Bigger, Badder Bugs

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In this paper we motivate the 'principles of trust', chance-credence principles that are strictly stronger than the New Principle yet strictly weaker than the Principal Principle, and argue, by proving some limitative results, that the principles of trust conflict with Humean Supervenience.

1. Introduction

Humean Supervenience is the speculative, albeit appealing, thesis that the nomic supervenes on the categorical.[1] This paper asks whether Humean Supervenience is compatible with there being a tight enough connection between chance and rational credence, and offers new reasons for thinking not.

Past work is instructive.[2] There is, on the one hand, some familiar bad news for Humeans: Humean Supervenience is incompatible with the Principal Principle. In fact, Humean Supervenience is incompatible with the weakening of the Principal Principle one gets from a restriction to initial chance and rational initial credence. If Ch is the initial chance function, Cr is the class of rational initial credence functions, p is a proposition, and $\langle Ch(p) = x \rangle$ is the proposition that the initial chance

[1] Like Briggs (2009a), we take Humean Supervenience to be necessary and a priori if true, distinguishing it from the thesis that the nomic supervenes on the distribution of the categorical properties which are intrinsic to point-sized regions or objects, which may be, as Vranas (2002) and Lewis (1994) argue, contingent or a posteriori, or both. Although, for reasons discussed in note 13, that assumption may not be necessary.

[2] The literature discussing Humean Supervenience and chance-credence principles is vast; see, for example, Arntzenius and Hall (2003), Bigelow, Collins and Pargetter (1993), Briggs (2009a, 2009b), Hall (1994, 2004), Halpin (1994, 1998), Hicks (2017), Ismael (2008), Levinstein (2023), Lewis (1980, 1994), Pettigrew (2012, 2015, 2016), Schaffer (2003), Thau (1994), Vranas (2002), and Ward (2005).

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of p equals x, then we have the following, a principle that asserts that rational initial credence *reflects* initial chance:[3]

Reflection. $\forall \pi \in Cr : \pi(p | \langle Ch(p) = x \rangle) = x.$

As the so-called 'big, bad bug' shows, Humean Supervenience and Reflection are not both true if chance has the features that science takes it to have. (See §3 for more.)

There is, on the other hand, some familiar good news for Humeans: Humean Supervenience is compatible with the New Principle.[4] Restricting the New Principle to initial chance and rational initial credence gives us the following, a principle that asserts that rational initial credence *new-reflects* initial chance:

New Reflection. $\forall \pi \in Cr : \pi(p | \langle Ch(p) = x \rangle) = Ch(p | \langle Ch(p) = x \rangle).$

Chance having the features science takes it to have does not force a choice between Humean Supervenience and New Reflection. (See §4 for more.)

Past work leaves much undecided, however. New Reflection does not draw a tight enough connection between chance and credence. And a case can be made that Reflection is stronger than need be: that the connection between chance and rational credence can be tight enough, even if Reflection fails. An investigation of intermediate chance-credence principles, strictly stronger than New Reflection and strictly weaker than Reflection, is thus prompted.

This paper focuses primarily on three such principles: collectively, the *principles of trust*.[5] The first asserts that rational initial credence *simply trusts* initial chance:

Simple Trust. $\forall \pi \in Cr : \pi(p \mid \langle Ch(p) \geq x \rangle) \geq x.$[6]

[3] Let Ch_t be the chance function that holds at time t, and let q be any proposition. The Principal Principle is the following: $\forall \pi \in Cr : \pi(p \mid q \wedge \langle Ch_t(p) = x \rangle) = x$, if q is admissible with respect to $\langle Ch_t(p) = x \rangle$. See Lewis (1980).

[4] Let Ch_t be the chance function that holds at time t, and let q be any proposition. Then we have the New Principle: $\forall \pi \in Cr : \pi(p \mid q \wedge \langle Ch_t(p \mid q) = x \rangle) = Ch_t(p \mid q \wedge \langle Ch_t(p \mid q) = x \rangle)$. See Hall (1994), Lewis (1994), and Thau (1994).

[5] For discussion of intermediate chance-credence principles, including the principles of trust, see Dorst (2019, 2020), Dorst et al. (2021), Elga (2013), and Levinstein (2023); see also Schervish (1989).

[6] Equivalently, using upper bounds: $\forall \pi \in Cr : \pi(p \mid \langle Ch(p) \leq x \rangle) \leq x$.

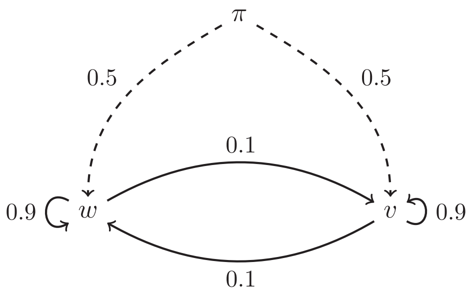

Fig. 1. π assigns each of w and v probability 0.5. At w, the chance of w equals 0.9, and the chance of v equals 0.1. At v, the chance of w equals 0.1, and the chance of v equals 0.9.

Reflection is about equality. Credence function π reflects chance just if, for every value x and proposition p, conditional on the chance of p being equal to x, $\pi(p)$ equals x. Simple Trust is about lower (or, equivalently, upper) bounds. Credence function π simply trusts chance just if, for each value x and proposition p, conditional on the chance of p being at least x, $\pi(p)$ is at least x.

Figure 1 illustrates the difference with a two-world model. Since w is the only world at which the chance of w equals 0.9, $w = \langle Ch(w) = 0.9\rangle$;[7] and since, for every x, $0.1 < x \leq 0.9$, w is the only world at which the chance of w is at least x, $w = \langle Ch(w) \geq x\rangle$. Similarly, $v = \langle Ch(v) = 0.9\rangle$, and, for any x, $0.1 < x \leq 0.9$, $v = \langle Ch(v) \geq x\rangle$. Credence function π simply trusts chance: for any x, $0.1 < x \leq 0.9$, $\pi(w \mid \langle Ch(w) \geq x\rangle) \geq x$ and $\pi(v \mid \langle Ch(v) \geq x\rangle) \geq x$. But π does not reflect chance: $\pi(w \mid \langle Ch(w) = 0.9\rangle) \neq 0.9$; rather, $\pi(w \mid \langle Ch(w) = 0.9\rangle) = 1$.

The second principle of trust strengthens Simple Trust by ensuring that a rational initial credence function updated on some information simply trusts initial chance updated on the same information, thus asserting that rational initial credence *resiliently trusts* initial chance:

Resilient Trust. $\forall \pi \in Cr : \pi(p \mid q \wedge \langle Ch(p \mid q) \geq x\rangle) \geq x$.[8]

The third, strictly stronger than the previous two, strengthens Simple Trust by extending it to the expectation of all random variables. If χ is a random variable, $\mathbb{E}_\pi(\chi)$ is the expectation of χ derived from some

[7] To ease the exposition, we ignore the distinction between a world and its singleton.

[8] Equivalently, using upper bounds: $\forall \pi \in Cr : \pi(p \mid q \wedge \langle Ch(p \mid q) \leq x\rangle) \leq x$.

rational initial credence function π, and $\mathbb{E}_{Ch}(\chi)$ is the expectation of χ derived from *Ch*, then we have the following, a principle that asserts that rational initial credence *totally trusts* initial chance:

Total Trust. $\forall \pi \in Cr : \mathbb{E}_\pi(\chi \mid \langle \mathbb{E}_{Ch}(\chi) \geq x \rangle) \geq x$.[9]

Imagine a forecaster who predicts the profit or loss that a given company will make in the next quarter. And to illustrate just one way in which totally trusting and reflecting come apart, imagine that we take the forecaster to be a timid expert, and that we predict, for each n, that the company will make a profit or loss of \$2n, conditional on the forecaster predicting that the company will make a profit or loss of \$$n$. If we divide our credence uniformly among the finitely many predictions that the forecaster might make, then we totally trust the forecaster's predictions: conditional on the forecaster predicting that the company will make a profit or loss of at least \$$n$, we predict that the company will make a profit or loss of at least \$$n$; conditional on the forecaster predicting that the company will make a profit or loss of at most \$$n$, we predict that the company will make a profit or loss of at most \$$n$. A more familiar form of deference is reflection-like: predicting, for each n, that the company will make a profit or loss of \$$n$, conditional on the forecaster predicting that the company will make a profit or less of \$$n$. If we regard the forecaster as a timid expert, then we do not defer to the forecaster in this reflection-like way. But, as we will see, totally trusting is a relation of tremendous interest, a real epistemic joint. Some properties that chance ought to have—some properties that chance must have, we claim, if the connection between chance and rational credence is tight enough—are had by chance only if Total Trust holds. (See §6 for more.)

Reflection is substantially stronger than Total Trust, as recent work on higher-order evidence underscores. A case can be made that rational initial credence, though not reflecting itself, totally trusts itself.[10] Hoping that Humean Supervenience will prove compatible with Total Trust, despite being incompatible with Reflection, is thus—prior to a proper investigation of the matter—not unreasonable.

But the new news is bad news for Humeans. The compatibility of Humean Supervenience and Total Trust is doubtful. In fact, in light of the limitative results proved below, it is doubtful that any rational initial credence function totally trusts initial chance if Humean Supervenience

[9] Equivalently, using upper bounds: $\forall \pi \in Cr : \mathbb{E}_\pi(\chi \mid \langle \mathbb{E}_{Ch}(\chi) \leq x \rangle) \leq x$.

[10] This case is made in Dorst (2019, 2020) and Dorst et al. (2021).

holds. One of the bigger, badder bugs below concerns Simple Trust. We develop an argument that no rational initial credence function simply trusts initial chance if Humean Supervenience holds. But the assumptions of that argument are stronger than the assumptions needed for the other bigger, badder bug: the argument that no rational initial credence function totally trusts chance if Humean Supervenience holds.

2. Inventory of formal tools

Let us begin with an inventory of the formal tools invoked below.

There is, to begin with, a set of possible worlds, W, assumed (for convenience) to be finite, and a set of propositions, identified with the power set of W.

There is also a set of random variables. A *random variable* χ is a function that maps each possible world w to some real number, $\chi(w)$, the value of χ at w. One special set of random variables is the set of indicator variables, the random variables whose only possible values are 0 and 1. The set of indicator variables is, in a certain sense, interchangeable with the set of propositions: for each indicator variable χ, there is a unique proposition that contains world w just if $\chi(w) = 1$; for each proposition p, there is a unique indicator variable that maps world w to 1 just if w is an element of p.

There is the aforementioned set of rational initial credence functions, Cr. Every credence function maps each proposition to some real number on the unit interval, and we assume that every rational initial credence function is a regular probability function: a function that satisfies the probability axioms and gives non-zero credence to every non-empty proposition.[11] Rational credence evolves: a rational agent's present credences are arrived at by conditioning their rational initial credence function on the information they have gathered heretofore. But to keep things simple, we set non-initial credence aside, hereafter letting 'credence' denote initial credence.

