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# Geometry for a Brain.

# Optimal Control in a Network of Adaptive Memristors

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**Abstract**

In the brain the relations between free neurons and the conditioned ones establish the constraints for the informational neural processes. These constraints reflect the system-environment state, i.e. the dynamics of homeocognitive activities. The constraints allow us to define the cost function in the phase space of free neurons so as to trace the trajectories of the possible configurations at minimal cost while respecting the constraints imposed. Since the space of the free states is a manifold or a non orthogonal space , the minimum distance is not a straight line but a geodesic. The minimum condition is expressed by a set of ordinary differential equation ( ODE ) that in general are not linear. In the brain there is not an algorithm or a physical field that regulates the computation, then we must consider an emergent process coming out of the neural collective behavior triggered by synaptic variability. We define the neural computation as the study of the classes of trajectories on a manifold geometry defined under suitable constraints. The cost function supervises pseudo equilibrium thermodynamics effects that manage the computational activities from beginning to end and realizes an optimal control through constraints and geodetics. The task of this work is to establish a connection between the geometry of neural computation and cost functions. To illustrate the essential mathematical aspects we will use as toy model a Network Resistor with Adaptive Memory (Memristors).The information geometry here defined is an analog computation, therefore it does not suffer the limits of the Turing computation and it seems to respond to the demand for a greater biological plausibility. The model of brain optimal control proposed here can be a good foundation for implementing the concept of "intentionality", according to the suggestion of W. Freeman. Indeed, the geodesic in the brain states can produce suitable behavior to realize wanted functions and invariants as neural expressions of cognitive intentions.

**Keywords:** neural geometry; Fisher information; analog computation, brain simulator, memristor; physic models of cognition

## 1. Introduction

We open by referring to the inspiring imaginary discussion between Alan Turing and Santiago Ramon y Cajal [1] on the meaning of neural computation:

*Santiago: I'm beginning to like the perspective of neural computation. Even more, I like the idea that each neuron computes.*
*Alan: You do? I'm not sure I like it myself.*
*Santiago: Why? Don't you think that the brain computes? I could understand it if someone else would say so, but you?*
*Alan: Well, you see, the term computation has too much a formal meaning for me, and it is hard for me to see what do they mean by saying "computation"? I suspect that they use it just as one of these buzz words.*
*Santiago: Maybe I'm too naive, but I think that the term "computation" is meant to emphasize some very simple but deep idea. Namely, that the brain is yet another machine that computes functions. Functions, which might be very complicated but basically relates the external world of sensory "input" to our output in terms of action in the world.*
*Alan: I think I share this view. I will not claim that the brain is a kind of special machine.*
*Santiago: Hey, beware, it is a very special machine, you know, but it is not so special that it uses some extra metaphysical powers. Some kind of "virtual spirits" as Descartes called them. Putting it differently, the brain is very complicated, but once we understand it, we can, at least ideally, implement it with a different hardware. For example, simulate it on an electronic computer. Maybe this is where the computation comes from.*
*Alan: This is an interesting point, but very confusing. You see, when we talk about computation what I have in mind is a Turing Machine. I mean that even if any computable function is computable by a Turing Machine, not everything that is computable by a Turing Machine is a computation. For example, just as you would like to simulate the brain, you can simulate other complex systems, say the collision of two galaxies. Surely the simulation is a computation and a very complicated one. But the collision itself is probably not a computation it is a physical phenomena that has nothing to do with computation.*

Actually Turing, in the last years of his short and creative life, showed a keen interest in what we now call "natural computation"[2,3]. A question on which classical cognitivism seems to get mislead is that of information. If the new cognitive science has to be based on what we know of the embodied brain, then it is appropriate to ask whether the Shannon-Turing computational model is the most suitable one to describe the characteristics of the informational flow in cognitive processes. In other words, in order to study cognitive emergence it is necessary to develop an approach to information which can include the *meaning* and *purpose* of behavioural scenarios. A theory of this kind must combine syntax and semantics, and a geometric approach such as the one outlined here seems to us very promising in different areas [4, 5, 6]. With a theoretical apparatus of this kind it might be possible to set up various matters not approachable within the classical symbolic theories of cognition.

Walter J. Freeman discusses the question of consciousness in the brain in this way[7]:

*Given that current efforts to deal with the problem of consciousness have led either to its dismissal (Dennett, 1991), trivialization (Crick, 1994), or transplantation to fields outside of biology, my view is that an alternate approach may be called for. Rather than pursuing forthrightly the elusive concept of consciousness, I suggest that an alternative target be formulated. For me that target lies in the area of goal-directed behaviors, overlapping largely but not completely with what are commonly called 'voluntary' actions (Smith, 1994), whether or not they are conscious These behaviors emerge from within brains, in contrast to evoked or reflex actions, and their flexibility and adaptiveness in the face of unexpected obstacles belies the possibility of genetic or environmental programming. The experimental, mechanistic question is: How can populations of neurons in brains generate the neuroactivity patterns directing these movements? A useful set of theoretical tools with which to seek answers is to be found in the self-organizing properties of nonlinear dynamic systems. The theoretical, philosophical question is: What principle or organizing concept can be adopted to supplant the notion of consciousness? My choice of focus is the term 'intentionality'.*

Freeman's idea of intentionality is the starting point of our work, in which we consider the best optimal adaptation in the neural space as the biomathematical image of the intentionality.

At the Berkeley conference on memristors (November 21 2008) Greg S. Snider of the Hewlett Packard Laboratory discusses the question of neural net as follows:

*In 1980 an artificial neural network was built that work but has high precision components, slow unstable learning, non adaptive (Train and ship) and external control. Now we want Low precision components, fast stable learning, Adapt to environment and autonomous. How we can get this? We can get make components dynamical, add feedback (positive & negative) and close the loop with the outside world. The ordinary differential equations or ODE to control the neural dynamic are a stiff and nonlinear system. Why not just program this on a computer? We know that stiff and nonlinear dynamical systems are inefficient on digital computer. An example is the IBM Blue Gene project with 4096 CPUs and 1000 Terabytes RAM. To simulate the Mouse cortex use 8 x $10^6$ neurons, 2 x $10^{10}$*

*synapses $10^9$ Hz, 40 Kilowatts and digital. The brain uses $10^{10}$ neurons, $10^{14}$ synapses 10 Hz and 20 watts with analog system. Analog is more efficient by several order of magnitude.*

Greg S.Snider suggests to use analog electrical circuit denoted CrossNet with memristor to tackle the simulation of the brain neural computation in the brain [8, 32, 33, for the breaking news see 34 ]. Snider suggests that physical (or analog) computers are more efficient to solve the problem of neural networks. In fact, in analog systems we do not have algorithms to program the neurons, but global constraints. We propose to substitute the digital program with the minimum action and geodesic in a non Euclidean space or a informed/deformed space. The minimum path in the neural space currents is the Freeman' intention, whose dynamics is in the geodesic trajectory. Geometric and physical description of the Freeman intentionality is beyond any algorithmic or digital computation. In other words, what we try to do is a return to the geometrical approaches at the beginning of the post-Hopfield neural networks age [9,10], without forcing the "metaphorical" nature of these toy models( Arbib & Amari, [ 11 ]) but with a special attention to the new scenarios of brain simulation.
To clarify better the new computation paradigm, we can refer the following principle:

*"Modeling brain dynamics requires us to define the behavioral context in which brains interact with the world* (…) *Animals and humans use their finite brains to comprehend and adapt to infinitely complex environment." (*Freeman & Kozma, 2008[12])

We show that this adaptive system has a geometric interpretation that gives us the possibility to implement the required parameters in ODE to achieve the desired behaviors [two classics are:13, 14; for a general review on ODE in neural dynamics see 15]
We will consider three classes of neuronal dynamics. The inertial neuronal systems, in which there are no adaptation and therefore any modification of the parameters; the conservative neural systems, whose behavior adapt to its environment, converging on towards a final goal. They are, as it is evident, two cases very simplified. The last one is the non conservative and non inertial neural systems where the noise is present to destroy the coherence and the synchronicity of the conservative system. The noise gives the possibility to explore new ways compared to the conservative systems, provides a very wide variety of behaviors and reveals as the real key to the understanding of the brain [16] The noisy geodesic is modelled here as a percolation movement from one state to another one of the neural behaviours.

