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ORIGINAL ARTICLE

Transcendental philosophy and logic diagrams

Jens Lemanski[1,2]

[1]Philosophisches Seminar, Universität Münster, Münster, Germany

[2]Faculty of Cultural and Social Sciences, FernUniversität in Hagen, Hagen, Germany

Correspondence
Jens Lemanski, Universität Münster: Münster, Nordrhein-Westfalen, Germany.
Email: jenslemanski@uni-muenster.de

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Abstract

Logic diagrams have seen a resurgence in their application in a range of fields, including logic, biology, media science, computer science and philosophy. Consequently, understanding the history and philosophy of these diagrams has become crucial. As many current diagrammatic systems in logic are based on ideas that originated in the 18th and 19th centuries, it is important to consider what motivated the use of logic diagrams in the past and whether these reasons are still valid today. This paper proposes that transcendental philosophy was a key inspiration for the development of logic diagrams and that such diagrams can be employed in transcendental arguments, even after the linguistic turn.

I. | INTRODUCTION

Since the 1990s, logic diagrams have been a topic of increasingly intensive research in many fields, including computer science, mathematics, psychology, biology and philosophy, to name but a few.[1] The breakthrough book that initiated research into this area was Sun-Joo Shin's *The Logical Status of Diagrams*,[2] which was published in 1994. Shin dispelled existing prejudices against diagrams in logic by demonstrating that they are not just a didactic or heuristic device but, like any representational system, have their own syntax and semantics. This enabled her to develop the first formal system using logic diagrams, which she based on Venn diagrams. In the

[1]Legg (2013).
[2]Shin (1994).

following years, other researchers have shown that similar systems can also be developed using Euler diagrams,[3] which are now considered the precursors of Venn diagrams.[4]

The research conducted in the 1990s demonstrated that the diagrammatic systems of Shin and her successors fully satisfy the demands of modern logic although their representational systems, that is, the logic diagrams, partly originated in the Middle Ages and partly in the early modern period.[5] Given the various research approaches that use modern methods and criteria to examine and further develop 'old' logic diagrams today, the history of logic diagrams is an important and fundamental source of knowledge.[6] Logic historians not only provide material on different systems of representation, but they are often also able to link philosophies with their associated reflections.

More recent findings have suggested that it depends on philosophical attitudes whether logic diagrams are used or not. In the 18th century, rationalists such as the Leibnizians and Wolffians publicly opposed the use of diagrams in logic, and in the early 19th century, it was mainly Hegelians who continued this opposition.[7] Later, roughly between 1880 and 1990, it was primarily mathematicians who supported the so-called 'crisis in intuition', dominating in mathematics, physics and logic.[8] In the 20th century, philosophy was also dominated by purely verbal approaches that only some phenomenologists, semioticians and structuralists thwarted.[9] Thus, the so-called 'golden age of logic diagrams'[10] was limited to the late 18th and the 19th centuries. In the early 19th century in particular, it was primarily Kantians or Kant-influenced scholars, within their different schools, who used and further explored logic diagrams.[11]

In this paper, we explore the philosophical connection between logic diagrams and Kantian philosophy. We will focus mainly on so-called 'Euler diagrams', which also played a major role in the renaissance of logic diagrams in the 1990s. We ask whether there is a definite connection between transcendental philosophy and diagrammatic representations such as Euler diagrams. We argue that not every transcendental philosophy uses logic diagrams and that not every use of logic diagrams presupposes the acceptance of transcendental philosophy. Nevertheless, we also present examples that provide evidence of a certain affinity between logic diagrams and transcendental philosophy, both historically and currently. As a result, a number of additional scholars, not to be underestimated,

[3] Hammer (1996).
[4] Moktefi and Lemanski (2022).
[5] Hodges (2023) and Moktefi and Shin (2012).
[6] Gardner (1982).
[7] Pluder (2022a).
[8] Blasjo (2019).
[9] Greaves (2001).
[10] Englebretsen (2019).
[11] Lu-Adler (2017).

could also be considered transcendental philosophers, advocating the use of logic diagrams. This affinity has not yet been sufficiently investigated.

To highlight this affinity, Section II outlines what is meant by the term 'transcendental philosophy' and the role Kant's philosophy had in the golden age of logic diagrams. We see that Kant did not provide a definitive answer to the question of how significant diagrams play were for his philosophy although he did provide many starting points that have since been elaborated upon further. Thus, in Section III, we consider Arthur Schopenhauer, a German-speaking philosopher, influenced by Kant who used logic diagrams and who argued for an extension of transcendental argumentation based on logic diagrams. Section IV then presents the work of Thomas Wirgman, an early Kantian from the English-speaking world and shows how he interpreted transcendental philosophy with logic diagrams. Finally, in Section V, we present rational representationalism, a current trend that is following on from the diagrammatic transcendental philosophy of the 19th century but, at the same time, also seeking to engage with analytic philosophy.

It cannot be assumed that all researchers in the field of logic diagrams are familiar with Kant and transcendental philosophy and, vice versa, not all scholars in the field of transcendental philosophy have experience with logic diagrams. Therefore, we have aimed to provide a general and simplified account of the topic and refer to the relevant sources for detailed inquiries. In addition, this text will be purely descriptive and interpretive without any criticism. The goal here is to demonstrate the role that logic diagrams have played and continue to play in transcendental philosophy, rather than to evaluate specific arguments or logical techniques.

II. | KANT AND TRANSCENDENTAL PHILOSOPHY

Immanuel Kant's groundbreaking works, beginning with the *Critique of Pure Reason* in 1781, were quickly recognised by his contemporaries as examples of transcendental philosophy. A well-known conversation between Schiller and Madame de Stäel has been widely cited as evidence of this. In this conversation, it was said that anyone who understood the term 'transcendental' would also understand Kant's philosophy.[12] Even during Kant's own lifetime, however, it was well-known that the word 'transcendental' had a much longer history than his philosophy. In fact, instances of the term can be found as far back as the Middle Ages.[13] Nonetheless, Kant played such a significant role in popularising it that the term has become almost synonymous with his own philosophy.

[12] Aertsen et al. (1998).

[13] Aertsen et al. (1998).