There is also the *chance assignment*, a function that maps each world w to the initial chance function that holds at w, namely, Ch_w. We assume that every possible initial chance function is a probabilistic credence function. Chance evolves: the present chances are arrived at by conditioning the initial chance function on the history of the world heretofore.

[11] Assuming that every rational initial credence function is regular simplifies many of the arguments below. But the assumption is not essential.

But to keep things simple, we set non-initial chance aside, hereafter letting 'chance' denote initial chance.

Uncertainty about chance is uncertainty about chance *de dicto*. If an agent is uncertain whether the chance of p equals x, they are not uncertain, for any world w, about whether $Ch_w(p) = x$. What they are uncertain about is whether $Ch(p) = x$: whether the chance of p, whatever it is, equals x. Claims about chance are thus, unless otherwise noted, claims about chance *de dicto*. The proposition that the (*de dicto*) chance of p equals x, $\langle Ch(p) = x \rangle$, is a set that includes world w just if $Ch_w(p) = x$; the proposition that the (*de dicto*) chance of p is at least x, $\langle Ch(p) \geq x \rangle$, is a set that includes world w just if $Ch_w(p) \geq x$.

Random variables are not bearers of chance; only propositions are. But random variables have (*de dicto*) chance-expectations, and our space of propositions includes propositions concerning the chance-expectations of random variables. The *chance-expectation* of χ, $\mathbb{E}_{Ch}(\chi)$, is a Ch-weighted average of the possible values of χ, $\sum_{v \in W} Ch(v)\chi(v)$. The proposition that the chance-expectation of χ equals x, $\langle \mathbb{E}_{Ch}(p) = x \rangle$, is a set that includes world w just if $\sum_{v \in W} Ch_w(v)\chi(v) = x$; the proposition that the chance-expectation of χ is at least x, $\langle \mathbb{E}_{Ch}(p) \geq x \rangle$, is a set that includes world w just if $\sum_{v \in W} Ch_w(v)\chi(v) \geq x$.

3. The big, bad bug

With the inventory of formal tools behind us, let us rehearse the big, bad bug: an argument that the conjunction of Humean Supervenience and Reflection is inconsistent with scientific practice.

Humean Supervenience is a constraint on the chance assignment. Possible worlds can be partitioned by their Humean mosaics.[12] A cell of the partition is a *mosaic*. A chance assignment verifies Humean Supervenience just if it maps any pair of worlds in the same mosaic to the same chance function.[13]

Reflection is another constraint on the chance assignment. The chance assignment verifies Reflection only if some rational credence function reflects the chances it engenders. A chance assignment is

[12] Or so one must assume to take Humean Supervenience seriously.

[13] Here we rely on the assumption that Humean Supervenience is necessary if true. For a defence of the assumption, see Briggs (2009a, pp. 443–4). But in so far as we are interested in Resilient Trust or Total Trust, the assumption is not essential. If Humean Supervenience is contingent, then we can focus on the following claim entailed by Resilient Trust: every rational initial credence function updated on Humean Supervenience simply trusts chance updated on Humean Supervenience.

immodest just if it verifies the following, a principle that asserts that each possible chance function gives itself chance one:

Immodesty. For any worlds v and w, if $Ch_v \neq Ch_w$, then $Ch_v(w) = 0$.

And, if we ignore degenerate chance assignments (as we will, hereafter), Reflection implies Immodesty: a regular probability functions reflects the chances engendered by a non-degenerate chance assignment only if the chance assignment is immodest.[14]

There are chance assignments that verify both Humean Supervenience and Immodesty, but there is a third constraint. An adequate chance assignment must accord with scientific practice. It is not easy to say what it takes to accord with scientific practice, but a necessary condition is ready to hand. Consider *the best-system function*: a function that maps each mosaic to the theory or theories that best systematize the mosaic, as judged by the method of theory choice implicit in science. Any theory that could be among the outputs of the best-system function determines a chance function over the space of possible worlds. A chance function *systematizes* a mosaic just if it is determined by all of the theories to which the best-system function maps the mosaic. To accord with scientific practice, a chance assignment must verify:

Possible Systematization. Every chance function is compossible with every mosaic it systematizes.

Verifying Possible Systematization is easy if Humean Supervenience fails, since different chance functions then can hold at worlds in the same mosaic. But if Humean Supervenience holds, then a chance function is compossible with a mosaic only if it is necessitated by the mosaic. Humean Supervenience and Possible Systematization thus together imply:

Necessary Systematization. Every chance function is necessitated by every mosaic it systematizes.

[14] Reflection is a norm of local chance reflection. There is also a norm of global chance reflection: $\forall \pi \in Cr : \pi(p \mid \langle Ch = Ch_w \rangle) = Ch_w(p)$. The global norm straightforwardly implies Immodesty; see Dorst (2020, p. 616, Fact 3.1). And although, strictly speaking, the local and global norms are not equivalent, the difference between them can be ignored. For, as Gallow (2023) proves, they come apart only in the degenerate case in which the chance assignment is 'half-cyclic'.

A chance function is *system-modest* just if it assigns positive chance to a mosaic systematized by a distinct chance function. If some mosaic is systematized by a system-modest chance function, then Immodesty and Necessary Systematization are not both true. Possible Systematization, Humean Supervenience, and Immodesty together imply:

> *Immodest Systematization*. No mosaic is systematized by a system-modest chance function.

And therein lies the problem, for Immodest Systematization is false. There is room for disagreement about when a chance function systematizes a mosaic. The method of theory choice implicit in science is not entirely transparent to us. But nor is it entirely opaque. We know enough about it to know that some mosaics are systematized by system-modest chance functions.

There are realistic ways of illustrating the failure of Immodest Systematization. Lewis (1994, p. 482) appeals to radioactive decay, noting that a mosaic systematized by a chance function that encodes one half-life for a given radioactive particle gives positive chance to mosaics systematized by distinct possible chance functions that encode distinct half-lives for the same radioactive particle. But partly to make the problem clearer and partly to set the stage for the limitative results below, we will appeal to, as we call them, 'flip models'.

Each flip model is associated with some natural number, n. The mosaic of a world in an n-flip model is a binary sequence of length n, envisaged, picturesquely, as the outcomes of the flips of some quantum coin: *HTHHTH* … We assume that every binary sequence of length n is the mosaic of some world in the n-flip model; we assume—identifying worlds and mosaics and thereby hard-coding the truth of Humean Supervenience—that no binary sequence of length n is the mosaic of more than one world in the n-flip model; and we assume that each world w has some precise chance function, Ch_w.[15] We can thus refer to an n-flip model as a pair $\langle W, \mathcal{P} \rangle$, where W is the set of binary sequences of length n, and $\mathcal{P}$ is a function from W to probability functions over W, that is, $\mathcal{P} : W \to \Delta(W), w \mapsto Ch_w$.

[15] For some w, Ch_w may be deterministic, that is, it may specify the result of each flip with probability 1.

We call a chance function IID when it treats the coin flips as independent and identically distributed. Formally, if H_j is the proposition that the jth flip lands heads, then:

IID. Chance function Ch is IID just if, for any j and k, $j < k \leq n$:
(1) $Ch(H_j \wedge H_k) = Ch(H_j)\,Ch(H_k)$, and
(2) $Ch(H_j) = Ch(H_k)$.

One expects the chances associated with coin flips to be distributed binomially, and it is the IID chance functions that deliver binomial distributions. Let $IID(x)$ be the IID chance function *centred* on x, the chance function that deems each flip independent and accords each flip chance x of landing heads; and let $\langle Ch = IID(x)\rangle$ be the proposition that holds at world w just if $Ch_w = IID(x)$. If w is a world in the n-flip model at which $\langle Ch = IID(x)\rangle$ holds, and v is a world in the n-flip model at which k of the n flips land heads, then $Ch_w(v) = x^k(1-x)^{n-k}$; hence, if $\langle \#H = k\rangle$ is the proposition that exactly k of the n flips land heads, $Ch_w(\langle \#H = k\rangle) = \binom{n}{k}x^k(1-x)^{(n-k)}$.

Some venerable approaches to chance entail that every world in a flip model is systematized by an IID chance function. For example, according to frequentism, whenever exactly k of the n flips at world w land heads, $Ch_w = IID(k/n)$. Frequentism is not obvious, however. Consider the following, from the 20-flip model:

w_i : *HHHHHHHHHHTTTTTTTTTT*

It may be that the best-system function maps w_i to the deterministic theory that a flip lands heads if and only if it is among the first ten flips, in which case the chance function that systematizes w_i does not treat the coin flips as identically distributed.

But we know that many worlds in flip models are systematized by IID chance functions—IID chance processes are ubiquitous in science, the norm from which exceptions deviate. We know that the following, from the 20-flip model, is systematized by $IID(1)$:

w_j : *HHHHHHHHHHHHHHHHHHHH*

We know that the following, from the 20-flip model, is systematized by $IID(0)$:

w_k : *TTTTTTTTTTTTTTTTTTTT*

And we know that many of the worlds wherein exactly half of the flips land heads are systematized by *IID*($^1/_2$), the following being a good candidate:

w_l : *HTHTHHTTTHHHTTTHTTHH*

Arguably, we know something stronger. The great virtue of focusing on flip models is that it allows us to state precise claims about what science requires of the chance assignment, and a case can be made that we know the following, a principle that asserts that *IID*(x) systematizes some world in the n-flip model whenever x is the actual proportion of heads to flips at some world in the model:

> *Proportional Systematization*. For any m and n, $0 \leq m \leq n$, there is some world in the n-flip model systematized by *IID*($^m/_n$).

Proportional Systematization is plausible and interesting, and it will play an important role in one of the bigger, badder bugs to come.

But if our aim is only to bring out the falsity of Immodest Systematization, nothing so strong is needed. Indeed, the following suffices:

> *Non-trivial Systematization*. In some n-flip model, some world is systematized by *IID*(x), $0 < x < 1$, and some world is systematized by some chance function distinct from *IID*(x).

Non-trivial Systematization is an extremely weak claim about what science requires of a chance assignment, yet it is inconsistent with Immodest Systematization. If some world in the n-flip model is systematized by *IID*(x), and some world is systematized by a chance function distinct from *IID*(x), then every world systematized by *IID*(x) is systematized by a system-modest chance function, since *IID*(x) gives positive chance to every world in the n-flip model.

Taking a step back, we can see the structure of the challenge facing Humeans. The big, bad bug has three parts. There is a scientific part, a purported claim about what science requires of the chance assignment. There is an epistemological part, the claim that the connection between chance and rational credence is tight enough only if Reflection holds. And there is the mathematical part, a proof that Humean Supervenience is inconsistent with Reflection, given the purported claim about what science requires of the chance assignment. Humeans wax poetic about the epistemological virtues of their metaphysics, the optimific balance of strength, simplicity and fit that chance and laws as they envisage them achieve. But the big, bad bug is an impossibility result, and waxing poetic is not adequate response to an impossibility result. What Humeans need is a tenability result: a *proof* that Humean Supervenience is consistent with some not-too-loose connection between chance and rational credence, given some not-too-weak claim about what science requires of the chance assignment.