## 2. Geodesic and constrains in the states' space

We define the state space as a vector space where any point is located in the N dimension,

$$q = (q_1, q_2, ...., q_N) = 0 \qquad (1)$$

Any component of q is a state of one neuron. Now we know that in general neurons are not independent but are constrained by a set of functions or invariants

$$f_j(q_1, q_2, ....., q_N) = 0 \qquad j = 1, ....., m \qquad (2)$$

Now in N dimension we have the space of the state , the sub space of the m dimension is the space of the interior coordinates and the complementary space of N – m dimension has the exterior coordinates of the free states. The possible trajectories are in the N – m exterior coordinates..

Because in (2) the time is not present in explicit way we have that

$$\frac{df_j}{dt} = \frac{\partial f_j}{\partial t} + \sum_k \frac{\partial f_j}{\partial q_k} dq_k = \sum_k \frac{\partial f_j}{\partial q_k} dq_k = \left[\frac{\partial f_j}{\partial q}\right]^T dq = 0 \qquad (3)$$

For the (3) the functions in (2) are invariants. Given the function

$$q_k = q_k(x_1, ... x_{N-m}) \qquad (4)$$

Where $x = (x_1, ... x_{N-m})$ are the free variable , we have

$$dq_k = \sum_j \frac{\partial q_k}{\partial x_j} dx_j \qquad (5)$$

So

$$\sum_k \frac{\partial f_j}{\partial q_k} dq_k = \sum_k \frac{\partial f_j}{\partial q_k} \sum_j \frac{\partial q_k}{\partial x_j} dx_j = \sum_{k,j} \frac{\partial f_j}{\partial q_k} \frac{\partial q_k}{\partial x_j} dx_j = 0$$

We conclude that

$$Df = \left[\frac{\partial q}{\partial x_j}\right]^T \left[\frac{\partial f_j}{\partial q}\right] = \left(\frac{\partial q_1}{\partial x_j}\frac{\partial}{\partial q_1} + \frac{\partial q_1}{\partial x_j}\frac{\partial}{\partial q_1} + ..... + \frac{\partial q_N}{\partial x_j}\frac{\partial}{\partial q_N}\right) f_j = 0 \qquad (6)$$

D is the derivative in the direction of the tangent of the manifold which dimension is

N – m. The functions f have derivatives zero ( invariants ) when we move inside the manifold. Given a new function F we have that

$$DF = \left[ \frac{\partial q}{\partial x_j} \right]^T \left[ \frac{\partial F_j}{\partial q} \right] = \left( \frac{\partial q_1}{\partial x_j} \frac{\partial}{\partial q_1} + \frac{\partial q_1}{\partial x_j} \frac{\partial}{\partial q_1} + \ldots\ldots + \frac{\partial q_N}{\partial x_j} \frac{\partial}{\partial q_N} \right) F$$

It measures the change of F respect to the invariant functions for which the derivative D is equal to zero. The components of the tangent of the manifold of N- m dimension are $\frac{\partial q_k}{\partial x_j}$

Now because $(I - A(A^T A)^{-1} A^T)A = 0$ (7)

Where $A = A_{k,j} = \left[ \frac{\partial f_j}{\partial q_k} \right]$

we have

$$\frac{\partial q_k}{\partial x_j} = (I - A(A^T A)^{-1} A^T) = I - \left[ \frac{\partial f_j}{\partial q_k} \right] \left( \left[ \frac{\partial f_j}{\partial q_k} \right]^T \left[ \frac{\partial f_j}{\partial q_k} \right] \right)^{-1} \left[ \frac{\partial f_j}{\partial q_k} \right]^T \qquad (8)$$

For which we have the ODE ( ordinary differential equations)

$$\frac{\partial q_k}{\partial x_j} = h_k(q_1, q_2, \ldots\ldots, q_N) \qquad (9)$$

So the tangent vector can be computed by the matrix A of the derivative of the functions in (2)

The metric tensor in the manifold is

$$g = \left[ \frac{\partial q}{\partial x_j} \right]^T \left[ \frac{\partial q}{\partial x_j} \right] = (I - A(A^T A)^{-1} A^T)^T (I - A(A^T A)^{-1} A^T)$$
$$= (I - A(A^T A)^{-1} A^T) \qquad (10)$$

And the geodesic is

$$ds^2 = \sum_{i,j} g_{i,j} dx_i dx_j \qquad (11)$$

Now for the geodesic in the space of the states is

$$\delta A = \delta \int \sqrt{g_{h,k} dx^h dx^k} = \delta \int \sqrt{g_{h,k} \frac{dx^h}{dt} \frac{dx^k}{dt}} dt = 0 \qquad (12)$$

Where A is the general action.
Now for the Euler differential equations we have that the minimum condition can be found when

$$\frac{d}{dt} \frac{\partial \sqrt{g_{h,k} v^h v^k}}{\partial v^j} - \frac{\partial \sqrt{g_{h,k} v^h v^k}}{\partial x^j} = 0$$

$$\frac{dx^h}{dt} = v^h \qquad (13)$$

We show in a graphic the geodesic as a minimum cost in this way

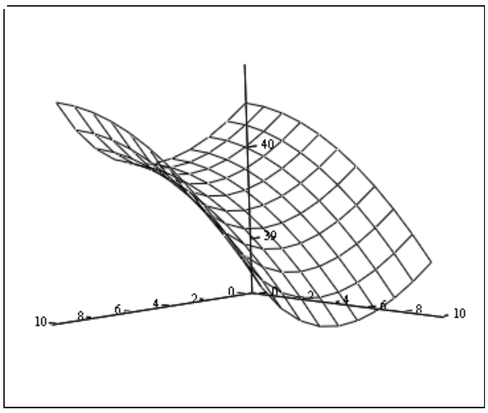


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Figure 1 Geodesic as the trajectory at the minimum value for the action A

When we know the invariants we can compute the tangent vector and the geodesic for which we have the stationary condition (the local variation of the geodesic is equal to zero).

### 2.1 Toy example of geodesic and electrical circuit

According to Krone [17], the tensor image of the electrical circuit is given by the trivial electrical circuit

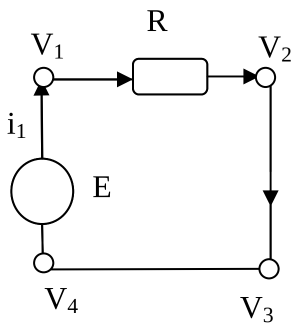



Figure 2 Toy electrical circuit with one generator E and one resistor R

Now we compute the power W that is dissipated at the resistance R in this way

$$W = R(\frac{dq}{dt})^2 = Ri^2$$

Where i is the current in the circuit and q are the states that move inside the circuit. Now we define the infinitesimal distance ds in this way:

$$(\frac{ds}{dt})^2 = W = R(\frac{dq}{dt})^2 = Ri^2$$

And

$$ds = \sqrt{W}\,dt = \sqrt{R(\frac{dq}{dt})^2}\,dt \qquad (14)$$

Now we know that in the electrical circuit the currents flow in the circuit so as to dissipate the minimum power, The geodesic line in the one dimension current space *i* is the trajectory in time.

For the minimum dissipation of the power or cost C, we have

$$\delta C = \delta \int ds = \delta \int \sqrt{W}\, dt = \delta \int \sqrt{R(\frac{dq}{dt})^2}\, dt = 0 \qquad (15)$$

Now we can compute the behaviour of the states for which we have the geodesic condition of the minimum cost. We know that this problem can be solved by the Euler differential equations or ODE

$$\frac{d}{dt}\frac{\partial(\sqrt{R(\frac{dq}{dt})^2})}{\partial \frac{dq}{dt}} - \frac{\partial(\sqrt{R(\frac{dq}{dt})^2})}{\partial q} = 0 \qquad (16)$$

$or$

$$\frac{d}{dt}\frac{\partial R(\frac{dq}{dt})^2}{\partial \frac{dq}{dt}} - \frac{\partial R(\frac{dq}{dt})^2}{\partial q} = 0$$

When R is independent on the states so R has no memory , we have that the previous equation can be written in this way

$$\frac{d\,(R\,\frac{dq}{dt})}{dt} = 0$$

When R is independent on the time we have

$$\frac{d\,(\frac{dq}{dt})}{dt} = \frac{d^2 q}{dt^2} = 0 \qquad (17)$$

$$q(t) = at + b$$

So the current is constant and is equal to

$$i = \frac{dq}{dt} = a = \frac{E}{R}$$

The geodesic is a straight line in the space of the states.