Furthermore, despite widespread recognition of the term ‘transcendental philosophy’, there has been considerable debate over what it means exactly. Scholars have identified several different interpretations of the term in Kant's writings and three strategies for clarifying its meaning have emerged[14]:

1. The meaning of transcendental and transcendental philosophy is defined in distinction to other terms so that contrary, contradictory or even subcontrary relations are sought between the term to be defined and an oppositional term, for example, transcendental/metaphysical, transcendental/transcendent, transcendental/psychological, etc.
2. The term ‘transcendental’ is defined directly through Kant's own definition of the term as given in sentences such as ‘transcendental philosophy is …’ or ‘… is called transcendental’. These sentences then serve as the primary determinant of the term's meaning.
3. The meaning of the term ‘transcendental’ is derived from its use in context, that is, sentences that include adjectives, adverbs or nouns with the root word ‘transcendental’ are used to illuminate the term's meaning.

Of these three approaches, the second is particularly notable. Two passages from the *Critique of Pure Reason*, in which Kant provides explicit definitions of the term ‘transcendental’, are widely regarded as important not only for Kant himself, but also for his successors. They are

(2.1) ‘I call all cognition transcendental that is occupied not so much with objects but with our manner of cognition of objects insofar as this is to be possible a priori. A system of such concepts would be called transcendental philosophy’.[15]

(2.2) ‘And here I make a remark the import of which extends to all of the following considerations, and that we must keep well in view, namely that not every a priori cognition must be called transcendental, but only that by means of which we cognize that and how certain representations (intuitions or concepts) are applied entirely a priori, or are possible (i.e., the possibility of cognition or its use a priori). Hence, neither space nor any geometrical determination of it a priori is a transcendental representation, but only the cognition that these representations are not of empirical origin at all and the possibility that they can nevertheless be related a priori to objects of experience can be called transcendental’.[16]

[14]Aertsen et al. (1998).
[15]Kant (1998: B25).
[16]Kant (1998: B 80f.).

Both definitions reveal some variations, but they also share a common element that many consider the central aspect of transcendental philosophy. This aspect can be summarised using a popular formula present (at least partly) in both definitions, as well as other places in Kant's work: Transcendental philosophy seeks to uncover *the conditions of possibility for knowledge, cognition, experience and so on*. This definition of transcendental philosophy is the basis of numerous studies that present it in even more detail, for example.[17] Indeed, this definition serves as a basis for characterising so-called *transcendental arguments*.

> In Kant's conception, an argument of this kind begins with a compelling premise about our thought, experience or knowledge, and then reasons to a conclusion that is a substantive and unobvious presupposition and necessary condition of this premise.[18]

In the following discussion, we will continue to work with this common element, but we do not want to exclude the possibility that other significant aspects of transcendental philosophy may also be instrumental in determining the relationship between transcendental philosophy and logic diagrams.

Let us take the definition that the method of transcendental philosophy is the search for the conditions of the possibility of something and that this something is, for example, *cognition*. In quotations 2.1 and 2.2, cognition is defined more precisely and Kant refers to the form of cognition to be examined in many other passages (e.g., B 28). Specifically, he is concerned with a priori cognition that is valid independently of all experience but is only actualised in the subject through experience. This means that, because we are subjects, we must already possess the conditions of possibility required to experience a circle but we can only actualise this possibility through direct experience of the circle. Furthermore, cognition should be synthetic, meaning that two separate pieces of information should be connected in such a way that they result in a new piece of information. For example, the proposition that a circle is round would not be synthetic because it is already part of the necessary properties of a circle.

Kant distinguished between two essential forms of a priori knowledge, intuition and concept, which are associated with two areas of transcendental philosophy. Intuition or sensibility is examined in transcendental aesthetics, whereas concept or understanding is examined in transcendental logic. Both areas then have to be classified in more detail: aesthetics deals with space and time, whereas logic is divided into categories, principles and inferences

[17]Aertsen et al. (1998), Piche (2016), and Gardner and Grist (2015).
[18]Pereboom (2022).

according to the Aristotelian *Organon*.[19] For the purposes of this paper, this basic distinction between intuition and concept, corresponding to the two fields of transcendental aesthetics and logic, is sufficient. Only through the interplay of both main areas can specific and meaningful cognition be achieved, as Kant famously stated: 'Thoughts without content are empty, intuitions without concepts are blind'.[20]

Kant's transcendental philosophy is often considered revolutionary because it is considered as a fusion of rationalism and empiricism that not only draws from the philosophies of Leibniz and Wolff but also from Locke and Hume others well. The now well-established definition of transcendental philosophy or transcendental arguments allows a specific argumentative connection to be made with this historical lineage. From the standpoint of transcendental philosophy, both predecessor schools are one-sided: Rationalism privileges the concept and neglects intuition, while empiricism privileges intuition and neglects concepts. For transcendental philosophy, however, cognition is mainly a conjunction of both areas.

Diagrams provide an interesting arena within which to explore the interplay between intuition and concept, something that Kant reflected upon primarily in mathematics but which he also applied to logic. Thus, while Kant did not reflect on the meaning of logic diagrams specifically, one can transfer his arguments from the philosophy of mathematics to the philosophy of logic diagrams. It is noteworthy that, around the year 1790, Kant was intensively reflecting on Euclidean figures in mathematics and utilising Eulerian diagrams in formal logic. (For the difference between transcendental and formal or general logic in Kant[21])

Let us begin by examining logic diagrams. Kant attributed the origin of his logic diagrams specifically to Euler. Figure 1 displays the four fundamental Euler diagrams for the judgements provided in syllogistics: a = All B are A (aBA); e = No A is B (eAB); i = Some A is B (iAB); o = Some A is not B (oAB). Kant mistakenly identified aAB with aBA, but any logician would quickly recognise the agreement with Euler. Kant then deduced inferences from the four basic Euler diagrams, such as Darii in Figure 2 (aCB, iAC, thus iAB). He employed these and many other diagrams in his logic lectures, particularly between 1770 and 1790. In general, Kant's diagrams align with Euler's; however, Kant also endeavoured to implement them in other fields beyond syllogistics. Moreover, Kant's manuscripts feature some logic diagrams whose meaning and function remain unclear.

A small selection of Kant's extensive range of logic diagrams became known through the works of his students and fellows, including Friedrich August Nitsch, Johann Gottfried Kiesewetter and Georg Samuel Albert Mellin. Others

[19] Santozki (2006: sect. 4.1.1.).

[20] Kant (1998), Kovač (2020).

[21] See Kovač (2020).

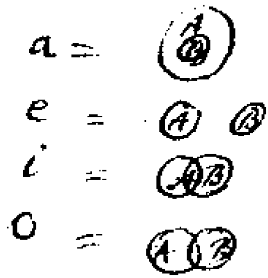

FIGURE 1 Kant's Euler diagrams taken from I. Kant's Manuscripts, see Kant and Meier 1752: 94.

