Partialhood

forthcoming in *Oxford Studies in Metaphysics*

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May 9, 2022

My bedroom window is a part of my house, but it is not a partial house. A half-built house is a partial house, but there is no house it is a part of.[1] Being a part of something—parthood—is a familiar topic of philosophical inquiry. Being a partial something—partialhood—is not. The neglect of partialhood is a shame because it is intrinsically interesting as well as metaphysically and semantically important. My overarching goal is to argue for this importance. After all, the mere fact that a relation has been ignored is not good evidence that it shouldn't be.

In order to show that partialhood is important, we must first understand it: I tackle this in §1-§3. In §1 I link partialhood to ordinary tractable discourse. This provides us with set of clear judgments against which to develop a view. In §2, I argue that partialhood cannot be easily reduced, undermining the idea that understanding it is a shallow affair. In §3, I show that despite this failure of reduction there are a number of informative things we can say about it. In particular, partialhood enters into some key inferential relations and has systematic extrinsic instantiation conditions.

Having explicated these features of partialhood, I then utilize them. In §4 and §5 I use partialhood to yield new insight into two longstanding issues. In §4 I argue that partialhood provides us with kind-categorizations that help us identify the metaphysical basis of the mass/count distinction. A familiar thought is that mass nouns are distinguished by designating distributive kinds—where a kind K is distributive just in every part of a K is also a K. For example, instances of water seem to be such that they can be arbitrarily subdivided into further instances of water. There is a problem: if we subdivide water enough we get hydrogen molecules, which are not water. I argue that we can solve this problem by understanding distributivity in terms of partialhood rather than parthood. In §5 I argue that partialhood allows us to give an elegant account of the progressive. Progressive sentences, e.g. 'I am making a cup of tea' express that an event is underway or happening. A common analysis takes progressive sentences to express relations between event-tokens and event-types. Theorists

[1] There is a true reading of 'that is part of a house', where 'that' designates a half-built house. This is not the more familiar notion of parthood that philosophers ordinarily discuss, which is a relation that can only hold at t if both of its relata exist at t. When I discuss parthood, I'll stick with the more familiar notion.

have tried to understand this relation in modal, telic, and mereological terms. I advance a new hypothesis: the relation just is partialhood. This debate is usually undertaken in linguistic terms, but there's a metaphysical correlate: to give an account of the progressive is to explain what it is for an event of a certain kind to be in progress, or happening (cf. Kroll 2015: 2931). On my view, for an event of kind K to be happening is for there to be a token event e that is a partial K.

§1 Partialhood in Everyday Discourse

I introduced our topic using the term 'partial' and, at least in some central cases, we can use this terminology to generate clear judgments. However, our judgments about sentences containing 'partial' are limited, and those sentences don't bear uncontroversial entailment relations that would allow us to construct a theory of partialhood. We need another way in.

§1.1 Speaking of Partials

We can find our way in by considering the ways that we speak about partial entities. Co-opting an example from Salmon (1997), imagine that there are two whole oranges and one half-orange on our table, and that the whole oranges weigh a pound each, while the half-orange weights a half-pound. The half-orange is a partial orange. There are a variety of natural language sentences with truth-conditions sensitive to the partial orange.[2] In particular, the partial orange can affect our counts, our measurements, and we can designate it directly. These phenomena are captured in (1), (2), and (3), respectively.

(1) Two and a half oranges are on the table.

(2) Two and a half pounds of oranges are on the table.

(3) That half-orange is on the table.

In (1), we count the oranges on the table. Its truth is witnessed by three entities: the two whole oranges and the half-orange.[3] In (2), we measure, in pounds, the weight of the oranges on the table. The truth of (2) is partly dependent on the weight of our half-orange.[4] In (3), we use the compound

[2] I am not suggesting that the partiality-sensitive reading of such sentences is the *only* reading. In fact, given the fact that such sentences all contain plausibly context-sensitive items (nouns) and the existence of loose speech, there are plausibly readings of these sentences that are not partiality-sensitive. All that matters for my purposes is that some readings are.

[3] The compositional semantics of these sentences is controversial, though nothing in this discussion will depend on the details of any particular proposal. I have made my own proposal in Liebesman (2016), which is influenced by the discussion in Salmon (1997). Alternate proposals can be found in Nicholas (2016), and Ionan, Matushansky, and Ruys (2006). The latter proposal is endorsed and elaborated in Snyder and Barlew (2019).

[4] Measure sentences such as (2) have received extensive attention in the semantics literature. See Rothstein (2017) for an overview and references.

'half-orange' as part of a complex demonstrative to designate the half-orange, ensuring that the half-orange is directly involved in the truth-conditions of (3).[5]

Focussing on counts, the distinction between parthood and partialhood becomes clear. If there are two whole oranges and one half orange on the table, then there are two and a half oranges on the table. However, if there are two whole oranges alongside a few discarded orange seeds, then there are still exactly two oranges. Despite being former parts of an orange, the seeds do not affect our orange-count and, thus, are not partial oranges.

Similar remarks apply to measurement. Assume that each orange weighs 1lb. and their weight is distributed evenly. If there are two whole oranges and one half orange on the table, then there are two and a half pounds of oranges on the table. However, if there are two whole oranges alongside some discarded seeds, then there are exactly two pounds of oranges on the table. Despite being former parts of an orange, the seeds don't affect the measurement and thus are not partial oranges.

With these examples in mind, we'll define 'partial K' as the ability to affect either K counts or K measurements. Since any K can affect a K-count or K-measurement, this has the immediate result that every K is a partial K. By analogy with improper parthood (where everything is a part of itself), we can think of ourselves as theorizing about improper partialhood. Of course, we could define a notion of proper partialhood but, for our purposes, it is the less interesting relation. One can think of our definition as stipulative (or precising) rather than an attempt to capture the meaning of the English word 'partial'. 'Partial' is useful, but it is useful in the same way that 'in virtue of' provides us a way into thinking about grounding, or 'because' a way into explanation. For the sake of readability I will sometimes use 'partial', but one should understand it as shorthand for the official notion defined below. Of course, merely claiming that to be a partial K is to be able to affect K-counts or K-measurements is hardly a satisfying definition. I'll now turn to something more precise.

§1.2 The Official Definition

In order to give a more precise account what it is to be able to affect K-counts or K-measurements, I'll make some assumptions about the semantics of number words. Following Landman (2003), Scontras (2014), and Rothstein (2017), suppose that number words like 'two' have the same semantic type as adjectives like 'happy'. Each can modify a first-order predicate to produce another first-order predicate. For instance, 'happy' modifies 'dog' such that 'happy dog' is true of all and only the happy dogs. On the adjectival view of 'two', 'two' modifies 'dog', such that 'two dogs' is true of pluralities containing two members, both of which are dogs.

Ability to affect K counts suffices to be a partial K. Here's what this means. Assume o is a partial K. If we have a predicate 'K' that designates K, then

[5] Terms like 'half-orange' have not received extensive attention, though see Ionan, Matushansky, and Ruys (2006).

there is some number word or phrase 'N' such that 'N K' is true of *o*, or of some plurality containing *o*. Going back to our partial house: 'Two and a half houses' is true of the plurality containing the two whole houses and the partial house.

Our assumption is that number terms—both simple terms like 'two' and complex terms like 'twenty-two' and 'two and a half'—are adjectival.[6] We'll take them to be interpreted as modifiers, functions from first-order properties to first-order properties. We'll call such functions 'numbers'.[7] So, [['two']] is a number: a function from first-order functions like λx.dog(x) to first-order functions over pluralities. Given this, [['two dogs']] = λxx.xx are two dogs. While I'll borrow plural variables (e.g. 'xx') from plural logic, I'll merely stipulate that they designate pluralities or singular entities while being silent on the nature of pluralities. I'll also borrow the '$\prec$' symbol from plural logic and take it to have its usual interpretation as *inclusion*. I'll assume that kinds are non-function individual entities but each kind can be mapped to a corresponding property. I'll use the superscripted '$\cup$' to map a kind to its corresponding property, so $^{\cup}$dog-kind is the property associated with the kind *dog*.[8] Using 'N' to range over numbers (understood as above), we can give a sufficient condition for partialhood for an arbitrary kind K as follows. We'll take 'Π' to express the partialhood relation, i.e. 'Π(x,K)' means that x is a partial K.

$$(\exists \mathrm{xx}((\mathrm{x} \prec \mathrm{xx}) \land \exists \mathrm{N}(\mathrm{xx} \in \mathrm{N}(^{\cup}\mathrm{K})))) \to \Pi(\mathrm{x},\mathrm{K})$$

Our half-orange is a partial orange, because there is some number—two and a half—such that that number applied to $^{\cup}$orange-kind is true of a plurality that contains the half-orange. Of course not just any modifier picks out a number in our sense, but I will assume that there are independently established facts about what is and isn't a number.

All of this has proceeded from the assumption that number words are adjectival, and has utilized some potentially contentious logical and metaphysical resources, but that assumption and those resources are inessential. In principle a similar account could be constructed for any extant semantics of number-words and fraction terms.[9]

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[6] See Ionin and Matushansky (2006) for a discussion of complex cardinals like 'twenty-two'.

[7] This is somewhat tendentious, given disagreement about the semantics of complex terms. For instance, on the view developed in Barlew and Snyder (2018), 'two and a half' expresses a complex modifier which they would not obviously take to correspond to a single number. Tendentious as it may be, the terminology is irrelevant to my argumentation. For recent insightful discussions of the ontology of numbers that are tethered to the semantics literature, see Hofweber (2005) and (2014), Balcerak Jackson (2013) and (2014), Balcerak Jackson and Penka (2017), Felka (2014) and (2016), and Snyder (2017).

[8] My usage of '$\cup$' follows Chierchia (1998).