# 3. Inertial , non inertial ( conservative ) and dissipative geodesic.

### 3.1 Inertial and non inertial geodesic

Given the tangent vector to the manifold in a subspace of the N - m number of the free variables x ,

$$\left[\frac{\partial q}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial q_1}{\partial x_1} & \frac{\partial q_1}{\partial x_2} & \cdots & \frac{\partial q_1}{\partial x_{N-m}} \\ \frac{\partial q_2}{\partial x_1} & \frac{\partial q_2}{\partial x_2} & \cdots & \frac{\partial q_2}{\partial x_{N-m}} \\ \frac{\partial q_3}{\partial x_1} & \frac{\partial q_3}{\partial x_2} & \cdots & \frac{\partial q_3}{\partial x_{N-m}} \\ \frac{\partial q_4}{\partial x_1} & \frac{\partial q_4}{\partial x_2} & \cdots & \frac{\partial q_4}{\partial x_{N-m}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial q_n}{\partial x_1} & \frac{\partial q_n}{\partial x_2} & \cdots & \frac{\partial q_n}{\partial x_{N-m}} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N-m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2.N-m} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,N-m} \\ a_{4,1} & a_{4,2} & \cdots & a_{4,N-m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,N-m} \end{bmatrix} \tag{18}$$

Where $a_{i,j}$ are constant values, the tensor metric gives us a metric of an inertial system. We remember that an inertial system is used here to describe the ODE types based on standard physical terminology, which is well defined. The metric tensor is

$$g_{i,j} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N-m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2.N-m} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,N-m} \\ a_{4,1} & a_{4,2} & \cdots & a_{4,N-m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,N-m} \end{bmatrix}^T \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N-m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2.N-m} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,N-m} \\ a_{4,1} & a_{4,2} & \cdots & a_{4,N-m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,N-m} \end{bmatrix} \tag{19}$$

An example of non trivial inertial system is the geodesic ( Kinetic energy ) for s mechanical rotatory system with the inertial moment $I_{i,j}$ . So we have

$$g_{i,j} = mI_{i,j}$$

Where m is the mass of the system. The geodesic is

$$T = \left(\frac{ds}{dt}\right)^2 = \sum_{i,j} g_{i,j} \frac{dx^i}{dt}\frac{dx^j}{dt} = m\sum_{i,j} I_{i,j} v^i v^j \qquad (20)$$

*where* $v^i$ *is the vector of the different angular velocities of the system, and T the Kinetic energy.[18]*

When we have

$$\left[\frac{\partial q}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial q_1}{\partial x_1} & \frac{\partial q_1}{\partial x_2} & \cdots & \frac{\partial q_1}{\partial x_{N-m}} \\ \frac{\partial q_2}{\partial x_1} & \frac{\partial q_2}{\partial x_2} & \cdots & \frac{\partial q_2}{\partial x_{N-m}} \\ \frac{\partial q_3}{\partial x_1} & \frac{\partial q_3}{\partial x_2} & \cdots & \frac{\partial q_3}{\partial x_{N-m}} \\ \frac{\partial q_4}{\partial x_1} & \frac{\partial q_4}{\partial x_2} & \cdots & \frac{\partial q_4}{\partial x_{N-m}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial q_n}{\partial x_1} & \frac{\partial q_n}{\partial x_2} & \cdots & \frac{\partial q_n}{\partial x_{N-m}} \end{bmatrix} = \begin{bmatrix} a_{1,1}(x) & a_{1,2}(x) & \cdots & a_{1,N-m}(x) \\ a_{2,1}(x) & a_{2,2}(x) & \cdots & a_{2.N-m}(x) \\ a_{3,1}(x) & a_{3,2}(x) & \cdots & a_{3,N-m}(x) \\ a_{4,1}(x) & a_{4,2}(x) & \cdots & a_{4,N-m}(x) \\ \cdots & \cdots & \cdots & \cdots \\ a_{n,1}(x) & a_{n,2}(x) & \cdots & a_{n,N-m}(x) \end{bmatrix} \qquad (21)$$

The metric tensor is

$$g_{i,j} = \begin{bmatrix} a_{1,1}(x) & a_{1,2}(x) & \dots & a_{1,N-m}(x) \\ a_{2,1}(x) & a_{2,2}(x) & \dots & a_{2.N-m}(x) \\ a_{3,1}(x) & a_{3,2}(x) & \dots & a_{3,N-m}(x) \\ a_{4,1}(x) & a_{4,2}(x) & \dots & a_{4,N-m}(x) \\ \dots & \dots & \dots & \dots \\ a_{n,1}(x) & a_{n,2}(x) & \dots & a_{n,N-m}(x) \end{bmatrix}^T \begin{bmatrix} a_{1,1}(x) & a_{1,2}(x) & \dots & a_{1,N-m}(x) \\ a_{2,1}(x) & a_{2,2}(x) & \dots & a_{2.N-m}(x) \\ a_{3,1}(x) & a_{3,2}(x) & \dots & a_{3,N-m}(x) \\ a_{4,1}(x) & a_{4,2}(x) & \dots & a_{4,N-m}(x) \\ \dots & \dots & \dots & \dots \\ a_{n,1}(x) & a_{n,2}(x) & \dots & a_{n,N-m}(x) \end{bmatrix} \quad (22)$$

And in this case the geodesic is a non inertial geodesic with invariant given by the geodesic (20).

### 3.2 Geodesic in non conservative systems [18]

To take into account a minimum of biological plausibility it is necessary to introduce a simplified model of membrane.

Ion channel noise asserting that conformational changes in ion channels are actually exposed to two different kinds of fluctuations namely the intrinsic noise and the topological noise. The intrinsic noise is associated with the stochastic nature of the movement of gating particles between the inner and the outer faces of the membrane. The topological noise, on the other hand, is associated with the fluctuations emerging from the uncertainty in accessing the permissible topological states of open gates. In a toy membrane just having three potassium channels (twelve gates), for example, nine open gates can be configured into a variety of topological states with the possible results that none of the channels is open, one is open, or two are open. Now the join probability for the ion channels is given by

$$P = P(q_1, q_2, .....q_n)$$

The join probability to open or close the channel is computed in the space of the ionic states

$$q = (q_1, q_2, .....q_n)$$

That move inside the channels. So given a vector q in the state of the channels , we can compute the probability for the given configuration q of states in the channels. At any configuration we associate a probability. So we have that there exist configurations with very low probability and configurations with high probability.

Now given the join probability P , we compute the variation of the probability with respect to the state $q_j$ . So we have

$$D_j P = \frac{\partial P}{\partial q^j}$$

Now given the current

$$i_j = \frac{dq_j(t)}{dt}$$

We can compute the flux of states for the current as a random variable

$$\Phi_j = P\frac{dq_j(t)}{dt}$$

Now we assume that we have this invariant form

$$\Phi_j + \lambda D_j P = 0$$

When the probability P is a constant distribution value , the flux of the state is equal to zero. But when the variation of P for the change of state is positive the flux is negative , when the variation of P is negative the flux is positive. So the flux has an inverse value to the probability . When the probability is a constant value the flux is zero. So the flux is controlled by the probability in an inverse way.

$$i_j = \frac{dq_j}{dt} = -\lambda\frac{1}{P}D_j P = -\lambda\frac{\partial \log P}{\partial q_j} \quad (23)$$

Now the current across the channels is the superposition of two currents: one is the ordinary current I and the other is the random current i. In conclusion the total current is

$$J_i = I_i + i_i = I_i - \lambda_{j,i}(q)\frac{\partial(\log P)}{\partial q^j} \quad (24)$$

It can be written in this way

$$J_i = I_i - \lambda_{i,j} A^j \quad (16)$$

Now the noise can be formally presented as a field which value is

$$A^j = \frac{\partial(\log P)}{\partial q^j}$$

For the definition of the current we have

$$J^j = \frac{dQ^j}{dt} = (I_j - \lambda A_j)$$

In conclusion we can change the previous definition of the current and also the geodesic expression in this way

$$W = Z_{k,h} \frac{dQ^k}{dt} \frac{dQ^h}{dt} = ds^2 \quad (25)$$

The new geodesic can be written also in this way

$$\begin{aligned} dE^2 &= \left(\frac{ds}{dt}\right)^2 = Z_{k,h}(I^h - \lambda^h A^h)(I^k - \lambda^k A^k) \\ &= Z_{h,k}(q) I^j I^k - 2\lambda Z_{h,k}(q) I^h A^k + \lambda^2 Z_{h,k}(q) A^h A^k \quad (26) \\ &= W_1 + W_{1,2} + W_2 \end{aligned}$$

So we have three different powers. $W_1$ is the ordinary power for the ionic current without noise. The second $W_{1,2}$ is the flux of power from current to the noise current. The last $W_2$ is the power in the noise currents.