4. New Reflection

The challenge facing Humeans would be less formidable if New Reflection drew a tight enough connection between chance and rational credence. But it doesn't.

Indeed, New Reflection bears on the connection between chance and rational credence only indirectly. What it directly bears on is the connection between rational credence and, as we will call it, 'informed chance'. For each possible chance function Ch_w, there is the proposition that Ch_w holds, $\langle Ch = Ch_w \rangle$, and the informed chance function at world w, Ch^+_w, is $Ch_w(- \mid \langle Ch = Ch_w \rangle)$, the chance function at w conditioned on $\langle Ch = Ch_w \rangle$. Our space of propositions includes propositions concerning the (*de dicto*) informed chances of propositions. The proposition that the informed chance of p equals x, $\langle Ch^+(p) = x \rangle$, is a set that includes world w just if $Ch^+_w(p) = x$; the proposition that the informed chance of p is at least x, $\langle Ch^+(p) \geq x \rangle$, is a set that includes world w just if $Ch^+_w(p) \geq x$.

New Reflection is equivalent to the following, a principle that asserts that rational credence *reflects* informed chance:

Informed Reflection. $\forall \pi \in Cr : \pi(p \mid \langle Ch^+(p) = x \rangle) = x.$

The connection New Reflection draws is thus just as tight as the connection Reflection draws, but whereas Reflection connects rational credence and chance, New Reflection connects rational credence and informed chance.

If chance is immodest, then chance and informed chance coincide: $Ch_w = Ch^+_w$ for each world w. But if Humean Supervenience holds, then chance is modest,[16] and if chance is modest, then chance and informed chance can come apart.

[16] Humean Supervenience, Possible Systematization, and the negation of Immodest Systematization together entail the negation of Immodesty.

A frequentist, 2-flip model provides a simple illustration. There are four worlds, *HH*, *HT*, *TH*, and *TT*. If frequentism holds at each, then $Ch_{HH} = IID(1)$, $Ch_{HT} = IID(1/2) = Ch_{TH}$, and $Ch_{TT} = IID(0)$. But the chance of both flips landing heads is 1/4 only if exactly one flip land heads. So chance and informed chance come apart: $Ch_{HT}(HH) = 1/4 < Ch^{+}_{HT}(HH) = 0$.

The connection New Reflection draws between rational credence and informed chance induces an indirect connection between chance and rational credence. But the induced connection is not tight enough if chance and informed chance can come apart, as we can see by considering anti-expertise.

Say that credence function π treats *de dicto* probability function P as an *anti-expert* with respect to some proposition-value pair, (p, x), just if $\pi(p \mid \langle P(p) \geq x\rangle) < x$ and $\pi(p \mid \langle P(p) < x\rangle) \geq x$; and say that P is *free of anti-expertise* just if no rational credence function treats P as an anti-expert with respect to any proposition-value pair. While Reflection entails that chance is free of anti-expertise,[17] New Reflection does not. In fact, it is consistent with New Reflection that chance is rife with anti-expertise.

Chance is, as Lewis says, a guide to life:

> It is reasonable to let one's choices be guided in part by one's firm opinions about objective chances or, when firm opinions are lacking, by one's degrees of belief about chance. … The greater chance you think the ticket has of winning, the greater should be your degree of belief that it will win; and the greater is your degree of belief that it will win, the more, *ceteris paribus*, it should be worth to you and the more you should be disposed to choose it over other desirable things. (Lewis 1980, pp. 287–8)

But because it is consistent with New Reflection that chance is rife with anti-expertise, it is consistent with New Reflection that chance is an anti-guide to life. It is consistent with New Reflection that rational agents often take truth and chance to be anti-correlated, regarding as evidence against p information that increases what they think the chance of p is. It is thus consistent with New Reflection that rational agents often prefer a lesser chance to a greater chance of getting the things they desire. And that, we think, is absurd. Chance is not an anti-guide to life;

[17] Chance is free of anti-expertise if and only if Simple Trust holds, and Reflection entails Simple Trust.

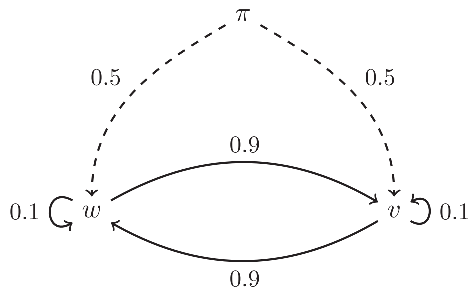

Fig. 2. π assigns w and v probability 0.5. $Ch_w(w) = Ch_v(v) = 0.9$, and $Ch_v(w) = Ch_w(v) = 0.1$. π new reflects Ch.

and from that we conclude that every tight enough chance-credence principle entails that chance is free of anti-expertise.

A two-world model provides an illustration. Suppose that each of w and v accords the other more chance than it accords itself: $Ch_w(v) = Ch_v(w) = 0.9$, and $Ch_w(w) = Ch_v(v) = 0.1$. The agent prefers w to v. The agent divides their credence equally between the two worlds and new-reflects chance: $\pi(w) = \pi(v) = 0.5$, and for any p, $\pi(p \mid \langle(Ch(p) = x\rangle) = Ch(p \mid \langle(Ch(p) = x\rangle))$. The agent then regards chance as an anti-expert: the agent thinks that evidence that the chance of w is low is evidence that w is true, and thus prefers a lesser chance of getting what they prefer, a lesser chance of w, to a greater chance. See Figure 2 for a depiction of this scenario.

Reflection is tight enough—Reflection entails that chance is free of anti-expertise. But Reflection implies Immodesty, and as the big, bad bug shows, Humean Supervenience is incompatible with any chance-credence principle that entails Immodesty. The principles of trust thus prove their interest; for all of them entail that chance is free of anti-expertise, and none of them imply Immodesty.

5. Simple Trust

Simple Trust, the weakest of the principles of trust, is equivalent to the claim that chance is free of anti-expertise. So if every tight enough chance-credence principle entails that chance is free of anti-expertise, Simple Trust holds.

Simple Trust can also be motivated by appeal to accuracy. Say that credence function π treats *de dicto* probability function P as *expectedly inaccurate* just if, for some acceptable way of measuring accuracy, π

expects itself to be more accurate than P; and say that P is *free of expected inaccuracy* just if no rational credence function treats P as expectedly inaccurate. Chance ought to be free of expected inaccuracy. The indicator function at world w specifies the value of each indicator variable at w, thus—given the aforementioned interchangeability of indicator variables and propositions—specifying the truth-value of each proposition at w. Chance is highly inaccurate at world w just if the divergence between Ch_w and the indicator function at world w is great, and while proponents and opponents of Humean Supervenience disagree about the prevalence of worlds at which chance is highly inaccurate, all sides agree that no rational (initial) credence function gives high credence to worlds at which chance is highly inaccurate.

Chance is free of expected inaccuracy only if Simple Trust holds, however. In fact, the implication goes both ways. As Levinstein (2023) shows, if the received view is correct about what the acceptable ways of measuring accuracy are—if the acceptable ways of measuring accuracy are the additive, strictly proper, truth-directed measures that satisfy certain continuity and limit assumptions—then Simple Trust is equivalent to the claim that chance is free of expected inaccuracy.[18]

6. Total Trust

It is doubtful that Simple Trust is itself tight enough, however, for two reasons.

The first concerns accuracy. The accuracy argument for Simple Trust, when generalized, becomes an argument for Total Trust. The specification function at world w generalizes the indicator function at world w, specifying the value of all random variables at w. A probability function P induces an estimate function, $\mathbb{E}_P$, which maps each random variable χ to some real number, $\mathbb{E}_P = \sum_W P(w)\chi(w)$, and just as divergence is distance between probability and indication, estimate inaccuracy—the generalization of inaccuracy to all random variables—is divergence between estimate and specification. The estimate inaccuracy for a set of random variables of probability function P at world w is a measure of how far apart $\mathbb{E}_P$ is from the specification function for those variables at w.[19]

[18] For the precise conditions required on measures of accuracy, see Levinstein (2023).

[19] For technical details, see Dorst et al. (2021) and Campbell-Moore (MS).

Say that credence function π treats *de dicto* probability function P as *expectedly estimate inaccurate* just if, for some acceptable way of measuring estimate inaccuracy, π expects itself to be more estimate-accurate than P for some random variable; and say that P is *free of expected estimate inaccuracy* just if no rational credence function expects itself to be more expectedly estimate-accurate than P for any random variable. Chance ought to be free of expected estimate inaccuracy, for the same reasons that chance ought to be free of expected inaccuracy. But, as Dorst et al. (2021) show, generalizing the result proved in Levinstein (2023), if the received view is correct about what the acceptable ways of measuring estimate inaccuracy are—if the acceptable measures of estimate inaccuracy are strictly proper, truth-directed measures that satisfy certain continuity and limit assumptions—then Total Trust is equivalent to the claim that chance is free of expected estimate inaccuracy.[20]

The second reason concerns choice. If chance is a guide to life, then deferring a choice to chance—letting chance choose on one's behalf, as it were, giving chance power of attorney—ought always to be rational. But deferring a choice to chance is always rational only if Total Trust holds. In fact, the implication holds both ways. As Dorst et al. (2021) show, Total Trust is equivalent to the claim that deferring a choice to chance is always rational.

Choice technicalities: A choice is a set of pairwise exclusive options, $\mathcal{O} = \{o_1, \dots, o_n\}$. Each option is a random variable, a function that maps each world to some real number which represents how desirable the agent finds the option at the world. The expected value of option o, relative to credence function π, $V(\pi, o)$, equals $\sum_W \pi(w)o(w)$.

Deferring a choice among $\mathcal{O}$ to chance is a strategy: the chance-expected value of option o at world v is $\sum_w Ch_v(w)o(w)$, and deferring a choice among $\mathcal{O}$ to chance is a function that maps each world v to some option that maximizes chance-expected value at v. If $s(w)$ is the value at w of the option to which world w is mapped by the strategy of deferring a choice to chance, then the expected value of deferring a choice among $\mathcal{O}$ to chance, relative to credence function π, is $\sum_W \pi(w)s(w)$.

Credence function π *permits* deferring a choice among $\mathcal{O}$ to P just if, for each o in $\mathcal{O}$, $V(\pi, o) \leq V(\pi, s)$. It is *rational* to defer a choice among $\mathcal{O}$ to P just if every rational credence function permits deferring a choice among $\mathcal{O}$ to P. And it is *always* rational to defer a choice to P just if, for any $\mathcal{O}$, it is rational to defer a choice among $\mathcal{O}$ to P. *End of technicalities.*

[20] For the precise statement and proof of this result, see Dorst et al. (2021)

It is doubtful that the connection between chance and rational credence is tight enough if it is not always rational to defer a choice to chance. Deferring a choice to chance is playing the chances, selecting an option that maximizes chance-expected value, and if chance is a guide to life, then it should always be rational to play the chances. But if it is always rational to defer a choice to chance, then Total Trust holds: the claim that every rational credence function totally trusts some *de dicto* probability function P is equivalent to the claim that it is always rational to defer a choice to P.[21]

It is an interesting question whether Total Trust is itself tight enough. One worry stems from expectation-matching.[22] Another worry stems from stochastic dominance.[23] One could insist that nothing short of Reflection is tight enough. Indeed, the big, bad bug is often cited as a reason against Humean Supervenience. However, many Humeans remain, and many of them have taken solace in the New Principle. These Humeans are our dialectical targets. The New Principle is not tight enough, as it does not require chance to be a guide to life. More is needed, and we submit that even those willing to abandon Reflection should still demand that chance be totally trustworthy.