FIGURE 2 Kant's Euler diagram for Darii taken from Kant's manuscripts, see Kant and Meier 1752: 94.

became familiar through the textbook on Kant's logic edited by another of his students, Benjamin Jäsche. In his logic, Kant states that every concept has a sphere (as its extension) that may or may not contain another in a judgement, something which is intuited by a circle in an Euler diagram; thus, according to the well-known Kantian formula provided above, the concept is enriched by the intuition. As Kant never contemplated the relationship between concept and intuition in logic diagrams, it is worthwhile considering his mathematical discussion. When reflecting on diagrams in mathematics, Kant primarily had Euclidean geometry in mind. The examples Kant discussed that are relevant here include Proposition 21.[22] and Proposition I,5 of the Euclidean *Elements*.[23] For further details, see.[24] While empiricists view the intuitive power of Euclidean theorems as primarily in their empirical forms, rationalists deny the evidential power of empirical forms and argue that it is the innate laws of thought that make mathematics alone true. The empirical form in this case is, for example, the diagram drawn in a Euclidean proof. During Kant's time, the German-speaking world was dominated by rationalists, which was why Kant

[22]Kant (1998: B 474f.).

[23]Kant (1900: X, 489).

[24]Koriako (1999), Friedman (2012), Posy and Rechter (2019).

opposed the one-sided view that considered diagrams or empirical forms in general useless.

Kant had a special role in this debate. On one level, he attempted to prove both rationalists and empiricists right and to mediate between the two. He argued that both concept and intuition play a crucial role in a Euclidean proof, as the diagram cannot speak for itself without a concept and the proof expressed in terms could not be applied without a diagram. This argument can be translated into the famous formula: 'Diagrams without concepts are blind, and concepts without diagrams are empty'.

On the other level, Kant overcame the argument that the proving forms must be empirical using transcendental arguments. The conditions of the possibility of knowing a Euclidean figure are a priori and not a posteriori or empirical. This implies that in sensuality, we presuppose the human cognition of space and time, and that, in logic, we presuppose certain basic concepts or categories such as quantity, quality, relation and modality in order to be able to produce any Euclidean proof. To put it simply, the human mind must be such that it can recognise and imagine geometric shapes in space. Without this a priori prerequisite, no real diagram could be constructed and no conceptual explanation in the form of a Euclidean theorem could be given.

Therefore, diagrams are not, essentially, empirical forms because diagrammatic reasoning is an ability of the human mind and a 'pure intuition'. Whereas rationalists argue that our senses can be deceptive and, thus, sensorially perceived diagrams are inaccurate and uncertain, Kant points out that it is also possible for an imagined diagram to contribute to knowledge. Therefore, the imaginability of a fact in a diagram is a condition of the possibility of cognition. The precondition for the cognition of a mathematical concept is the possibility of representing the corresponding intuition a priori.[25] Kant had already realised in his dissertation that the axioms of Euclidean philosophy must be diagrammatically constructible if we are to be able to derive theorems from them intuitively rather than logically:

> This pure intuition is in fact easily perceived in geometrical axioms, and any mental construction of postulates or even problems. That in space there are no more than three dimensions, that between two points there is but one straight line, that in a plane surface from a given point with a given right line a circle is describable, are not conclusions from some universal notion of space, but only *discernible* in space as in the concrete.[26]

[25]Kant (1998: B 741).

[26]Kant (1894: §15).

Like many of his contemporaries, Kant did not use the term 'diagram'. During his lifetime, diagram was simply another word for what was known as a figure in geometry. Although the term diagram has been around for a long time, its current definition did not emerge until the 19th century. Indeed, the term 'logic diagram' was only popularised by Charles Sanders Peirce. The diagram conceptually reflected in Kantian linguistic style can be best categorised as a schema. Put simply, a schema connects a fundamental concept of understanding (category) with an intuition that may be either empirical or a priori. The empirical element is the actual physical diagram, which cannot claim generality and is, thus, a picture. However, the a priori diagram (also referred to as an 'imagined diagram') is a pure schema that possesses generality.[27] Much more could be said about Kant here, but it should be sufficient to have demonstrated that Kant opened up a transcendental perspective on diagrams in his epistemology and philosophy of mathematics that is reflected in his use of logic diagrams.

III. | SCHOPENHAUER'S LOGIC DIAGRAMS IN TRANSCENDENTAL ARGUMENTATION

In Section II, we saw that transcendental philosophy after Kant has made use of numerous logic diagrams although Kant himself only reflected on diagrams in mathematics. As logic diagrams were, in Kant's time, simply applications of Euclidean figures to logic, the arguments from the discussion of mathematical diagrams can be transferred to logic diagrams. Kantian transcendental philosophy holds that diagrams play an important role in the process of cognition, as intuition and concepts must interact during this process.

During his lectures on logic from around 1790, Kant made extensive use of logic diagrams. Some were based on Euler's diagrams but he also created new diagram forms that have not yet been researched. By 1790, the first textbooks on Kantian logic had already emerged with an increasing focus on diagrams.

These represent a major achievement in transcendental philosophy, as earlier rationalists had censored Euler's book removing logic diagrams from them. Following Kant, German-speaking pioneers in the use of logic diagrams included Johann Gottfried Kiesewetter, Johann Gebhard Ehrenreich Maaß and Georg Samuel Albert Mellin. In 1800, Jäsche compiled and edited the *Lehrbuch zur Logik*, which was based on Kant's manuscripts and included several diagrams from Kant's lectures.[28] Early recipients who had further elaborated on and published these diagrams by the 1820s include Karl Christian Friedrich Krause, Wilhelm Traugott Krug, Jakob Friedrich Fries and Arthur Schopenhauer. Krause and Maaß deserve special mention here given their intensive engagement with

[27]Kant (1998: B 180).

[28]Kant (1992).

Euler-Kant diagrams in their publications. However, it should be noted that while Maaß was critical of both the diagrams and Kant, Krause expressed support for Kantian philosophy and logic diagrams. Nevertheless, it is difficult to identify an explicitly transcendental philosophical approach in these writings.