## 3.3 The Fisher Information in Neurodynamics

Now we compute the average of the power as the cost function which value must be minimum. So we have

$$E = \int P[Z_{i,j}(I_i - \lambda A_i)(I_j - \lambda A_j)]dtd^n x$$
$$= \int PZ_{i,j}[(I_i I_j + \lambda^2 A_i A_j) - 2\lambda I_i A_j]dtd^n x$$
$$= \int PZ_{i,j}[(I_i I_j + \lambda^2 \frac{\partial \log P}{\partial q_i}\frac{\partial \log P}{\partial q_j}) + W_{1,2}]dtd^n x$$
$$= \int P[Z_{i,j} I_i I_j + W_{1,2})dtd^n x + Z_{i,j}\lambda^2 \int P \frac{\partial \log P}{\partial q_i}\frac{\partial \log P}{\partial q_j})]dtd^n x \quad (27)$$

Let us consider a parameterized family of probability distributions

$$S = \left\{P(x,t,q_1,q_2,...,q_n)\right\}$$

where x and t are random variables and

$$q = (q_1, q_2, ..., q_n)$$

is a real vector parameter to specify a distribution. The family is regarded as a n-dimensional manifold having q as a coordinate system is a Riemannian manifold and

$$G(q) = g_{i,j}(q) = \int P \frac{\partial \log P}{\partial q_i}\frac{\partial \log P}{\partial q_j})]dtd^n x \quad (28)$$

Play the role of a Fisher information matrix

The idea of information geometry [19] is that a family of probability distributions that are labelled by some continuous parameters as state q can be thought as a space, each distribution being a “point”, while the parameters q play the role of coordinates. A question that immediately arises is whether there is a natural way to measure the extent to which neighbouring “points” can be distinguished from each other. The answer is that such a measure exists, that it is unique, and most remarkably, that it has all the properties one would wish to impose upon a measure of distance. Thus, *distinguishability is the distance*, which raises the possibility that perhaps the familiar notion of physical spatial distance might itself be explained in this way:

$$ds^2 = \sum_{i,j} g_{h,k} dq^h dq^k$$

*where*

$$g_{i,j}(q) = \frac{1}{\lambda^2} E(A^i A^j)$$
$$= E\left(\frac{\partial \log P(x,t,q)}{\partial q_i} \frac{\partial \log P(x,t,q)}{\partial q_j}\right) = \int P \frac{\partial \log P(x,t,q)}{\partial q_i} \frac{\partial \log P(x,t,q)}{\partial q_j} dxdt \quad (29)$$

We connect the previous distance or metric with the Kullback–Leibler divergence in this way

$$ds^2 = \sum_{i,j} g_{h,k} dq^h dq^k = 2KL(P(x,t,q) : P(x,t,q+dq))$$

*where*

$$KL(p:q) = \int p \log\left(\frac{p}{q}\right) dxdt$$

It is the well known Kullback–Leibler divergence [ 26 ].

When the noise is equal to zero the Fisher information assumes the maximum value and the geodesic is equal to the classical geodesic. On the other hand, in the case with noise, we assume that the field

$$A^j = \frac{\partial(\log P)}{\partial q^j}$$

interacts with the neural network and destroys the coherence. The information approaches zero and the cost function will be reduced. For the random potential A we have this example of random geodesics

## 3.4 Percolation and geodesics

Resistor networks, from which resistors are removed at random, provide the natural generalization of the lattice models for which percolation thresholds and percolation probabilities can be considered[20, 21]. Given the membrane resistances given by the graph

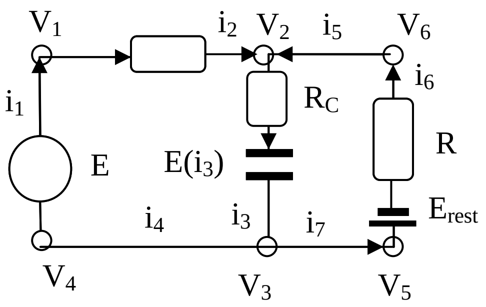



Figure 3 Membrane resistances and equivalent electrical circuit
Any resistance can be separate in two parallel resistances as we do in figure for the resistance R. The resistance R is separated in two parallel resistances $R_1$ and $R_2$. The resistance $R_1$ is a resistance that can be removed or changed at random in the percolation phenomena , $R_2$ is the crisp resistance. The total resistance R now is given by the traditional expression

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

The total current is I + i where I is the current in the crisp resistance $R_2$ and i in the random value resistence $R_1$ .

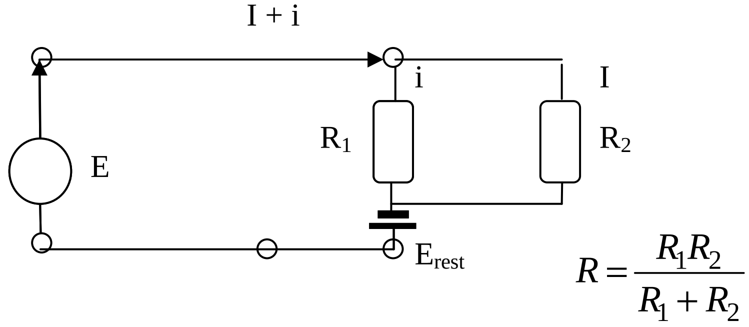



Figure 4 random $R_1$ and crisp resistance $R_2$ generate the equivalent resistance R in a parallel configuration.

For the additivity of the power we have

$$W = R_1 i^2 + R_2 I^2 = R(I+i)^2 = (\frac{R_1 R_2}{R_1 + R_2})(I+i)^2 \qquad (27)$$

The geodesic can be written in this way

$$(\frac{ds}{dt})^2 = \sum_{i,j} Z_{i,j}(I_i + i_i)(I_j + i_j) = \sum_{i,j} \frac{Z_{1,i,j}Z_{2,i,j}}{Z_{1,i,j}+Z_{2,i,j}}(I_i + i_i)(I_j + i_j) \quad (28)$$
$$= \sum_{i,j} Z_{2,i,j} I_i I_j + \sum_{i,j} Z_{1,i,j} i_i i_j$$

The geodesic is composed by two parts: one is the synchronic and crisp geodesic the other is the noise change of the crisp and synchronic geodesic. So we have the graph image

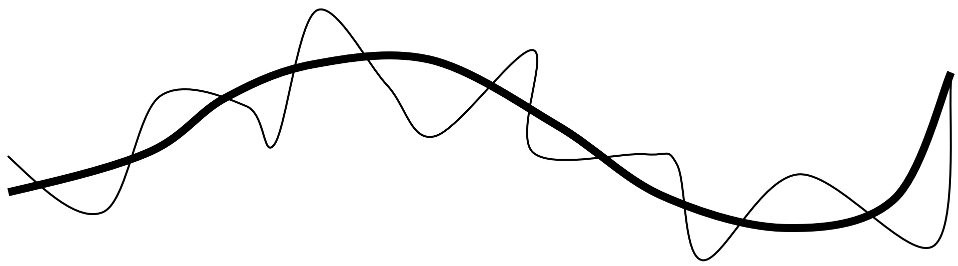


Figure 5 In evidence we have the geodesic that are solutions of the ODE. For the noise the geodesic is transformed in a more complex geodesic that is related to the Fisher information. The total effect is the percolation random geodesic.

