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ORIGINAL PAPER

Baumann on the Monty Hall problem and single-case probabilities

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Abstract Peter Baumann uses the Monty Hall game to demonstrate that probabilities cannot be meaningfully applied to individual games. Baumann draws from this first conclusion a second: in a single game, it is not necessarily rational to switch from the door that I have initially chosen to the door that Monty Hall did not open. After challenging Baumann's particular arguments for these conclusions, I argue that there is a deeper problem with his position: it rests on the false assumption that what justifies the switching strategy is its leading me to win a greater percentage of the time. In fact, what justifies the switching strategy is not any statistical result over the long run but rather the "causal structure" intrinsic to each individual game itself. Finally, I argue that an argument by Hilary Putnam will not help to save Baumann's second conclusion above.

Keywords Monty Hall · Probability · Rigidity · Causal structure

1 Introduction

Suppose that you are presented with three cards down, two queens and one king. Your probability of drawing the king seems to be 1/3. But what does this common-sense conclusion *mean*? A natural answer is the *statistical* interpretation: to say that I have a 1/3 probability of drawing a king in this particular game is just to say that I will draw a king in 1/3 of a sufficiently large number of such games.[1] Suppose, however, that I will *not* continue to repeat this game, that I will draw a card in only this one instance. Then what of the statistical interpretation?

[1] See Moser and Mulder (1994, pp. 115–116, 118).

[redacted]

We might go one of two ways here. First, we might adopt the "counterfactual" statistical interpretation and argue that the statistical interpretation may still make sense of probability in this one game. To say that I have a 1/3 probability of drawing a king is just to say that drawing a king in this particular game falls into the category of outcomes that *would* happen in 1/3 of a sufficiently large number of games. Second, we might adopt the "skeptical" view: the statistical interpretation can*not* make sense of probability-statements applied to individual games. Rather, it meaningfully applies only to an extended series of games. Therefore it just does not make *any* sense to say that I have a 1/3 probability of drawing a king in this particular game.

In a recent article, Peter Baumann (2005) uses variations on the Monty Hall game to support the skeptical version of the statistical interpretation. He argues that the Monty Hall game helps to show that probability-statements applied to single, isolated games are nonsensical or meaningless. Baumann then uses this conclusion to draw another: that it is not necessarily rational to switch doors in a single game, only over an iterated series of games.[2] I shall argue that both of Baumann's conclusions are mistaken.[3]

2 The standard Monty Hall Problem

The "original" or "standard" Monty Hall Problem involves two agents—Monty Hall, the game show host, and myself, the game show contestant.[4] After I walk up the steps, Monty Hall presents me with three closed doors—1, 2, and 3. Behind one of these three doors is a brand new car; behind each of the other two doors, nothing.[5] Monty Hall then asks me to choose a door. So I do—door 1. Monty Hall then opens door 3, which is revealed to have nothing behind it.[6] Monty Hall turns back around and asks me whether I want to stick with door 1 or switch to door 2. As I ponder the issue, he offers me $100 *not* to switch. So I have the option of sticking with door 1 (and winning $100) or switching to door 2. I will win whatever is behind the door that I choose at this point.

[2] Baumann actually draws a second inference as well: application of probability-statements to a single Monty Hall game leads to Moore-paradoxicality. Since I shall argue that the underlying basis for this inference (i.e., that application of probability-statements to single instances is meaningless) is false, my arguments will help to show that this second inference is not necessarily true as well.

[3] As will become clear in Sect. 6, my position coincides with Horgan's (1995).

[4] The Monty Hall Problem has been written about primarily in math journals and popular magazines and newspapers. For a comprehensive list of such discussions, see the bibliography in Barbeau (1993) and Barbeau (2000, pp. 87–89). It is also starting to make its way into other kinds of literature as well—e.g., law (see Yin, 2000, pp. 177–178, 189) and philosophy (see the authors discussed in this paper as well as Bradley & Fitelson, 2003). Mary vos Savant (1990a) first popularized the problem. Vos Savant, however, cannot be credited with originating it. Rather, the credit for origination arguably goes to Gardner (1959a, 1959b). Saunders (1990) also discussed the problem just prior to vos Savant.

