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Homogeneous Model in Finite Element Analysis for Natural Frequency Calculation of Axisymmetric Shells

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Purpose. The article aims to provide practical recommendations for calculating natural frequencies in axisymmetric shells using finite element methods. It focuses on the need to develop a simplified model that can be used in any modern finite element software package. The study analyzes the impact of the simplified homogeneous model on the deviation and error of natural frequencies compared to real structures. **Design / Method / Approach**. The research is based on creating a simplified shell geometry by determining parameters such as shell thickness and density. These parameters are derived under the condition of equivalence in the moment of inertia and mass of the cross-sectional element. These parameters can vary along the height of the shell. **Findings.** The natural frequencies of the experimental shell with complex geometry were calculated and compared with those of the simplified model. The deviations and errors in the calculated frequencies were determined. It was demonstrated that the simplified model allows the calculation of natural frequencies with a deviation of no more than 1% from the experimental model, while significantly reducing computation time and the required computer resources. **Theoretical Implications.** The research expands the understanding of challenges in calculating the natural frequencies of complex objects using finite element methods under limited computational resources. **Practical Implications**. Practical recommendations are provided for engineers and designers when performing modeling tasks in the mechanics of deformable solid bodies. **Originality / Value.** The article presents an original analysis of a real case where a simplified model was implemented to determine the natural harmonics of a liquid rocket engine nozzle, making it a valuable tool for studying complex structures. **Research Limitations / Future Research.** The study is limited to the analysis of determining the natural frequencies of axisymmetric shells and does not cover all possible geometric features. Future research may focus on developing simplified models based on the equivalence of stiffness and mass parameters. **Article Type.** Case study, practical article.

Keywords:

natural frequencies, finite element method, axisymmetric shells, simplified homogeneous model, shell geometry, computation time reduction

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Thin-walled shell structures are important structural elements in many engineering fields. These structures are prone to resonance at their natural frequencies under dynamic loads, making the task of determining the natural frequencies relevant. This is particularly relevant when solving nonlinear dynamics problems. To solve nonlinear dynamic problems, semi-analytical methods are most often used, which involve low-dimensional analytical-numerical models with a small number of degrees of freedom. Such approaches are presented in the works of (Popov at al., 1998), (Jansen, 2002), (Amabili, 2008) and (Strozzi & Pellicano, 2013). The use of the spectral element method (SEM) for determining the natural frequencies in shells is presented in the work (Mukherjee at al., 2021). The simplification of the problem by replacing the shell element with a beam element is presented in the work of (Cela at al., 2000). When using the finite element method, the problem is also solved using reduced-order elements (Rahman at al., 2011). However, the analysis of transient processes based on finite elements is computationally expensive. In the work of Dey and Ramachandra (Dey& Ramachandra, 2015) using the example of a composite cylindrical shell under pulsating load, it was demonstrated that it is not possible to obtain a solution to the problem using the finite element method.

There is a contradiction between the resources available to the researcher and the accurate development of the model for the part being studied in all finite element numerical modeling packages (Abaqus, 2009; COMSOL, 2023; Hexagon, 2022). On one hand, the quality of the modeling and result itself depends heavily on the number and quality parameters of the created finite elements. On the other hand, computer resources in some cases happen to be insufficient for operations with the matrices of the finite element model in case of complex geometry of the part. This problem requires not only significant resources from the workstation but also a lot of time to solve on condition that the model is built. Moreover, there is a possibility that the problem cannot be solved with the tools and resources available to the researcher. This is especially true for all tasks related to the calculation of natural frequencies of parts in the context of solid mechanics problems. Such tasks are always resource-intensive and require a significant amount of computational memory and time. A researcher without a sufficiently powerful workstation always needs to find a compromise solution. This paper examines one such approach, namely the use of a homogeneous model for parts with complex geometry. The potential of this approach is demonstrated using the example of calculating the natural frequencies of a supersonic part of rocket engine nozzle, which has many small cooling channels. A numerical comparison of the results obtained for both the complex geometry of the part and the simplified one is performed. Recommendations are provided for developing such homogeneous models for problems involving the determination of natural frequencies of symmetric shell structures.