## 4. Geodesic in neural space

In this chapter we show that any electrical neural activity can be represented in manifold state space where the minimum path (geodesic) between two points in the multi-space of the currents is function of the neural parameters as resistors with or without memory. Now we begin with the simplest case given by electrical activity of one little part of the membrane of axons, dendrites or soma. In this case we ignore the presence of the voltage-gated channels in the membrane.

### 4.1 Membrane electrical activity and geodesic

When we ignore the presence of voltage-gated channels in the membrane we can model the membrane electrical activity by this circuit

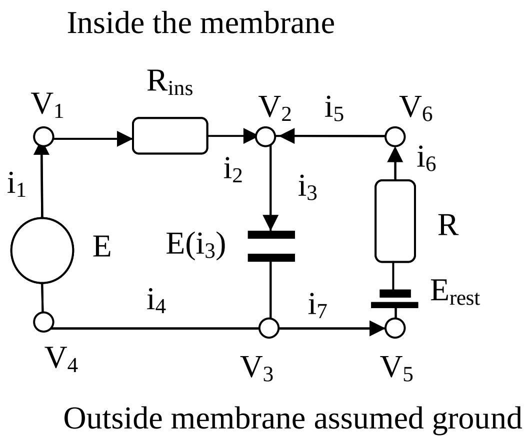



Figure 6 Input voltage source E , Internal to the membrane resistance $R_{ins}$ , Capacitor voltage source $E(i_3)$, R is the membrane resistance , $E_{rest}$ is the rest voltage source generator.

We remark that in the figure 6 we have three voltages sources. The first generator source is the impulse at the begin of the axon, the second $E(i_3)$ is

$$(V_3 - V_2) = E(i_3) = \frac{q_3(t)}{C} = \frac{1}{C}\int i_3(t)dt \qquad (29)$$

The third $E_{rest}$ is the voltages actively generated by ATP when the axon is at rest state. Any circuit defines the relation between the voltages and the currents in this way

$$v_j = f_j(i_1, i_2, ...., i_n)$$

Because the expression of the dissipate power W is

$$W = i^1 v_1 + i^2 v_2 + i^3 v_3 + ..... + i^n v_n$$

We have

$$W = i^1 f_1 + i^2 f_2 + i^3 f_3 + ..... + i^n f_n$$

When the edges are separated one from the others we have n free variables so the power can be written in this way

$$W = R_1 \, i_1^2 + R_2 \, i_2^2 + R_3 \, i_3^2 + \ldots.. + R_n \, i_n^2 \qquad (30)$$

*for*

$$v_j = R_j i_j^2$$

Now when we connect the edges in a circuit we reduce the independent currents from n to p < n.

$$i_j = \alpha_{1,j} \, i_1 + \alpha_{2,j} \, i_2 + \alpha_{3,j} \, i_3 + \ldots.. + \alpha_{p,j} \, i_p \qquad (31)$$

So all the n currents are linear combinations of the independent currents p. In this situation the power takes the form

$$W = R_1 \, (\alpha_{1,1} i_1 + \ldots + \alpha_{p,1} i_p)^2 + R_2 (\alpha_{1,1} i_1 + \ldots + \alpha_{p,1} i_p)^2 + R_3 \, (\alpha_{1,2} i_1 + \ldots + \alpha_{p,2} i_p)^2 + \ldots..$$
$$+ R_n \, (\alpha_{1,n} i_1 + \ldots + \alpha_{p,n} i_p)^2 \qquad (32)$$

So the power can be written in this form

$$W = \sum_{h,k} g_{h,k} i^h i^k = g_{h,k} i^h i^k \qquad (33)$$

Where the metric tensor g is

$$g = \alpha^T R\alpha$$

Where

$$\alpha = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \ldots & \alpha_{1,p} \\ \alpha_{2,1} & \alpha_{2,2} & \ldots & \alpha_{2,p} \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_{n,1} & \alpha_{n,2} & \ldots & \alpha_{n,p} \end{bmatrix}, \ \mathrm{R} = \begin{bmatrix} R_1 & 0 & \ldots & 0 \\ 0 & R_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & R_n \end{bmatrix} \qquad (34)$$

Because

$$i^j = \frac{dq^j}{dt} \quad j = 1,2,3,\ldots\ldots, p$$

*and*

$$W = \frac{dE}{dt}$$

Where E is the energy and W the power, the geodesic equation can be written in this way

$$dE = \sum_{h,k} g_{h,k} dq^h dq^k = \sum_{h,k} \left( \sum_{j,i} \alpha_{k,j} R_{j,i} \alpha_{i,k} \right) dq^h dq^k \tag{35}$$

We know that W in an electrical circuit takes the minimum value. So the power is comparable to the Lagrangian in mechanics (Hamilton principle) or the Fermat principle in optics ( minimum time). In the context of Freeman's neurodynamics, we hypothesize that the minimum condition in any neural network gives us the meaning of "Intentionality". . A neural network changes the reference and the neurodynamics in a trajectory with minimum dissipation of power or geodesic that we can see in figure where the minimum value is the solution of the ODE in (1)

In conclusion any neural network or the equivalent electrical circuit generates a deformation of the currents space and geodesic trajectories.

### 4.2 Relation between voltage sources and currents

The relation between voltages sources and currents for the circuit in figure 2 is

$$\begin{bmatrix} i_1 \\ i_5 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_3 - i_1 \end{bmatrix} I = (\alpha^T Z \alpha)^{-1} E = \begin{bmatrix} R_{ins}+3 & 1 \\ 1 & R+3 \end{bmatrix}^{-1} \begin{bmatrix} E \\ E(i_3)+E_{rest} \end{bmatrix}$$

$$= \frac{1}{3R+3R_{ins}+8} \begin{bmatrix} R+3 & -1 \\ -1 & R_{ins}+3 \end{bmatrix} \begin{bmatrix} E \\ E(i_3)+E_{rest} \end{bmatrix}$$

$$= \frac{1}{3R+3R_{ins}+8} \begin{bmatrix} (R+3)E-(E(i_3)+E_{rest}) \\ (R_{ins}+3)(E(i_3)+E_{rest})-E \end{bmatrix} \tag{41}$$

Now when we know E and $E_{rest}$ we have a recursive learning to compute the vector currents

$$\begin{bmatrix} i_1 \\ i_3 \end{bmatrix}$$

The analog electrical circuit computes the current vectors without any algorithm but by the electrical circuit. Now for all parts of the neural network is possible to give the analog electrical circuit in this way

### 4.3 Geodesic in presence of voltage-gated channels in the membrane

When we have the voltage gate channels in the membranes, we can introduce resistor with memory or memristors that represented by a variable resistor in figure 7:

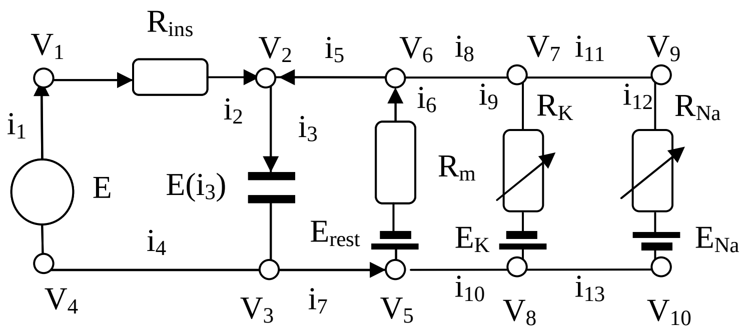
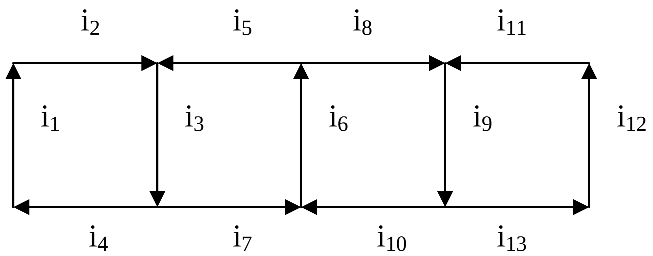



Figure 7 Electrical membrane circuit with voltage-gated channels in the membrane

The graph (topology) of the network is




The independent currents are

$$(i_2, i_5, i_8, i_{11})$$

The metric tensor is

$$g = \begin{bmatrix} R_{ins} + 3 & 1 & 0 & 0 \\ 1 & R_m + 3 & R_m & 0 \\ 0 & R_m & R_K + R_m + 2 & R_K \\ 0 & 0 & R_K & R_K + R_{Na} + 2 \end{bmatrix} \qquad (42)$$