[9] For those more inclined to take number words to express second-order relations, here is how to construct a similar definition. If we take 'two and a half oranges' to be a generalized quantifier, its semantic value is a set of sets/property of properties (or a function from first-order functions to truth-values, type $<<e,t>,t>$). (As is custom, I use 'generalized quantifier' to pick out both expressions like 'two dogs' and their semantic values.) Following Barwise and Cooper (1981), we can define the notion of a witness for a generalized quantifier in three steps. First, we define the notion of 'living on':

A generalized quantifier Q *lives on* a set A iff $\forall$B(B $\in$ Q $\leftrightarrow$ A $\cap$ B $\in$ Q)

Note that our definition of partialhood thus far proceeded only from the observation that we can count partials. We also observed that we can designate them using terms like ‘half-orange’, and that they affect measurements. Measurement provides us with a second sufficient condition for partialhood. Intuitively, the idea is that a half-apple can affect apple measures. To make this more precise, we’ll understand measures as functions from object/kind pairs to numbers. If h is our half-apple, A is *apple-kind*, and P is the measure function for weight-in-pounds, then $\mathrm{P}(<h,\mathrm{A}>)=.5$.[10] Allowing ourselves quantification over measures with ‘M’, we can articulate our second sufficent condition as follows:

$$\exists \mathrm{M}(\mathrm{M}<\mathrm{x},\mathrm{K}> \; > 0) \rightarrow \Pi(\mathrm{x},\mathrm{K})$$ [11]

As was the case with numbers, not just any function of the right logical type qualifies as a measure. As was the case with numbers, I assume distinguishing which functions are genuine measures is an independent task in philosophy of mathematics.[12] Of course, we do have some clear examples of genuine measures, such as weight-in-pounds. Similarly, it is easy to construct a non-measure: consider a function that uniformly maps entities to the number 1,234. To use Krifka’s (1990: 494) suggestive language, a genuine measure is a “function from concrete entities to abstract entities such that certain structures of the concrete entities, the empirical relations, are preserved in certain structures of the abstract entities”. Not just any function will reflect such ‘empirical relations’.[13]

Taking our two sufficient conditions to be jointly necessary, here is our official definition. Henceforth, we’ll take ‘partial’ to be a term of art that is defined as

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Using the notion of *lives on*, Barwise and Cooper define the notion of a witness set as follows:

> A witness set of a quantifier Q living on A is any subset w of A such that w $\in$ Q.

Finally, we can define witness:

> x is a witness for Q iff x is a member of some witness set for Q.

Utilizing Q to quantify over generalized quantifiers (semantic values not linguistic items), we can articulate our sufficient condition for partialhood.

$$\exists \mathrm{Q}(\mathrm{x} \text{ is a witness for } \mathrm{Q}(\mathrm{K})) \rightarrow \pi(\mathrm{x.K}).$$

[10] These measures are not plausibly the compositional semantic values of measure terms like ‘pound’. However, they are closely related to a plausible proposal, see Scontras (2014: 37).

[11] This may need to be modified a bit to accommodate all of our intuitions. In particular, due to the plural marking on ‘apples’, it may be odd to claim that I have a half-pound of apples in my pocket if I just have one half apple. We can accomodate this by taking measure functions to be functions from plurality/kind pairs to numbers, and taking it to suffice for partialhood that an entity non-trivially contributes to the measure of a plurality. Here’s how to modify the condition:

$$\exists \mathrm{xx}(\mathrm{x} \prec \mathrm{xx} \wedge (\exists \mathrm{M}((\mathrm{M}<\mathrm{xx},\mathrm{K}> - \mathrm{M}<\mathrm{xx}\text{-}\mathrm{x},\mathrm{K}>) > 0))) \rightarrow \Pi(\mathrm{x},\mathrm{K})$$

[12] See Wellwood (2019) for some proposed constraints on which functions can be expressed by some measure constructions in natural language, which may give us insight into this question.

[13] In a similar vein, Wellwood (2019) writes: “. . . in order to be properly called a ‘measure’, a function must preserve certain structural properties of the measuranda in the structure of scales.” Wellwood cites Berka (1983) as influencing this conception of a measure.

follows:

$$\Pi(\mathrm{x,K})\ (\text{x is a partial K}) =_{def}$$
$$\exists \mathrm{xx}((\mathrm{x} \prec \mathrm{xx}) \wedge \exists \mathrm{N}(\mathrm{xx} \in \mathrm{N}(^{\cup}\mathrm{K}))) \vee$$
$$\exists \mathrm{M}(\mathrm{M}\langle \mathrm{x,K}\rangle > 0)$$

How are our two sufficient conditions related? In particular, one may wonder whether it is true that an object could affect a K-count just in case it could affect a K-measure. If certain kinds can only be measured but not counted, or *vice-versa*, this won't be the case. Many theorists hold that kinds designated by mass nouns can't be counted. As we'll see in §4, I'm skeptical, but, in general, the measurement condition will be useful when discussing mass nouns and the counting condition will be useful when discussing count nouns. What about kinds that can be both counted and measured, such as apples? In such cases, the ability to affect counts and the ability to affect measures do seem go together. Imagine again that all oranges weigh exactly 1 lb. If there are more than two and less than three oranges (count), then there will be more than 2 lbs. but less than 3 lbs. of oranges (measure). If there are more than 2 lbs. but less than 3 lbs. of oranges (measure), then there will be more than 2 but less than 3 (count). I don't know of a fully general argument that ability to affect measures and ability to affect counts always go together in this way but nothing in this discussion will hinge on that.

I have stressed repeatedly that I will remain neutral on the semantics of number terms and that my proposal is, in principle, compatible with all extant treatments of words like 'two' and phrases like 'two and a half'. Our definition makes it clear how I can maintain this neutrality. In order to determine whether some object *o* is a partial K we need merely consult counting/measuring sentences themselves. If *o* makes true some K-counting or K-measuring sentences then it is a partial K. Exactly how *o* is involved in those truth-conditions will be a difficult matter for compositional semantics, but one we need not answer in order to utilize such sentences to theorize about partialhood.

§1.3 Two Assumptions and a Clarification

Two assumptions I'll make throughout are worth flagging. I'll also clarify the notion of kinds that I'll work with.

The first assumption is that sentences like (1)-(3) are often true, and that (1)-(3) have true readings in the scenarios described. This is not controversial in the literature on fractions or measurement, and such sentences are common in ordinary discourse. As always, one could hold an error theory about them, but I'll assume away such a theory. Furthermore, note that the existence of such true readings is compatible with the existence of other readings.

The second assumption is more controversial, though it is inessential. It is that, at least for many kinds K, non-Ks can be partial Ks. Consider, again, our half-orange. I will assume that it is not an orange. This assumption is not controversial in the literature: it is vindicated by every extant semantics for fraction terms. One reason to hold it is that abandoning it would lead to bizarre results given standard accounts of quantification. If the half-orange is an orange, then, on standard views of 'Three' it will be true that whenever there are two and a half oranges there are also three oranges![14] Of course, one may insist that the half-orange is an orange, it simply isn't a whole orange, and then modify their theory of counting accordingly such that enumerating oranges is equivalent to enumerating whole oranges. For the most part I'll assume away such a view, but, as I will make clear, it is compatible with my discussion. What's important is that there is a distinction between the whole orange and the half-orange. On my favoured view, one is an orange while the other isn't. On another view, one is a whole orange while the other isn't, despite the fact they are both oranges. I'll pepper the paper with footnotes about what happens when this assumption is dropped.

In addition to my two assumptions, I'll add a clarification about my notion of a kind. I'll assume that kinds are at least as plenitudinous and as fine-grained as intensions.[15] Of course, that's compatible with making many distinctions among kinds, e.g. between natural and non-natural kinds. One unfriendly to this view of kinds could substitute intensions throughout. I'll also assume that nouns designate kinds with the following membership conditions:[16]

> A count noun 'N' designates the kind K such that x is a member of K iff x is an N.
> A mass nouns 'N' designates the kind K such that x is a member of K iff x is some N.

As we've defined partialhood, for any arbitrary kind K, each of its members is a partial K. Given this, the first slot of partialhood can be saturated by any sort of entity (concrete object, abstract object, event, etc.) and the second slot can be saturated by (at least) any membered kind. Given our assumed plenitude of kinds, this is quite broad. A more interesting question, that will play a key role in our discussion, is what sorts of things can be mere partials, and what kinds admit to them, where a merely partial K is a partial K that is not a K. To preview, I am going to argue in §4 that (roughly) kinds designated by mass nouns don't admit mere partials (there is no partial water) while kinds designated by count nouns do.

[14]See Liebesman (2015b) for a more thorough defense of the claim that the half-orange is not an orange.

[15]In other work, I discuss some of these issues in more detail. In Liebesman (2011) I discuss the differences between kinds, sets, properties, and pluralities. In Liebesman and Sterken (2021) we discuss ways to justify the plenitude of kinds that are based on the semantics of generics.

[16]I will not assume that these kinds are the semantic values of their corresponding nouns given that for the most part I will set aside most issues pertaining to the compositional semantics of number words, counting sentences, and measure sentences.

§2 Reduction?

Can we reduce partialhood to independently well-understood properties and relations? In the next two subsections I'll argue against several attempted reductions. To be clear, I will not argue that partialhood is metaphysically fundamental. In the sense that I am using 'reduction', relations with (instances that have) non-trivial grounds may not be reducible. Rather, I will merely argue that there is no straightforward (relatively short) biconditional with 'x is a partial y' on one side, that uses only familiar relatively well-understood terminology on the other. The absence of such a reduction is compatible with the non-fundamentality of partialhood. Though I don't have a decisive argument against every possible reduction, arguments against the most natural reductions, coupled with principled arguments that certain sorts of reductions are impossible, leads me to pursue a non-reductive account in §3.

§2.1 Mereological Reductions

Given that 'partial' has 'part' as a part, it is tempting to try to reduce partialhood to parthood. Though nobody has attempted to reduce partialhood in the literature, there are examples of specific types of partialhood being reduced to mereological (or related) notions. Yablo (2015), for instance, reduces the partial truth of a sentence to its having a true part.

Inspired by Yablo, we can begin with the simplest mereological reduction of partialhood. On this view, *e* is a partial K just in case there exists a K that *e* is a part of. As we've already seen, this fails in both directions. My bedroom window is part of my house but not a partial house, and a halfway-built house is a partial house but not part of any house. Any plausible mereological reduction will have to be more subtle.

How can we give a more subtle mereological reduction? The most promising route that I know of departs from the reflection that a merely partial house has *some*, but not *all* of what it takes to be a house. Since, we're considering a partialhood relation on which all Ks are partial Ks, we can jettison the 'not all' component, and focus on the notion of having *some* of what it takes to be a house. Yablo (2015) has given us a mereology of content such that we can try to make sense of this in mereological terms. Our more subtle reduction has two steps. On the first step, we reduce all partialhood to partial truth. On the second step, we reduce partial truth to parthood.

Step 1: e is a partial K just in case *e is K* is a partial truth.

Step 2: *e is K* is a partial truth just in case part of *e is K* is true.

In order to evaluate this reduction, we need a method to identify parts of truths. On Yablo's view, t is a part of t′ just in case t′ entails t, and the subject matter of t is part of the subject matter of t′.[17] Of course, we now need

[17] In presenting Yablo's view, for rhetorical simplicity, I'll move back and forth between

a theory of subject matter and subject matter mereology, and Yablo gives us both. However, we needn't delve into the details as a single example will suffice. Emma is a dog, and dogs are not partial cats. However, on the attempted reduction, any dog is a partial cat. To see this, consider (4) and (5).

(4) Emma is a mammal

(5) Emma is a cat.

