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The Ramsey Test and the Indexicality of Conditionals — A Proposed Resolution of Gärdenfors' Paradox*

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Abstract

Working within the AGM approach, Peter Gärdenfors has proved that — given certain auxiliary assumptions — there are no non-trivial belief revision systems that satisfy both the Ramsey test and the Preservation condition. There are various ways of reacting to Gärdenfors' paradoxical result. Gärdenfors himself has described his result as a dilemma: we must give up either the Ramsey test or the Preservation condition. Isaac Levi has pointed to an implicit assumption in Gärdenfors' approach: conditionals express truth-value bearing propositions and are therefore eligible as members of belief sets. Here, we shall focus on another implicit assumption: a conditional sentence A > B expresses *one and the same* proposition relative to every belief state. We shall argue that contrary to the this assumption, epistemic conditionals are naturally interpreted as *context-sensitive:* an epistemic conditional expresses a proposition, but only relative to a belief state.
According to the approach advocated here, a sharp distinction is made between the semantic level containing propositions and belief states and the linguistic level containing sentences and sets of sentences. Belief revision is viewed as an operation on belief states; and it is primarily propositions rather than sentences that are accepted relative to belief states. At the semantic level, Gärdenfors' result applies to the Ramsey test in the form:

(P-R) The proposition $P \Rightarrow Q$ is accepted in the belief state S iff Q is accepted in the state $S*P$ which is the result of revising S with the proposition P.

Thinking of the connective > as corresponding to a binary operation $\Rightarrow$ on propositions is, however, tantamount to assuming that epistemic conditionals are context-independent. Once, we give up this assumption, we see that the proper way of formulating the Ramsey test for propositions is not (P-R) but rather:

(P-Ramsey) $P \Rightarrow_S Q$ is accepted in the belief state S iff Q is accepted in $S*P$.

But, as we shall see, (P-Ramsey) is compatible with Gärdenfors' postulates for belief revision. We also show how to accommodate the full Ramsey test for conditional *sentences*, thus allowing for unlimited iteration of conditionals. Our conclusion is that there is no real conflict between Preservation and the Ramsey test — once we take the context dependency of epistemic conditionals into account.

1. Introduction

The Ramsey test gives the following intuitive criterion for the rational acceptance of (indicative) conditionals:[1]

(RT) A conditional proposition "If A, then B" is (rationally) accepted in a given state of belief S just in case B should be accepted if S were revised with A as a new piece of information.

That is, in order to decide whether to accept a given conditional, I ask myself whether it would be reasonable to accept the conclusion, if I learned that the antecedent was true. If the answer is yes, I now have good reasons to accept the conditional. Otherwise, I should reject it.

To give an example of the application of the Ramsey test, consider the conditional:

(1) If the butler did not do it, then the gardener did.

According to the test, this conditional is acceptable, with respect to my present state of belief, just in case it would be reasonable for me to believe that the gardener did it, after having received the information that the butler didn't.[2]

Given a degree of regimentation, we may formulate the Ramsey test as follows:

(RT) The conditional "If A, then B" (in symbols, A > B) is accepted in a given belief state S if and only if B is accepted in the belief state S∗A that is the result of revising S with the new information A.

Or, more briefly:

(RT) A > B is accepted in S iff B is accepted in S∗A.

In this formulation of the test, we speak of (i) *belief states*, (ii) *acceptance* of a sentence A in a belief state S, (iii) conditionals A > B, and (iv) the operation ∗ of revising a belief state S with a sentence A *(belief revision).* We do not, in this our official formulation of the test, commit ourselves to any particular formal representation of belief states or any particular formal representation of acceptance.

However, the Ramsey test has turned out to be problematic. Peter Gärdenfors has shown that the test — or at least a formal version of it — is incompatible with certain intuitively appealing postulates for belief revision. Gärdenfors presents this result, which we shall refer to as *Gärdenfors' paradox*, in his recent book (1988).[3] Gärdenfors' axioms for belief revision are also presented in an already classical paper which he has written together with Carlos Alchourrón and David Makinson.[4]

Alchourrón, Gärdenfors and Makinson (AGM, for short) represent belief states by certain sets of sentences, so-called *belief sets.* They also assume that belief sets are closed under the rules of deductive logic. On the AGM approach a belief state is nothing but a logically closed set of sentences. To say that a sentence A is accepted in a belief state S reduces to the claim that A is a set-theoretic member of the corresponding belief set. The AGM approach represents belief revision as an operation on belief sets, that is, a function that transforms belief sets into new belief sets, relative to new information. If G is the original belief set and A is the new information, then the new belief set is denoted by G∗A. We refer to this new belief set as G *revised with* A. Note that the new information A may or may not be logically compatible with G. In the former case, the AGM approach takes G∗A simply to be the *expansion* of G with A, i.e., the set of logical consequences of G ∪ {A}. In the latter case, when A is incompatible with G, G∗A must involve a genuine revision of G. Some of the old beliefs must then be given up in order to make room for the new information.

In his discussion of the Ramsey test, Gärdenfors modifies the basic AGM approach by adding a conditional connective > to the object language. Thus, conditionals are treated just like any other sentences of the object language as possible members of belief sets. That a conditional is accepted relative to a belief set is represented by the set-theoretic membership of the conditional in the set in question. Hence, within Gärdenfors' framework the Ramsey test takes the form:

(GRT) $A > B \in G$ iff $B \in G*A$. *(Gärdenfors' Ramsey Test)*

Gärdenfors' paradox has both a formal and an informal dimension. First, we have a technical result — Gärdenfors' impossibility theorem — stating that the AGM approach cannot "on pain of triviality" be augmented in the way described with conditionals satisfying (GRT). To be exact, Gärdenfors' theorem says that there is no AGM-system for belief revision satisfying (GRT) together with the conditions:[5]

(Success) $A \in G*A$;
(Consistency) if $A \nvdash_L \bot$ and $\bot \notin G$, then $\bot \notin G*A$;
(K*P) if $\neg A \notin G$, then $G \subseteq G*A$.
(Non-Triviality) There exist two sentences B, C and three consistent belief sets G, H and K such that:

(1) $B \in G$ and $G \cup \{\neg C\}$ is consistent;
(2) $C \in H$ and $H \cup \{\neg B\}$ is consistent;
(3) $G \subseteq K$ and $H \subseteq K$.