[5] Some variations suggest that behind these other two doors are goats. But I reject the notion that goats are worthless or undesirable.

[6] Did Monty Hall *know* that door 3 had nothing behind it? Or did he just open it on a lark, in which case he risked opening the door with the car? Also, could Monty Hall have opened the door that I initially chose—in this case, door 1? Or did he have to leave it closed and open another door? I shall explain in Sect. 6 why I believe that 2/3 probability attaches to door 2 only if we assume that Monty Hall *deliberately* opens one of the two doors that I did not initially choose.

3 The Sticker vs. the Switcher

The question that I am faced with is this: do I have good reason to switch to door 2? Initially, it seems that the answer is no. That is our intuition.[7] And there are two arguments for our intuition. First, when I initially chose door 1, I had a 1/3 probability of choosing the "right" door—i.e., the door with the car. But now that there are only two doors left, I have a 1/2 probability of choosing the right door. So there is no good reason to switch to door 2. The probability that the car is behind door 2 (i.e., 1/2) is just as good—or bad—as the probability that the car is behind door 1. So I might as well stick with door 1 and at least win $100. Better 1/2 probability of winning a car *plus* $100 than 1/2 probability of winning a car plus *no* money.[8] Second, it seems nonsensical to suggest that the probability of a given door's being correct is greater if I switch to it than if I chose it initially. *When* I choose a door—initially or after being given the option of switching—seems entirely irrelevant to whether or not the car is behind it and therefore to the probability of that door's being the right one. Call a proponent of either argument a "Sticker."[9]

Against the Sticker, the "Switcher" provides the following counter-proposal. The Switcher argues that I should switch from door 1 to door 2. For assume that I were to play the Monty Hall game many—say, one thousand—times. Assume also that Monty Hall uses a randomizing device to determine which door to put the car behind in each game. So for any particular game, none of the previous games gives me any information about which door the car might be behind.[10] The fact of the matter is that if I were to follow a uniform switching strategy for all one thousand games, I would win a car 2/3 of the time—that is, around 667 times. Conversely, if I were to follow a uniform sticking strategy for all one thousand games, I would tend to win a car only 1/3 of the time—that is, around 333 times. It follows from these statistical results that, after Monty Hall opens door 3 and reveals nothing behind it, the probability that door 1 has the car remains at 1/3 and the probability that door 2 has the car rises from 1/3 to 2/3. So it is rational for me to switch rather than to stick in any given game. (In case Switchers out there dislike this defense of their position, I will provide what I take to be two *stronger* arguments for switching in Sects. 6 and 7.)

4 Baumann's Two-Player Argument

Baumann challenges the application of the "Switching Argument"[11] in Sect. 3 to individual games and instead defends our initial intuition that there is no advantage to

[7] See Seymann (1991, p. 288) and vos Savant (1990b, 1991a, 1991b).

[8] Moser and Mulder (1994, pp. 120–127) suggest that this argument—what they call the "Staying Argument and what Horgan (1995, p. 210) calls the "Symmetry Argument"—works perfectly well for a single game but less and less well for larger and larger series of games.

[9] Most of the original responses to vos Savant (1990b, 1991a) took this position. As vos Savant (1991a) points out: "I'm receiving thousands of letters, nearly all insisting that [the Sticker is correct], including one from the deputy director of the Center for Defense Information and another from a research mathematical statistician from the National Institutes of Health! Of the letters from the general public, 92% are [for the Sticker position]; and of the letters from universities, 65% are [for the Sticker position]. Overall, nine out of 10 readers completely [adopt the Sticker position]."

[10] Moser and Mulder (1994, pp. 114–115, 117, 124) discuss random prize placement and the "evidential arbitrariness" of the contestant's choice at greater length.