(5) is false. It, however, entails (4), which is true. If (4) is part of (5), then the result is that (5) is a partial truth, and by the proposed reduction, Emma is a partial cat. In fact, on Yablo's view (4) is a part of (5): (4) is entailed by (5), and the subject-matter of the former—roughly, mammalhood—is part of the subject-matter of the latter—cathood. Given that Emma is not a partial cat, this more subtle mereological reduction fails.[18]

One could attempt to save the above reduction by modifying our understanding of the mereology of truths. The driving idea is that e is a partial K just in case *e is K* has a true part, and the optimist about the reduction will attempt to seek out a mereology for content that vindicates this idea. The problem is that, for principled reasons, it is hard to see how to give such a mereology. Consider two options.

On the first option, we take propositions to be structured entities composed of either entities, properties, and relations, or senses-thereof.[19] On this option it is easy to make sense of the mereology of truths: truths are propositions, and they have components and structure.[20] However, on this view, the mereology of truths is not nearly rich enough to support a reduction of partialhood: it will undergenerate. Consider the proposition that *e is a house*, where *e* is a half-built house. That proposition doesn't have any proper parts that are truth-apt! On the view envisioned, its only proper parts are an entity and a property. Since the proposition itself is false, and it doesn't have any true proper parts, it would follow that e is not a partial house. However, this is false.

On the second option, we make sense of the mereology of truths not by focusing on their constituents, but rather by focusing on what they say about the world. This is Yablo's strategy. We can think of a proposition like *Emma is a cat* as telling us all sorts of things about Emma that add together (compose) to generate the proposition: e.g. that Emma is a mammal and that Emma is physical. The problem is that it seems that pursing this sort of mereology for contents, along with a mereological reduction of partialhood, will lead to a severe overgeneration. We've already, in effect, seen this with Emma. What could the proper parts of *Emma is a cat* be? Well, they must at least be asymmetrically

speaking of truths and true sentences. Taking truths to be true propositions, this means we'd need methods for identifying the parts both of sentences and propositions, and, in fact, Yablo provides us both methods.

[18] Of course, none of this is a problem for Yablo, who doesn't pursue the reduction!

[19] See King (2007), Soames (2010), and Hanks (2015) for recent versions of this view.

[20] Of course whether the constituents of propositions on structured accounts really are parts is a controversial topic. I'm not intending to take a stand, I'm merely suggesting that the view on which they are parts is easy to make sense of.

entailed by *Emma is a cat*, and they must together tell us that *Emma is a cat*. The most natural candidates will all be property ascriptions to Emma that ascribe her properties like *being a mammal*. The problem is that, by this very recipe, there will be all sorts of entities in these categories that aren't cats, and, in fact, share with them only the property of being a mammal. We have no good reason to expect that these will be partial cats.

Stepping back, the project of identifying partialhood requires identifying some intimate connection between entities and kinds, and we have no reason to think that the partial truth of *e is K* guarantees this intimate connection: after all there are lots of components to being a K and many of these will be instantiated by things that are not partial Ks. As usual, these considerations are not decisive, and I don't see how to make them so in the absence of particular proposals. They do suggest, however, that identifying the parts of the proposition *that e is K* is a very different task from identifying the partial Ks.

§2.2 Modal and Telic Reductions

Focus, again, on a half-built house. It is natural to think that what makes it a partial house is that it is *directed towards* a complete house.[21] We can separate out two components of this natural thought in order to attempt a reduction. First, we associate an object with an event. In the case of the partial house, we associate it with a building event. Second, we take that event to be directed at an end. In the case of the partial house, it is directed at the creation of a house.

For a full-fledged reduction, we would have to develop each of these components in much more detail. On the first: each object is associated with many events in many ways, so we'd need to identify the relevant function from objects to events. On the second: the notion of being *directed* is usually not taken to be an acceptable primitive. One could attempt to analyze this notion in either modal or telic terms.

Without yet answering these questions, here is the attempted reduction, where 'f' designates the function from objects to their relevant associated events (e.g. the house-building event):

> o is a partial K just in case f(o) is directed at the existence of a K

We can then go on to analyze the notion of being 'directed at' in modal or telic terms:

> e is directed at the existence of a K just in case e develops in a K-creation event in every e-inertia world.

> e is directed at the existence of a K just in case e has the existence of a K as its telos.

The notion of an e-inertia world is that of a world in which e develops without interference. In the case of building a garage, the idea is that we look

[21] Cf. Kroll (2015) on the directedness intuition about the progressive.

to the worlds where the building continues unimpeded. The notion of inertia worlds comes from Dowty (1978) and making sense of their properties has been a major theme in the literature on the progressive.

These are the barest sketches of modal and telic reductions of partialhood. We'd want to hear much more about all of their components before taking them to be genuinely reductive, let alone successful. Yet, even with these meager materials we have enough to argue against the reductions.

The first problem pertains to the directedness intuition itself. Not all partial Ks conform to the intuition. Consider, for instance, the following case:

> **Remainder**: I eat half of an orange, and put the remaining half on the table, alongside two whole oranges. Two and a half oranges are then on the table.

The half-orange affects orange counts and measures but it is not associated with any event that is directed at the existence of a K. A natural suggestion is to make the analysis disjunctive, allowing for destruction:

> o is a partial K just in case either f(o) is directed at the existence of a K or f(o) is the destruction of a K

Adding this disjunct will complicate f, and, as I'll return to, it will likely overgenerate partial entities. However, there's a bigger problem: some partial Ks are associated neither with a K-creation or a K-destruction:

> **Completed Partials**: Sales of bagels are flagging! In these carb-conscious days, nobody wants to eat an entire bagel. The board of directors at the Bagel Corporation gets together to attempt to salvage their business. They know that everybody loves the taste of bagels, but carbs are unpopular. They hit upon a solution: to market half-bagels. (Slogan: 'Half the carbs of a bagel!') Half-bagels catch on and eventually the corporation opens a factory. Each creation event in the factory is aimed squarely at producing a half-bagel. After all, bagels don't sell.

Half-bagels affect our bagel counts (imagine there's one alongside two wholes),[22] but, despite being merely partial bagels, they are not associated with events

[22] Some have claimed that half-bagels no longer affect the bagel count when they are conventional artifacts themselves. We can easily modify the scenario to sidestep the worry. Imagine that instead of there being a half-bagel factory, I merely set out to make two and a half bagels because I don't have enough dough for a third. All of the same remarks apply, and this time there aren't any conventionalized half-bagels. Also note that we can give similar cases not involving agents at all. For instance, imagine that for some bizarre reason a half-orange grows naturally, and not because its growth was cut short. We can also give cases involving a wide variety of different kinds, e.g. I can deliberately build a half-house or write a half-sonnet. Given the plethora of similar cases, those resistant to any version of the bagel example can simply switch examples.

aimed at the creation of whole bagels, nor are they the result of whole bagel-destruction. This is why they are *completed* partials. The possibility of completed partials shows that the directedness intuition itself fails to cover all cases, even when we add a disjunct for destruction. There may be entities that affect a K-count, even if those entities simply are not associated either with events aimed at K-creation, or K-destruction.

The second problem for the attempted modal/telic reductions is that they all overgenerate. The problem is that for something to affect a K-count, it does not suffice that it is the theme of a K-creation event. Imagine that I have just planted an orange tree. This plant is associated with an event that would result in the existence of an orange, if the event continues unimpeded. However, an orange tree is clearly not a partial orange. The point generalizes. Some rising dough may be part of a process aimed at bagel-creation, but that dough is not a partial bagel. In order for something to count as a partial K, it must meet more stringent requirements than merely being the theme of a K-creation event.[23] Overgeneration problems also afflict the destruction disjunct that we added in an attempt to account for such cases. Imagine that all that's left of an orange is some discarded pith. This pith is obviously associated with a K-destruction event, but it does not count as a partial orange.

There's an obvious rejoinder to this second problem: attempt to better circumscribe the relevant function from objects to events. After all, in giving the objection I've used the unhelpful 'associated with' rhetoric. What we need is a function that maps the half-orange to a destruction event, and the pith to no such event. I agree that this is what the reduction would need to succeed. What I don't see is any way to even begin giving a reductive account of such a function. It seems that the only relevant thing that separates the half-orange from the orange pith *vis-a-vis* the destruction event is that the former is a partial orange, while the latter is not.

The third problem, and perhaps the most obvious one, is that as of yet, we have no reason to think that the sketched reductions are genuine reductions at all. There are multiple reasons for this. One reason is that we haven't said anything about the f function, and, as we just stated, there are reasons to think that adequately characterizing it requires utilizing the notion of partialhood itself. Another reason is that the modal reduction requires the notion of inertia worlds, which themselves are best characterized in terms of continuing without interruption. It remains to be seen whether this can be understood in terms that the would-be reductionist would find acceptable. In fact, Kroll (2015) argues that it can't. A final reason is that the telic reduction employs the notion of a telos which many will find unacceptable for a would-be reduction.[24]

[23] An analogous problem arises for the inertia worlds account of the progressive, see Higginbotham (2004) for a proposed solution.

[24] An anonymous referee points out that one could try give a reduction of partialhood in terms of counting/measuring. Since my definition is undertaken in those terms, I think such a reduction would fail: it would attempt to reduce a relation defined in terms of counting/measuring to a notion again involving counting/measuring.

§3 Logic and Difference Makers

Absent a promising reduction, I'll illuminate the nature of partialhood by identifying a number of its key properties. I'll first identify several of its logical properties in §3.1-§3.3, then, in §3.4 I'll consider some factors that determine its instantiation.

§3.1 Triviality

(Improper) parthood is reflexive. Taking '$\leq$' to designate improper parthood, the following holds: $\forall$x(x$\leq$x). Partialhood is obviously not reflexive. After all, the second slot of partialhood is saturated only by kinds, so no non-kind x is such that $\Pi(x, x)$. What about kinds: for an arbitrary kind K is it a partial K? Hardly! Consider the kind *homo sapiens*. That kind is not a partial *homo sapien*.

Is partialhood irreflexive, i.e. is it the case that $\forall$x$\neg\Pi$(x,x)? Given our earlier observations about the varieties of entities that can saturate the first slot of the partialhood relation, and the variety of kinds that can saturate the second slot, we should hesitate to endorse the irreflexivity of partialhood. In fact, there are several types of potential counterexample to irreflexivity. As we have already seen, for any kind K, if x is a member of K it follows that x is a partial K. So, if there is any kind K that is a member of itself, then Π(K,K). The kind *kind* is a plausible example of a kind that is a member of itself.