The first author to explicitly strive for a transcendental argument that incorporated diagrams as a crucial component was Schopenhauer. In his main work of 1818, *The World as Will and Representation*, he hinted at a logic based on Euler diagrams but he did not publish much of the material as he believed that logic was a specialised subject that only concerned academic philosophers. He did not want to bore his broad audience with such specific and unnecessary topics, especially since he believed that everyone could argue logically by nature. Given his brevity and the misunderstanding of logic and mathematics evident in his published works, Schopenhauer was discredited until recent years. In the 2010s, researchers from various disciplines such as philosophy, linguistics, history of science and AI, revised this image of Schopenhauer[29] and we now know that Schopenhauer studied logic and mathematics based on diagrams intensively in his lectures in Berlin.

Schopenhauer himself considered his work on the use of diagrams in logic and mathematics to be some of his most important contributions to theoretical philosophy. No one, he writes, has ever worked as intensively on logic diagrams as he has.[30] He used mainly Euler-type diagrams, but also partition diagrams and tree diagrams, and he seems to have had a square of opposition in mind in some passages of the text although this is no longer traceable in the manuscripts.[31] Furthermore, he perceived his diagrammatic method for mathematics as a definitive progression over all of his forerunners even if he also acknowledged that it required further refinement.[32] In the following, we will concentrate only on his use of Euler diagrams.

Schopenhauer became acquainted with Euler diagrams through the work of Gottlob ‘Aenesidemus’ Schulze and, at that time, was simultaneously studying mathematics using textbooks based on the Kantian philosophy of intuition.[33] Schopenhauer also studied mathematics under Professor Bernhard Friedrich Thibaut in Göttingen and was familiar with Franz Ferdinand Schweins' textbook, which he had worked with extensively in his school days. As early as 1818, he reviewed both Thibaut's and Schweins' textbooks favourably, as both attributed a decisive role to diagrams in mathematics.[34]

[29]Fouquere and Quatrini (2012), Moktefi and Lemanski (2018), Pluder (2020).

[30]Schopenhauer (2022: 244).

[31]Lemanski and Demey (2021).

[32]Schopenhauer (2022: 443f.).

[33]D'Alfonso (2018).

[34]Lemanski (2022).

Schopenhauer's treatises on logic and mathematics, published posthumously in the so-called *Berlin Lectures* of the 1820s, are heavily based on diagrams. Schopenhauer assumed in his philosophy of language, that concepts have an extension or sphere that can be symbolised by a circle.[35] Indeed, concepts were fundamental to Schopenhauer's compositional approach: They become judgements that give rise to inferences and, eventually, whole systems of inferences that form a philosophy. Thus, in his chapter on judgements, he developed a question of a transcendental philosophical nature: What is the condition of the possibility of judgements? He assumed that every judgement is a connection or disconnection of at least two concepts.[36] However, if concepts can be represented by spheres or circles, then concepts in judgements cannot behave differently from circles in Euclidean space. In this regard, until this point, Schopenhauer is still in complete agreement with Kant, for example.[37]

Schopenhauer was concerned with 'deriving the possibility of judgement' from its conditions.[38] In order to investigate the potential conceptual relationships within judgements, Schopenhauer embarked upon a project inspired by Schweins that he encountered in his studies of *Mathematik für den ersten wissenschaftlichen Unterricht*, Vol. II.[39] Schweins' geometric approach aligns with Schopenhauer's logic in being strictly compositionalist. Points become lines, straight lines become triangles, quadrilaterals and polygons, while curved lines, if their curvature is uniform, become circles. These figures can then be used to construct surfaces and, ultimately, solids. Schweins first presented the properties and components of a circle; then, the relationships that two or more circles have with each other and their components. For example, based on the main properties that Schweins enumerated, when considering two circles in relation to each other, they can either be inside each other (Figure 3: fig. 74), apart (Figure 3: fig. 76) or connected by two average points (Figure 3: fig. 79).

Schopenhauer then selected from among these spatial relations of circles those in which he saw an isomorphism to the relations of concepts in the judgement. All judgements must correspond to geometric diagrams but not all geometric diagrams depict judgements, at least not in natural language. Therefore, the condition of the possibility of knowledge in the form of judgements is based on the possible circular positions in Euclidean space. Schopenhauer viewed this as a clear extension of transcendental philosophy and he criticised Kant's method in transcendental logic for being based on

[35] Schopenhauer (2022: 226).

[36] Schopenhauer (2022: 230).

[37] Kant (1992: 593).

[38] Schopenhauer (2022: 241).

[39] Schweins (1810).

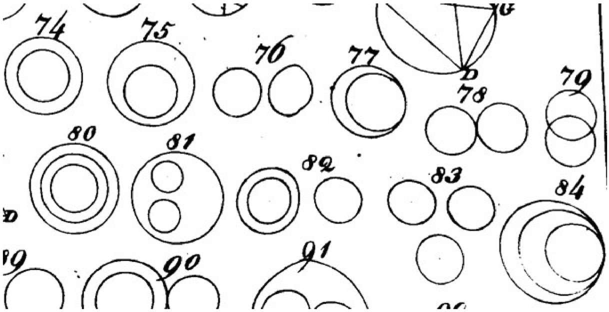

FIGURE 3 Schweins' Diagrams in Schweins (1810: table II).

arbitrariness rather than on transcendental reasoning. He argued that Kant had merely compiled the basic judgements from 18th-century logic text-books.[40] The fact that this criticism is still being discussed today is evident in some current studies.[41] Schopenhauer boasted of having found a transcendental method for discovering all forms of judgement in natural language by reflecting on the conditions of intuition in Euclidean geometry. Thus, he stated, the ‘clue’ (Leitfaden) to the discovery of all basic judgements ‘are the schemata’,[42] a reference to the concept of a guideline or clue that Kant employed in his transcendental logic.[43]

This transcendental method can be imagined as follows: taking into account the isomorphism of circles and concepts, and with the help of Schweins, Schopenhauer constructed all possible spatial relations between circles and checked whether a change in the spatial relation has semantic meaning, that is, whether it can depict a possible judgement. This is the case, for example, with nos. 74, 76 and 79 depicted in Figure 3. As Schopenhauer is quick to note, these forms correspond to the well-known Euler diagrams for syllogistics. The exact correspondence can be observed by comparing the spatial positions of circles in Figure 3 with Figure 1. In total, Schopenhauer identified six basic forms of judgement, known in the literature as the ‘relational diagrams’ that correspond to the basic forms of oppositional geometry.[44]

Figure 4 presents, as an example Schopenhauer's relational diagram number 3, which corresponds to Schweins's no. 79 in Figure 3. Figure 4 depicts the concept red (Roth) in the left circle and the concept flower (Blume) in the right. Schopenhauer identified the diagram in such a way that the shape

[40] Cartwright (2010: 206f.).