The geodesic trajectory is

$$W = (R_{ins} + 3)i_2^2 + (R_m + 3)i_5^2 + (R_K + R_m + 2)i_8^2 + (R_K + R_{Na} + 2)i_{11}^2 + \\ + 2i_2 i_5 + 2R_m i_5 i_8 + 2R_K i_8 i_{11} \qquad (43)$$

The geodesic takes the minimum value for the currents inside the circuit. For the axon we repeat the same simple unity many times as follows:

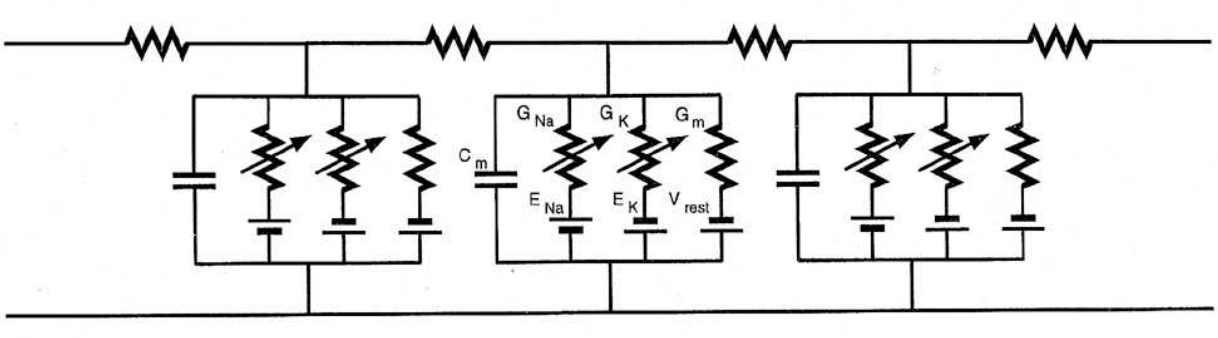



Figure 8. Axon and electrical circuit

The topology of the axon with three elements is

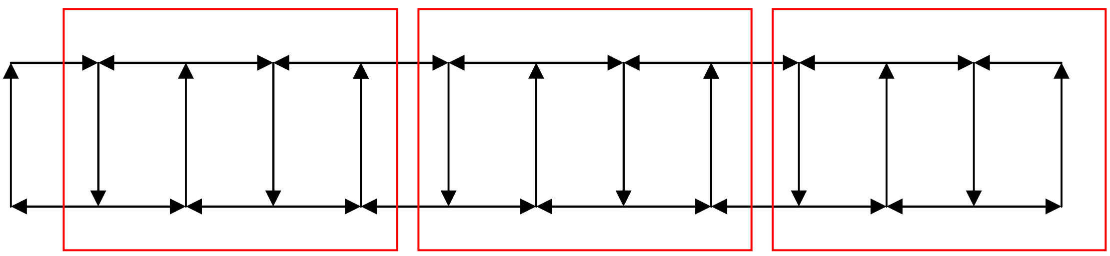


Figure 9 Topology of the three steps of the axons element. At any loop we have one current so we have 1+3+1+3+1+3 = 4*3 =12 loops and 12 free currents.

In figure 9 we have three unities of the axon. The space of the currents has dimension equal to

1+3+1+3+1+3 = 12

If $G_1$ is the space of the currents in the first four loops, the total space for the three steps is given by the direct sum of three equal spaces

$G = G_1 \oplus G_2 \oplus G_3$

The metric tensor is in the 12 space of the currents and is given by the matrix

$$g = \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix}$$

*where*

$$g_1 = \begin{bmatrix} R1_{ins} + 3 & 1 & 0 & 0 \\ 1 & R1_m + 3 & R1_m & 0 \\ 0 & R1_m & R1_K + R1_m + 2 & R1_K \\ 0 & 0 & R1_K & R1_K + R1_{Na} + 2 \end{bmatrix} \tag{44}$$

$$g_2 = \begin{bmatrix} R2_{ins} + 3 & 1 & 0 & 0 \\ 1 & R2_m + 3 & R2_m & 0 \\ 0 & R2_m & R2_K + R2_m + 2 & R2_K \\ 0 & 0 & R2_K & R2_K + R2_{Na} + 2 \end{bmatrix}$$

$$g_3 = \begin{bmatrix} R3_{ins} + 3 & 1 & 0 & 0 \\ 1 & R3_m + 3 & R3_m & 0 \\ 0 & R3_m & R3_K + R3_m + 2 & R3_K \\ 0 & 0 & R3_K & R3_K + R3_{Na} + 2 \end{bmatrix}$$

So the geodesic for three steps in the axon is

$$W = \left(\frac{ds}{dt}\right)^2 = W_1 + W_2 + W_3$$

*where*

$$\begin{aligned}
W_1 &= (R1_{ins} + 3)i_2^2 + (R1_m + 3)i_5^2 + (R1_K + R1_m + 2)i_8^2 + (R1_K + R1_{Na} + 2)i_{11}^2 + \\
&\quad + 2i_2 i_5 + 2R1_m i_5 i_8 + 2R1_K i_8 i_{11} \\
W_2 &= (R2_{ins} + 3)i_2^2 + (R2_m + 3)i_5^2 + (R2_K + R2_m + 2)i_8^2 + (R2_K + R2_{Na} + 2)i_{11}^2 + \\
&\quad + 2i_2 i_5 + 2R2_m i_5 i_8 + 2R2_K i_8 i_{11} \\
W_3 &= (R3_{ins} + 3)i_2^2 + (R3_m + 3)i_5^2 + (R3_K + R3_m + 2)i_8^2 + (R3_K + R3_{Na} + 2)i_{11}^2 + \\
&\quad + 2i_2 i_5 + 2R3_m i_5 i_8 + 2R3_K i_8 i_{11}
\end{aligned} \tag{45}$$

For axon with myelin we have a similar structure but only at the node.

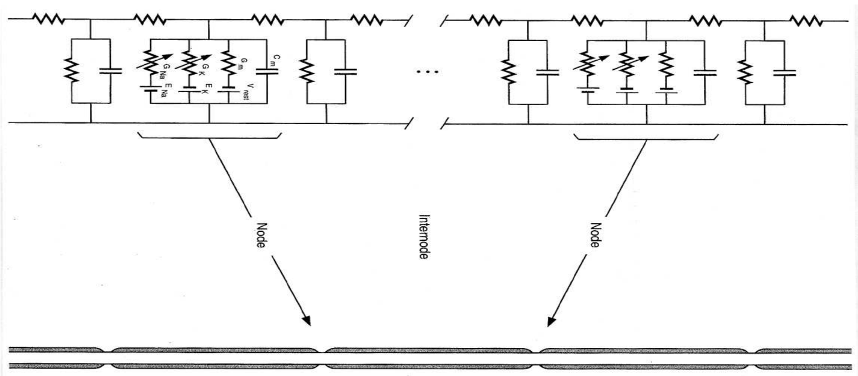



Figure 10. Axons with myelin and electrical circuit

## 4.4 Geodesic image of the synapses and dendrites

We remember that any synapse is a little part of the three of dendrite that we show in figure 11:

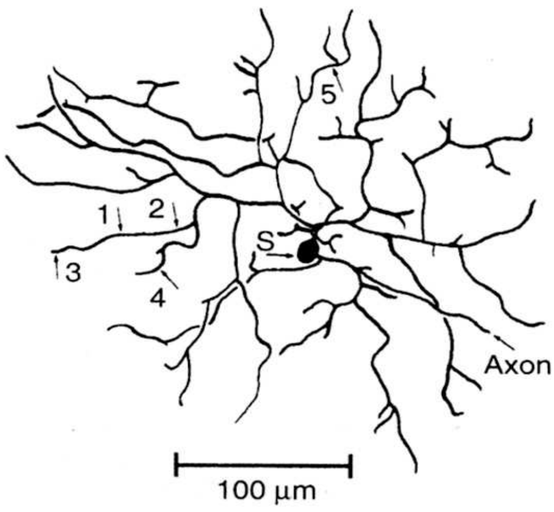



Figure 11. Dendritic tree with synapses, soma and axon.