Then, there is Gärdenfors' own interpretation of his formal result. He focuses the attention on the following *Preservation Criterion* for belief revision:

(P) If a sentence (or proposition) B is accepted in a given belief state S and A is consistent with the beliefs in S, then B is still accepted in S∗A.

(K*P) is of course intended as a formal counterpart of (P) within the AGM approach. Gärdenfors interprets the impossibility theorem as a dilemma: either the Ramsey test or the Preservation Criterion has to be given up. Here are some quotes from Gärdenfors (1988):

The main purpose of this section is to prove that, on pain of triviality, *the Ramsey test and the preservation criterion are inconsistent with each other*. (p. 157)

The theorem and its corollary show that the Ramsey test and the preservation condition (K*P) ... cannot both be rational criteria for belief revision. (p. 159)

Theorems 7.10 and 7.14 and their corollaries present us with a dilemma: When investigating belief revision systems, we must give up either the Ramsey test or the preservation condition. (p. 162)

Here, I shall argue that Gärdenfors result doesn't establish a genuine conflict between the Ramsey test and the Preservation condition. The appearance of a conflict is a result of the particular way acceptance, belief states, and revision of belief states are represented formally within Gärdenfors' approach. I shall argue that this representation is based on certain questionable presuppositions. Among these presuppositions are: (i) that all the propositions that

may be accepted in a belief state are expressed by sentences of the object language (the *expressibility assumption*); (ii) that all sentences of the object language, including conditionals express truth-value bearing propositions (the *propositional assumption*); (iii) that every sentence of the object language expresses one and the same proposition in a context-independent manner. In particular, that the proposition that is expressed by a conditional sentence is independent of the belief state of the person who considers the conditional in question (the *non-indexicality assumption*).

In order to bring out the assumptions underlying Gärdenfors' claim, we distinguish between propositions and sentences. Primarily, it is propositions rather than sentences that are accepted in belief states. A sentence A is accepted in a belief state S just in case the proposition that A expresses relative to S is accepted in S.

A crucial step in Gärdenfors' proof of his impossibility theorem is the derivation of the following monotonicity condition from (G-Ramsey):

(G-Monotonicity) If $G \subseteq H$, then $G*A \subseteq H*A$.

The informal condition that (G-Monotonicity) is intended to capture is presumably:

(Monotonicity) If S and T are belief states and all propositions that are accepted in S are also accepted in T, then all propositions that are accepted in $S*P$ are also accepted in $T*P$.

It is a trivial matter to derive (G-monotonicity) from (G-Ramsey). However, to derive Monotonicity from the Ramsey test is far from trivial. Let us see how it is done. Suppose that all propositions that are accepted in S are accepted in T. Let Q be a proposition that is accepted in $S*P$. We want to show that Q is also accepted in $T*P$. By the *expressibility assumption*, there are sentences A and B that express P and Q, respectively, relative to the state S. By the *propositional assumption*, there is a proposition R that is expressed by the sentence $(A > B)$ relative to S. By the *non-indexicality assumption*, A, B and $A > B$ express the same propositions P, Q and R relative to all belief states. Hence, we can write $S*P$ and $T*P$ as $S*A$ and $T*B$, respectively. It also follows that B is accepted in $S*A$. Hence, by the Ramsey test, $A > B$ is accepted in S. That is, R is accepted in S. By the supposition, R is also accepted in T. This, in turn means that $A > B$ is accepted in T (the non-indexicality assumption, again). Hence, by the Ramsey test, B is accepted in $T*A$, which means that Q is accepted in $T*P$.

In proving Monotonicity from the Ramsey test, we needed all the three presuppositions that are left implicit in Gärdenfors' approach. Our conclusion is that Monotonicity is not in any intuitive sense a logical consequence of the Ramsey test.

Isaac Levi (1988) has argued that Gärdenfors' dilemma is really a tri-lemma. Levi points to an implicit assumption in Gärdenfors' approach to the Ramsey test: that conditionals express truth-value bearing propositions and therefore are eligible as members of belief sets (the propositional assumption). Contrary to this assumption, Levi maintains that conditionals lack truth values. For this reason, he also denies that they can be members of belief sets.

Here, we shall focus our attention on another assumption that is implicit in Gärdenfors' treatment of conditionals:

A conditional sentence A > B expresses *one and the same* proposition relative to every belief state. *(The Non-Indexicality Assumption)*

We shall argue that — once this assumption is given up — there is no genuine conflict between the Ramsey test and the Preservation condition. That is, it is possible without threat of paradox to keep both the original *Ramsey test*:

(RT) A > B is accepted in a belief state X iff B is accepted in X∗A

and the *Preservation* condition in the form:

(P) If A is consistent with X, then X is included in X∗A,

without giving up the assumption that conditionals express truth-value bearing propositions. When in (P) we say that one belief state X is *included in* another state Y, we mean that all *propositions* that are accepted in X are also accepted in Y. This does not necessarily mean, however, that all the *sentences* that are accepted in X are accepted in Y. In the presence of context-dependent sentences, that may express different propositions relative to different belief states, inclusion between the propositions accepted does not imply the corresponding inclusion between sentences. Hence, the above form of Preservation does not imply:

If A is consistent with X, then every sentence that is accepted in X is also accepted in X∗A.

The latter condition is plausible only if all the sentences of the object language are context independent.

2. The context-sensitive nature of epistemic conditionals

The assumption that the sentences of the object language express determinate propositions in a *context independent* way is implicit in the AGM approach. If one and the same sentence could express different propositions relative to different belief states, then set-theoretic statements concerning belief sets, for instance $G \subseteq H$ or ($A \in G$ & $A \in H$), would not have their intended interpretation. Suppose namely that G and H are belief sets representing the belief states X and Y, respectively. Then, the following condition is supposed to obtain:

(∗) G is included in H if, and only if, X is included in Y.

If, however, the object language contains context-sensitive sentences, this connection might fail. To see that the right-to-left direction might fail, suppose that every proposition that is accepted in X is also accepted in Y. Let A be a sentence in G and let $[\![A]\!]^X$ be the proposition that A expresses relative to X. Since G represents the state X, $[\![A]\!]^X$ is accepted in X.