[11] Moser and Mulder's (1994, p. 111) name. Horgan (1995, p. 211) calls it the "Statistical Argument."

switching over sticking in any single game. Baumann's point is *not* that I should never switch or that it never matters whether or not I switch. Rather, Baumann's point is that the Switching Argument works only if I am playing a sufficiently large number of Monty Hall games, not if I am playing only one Monty Hall game; that while a uniform strategy of switching would lead me to win approximately 2/3 of a sufficiently large number of games and is therefore the rational strategy for the entire series, it is not necessarily the more rational strategy in the standard Monty Hall situation where I am playing only one game.[12]

In support of this conclusion, Baumann offers two main arguments, the "Two-Player Argument" (2005, pp. 71–75) and the "Counterfactual Counterpart Argument" (2005, pp. 75–76). Baumann's Counterfactual Counterpart Argument is similar enough to his Two-Player Argument that I believe I can capture both well enough by discussing only the latter. Baumann's Two-Player Argument proposes a variation on the standard Monty Hall situation. Again, in the standard Monty Hall situation, I initially choose a door, Monty Hall opens another door with nothing behind it, and Monty Hall then asks me if I want to switch to the other unopened door or stick with the door that I initially chose. Baumann suggests that we might well imagine under the very same circumstances another player also chooses a door and is offered a choice of switching or sticking after Monty opens an empty door. Baumann then uses this variation on the standard Monty Hall game to show that probabilities cannot be meaningfully applied to a single game.

Although Baumann presents it as one argument, his Two-Player Argument actually splits into two different arguments, what I shall refer to simply as Baumann's "First Argument" and "Second Argument." In Baumann's First Argument (2005, pp. 72–75), he offers a chart (2005, p. 72) that is supposed to show that if two players, A and B, are competing in a single Monty Hall game in which the prize is behind door 1, then the probability that one of them—say, A—will win by switching is 6/9 and by sticking is 5/9:

Situation	A chooses	B chooses	Monty Hall opens door	Switch	Stick
1	1	1	2 or 3	L	W
2	1	2	3	L	W
3	1	3	2	L	W
4	2	1	3	W	L
5	2	2	3	W	L
6	2	3	1	W	W
7	3	1	2	W	L
8	3	2	1	W	W
9	3	3	2	W	L
				6W, 3L	5W, 4L

[12] Moser and Mulder (1994) similarly argue that the Switching Argument works perfectly well for a large enough series of games but less and less well for smaller and smaller series of games and (therefore) not at all for a single game. Horgan (1995, pp. 218–220), on the other hand, argues that it is *impossible* for single-case probabilities to differ from long-run probabilities, that there must instead be an *isomorphism* between them. So if the probability of winning by switching is 2/3 over the course of a sufficiently large series, then it must also be 2/3 in a single game. As I stated in footnote 3, my position in Sect. 6 will fall much closer to Horgan's.

Baumann then argues as follows:

(1) The result—6/9 and 5/9 probabilities for switching and sticking respectively—is "absurd" because the total probability adds up to 11/9 when it should add up to 9/9 or 1.
(2) ∴ Probabilities cannot be meaningfully applied to a single game.[13]

Baumann's First Argument fails for two reasons. First, if probabilities do not "add up" in a single game, then they should not any more "add up" in an extended series. It is difficult to see—and Baumann fails to make it clear—how the problematic probabilities in a single game would be "ironed out" merely by the repetition of games. Second, Baumann's chart is simply mistaken. If the prize is behind door 1, then A does *not* win by sticking with door 2 in Situation 6 or by sticking with door 3 in Situation 8. This is plain error. Instead, A should *switch* to door 1 in both cases. Once we correct these errors in the chart, we find that the probabilities are not 6/9 and 5/9 but rather 6/9 and 3/9. So (2) no longer follows, and Baumann's First Argument collapses.

Baumann's confusion on this point about the right strategy to follow when the prize is revealed behind door 1 may derive from a previous—equally problematic—point that he makes. In setting up the two-player variation on the Monty Hall game, Baumann allows for the possibility that (a) A and B choose two different doors; (b) Monty Hall opens the third, unchosen door; and (c) the prize is behind this third door:

> If [A and B] choose different doors, then Monty Hall will open the unchosen door, even if it has a prize behind it. If the players see that there is a prize behind the opened door, they go for it, no matter what their strategy. If both players pick the door with the prize behind it, then they both get the full prize. (2005, p. 72)

But how can either player "go" for the prize when it has already been revealed to be behind a door that they did not choose? The game is over at this point, and there is no longer any question about probabilities (0 for both closed doors, 1 for the opened door).