Tasks and objectives

The objectives of the paper are to simplify the complexity of part's geometry, build a finite element model that enables creation of a high-quality model and further solution of the problem of determining the natural frequencies with the use of lesser computational resources.

Materials and methods

The task of determining the natural frequencies of any part depends on the distribution of mass and stiffness across the object under analysis. Therefore, when developing an alternative model, the geometry is simplified while maintaining the equivalence of the design's mass and stiffness. In this process, a homogeneous equivalent simplified model is used. This approach is not universal and is implemented for axisymmetric shell structures with complex geometry. The studied shell is replaced with a simplified homogeneous model variant, which parameters are determined according to the following algorithm:

1. The geometry of the shell’s cross-section is analyzed, considering possible variations in height. The symmetrically repeating part of the cross-section is identified.

2. The center of gravity of this part is located. If the geometry is complex, it is replaced with a combination of simple geometric elements. For this cross-section, its area and momentum of inertia are calculated.

3. The shell with a complex cross-section is replaced by a homogeneous model, with its thickness and density determined based on the equivalence of the momentum of inertia and mass.

This algorithm is demonstrated using the example of determining the natural frequencies of a liquid rocket engine nozzle, which has a complex geometry of small cooling channels. Main view of the nozzle is shown in Fig. 1.

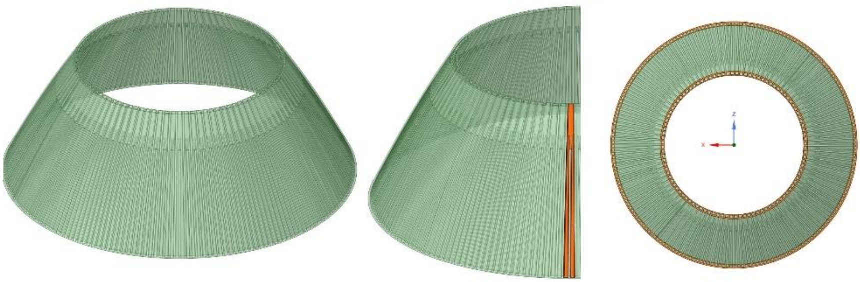

Figure 1 – Main view of the nozzle (Source: author)

This part has the following characteristics: it is a conical shell with a thickness of 3 mm, diameters of 170.8 mm and 101.5 mm, and a height of 70.8 mm. The number of nozzle’s cooling channels at the smaller diameter is N=100, with dimensions of 1.68×2.44 mm. These channels are symmetrically arranged with a repeat angle of 3.60° around the circumference. At a height of 54.8 mm from the base, the channels double in number. At the base diameter of the cone, the number of channels is N=200, symmetrically arranged with a repeat angle of 1.80° around the circumference. The dimensions of these channels are 1.56×1.98 mm. It is evident that the symmetrically repeating part of the cross-section of the channel, in the first approximation, resembles an I-beam. Thus, geometric parameters of this I-beam, such as the area, center of gravity, and momentum of inertia, can be determined. The homogeneous model represents a shell of constant thickness, the value of which is determined by the condition that the momentum of inertia of the symmetrically repeating part equals the momentum of inertia of a rectangle. The base of this rectangle is equal to the base size of the I-beam. To satisfy the condition of mass invariance between the homogeneous model and the analyzed design, we calculate the thickness of the rectangle under the condition that the area of the rectangle equals the area of the I-beam.

The thicknesses determined by the conditions of momentum of inertia equality, δi, and area equality, δF, differ. To create an equivalent homogeneous model, the material density of the homogeneous model is adjusted. The material density of the analyzed geometry is reduced by a factor, which is the ratio of the thicknesses δi/ δF. The result of these operations is development of an equivalent homogeneous model with a constant thickness δi and an adjusted density that meets the condition of mass invariance. These parameters may vary according to the height of the shell.