[41] Lu-Adler (2016).

[42] Schopenhauer (2022: 244).

[43] Kant (1998: B 102).

[44] Lemanski and Demey (2021).

FIGURE 4 Schopenhauer's relational diagram no. 3 taken from the manuscripts of the *Berlin Lectures*, Staatsbibliothek zu Berlin/ PK, Na 50, NL Schopenhauer, 1428, Bl. 123v.

presents the i- and o-judgements of the Euler diagrams as shown in Figure 1, i.e. 'some flowers are red' and 'some flowers are not red', as well as their conversions.

Whether a circle lies tangentially (Figure 3, nos. 77 and 78) to another or non-concentrically in another (Figure 3, no. 75) has no semantic significance and is, therefore, ignored by Schopenhauer. Some modern logical calculi, such as region connection calculus, view this differently. Nevertheless, Schopenhauer's method shares similarities with these modern logical systems in that it is agreed that diagrams provide a possible condition for representing and proving logical reasoning.

While Schopenhauer supplemented Schwein's approach, he also critically examined whether the spatial relationship of two circles to a third is significant for a judgement. However, the full details of this investigation are not needed here. What is crucial for our inquiry is that Schopenhauer's approach not only extended Kant's ideas but also employed logic diagrams for arguments that were previously not considered transcendental in nature.

IV. | WIRGMAN'S EXPLANATION OF TRANSCENDENTAL PHILOSOPHY THROUGH LOGIC DIAGRAMS

In the previous sections, we learned that diagrams played an important role in Kant's theory of cognition, particularly in mathematics. According to Kant, cognition is not solely based on language or purely mental processes as rationalists assert, but also on intuition. However, this intuition need not always be empirical, as empiricists argue, it can also be a priori or imagined. In some cases, diagrams or diagrammatic reasoning are, therefore, the condition that makes cognition possible, as in theorems of Euclidean geometry and logical facts.

Schopenhauer took this a step further by applying logic diagrams to transcendental philosophy itself. He identified a gap in Kant's argumentation that is still debated today and developed a transcendental argument to demonstrate that logic diagrams are essential to establishing a guideline for propositional knowledge. As such, logic diagrams play a substantive role in transcendental philosophy and work as a condition of the possibility of knowledge in a transcendental argument.

Thomas Wirgman offers a third approach in which logic diagrams play a vital role in transcendental philosophy. Although Wirgman was not a professional philosopher and was known for his obscure interpretation and reverential copying of Kant, his education and influence demonstrate the significant role he played in promoting Kantian transcendental philosophy and the use of logic diagrams in Britain. Wirgman studied with Nitsch in London in the mid-1790s and, as a later member of the Kantian Circle, was also in contact with Coleridge, Madame de Stäel and other well-known intellectual figures of the early 19th century. Despite all the traditional criticism of his personality and his peculiar writing style, Wirgman was nevertheless an independent and intelligent interpreter of Kant who should not be underestimated.

Wirgman's teacher, the mathematician Nitsch, studied logic with Kant himself, in the early 1790s, when Kant's manuscripts were replete with logic diagrams. Nitsch also published Euler-type diagrams in his writing that Wirgman later adopted in a slightly modified form. However, Wirgman took them a step further and attempted to interpret the fundamental concepts of transcendental philosophy through logic diagrams. Almost all of Wirgman's diagrams resemble Euler's, even though they are frequently deployed in fields beyond logic. According to him, it was possible to understand transcendental philosophy in its entirety through pictures and intuitions, which is why logic diagrams can be said to represent the visual and sensory form of Kantian philosophy.

Wirgman began publishing this approach from the 1810s onwards and found many influential recipients both during this period and in the decades that followed. One notable recipient of Wirgman's diagrams was Dugald Stewart, who found the Kantian criticisms of his philosophy that Wirgman presented to him in letters, so compelling that he revised his logic in response.[45] William Stirling Hamilton of Preston was another recipient of Wirgman's work, who later authored the seminal textbook on logic and metaphysics that almost all English-speaking logicians up to and including John Venn studied. Similarly, Augustus De Morgan engaged with Wirgman's writings, albeit not without his typical sarcasm.[46]

Wirgman's approach began in 1812 with several very idiosyncratic but thoroughly erudite articles in the *Encyclopaedia Londinensis*, including one on Kant and another on logic. The latter article included a discussion of various logical approaches from the medieval Arabic-speaking world and his contemporaneous, modern English milieu, and acknowledged the attempts at a 'universal logic' made by Leibniz and Lambert.[47] Wirgman not only provided a detailed account of the *Jäsche Logic* organised according to the Kantian table of judgements, enriched with information from the *Critique of Pure Reason* and the

[45]Bow (2022).

[46]Wellek (1931: 212).

[47]Wirgman (1815: 1–4).

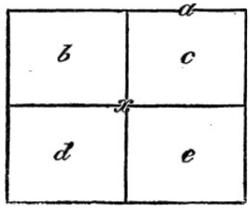

FIGURE 5 Wirgman's interpretation of Kant's diagram for disjunctive reasoning, Kant (1992: 109) taken from Wirgman (1815: 23).

False Subtlety of the Four Syllogistic Figures, he also repeatedly emphasised Stoic logic and included a chapter on probabilistic reasoning. In addition, he provided a paraphrase of Kantian logic that was more in line with the mathematical style of the time than Kant's own formulations. To illustrate, we present the diagram for disjunctive judgements from the *Jäsche Logic* (Figure 5):

'In the Disjunctive Judgment, Kant is either a European, an Asiatic, an African or an American; let *x* represent the intuition Kant; let *a* represent the whole sphere of the Conception under consideration, namely, the world; and *b*, *c*, *d*, *e*, the members of Disjunction. Hence, as *x* is contained under *a*, it must consequently be found either in *b*, *c*, *d*, or *e*, which taken together complete the sphere of *a*'.[48]

Furthermore, it is peculiar that, in these articles, Wirgman attempts to partially axiomatise Kantian philosophy. Just like in the *Jäsche Logic*, Wirgman distinguishes between intuitive principles or axioms and conceptual principles or acroams[49] and considers synthetic principles axioms when they are intuitive.[50] For example, Wirgman considers the following proposition an axiom: 'All that one can know are representations, but not the objects which produce them'.[51] Thus, representations are the necessary conditions of the possibility of knowledge. From this proposition, he derives both a general and a special proposition of representation, the general being supported by an apagogical proof. In this, attentive readers may recognise a clear similarity to the Kantians or Kant-influenced philosophers such as Karl Leonhard Reinhold or Schopenhauer.