[21] Dorst et al. (2021) offer an example to help illustrate the difference between Simple Trust and Total Trust. Suppose that there are three worlds, w, v, and u. Suppose that there are two options, $o_0(w) = o_0(v) = o_0(u) = 0$, $o_1(w) = 29$, $o_1(v) = -3$, and $o_1(u) = -13$. And consider the following chance assignment: $Ch_w(w) = 0.45$, $Ch_w(v) = 0.10$, and $Ch_w(u) = 0.45$; $Ch_v(w) = 0.15$; $Ch_v(v) = 0.70$, and $Ch_v(u) = 0.15$; and $Ch_u(w) = 0.30$, $Ch_u(v) = 0.10$, and $Ch_u(u) = 0.60$. At each of the three worlds, the chance-expected value of o_1 exceeds zero, and hence exceeds the chance-expected value of o_0. But some probabilistic credence functions that simply trusts (and indeed resiliently trusts) this chance assignment nevertheless strictly prefer o_0 to o_1. One example is $\pi(w) = 0.17$, $\pi(v) = 0.56$, and $\pi(u) = 0.27$.

[22] Matching one's credences to one's expectation of the chances is a central part of science and a ubiquitous part of daily life. It is thus natural to insist that a chance-credence principle entail Chance Expectation: $\forall \pi \in Cr : \pi(p) = \sum_W \pi(w) Ch_w(p)$. Reflection entails Chance Expectation, but Total Trust does not, as the following two-world model illustrates: $\pi(v) = \pi(w) = 0.5$; $Ch_v(v) = 0.9$; $Ch_v(w) = 0.1$; $Ch_w(w) = 0.8$; and $Ch_w(v) = 0.2$; cf. (Dorst et al. 2021, p. 124, n. 18).

[23] The proposition that the value of option o exceeds x $\langle o \geq x \rangle$, is a set that includes world w just if $o(w) \geq x$. The proposition that option o_i chance-wise stochastically dominates option o_j, $\langle o_i \succ o_j \rangle$, is a set that includes world w just if (a) for every x, $Ch_w(\langle o_i \geq x \rangle) \geq Ch_w(\langle o_j \geq x \rangle)$, and (b) for some x, $Ch_w(\langle o_i \geq x \rangle) > Ch_w(\langle o_j \geq x \rangle)$. Reasoning by chance-wise stochastic dominance is ubiquitous and intuitive. It is thus natural to insist that a chance-credence principle entail Chance-wise Stochastic Dominance: $\forall \pi \in Cr$: if $\pi(\langle o_i \succ o_j \rangle) > 0$, then $\sum_W \pi(w \mid \langle o_i \succ o_j \rangle)\, o_i(w) \geq \sum_W \pi(w \mid \langle o_i \succ o_j \rangle)\, o_j(w)$. Reflection entails Chance-wise Stochastic Dominance, but Total Trust does not, as the following four-world model illustrates: $\pi(u) = \pi(v) = \pi(w) = \pi(x) = 1/4$; $\pi = Ch_u$; $Ch_v(u) = 2/9$, $Ch_v(v) = 1/3$, $Ch_v(w) = 2/9$, and $Ch_v(x) = 2/9$; $Ch_w(u) = 2/11$, $Ch_w(v) = 3/11$, $Ch_w(w) = 4/11$, and $Ch_w(x) = 2/11$; $Ch_x(u) = 2/13$, $Ch_x(v) = 3/13$, $Ch_x(w) = 4/13$, and $Ch_x(x) = 4/13$; $o_i(u) = 1$, $o_i(v) = 2$, $o_i(w) = 0$, and $o_i(x) = 4$; and $o_j(u) = 4$, $o_j(v) = 0$, $o_j(w) = 1$, and $o_j(x) = 1$. Although π totally trusts chance, $\sum_W \pi(w \mid \langle o_i \succ o_j \rangle)\, o_i(w) = 1.5 < \sum_W \pi(w \mid \langle o_i \succ o_j \rangle)\, o_j(w) = 2$.

7. A bigger, badder bug

Our first limitative result concerns Simple Trust. Consider the following, a principle that asserts that every proposition is compossible with every possible proposition that sets a positive lower bound on its chance:

> *Threshold Compossibility*. For every value $x > 0$, if $\langle Ch(p) \geq x \rangle$ is possible, then $p \wedge \langle Ch(p) \geq x \rangle$ is possible.

Simple Trust entails Threshold Compossibility. In fact, no regular probability function simply trusts chance if Threshold Compossibility fails.[24] And as flip models make clear, the conjunction of Humean Supervenience and Threshold Compossibility is incompatible with plausible claims about what science requires of the chance assignment. For example, as we prove in this section, in any n-flip model, $n > 4$, Threshold Compossibility is incompatible with Proportional IID.

The proof proceeds by cases. Let a *k-heads world* be a world at which k flips land heads, and consider the following, a principle that asserts that $IID(k/n)$ holds at some world w in an n-flip model only if w is a k-heads world:

> *Matching*. For any world w in an n-flip model, if $Ch_w = IID(k/n)$, then $w \in \langle \#H = k \rangle$.

If Proportional IID holds, and Matching fails, then the chance that some world accords itself is exceeded by the chance accorded to it by some other world. To see this, take an arbitrary counter-instance to Matching: suppose that $Ch_w = IID(k/n)$, and suppose that w is a j-heads world, $j \neq k$. Since Proportional IID holds, there is some world v in the n-flip model at which $IID(j/n)$ holds. For any z, $0 \leq z \leq n$, the chance of w at a world at which $IID(z/n)$ holds equals $(z/n)^j(1 - k/n)^{n-j}$, which takes its unique maximum at $z = j$. The chance of w at v thus exceeds the chance of w at w, and Threshold Compossibility therefore fails. The proposition that the chance of w is at least as high as the chance of w at v is, although possible, not compossible with w.

Threshold Compossibility also fails, however, in any n-flip model, $n > 4$, if Proportional IID and Matching hold, as we see clearly in the 6-flip model. Let $\langle \#H = 2 \rangle \vee \langle \#H = 4 \rangle$ be the proposition that the coin

[24] If π is a rational credence function, and $\langle Ch(p) \geq x \rangle$ is possible, then $\pi(p \mid \langle Ch(p) \geq x \rangle)$ is defined. If $\pi(p \mid \langle Ch(p) \geq x \rangle)$ is defined, and $p \wedge \langle Ch(p) \geq x \rangle$ is impossible, then $\pi(p \mid \langle Ch(p) \geq x \rangle) = 0 < x$.

lands heads either exactly two or exactly four times; let w_2 be a 2-heads world at which $IID(2/6)$ holds; let w_3 be a 3-heads world at which $IID(3/6)$ holds; and let w_4 be a 4-heads world at which $IID(4/6)$ holds. Because of the bell-shape of the binomial curve, $Ch_{w_3}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle)$, the sum of the fairly high chance w_3 accords to 2-heads worlds and the fairly high chance w_3 accords to 4-heads worlds, exceeds both $Ch_{w_2}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle)$, the sum of the high chance w_2 accords to 2-heads worlds and the low chance w_2 accords to 4-heads worlds, and $Ch_{w_4}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle)$, the sum of the low chance that w_4 accords to 2-heads worlds and the high chance that w_4 accords to 4-heads worlds.

$$Ch_{w_2}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle) = \binom{6}{2}\left(2/6\right)^2\left(4/6\right)^4 + \binom{6}{2}\left(2/6\right)^4\left(4/6\right)^2 \approx 0.41$$

$$Ch_{w_3}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle) = \binom{6}{2}\left(3/6\right)^2\left(3/6\right)^4 + \binom{6}{2}\left(3/6\right)^4\left(3/6\right)^2 \approx 0.47$$

$$Ch_{w_4}(\langle \#H = 2\rangle \vee \langle \#H = 4\rangle) = \binom{6}{2}\left(4/6\right)^2\left(2/6\right)^4 + \binom{6}{2}\left(4/6\right)^4\left(2/6\right)^4 \approx 0.41$$

For a visual depiction, see Figure 3.

We can thus produce a counterexample to Threshold Compossibility by taking any non-empty subset of $\langle Ch = IID(2/6)\rangle \vee \langle Ch = IID(4/6)\rangle$, which includes exactly as many elements of $\langle Ch = IID(2/6)\rangle$ as $\langle Ch = IID(4/6)\rangle$. One example is the disjunction of w_2 and w_4:

$$Ch_{w_2}(w_2 \vee w_4) \approx 0.027$$
$$Ch_{w_3}(w_2 \vee w_4) \approx 0.031$$
$$Ch_{w_4}(w_2 \vee w_4) \approx 0.027$$

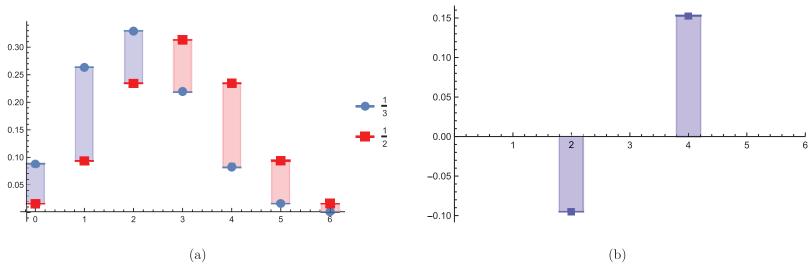

Fig. 3. Figure 3(a) displays the probabilities assigned to $0, 1, 2, 3, 4, 5, 6$ occurrences of heads for $IID(3/6)$ and $IID(2/6)$. Figure 3(b) isolates the difference assigned to two occurrences and six occurrences of heads. Although $IID(3/6)$ assigns lower probability to there being exactly two occurrences of heads than $IID(2/6)$ does, it assigns significantly higher probability to there being exactly four occurrences of heads.