Baumann's Second Argument (2005, pp. 72–75, 76–77) proceeds as follows:

(3) Assume that probabilities can be meaningfully applied to a single game.
(4) Assume that A and B choose their respective doors and Monty Hall then opens an empty door. So Situations 6 and 8 have been eliminated, in which case we are considering now only seven of the nine possible situations.
(5) ∴ The probability for switching is 4/7 and the probability for sticking is 3/7.
(6) Consider Situation 2 in the chart above: A chooses door 1, B chooses door 2, and Monty Hall opens an empty door 3.
(7) ∴ The probability for door 2 is both 3/7 and 4/7, and the probability for door 1 is both 3/7 and 4/7.
(8) This result is absurd. First, one door cannot have two different probabilities. Second, the total probability for both closed doors must add up to 1.

[13] Although he does not make it explicit, I believe that Baumann has the following more elaborate version of the First Argument in mind:
(2*) ∴ Probabilities cannot be meaningfully applied to a single game with two players.
(3*) (1) and (2*) do not depend on there being two players rather than only one player.
(4*) ∴ Probabilities cannot be meaningfully applied to a single standard Monty Hall game with only one player.

(9) This absurd result cannot be avoided by suggesting that, say, door 2 has one probability for A (4/7) and another probability for B (3/7). The probability for door 2 must be the same for both players. For the probability attached to a given door is determined entirely by relevant information. And both A and B share all of the same relevant information.

(10) ∴ (3) is false.

The problematic step in this argument, however, is (9). Contrary to Baumann, the addition of a second player to the game leads the probability for both closed doors to rise to 1/2. The addition of a second player changes the probabilities because it changes the initial assumptions that we may make. Consider again the one-player scenario. When A chooses door 1 and Monty Hall then opens door 3, we assume that door 3 was opened because it is part of the pair of doors that is opposed to the door that A initially chose. And because there was a 2/3 probability that the car was behind one of the two doors in this pair, either door 2 or door 3, the probabilities rise to 2/3 for the only door remaining in the pair, door 2. (I shall defend this point further in Sect. 6.) But when A competes against B, we may no longer assume that door 3 was opened because it is part of the pair of doors that is opposed to the door that *A* initially chose—i.e., doors 2 and 3. For it might just as easily have been opened because it is part of the pair of doors that is opposed to the door that *B* initially chose—i.e., doors 1 and 3. Since we have no reason to place door 3 in either pair, it follows that doors 1 and 2 oppose each other individually; neither opposes the other as part of a pair with door 3. Therefore the probability for either door does not rise to 2/3. Instead, the probabilities for both doors rise to 1/2. (Again, this argument will become clearer in Sect. 6.) So the proper conclusion is not Baumann's—i.e., that probabilities cannot meaningfully apply to single games and therefore that a player has no reason to switch in a single game. Rather, the proper conclusion is that a player has no reason to switch in a single game *when there is an additional player* because the presence of the second player *changes* the situation in such a way that the probabilities for the remaining closed doors are not 1/3 and 2/3 but rather 1/2 and 1/2.

5 Baumann's Rigidity Argument

Return to the single-player situation. Call the door that I initially choose "$\text{door}_{\text{chose}}$," the empty door that Monty Hall then opens "$\text{door}_{\text{empty}}$," and the other door (in addition to $\text{door}_{\text{chose}}$) that remains closed "$\text{door}_{\text{otherclosed}}$." According to the Switcher—and Baumann himself—it makes sense to say that I have a 2/3 probability of winning by switching to $\text{door}_{\text{otherclosed}}$ over the course of a series. But this proposition already seems to threaten Baumann's thesis that single-case probabilities cannot be applied to a single game. For the proposition that I have a 2/3 probability of winning by switching to $\text{door}_{\text{otherclosed}}$ over a series of games would seem to imply that I have a 2/3 probability of winning by switching to $\text{door}_{\text{otherclosed}}$ in a particular game.

As I understand him, Baumann deflects this threat by arguing that (a) $\text{door}_{\text{otherclosed}}$ is a definite description and (b) probability statements using definite descriptions apply to series of games, not to individual games. But the threat is not vanquished. Assume that $\text{door}_{\text{otherclosed}}$ in a particular game is door 2. Then if the former has a probability of 2/3, so does the latter. This time, Baumann cannot offer the same response. For "door 2" is not a definite description but a rigid designator. And because the probability status of a rigid designator's referent changes from game to game, rigid designators may *not* be used in probability statements applied to a series of games. Instead, they may be used only in probability statements applied to doors in individual games. So Baumann's thesis that probabilities cannot meaningfully apply to single games seems to be in danger once again.