It is unfortunate that Wirgman only rarely reflected on logic diagrams. However, he offered two interesting passages on transcendental philosophy, which I explore here first. In his encyclopaedia article on logic, Wirgman explained the hypothetical method that bears a clear resemblance to today's

[48]Wirgman (1815: 23).

[49]Kant (1992: 606).

[50]Wirgman (1815: 23).

[51]Wirgman (1815: 18).

methodological interpretation of transcendental philosophy. The hypothetical method is primarily concerned with clarifying 'the possibility of the presupposition itself'.[52] If the possibility is established, it can be assumed to be the reality provided a reason is given. This is the case, for example, with the Copernican hypothesis, as Kant explains in the second introduction to the *Critique of Pure Reason*. Although Wirgman does not elaborate further, this interpretation of the Copernican revolution is linked to the understanding of contemporary transcendental philosophy in Kantian scholarship.[53]

In his encyclopaedia article on Kant, Wirgman made the explicit statement that transcendental philosophy 'possesses as much internal evidence as the Elements of Euclid'.[54] Wirgman organised transcendental philosophy into three faculties of the human mind: sense, understanding and reason. In the article on logic, these three faculties were presented as a derivation of the specific proposition of knowledge. Furthermore, these three faculties build on each other compositionally, with the result that Wirgman's interpretation of Kant holds that the condition necessary for the possibility of knowledge must always be based on the senses. Wirgman used logic diagrams to explain all three faculties in the encyclopaedia articles and his later writings. However, at this point, we will limit the following interpretation to the logic diagrams for sensibility and show why Wirgman uses Eulerian diagrams in the first place.

Wirgman follows Kant's division of sensibility into internal and external senses, with the former representing time and the latter space. According to Wirgman, these two senses form the transcendental foundation and the 'absolute limit of all knowledge'[55] because every object about which we can make a true statement must be able to be grasped with the senses a priori or a posteriori. As early as 1812, Wirgman argued that the senses are also compositional because space is a necessary condition for the possibility of temporal intuition, but not vice versa.

> Time has a larger sphere than Space; for whatever is in Space is in Time also. But we cannot say, conversely, that whatever is in Time is in Space also; for instance, a Thought is in Time, as it has beginning, middle, and end; but it is not an extended body, consequently not in space.[56]

In this quotation, Wirgman applied Kantian formal logic in order to explain transcendental aesthetics. Every concept has both content and extension, and every concept is both contained in the idea of things (content) and

[52] Wirgman (1815: 16).
[53] Brandt (2007), Moledo (2017), Olson (2018).
[54] Wirgman (1812: 605).
[55] Wirgman (1812: 608).
[56] Wirgman (1812: 608).

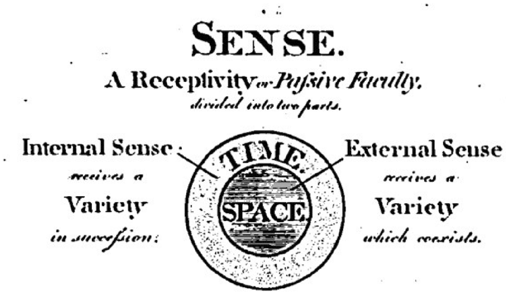

FIGURE 6 Wirgman's diagram of transcendental aesthetics given in Wirgman (1812: 608).

contains properties of those things (extension).[57] Consider this in relation to the rule of subalternation, which states the particular can be deduced from the universal but not the other way around: 'A Judgement is termed subalternate when it is contained under another, as particular under universal'.[58] This extension can be represented as a space or sphere, which is symbolised by circles in spatial logic diagrams such as Euler diagrams. The inference from the smaller sphere to the larger sphere is universal according to the subalternation rule while the conversion is only partial.

In 1812, Wirgman presented his approach using the Euler diagram depicted in Figure 6. In the 1820s, he published several works on the philosophy of religion, the philosophy of mind and natural science. From these works, it is evident that Wirgman saw a close connection between transcendental philosophy and logic diagrams: All his works are replete with them and he often used the term 'transcendental philosophy' interchangeably with 'Kantian philosophy' in his titles and writings. Indeed, his short work, *Principles of the Kantesian or Transcendental Philosophy*, which was later dubbed *British Euclid* in subsequent editions, is a notable example of this.

In his work *Mental Philosophy*, a pedagogical dialogue published in 1838, Wirgman argued that time and space are 'universal and necessary elements of the mind',[59] that is, they are the conditions of the possibility of cognition. Although they are a priori, concrete cognition arises when objects are perceived through at least one of the five senses. Wirgman distinguished these senses into two types: outward senses, such as the eye and hand, which can take in a lot of information at once, and inward senses, such as the ear, tongue and nose, which can only perceive one piece of information at a time. In this book, Wirgman assigned each of the five senses to one of the two types of senses and illustrated the logical relationship between them using the same type of Euler diagram

[57] Wirgman (1815: 20).
[58] Wirgman (1815: 24).
[59] Wirgman (1938: 12).

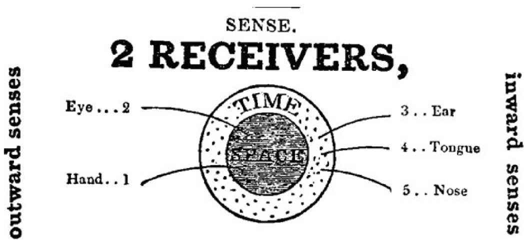

FIGURE 7 Wirgman's diagram of the two receivers taken from Wirgman (1938), appendix.

used for *a*-propositions. Here, the Kantian senses are depicted as receivers, as given in Figure 7, the categories of understanding as 12 builders, and the six rules as regulators of reason.

Throughout his writings, Wirgman employed Euler diagrams that utilised the cognitive potential of space, not only to represent the faculty of sensibility but also to explain other faculties. For instance, the Euler diagrams set up in the Kant article are utilised to explain the conceptual extension in the Kantian table of categories and judgements. In his encyclopaedia article on logic, these conceptual extensions were then transferred to the doctrine of immediate inferences.