Then, by the supposition, $[\![A]\!]^X$ is also accepted in Y. But, from this we cannot infer that $A \in H$. For that we would need $[\![A]\!]^Y$ to be accepted in Y which may not be the case, since $[\![A]\!]^X$ and $[\![A]\!]^Y$ may be different propositions. Hence, we cannot conclude that $G \subseteq H$.
To see that also the left-to-right direction of $(*)$ might fail, suppose that $G \subseteq H$ and that the proposition P is accepted in X. G represents X, so there must be a sentence $A \in G$ such that $[\![A]\!]^X = P$. Since $G \subseteq H$, $A \in H$. It follows that $[\![A]\!]^Y$ is accepted in Y. We cannot, however, conclude that P is accepted in Y, since $[\![A]\!]^Y$ may be different from P. Once we allow sentences that may express different propositions with respect to different belief states, then both directions of $(*)$ fail. For context-dependent sentences A, even *Success* fails: A may not be a member of $G*A$.
What reasons could we possibly have for saying that conditionals are context-sensitive: that they express different propositions with respect to different belief states?[6] Consider the principle:

(M) If the conditional $A > B$ is accepted in the belief state X and X is included in Y, then $A > B$ is also accepted in Y.

If we assume that a conditional $A > B$ expresses a proposition $[\![A > B]\!]$, quite independently of context, then the principle (M) seems to be valid. To say that $A > B$ is accepted in X then means that the proposition $[\![A > B]\!]$ expressed by $A > B$ is a member of X. But if X is included in Y, then, of course, it follows that $[\![A > B]\!]$ is accepted in Y. So, $A > B$ is accepted in Y.
However, the principle (M) does not seem to be intuitively correct for epistemic conditionals. Suppose that, in my present belief state, I accept the conditional:

if there is a storm tomorrow, the old oak tree will still stand.

Then, I learn that the old oak tree is infested with termites. We suppose that this information is consistent with my old beliefs and that my belief change is *preservative* in the sense that I do not give up any of my old *factual* beliefs when I receive the new information. However, it is still quite likely that I should give up the conditional upon receiving this information. Thus, learning a new fact might lead me to give up one of the conditionals that I previously accepted, even if I do not give up any of my old factual beliefs.
If we assume that epistemic conditionals express propositions, then the natural conclusion to draw from the apparent failure of (M) is that such conditionals express different propositions with respect to different belief states. Instead of assigning propositions to conditional sentences in a context-independent way, we need to *relativize* the assignment of propositions to belief states. Only relative to a belief state does an epistemic conditional $A > B$ express a determinate proposition. We should speak of the proposition $[\![A > B]\!]^X$ expressed by the conditional $A > B$ *relative to* the belief state X. It is then natural to say that the conditional

A > B is *accepted* in the belief state X if, and only if, the proposition $[\![A > B]\!]^X$ expressed by A > B relative to X is a member of X. In other words,

A > B is accepted in X iff $[\![A > B]\!]^X$ is accepted in X.

Now, we see how the principle (M) might fail. Suppose that A > B is accepted in the belief state X and that X is included in Y. Then, $[\![A > B]\!]^X$ is accepted in X, so $[\![A > B]\!]^X$ is also accepted in Y. But in order to infer that A > B is accepted in Y, we need instead: $[\![A > B]\!]^Y$ is accepted in Y. This does not follow, since $[\![A > B]\!]^X$ and $[\![A > B]\!]^Y$ may be different propositions.

The idea that conditional sentences express different propositions relative to different belief states is quite a natural one. Consider the following two sentences:[7]

(1) If Bizet and Verdi were compatriots, Verdi was French.
(2) If Bizet and Verdi were compatriots, Bizet was Italian.

(1) could be used to make a true statement by a contemporary speaker who knows that Bizet was French, but does not know the nationality of Verdi. For such a speaker, the claim made by (2) would be false. The situation is the opposite for a speaker who knows that Verdi was Italian but does not know the nationality of Bizet.

Now, what does A > B mean? The analysis of conditionals given here is close to those of Stalnaker and Lewis, except for containing an *additional parameter:* a belief state. The intuitive idea is expressed by Stalnaker as follows:[8]

A conditional statement, *if A, then B,* is an assertion that the consequent is true, not necessarily in the world as it is, but in the world as it would be if the antecedent were true.

In possible worlds terms we can express this idea roughly as:

A conditional sentence A > B is true at a world w just in case B is true at all the A-worlds that are most similar to w.

However, here we shall think of the notion of similarity involved in the truth condition for conditionals as an *epistemic notion* which is determined by the agent's belief state. Making this dependence on a belief state explicit, we get:

A conditional sentence A > B is *true at a world* w *relative to a belief state* X just in case B is true at all the A-worlds that are most X-similar to w,

where X-similarity is a concept of similarity between possible worlds that is determined by the belief state X. According to this type of semantics, the truth-value of a conditional A > B is dependent both on the state w of the world and the belief state X. Relative to a belief state X, A > B can be said to express the proposition:

$[\![A > B]\!]^X$ = {w: A > B is true at w relative to X}.

3. Gärdenfors paradox and its resolution

The approach described here differs from AGM and that of Levi (1988) in making a sharp distinction between the semantic level involving propositions and belief states and the linguistic level involving sentences and sets of sentences. Belief revision is seen as an operation on belief states; and it is primarily propositions rather than sentences that are accepted relative to belief states. We may think of a person's *belief state* as the set of all propositions that he accepts. We do not suppose in general that belief states are logically closed.

It is convenient for our purposes to identify propositions with certain sets of possible worlds. If W is the set of possible worlds, then the set **P** of all the propositions that the agent might entertain is a family of subsets of W. A proposition $P \in \mathbf{P}$ is *true at* a possible world w if, and only if, $w \in P$. We suppose that **P** is a Boolean set algebra, i.e., it contains W and is closed under the Boolean set-operations $\cap$, $\cup$ and -. *Belief states* are certain sets of propositions, i.e., we have a family $\mathbf{K} \subseteq \wp(\mathbf{P})$ of all possible belief states. A proposition P is accepted in a belief state X if, and only if, $P \in X$. A belief state X *entails* a proposition P iff $\cap X \subseteq P$.

Once we are reminded of the context dependent nature of conditionals and other epistemic constructions, the representation of belief states by sets of sentences and acceptance by set-theoretic membership in such sets becomes less appealing. If we distinguish between propositions, belief states, acceptance, on the one hand, and sentences, belief sets and membership, on the other, we see that the most perspicuous way of formulating the conditions of Success, Consistency and Preservation is in terms of the former notions:

(P-Success) The proposition P is accepted in $X*P$.