That partialhood is not reflexive and also not irreflexive is not particularly illuminating. What's more illuminating is that there is a somewhat nearby property that partialhood does have. When it comes to parthood, identity is a limiting case: everything is part of itself. Partialhood, being a cross-categorial relation, doesn't share this property. However, if we take a half-orange to be a paradigmatic instance of being a partial orange, then we can take a whole orange to be a limiting case of being a partial orange. In other words, we're not taking partial to express *merely partial* but, rather, *at least partial*.[25] This coheres with our official definition. Obviously, any orange can affect orange counts. We can name this property **Triviality**, the idea being that oranges are trivially partial oranges:

Triviality: $\forall$x$\forall$K(x$\in$K $\rightarrow$ Π(x,K))

§3.2 Absorption

Most endorse the transitivity of parthood: $\forall$x$\forall$y$\forall$z((x$\leq$y $\wedge$ y$\leq$z) $\rightarrow$ x$\leq$z).[26] Partialhood is not plausibly transitive, and taking it to be transitive wouldn't

[25] This language suggests a way to explain away the counterintuitiveness of claiming that an orange is a partial orange: it arises from scalar implicature. Given I've officially eschewed 'partial' in favour of my official definitions, the counterintuitiveness doesn't matter for our purposes anyway.

[26] Some, of course, reject transitivity. See Johansson (2004) and Varzi (2006) for discussion.

be particularly illuminating: in general, more specific kinds are not members of less specific ones. Furthermore, in the cases in which kinds are members of other kinds, transitivity plausible fails. Orange-kind is a member of the second-order kind *kinds of fruit*, but a partial orange is not a partial kind-of-fruit.

One intuition underlying transitivity is that parthood transmits upward. So, if x has parts zz, then any y that has x as a part will also have zz as parts, and so on. Transposing this to kind-membership, it is plausible that membership transmits upwards in a similar way: if x is a member of a kind K, and K is a sub-kind of K′, then x is a member of K′. So, for instance, Fluffy is a cat, as well as a mammal. In fact, we may use a modalized version of this relation to give one (fairly weak) understanding of the sub-kind relation: K is a sub-kind of K′ just in case, necessarily, if x is a member of K then x is a member of K′.

Partialhood, the thought goes, transmits up the taxonomic hierarchy just like kind-membership, e.g. a partial blood orange seems to also be a partial orange, given that blood-orange-kind is a subkind of orange-kind. We'll call the principle based on this thought **Absorption**, the idea being that a super-kind absorbs the partial members of its sub-kinds. In stating it, we'll take 'SK' to express the sub-kind relation.

Absorption: $\forall x \forall K \forall K'(\Pi(x,K) \wedge SK(K,K')) \rightarrow \Pi(x,K')$

Absorption is not obvious. There are two main types of worrying cases, though, ultimately, neither undermines the principle.

The first type of worrying case comes from the fact that an entity may bear a very different relationship to a sub-kind than to its super-kind. For instance, consider the kind *house* and the kind *thing*, where we're understanding *thing* in its most general sense. Necessarily, every house is a thing, so on our skeletal understanding of the *sub-kind* relation, *house* is a sub-kind of *thing*. A half-built house is a partial house, but not a house. According to **Absorption**, it is a partial thing as well. In addition, a half-built house is, of course, a thing. So, a half-built house is a merely partial house, but a thing. The half-built house bears a very different relation to the two kinds.

This is not a counterexample to **Absorption**. After all, according to **Triviality**, every thing is a partial thing. So, **Absorption** is compatible with entities bearing very different relations to sub-kinds and their super-kinds. We would have a counterexample to **Absorption** if we had a partial member x of kind K, and a super kind K' whereby x differed with regard to K and K′not by being a non-K and a K′, but, rather, by being a non-K and a non-partial K′. The *house*/*thing* example is not such a case.

The second type of worrying case comes from the fact that an arbitrary kind K will stand in the sub-kind relationship (as we're understanding it) to many unrelated kinds. Consider, for instance, the kind *house or rabbit.* If this kind exists, it is a super-kind of *house.* According to **Absorption**, any partial *house* is a partial *house or rabbit.* However, the worry continues, this is an odd result.

There are a number of responses to this second worry. Response 1: the fact that is odd to claim that something is a partial house or rabbit may be plausibly explained pragmatically. Response 2: when it comes to gerrymandered and/or disjunctive kinds we usually don't have especially clear intuitions that can be used to support this worry. Response 3: insofar as one is moved by this example, there are more restrictive notions of kinds, as well as less skeletal notions of the *sub-kind* relation that one may wish to adopt. If, for instance, we adopt a notion of kinds on which disjunctive categories do not constitute kinds then the worry disappears.

§3.3 The Partialhood-to-possibility Principle

If I'm drawing three squares on the board and I stop after finishing two and completing half of the third, then (6) is true.

(6) Two and a half squares are on the board.

If, however, I'm attempting to draw a round square (perhaps under the sway of some bad philosophy) and I stop after drawing the bottom-half of a circle, then (7) is not true.

(7) A half round-square is on the board.

This suggests that, at least in this case, possibility constrains partialhood. This suggestion can be captured in the Partialhood-to-possibility principle.

The Partialhood-to-possibility Principle (PPP): $\forall x(\Pi(x,K) \rightarrow \Diamond\exists y(Ky))$

We'll understand the possibility operator in **PPP** as expressing metaphysical possibility. Is **PPP** plausible? Just as with **Absorption**, I can't give conclusive proof of such a generalization. However, I think **PPP** is on shakier footing than **Absorption** because there is a more compelling potential counterexample.[27]

Imagine that I'm a logic student proving things. I prove a couple of fairly mundane theorems, and then, thinking that I've had a brilliant insight, I attempt to prove the negation of Fermat's last theorem. Given that Fermat's last theorem is true, I can't prove its negation. However, if I've written out a number of steps (alongside my two successful proofs), then there is a temptation to treat (8) as true.

(8) A half-proof of Fermat's last theorem is on the board.

The only witness for (8) is the doomed attempt to disprove Fermat's last theorem. How can this attempt constitute a half-proof given that it aims at the impossible? I don't have a decisive response to this sort of example, but I think there are lots of options for the proponent of **PPP**. First, as noted by Hawthorne and Magidor (2018), 'proof' does not always mean successful proof.[28] In fact, it is natural in some contexts to say that a student wrote three proofs, but only two succeeded. If that's the case, then the example presents no difficulty for

[27] Thanks to Jared Henderson for this example.

[28] They emphasize that this is just one instance of a larger pattern: the same remark applies to 'memory' and 'explanation'.

PPP. After all, the steps certainly constitute a possible failed proof which, on the view we're considering, is a type of proof. Second, note that the steps may possibly constitute some sort of successful proof, even if they don't possibly constitute a successful proof of the negation of Fermat's last theorem.

There's more to be said about this example, but I won't wade deeper into the dialectic. The reason, which will become clear in §5.3.5, is that the mere *prima facie* plausibility of **PPP** gives us some reason to utilize partialhood in our semantics for the progressive. For those purposes, the important claim is not that **PPP** is true but, rather, that it is supported (or undermined) by precisely parallel considerations that support (or undermine) a similar principle governing the progressive.

§3.4 Externality and Difference-Makers

Partialhood is an external relation in the sense of Armstrong (1978: 84-5): its instantiation is not determined by the intrinsic natures of its relata. This is not surprising. After all, for myriad kinds K, membership in K is not determined by the intrinsic properties of its members. Given **Triviality**, the externality of kind-membership will entail the externality of partialhood. However, the externality of partialhood goes beyond the externality of kind-membership in the case of mere partials. Just what extrinsic factors determine whether partialhood is instantiated? For the reasons given in §2, I won't give necessary or sufficient conditions, but there are several sorts of cases that allow us to identify factors that differentiate instances of mere partialhood from non-instances. I'll call these factors 'difference-makers'.[29]

How can we identify difference-makers for partialhood? Consider the linguistic notion of a minimal pair. Minimal pairs are words/phrases that differ only in the element to be studied. For example, 'zeal' and 'seal' are a phonological minimal pair. We can extend the notion. Metaphysical minimal pairs are pairs of situations that differ only in an aspect to be studied. We can give several metaphysical minimal pairs that allow us to identify difference-makers for partialhood. In each, we have two situations differing only in a feature F, such that partialhood is instantiated in one but not the other. F, then, is a difference-maker for partialhood. F, however, need not provide a sufficient condition for the instantiation of partialhood—it may only be sufficient when combined with other features shared by the pair. F will also not generally provide a necessary condition: after all, if there are several different difference-makers for partialhood then none will be necessary.

The first difference-maker can be identified by focusing on a case already discussed on 2.2. Recall that in **Remainder**, a half-orange is what remains after destroying the other half. In §2.2 the importance of this case was in demonstrating that something could be a partial K without having a complete K as its telos. Now, we can focus on a different aspect of similar cases. There

[29] Apologies for co-opting a term used to mean something slightly different in the causation literature.

are pairs of cases in which x and y are intrinsic duplicates, but x is a partial K and y is not because x is what remains of a K while y is not.

> **House-Remainder**: Half of a Victorian house is destroyed. The house-remainder is on a street with two complete Victorian houses. 'Two and a half Victorian houses are on the street' is true.

> **Duplicate-remainder**: An anglophile constructs a perfect duplicate of a Victorian house, albeit one constructed more than a century after the Victorian era. Sadly, half of the house is destroyed. The house-remainder is on a street with two complete Victorian houses. 'Two and a half Victorian houses are on the street' is false.

The difference between **House-Remainder** and **Duplicate-Reminder** lies in extrinsic features of the building remainder. In the former, the remainder *was part of a Victorian house*, while in the latter it wasn't. The lesson is that, at least for these cases, the fact that an object x *was part of a K* is a difference-maker for being a partial K. And, of course, once we appreciate this, it is easy to construct similar cases in which this extrinsic property is the difference-maker for partialhood. As I already stressed, the extrinsitcality of partial kind membership is no surprise given the extrinsiticality of kind membership more generally. For something to be a member of the kind *Victorian house* it must have been built during the reign of Queen Victoria. Given that the duplicate is not part of a house that was built during this period, it is not a partial Victorian house.

Even if being aimed at a complete K (where aiming is fleshed out in terms of either modality or telicity) cannot provide the basis for a reduction of partialhood, it can provide us with a difference-maker.[30] Consider **Trajectory** and **Accident**.

> **Trajectory**: I'm a sculptor, specializing in busts. I have completed two, and I'm midway through making a third. I've shaped my clay into a mass that to the untrained observer is still formless, but is a crucial step on its way to becoming a finished bust. 'Two and a half busts are complete' is true.

> **Accident**: After a particularly violent storm, two busts wash up on shore. Due to meteorological contingencies, the storm has shaped some clay from the sea into an object that is intrinsically identical to the formless mass from **Trajectory**. Nonetheless 'two and a half busts are on the shore' is false.