In response, Baumann offers his Rigidity Argument (2005, pp. 75–76). The Rigidity Argument suggests that even if $door_{otherclosed}$ is door 2 in a particular game, the door still has two different probabilities under these two different descriptions. While the probability for the door under the definite description $door_{otherclosed}$ is 2/3, there is no probability for the door under the rigid description door 2. Door 2 has no probability under this description because of the arguments that were given in Sect. 4—Baumann's First and Second Arguments. Again, both arguments purport to show that probability cannot meaningfully apply to single games.

Assuming that this interpretation of the Rigidity Argument is correct, it does not work. First, I have already given compelling reasons to reject Baumann's First and Second Arguments. To the extent that the Rigidity Argument depends on these arguments, we may reject it as well. Second, Baumann is wrong to suggest in the first place that probability statements using definite descriptions apply only to series of games, not to individual games. It makes perfect sense to say that the probability for $door_{otherclosed}$ in this particular game, which will not be repeated, is 2/3. And if probability statements like this may be meaningfully applied to single games, then it is difficult to see why probability itself cannot meaningfully apply to single games.

6 The Second Switching Argument

I believe that I have diagnosed the main problem with Baumann's arguments for the conclusion that it is not necessarily rational to switch rather than to stick in any single Monty Hall game. But this diagnosis only scratches the surface. There is yet a deeper problem with Baumann's position: he fails to recognize the *stronger* argument for switching.[14] It is not the one presented in Sect. 3—i.e., the Switching Argument, which says that I should switch in a single instance because this kind of strategy will lead me to win 2/3 of the time in the long run. Rename this the "*First* Switching Argument." Indeed, if the First Switching Argument *were* the stronger argument for switching, Baumann's point that this argument does not apply when there is *no* long run might be apt. But since the stronger argument for switching—call it the "Second Switching Argument"—does not depend on whether or not the Monty Hall game is played many times or only one time, Baumann's entire position founders on this false premise.

One question that the First Switching Argument immediately prompts is: just *why* does a uniform switching strategy across a sufficiently large number of games lead me to win 2/3 of the time in the first place? Why not only 1/2 the time? Or any *other* fraction? The First Switching Argument presents this statistical result as if it is a brute fact, a mere accident without any deeper explanation.[15] But there must be a deeper explanation. And this deeper explanation constitutes the *stronger* argument

[14] So do Moser and Mulder (1994, esp. pp. 121, 124–125, 126–127).

[15] See Hoffman (1998, p. 239).

for switching—again, the Second Switching Argument. The Second Switching Argument also helps to refute both of the arguments in Sect. 3 for sticking.

There are two parts to the Second Switching Argument. The first part challenges the statistical interpretation of probability-statements that was offered in Sect. 1. Again, the statistical interpretation says that if one king and two queens are face down, to say that I have a 1/3 probability of drawing the king is just to say that I will draw a king in 1/3 of a sufficiently large number of such trials. My proposed alternative to the statistical interpretation is what I shall refer to as the "intrinsic interpretation." According to the intrinsic interpretation, the statistical interpretation has everything backwards. It is not that the 1/3 probability of drawing a king in a single instance derives from the statistical results over an extended series—as if these statistical results were nothing more than an accident or brute fact. Rather, the statistical results over an extended series derive from the 1/3 probability in each particular instance. And this intrinsic 1/3 probability itself derives from each instance's "causal structure":[16] the fact that (a) only one of the three cards is a king; (b) I have no evidence, and therefore must simply guess, which face-down card this is; and therefore (c) only one of the three possible guesses available to me can be successful.