(P-Consistency) If P and X are consistent, when considered separately, then $X*P$ is also consistent.

(P-Preservation) If Q is accepted in a given belief state X and P is consistent with X, then Q is still accepted in $X*P$.

Now, if we formulated the Ramsey test in an analogous fashion as:

(P-R) $P \Rightarrow Q$ is accepted in X iff Q is accepted in $X*P$,

where $\Rightarrow$ is a binary operation on propositions corresponding to the conditional connective >, we would indeed be confronted with Gärdenfors' theorem. We could then derive the following monotonicity condition:

(P-Monotonicity) If $X \subseteq Y$, then $X*P \subseteq Y*P$.

And P-Monotonicity is easily seen to be incompatible with the above conditions on belief revision, given the additional requirement:

(Non-Triviality) There exist two propositions P, Q and three consistent belief states X, Y and Z such that:

(1) $P \in X$ and $X \cup \{-Q\}$ is consistent;

(2) $Q \in Y$ and $Y \cup \{-P\}$ is consistent;

(3) $X \subseteq Z$ and $Y \subseteq Z$.

That is, we have:

THEOREM 1 (*Gärdenfors impossibility theorem):*
Suppose that S = <W, **P**, **K**, $*$> is a belief revision system that satisfies the above postulates. Then, there is no operation $\Rightarrow$: $\mathbf{P} \times \mathbf{P} \rightarrow \mathbf{P}$ satisfying the following version of the *Ramsey test:*

(PR) $(P \Rightarrow Q) \in X$ iff $Q \in X*P$.

Proof: Suppose to the contrary that there is an operation $\Rightarrow$ in S satisfying (PR). Using the latter condition, we prove:

(P-Monotonicity) If $X \subseteq Y$, then $X*P \subseteq Y*P$.

By the non-triviality of S, there are propositions P and Q and consistent belief states X, Y, Z satisfying conditions (1) - (3) above. Let R be the proposition $-P \cup -Q$.
It follows from (1) and (2) that $X \cup \{R\}$ and $Y \cup \{R\}$ are consistent. Hence, by Preservation, $X \subseteq X*R$ and $Y \subseteq Y*R$. But, $P \in X$ and $Q \in Y$, so $P \in X*R$ and $Q \in Y*R$.
Since $X \subseteq Z$ and $Y \subseteq Z$, Monotonicity insures that $X*R, Y*R \subseteq Z*R$. It follows that P, Q $\in Z*R$. Success yields that also $R \in Z*R$. But {P, Q, R} is inconsistent, so $Z*R$ must be inconsistent. On the other hand, each of Z and R is consistent, so the condition of Consistency implies that also $Z*R$ is consistent. We have thus reached a contradiction. □

In the above theorem, we showed that the conditions of Success, Consistency, Preservation and Non-Triviality are inconsistent with the existence of a binary operation on propositions that satisfies the Ramsey test.
However, thinking of the conditional connective > as corresponding to a binary operation $\Rightarrow$ on propositions is tantamount to assuming that conditional sentences are context-independent. Given such an operation $\Rightarrow$, we could formulate the following semantic clause for conditionals:

(i) $[\![A > B]\!] = [\![A]\!] \Rightarrow [\![B]\!]$,

where $[\![A > B]\!]$ is the proposition expressed by A > B. But if we instead think of conditionals A > B as expressing propositions only relative to belief states, we would rather like to have something like the following semantic clause:

(ii) $[\![A > B]\!]^X = [\![A]\!]^X \Rightarrow_X [\![B]\!]^X$,

where $[\![A > B]\!]^X$ is the proposition expressed by A > B relative to the belief state X and $\Rightarrow_X$ is a ternary proposition-forming operator taking two propositions and a belief state as argu-

ments. We may think of $\Rightarrow_X$ as a context-dependent propositional operator. For such an operator, the Ramsey test takes the form:

(P-Ramsey) $(P \Rightarrow_X Q) \in X$ iff $Q \in X*P$.

With P-Ramsey our proof of Gärdenfors' theorem does not go through, since Monotonicity is no longer derivable. To see this suppose that $X \subseteq Y$ and $Q \in X*P$. Then, by P-Ramsey, $(P \Rightarrow_X Q) \in X$, from which we conclude $(P \Rightarrow_X Q) \in Y$. However, from this we cannot reach the desired conclusion $Q \in Y*P$. To get there we would need $(P \Rightarrow_Y Q) \in Y$, instead.
As a matter of fact, we can prove that there are non-trivial belief revision systems of the type $<W, \mathbf{P}, \mathbf{K}, *, \Rightarrow_X>$ that satisfy the propositional versions of the Gärdenfors axioms for belief revision together with the condition P-Ramsey. That is, we have:

THEOREM 2. There are systems $S = <W, \mathbf{P}, \mathbf{K}, *, \Rightarrow_X>$ satisfying Success, Consistency, Preservation, Non-Triviality, P-Ramsey together with the conditions:

(Closure) If X entails P, then $P \in X$;
(W) $X*W = X$;
(Revision by Conjunction) If $X*P \cup \{Q\}$ is consistent, then $X*(P \cap Q)) = (X*P)+Q$,

where for any X and P, X+P is the *expansion* of X with P, i.e., the set:

$\{Q \in \mathbf{P}: \cap X \cap P \subseteq Q\}$.

Sketch of proof: Let W and **P** be given and let **K** be all subsets of **P** that are closed under entailment. We associate with every consistent $X \in \mathbf{K}$ a system $\$_X$ of spheres in the sense of Grove (1988) around $\cap X$. If X and P are consistent, then we define $X*P$ to be $\{Q \in \mathbf{P}: S \cap P \subseteq Q\}$, where S is the smallest sphere in $\$_X$ such that $S \cap P \neq \emptyset$. Otherwise, we let $X*P$ be $\wp(W)$. It is easily verified that the conditions Success, Consistency, Closure, *(W)*, and Revision by Conjunction are satisfied. Preservation follows from *(W)* together with Revision by Conjunction. We can easily see to it that **P** contains two propositions P and Q such that $P \cap Q \neq \emptyset$, $P \cap -Q \neq \emptyset$, $-P \cap Q \neq \emptyset$ and $-P \cap -Q \neq \emptyset$. Two such propositions are said to be completely independent. Let then $X = \{R: P \subseteq R\}$, $Y = \{R: Q \subseteq R\}$ and $Z = \{R: P \cap Q \subseteq R\}$. Then, X, Y, Z are consistent belief states such that:

(1) $P \in X$ and $X \cup \{-Q\}$ is consistent;
(2) $Q \in Y$ and $Y \cup \{-P\}$ is consistent;
(3) $X \subseteq Z$ and $Y \subseteq Z$.