Being aimed at a complete K is a difference-maker, but being aimed at a complete K is surely not sufficient for partialhood. There are cases in which an

[30]Importantly, note that trajectories may be fairly coarse grained. An object may be on the trajectory to become a statue even it is not on the trajectory to have a perfectly specific weight or height.

agent aims at producing a K, but it is not in their control. In such cases, we often judge that the product of their efforts is not a partial K. In fact, we've already seen one such case: the attempted production of a round square. By contrast, when producing a K is in an agent's control, the product of their (incomplete) efforts is often judged to be a partial K. Control, then, provides us with another difference-maker:

> **Control**: To distract myself from work, I've taken up a hobby: I paint landscapes of the Rocky Mountains. Today I completed two landscapes, and I'm halfway through the third. 'Two and a half landscapes (of the Rockies) are on the table' is true.
>
> **Lack**: My young daughter likes to mimic me and play with my paint supplies. One day, a combination of finger-painting and paint-slinging leads her to create a canvas that is intrinsically identical to my half-landscape. Nonetheless, when she puts it on the table next to two of my complete landscapes, 'Two and a half landscapes (of the Rockies) are on the table' is false.

In sum, we've identified three difference-makers for whether x is a partial K:

- Whether x was part of a K.
- Whether x is on the trajectory to become a K.
- Whether it is within control of a relevant agent whether x will become a K.

To stress again, none of these provide necessary or sufficient conditions for partialhood. They do, however, partially reveal the nature of partialhood by identifying features crucial to its instantiation. This will be important in evaluating applications of partialhood. If one hypothesizes that a linguistic item expresses partialhood, or that partialhood plays a role in a metaphysical theory, then we'd expect to find these types of cases.

Do these three difference-makes exhaust all possible partialhood difference-makers? That is extremely doubtful. Imagine attempting to list all of the extrinsic features relevant to kind membership. The diversity of kinds as well as the diversity of possible conditions makes it implausible that there will be a single relatively tractable list. Nonetheless, we can identify extrinsic features that cover a number of cases. Consider, for instance, the property of being *maximal*, where a kind K is maximal just in case large (enough) parts of a K are not themselves K. According to Sider (2003), this is an extrinsic condition on kind membership for a variety of familiar kinds. What we're doing with partialhood is similar: we are identifying difference-makers for a wide variety of kinds.

§4 Using Partialhood Part 1: Mere Partials and the Mass/Count Distinction

Lots of kinds have merely partial members. Houses, oranges, statues, and cars, to re-use our examples, all have partial members that aren't also their members. Not all kinds do, though. Is there merely partial water, i.e. partial water that is not water? It seems doubtful. Are there merely partial humans? Again, I doubt it. Kinds that don't admit merely partial members have the following property, called **NMP** (for no mere partials):[31]

NMP: $\forall x(\Pi(x,K) \rightarrow x \in K)$

There are two different sorts of kinds that exemplify **NMP**. On the one hand, there are kinds whose members are arbitrarily divisible into further instances of the kind. Water is one such kind. If you take a quantity of water, a puddle for example, and divide it in half, you have two quantities of water.[32] And you can repeat this trick. In 4.1 I'll discuss such kinds, and argue that identifying them allows us to articulate a revised version of distributivity that can be used to identify the metaphysical basis of the mass/count distinction. On the other hand, there are kinds whose members aren't divisible into further instances of the kind, yet still have no merely partial members. Intuitively, there's some *integrity* to being a human that prevents there from being a partial human: being human is an all-or-nothing game.[33] In 4.2 I'll briefly discuss such kinds.

§4.1 Distributivity and the Mass/Count Distinction

How can we account for the semantic difference between mass ('water', 'gold') and count nouns ('dog', 'chair')? A familiar thought is that mass nouns are *distributive* while count nouns are not.[34] Distributivity is standardly defined mereologically (Cheng 1973).[35] A kind K is distributive just in case any member of K is such that all of its proper parts are K:[36]

[31] If one takes all of these 'mere partials' to be members of their relevant kinds, then we'd have to invoke the distinction between whole members and non-whole members in order to formulate a workable version of **NMP**.

[32] I follow Cartwright (1970) in using phrases of the form 'quantity of MN' when I want to designate something in the extension of a MN while using count-terminology. Zimmerman (1995: 58) notes that this is imperfect, and I agree.

[33] As will become clearer in 4.2, this must be a different sort of integrity than Moltmann (1997) and (1998) has in mind. That notion applies to anything in the extension of a count noun, whereas my notion applies to only a subset of kinds designated by count nouns.

[34] Some use the term 'divisive' for the same property.

[35] Unlike Cheng, I define the distributivity directly for kinds rather than for predicates. Ultimately, then, distributivity picks out a metaphysical property that, I hypothesize, is reflected in the mass/count distinction. Given that it is a metaphysical property it also makes sense to ask what purely metaphysical work it could be put to. One natural thought it to use it to make the distinction between object kinds and stuff kinds. I won't consider that possibility here, though see Steen (2016) for a survey of ways to make sense of that distinction.

[36] '$<$' expresses proper parthood: $a<b$ iff $(a\leq b \wedge a\neq b)$.

K is distributive $=_{def}$ $\forall x \forall y((x \in K \wedge y < x) \rightarrow y \in K)$

We can then go on to hypothesize that mass nouns designate distributive kinds while count nouns designate non-distributive kinds. The well-rehearsed problem with this hypothesis is that it is false.[37] Every quantity of water has some parts that aren't water: hydrogen molecules. Water is hardly unique. Fruitcake, for example, contains raisins (Taylor 1977). Any workable form of distributivity has to allow for the fact that, even with kinds designated by paradigmatic mass nouns, decomposition often (always?) reaches a point at which there are minimal parts: parts such that their parts are no longer members of the kind. If we want a principle that captures the distributivity intuition—that mass kinds can be arbitrarily decomposed into further members—then we need a weaker principle.[38] A natural strategy is to further restrict the antecedent. For a decomposition of a member of K to yield further members of K, the decomposition must somehow be *significant enough*: hydrogen molecules don't to the job for water.[39] Partialhood can provide the restriction:

K is Π-distributive $=_{def}$ $\forall x \forall y((x \in K \wedge y < x \wedge \Pi(y,K)) \rightarrow y \in K)$

For a kind K to be Π-distributive, not all parts of its members must be Ks, but all of its parts that are partial Ks must also be Ks. Notice that Π-distributivity is entailed by NMP. Our more general hypothesis is that mass nouns (or at least certain mass nouns—more on that soon) satisfy NMP. In the context of saving distributivity, the important entailment of NMP is Π-distributicity.

Here's the intuition underlying the move to Π-distributivity. Recall that, by definition, partialhood is linked to counting and measuring. For something to be a partial K is for it to be able to affect K-counts or K-measures. Now imagine that we're counting quantities of water or measuring water. What can affect our count/measure? It seems that only quantities of water can. If you try to sub-divide quantities of water, as we sub-divide oranges, you are just left with more quantities of water, unless you are left with something that doesn't affect the count/measure at all, like a hydrogen molecule. To be sure, if a hydrogen molecule weighs something, it may contribute to a water-measure by being a part of some water. However, as we've defined partialhood, for an entity *o* to contribute to a K-measure M is for $M<o,K>$ to be greater than 0; in this case for the hydrogen molecule to measure greater than 0 liters of water (or whatever measure you choose). Hydrogen molecules can't do that.

[37] See Quine (1960), ter Mulen (1981), Koslicki (1999), Gillon (2002), Pelletier and Schubert (2002), and Champollion (2017) for discussions of distributivity that focus on the problem of minimal parts, or the observation that even with kinds designated by mass nouns, there seems to be a level of division below which kind membership is lost.

[38] This is somewhat controversial. Bunt (1985), for instance, thinks that while distributivity may be strictly speaking false, it is true as far as the semantics of mass nouns is concerned. Like Moltmann (1998) and Gillon (2002), I have a hard time making sense of this position.

[39] This 'significant enough' idea is familiar in the literature. Moltmann (1989), (1991), for instance solves the problem (for atelic predicates anyway) by requiring that the parts be relevant, where relevance is highly context-sensitive. One could see my proposal as a fleshing out of Moltmann's idea.

If one thinks that the kinds designated by mass nouns can be measured but not counted, then evaluating NMP for mass kinds will solely involve testing the measure-facts. However, I am skeptical. Already, I used 'quantity of water' interchangeably with 'water' and, given the right context, I think they can designate the same kind, such that x is a quantity of water iff x is water. ('Quantity of', of course, is context-sensitive and sometimes picks out contextually salient quantities, but I think it needn't always.[40])

My hypothesis, to an initial approximation, is that mass nouns designate Π-distributive kinds, and count nouns need not. Before we can test this hypothesis we need to address two complications which will lead us to refine the hypothesis. The first, which was in effect brought up in our discussion of NMP, is that some kinds designated by count nouns will satisfy Π-distributivity for the wrong reason: their instances don't admit any proper parts that are partials at all, so, trivially, all of their proper parts that are partials will be instances. The second is that the class of mass nouns is quite diverse and includes abstract kind-nouns ('honour') and mass nouns for intuitively discrete kinds ('furniture'), and one may be reasonably skeptical that there are informative true generalizations over all mass nouns.[41]

The first complication for the hypothesis that mass nouns designate Π-distributive kinds while count nouns do not is that there are cases where Π-distributivity may be trivially satisfied. Assume, for instance, that, necessarily, there are no merely partial humans. In that case, *human* is Π-distributive. Consider an arbitrary human: every one of its proper parts that is a partial human is also a human. There are none!

Notice that this complication arises already for distributivity, before we even move to Π-distributivity. Take any kind with only simple (partless) members. That kind will satisfy distributivity: consider one of its arbitrary members: every one of that members' parts will also be a member of the kind. There are none! The solution in both cases is to require that there are some entities that satisfy the antecedent. So, let's call a kind K *non-vacuously* Π-*distributive* just in case K satisfies Π-distributivity as given above, and, in addition, at least one of K's members has a proper part that is a partial K. Now we can articulate a refined version of our generalization: mass nouns are non-vacuously Π-distributive while, count nouns are not.

The second complication is that the class of mass nouns is diverse, and distributivity may be more plausible for concrete kind terms like 'water' than abstract kind terms like 'honour' and quantized mass terms like 'furniture'. I'll respond here by weakening the hypothesis. Rather than claiming it is true of every syntactically mass noun, I hypothesize that it is true of core examples

[40]See Chierchia (2010) for a semantic proposal for 'quantity of' that incorporates this context-sensitivity.

[41]Compare Koslicki (1999: 63): "a more fruitful approach would aim at isolating smaller subdivisions within the original linguistic classification, as guided by metaphysical considerations." Similarly, Zimmerman (1995: 53) and Steen (2016) avoid generalization about the metaphysics underlying all mass nouns, and, rather focus on only one specific kind (concrete mass expressions).

of mass nouns for concrete kinds. What are such 'core examples'? Following Chierchia (1998) and Grimm (2018) note that certain kinds are always designated by mass nouns when a language has mass nouns:[42] we'll take nouns for such kinds to be core examples. In Grimm's taxonomy, core examples will include mass nouns for naturally occurring liquids like 'water' and 'blood'. If there is any metaphysical basis for the mass/count distinction, core examples are a good place to find it: after all, they abstract away from interlinguistic contingencies and focus on the kinds that are universally designated by mass nouns.