The dendrite tree is useful to simulate a complex tensor metric in this general form

$$g = \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,n} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ g_{n,1} & g_{n,2} & \cdots & g_{n,n} \end{bmatrix} \qquad (46)$$

The electrical activity of the synapse is given by the circuit

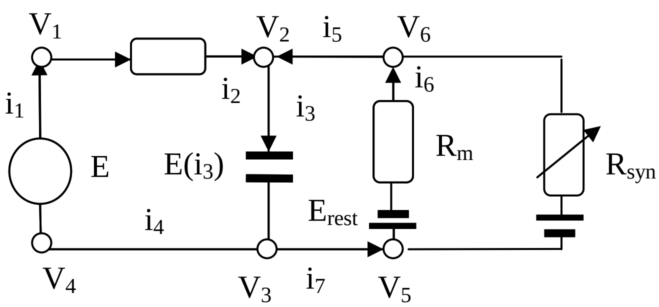



Figure 12 Electrical circuit for synapses.

The metric tensor is

$$g = \begin{bmatrix} R_{ins}+3 & 1 & 0 \\ 1 & R_m+3 & R_m \\ 0 & R_m & R_{syn}+R_m+2 \end{bmatrix}$$

The geodesic trajectory is

$$W = \left(\frac{ds}{dt}\right)^2 = (R_{ins}+3)i_2^2 + (R_m+3)i_5^2 + (R_m+R_{sys}+2)i_8^2 + 2i_2i_5 + 2R_m i_5 i_8 \quad (47)$$

Now we show the connection between the synapses and the dendrite by this electrical device

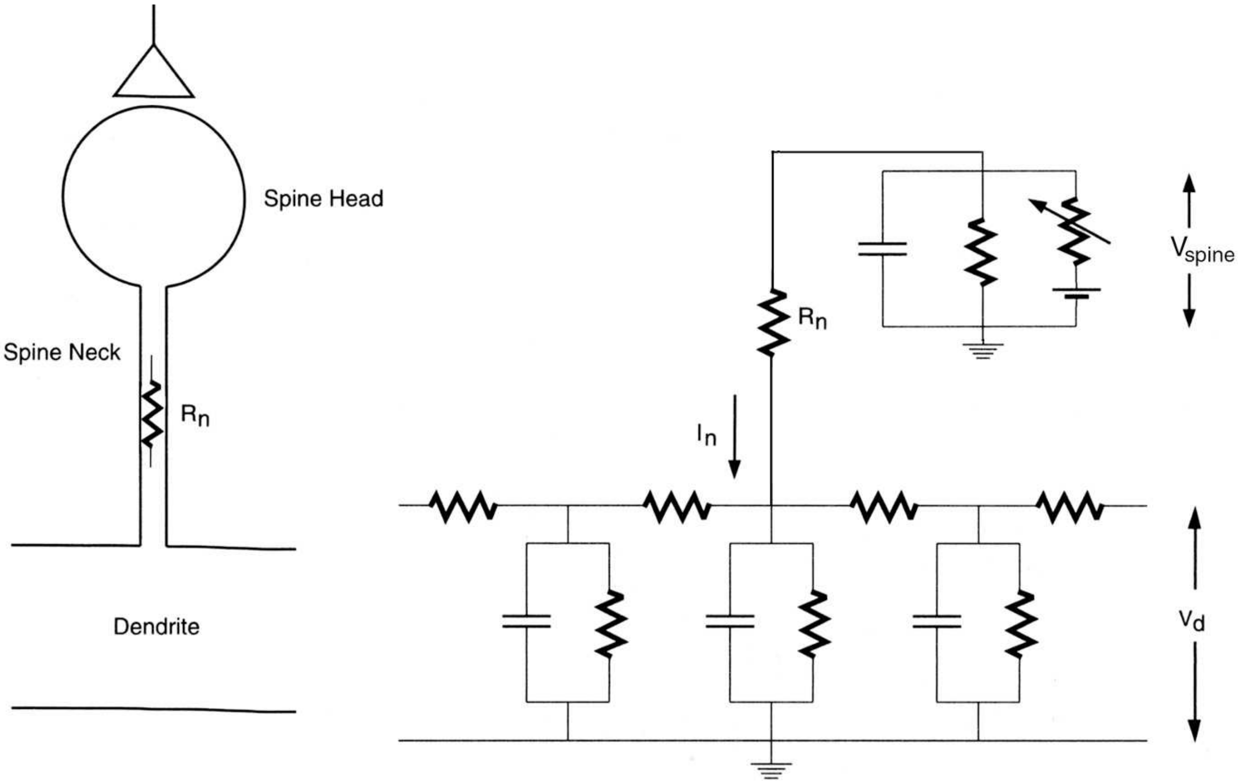



Figure 13 Electrical connection between synapses and dendrite.

## 4.5 Geodesic image of shunting inhibition

For the shunting inhibition we have

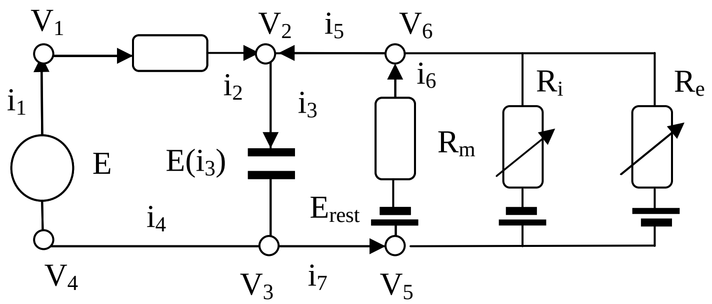



Figure 14 Electrical scheme for shunting inhibition. $R_e$ is the excitatory resistance, $R_i$ is the inhibitory resistance, $R_m$ is the resistance of the membrane.

The geodesic is

$$W = (R_{ins} + 3)i_2^2 + (R_m + 3)i_5^2 + (R_i + R_m + 2)i_8^2 + (R_i + R_e + 2)i_{11}^2 + \\ + 2i_2 i_5 + 2R_m i_5 i_8 + 2R_i i_8 i_{11} \qquad (48)$$

## 5. Implementation of a system in a neural network

$$\begin{cases} q_1 = x_1 x_2 \\ q_2 = x_1 + x_2 \end{cases} \qquad (49)$$

The metric tensor is

$$J = \begin{bmatrix} \dfrac{\partial q_1}{\partial x_1} & \dfrac{\partial x_1}{\partial x_2} \\ \dfrac{\partial q_2}{\partial x_1} & \dfrac{\partial x_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} x_2 & x_1 \\ 1 & 1 \end{bmatrix} \qquad (50)$$

And

$$g = \begin{bmatrix} \dfrac{\partial q_1}{\partial x_1} & \dfrac{\partial q_1}{\partial x_2} \\ \dfrac{\partial q_2}{\partial x_1} & \dfrac{\partial q_2}{\partial x_2} \end{bmatrix}^T \begin{bmatrix} \dfrac{\partial q_1}{\partial x_1} & \dfrac{\partial q_1}{\partial x_2} \\ \dfrac{\partial q_2}{\partial x_1} & \dfrac{\partial q_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} x_2 & x_1 \\ 1 & 1 \end{bmatrix}^T \begin{bmatrix} x_2 & x_1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 + x_2^2 & 1 + x_1 x_2 \\ 1 + x_1 x_2 & 1 + x_1^2 \end{bmatrix} \qquad (51)$$

The geodesic is

$$(\frac{ds}{dt})^2 = (1 + x_2^2)i_1^2 + 2(1 + x_1 x_2)i_1 i_2 + (1 + x_1^2)i_2^2 \qquad (52)$$

With the circuit of the membrane

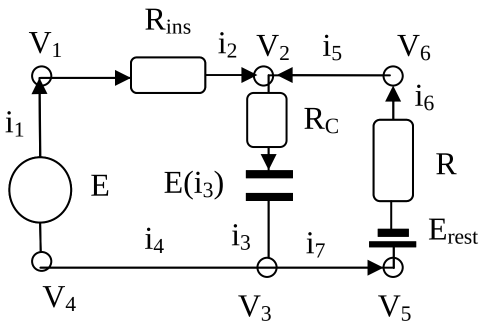



Figure 20. The electrical circuit of the neural unit for the non inertial change of reference.