Results

To verify the correctness of the recommended approach, a numerical experiment was conducted, and the results of the calculations for the real model were compared with those for the homogeneous model developed according to the algorithm. Figures 2 and 3 present the main view, finite element model, and model elements for both the actual and homogeneous equivalent models. The homogeneous model features variable thickness and density. Up to a height of 16.0 mm from the base of the conical shell, its thickness is 2.358 mm. Then, it linearly decreases to 2.2 mm and remains constant. The density is adjusted by factors of 1.461 and 1.506 in the corresponding sections.

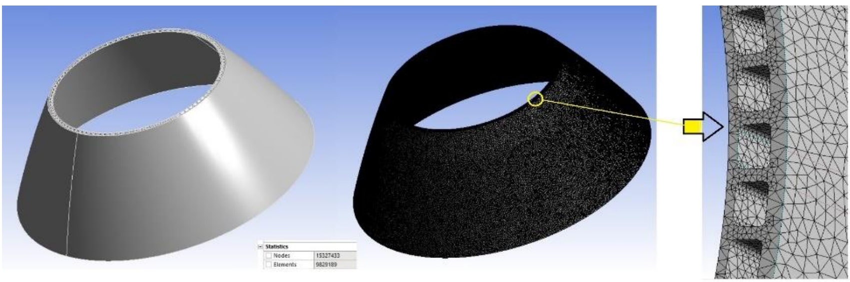

Figure 2 – Main view and finite element model of the experimental shell (Source: author)

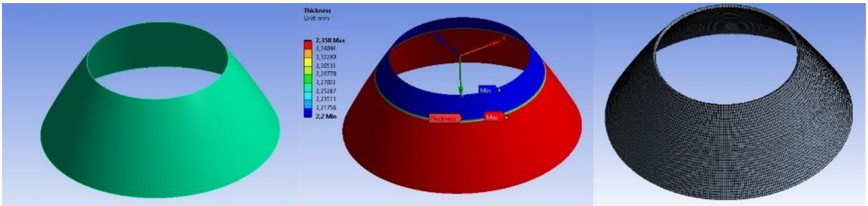

Figure 3 – Main view and finite element model of the homogeneous model (Source: author)

For comparison, the first 6 natural frequencies were calculated. The results of the natural frequency values for the models for the 1st, 3rd, and 5th harmonics are presented in Figures 4–6.

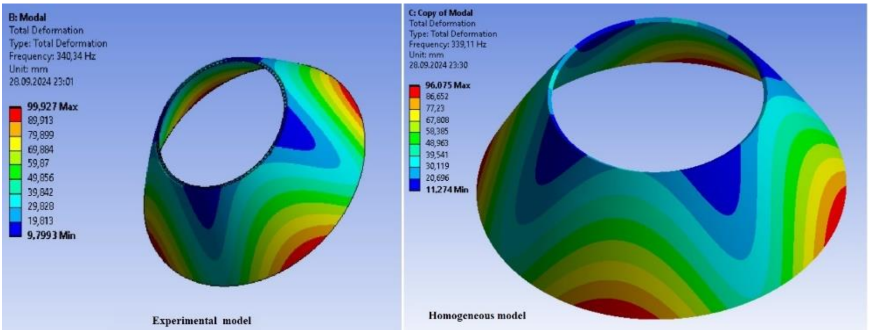

Figure 4 – Comparison of the values of the 1st harmonic for the experimental and homogeneous models (Source: author)