The fact that we can't find a single semantic property that characterizes all syntactically mass nouns should not be surprising. After all, we know that semantic and syntactic categories don't correlate perfectly. 'Underpants', for instance, is syntactically but not semantically plural. In a similar vein, I think we should take our generalization to express a weaker claim about central cases that nonetheless captures some key intuitions about the mass/count distinction. Whether our hypothesis has additional utility (in, for instance, predicting facts about the distribution of mass nouns) is a topic for another day, but, to push the analogy, the fact that 'glasses' is not semantically plural does not lead us to think that our semantic characterizations of plurality are explanatorily impotent when it comes to the distribution of plurals.

Let's test this hypothesis against some central cases. Consider 'water'. According to our hypothesis, the kind designated by 'water' is non-vacuously Π-distributive. This, in turn, requires two things (1) Π-distributivity, and (2) non-vacuity: that at least one member of *water* has a proper part that is a partial K. I'll argue for these properties in reverse order.

Begin with non-vacuity. This requires that there is a quantity of water with a proper part that is a partial K. This is easy to demonstrate. Consider an arbitrary quantity of water: a particular puddle. The left half of that puddle is also a quantity of water. By **Triviality** that left half is a partial quantity of water. By our earlier hypothesis 'water' and 'quantity of water' designate the same kinds, so it follows that the left half is partial water. Similarly, consider a 1-litre quantity of water. The bottom half of that quantity is itself a quantity of water that measure .5 liters of water. Each case illustrates that some quantity of water has a proper part that is itself water, which by **Triviality** entails that some quantity of water has a proper part tat is itself partial water.

Next, consider Π-distributivity. Consider an arbitrary instance of water, which we'll call *a*. If *water* is Π-distributive, then each proper part of *a* which is partial water is such that it is water. We can demonstrate this by *reductio*. Assume *water* is not Π-distributive. There will then be a part of a quantity of water that is merely partial water. Mere partials, in turn, are in the extension of number phrases including terms expressing fractions. There will then be true readings of sentences expressing fractional quantification, e.g. '(exactly) two and a half quantities of water are on the road'. After all, the merely partial water will serve as a witness. However, when we use 'quantity' in the

[42] According to Lima (2014), in the Brazilian language Yudja all nouns can be used as count.

intended manner, there are never such truths. There cannot be two and a half quantities of water, when for something to be a quantity of water is for it to be water! There can be two or three. So, the assumption is false. It leads to the predictions that fractional quantifications could express true counts of quantities of water, but they can't. This argument does rely on an impossibility judgment. However, most find this judgment robust. We are unable to envision a scenario where such fractional counts are true, at least when we use 'quantity' in the intended manner. Switching to measuring, merely partial water would be something that could affect water-measurement without itself being some water. Apples provide us with such a case: a half-apple affects our measure, in pounds, of apples without itself being an apple. Again, we don't find this with water. These arguments are, of course, dependent on negative existential claims but I do not know of any *prima facie* compelling counterexamples.

Now let's turn to count nouns. On the hypothesis, they need not be non-vacuously Π-distributive. There are two ways to have this property. They could fail to be Π-distributive, or they could be vacuously Π-distributive. Nouns like 'orange', 'bagel', and 'house' fall into the first category. They are not Π-distributive. This is entailed by the fact that there are truths expressed by fractional quantification (on exact readings), like 'two and a half bagels/oranges/houses are on the table'. Other count nouns like 'dog' and 'human' are Π-distributive, but vacuously so. They have no proper parts that are partial members. We can see this by considering the two ways to be a partial kind-member. On the one hand, a proper part of a dog may be a partial dog in virtue of being a dog. Since dogs do not have other dogs as parts, dogs do not have partial dogs as parts in this way. On the other hand, a proper part of a dog may be a partial dog in virtue of being merely partial. This would entail that there are true readings of fractional quantificational claims like 'two and a half dogs are in the yard', but there aren't. Also, notice, that if one disputes this judgment then one would claim that 'dog' is not Π-distributive, which would still distinguish 'dog' from 'water'.[43]

§4.2 All-or-nothing Kinds

Our discussion of the mass/count distinction has led us to make a tripartite distinction between kinds. First, we have those that are non-vacuously Π-distributive. We've hypothesized that paradigmatic mass nouns designate kinds that fall in this category. Second, we have those that are not Π-distributive. Count nouns like 'orange' and 'house' designate kinds that fall in this category, as do many other count nouns designating artifactual kinds and some designating natural kinds (especially when the members of those natural kinds have

[43]Space precludes extensive comparisons between my solution to the problem of minimal parts and those of others. However, I think it is compatible with some of the more prominent solutions. I've already mentioned the potential compatibility with Moltmann's solution, and I see nothing incompatible between my solution and that of Champollion (2017: ch. 5). Another way to put this is that there are lots of different ways to weaken distributivity, and I suspect that several of them are true of (central) mass nouns. Which a theorist focuses on will depend on their interests and mine clearly differ from those of Moltmann and Champollion.

natural decompositions, like oranges). Third, we have kinds that are vacuously Π-distributive. I suggested that count nouns like 'human' designate such kinds. In this subsection I will investigate this third category—all-or-nothing kinds—so-called because because not only do they not have mere partials, they don't have proper parts that are partials at all.

I'll begin with a thought experiment that challenges the idea that *any* kinds are all-or-nothing kinds. The idea is that if we construct Alien Factory scenarios, even biological kinds like *rabbit* admit mere partials.

> **Alien factory**: a benevolent alien took an incognito vacation to the Canadian Rockies and was deeply impressed with the colour-changing white-tailed jackrabbits.[44] Not wanting to pluck any actual jackrabbit from its natural environment, the alien decided to make molecule-for-molecule duplicates of their favourite jackrabbit. The alien constructed a factory for this purpose, and has completed two duplicates, as well as being halfway done with a third. Pleased with his progress, the alien truly utters 'Two and a half rabbits are on the factory floor.'

Alien factory can be generalized to any physical kind, and this suggests that the best candidate kinds for being all-or-nothing kinds are, in fact, not Π-distributive. How should we respond to this thought experiment?

A first response is to take the thought-experiment at face value and claim that biological kinds are not Π-distributive. This response incurs an explanatory debt: in most familiar contexts it is bizarre to speak of fractional members of biological kinds. How can this be explained? We could try to invoke pragmatic considerations, but I don't know of any that wouldn't overgenerate.

A second response, which I take to be the most promising, is to take 'jackrabbit' to designate a different kind in **Alien Factory** contexts than in ordinary contexts. This response can be buttressed by considering that in **Alien Factory** we seem to be thinking of white-tailed jackrabbits as a purely physical kind, with membership conditions given in terms of their specific physical constitution, while in ordinary contexts we think of them as a biological kind. We can bring this out by thinking of cases where the two come apart. Imagine that the alien factory produces a duplicate of a dead jackrabbit. In **Alien Factory** it is perfectly intuitive to claim that there is a (completed) jackrabbit on the factory floor, though in ordinary contexts it is natural to deny this.[45]

Adjudicating between these responses would take us too far afield. Assuming, though, that a version of the second response could be made to succeed,

[44] These are a real: they are white in the winter and brown the rest of the year!

[45] A third response is to take the partialhood relation to have another argument place for something like standards. I'm skeptical of this response because one can account for the context-sensitivity of counting with fractions by taking into account independently motivated context-sensitivity. This obviates the need to introduce an additional parameter. In particular, sentences that express such counts all contain nouns that restrict the quantifiers, and, following Stanley and Szabó (2001), we have independent reason to think that these nouns are context-sensitive.

the result is that count nouns are, in fact, bifurcated into two classes: those that designate all-or-nothing kinds and those that don't. Here's an interesting and difficult question: what does this distinction track? One doomed idea is that it tracks the natural/non-natural kind distinction. We've already seen that this is false: *orange* and *apple* are plausibly natural kind terms but they do not designate all-or-nothing kinds. Interestingly, the distinction seems to track the distinction between fauna and non-fauna. I'm not sure why this is but it does suggest that partialhood is more entwined with distinctions among kinds than merely tracking the mass/count distinction. Properly understanding this would require surveying a host of kinds to come up with a more comprehensive list of those that are all-or-nothing and those that aren't. I leave that ambitious task for another day.

§5 Using Partialhood Part 2: The Progessive

§5.1 P = Π

Sentences in the progressive express that events are underway, or in progress.

(9) Elise is building a house.

(10) John is swimming.

(9) expresses that house-building is underway, and Elise is the builder; (10) expresses that swimming is underway and John is the swimmer. Following Landman (1992) and Higginbotham (2004), the progressive can be understood as expressing a relation between a event-token and an event-kind. (9), for instance, can be understood in terms of a relation between a particular building-in-progress, and the event-kind *building a house*.[46] On this view, (9) has the following semantic form:

(11) $\exists$e PROG(e, $\lambda e'$.Build(Elise, house, e'))

To a reasonable approximation, we can gloss (11) as *there exists an event token, such that it stands in the PROG relation to the following event-type: events in which Elise built a house.*[47] The task of analyzing the progressive now becomes the task of understanding the PROG relation. Familiar analyses are modal. Dowty's (1977) influential view, for instance (and to an approximation)

[46] This is the most straightforward way to utilize partialhood in understanding the progressive, and it has the virtue of being easily incorporated into a compositional theory. That said, one could treat the progressive as a sentential operator on the model of Dowty (1979) and understand the sentential operator in terms of partialhood. A sketch: Prog(ϕ) $\leftrightarrow$ $\exists$e(Π(e,ϕ)). The sketch identifies (at least some) propositions with event-kinds. One could reject this as long as an alternative mapping from propositions to event-kinds is provided.

[47] Notice that I'm taking the λ-abstract in (11) to designate an event-kind. Perhaps, however, it merely designates a property and we need another operation to map the property to the kind (cf. Cheirchia 1998). If so, the only complication to (11) is that we'd have to add that operator.

is that an event token e stands in the PROG relation to an event type e′ just in case if e were to continue unimpeded it would become an e′. In the remainder of this section, I'm going to defend an alternative to modal views: that PROG just is partialhood.[48]

PROG = Π.

For short, I'll dub this hypothesis 'P = Π'.[49] I'll defend the hypothesis in two ways. In giving our metaphysics for partialhood, we identified several difference-makers for partialhood. If P = Π, then PROG will have these same difference-makers and be instantiated in the same sorts of cases. In 5.2 I'll argue that this prediction is accurate. We also identified some central logical properties of partialhood. If P = Π, we'd expect these logical properties to capture entailment patterns characteristic to PROG. In 5.3, I'll argue that this prediction is accurate.