The second part of the Second Switching Argument proceeds to offer the deeper causal structure in each Monty Hall game, the causal structure that underlies the statistical fact that a uniform switching strategy will lead to my winning 2/3 of a sufficiently large number of Monty Hall games. It is not as simple to explicate as some pretend. But here is my humble attempt:

(11) Before any door is opened, the probability that any given door is the winner is 1/3. For the reason given just above, this probability is intrinsic to the situation and does not depend on whether or not I play the Monty Hall game a sufficiently large number of times.
(12) Probability is information-sensitive. The probability for a given door changes only if potentially new information is received about that door. Conversely, then, the probability for a given door remains the same if no potentially new information is received about that door.
(13) Assume that I initially choose door 1 and Monty Hall subsequently opens a door other than door 1 that is empty.
(14) If Monty Hall refrained from opening door 1 *just because* I initially chose it, then I am *not* entitled to the inference that Monty Hall refrained from opening door 1 for *another* reason—namely, the reason that the car is behind it.
(15) ∴ If I may not infer that door 1 was left closed because it contains the car behind it, then I have not learned anything potentially new about door 1.
(16) ∴ If I have not learned anything potentially new about door 1, then the probability for door 1 does not change. It remains at 1/3. [(11), (12)]
(17) Assume that Monty Hall opened door 3 and revealed it to be empty. So the probability for door 3 is now 0.
(18) The total probability for all closed doors equals 1.
(19) ∴ If Monty Hall refrained from opening door 1 just because I chose it, the probability for door 2 rises from 1/3 to 2/3. [(16), (17), (18)]

[16] Moser and Mulder's (1994) term. Horgan (1995) uses it as well. The causal structure of a Monty Hall game is the set of conditions that ultimately explains why sticking and switching have the probabilities that they do.

(20) ∴ While I have not learned anything potentially new about door 1, in which case the probability for door 1 remains at 1/3, I have possibly learned something potentially new about door 2, in which case the probability for door 2 changes. If Monty Hall refrained from opening door 1 just because I initially chose it, then I am not entitled to the inference that Monty Hall opened door 3 rather than door 1 for the reason that door 1 has the car. But I *am* entitled to the inference that Monty Hall opened door 3 *rather than* door 2 for the reason that door 2 has the car. As we have just seen, this inference has a strength of 2/3 probability.[17]

(21) We may assume that Monty Hall refrained from opening door 1 just because I initially chose it—especially if he told me when I first entered the contest that after I initially chose a door, he would first open an empty door and then ask me if I want to switch to the other unopened door.

(22) ∴ After choosing door 1 and learning that door 3 is empty, it is rational for me to switch to door 2, which has a 2/3 probability of being the winner, even if I must forsake a guaranteed $100, rather than stick with door 1, which continues to have only a 1/3 probability of being the winner, even though this latter option will win me a guaranteed $100.[18]

If my argument here is correct, then the key point of contention between the Switcher and the Sticker has been resolved in favor of the Switcher. Again, while the Sticker believes that the probability for the door that I initially chose rises from 1/3 to 1/2 after Monty Hall opens an empty door, the Switcher believes that the probability for the door that I initially chose remains at 1/3 and (therefore) that the probability for the other door that Monty Hall leaves closed rises to 2/3.

My position might be even further strengthened—or at least further illuminated—by considering a variation of the standard Monty Hall situation, a variation that *lacks* agency or intent behind the opening of door 3. Suppose that, right after I choose door 1, a sudden, random, powerful gust of wind blows door 3 open and reveals nothing behind it.[19] So door 3 was opened *not* because I just chose another door (door 1). Rather, door 3 was opened by chance; doors 1 and/or 2 could just as easily have been opened as well/instead. In this admittedly peculiar situation, does the probability for door 2 still rise to 2/3? Or does the probability for door 2, like the probability for door 1, rise instead to 1/2?

A Switcher who supports the "2/3 View" will argue that my initially choosing a door—say, door 1—sets up an automatic opposition between door 1 and the other two doors such that it does not matter how a door in the pair of doors opposed to door 1 is opened—whether by a random gust of wind or by an agent like Monty Hall. Even if it is opened not intentionally but by a random gust of wind—or even if Monty Hall does not know what door I have initially chosen and therefore opens door 3 for a reason independent of the fact that I have chosen door 1—the probability for door 2 would still rise to 2/3.

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[17] Think of it this way: the standard Monty Hall situation (in which I choose $\text{door}_{\text{chose}}$, $\text{door}_{\text{empty}}$ is opened, and I am then given the option of switching to $\text{door}_{\text{other closed}}$) is arguably equivalent to first choosing *two* doors, Monty Hall's opening an empty door in the pair, and then being left with the unopened door in the pair.