Thus, Non-Triviality is satisfied.
We are next going to define the operation $\Rightarrow_X$. For this purpose, we associate with each world w and each belief state X a system of spheres $\$_{X,w}$ that is weakly centered around w. We impose the following constraint:

if $w \in \cap X$, then $\$_{X,w} = \$_X$.

That is, if w is a world that is compatible with all the beliefs in state X, then the sphere system around w coincides with that around X.
We define $\Rightarrow_X$ by letting:

$$P \Rightarrow_X Q = \{w: (\exists S \in \$_{X,w})(\varnothing \neq S \cap P \subseteq Q)\}.$$

It remains to show that P-Ramsey holds, i.e.,

$$P \Rightarrow_X Q \in X \text{ iff } Q \in X{*}P.$$

Suppose that $P \Rightarrow_X Q \in X$. Then, $\cap X \subseteq P \Rightarrow_X Q$. That is, (i) for all $w \in \cap X$, $w \in (P \Rightarrow_X Q)$. If $\cap X = \varnothing$, then $X{*}P = \wp(W)$. Hence, the desired conclusion holds in this case. Suppose that (ii) $\cap X \neq \varnothing$. By the constraint: for all $w \in \cap X$,

$$w \in (P \Rightarrow_X Q) \text{ iff } (\exists S \in \$_X)(\varnothing \neq S \cap P \subseteq Q).$$

But (i) and (ii) yields that for some $w \in \cap X$, $w \in (P \Rightarrow_X Q)$. Hence, $(\exists S \in \$_X)(\varnothing \neq S \cap P \subseteq Q)$. But this means that $Q \in X{*}P$.
For the other direction, suppose that $Q \in X{*}P$. Let $w \in \cap X$. By the constraint,

$$w \in (P \Rightarrow_X Q) \text{ iff } (\exists S \in \$_X)(\varnothing \neq S \cap P \subseteq Q).$$

That is,

$$w \in (P \Rightarrow_X Q) \text{ iff } Q \in X{*}P.$$

It follows that $w \in (P \Rightarrow_X Q)$. We have shown that, $\cap X \subseteq (P \Rightarrow_X Q)$, which means that $(P \Rightarrow_X Q) \in X$. □

In the above proof we outlined a semantics for belief revision and conditionals based on systems of spheres. First, every belief state X was associated with a system of spheres $\$_X$ in terms of which the belief revision operation X∗... was defined. Secondly, each world w was associated with a system of spheres $\$_{X,w}$ relative to X. In terms of the latter system of spheres, we could define the propositional operator (... $\Rightarrow_X$ ...). A condition was imposed connecting the two kinds of sphere systems:

if w is compatible with all the beliefs in X, then $\$_{X,w} = \$_X$.

From this condition, we proved:

(P-Ramsey) $P \Rightarrow_X Q \in X$ iff $Q \in X{*}P$.

This modelling showed (P-Ramsey) to be compatible with propositional versions of Gärdenfors' axioms for belief revision.
Suppose now that we have a formal language L with sentences built up from atomic ones using Boolean connectives $\perp$ and $\rightarrow$ and the conditional connective >. L_0 is the fragment of L without >. We let S = <W, **P**, **K**, ∗, $\Rightarrow_X$> be a belief revision system satisfying Success, Consistency, Preservation Non-Triviality and P-Ramsey. We let ⟦...⟧ be an interpretation

function that assigns propositions $[\![A]\!]^X$ to sentences A in L relative to belief states X. This function is assumed to satisfy the requirements:

(a) If A is a sentence of L_0, then for all X, $Y \in \mathbf{K}$, $[\![A]\!]^X = [\![A]\!]^Y$. Hence, for sentences of L_0 we may write $[\![A]\!]$ instead of $[\![A]\!]^X$.

(b) For every $P \in \mathbf{P}$, there exists a sentence A of L_0 such that $P = [\![A]\!]^X$.

(The Expressibility Assumption)

(c) $[\![\bot]\!]^X = \varnothing$.

(d) $[\![A \rightarrow B]\!]^X = (W - [\![A]\!]^X) \cup [\![B]\!]^X$.

(e) $[\![A > B]\!]^X = [\![A]\!]^X \Rightarrow_X [\![B]\!]^{(X * [\![A]\!]^X)}$

Writing $X*A$ for $X*[\![A]\!]^X$, we can simplify (e) to:

$$[\![A > B]\!]^X = [\![A]\!]^X \Rightarrow_X [\![B]\!]^{(X*A)}.$$

Assumption (a) says that the sentences of the basic language L_0 are context-independent. The expressibility assumption is the requirement that the basic language has sufficient expressive power to express all the propositions that the agent might accept. Together these two conditions makes it possible to represent belief states, in a context-independent way, by sets of sentences of L_0.

We say that a sentence A is *accepted* in the belief state X just in case the proposition $[\![A]\!]^X$ that is expressed by A relative to X is accepted in X. That is:

A is accepted in X iff $[\![A]\!]^X \in X$.

Then, we get:

A > B is accepted in X iff
$[\![A > B]\!]^X \in X$ iff
$([\![A]\!]^X \Rightarrow_X [\![B]\!]^{(X*A)}) \in X$ iff
$[\![B]\!]^{(X*A)} \in X*[\![A]\!]^X$ iff
$[\![B]\!]^{(X*A)} \in X*A$ iff
B is accepted in X∗A.

That is, we get our original formulation of the Ramsey test:

(RT) A > B is accepted in the belief state X iff B is accepted in X∗A.

Gärdenfors' paradox arises when we do not distinguish clearly between sentences and the propositions expressed by sentences and do not pay attention to the context dependent nature of epistemic conditionals. The principles of Success, Consistency, and Preservation, — formulated as above for propositions and belief states — are consistent with the Ramsey test.