§5.2 The Case from Cases

Completed partials gave us a case that presents a problem for any modal or telic analysis of partialhood. The case shows that there can be an entity e that is a partial K, despite the fact that e is neither telicly nor modally directed at K. In our specific case, we considered a factory that deliberately produces half-bagels, without in any sense aiming at complete bagels. If P = Π, we'd expect to find similar cases in the domain of events that are reflected in the truth-conditions of progressive sentences. In other words, we'd expect to find an e and K such that PROG(e,K), and there is no sense in which e is aimed at a K. In fact, we do find such events:

> **Overstuffed Sandwich**: The caterers at the conference were overenthusiastic and made the sandwiches too large. I take one sandwich for lunch, but I have no intention of eating the entire thing. However, I do eat some of it.

In **Overstuffed Sandwich**, my sandwich-eating event is analogous to the half-bagel in **Completed Partials**. Just as the half-bagel is not aimed at a complete bagel, my eating-event is not aimed at a complete sandwich-eating. Nonetheless, (12) is true in **Overstuffed Sandwich**, which is analyzed as (13).

(12) I am eating my sandwich.

(13) $\exists$e PROG(e, λe′.eating(me, my sandwich, e′))

[48] I'm inspired by the mereological analysis of the progressive offered in Bennett and Partee (1972). On my view, they were close to correct but they used parthood where they needed partialhood.

[49] I've emphasized that this hypothesis is not a modal analysis of the progressive. What is its relationship to other non-modal analyses? Perhaps the most influential non-modal analysis utilizes event semantics and a primitive event predicate 'HOLD' (Parsons 1990 and Forbes 2006). We may be able to bring these analyses together by analyzing 'HOLD' in terms of partialhood.

Recall that we identified three difference-makers for partialhood. A difference-maker was not a necessary or sufficient condition (I remain skeptical of identifying those). Rather, when we have two cases that differ only with regard to a feature F, and partialhood is instantiated in one but not the other, F is a difference-maker for partialhood. If P = Π, PROG will have the same difference-makers.

The first difference-maker for partialhood is that x was a part of K. In other words, we had two cases featuring intrinsically identical objects o and o′, such that the only relevant difference between the cases was that o was part of a K, while o′ was not, and Π(o,K) but not Π(o′,K). Similar cases arise in the case of events:

> **Game**: A baseball game is played from 4-7 PM. From 4-4:03 PM, there's a sub-event e of the game. During e the players undertake some familiar game-related actions such as throwing a pitch and swinging at it. It is true at 4:02 PM that the players are playing a baseball game. It is also true that, at 4:02 PM the players are playing a baseball game.
>
> **Warmup**: Some players are warming up before a game that will start at 7. From 4-4:03 PM they undertake some familiar game related actions such that the event e' that occupies that interval is intrinsically identical to e (as described in **Game**). However, it is false at 4:02 PM that the players are playing a baseball game. It is also false that, at 4:02 PM the players are playing a baseball game.

Just as in **House Remainder** and **Duplicate Remainder**, we have two intrinsically identical objects that differ only in their mereological relations to a K such that one is a partial K and the other is not. As predicted, then, the difference maker for partialhood is a difference-maker for PROG.

There is a complication worth noting. The extrinsic difference in **House Remainder** and **Duplicate Remainder** had to do with past mereological relations: one object *was* part of a Victorian house while the other *wasn't* (and isn't). **Game** and **Warmup** are different: one event *is* part of a game while the other isn't. This disanalogy, however, flows from a more general disanalogy between objects and events: unlike objects, an event exists whenever any part of it exists. If that general disanalogy holds, then we'd expect the same disanalogy at in our cases. (And, of course, if it doesn't then it doesn't afflict our cases.)

The second difference-maker for partialhood is that x is on the trajectory to become K. Here are event-centric cases:

> **House-building**: I have laid down a building-foundation and, if I continue unimpeded, I will produce a house. Just after the foundation is laid, it is true that I am building a house.
>
> **Garage-building**: I've laid down a building-foundation, in fact this event is intrinsically identical to the foundation-creation event in

> **House-Building**. However, if I continue unimpeded, I will produce a garage rather than a house. It is false that I am building a house.

Note that **House-building** and **Garage-building** need not differ in terms of my intentions. Perhaps I am blindly following instructions being shouted at me and I have no intentions whatsoever about the fruits of my labour.[50] Nonetheless, it is true in **House-building** and false in **Garage-building** that I am building a house.

Intentions, of course, may matter for the instantiation of partialhood and PROG. The third difference-maker for partialhood is that it is in control of the relevant agent whether x will become a K. Variants of **Control** and **Lack** demonstrate that this is also a difference-maker for PROG.

> **Painting-Control**: I've taken up a hobby: I paint landscapes of the Rocky Mountains. Today I'm halfway through the a painting, and it is well within my power to complete the landscape. It is true that I am painting the Rocky Mountains.

> **Painting-Lack**: My two-year old daughter likes to mimic me and play with my paint supplies. One day, a combination of finger-painting and paint-slinging leads her to create a canvas that is intrinsically identical to my half-landscape. Nonetheless, it is false that she is painting the Rocky Mountains.

This series of cases presents strong *prima facie* evidence for P = Π. If P = Π, we'd expect to find the same types of cases in the event realm (modulo modifications due to the differences between objects and events) that we found in the object realm when discussing partialhood. That is, in fact, what we found. Of course, this is hardly decisive. There are a host of semantic properties that have been attributed to the progressive, and if P = Π, we'd expect that our account of partialhood will predict these properties. I'll turn to these next.

§5.3 Entailments and other Predictions

If P = Π, the logical properties of partialhood will be reflected in the semantics of the progressive, and *vice-versa*. In this subsection I'll argue that this is the case.

[50] The fact that trajectories may be coarse-grained, which I emphasized in fn. 29, may account for another puzzling fact about the progressive stressed by Bonomi (1997): that the progressive fails to distribute over disjunction. It may be true, for instance, that I am going to Boston or New York, without it being true that I am going to Boston, or being true that I am going to New York. Similarly, it may be true that I am building a statue between three and four feet high, without it being true for some more specific height that I am building a statue of *that* height. This can be explained by the fact that the height-trajectory itself is not so fine-grained.

§5.3.1 The Imperfective Paradox

Perhaps the most familiar challenge in giving a semantics for the progressive is accounting for the fact that progressive sentences do not entail their perfective counterparts, at least when the verb is telic. For instance, (14) does not entail (15)

(14) Elise was building a house.

(15) Elise built a house.

Assuming P = Π, and ignoring tense for the moment, our analysis of (14) is given in (29), which should not entail (15).

(16) $\exists e\Pi(e, \lambda e'.\text{build}(\text{Elise, house, e}'))$

Given that there can be a partial K without there existing any K whatsoever, it is easy to see that (14) does not entail (15).[51]

§5.3.2 Perfective to Progressive

Though progressive sentences with telic verbs do not entail their past-tensed perfective counterparts, the reverse does seem to be true (Szabó 2004). (15) does seem to entail (17)

(15) Elise built a house

(17) Elise was building a house.

Getting back to ignoring tense, we analyze (15) as (18)

(18) $\exists e(\text{Build}(\text{Elise, house, e}))$

Recall that partialhood satisfies **Triviality**: every K is a partial K. So, every Elise house-building event is a partial Elise house-building event:

$\forall x(x \in \lambda e'.\text{build}(\text{Elise, house, e}) \rightarrow \Pi(x, \lambda e'.\text{build}(\text{Elise, house, e}))$

It follows, then, that (15) entails (19)

(19) $\exists x\Pi(x, \lambda e'.\text{build}(\text{Elise, house, e}))$

(19) is, of course, just our analysis of (17), so (15) entails (17).

Something may strike one as odd about this account of the (past) perfective to (past) progressive entailment. Our intuition that (15) entails (17) may come from the fact that we think of house-building processes as unfolding over time,

[51] Recall that I have been assuming that there are mere partials, i.e. partial Ks that are not K. If this is not true then a progressive sentence does entail its perfective counterpart. However, on such a view there is a substantial distinction between being a K and being a full/complete/finished K, so while (14) would entail (15), it would not entail 'Elise finished building a house'/'Elise completed building a house.'.

and that if Elise built a house, there must have been some pre-completion time during which she was building a house. Our explanation does nothing to vindicate this thought. After all, it is the entire build-event, including its moment of completion, that witnesses the truth of (17).

There are two ways to respond to this worry. The first is to note that the truth of (17) will be overdetermined whenever there is a partial build event that precedes the completion of the building. I'm far from averse to invoking a second explanation for the entailment! The second is that taking a complete build-event to witness the (past) progressive is desirable independently. To see why, we can consider a momentary event. Imagine that a wizard snaps her finger, and Elise finds herself in sitting position at time t, then the same wizard snaps again and Elise is not is a sitting position at t+1. At t, (20) is true.

(20) Elise is sitting.

This shows that progressive sentences can be true even if the events that make them true do not unfold over time. It also shows that events at their final moments can witness the truth of progressive sentences.

§5.3.3 Subkind to Superkind

In general, if I am ϕ-ing, and ϕ-ing is a subkind of/way of ψ-ing, then I am also ψ-ing. This is demonstrated by the following pairs.

(21) I am walking across the street.

(22) I am moving across the street.

(23) I am boiling an egg.

(24) I am cooking an egg.

Recall that, according to **Absorption**, partialhood transmits up the taxonomic hierarchy. We can then predict the inference. (21) is analyzed as (25) and (22) is analyzed as (26). As long as λe′.walk-across-the-street(Me, e) is a sub-kind of λe′.move-across-the-street(Me, e), **Absorption** predicts that (25) entails (26), and when we add P = Π, we predict that (21) entails (22).

(25) $\exists$xΠ(x, λe′.walk-across-the-street(Me, e))

(26) $\exists$xΠ(x, λe′.move-across-the-street(Me, e))

§5.3.4 Progressive-to-Perfective for Atelic Predicates

When the verb in a progressive sentence is telic, the sentence does not entail its perfective counterpart. However, when the verb in a progressive sentence is atelic, it does. (27) entails (28).

(27) Emma was running.

(28) Emma ran.

How can we secure this entailment? Recall that in §4.1 we hypothesized that mass nouns designate kinds that satisfy **NMP**. A familiar claim is that there are analogies between the mass/count distinction in the nominal domain and the telic/atelic distinction in the verbal domain.[52] In particular, there is thought to be an analogy between mass nouns and atelic verbs, as well as between count nouns and telic verbs. The intuition is that atelic verbs like 'walk' designate events that are internally homogeneous, much like 'water' designates entities that are internally homogeneous. By contrast, telic verb phrases like 'build a house' are internally heterogeneous, and they are externally bounded. If our hypothesis about the mass/count distinction is correct, then the insight should carry over to the telic/atelic distinction. Here, then, is another hypothesis: atelic verbs designate non-vacuously Π-distributive (event) kinds, while telic verbs do not.