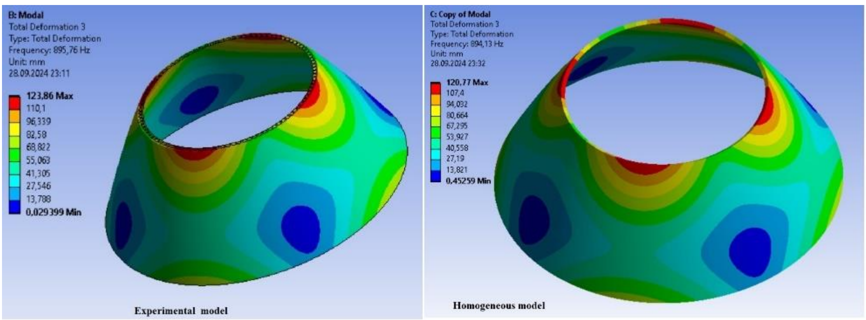

Figure 5 – Comparison of the values of the 3rd harmonic for the experimental and homogeneous models (Source: author)

The frequency differences between each harmonic of the experimental and homogeneous models are 1.23 Hz, 1.63 Hz, and 8.7 Hz, respectively. These values correspond to deviations in the natural frequencies of the models by 0.36%, 0.18%, and 0.93%, respectively. The error in determining the natural frequency using the homogeneous model does not exceed 1% for all calculated natural frequencies. All calculations were performed in the ANSYS system.

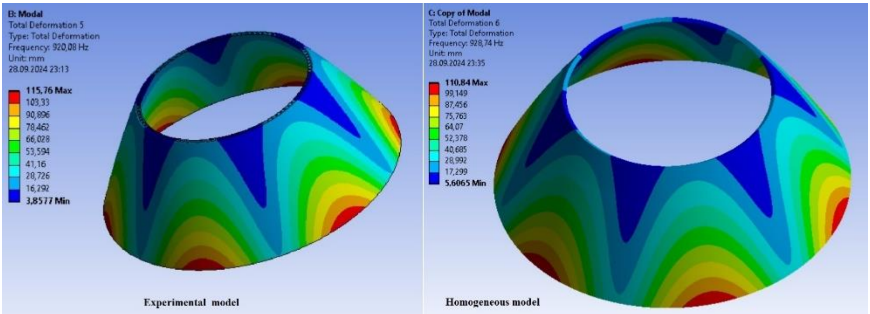

Figure 6 – Comparison of the values of the 5th harmonic for the experimental and homogeneous models (Source: author)

A comparison of parameters of the finite element models for the experimental and homogeneous models, as well as the resources used, is presented in Table 1.

Table 1 – Comparison of resources used and parameters of the finite element models in the experimental and homogeneous models (Source: author)

Parameters	Experimental shell	Homogeneous shell
1. Number of nodes in the model	15327433	13320
2. Number of elements in the model	9829189	12960
3. Quality of the finite element mesh:		
Element Quality	0.7842	0.8748
Aspect Ratio	1.965	1.6734
Jacobian Ratio	0.9624	0.9855
4. Memory resources used the solution	92.3 Gb	1.580 Gb
5. Size of the results file	6.624 Gb	10.5 Mb
6. Calculation time for the task	3720 s	4 s

Conclusions

The results of the comparative numerical modeling showed the following:

- when calculating the natural frequencies of axisymmetric shells with complex geometry, it is advisable to use a homogeneous model. The homogeneous model is a shell without channels, without elements of complex geometry, and has a constant thickness in the cross-section.

- the simplification of the geometry of the experimental shell is achieved by determining the parameters of the homogeneous model, which meet the requirements of equivalence of momentum of inertia and mass. The parameters of the model are the shell thickness and density. The equivalence condition defines the variable thickness and density along the height of the homogeneous model.

- the obtained results showed that the deviation of the natural frequency of the experimental model from that of the homogeneous model does not exceed 1%.

- the simplification of the model allows for calculations without using powerful workstations. The number of nodes and elements in the model is reduced by more than 1000 times, enabling the creation of a significantly higher-quality mesh and reducing the computational error of the model. The use of computational resources and the time for problem solution are also significantly reduced.

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