The calculations above pertain only to the 6-flip model. But similar reasoning shows that in any n-flip model, $n > 4$, Threshold Compossibility fails if Proportional IID and Matching both hold.[25]

Proportional IID enjoys considerable plausibility. If it is possible that a quantum coin flipped n times lands heads exactly m times, then it seems possible that each flip of a quantum coin flipped n times be independent and have chance m/n of landing heads. A Humean who denies Proportional IID thus denies the possibility of something that seems possible. Of course, Humeans are committed to denying the possibility of things that seem possible already. It seems possible that an indeterministic quantum coin lands heads on each of its n flips. But there is only one n-heads world in the n-flip model. So if a Humean thinks that the n-heads world in the n-flip model is deterministic, a world in which it is nomically necessary that every flip lands heads, then the Humean must deny that it is possible that an indeterministic quantum coin land heads on each of its n-flips. But denying Proportional IID is not just denying the possibility of something that seems possible. It is one thing to set limits on how far apart the underlying chances and frequencies can be. It is another thing to set limits on how close together they can be. The

[25] For each m, let w_m be a m-heads world in the n-flip model at which $IID(m/n)$ holds. If $n > 4$ is even, then $w_{(n-2)/2} \vee w_{(n+2)/2}$ is not compossible with the claim that the chance of $w_{(n-2)/2} \vee w_{(n+2)/2}$ is at least x, where x is the chance of $w_{(n-2)/2} \vee w_{(n+2)/2}$ at $w_{n/2}$. If $n > 4$ is odd, then $w_{(n-3)/2} \vee w_{(n+1)/2}$ is not compossible with the claim that the chance of $w_{(n-3)/2} \vee w_{(n+1)/2}$ is at least x, where x is the chance of $w_{(n-3)/2} \vee w_{(n+1)/2}$ at $w_{(n-1)/2}$.

chances that feature in our best scientific theories are often arrived at by fitting a curve to the actual frequencies.

And the full strength of Proportional IID is not needed to render Threshold Compossibility and Humean Supervenience incompatible. Say that x is a possible IID centre in an n-flip model just if $IID(x)$ holds at some world in the n-flip model. The thrust of the point then can be put, vaguely but helpfully, as follows: *Threshold Compossibility fails in an n-flip model whenever the possible IID centres are sufficiently clustered.* Proportional IID entails that the possible IID centres are sufficiently clustered, but weakenings do likewise. For example, if there are three possible IID centres inclusively between $8/20$ and $12/20$ in the 20-flip model, then Threshold Compossibility fails.

Reconciling Simple Trust and Humean Supervenience is harder than reconciling Threshold Compossibility and Humean Supervenience—Threshold Compossibility does not entail Simple Trust. But appreciating the challenge of reconciling Threshold Compossibility and Humean Supervenience helps us see how formidable the challenge facing Humeans is. Science requires that there be many possible IID centres, and apparently weak claims about the diversity and distribution of possible IID chance in flip models renders Threshold Compossibility false.

8. Another bigger, badder bug

The next limitative result concerns Total Trust. Consider the following, a principle that asserts that there are at least two non-trivial possible IID centres in big enough flip models.

> *Non-trivial Diversity*. If n is big enough, then for some x and y, $0 < x < y < 1$, $IID(x)$ and $IID(y)$ each hold at some world or other in the n-flip model.

There is a claim about the extent of IID chance: a claim, clarified and made precise below, about the proportion of worlds in flip models at which IID chance functions hold. The claim is weak—it is very plausible that its truth is part of what science requires of the chance assignment. And as we prove (in the Appendix), Non-trivial Diversity and Total Trust are not both true if this weak claim about the extent of IID chance holds.

The tension Total Trust engenders in flip models between the extent of IID chance and the diversity of possible IID centres is easy to see if we consider a very strong claim about the extent of IID chance.

Call w and v *mirrored* in an n-flip model just in case the sequence of heads and tails in w and v is exactly switched. That is, H_j (heads on the jth flip) holds at w just in case T_j holds at v. For example, in a five flip model, the world *HHTTH* and the world *TTHHT* are mirrored. The following constraint requires a symmetry between mirrored worlds when one has an IID chance function.

Symmetry. An n-flip model is *symmetric* just if, for all $w \in W$, if $Ch_w = IID(x)$, and v mirrors w, then $Ch_v = IID(1 - x)$.

Let $\#w$ be the number of occurrences of heads at w. That is, $\#w = k$ just in case $w \in \langle \#H = k \rangle$. We then have the following result:

Initial Triviality. If $\langle W, \mathcal{P} \rangle$ is an n-flip model totally trusted by some π, all members of $\mathcal{P}$ are IID, and $\langle W, \mathcal{P} \rangle$ is symmetric, then if $0 < \#w < n$, $Ch_w = IID(1/2)$.

So, for example, if Total Trust holds, all of the possible chance functions in the 1000-flip model are IID, and the 1000-flip model is symmetric, then Non-trivial Diversity fails; for $IID(1/2)$ then holds at every world in the 1000-flip model, except perhaps the 0-heads and the 1000-heads worlds.[26]

Here is a sketch of the proof:

Proof (Sketch). The proof appeals to a background fact (Appendix, Theorem 1): if $\langle W, \mathcal{P} \rangle$ is an n-flip frame, then some regular probability function π totally trusts $\langle W, \mathcal{P} \rangle$ if and only if the members of $\mathcal{P}$ totally trust one another.

Suppose each element of $\mathcal{P}$ is IID, and suppose that $\langle W, \mathcal{P} \rangle$ is symmetric. We show that if the elements of $\mathcal{P}$ resiliently trust one another, then $Ch_w = Ch_v$ for all $Ch_w, Ch_v \in \mathcal{P}$ unless there are either 0 or n occurrences of heads at w or v.

Let E be the proposition that there are either $n - 1$ or n total occurrences of heads and H^n be the proposition that all flips are heads. By Symmetry and the fact that all chance functions are IID,

[26] The idea for this result depends on the fact that in a binomial distribution the probability of all flips coming up heads decreases very rapidly for $IID(x)$ as x decreases. Suppose, then, that Ch_w is $IID(x)$ for some low x. If Ch_w conditions on the fact that the chance of heads is actually *high*, it still won't assign high probability to all heads. That is, $Ch_w(\text{All heads} \mid Ch(H) \text{ is high})$ will still be too low.

all worlds with the same number of occurrences of heads have the same chance function. Let Ch_j refer to the chance function at all worlds with j-occurrences of heads and let $Ch_j(H) = p_j$.[27] Finally, let $Ch_{n-1}(H^n \mid E) = x$.

We can then derive that:

$$Ch_1(H^n \mid E, \langle Ch(H^n \mid E) \geq x\rangle) = Ch_1(H^n \mid E) = \frac{p_1^n}{np_1^{n-1}(1-p_1) + p_1^n} \quad (1)$$

and

$$\begin{aligned} Ch_{n-1}(H^n \mid E, \langle Ch(H^n \mid E) \geq x\rangle) &= p_{n-1}(H^n \mid E) \\ &= \frac{p_{n-1}^n}{np_{n-1}^{n-1}(1-p_{n-1}) + p_{n-1}^n} \quad (2) \\ &= \frac{(1-p_1)^n}{n(1-p_1)^{n-1}p_1 + (1-p_1)^n} \quad (3) \\ &= x \end{aligned}$$

(Lines (1) and (2) follow from the fact that H is distributed according to a binomial distribution, and line (3) follows from Symmetry.)

If all functions in $\mathcal{P}$ *totally* trust one another, then they *resiliently trust* one another. So we check what is required to make line (1) greater than or equal to line (3). With some simple algebra, we find that this requires $p_1 \geq 1/2$ and $p_{n-1} \leq 1/2$. Given Resilient Trust, this entails that $p_1 = \ldots = p_{n-1} = 1/2$. □

We prove a variant of this result in the Appendix (Theorem 12). Of course, even if the chance functions at many or most of the worlds in the n-flip model are IID, it is doubtful that every possible chance function in the n-flip model is IID. Initial Triviality thus puts little pressure, if any, on a Humean. But all we need to render Non-trivial Diversity and Total Trust incompatible is a weak claim about the extent of IID chance: the claim, clarified and made precise immediately below, that the extent of IID chance in n-flip models does not decrease as n increases.

[27] For what we've said so far, some worlds with the same number of heads might still have (up to two) different IID chance functions. This slightly complicates the proof in tedious ways, so we omit details.

For simplicity, we consider only n-flip models where n is even, and we assume that there is at least one $(n/2)$-heads world at which $IID(1/2)$ holds. We put these two ideas together with the following axiom:

Fifty/Fifty. If $\langle W, \mathcal{P} \rangle$ is an n-flip model, then n is even, and at some $w \in \langle \#H = n/2 \rangle$, $Ch_w = IID(1/2)$.

It will now be useful to introduce some more definitions. For a given n-flip model, we say that a number m is in the *IID region* of n if there is some m-heads world at which an IID chance function holds. In notation, we write $IID(Ch_w)$ to mean Ch_w is IID, and we define IID reg$(n) \coloneqq \{m : \exists w \text{ such that } \#w = m \text{ and } IID(Ch_w)\}$.

We say that m is in the *even odds region* of n just if there is some m-heads world in the n-flip model at which $IID(1/2)$ holds. In notation, EO-region$(n) \coloneqq \{m : \exists w \text{ such that } Ch_w = IID(1/2) \text{ and } \#w = m\}$. And we let $\ell(n)$ be the *smallest number* in the even odds region of n: $\ell(n) \coloneqq \min_m m \in$ EO-region(n).

The next axiom codifies the earlier thought that IID chance functions are possible at worlds with a reasonable mixture of heads and tails. The specific assumption we need is:

Sufficiency. If $\langle W, \mathcal{P} \rangle$ is an n-flip model, then (1) for all k such that $n/4 \leq k \leq n/2$, k is in the IID region of n, and (2) if 0 is not in the even odds region of n, then $\ell(n) - 1$ is in the IID region of n.

The first part of this axiom ensures that an IID chance function holds at some k-heads world if k is between $n/4$ and $n/2$. This seems very reasonable, especially in large models. There is, taking such a case, some 250,000-heads world in the 1,000,000-flip model without any discernible pattern beyond the fact that tails occurs three times as often as heads.[28] The second part ensures that there is some world with an IID chance function centred on something other than $1/2$, unless the model is completely trivial and assigns an IID chance function centred on $1/2$ even in the n-heads world.

The next assumption establishes a particular type of lower bound on the percentage of worlds with IID chance functions.

[28] When combined with Symmetry, Sufficiency guarantees that there an IID chance function holds at some k-heads world, if k is between $n/2$ and $3n/4$.

Boundedness. There exist $d > 0$ and $N \in \mathbb{N}$ such that for all $n \geq N$, if $\langle W, \mathcal{P} \rangle$ is an n-flip model and m is in the IID region of n, then

$$\frac{|\{w : \#w = m \text{ and } IID(Ch_w)\}|}{|\langle \#w = m \rangle|} \geq d$$

Here's the intuition. We let the Humean pick some number n that she counts as 'big'. We also let her pick some really small lower bound. For concreteness, say big numbers are at least 100 and the lower bound is 1%. We give her a big n-flip model and ask her for which $m \leq n$ there is at least one m-heads world at which an IID chance function holds. This axiom then requires that at least one percent of the m-heads worlds have IID chance functions. She is free to make 'big' be as large as she likes, and she is free to make d be as small as she likes so long as it is bigger than 0.[29]

This axiom is technical, but innocuous. Worlds at which IID chance functions hold are *disorganized*. There is not much to say about them beyond roughly what the frequency of heads to tails is. (If there were more to say, then there would be a nice law characterizing the pattern.) As n grows large, more and more worlds are disorganized—most sequences appear totally random. Think of a television screen with its mix of black and white pixels. There are a few arrangements of such pixels that result in discernible patterns, something you could relatively easily describe. But for the vast majority, the screen is just random noise. Denying Boundedness is akin to thinking that discernible patterns are more common as the size of the television screens increases, which is exactly the opposite of what seems clear. Discernible patterns are less common as the size increases.