Given this hypothesis, our analysis predicts that (27) entails (28). Here's how. We analyze (27) as (29) (again temporarily ignoring tense):

(29) $\exists e\Pi(e, \lambda e'.\text{run}(\text{Emma}, e'))$

Since $\lambda e'.\text{run}(\text{Emma}, e')$ is an event-kind designated by an atelic verb, it satisfies **NMP**:

$$\forall x(\Pi(x,\lambda e'.\text{run}(\text{Emma}, e')) \rightarrow x \in \lambda e'.\text{run}(\text{Emma}, e'))$$

(29) and this instance of **NMP** jointly entail (30).

(30) $\exists e(e \in \lambda e'.\text{run}(\text{Emma}, e'))$

(30), is just our analysis of (28), so we have secured the entailment from (27) to (28).

In fact, when a predicate is atelic we have an equivalence between the progressive and the corresponding clause. We've already seen that (29) entails (30), and it is easy to see that (30) entails (29). By **Triviality**, every member x of any kind K is such that Π(x,K). So, from (30) and **Triviality**, (29) follows.

Of course, (29) and (30) leave out the tense in (27) and (28) so the former are not adequate analyses of the latter. To give working anlayses, we'll take 'τ' to express a function from events to the intervals at which they occur. Analyzing past tense quantificationally, and taking 'p' to be a free variable that designates the time-of-utterance, (31) and (32) are the proper analyses of (27) and (28). Assuming that atelic predicates satisfy NMP, (31) and (32) are mutally entailing as well.

(31) $\exists e\exists t(t<p \wedge \tau(e)=t \wedge \Pi(e, \lambda e'.\text{run}(\text{Emma}, e')))$

(32) $\exists e\exists t(t<p \wedge \tau(e)=t \wedge e \in \lambda e'.\text{run}(\text{Emma}, e'))$

Even though (29) and (30) leave out the tense in (27) and (28), (29) and (30) are perfectly good analyses of their untensed counterparts (33) and (34).

[52] Bach (1986) is the seminal discussion. See Rothstein (2004: ch. 1) for an overview and references, and Champollion (2017) for a recent attempt to characterize these analogies.

While (33) and (34) cannot grammatically occur unembedded in English, they can sometimes occur as complements of some perceptual verbs. Given that such clauses can only occur embedded, our access to their truth-conditions is indirect, as emphasized by Zucchi (1999).

(33) Emma running.

(34) Emma run.

This upshot of all of this is that if P = Π, we predict that progressive sentences with atelic verbs entail their non-progressive counterparts by virtue of a strong claim: that the two are equivalent. This yields a worry: that the equivalence doesn't always hold.[53] We can't test this directly for (33) and (34) because they cannot occur unembedded, but we can try to test the equivalence by embedding them. Switching examples (to strengthen the worry), consider (35) and (36).

(35) I saw Emma waltzing.

(36) I saw Emma waltz.

Since, on our view, 'Emma waltz' and 'Emma waltzing' are equivalent, it may seem that we predict the equivalence of (35) and (36).[54] However, this strikes some as wrong. Imagine that I glance Emma raising her leg in the very first moment of performing a waltz. It seems that (35) is true. However, some intuit that (36) is false, because I did not see Emma complete a full waltz-step.

While I share these intuitions, I don't think they undermine the equivalence between progressive statements and their non-progressive counterparts, when the verbs are atelic. To see why, first note that we should distinguish between clauses with no aspect and clauses with perfective marking. As Szabó (2004) emphasizes, this is supported by the fact that there is overt perfective marking in Slavic and Romance languages. Second, note that our intuitions vary based on the perceptual verb used (Szabó 2004: 52). Compare (36) to (37).

(37) I was watching Emma waltz.

(37) can be true even when I only see Emma raise her leg. This suggests that we cannot easily test the truth-conditions of the complements by considering such attitude verb constructions. Szabó (2004) has a hypothesis: such embedded clauses inherit their aspect from the verb in the main clause. Now we have an explanation for the inequivalence between (35) and (36) that doesn't rely on the inequivalence between 'Emma waltzing' and 'Emma waltz': in (36), 'Emma waltz' has a tacit perfective marker. Stepping back, this shows that we can't easily probe the truth-conditions of clauses like (33) and (34) by observing the behaviour of their embedded occurrences: those embedded occurrences, after all, will be aspectually marked by the main clause verb. This response does raise a

[53]Thanks to Nick Kroll for pushing this worry.

[54]Perhaps the equivalence is broken by the intensionality of 'saw'. I'll set aside that response, because I favour the one I give above.

question. What does perfective marking on an atelic verb do? I certainly don't have a fully worked-out story, but in the case of 'waltz' there is a natural thought: perfective marking imposes the requirement that a waltz-step is complete.

The entire discussion in this subsection proceeded from the hypothesis that atelic verbs designate non-vacuously Π-distributive kinds. The primary support for that claim was the well-known analogies between the mass/count distinction and the telic/atelic distinction, as well as that Π-distributivity makes sense of these analogies. Ideally, however, we'd ultimately support this hypothesis with more direct argumentation. Providing such argumentation is beyond the scope of this discussion, but I will briefly mention two sorts of relevant data. First, we may be able to directly use counting or measuring sentences to support the hypothesis. This is somewhat difficult, given that we can't obviously count or measure using verb-phrases. So, to test the hypothesis that a given verb designates a Π-distributive kind, we must find a noun that co-designates with that verb. The natural way to do that is to use gerundive nominalization or other nominalization devices.[55] Second, we may be able to rely on comparatives, which have truth-conditions sensitive to measure in order to test our hypothesis. For instance, if a sentence like 'a spoke longer than b' is made true partly by the contribution of event e, then we'd predict that e is a speaking event.[56]

§5.3.5 Progressive to Possibility

Finally, we have a somewhat more contentious clam: that there can only be a K event-in-progress, if it is possible for there to be a K-event:

$$\forall x(\mathrm{PROG}(x,K) \rightarrow \Diamond\exists y(Ky))$$

A host of cases support this generalization. Consider, for instance, the fact that—no matter how hard I try—it cannot be true that I am drawing a round square. Recall that we adopted **PPP** as a tentative hypothesis about partialhood. If P = Π, and **PPP** holds, then there is no trouble predicting the modal entailment of the progressive: after all, being a partial K entails the possibility of a K according to **PPP**.

PPP: $\forall x(\Pi(x,K) \rightarrow \Diamond\exists y(Ky))$

Of course, there is a complication. In my discussion of **PPP** I mentioned a potential counterexample: it seems that there can be a partial proof of an impossible claim, even if it is not possible that there is a proof of that claim. I also mentioned a number of responses to the counterexample.

In this subsection I will not attempt to argue for either **PPP** or for the claim that the progressive entails the possibility claim. Rather, I will make the more attenuated claim that the two theses are subject to the same *prima facie* counterexamples and the same responses. This, in and of itself, gives further evidence for the hypothesis that P=Π.

[55] See Liebesman (2015a) for arguments that such nominalizations co-designate with their verb counterparts.

[56] See Wellwood (2019) for discussion of how we measure certain events using comparatives.

Szabó (2008) has given the most familiar *prima facie* counterexample to the claim that the existence of a K event-in-progress entails the possibility of a K-event. (38), it seems, can be true. However, it seems to be impossible to fully enumerate the primes.

(38) Frank is enumerating the primes.

Even granting that it is impossible to enumerate all of the primes, there are plausible responses to Szabó's examples. As Kroll (2015: 2946) points out, 'enumerating the primes' may designate events in which *some* of the primes are enumerated rather than all. In fact, our intuitions make this plausible. As Kroll puts it (in reference to Frank's mother):

> Suppose you ask: "All of the primes or some of the primes?" If the mother were to respond with "All of the primes," it seems to me that she would be saying something false. On the other hand, if the mother were to respond with "Some of the primes," she would be saying something true (2015: 2946).

The basic move, then, is the claim that 'enumerating the primes' does, in fact, designate an event kind with possible instances. Notice that this is parallel to our defence of **PPP**. There, the worry was that there could be a partial proof of an impossible claim, even though it was impossible for there to be a proof of that claim. However, if 'proof' designates attempted proofs rather than successful proofs, the worry is dissolved.

A second sort of case given by Szabó (2008) and Wulf (2009) is intended to raise a problem for the claim that the existence of a K event-in-progress entails the possibility of a K-event. (39) (Szabó (2008)) and (40) (Wulf (2009)) exemplify this type of case:

(39) Antoni is building the cathedral.

(40) Shannon was making a pumpkin pie, but someone had already used the last can of pumpkin.

Focus on (39). The idea is that Antoni knows he can't finish building the cathedral. So, the objection goes, (39) does not entail (41).

(41) Antoni can build the cathedral.

There are numerous responses one can make to this objection. Mayerhofer (2014: 101) focuses on the fact that modals are context-sensitive. His thought is that when we are careful to specify the relevant features of the intended contexts, then either both (39) and (41) are true, or they are both false. For instance, we can imagine that Antoni is the only one working on the cathedral. In this case Mayerhofer claims that (39) no longer seems true. However, we can also imagine that Antoni is working with a team, in which case (39) and (41) both seem true.

Imagine a parallel case for partialhood: an architect wants to build a 321-story skyscraper (about double the size of the next-tallest building in the world).

However, given our current architectural information, we cannot build such a thing: it would collapse. Once our ambitious architect is 107 stories into the construction, (42) seems true. But, we may think, (43) is false.

(42) A third of a 321-story skyscraper has been built.

(43) A 321-story skyscraper can be built.

The same moves Mayerhofer made can be made here. We can describe the context in such a way that 'can' expresses a variety of flavours of modality. The responder can then argue that, on any disambiguation either (42) and (43) are both true, or they are both false.

Let me step back a bit. My aim in this subsection has been merely to argue that PROG and partialhood have the same status with regard to possibility entailments. That point doesn't tell us anything about the support for **PPP**. However, recall that **PPP** was explicitly stated in terms of metaphysical modality. Given that, only the proof case provides even a *prima facie* counterexample against **PPP**. Similarly, if our modal view of the progressive merely takes the K event-in-progress to be a metaphysically possible K, then only the *enumerating the primes* case provides even a *prima facie* counterexample. My tentative judgment, then, is that **PPP** does hold, and that this is seen both with partialhood and in the progressive. There's a residual question: would such a weakened understanding of the modal implications of the progressive satisfy those who give modal accounts of the progressive? I'm not sure of the answer, but in the absence of a particular proposal, it is impossible to evaluate its significance for the hypothesis that $\mathrm{P} = \Pi$.

§6 Conclusion

Partialhood is interesting. While it can't be given a mereological, modal, or telic reduction, we can still say lots of substantive things about it. Partialhood is useful. It can help us understand the metaphysical basis for the mass/count distinction and it allows us to give a simple and elegant account of the progressive, which doubles as a simple and elegant account of what it is for something to be happening. Other applications are doubtlessly possible. Wherever parthood doesn't quite work, we should investigate partialhood. All of this shows that I've established my overarching claim. Partialhood is important.[57]

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