It is necessary to examine the contents of this diagrammatic approach more closely. In many places, the extent to which the diagrams actually reflect Eulerian and Kantian functions is questionable, as is the extent to which Wirgman actually adhered to Kant's ideas. In the literature, Wirgman is often depicted as a copyist of Kant[60] but his application of logic diagrams to areas of transcendental philosophy that are not usually interpreted logically probably remains unique to this day. Moreover, it should be noted that Kant himself used logic diagrams in his manuscripts to illustrate topics that were never treated diagrammatically in his published writings. Therefore, Wirgman followed Kant's intuitive diagrammatic approach without having known it directly at all, for example.[61]

Even though Wirgman's later writings, in which he moved further and further away from Kant, may initially deter many readers with their enthusiasm and idiosyncrasies, they also offer material worthy of discussion. At this point, it suffices to show that, based on the example of transcendental aesthetics, diagrams not only played a role in transcendental philosophy (Kant), logic diagrams can also be used for transcendental arguments (Schopenhauer) and transcendental philosophy can be understood diagrammatically (Wirgman).

[60]Wellek (1931).

[61]Schulthess (1981: 119).

V. | RATIONAL REPRESENTATIONALISM AND LOGIC DIAGRAMS AS A TRANSCENDENTAL PRECONDITION FOR LANGUAGE

The use of diagrams in philosophy has been a topic of increasing interest in recent years, with most approaches being derived from phenomenology and semiotics, and often neglecting the logical aspects or logic diagrams. Despite this, rational representationalism is associated with transcendental philosophers, such as Kant and Schopenhauer, and merges their ideas with those of Wittgenstein, Apel, Blumenberg and Shin. The development of rational representationalism can also be traced to various works based on Schopenhauer and Kant,[62] some of which have provided a critical perspective.[63] The most comprehensive presentation of rational representationalism in English was provided in 2021,[64] and this will be the main source for what follows. Although rational representationalism may incorporate various forms of logic diagrams and links to transcendental philosophy, it has yet to fully articulate the nature of this connection. Therefore, this section explores the precise relationship between logic diagrams and the transcendental approach, thus contributing to a more detailed understanding of rational representationalism.

Rational representationalism seeks to reconcile two previously opposing schools of thought, modern rationalism and representationalism. Moreover, it aims to connect transcendental philosophy and analytical philosophy or philosophy post-linguistic turn. From one perspective, rational representationalism opposes the anti-representationalist tendencies prevalent in contemporary philosophy, epitomised, in particular, by Richard Rorty and his intellectual successors. Yet, seemingly conversely, it also advocates for constructive definitions of representationalism. Let us begin by examining the interpretations of representationalism put forth by its critics and proponents.

According to critics, representationalism posits that we perceive the external world directly, that representationalist philosophers 'find it fruitful to think of mind or language as containing representations of reality'.[65] Thus, in his early philosophical work, Rorty established the linguistic turn as dogma and, in later years, criticised analytical philosophy continuing representationalism without offering a truly positive philosophy to replace it (the value of Rorty's hermeneutic approaches is not discussed here). In response, anti-representationalist and rationalist philosophies such as inferentialism, which replaces intuitive representation with linguistic explication, emerged.

[62] Matsuda (2016), Dobrzański (2017), Beziau (2020).

[63] Schumann (2020).

[64] Lemanski (2021).

[65] Rorty (1990: 2).

According to its proponents, the definition of representationalism does not significantly deviate from that proposed by its critics. However, they do offer more precise definitions and explanations. The definition proposed by Plagnol,[66] which has been slightly modified for the purpose of our discussion, can be used as a demonstration: 'A set of entities *E* is a representation system for the world *W* if *W* can be reconstructed at least ideally from *E* according to a representation function *F* that links some elements of *W* (contents) with some elements of *E* (representations)'. Proceeding with this definition, with some additional assumptions, for example, that an entity *E* cannot represent any other entity than *E*,[67] now enables us to avoid becoming distracted by other issues that do not need to be discussed in detail here.

Rational representationalism views representation as the essential task of philosophy because philosophy is not just about good arguments, it is also about developing a rational understanding of the world and reality. In this sense, rational representationalism defines philosophy as *sapientia mundi* (wisdom of the world), a qualitative representation of the world distinct from the quantitative approaches of natural science.[68] An anti-representationalist philosophy is destined to embrace innatism or strict rationalism that relies solely on innate concepts, raising doubts about their connection to the external world. As per the famous Kantian formulae, conceptual philosophy without the world is empty and the world without concepts is blind.

However, the representationalist approach persisted in classical approaches from Bacon's *Advancement of Learning* to Carnap's *The Logical Structure of the World*, seeking to represent the world, primarily, through the application of rational arguments and techniques. In contrast, in the modern and pure rationalist approach of philosophy post-linguistic turn, rational methods of logic dominate and all philosophy is believed to be based on the linguistic constitution rather than the mental states of the subject. As Rorty pointed out, language is considered to be the subject that is perceiving the world and, according to the philosophers that followed him, modern rational methods are to be given primacy. However, rational representationalism seeks to unite both of these approaches.

At first glance, it may appear unsurprising that rational representationalism would acknowledge the use of rational methods of logic as a means of representation, that rational methods, such as logic diagrams, would be used to represent representations.[69] However, it became more challenging to maintain a representationalist philosophy that refers to the world after the paradigm shift that accompanied the linguistic turn. For if philosophy and science are to be expressed in terms of linguistic and rational relationships, then surely the world

[66]Plagnol (2022: 738f.).

[67]Plagnol (2022), p. 739.

[68]Pluder (2022b).

[69]Matsuda (2016), Lemanski and Demey (2021).

is something that exists outside language. Thus, any such reference to representation, no matter how rational, would be a regression from the paradigm that regards language as the central element of philosophising. This problem was identified by philosophers associated with inferentialism[70] and transcendental philosophy.[71]

This is where the transcendental argumentation of rational representationalism and its affinity to logic diagrams (and gestures) comes into play. Rational representationalism argued that there are 'foreign bodies' in language,[72] that pure rationalism or anti-representationalism had become confused by the 'idea of an idea' or the 'concept of a concept' and had not established the existence of different forms of language. With reference to metaphorology, rational representationalism has examined this linguistic dogma of rationalism more closely and found that a concept is not just a concept but a metaphor. Concepts are defined as linguistic elements based on real or 'worldly actions'. Just as we take objects in hand and perform real actions with them, concepts take on meanings and perform linguistic actions with them. The concept, *conceptus*, *Begriff* comes from conceiving, *concipere*, *begreifen* and has the function of grasping, as previously recognised by the Stoic philosophers.[73] Thus, in rational representationalism, a concept is, itself, a metaphor that refers to the world.