The final axiom is required for technical reasons:

Monotonicity. If $\langle W, \mathcal{P} \rangle$ is an n-flip model and Ch_w, Ch_v in $\mathcal{P}$ are both IID with $Ch_w(H) < Ch_v(H)$, then $Ch_w(\langle Ch(H^n) \geq 2^{-n} \rangle) < Ch_v(\langle Ch(H^n) \geq 2^{-n} \rangle)$.

If all worlds in the model are IID, then Monotonicity is redundant. In that case, $\langle Ch(H^n) \geq 2^{-n} \rangle = \langle Ch(H) \geq 1/2 \rangle$. This axiom rules out strange situations where many non-IID worlds with relatively few heads for some reason give fairly high probability to the claim that all flips land heads.

[29] We can actually weaken this axiom so that it only applies to m in the even odds region of n instead of in the IID region of n, but it strikes us as a bit less natural when stated that way.

We can now state our most powerful result (see the Appendix for proof).

> *Serious Triviality*. Let $\langle W_1, \mathcal{P}_1\rangle, \langle W_2, \mathcal{P}_2\rangle$, ... be a sequence of models with $|W_i| < |W_{i+1}|$. Assume each validates Sufficiency, Fifty/Fifty, Monotonicity, and Symmetry. Moreover, assume that Boundedness holds of the sequence. Then there exists an $N \in \mathbb{N}$ such that if $i \geq N$ and some regular probability function totally trusts $\langle W_i, \mathcal{P}_i\rangle$, then for all $Ch_w \in \mathcal{P}_i$ such that $IID(Ch_w)$, we have $P_w = IID(1/2)$.

Serious Triviality tells us that the weak claim about the extent of IID chance—the conjunction of Sufficiency, Fifty/Fifty, Monotonicity, Symmetry, and Boundedness—implies that Total Trust and Non-trivial Diversity are not both true. If any rational credence function totally trusts chance and the weak claim about the extent of IID chance holds, then for large n, every possible IID chance function in the n-flip model is centred on 1/2, except possibly the 0-heads and n-heads worlds.

Science requires both that the extent of IID chance be considerable and that the diversity of possible IID centres be many. The case for Total Trust is strong. But as the proof of Serious Triviality reveals, no chance assignment that is totally trusted by a rational credence function provides both the extent of IID chance and the diversity of possible IID centres that science requires.

9. Conclusion

The big, bad bug shows that Humean Supervenience is inconsistent with Reflection, given a hard-to-deny claim about what science requires of the chance assignment. A promising Humean response is to reject Reflection in favour of some principle that draws a looser but still tight enough connection between chance and credence. The connection that New Reflection draws is, we argue, not tight enough, so we are led to the principles of trust, intermediate principles, which are strictly weaker than Reflection yet strictly stronger than New Reflection. The suspicion that Humean Supervenience is not consistent with a tight enough connection between chance and credence would be greatly reduced with a tenability result: a proof that Humean Supervenience is consistent with some or all of the principles of trust, given some not-too-weak assumptions about what science requires of the chance assignment. But what we have instead are bigger, badder bugs: proofs that Humean Supervenience is inconsistent with principles of trust, given stronger but still hard-to-deny claims about what science requires of the chance assignment.

Our limitative results pertain to particularly simple flip models: finite, fixed flip models, wherein each world has the same number of flips. Some of our results extend to finite, variable flip models, wherein different worlds have different numbers of flips.[30] But there is more work to do investigating both finite, variable flip models and infinite flip models.[31]

And there is work to do extending the argument beyond flip models. Realistic hypotheses about the world we find ourselves in are, in various ways, unlike a world exhausted by a sequence of coin flips. Even if a realistic hypothesis about our world could be encoded in a binary sequence, it is unlikely that our best scientific theories would treat each bit in the binary sequence as the outcome of some IID chance process. But the difference between the worlds in flip models and realistic hypotheses about the world we find ourselves in does not obviously provide solace to Humeans. Our experience suggests that reconciling Humean Supervenience and the principles of trust becomes harder, not easier, as the size and the complexity of the model increases.

The way forward is gradual and mathematically precise, proceeding from less to more realistic models. Our limitative results are just some of the very many out there—there is a continent to explore. There are many claims about what science requires of the chance assignment worth considering and many intermediate chance-credence principles besides the principles of trust. The continent is sure to contain stronger limitative results than the ones proved here. Whether the continent also contains philosophically interesting tenability results remains to be seen. Is there any proof that Humean Supervenience is consistent with some tight enough connection between chance and credence, given the truth of hard-to-deny claims about what science requires of the chance assignment?[32]

[30] For example, the fact that Simple Trust and Proportional IID are not both true in a finite, fixed flip model implies that Resilient Trust and Proportional IID are not both true in a finite, variable flip model.

[31] There is also work to do investigating flip models in which some worlds lack a precise chance function.

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Appendix

Here we prove a variant of the Initial Triviality result (Theorem 12) and prove the Serious Triviality result (Theorem 15).

A.1. Notation and terminology

As before, we use $\langle W, \mathcal{P}\rangle$ to refer to a generic n-flip model. We will switch to using $P_w \in \mathcal{P}$ to refer to the chance function at a world (instead of Ch_w) and P to refer to the (*de dicto*) chance function—whatever it is—(instead of Ch) partly for reasons of notational compactness and partly because the results hold generally for all such models even when P and P_w are interpreted differently.

As before, we will talk loosely and say that a function P_w is IID when it treats the flips in a sequence as IID. Even more loosely, we'll say a world w is IID just in case P_w is IID.

As in the main text, we will write $P_w = IID(x)$ to mean P_w is *IID* and assigns probability x to heads. It will also sometimes be convenient, when P_w is IID, to write $P_w(H) = x$ or $P_w(H) \geq x$. As in the main text, we will also write $IID(P_w)$ to mean that P_w is IID.

We'll say that $\langle W, \mathcal{P}\rangle$ *validates* Total/Simple Trust just in case all members of $\mathcal{P}$ totally/simply trust P. More explicitly, $\langle W, \mathcal{P}\rangle$ validates Simple Trust if for all w, $P_w(p \mid \langle P(p) \geq x\rangle) \geq x$ for all x, and similarly for Total Trust.

As a reminder, we also have the following notation:

- $\#w$ refers to the number of heads at w.
- $\ell(n)$ is the smallest number k in an n-flip model obeying Fifty/Fifty such that for all w where $\#w = k$, w has an IID chance function centred on $1/2$.
- H^n refers to the proposition that all n flips in an n-flip model land heads.

We also remind the reader of the following principles for reference below (now with P and P_w replacing Ch and Ch_w.

Symmetry. An n-flip model is *symmetric* just if, for all $w \in W$, if $P_w = IID(x)$, and v mirrors w, then $P_v = IID(1-x)$.

Fifty/Fifty. If $\langle W, \mathcal{P}\rangle$ is an n-flip model, then n is even, and at some $w \in \langle \#H = n/2\rangle$, $P_w = IID(1/2)$.

Sufficiency. If $\langle W, \mathcal{P}\rangle$ is an n-flip model, then (1) for all k such that $n/4 \leq k \leq n/2$, k is in the IID region of n, and (2) if 0 is not in the even odds region of n, then $\ell(n) - 1$ is in the IID region of n.

Monotonicity. If $\langle W, \mathcal{P}\rangle$ is an n-flip model and P_w, P_v in $\mathcal{P}$ are both IID with $P_w(H) < P_v(H)$, then $P_w(\langle P(H^n) \geq 2^{-n}\rangle) < P_v(\langle P(H^n) \geq 2^{-n}\rangle)$.

Boundedness. There exists $d > 0$ and $N \in \mathbb{N}$ such that for all $n \geq N$, if $\langle W, \mathcal{P}\rangle$ is an n-flip model and m is in the IID region of n, then

$$\frac{|\{w : \#w = m \text{ and } IID(P_w)\}|}{|\langle \#w = m\rangle|} \geq d$$

A.2. Results

Our main question concerns when a regular probability function π can totally trust chance. As it turns out, to answer that question, we just need to find out when the frame $\langle W, \mathcal{P}\rangle$ validates total trust, as the following theorem establishes.

Theorem 1 (Dorst et al.). A regular probability function π totally trusts a frame $\langle W, \mathcal{P}\rangle$ only if $\langle W, \mathcal{P}\rangle$ validates total trust.

The proof is involved, so we omit it here and refer the interested reader to Dorst et al. (2021, Theorem 4.1). As we'll see, our results below entail that the functions in $\mathcal{P}$ can't even *simply* trust one another. We conjecture that no regular probability function can simply trust them.

For what follows, it's important to keep in mind that if $IID(P)$, then according to P, H follows a Bernoulli Distribution with parameter $P(H)$. In turn, if X is a random variable representing the total number of heads, then X is distributed according to a binomial distribution with parameter $P(H)$. If $P(H) = p$, the probability of any given world with $\#w = k$ is $p^k(1-p)^{n-k}$. So, if $0 < p < 1$, then for all $w \in W$, $P(w) > 0$.

We now prove some basic facts about models that validate Simple Trust. (Dorst 2020 provides a more general result implying part 2 of the following proposition.)

Proposition 2. Suppose $\langle W, \mathcal{P}\rangle$ validates Simple Trust. Then

(1) If $\langle W, \mathcal{P}\rangle$ validates Fifty/Fifty, then for all $w \in W$, $P_w(w) > 0$, and

(2) For all $w, v \in W$, $P_w(w) \geq P_v(w)$

Proof. To prove (1), let $P_h = IID(1/2)$ be in $\mathcal{P}$. (Existence is guaranteed by Fifty/Fifty.) For all $w \in W$, it's clear $P_h(w) > 0$. Suppose $P_w(w) = 0$

for some $w \in W$. Then $w \in \langle P(w) \leq 0\rangle$, so $P_h(w \mid \langle P(w) \leq 0\rangle)$ is defined and > 0. Contradiction.

To prove (2), suppose $P_w(w) < P_v(w) = x$. Then $w \notin \langle P(w) \geq x\rangle$. So, $P_v(w \mid \langle P(w) \geq x\rangle) = 0 < x$. □

Proposition 3. Suppose $\langle W, \mathcal{P}\rangle$ validates Simple Trust. Let $P_w, P_v \in \mathcal{P}$ be IID with $\#w < \#v$. Then $P_w(H) \leq P_v(H)$.

