente einer Kritik der formalen Vernunft. Berlin, New York, 1986.

Ueberweg 1871 — *Ueberweg F*. System of Logic and History of Logical Doctrines / transl. by Th. Lindsay. London 1871.

van Inwagen, Sullivan 2016 — van Inwagen, P., Sullivan, M. Metaphysics // The Stanford Encyclopedia of Philosophy (Spring 2016 Edition) / ed. by Edward N. Zalta. URL = <http://plato.stanford.edu/archives/spr2016/entries/metaphysics/> (accessed 28.06.2016).

Venn 1894 — *Venn J*. Symbolic Logic. 2nd rev. ed. London, 1894.

Vives 1531 — *Vives I. L*. De censura veri et falsi // De disciplinis Libri XX, Tertio tomo de artibus libri octo. Antverpia, 1531.

Vlrich 1792 — *Vlrich I. A. H*. Institvtiones logicae et metaphysicae. Scholae svae scripsit perpetva Kantianae disciplinae ratione habita. Ienae, 1792.

von Eberstein 1794 — *Freyherrn von Eberstein W. L. G*. Versuch einer Geschichte der Logik und Metaphysik Deutschen von Leibnitz bis auf gegenwärtige Zeit. Vol. 1. Halle, 1794.

Weigelius 1669 — *Weigelius E*. Idea Matheseos universæ cum speciminibus Inventionum Mathematicarum. Jenae, 1669.

Weigelius 1693 — *Weigelius E*. Philosophia Mathematica. Archimetria. Jenæ, 1693.

Weisius, Langius 1712 — Weisius, C., Langius, I. C. Nvclevs Logicae Weisianae. Gissa-Hassorum, 1712.

Wolters 1980 — *Wolters G*. Basis und Deduktion. Studien zur Entstehung und Bedeutung der Theorie der axiomatischen Methode bei J. H. Lambert (1728–1777). Berlin, New York, 1980.

Ziehen 1920 — *Ziehen T*. Lehrbuch der Logik auf positivistischer Grundlage mit Berücksichtigung der Geschichte der Logik. Bonn, 1920.