However, rational representationalism goes beyond 'traditional' metaphorology and demonstrates that the vocabulary of our rational theories is riddled with 'foreign bodies' like these. Metaphors are present in logic everywhere and the rationalist attempt to replace them did not realise that translating one metaphor often results in the creation of another metaphor or the expression of the metaphor through algebraic notations instead of linguistic pre-conditions.[74] Of course, this does not mean that logic diagrams are better than linguistic expressions or algebraic notations (in fact, in some theories and contexts, they appear far worse) but diagrams can help us understand what we are doing when we reason rationally. Furthermore, rational representationalism does not argue that language only consists of metaphors as some other philosophies of language maintain, only that our rational theories contain 'absolute metaphors'.[75] These linguistic forms exist in 'the logical space of concepts' and give the impression of that they are concepts. However, they are actually only metaphors, just as 'metaphor' itself is a

[70]McDowell (1996: VI.8).

[71]Apel (1972).

[72]Lemanski (2021: 424).

[73]Blumenberg (2010: chap. 1).

[74]Apel (1972).

[75]Blumenberg (2010: Intr. and chap. 8).

metaphor, that is, a transfer (*metaphora*, *translatio*) from the world to the logical space of concepts.[76]

As not even our theories of rationality can do without such metaphors, rational representationalism argues transcendentally that the condition of the possibility of logical language is the use of metaphors. The strategy for dealing with them is not to transfer them to other language contexts that, in turn, may have their own metaphors, but to make their representational character explicit.[77] In the attempt to avoid having to resort to the idea that the world as an empirical given, logic diagrams (and gestures) play a decisive role.

It is not the empirical diagram that supports this understanding of rational metaphors. Being able to understand these metaphors as transfers from the world to the logical space of reason is what is necessary to understand their use in language. Put simply, we do not need to analyse containers in the world in order to make logical statements about containment and non-containment.[78] Rather, we need to become aware of the conditions of the possibility of the spatial perception and imagination with which we perceive the world. For example, the metaphor of being contained can be understood through the Euler diagrams that we have already seen, such as Figure 1. However, these diagrams do not have to be constructed empirically, with pen and paper, for example. The mental version or 'inner-eye diagram' is sufficient for intrapersonal communication, while the external diagram is a communication medium for interpersonal and human–machine interactions.[79] Rational representationalism also draws on recent studies that have argued for diagrammatic conditions in animal reasoning.[80] This is an indication that representations are the conditions of possibility for being able to act rationally all any level. Although, conversely, representations cannot be reflected without rational abilities.

In adopting the approach of rational representationalism, the transcendental philosophical approach shifted in contrast to Kant, Schopenhauer and Wirgman. In accordance with original transcendental philosophy, rational representationalism is also concerned with the condition of the possibility of knowledge. However, knowledge is not something that is thought to depend on the cognitive constitution of the subject of cognition. It is thought to depend on the language of those who strive for cognition or communication. In our rational–logical language, we find numerous metaphors (concept, conclusion, inference, inferring, containing etc.) and in order to understand them we need representations, which can be found, for example, in logic diagrams or gestures.

[76] Lemanski (2021: chap. 3.2).

[77] Dobrzański (2017: chap. I.8).

[78] Lakoff (1987).

[79] Gardner (1982), Lemanski (2021).

[80] Camp (2009).

VI. | CONCLUSION

The four approaches examined here have both clear differences and commonalities. Kant's initial approach was foundational, the two approaches of Schopenhauer and Wirgman followed in direct succession to Kant, and rational representationalism is a current theory that is emerging now. Kant's contribution to modern transcendental philosophy was significant as he elevated the role of diagrams to be on par with linguistic, logical and rational approaches. In doing so, he strengthened the role of diagrams and challenged the dominant rationalists of the time. However, he also tempered his diagrammatic approach by weakening the empirical dimension of diagrams through the transcendental approach, which was in opposition to the approach of the empiricists. Schopenhauer consistently applied diagrams to all areas of logic and mathematics, demonstrating that they are instrumental in establishing transcendental arguments even in areas Kant omitted. For Schopenhauer, the condition of the possibility of propositional knowledge was based on the conditions of the possibility of circular relations in Euclidean space.

Wirgman went a step further presenting logic diagrams as a condition for understanding elements of transcendental philosophy. In his view, Kantian transcendental philosophy is organised around intuition and, thus, logic diagrams. His aim was to demonstrate and advance the condition of the possibility of a transcendental philosophy. Similarly, rational representationalism also embedded logic diagrams in transcendental philosophical argumentation. However, they did not examine the subject of knowledge focusing, instead, on logical or rational language. As a result, the possibility of a rational philosophy after the linguistic turn became tied to the analysis of linguistic conditions. These conditions are characterised by metaphors that cannot be easily translated into pure concepts or algebraic (or other symbolic) notations. One method for understanding these metaphors is to retrace them to forms such as those given in diagrams.

If we define transcendental philosophy as the question of the condition of the possibility of cognition, knowledge, experience, etc., the four approaches can be divided into two pairs. Kant and Schopenhauer focused on the condition of the possibility of a priori cognition, where the subject is at the forefront of the epistemological process. In contrast, Wirgman and rational representationalism were more concerned with the condition of the possibility of understanding. In Wirgman's view, Kantian transcendental philosophy itself is central to the process of understanding, seemingly in the form of a diagrammatic hermeneutics. Rational representationalism, in contrast, is more about understanding the metaphors of rationalism that lead to the realisation that rationalism already explicates representational forms in its language that cannot be avoided.

The transcendental philosophical formula requires further explanation of the types of conditions involved. Is it a sufficient, necessary or sufficiently necessary condition in each of the respective transcendental philosophical approaches and, furthermore, what role do diagrams play in each case? However, since these questions require another intensive discussion of the four approaches, they are omitted here. There are also, very likely, many other connections between logic diagrams and transcendental philosophy that have remained unexplored since the early 20th century. Indeed, additional approaches could well be found in the work of, to name a few, Alois Riehl from Austria, Theodor Ziehen from Germany and Alf Nyman from Sweden.

This study is limited to one definition of transcendental philosophy and one type of diagram, Euler diagrams. Other criteria will most likely reveal further exciting parallels. However, these must be pursued in further studies. Here, it should suffice to have shown, through a few initial examples, that diagrams are a valid and important topic in transcendental philosophy and vice versa.

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ORCID

Jens Lemanski https://orcid.org/0000-0003-3661-4752

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