Proof. Let $P_w(H) = p_w$ and $P_v(H) = p_v$. Suppose $\#w < \#v$ but $p_w > p_v$. Recall that if X is the number of heads, then according to both P_v and P_w, X is distributed according to a Binomial Distribution with parameters p_v and p_w respectively. So, if $\#w/n \leq p_v < p_w$, then $P_v(w) > P_w(w)$, which entails $\langle W, \mathcal{P}\rangle$ violates Simple Trust, (by part 2 of Proposition 2). Likewise, if $p_v < \#w/n \leq p_w \leq \#v/n$, $P_w(v) > P_v(v)$. Finally, suppose $p_v < \#w/n \leq \#v/n \leq p_w$. In this case, $P_v(w) > P_v(v) \geq P_w(v) \geq P_w(w)$, again violating Simple Trust by Proposition 2. □

Remark. Proposition 3 does not rule out the possibility of distinct IID chance functions at worlds w and v if $\#w = \#v$ in an n-flip model. Following the proof, we see that there could be a maximum of two different IID chance functions for worlds with the same number of heads, namely, one on each side of $\#w/n$. (This adds a wrinkle elided over to the proof sketch of Initial Triviality in the main text, but it's one that can be easily accommodated.) As we'll now see, there is one important exception.

Proposition 4. Suppose $\langle W, \mathcal{P}\rangle$ validates Simple Trust and Fifty/Fifty. Then if $w \in W$ is IID and $\#w = n/2$, $P_w = IID(1/2)$.

Proof. By Fifty/Fifty, some world $h \in W$ is IID such that $\#h = n/2$ and $P_h = IID(1/2)$. So, if P_w is also IID and $\#w = n/2$, then $P_w(w) \leq P_h(w)$. Given Proposition 2, $P_w(w) \geq P_h(w)$, so $P_w(H) = 1/2$. □

Remark. Note that Proposition 4 guarantees that for any n-flip model $\langle W, \mathcal{P}\rangle$ validating Simple Trust and Fifty/Fifty, $\ell(n)$ is defined and $\leq n/2$. Further, we have $\ell(n) \geq 1$ by part 1 of Proposition 2.

We can also put upper bounds on worlds with IID chance functions that have fewer than $\ell(n)$ total heads.

Fact 5. Suppose $\langle W, \mathcal{P}\rangle$ is an n-flip model validating Simple Trust and Fifty/Fifty. Suppose P_w is IID for some some world w with $\#w = \ell(n) - 1$. Then if $P_w \neq IID(1/2)$, $P_w(H) < \ell(n)/n$.

Proof. Suppose $\langle W, \mathcal{P}\rangle$ validates Simple Trust and $\ell(n) \geq 1$. Let $\#w = \ell(n) - 1$, and let $P_w = IID(p)$. Suppose $\ell(n)/n \leq p$. Let $v \in W$ with $\#v = \ell(n)$ and $P_v = IID(1/2)$. By hypothesis, $p \neq 1/2$. By Proposition 3,

p must be $< 1/2$. But in that case, since $\ell(n)/n \leq p < 1/2$, $P_w(v) > P_v(v)$, contradicting Proposition 2. □

We know that $P_w(H) \leq P_v(H)$ if $\#w < \#v$ and both have IID chance functions by Proposition 3. We also know, by Fact 5 that if $\#w < \ell(n)$ and w is IID, $P_w(H) < \ell(n)/n$.

It will be useful below to consider a special IID probability function P^ℓ over W but *not* in $\mathcal{P}$ such that $P^\ell(H) = \ell(n)/n$. The following lemma will serve to put an important constraint on P^ℓ; namely, if $\langle W, \mathcal{P}\rangle$ validates Simple Trust and Fifty/Fifty, then $P^\ell(H \mid \langle H^n \geq 2^{-n}\rangle) \geq 2^{-n}$.

> *Lemma* 6. Let $\langle W, \mathcal{P}\rangle$ be an n-flip frame validating Simple Trust with at least one IID function $P \in \mathcal{P}$ such that $P(H) \geq 1/2$. For any $x \in (0,1)$, let $P^{(x)} = IID(x)$.[33] Let $f(x) = P^{(x)}(H^n \mid \langle P(H^n) \geq 2^{-n}\rangle)$. Then f is strictly increasing over $(0,1)$.

Proof. Let $V \coloneqq \{w \in W \mid P_w(H^n) \geq 2^{-n}\}$. Note that the requirement that there be at least one IID chance function $P \in \mathcal{P}$ such that $P(H) \geq 1/2$ guarantees V is non-empty. Let $V(k) \coloneqq |\{w \in V : \#w = k\}|$. With f and $P^{(x)}$ defined as above, we then have

$$f(x) = \frac{x^n}{P^{(x)}(V)} \tag{4}$$

$$= \frac{x^n}{\sum_{k=0}^{n} V(k)x^k(1-x)^{n-k}} \tag{5}$$

f is clearly differentiable, so we just need to check that its derivative is positive. This is straightforward but tedious to do. □

Our next goal is to put lower bounds on $\ell(n)$ for a given model (Lemma 8). To do so we must first prove Lemma 7, which in turn appeals to the famous Inequality of Arithmetic and Geometric Means.

> *AM-GM Inequality.* For any list of n non-negative reals $x_1, \ldots, x_n$,
>
> $$\frac{1}{n}\sum_{i=1}^{n} x_i \geq \left(\prod_{i=1}^{n} x_i\right)^{1/n}$$
>
> with equality if and only if $x_1 = x_2 = \cdots = x_n$.

> *Lemma* 7. Suppose $n, k \in \mathbb{N}$ with $n > k$. Then $\frac{n}{2^{(n+k)/n}} \geq \frac{n-k}{2}$.

[33] Note that $P^{(x)}$ is not necessarily in $\mathcal{P}$.

Proof. Simple algebra shows that the lemma holds if and only if for all $n \geq k + 2$, we have:

$$\frac{n}{n-k} \geq 2^{k/n} \tag{6}$$

To prove line (6), first consider a list of numbers $x_1, \ldots, x_n$ with:

$$x_i = \begin{cases} 2 & i \leq k \\ 1 & i > k \end{cases}$$

We have:

$$\frac{1}{n}\sum x_i = \frac{n+k}{n}$$

and

$$\left(\prod x_i\right)^{1/n} = 2^{k/n}$$

So, by the AM-GM Inequality, $2^{k/n} < \frac{n+k}{n}$.

To prove line (6) holds, we just need to determine when $\frac{n+k}{n} \leq \frac{n}{n-k}$, and it is easy to see this holds whenever $n > k$. □

Let $\langle W, \mathcal{P} \rangle$ be an n-flip frame. Suppose $w \in W$ is a world with $\#w = \ell(n) - 1$ with IID chance function P_w. By Fact 5, if $P_w(H) \neq 1/2$, $P_w(H) < \ell(n)/n$. Let P^ℓ be defined over W (but not necessarily in $\mathcal{P}$) such that $P^\ell = IID(\ell(n)/n)$. By Lemma 6, we know

$$P_w(H^n \mid \langle P(H^n) \geq 2^{-n} \rangle) < P^\ell(H^n \mid \langle P(H^n) \geq 2^{-n} \rangle).$$

This will be important for the next lemma.

> *Lemma* 8. Suppose $\langle W, \mathcal{P} \rangle$ is an n-flip model validating Simple Trust, Fifty/Fifty, and Sufficiency, with $\ell(n) \geq 2$. Let $P^\ell(H) = \ell(n)/n$ be an IID probability function. Then (1) $P^\ell(\langle P(H^n) \geq 2^{-n} \rangle) > 0$, and (2) if $P^\ell(\langle P(H^n) \geq 2^{-n} \rangle) \geq 2^{-k}$ for $k \in \mathbb{N}$, then $\ell(n) \geq (n-k)/2$.

Proof. Part 1 follows trivially from the fact that $\ell(n) > 0$ and Fifty/Fifty.

We now establish part 2. Let $P_w \in \mathcal{P}$ be IID with $P_w(H) < 1/2$ and $\#w = \ell(n) - 1 > 0$. Such a P_w is guaranteed to exist by Sufficiency. By Proposition 2, $0 < P_w(H)$. Since P_w is also IID and $\langle W, \mathcal{P} \rangle$ validates Fifty/Fifty, $P_w(\langle P(H^n) \geq 2^{-n} \rangle) > 0$. By Proposition 5, $P_w(H) < \ell(n)/n$. Since $\langle W, \mathcal{P} \rangle$ validates Simple Trust, $P_w(H^n \mid \langle P(H^n) \geq 2^{-n} \rangle) \geq 2^{-n}$. So, by Lemma 6,

$$P^\ell(H^n \mid \langle P(H^n) \geq 2^{-n} \rangle) \geq 2^{-n} \tag{7}$$

Suppose $P^{\ell}(\langle P(H^n) \geq 2^{-n} \rangle) \geq 2^{-k}$. We have:

$$P^{\ell}(H^n \mid \langle P(H^n) \geq 2^{-n} \rangle) = \frac{\left(\ell(n)/n\right)^n}{P^{\ell}(\langle P(H^n) \geq 2^{-n} \rangle)} \leq \frac{\left(\ell(n)/n\right)^n}{2^{-k}} \tag{8}$$

So, from lines (7) and (8), it follows that:

$$2^k\left(\ell(n)/n\right)^n \geq 2^{-n}$$

which holds if and only if

$$\ell(n) \geq \frac{n}{2^{(1+k/n)}} \geq \frac{n-k}{2}$$

where the last line follows from Lemma 7. □

Having established a lower bound on $\ell(n)$, we now aim to establish an upper bound. The strategy is to consider a proposition true at just two worlds w and v (both IID), where $\#w = n/2 - k$ and $\#v = n/2 + k$. When k is sufficiently small, it will turn out that the proposition $\{w, v\}$ attains maximum probability amongst IID chances when $P = IID(1/2)$. This fact, which we establish in the next lemma, will then force IID chance functions at worlds with roughly $n/2$ occurrences of heads to assign heads probability $1/2$.[34]

> *Lemma* 9. Suppose n is even, $k \in \mathbb{N}$, and $k^2 \leq n/4$. Then the polynomial
>
> $$p^{n/2-k}(1-p)^{n/2+k} + p^{n/2+k}(1-p)^{n/2-k}$$
>
> achieves its maximum over the unit interval uniquely at $p = 1/2$.

Proof. Without loss of generality, assume $p \in [0, 1/2]$. When $p = 1/2$, the polynomial evaluates to $2/2^n$, so we need to show

$$p^{n/2-k}(1-p)^{n/2+k} + p^{n/2+k}(1-p)^{n/2-k} \leq 2/2^n$$

with equality if and only if $p = 1/2$. From simple algebra, we see that this holds if and only if:

$$p^{n/2-k}(2-2p)^{n/2+k} + (2p)^{n/2+k}(2-2p)^{n/2-k} \leq 2 \tag{9}$$

[34] Special thanks to Neal E. Young for help with the proof of this lemma.

Let $x = 1 - 2p$, so $x \in [0, 1]$. Line (9) holds just in case:

$$(1-x)^{n/2-k}(1+x)^{n/2+k} + (1-x)^{n/2+k}(1+x)^{n/2-k} \leq 2$$

This in turn holds if and only if:

$$(1-x)^{n/2-k}(1+x)^{n/2-k}\left[(1-x)^{2k} + (1+x)^{2k}\right] \leq 2 \tag{10}$$

Further, the left-hand side of line (10) decreases with n. Since $k^2 \leq n/4$, we just need to check that it holds for $n = 4k^2$.