4. The Ramsey test and revision of belief sets

Our resolution of Gärdenfors' paradox depended on viewing belief revision as primarily an operation on belief states and interpreting Gärdenfors' postulates on belief revision as applying to such an operation. We showed that such a propositional belief revision system could be provided with a context-dependent operator $\Rightarrow_X$ on propositions satisfying (P-Ramsey). Finally, we showed that the conditional connective > could be interpreted semantically in terms of $\Rightarrow_X$ in such a way that the Ramsey test (RT) became valid.

Now we want to see what happens if we view belief revision as an operation on belief sets, i.e., sets of sentences, instead. Starting out from a belief revision system S = <W, **P**, **K**, $*$, $\Rightarrow_X$> and an interpretation function $[\![...]\!]$ satisfying the conditions (a) - (e) above, we define a corresponding logic L, a set $\uparrow$(**K**) of belief sets corresponding to the set **K** of all belief states, and an operation $*$ of belief revision on belief sets. As a matter of fact, we define two notions of belief set, one for the basic language L_0 and one for the extended language L, and correspondingly two notions of belief revision. Within our framework, the two notions are interdefinable and the Ramsey test can be formulated in terms of both.

First, we define the *logic* L determined by S and $[\![...]\!]$. We say that a sentence A in L is an L-*consequence* of a set Γ of sentences in L (in symbols, $\Gamma \vdash_L A$) if for every belief state X $\in$ **K**, $\cap\{[\![B]\!]^X: B \in \Gamma\} \subseteq [\![A]\!]^X$. That is, $\Gamma \vdash_L A$ iff for every belief state X and every possible world w, if all the sentences in Γ are true at w relative to X, then A is also true at w relative to X. For sentences in L_0 the reference in this definition to the belief state X becomes superfluous. That is, if Γ is a set of sentences in L_0 and A belongs to L_0, then: $\Gamma \vdash_L A$ iff $\cap\{[\![B]\!]: B \in \Gamma\} \subseteq [\![A]\!]$.

Next, we need to decide on what we shall understand by a belief set. Each belief state is associated with two sets of sentences: First we have the set:

$$\{A \in L_0: [\![A]\!] \in X\}$$

of all *non-indexical* or *basic sentences* that corresponds to X. Then there is the set of all sentences of the extended language L that are accepted in X, i.e., the set:

$$\{A \in L: [\![A]\!]^X \in X\}.$$

Let us speak of the first set as the *descriptive belief set* corresponding to the belief state X, and the second set as the *acceptance set* corresponding to X. In view of the expressibility assumption, there is a one-to-one correspondence between belief states and descriptive belief sets. We also have a one-to-one correspondence between descriptive belief sets and acceptance sets. For each acceptance set G, the corresponding descriptive belief set is the set $G \cap L_0$ which we may refer to as the *descriptive core* of G, or core(G). Conversely, for each descriptive belief set K, we can define the corresponding acceptance set as:

$$E(K) = \{A \in L: [\![A]\!]^{[\![K]\!]} \in [\![K]\!]\},$$

where $[\![K]\!]$ is the belief state that corresponds to K, i.e., $[\![K]\!] = \{[\![B]\!]: B \in K\}$. In other words, E(K) is the set of all sentences of L that are accepted in the belief state corresponding to K. Of course, G = E(K) iff K = core(G).

For each belief state X, we let $\uparrow(X)$ be the acceptance set corresponding to X, i.e.,

$$\uparrow(X) = \{A \in L: [\![A]\!]^X \in X\}.$$

and we let $\uparrow(\mathbf{K})$ be the set of all acceptance sets, i.e.,

$$\uparrow(\mathbf{K}) = \{\uparrow(X): X \in \mathbf{K}\}.$$

The sentences of core(G) are context-independent, so we can speak in a context-independent way of the proposition $[\![A]\!]$ expressed by A, for each $A \in$ core(G). Furthermore, we have assumed that every proposition in **K** is expressed by some sentence in L_0 (The Expressibility Assumption). It follows that we can recover the belief state corresponding to a acceptance set G as the set of all propositions that are expressed by some member of core(G). That is the belief state corresponding to G is defined as:

$$[\![G]\!] = \{[\![A]\!]: A \in \text{core}(G)\} = [\![\text{core}(G)]\!].$$

Notice that::

$$[\![\uparrow(X)]\!] = X, \text{ and}$$

$$\uparrow([\![G]\!]) = \{A \in L: [\![A]\!]^{[\![G]\!]} \in [\![G]\!]\} = \{A \in L: A \in G\} = G.$$

We also have:

$$\text{if } [\![G]\!] = [\![H]\!], \text{ then } G = H.$$

In order to prove this, let $[\![G]\!] = [\![H]\!]$. Then, $G = \uparrow([\![G]\!]) = \uparrow([\![H]\!]) = H$.

We can now define two operations of belief revision, one operation $\oplus$ on descriptive belief sets and the other $*$ on acceptance sets. For any descriptive belief set K and any $A \in L$, we let:

$$K \oplus A = \{B \in L_0: [\![B]\!] \in [\![K]\!] * A\} = \{B \in L_0: [\![B]\!] \in [\![K]\!] * [\![A]\!]^{[\![K]\!]}\}.$$

That is, if K is a descriptive belief set and A is a sentence of L, then we define $K \oplus A$ as follows: first, we go to the belief state $[\![K]\!]$ corresponding to K. We then revise that state with the proposition $[\![A]\!]^{[\![K]\!]}$ that A expresses relative to that state. Finally, we let $K \oplus A$ be the set of all sentences of L that are accepted in the resulting belief state.

Similarly, we define for any acceptance set G:

$$G * A = \uparrow([\![G]\!] * A) = \{B \in L: [\![B]\!]^{([\![G]\!] * A)} \in [\![G]\!] * A\}.$$

We have:

$$[\![K \oplus A]\!] = [\![K]\!] * A, \text{ and}$$

$$[\![G * A]\!] = [\![G]\!] * A.$$

The two operations are interdefinable as follows:

$G{*}A = E(core(G) \oplus A)$
$K \oplus A = core(E(K){*}A)$.

For any pair of descriptive belief states K, K' we have:

$K \subseteq K'$ iff $[\![K]\!] \subseteq [\![K']\!]$.