[18] See Horgan (1995, p. 215). For other articulations of the Second Switching Argument, see Clark (2002, pp. 114–116), Gillman (1992, p. 3), and vos Savant (1991a).

[19] I owe this hypothetical to George Vuoso.

The main problem with the 2/3 View, however, is that it implausibly allows a purely subjective phenomenon—i.e., my choosing a door—to determine objective probabilities. Suppose, for example, that I had not yet chosen a door when the random gust of wind blew door 3 open and revealed nothing behind it. Call this the "Pre-Choice Situation." It seems fairly obvious that the probabilities attached to both doors 1 and 2 would now be 1/2. For the Pre-Choice Situation is really no different than if I were initially presented with two rather than three doors. But suppose now that I *had* chosen door 1 *in my head* but had not yet declared it publicly. As I was about to, the random gust of wind blew open door 3. Call this the "Private Choice Situation." It seems that the probabilities attached to both doors 1 and 2 would still be 1/2. For, first, I might just as easily have privately chosen door 2. Second, the Private Choice Situation differs from the Pre-Choice Situation only psychologically and therefore not in a way that would affect objective probabilities. Yet the 2/3 View above suggests otherwise—i.e., that as soon as I choose a door, an automatic opposition between this door and the other two is established such that the probability for door 2 would rise to 2/3 after door 3 is opened.

Of course, a defender of the 2/3 View might argue that I need to *publicly declare* my choice in order for the opposition to be established. But why should this matter if the random gust of wind is not responding or in any way affected by my public declaration? Given that a gust of wind rather than an agent opens door 3, I might as well have *not* publicly declared my choice. The public declaration of my choice seems to affect the objective probabilities attached to the doors only if this declaration not merely establishes an opposition between the door that I choose and the other two doors but also if this opposition is then "respected"—i.e., helps to motivate Monty Hall to open a door that I did not choose *because* I did not choose it. And this is just to say that Monty Hall's psychological state *does* matter. Door 2's probability rises from 1/3 to 2/3 only if we assume that Monty Hall *deliberately* opens door 3 *because* it is part of the pair of doors opposed to the door that I initially chose, door 1.

7 Horgan's Asymmetry Argument

In case one is not entirely satisfied for whatever reason with my version of the second part of the Second Switching Argument, Horgan (1995, pp. 214–217) offers a rather different argument for the same conclusion—i.e., (22). According to Horgan's "Asymmetry Argument," after I choose door 1 and before Monty Hall opens a door, there are four possibilities:

(23) The prize is behind door 1 and Monty Hall will open door 2.
(24) The prize is behind door 1 and Monty Hall will open door 3.
(25) The prize is behind door 2 and Monty Hall will open door 3.
(26) The prize is behind door 3 and Monty Hall will open door 2.

Horgan's argument continues:[20]

(27) The probability that the car is behind any given door—1 or 2 or 3—is 1/3. [(11)]
(28) ∴ The probability of (25) is 1/3, the probability of (26) is 1/3, and the probability of (23) and (24) together is 1/3.

[20] The reader should be aware that, for the sake of clarity, I have added premises and intermediate conclusions where I perceive minor gaps in Horgan's own formulation.

(29) Because we have no reason to think that (23) is more or less likely than (24), the probability of each is 1/6.
(30) ∴ The total probability of (24) and (25) amounts to 1/6 plus 1/3—i.e., 1/2. [(28), (29)]
(31) When Monty Hall opens door 3, the total probability of (23) and (26) drops to 0, and the total probability of the only remaining possibilities—(24) and (25)—rises to 1.[21]
(32) ∴ The total probability of (24) and (25) doubles. [(30), (31)]
(33) ∴ The probability of each possibility, (24) and (25), doubles.
(34) ∴ The probability of (24) is now 1/3 and the probability of (25) is now 2/3. [(28), (29), (33)]
(35) ∴ The more rational choice for me is to switch to door 2 (with 2/3 probability) rather than stick with door 1 (with 1/3 probability).