We have the geodesic

$$(\frac{ds}{dt})^2 = (R_C + R_{ins} + 2)i_1^2 + 2R_C i_1 i_2 + (R + R_C + 2)i_2^2 \quad (53)$$

Now when the resistances are functions of the states (adaptable conductance) in this way

$$\begin{cases} R_C + R_{ins} + 2 = 1 + q_2^2 \\ R_C = (1 + q_1 q_2) \\ R + R_C + 2 = 1 + q_1^2 \end{cases} \qquad (54)$$

We can compute the resistances that simulate the change of the currents inside the space $q_1$ and $q_{2}$,

## 6. Geodesic equation in non inertial state with memristors

In the neural network the cost function is expressed as follows:

$$C = \int ds^2 = \int Z_{i,j}(q) dq^i dq^j \qquad (55)$$

Where Z(q) is the matrix of the resistor with memory or adaptive conductance in neural network, or the impedance in neural channels. We remember that in physics the trajectories in the phase space of N body is a geodesic which expression is

$$ds^2 = (E - U(x))g_{i,j} dx^i dx^j \qquad (56)$$

Here E is the total energy and U the potential energy. In the optical system we have:

$$ds^2 = n(x) dx^i dx^j \qquad (57)$$

Where n(x) is the diffraction index. The metric tensor is function of the position x. In the electrical circuit we have

*Charge q:* comparable to the *position x*
*Current I:* comparable to the *velocity v*
*VoltageV:* comparable to the *momentum p*

The relation between current and voltages is

$$i^j = g^{i,j} v_j \qquad (58)$$

The relation between momentum and velocity is similar

$$p^j = g^{i,j} v_j \qquad (59)$$

Now for the neural network we determine the minimum cost we use the Euler equation in this way

$$\begin{gathered} \delta S = \delta \int ds^2 = \delta \int Z_{i,j}(q) dq^i dq^j = \delta \int (Z_{i,j}(q) \frac{dq^i}{dt} \frac{dq^j}{dt}) dt = \delta \int (Z_{i,j}(q) I^i I) dt = 0 \\ for \\ \frac{d}{dt} (\frac{\partial (Z_{i,j}(q) I^i I^j)}{\partial I^j}) - \frac{\partial (Z_{i,j}(q) I^i I^j)}{\partial q^k} = 0 \end{gathered} \qquad (60)$$

We substitute time with the square of the power and we have

$$\begin{gathered} \delta S = \delta \int ds^2 = \delta \int Z_{i,j}(q) dq^i dq^j = \delta \int (Z_{i,j}(q) \frac{dq^i}{ds} \frac{dq^j}{ds}) ds = \delta \int (Z_{i,j}(q) \lambda^i \lambda^j) dt = 0 \\ for \\ \frac{d}{ds} (\frac{\partial (Z_{i,j}(q) \lambda^i \lambda^j)}{\partial \lambda^j}) - \frac{\partial (Z_{i,j}(q) \lambda^i \lambda^j)}{\partial q^k} = 0 \\ where \\ s = \sqrt{W} \end{gathered} \qquad (61)$$

The intrinsic parameter s is the movement of one point in the space of the states on a geodesic line. The parameter s is the local movement of a point inside the geodesic line. We know that the geodesic line can be written in this way

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \dots \\ q_n \end{bmatrix} = \begin{bmatrix} s(t)\cos(\alpha_1(t)) \\ s(t)\cos(\alpha_2(t)) \\ \dots \\ ds(t)\cos(\alpha_n(t)) \end{bmatrix} \qquad (62)$$

Here q are the local reference for the point P that moves on the geodesic. In fact we have

$$q_1^2 + q_2^2 + \dots\dots + q_n^2 = s(t)^2(\cos(\alpha_1(t))^2 + \cos(\alpha_2(t))^2 + \dots\dots + \cos(\alpha_n(t))^2) = s(t)^2 \qquad (63)$$

For Riemann geometry, we have the differential equations for the neural network and the geodesic in the neural network when we substitute the time t with the distance s that the point P covers from one position A to another position B inside the trajectory of the geodesic is

$$\begin{aligned} &\frac{d^2 q^r}{ds^2} + \Gamma^j_{i,k}\frac{dq^i}{ds}\frac{dq^k}{ds} = 0 \\ &where \\ &\Gamma^j_{i,k} = \frac{1}{2} Z^{l,j}\left(\frac{\partial Z_{il}}{\partial q^k} + \frac{\partial Z_{kl}}{\partial q^i} - \frac{\partial Z_{ik}}{\partial q^l}\right) \end{aligned} \qquad (64)$$

Here Z is the matrix of the resistance with memory (adaptive synapses) in the neural network. When the neural network is in an inertial state the resistance has no memory and the curvature of the surface in the space of the states where the geodesic moves is equal to zero. Note that the geodesic differential equation is comparable to the K0 set in Freeman K-sets[22,23,24,25,26, 27].

It 's interesting to note that the Freeman & Kozma neural damped oscillator is here obtained from (60), as a result of the neural global dynamics in regime of auto-hetero-organization that characterizes the autonomy of the brain / self [28, 29]

For a given transformation of reference, we can build the associate geodesic, which allows to implement the transformation of reference in the neural network. The neural network as analog computer gives the solution of the ODE of the geodesic inside the wanted reference. The Freeman K set is the ODE of the geodesic that is the best trajectory in the space of the currents. We know that the neural network has a lot of noise that diminishes coherence and synchrony in the network behavior, which allows a broad search for new solutions and new goals by the neural network.

## 7. Conclusion: from neuro-geometry to cognitive scenario

We want to conclude with the Freeman reflections on the intentionality

*The biological basis of mental activity can be explored with two assumptions: that animals think in ways less complex than humans do, and that the neural mechanisms of mental processes are basically the same across species. The widespread concern for the property of consciousness does not offer a good biological target, because it remains a matter of conjecture whether it exists and what forms it might take in animals. An alternative concept is intentionality, which can be recognized and studied in its manifestations of goal-directed behavior. Three accepted meanings of the term are the 'aboutness' of beliefs and ideas according to analytic philosophers, intent as conceived by cognitive psychologists, and wholeness in the process of healing from injury in medicine and surgery. Intention is interpreted as, respectively, an attribute of mental representations, the expression of motivations and biological drives, or the proclivity of biological tissue to grow toward wholeness. An experimental search for representations in brains has failed to find evidence for them. Analysis of brain dynamics reveals neural mechanisms for the construction of large-scale space-time patterns of brain activity, which emerge by interactions of neurons in the limbic system, and which coordinate the operations of sensory and motor cortices. The conclusion is offered that the three characterizations of the concept of intentionality, namely unity, intent, and wholeness, can be incorporated into a nonlinear dynamical model of brain function centered in the construction of meaning rather than representations. In this view, consciousness is the dynamical process in which meanings are continually under construction in a chaotic trajectory through brain state space, and awareness is the subjective experience of the momentary focus of the activity that constitutes a meaning* [7]

It is known by now that the question of intentionality implies noisy long-range correlations in the brain [30, 31]. The approach here defined in terms of global neuro geometry is compatible with these theories. In this paper we argue that the multidimensional space of the states and currents reflect the geometric structure of the nonlinear neurodynamics corresponding to the optimal behaviour of the brain. In other words, we have attempted to define the "geometric correlates" for the intelligent agents. The memory in the neural network is not a passive element where we can store information. Memory is embodied in the neural parameters as synaptic conductances that give the geodesic trajectories in the non orthogonal space of the free states. The optimal nonlinear dynamics is a geodesic inside the deformed space ( deformed from orthogonal to non orthogonal ) that guides the neural computation. The adaptation gives to memory its character of reconstructive process, result of a complex interplay between cues, goals, knowledge [35]. Intentionality reflects this deformation, which gives us the correct reference on where the dynamics is in agreement with the goals of the intelligent systems. Noise is an important component of neural computation, which opens the possibility of exploring new reference systems and new points of views to achieve novel aims and goals. The trajectories obtained through optimal movements with noise can explore new optimal situations that cannot be obtained without the noise. Intention with noise as percolation process opens the possibly to change of the intent itself.

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