However, for acceptance sets G, H, we do *not* have:

$G \subseteq H$ iff $[\![G]\!] \subseteq [\![H]\!]$.

For acceptance sets G and H, it is important not to conflate ordinary set inclusion ($G \subseteq H$) with the relation (we write it, $G \sqsubseteq H$) that holds iff all the *propositions* that are accepted in the belief state $[\![G]\!]$ are also accepted in $[\![H]\!]$. Due to the Expressibility Assumption, we can define $\sqsubseteq$ as follows:

$G \sqsubseteq H$ iff $core(G) \subseteq core(H)$.

We have, of course,

$G \sqsubseteq H$ iff $[\![G]\!] \subseteq [\![H]\!]$.

Let us now see how to formulate the Ramsey test and Preservation within the present framework. First, we consider the Ramsey test:

(RT) $A > B$ is accepted in the belief state X iff B is accepted in $X{*}A$.

In terms of acceptance sets and revision of acceptance sets, this becomes:

$A > B \in G$ iff $B \in G{*}A$.

The same condition formulated in terms of descriptive belief sets K and the operation $\oplus$ is:

$A > B \in E(K)$ iff $B \in E(K \oplus A)$.

Consider next P-Preservation:

(P-Preservation) If Q is accepted in a given belief state X and P is consistent with X, then Q is still accepted in $X{*}P$.

This corresponds to:

If $A \in L_0$, K is a descriptive belief set and $K \cup \{A\} \nvdash_L \bot$, then $K \subseteq K \oplus A$,
(L_0-Preservation)

In other words,

If $A \in L_0$, G is an acceptance set and $(core(G) \cup \{A\}) \nvdash_L \bot$, then $G \sqsubseteq G{*}A$.

Suppose next that S = <W, **P**, **K**, $*$, $\Rightarrow_X$> satisfies P-Success, P-Consistency, P-Preservation, Closure, (W), Revision by Conjunction, Non-Triviality and P-Ramsey and that $[\![...]\!]$ satisfies the conditions (a) - (e) above. Then, the following conditions are also satisfied:

(1) The logic L determined by S and $[\![...]\!]$ contains all substitution instances of truth-functional tautologies and is closed under modus ponens (i.e., if $\vdash_L A$ and $\vdash_L A \rightarrow B$, then $\vdash_L B$).

(2) If $\vdash_L A \leftrightarrow B$ and $\vdash_L C \leftrightarrow D$, then $\vdash_L (A > C) \leftrightarrow (B > D)$.

(3) Descriptive belief sets and acceptance sets are L-closed sets in L_0 and L, respectively. *(Closure)*

(4) E is a one-to-one mapping between descriptive belief sets and acceptance sets such that for each descriptive belief set K, $K = E(K) \cap L_0$.

(5) If $A \in L_0$, then $A \in K \oplus A$. *(L_0-Success)*

(6) If $A \nvdash_L \bot$ and $K \nvdash_L \bot$, then $K \oplus A \nvdash_L \bot$. *($L_0$-Consistency)*

(7) If $A \in L_0$, K is a descriptive belief set and $K \cup \{A\} \nvdash_L \bot$, then $K \subseteq K \oplus A$, *(L_0-Preservation)*

(8) If $A \in L_0$ and $K \cup \{A\} \nvdash_L \bot$, then $K \oplus A = K+A$, where $K+A = \{B \in L_0 : K \cup \{A\} \vdash_L B\}$. ($L_0$-*Expansion)*

(9) If $\vdash_L A \leftrightarrow B$, then $K \oplus A = K \oplus B$.

(10) If $A, B \in L_0$ and $K \oplus A \cup \{B\} \nvdash_L \bot$, then $K \oplus (A \wedge B) = (K \oplus A)+B$. *($L_0$-Revision by Conjunction)*

(11) $A > B \in E(K)$ iff $B \in E(K \oplus A)$ *(The Ramsey test)*

(12) There exist two sentences B and C in L_0 and three consistent descriptive belief sets G, H and K such that: (i) $B \in G$ and $G \cup \{\neg C\}$ is consistent; (ii) $C \in H$ and $H \cup \{\neg B\}$ is consistent; and (iii) $G \subseteq K$ and $H \subseteq K$ *(L_0-Non-Triviality)*

In virtue of Theorem 2, there are belief revision systems satisfying the above conditions. It is also easy to see that $[\![...]\!]$ can be defined (recursively) in such a way that conditions (a) - (e) are satisfied. It follows that conditions (1) - (12) are mutually consistent.

The present approach has the formal advantage over Levi's (1988) of being able to account for iterated conditionals in a natural way. Levi's version of the Ramsey test does not provide a method for evaluating such conditionals. The present version of the test can, however, be applied to iterated conditionals without difficulty. Consider, for example, $(A > B) > (C > D)$. According to (RT), we have:

$(A > B) > (C > D) \in E(K)$ iff $C > D \in E(K \oplus (A > B))$ iff $D \in E((K \oplus (A > B)) \oplus C))$.

Or, in other words,

$(A > B) > (C > D) \in G$ iff $C > D \in G*(A > B)$ iff $D \in (G*(A > B))*C))$.

Semantically this means:

$[\![(A > B) > (C > D)]\!]^X \in X$ iff $[\![C > D]\!]^{X*(A > B)} \in X*(A > B)$ iff $[\![D]\!]^{(X*(A > B))*C} \in (X*(A > B))*C$,

where X is the belief state $[\![G]\!]$.

5. Can the paradox be reinstated?

It should be pointed out that the following form of Monotonicity:

For any descriptive belief sets K, K', and any $A \in L_0$,
if $K \subseteq K'$, then $K \oplus A \subseteq K' \oplus A$. ($L_0$-Monotonicity)

is sufficient in the presence of L_0-Success, L_0-Consistency, L_0-Preservation and L_0-Non-Triviality for the derivation of an inconsistency. However, it is impossible to derive L_0-Monotonicity from

(RT) For every descriptive belief set K and any $A, B \in L$,
$A > B \in E(K)$ iff $B \in E(K \oplus A)$.