One might argue that Horgan's Asymmetry Argument supports the 2/3 View in the last section. For it does not implicitly or explicitly depend on any opposition between door 1 and doors 2 and 3. This appearance, however, is illusory. Like my Second Switching Argument, Horgan's Asymmetry Argument simply *assumes* that Monty Hall must deliberately open one of the doors that I did not initially choose by admitting only four possibilities—i.e., (23) through (26) above—and thereby tacitly excluding the fifth possibility of Monty Hall's opening door 1 rather than doors 2 or 3.

8 Putnam's Argument

One might argue that even if I have shown how probabilities may be meaningfully applied to a single case *without* resorting to the counterfactual statistical interpretation (see Sect. 1), I have still failed to show that these probabilities dictate the rational course of action in a single case. Hilary Putnam (1987, pp. 80–85) credits Charles Sanders Peirce with what he takes to be a compelling argument for this conclusion.

With some non-substantive variations, "Putnam's Argument" goes like this. Consider two games—"Game A" and "Game B." In Game A, (a) there are *100* doors; (b) only one of them has a car; (c) if I choose the right door, then I win the car; and (d) if I choose one of the 99 wrong doors, then I win nothing. In Game B, (a) there are also 100 doors; (b) *99* of them have cars; (c) if I choose one of the *99* right doors, I win a car; and (d) if I choose the one wrong door, then I win nothing. Suppose that I am given the choice of playing either Game A or Game B. Suppose also that whichever game I choose to play, A or B, I will get to play only once. Which game should I choose to play?

Our intuition says that I should play Game B. For my probability of winning is much better—99/100 rather than 1/100. But Putnam argues that this intuition fails. For it derives from our experience across multiple situations and therefore applies only to multiple situations, not to a single situation. So, yes, if I were given *five* tries to win a car (or as many cars as I could), the rational choice would be to play Game

[21] As Horgan (1995, p. 217) puts it, "The information that Monty Hall opens door [3] is *richer* than the information that the prize is not behind door [3]." In other words, if Monty Hall opens door 3 after I have chosen door 1, door 2's probability increases to 2/3 rather than just 1/2 because Monty Hall is now excluding not merely the possibility that door 3 has the car (i.e., (26)) but also the possibility that door 1 has the car and that he has revealed nothing behind door 2 rather than behind door 3 (i.e., (23)).

B rather than Game A each time. Our intuition would be correct in that case. But whichever game I play, I have only one shot and therefore only 1/100—one door out of 100 doors—chance of winning. So, contrary to our intuition, it is no more rational for me to choose to play Game B than it is to choose to play Game A.

The problem with Putnam's Argument is that Putnam simply assumes that the statistical interpretation of probability is correct. But, as I have argued, the intrinsic interpretation is superior to the statistical interpretation. The fact is that I should choose the game that gives me a 99/100 probability of winning *not* because this strategy would lead me to win 99/100 times in the long run but because the causal structure of Game B is 98% more favorable than the causal structure of Game A. To be sure, 99 out of 100 doors with cars does not guarantee that I will win a car in one try. Nor, for that matter, would it guarantee that I would win a car in *several* tries. But it is rational to choose to play the game the causal structure of which will make it more likely *in that one single game* that I will walk away a winner rather than a loser.

9 Conclusion

I conclude two things. First, both versions of Baumann's Two-Player Argument fail to show that probabilities apply meaningfully only to a series of Monty Hall games, not to single, isolated games. Second, as a result, Baumann fails to show that it is not necessarily rational to switch to the other unopened door in a single Monty Hall game.

I have offered several arguments for these conclusions. But I think that the most convincing approach is merely to step back and look at the matter with a healthy dose of common sense, free from the complications of the Monty Hall problem. Quite simply, in our everyday lives, the rational strategy to follow for an extended series of homogeneous or sufficiently similar occasions is *ipso facto* the rational strategy to follow on each and every one of these occasions. So if it is rational to eat a healthy diet, then it is rational to eat healthy on the next occasion. If it is rational to drive safely, then it is rational to drive safely on this part of the highway. And if it is rational not to waste money, then it is rational not to buy this overly expensive product. This much more intuitive perspective provides even more—perhaps the most convincing—reason to reject Baumann's overall position that the probability-based rational approach for the series does not dictate the rational strategy for each individual game in the series.[22]

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