The right- and left-hand sides are equal in line (10) when $x = 0$. The left-hand side is differentiable, so to prove the theorem we just need to show the derivative is negative.

Taking the derivative of the LHS of line (10) when $n = 4k^2$ and simplifying is tedious, but we end up with:

$$-2k(1-x^2)^{2k^2-k-1}\left((1+x)^{2k}(2kx-1) + (1-x)^{2k}(2kx+1)\right)$$

Factoring out the $-2k(1-x^2)^{2k^2-k-1}$ out front, we see that we need to verify that:

$$(1+x)^{2k}(2kx-1) + (1-x)^{2k}(2kx+1) > 0 \tag{11}$$

for $k \geq 1$.

Using binomial expansion, we see that verifying line (11) is equivalent to verifying:

$$\sum_{i=0}^{2k}\binom{2k}{i}\left[x^i(2kx-1) + (-x)^i(2kx+1)\right] > 0 \tag{12}$$

The left-hand side of line (12), in turn, simplifies to:

$$4kx^{2k+1} + 2\sum_{i=0}^{k-1}\left[\binom{2k}{2i}2k - \binom{2k}{2i+1}\right]x^{2i+1}$$

It is straightforward to check that $\binom{2k}{2i}2k - \binom{2k}{2i+1} > 0$, which ensures the inequality of line (12) holds, as desired. □

We now can provide an upper bound on $\ell(n)$.

Lemma 10. Suppose $\langle W, \mathcal{P}\rangle$ is an n-flip model satisfying Simple Trust, Fifty/Fifty, Symmetry, and Sufficiency with $n \geq 4$. Then $\ell(n) \leq (n-\sqrt{n})/2$.

Proof. Let $j \leq \sqrt{n}/2$, with $j \in \mathbb{N}$. By Sufficiency and Symmetry, there exist $w, v \in W$ such that $\#w = n/2 - j$ and $\#v = n/2 + j$ and where P_w and P_v are both IID, and $P_v(T) = P_w(H)$.

Consider the proposition $X = \{w, v\}$. Let P_h be an IID chance function at a world h with $\#h = n/2$. By Fifty/Fifty, $P_h(H) = 1/2$. Lemma 9 entails that P_h assigns a strictly higher probability to X (namely, 2^{-n+1}) than any other IID probability function does.

Claim: $P_w(H) = 1/2$. For suppose not. Then $P_v(H) \neq 1/2$. In this case, $X \cap \langle P(X) \geq 2^{-n+1} \rangle = \varnothing$. So, since $P_h(\langle P(X) \geq 2^{-n+1} \rangle) > 0$, $P_h(X \mid \langle P(X) \geq 2^{-n+1} \rangle) = 0$, violating Simple Trust.

So, if $j \leq \sqrt{n}/2$, then $n/2 - j \leq \ell(n)$. Therefore, $\ell(n) \leq (n - \sqrt{n})/2$ as desired. □

> *Theorem* 11. Suppose $\langle W, \mathcal{P} \rangle$ is an n-flip model that validates Simple Trust, Symmetry, and Fifty/Fifty and $n \geq 6$. Suppose all functions in $\mathcal{P}$ are IID. Then for all $w \in W$, if $0 < \#w < n$, $P_w(H) = 1/2$.

Proof. Suppose $\ell(n) \geq 1$, and let $P^\ell(H) = \ell(n)/n$ with P^ℓ an IID probability function defined over W. Let X be a random variable such that $X(w) = \#w$ for $w \in W$. $X \sim B(n, \ell(n)/n)$ according to P^ℓ. The mode of $B(n, \ell(n)/n) = \ell(n)/n$, which means $P^\ell(\langle X \geq \ell(n)/n \rangle) \geq 1/2$. By Lemma 8, $\ell(n) \geq (n-1)/2n$. Since $\ell(n)$ is an integer with $n \geq 6$, $\ell(n) = n/2$. But by Lemma 10, $\ell(n) \leq \sqrt{n}/2 - 1$. So, $n/2 \leq \sqrt{n}/2 - 1$, which is impossible when $\ell(n) \geq 6$. So, $\ell(n) = 1$ for all $n \geq 6$. This completes the proof. □

> *Theorem* 12. Suppose $\langle W, \mathcal{P} \rangle$ is an n-flip model that validates Symmetry, Fifty/Fifty, and $n \geq 6$, and π is a regular probability function that totally trusts $\langle W, \mathcal{P} \rangle$. Suppose all functions in $\mathcal{P}$ are IID. Then for all $w \in W$ if $0 < \#w < n$, $P_w(H) = 1/2$.

Proof. This follows immediately from Theorems 1 and 11. □

We will now see how we can relax the assumption that all chance functions are IID and still cause trouble for the Humeans.

> *Lemma* 13. Suppose $\langle W, \mathcal{P} \rangle$ is an n-flip model satisfying Simple Trust, Fifty/Fifty, Monotonicity, Symmetry, and Sufficiency. Then $P^\ell(\langle P(H^n) \geq 2^{-n} \rangle) \leq 2^{-\sqrt{n}}$.

Proof. Given the assumptions, we know from Lemma 8, that if $P_w(\langle P(H^n) \geq 2^{-n} \rangle) > 2^{-k}$, $(n-k)/2 \leq \ell(n)$. From the assumptions and Lemma 10, we know $\ell(n) \leq (n - \sqrt{n})/2$. So, $k \geq \sqrt{n}$, meaning $P^\ell(\langle P(H^n) \geq 2^{-n} \rangle) \leq 2^{-\sqrt{n}}$. □

Note that $\langle IID(P) \text{ and } P(H) \geq {}^1\!/\!_2\rangle \subseteq \langle P(H^n) \geq 2^{-n}\rangle$. So what Lemma 13 entails is the following. Let P_w be an IID chance function that assigns probability under $^1\!/\!_2$ to H, but such that if $P_v \in \mathcal{P}$ is IID and $P_v(H) < {}^1\!/\!_2$, then $P_v(H) \leq P_w(H)$. It's easy to show, given the assumptions, that $P_w(\langle P(H^n) \geq 2^{-n}\rangle) < P^{\ell}(\langle P(H^n) \geq 2^{-n}\rangle)$.

Intuitively, at least when n is big, $P_w(H)$ should be *just* under $^1\!/\!_2$. After all, if just one more tail had been heads, then (if done in a way that maintained IID), the chance of heads would have been $^1\!/\!_2$. But Lemma 13 entails that $P_w(\langle P(H^n) \geq 2^{-n}\rangle) \leq 2^{-\sqrt{n}}$, which is small. (For example, when n is 10, this quantity is $P_w(\langle P(H^n) \geq 2^{-n}\rangle) < 0.12$. When $n = 100$, $P_w(\langle P(H^n) \geq 2^{-n}\rangle) < 0.001$.) This can only be the case if *either* $P_w(H)$ is extremely small, *or* very few worlds have IID chance functions. Indeed, as n grows, the proportion of worlds with approximately $^n\!/\!_2$ heads tends toward 1 (where 'approximately' here means within x% of $^n\!/\!_2$). So, either $P_w(H)$ must tend toward 0 or the percentage of worlds with IID chance functions must tend toward 0 very quickly. This is why, intuitively, when we add Boundedness, we end up with the Serious Triviality result in the main text.

> *Theorem* 14. Let $\langle W_1, \mathcal{P}_1\rangle, \langle W_2, \mathcal{P}_2\rangle, \ldots$ be a sequence of models with $|W_i| < |W_{i+1}|$. Assume each validates Simple Trust, Sufficiency, Fifty/Fifty, and Symmetry. Moreover, assume that Boundedness holds of the sequence. Then there exists an $N \in \mathbb{N}$ such that if $i \geq N$ and $P_w \in \mathcal{P}_i$ is IID, then $P_w = IID(^1\!/\!_2)$.

Proof. Suppose $\ell(n) \geq 2$. Let P^{ℓ} be IID with $P^{\ell}(H) = {}^{\ell(n)}\!/\!_n$. Let $\text{IID}(W) := \{w \in W : P_w \text{ is IID}\}$, and let $h(W) := \{w \in W : \ell(n) \leq \#w \leq n - \ell(n)\}$.

Note that, given Symmetry, if $w \in h(W) \cap \text{IID}(W)$, then $P_w(H) = {}^1\!/\!_2$. So,

$$d \cdot P^{\ell}(h(W)) \leq P^{\ell}(h(W) \cap \text{IID}(W)) \leq 2^{-\sqrt{n}} \tag{13}$$

where the first inequality follows from Strong Sufficiency with threshold d, and the second from Lemma 13.

We will now show that for large enough n, $d \cdot P^{\ell}(h(W)) > 2^{-\sqrt{n}}$, contradicting line (13). For fixed $\langle W, \mathcal{P}\rangle$, let $X(w) = \#w$. If $X \sim B(n, p)$, then X has increasing variance with p over $[0, {}^1\!/\!_2]$. By Lemma 10, $\ell(n) \leq {}^{(n-\sqrt{n})}\!/\!_2$. So the minimum possible value for $P^{\ell}(h(W))$ is achieved when $P^{\ell}(H) = {}^{(n-\sqrt{n})}\!/\!_{2n}$.

So, assume $P^{\ell}(H) = {}^{(n-\sqrt{n})}\!/\!_{2n}$. If $X \sim B(n, {}^{(n-\sqrt{n})}\!/\!_{2n})$, then $\sigma(X) = {}^{\sqrt{n-1}}\!/\!_2$, where $\sigma(X)$ represents the standard deviation of X. By Chebyshev's Inequality, we then know that $P^{\ell}({}^{(n-3\sqrt{n})}\!/\!_2 \leq X \leq {}^{(n+\sqrt{n})}\!/\!_2) > {}^3\!/\!_4$

(since the probability that X is within two standard deviations must be at least $3/4$). But the mode of X is $\ell(n)$, so $P^{\ell}(X < \ell(n)) < 1/2$. Therefore, $P^{\ell}(\ell(n) \leq X \leq (n+\sqrt{n})/2) > 1/4$. Thus $P^{\ell}(h(W) \cap \mathrm{IID}(W)) > d/4$. For sufficiently large n, $d/4 > 2^{-\sqrt{n}}$, which contradicts line (13). So, for large enough n, $\ell(n) = 1$. □

We now can state our final triviality result, referred to as Serious Triviality in the main text.

> *Theorem* 15. Let $\langle W_1, \mathcal{P}_1\rangle, \langle W_2, \mathcal{P}_2\rangle, \ldots$ be a sequence of models with $|W_i| < |W_{i+1}|$. Assume each validates Sufficiency, Fifty/Fifty, and Symmetry. Moreover, assume that Boundedness holds of the sequence. Then there exists an $N \in \mathbb{N}$ such that if $i \geq N$ and some regular probability function totally trusts $\langle W_i, \mathcal{P}_i\rangle$, then for all $P_w \in \mathcal{P}_i$ such that $IID(P_w)$, we have $P_w = IID(1/2)$.

Proof. This follows from Theorems 1 and 14. □

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