Thus, Gärdenfors' paradox is avoided.
At this point the reader might object and point out that there is another form of Monotonicity that actually follows from (RT), namely:

For any descriptive belief sets K, K', and any $A \in L$,
if $E(K) \subseteq E(K')$, then $E(K \oplus A) \subseteq E(K' \oplus A)$. (L-Monotonicity)

Couldn't this condition be used to construct a version of Gärdenfors' paradox? This seems in fact possible. The only thing we have to do is to replace the condition of L_0-Non-Triviality with the stronger condition:

(*) There exist two sentences B and C in L_0 and three consistent descriptive belief sets G, H and K such that: (i) $B \in G$ and $G \cup \{\neg C\}$ is consistent; (ii) $C \in H$ and $H \cup \{\neg B\}$ is consistent; and (iii') $E(G) \subseteq E(K)$ and $E(H) \subseteq E(K)$.

It is easy to see that this condition is sufficient to derive a contradiction. Let A be the sentence $\neg B \vee \neg C$. Since, B and C belong to L_0, the same holds for A. It follows from (i) and (ii) that each of G and H are logically compatible with A. Since, B and C belong to G and H, respectively, L_0-Preservation implies that $B \in G \oplus A$ and $C \in H \oplus A$. Hence, $B \in E(G \oplus A)$ and $C \in E(H \oplus A)$ (since, for any descriptive belief sets G, $E(G) \cap L_0 = G$). However, since $E(G), E(H) \subseteq E(K)$ (condition (iii')), L-Monotonicity implies that $E(G \oplus A), E(H \oplus A) \subseteq E(K \oplus A)$. It follows that $B, C \in E(K \oplus A)$. By L_0-Success, we also get $A \in E(K \oplus A)$. But this implies that $K \oplus A$ is inconsistent. On the other hand, $K \oplus A$ must be consistent, by L_0-Consistency.

It might seem as if we have indeed succeeded in reinstating the paradox. However, this is not really so. The above proof is nothing but a reductio proof of the negation of (∗) from the premises: L_0-Success, L_0-Consistency, L_0-Preservation and (RT). Given these assumptions, the situation envisaged in (∗) is impossible. This is no paradox, since we have no reasons to believe (∗) to be true.

To illustrate the situation envisaged in L_0-Non-Triviality, let B be the sentence "It is raining in Uppsala" and C the sentence "It is snowing in Lund". Suppose that our agent starts from a belief state in which has no opinion about the weather in either Uppsala or Lund. Let G, H and K be the descriptive belief sets that he would reach upon learning, respectively, B, C and $B \wedge C$. It is reasonable to assume that such G, H and K are going to satisfy conditions (i), (ii) and (iii). Thus, L_0-Non-Triviality looks like a reasonable requirement.

However, (∗) is far from reasonable. Applying the reasoning above to our example, we get $B \in E(G \oplus A)$ and $C \in E(H \oplus A)$. It follows, by the Ramsey test that $A > B \in E(G)$ and $A > C \in E(H)$. On the other hand, we also have that $A \in E(K \oplus A)$ and that $K \oplus A$ is consistent. Hence, it follows that either $B \notin E(K \oplus A)$ or $C \notin E(K \oplus A)$. Hence, by the Ramsey test again, either $A > B \notin E(K)$ or $A > C \notin E(K)$. Consequently, we cannot have both $E(G) \subseteq E(K)$ and $E(H) \subseteq E(K)$. Even though G and H may both be included in K, some of the conditionals accepted in G or H will have to disappear as we move to K. The impression that (iii') could obtain arises because we do not clearly distinguish between a (descriptive) belief set and the set of sentences that are accepted in this belief set.

Notes

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[1] The idea underlying the Ramsey test goes back to a suggestion that Frank Ramsey made in a footnote in one of his posthumously published papers (See Ramsey, 1931, p. 248.). The modern version of the test, a generalization of Ramsey's original idea, is essentially due to Robert Stalnaker (1968). His formulation of the test is as follows: "This is how to evaluate a conditional: First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true".

[2] While the Ramsey test may appear plausible for indicative conditionals like (1), it does not work so well for subjunctive ones. A person who has good reasons to accepts (1), need not have reasons to accept the corresponding subjunctive conditional: (2) If the butler hadn't done it, then the gardener would have. See Lindström & Rabinowicz (1992b), p. 2-3, for some reflections on the distinction between indicative and subjunctive conditionals. There we argue that the grammatical distinction between indicative and subjunctive conditionals is closely linked to a semantic distinction: "On the one hand, we have the *epistemic* (or doxastic) conditionals that express our dispositions to change our beliefs in the light of new information. These are the ones for which the Ramsey test appears plausible. On the other hand, there are the *ontic* ones that we use to make factual claims about the world. The epistemic conditionals have to do with hypothetical modifications of our *beliefs* about the world, while the ontic conditionals represent the hypotheses concerning what would be the case if the *world itself* were different — they have to do with the hypothetical modifications of the *facts* rather than with the modifications of our *beliefs* about the facts. This distinction between two kinds of conditionals parallels the well-known distinction between two kinds of probabilities: the epistemic probabilities ("credences") and the ontic or objective ones ("chances")." In this paper I am of course concerned with epistemic conditionals only.

[3] See also Gärdenfors (1986). The discussion around the Ramsey test is complicated and involves many different issues. Among these are issues concerning the interpretation of conditional sentences in natural language. And also philosophical questions concerning the interpretation of belief and the plausibility of the Ramsey test for various notions of belief. And finally there are the logical and philosophical issues that are actualised by Gärdenfors impossibility theorem. In this paper I am concentrating on the latter questions and I am deliberately ignoring the broader issues concerning the interpretation of belief and the plausibility of the test itself. For a broader discussion of the Ramsey test that also involves these questions, see Lindström & Rabinowicz (1992a) and (1992b). The latter paper also reviews and compares different approaches to Gärdenfors' paradox including the present one. I also recommend Hansson's (1992) and Morreau's (1992) recent discussions of the Ramsey test to be read in conjunction with the present one.

[4] Alchourrón, Gärdenfors, Makinson (1985).

[5] As will be seen from the discussion in Section 5, Non-Triviality is a far from innocuous requirement. At this point, however, we will not question it.

[6] The context-dependent nature of (indicative) conditionals, i.e., the idea that the same conditional sentence may express different propositions relative to different epistemic states, has been emphasised by Stalnaker (1975, 1984). It is also utilized in Morreau's (1992) epistemic semantics based on the Ramsey test for non-iterative conditionals.

[7] Cf. Quine (1962), p. 15.

[8] Stalnaker (1975). See Jackson (1991), p. 143.

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